Neural Comput & Applic (2017) 28:3801–3813
DOI 10.1007/s00521-016-2268-x

ORIGINAL ARTICLE

Global adaptive neural tracking control of nonlinear MIMO systems

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Received: 9 November 2015 / Accepted: 2 March 2016 / Published online: 22 March 2016

Abstract This paper addresses the globally stable tracking control problem of a class of uncertain multiple-input–multiple-output nonlinear systems. By employing the radial basis function neural networks to compensate for the system uncertainties, a novel switching controller is developed. The key features of the proposed control scheme are presented as follows. First, to design the desired adaptive neural controller successfully, an nth-order smoothly switching function is constructed originally. Second, the number of the neural networks and the adaptive parameters is reduced by adopting the direct adaptive approach, so a simplified controller is designed and it is easy to implement in practice. By utilizing the special properties of the affine terms of the considered systems, the singularity problem of the controller is completely avoided. Finally, the overall controller guarantees that all the signals in the closed-loop system are globally uniformly ultimately bounded and the system output converges to a small neighborhood of the reference trajectory by appropriately choosing the design parameters. A simulation example is given to illustrate the effectiveness of the proposed control scheme.

Keywords Globally stable tracking control · Uncertain multiple-input–multiple-output (MIMO) system · Direct adaptive backstepping control · Radial basis function neural networks (RBFNNs)

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1 Introduction

Due to the fact that neural networks (NNs) have good approximation capabilities over a compact set, they play an important role in control community, especially in uncertain nonlinear systems control. A large amount of progress in adaptive neural network control (ANNC) has been obtained in theory and applications, e.g., see [1–3].

In the early stage, some optimization techniques have been primarily used to derive parameter adaptive laws [4]. However, these control schemes cannot ensure the stability, robustness, and performance of the closed-loop systems. To overcome the above problems, some ANNC strategies have been established based on Lyapunov stability theory [5, 6]. In particular, an interesting ANNC has been originally established in [7] via the backstepping technique for strict-feedback systems. Since then, the so-called adaptive backstepping NN control (ABNNC) has been developed to solve the control problems of various systems, such as pure-feedback systems [8], output-feedback systems [9, 10], discrete-time systems [11–13], stochastic systems [14, 15], time-delay systems [16–19], and large-scale systems [20–23]. Meanwhile, several problems on ABNNC have also been resolved. For example, the problem of "explosion of complexity" has been overcome by introducing the adaptive dynamic surface technique in [24]. Recently, the globally stable ABNNC problem has been considered in [25, 26]. However, note that almost all the above control schemes just are applied to single-input–single-output (SISO) systems.

On the contrary, most practical systems are naturally described as nonlinear multiple-input–multiple-output (MIMO) models, such as flexible-joint robot systems [27], helicopter systems [28], missile systems [29]. Thus, it is more interesting to address the control problem of MIMO systems. Due to the inputs strong coupling in MIMO systems, some control methods cannot be directly extended to these systems under the weaker conditions, such as feedback linearization techniques [30], fuzzy logic control [31], and adaptive control [32]. However, of course, a few interesting results are still obtained for MIMO systems [33–36].

It is well known that the uncertain parameters and/or unknown nonlinear functions usually exist in the input coupling matrix, so it becomes very difficult to solve the control problem for uncertain MIMO systems. In the past decades, much effort has been made in ANNC for MIMO systems and some important results are obtained [37–43]. In [37] and [38], the ABNNC schemes have been established for MIMO systems in block-triangular forms, and the controller singularity problem has been completely avoided by using the integral-type Lyapunov function and the special properties of the affine terms [43]. Subsequently, ABNNC has been extended to solve the control problem for MIMO discrete-time systems [39, 40] and time-delay systems [41]. Moreover, there are other methods used to study the control problem of uncertain MIMO systems in the existing literature. For example, based on the principle of sliding mode control and the use of Nussbaum-type functions [44], an ANNC has been developed for MIMO systems with unknown nonlinear dead-zones in [42].

However, note that all of the above ANNC methods just ensure the closed-loop MIMO systems being semi-globally uniformly ultimately bounded (SGUUB). It is well known that these control schemes are effective under the condition that the RBFNNs approximation ability must be valid over the compact sets (or called NNs approximation domain [45]) all the time. In controller design, such a condition is difficult to be verified beforehand for the MIMO systems with high nonlinearity and uncertainty. Once the NNs inputs run out the NNs approximation domain, the ANNC law is invalid. In this case, the tracking performance may be deteriorated, even the stability of the control systems can be destroyed in practical implementation. Consequently, for uncertain MIMO systems, to develop an ANNC to guarantee the closed-loop system being globally uniformly ultimately bounded (GUUB) is an interesting topic. To the best of authors' knowledge, no reports on this issue have been found in the field of ANNC at present stage.

Motivated by the aforementioned discussions, in this paper we attempt to design an ANNC such that the closed-loop MIMO system is GUUB. The main contributions are listed as follows.

1. This paper is the first time to address the globally stable tracking control problem for MIMO systems. We design a switching ANNC law which includes a conventional ANNC law and an extra robust controller. The advantage of this controller is that the switching term $M(\cdot)$ is to switch off the conventional ANNC law once the NN inputs go beyond the neural networks approximation domain, and the extra robust controller begins to work at the same time. Such a controller guarantees that the closed-loop system remains GUUB.
2. To deign the desired controller, we originally construct an nth-order smoothly switching function which has continuous derivatives up to the nth order. This ensures the successful design of the switching ANNC law by using the backstepping technique.
3. By combining the direct adaptive approach with the backstepping technique, an ANNC scheme is developed, where only an NN is used to compensate for all the unknown parts in each backstepping design procedure, so the number of adaptive parameters is reduced. A simplified controller is obtained, and it is easy to implement in practice.

The rest of this paper is organized as follows. Section 2 presents some preliminaries. In Sect. 3, the design procedure of the globally stable ANNC is given and then the main results of this paper are addressed. In Sect. 4, two simulation examples are provided to verify effectiveness of the proposed control scheme. We conclude the work of this paper in Sect. 5.

Notation Throughout this paper, the following notations are adopted. R denotes the set of all real numbers; R^n denotes the real n-dimensional space; $|x|$ denotes the absolute value of a scalar x; $\|X\|$ denotes the Frobenius norm of an $m \times n$ matrix X, i.e., $\|X\| = \sqrt{\mathrm{Tr}(X^{\mathrm{T}}X)}$ where $\mathrm{Tr}(\cdot)$ represents the trace operator; C^i denotes the set of all functions with continuous ith partial derivatives; if no confusion arises, we always denote $(\tilde{\cdot}) = (\hat{\cdot}) - (\cdot)$, where $(\hat{\cdot})$ is the estimate of $(\cdot)$; $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$, respectively, denote the smallest and largest eigenvalues of a square matrix A; $\exp(\cdot)$ denotes the exponential function; $\tanh(\cdot)$ refers to the hyperbolic tangent function.

2 Preliminaries

2.1 System stability

Consider a general nonlinear system [46]

$$\dot{x} = f(x, t), \quad x(t_0) = x_0 \tag{1}$$

where $x(t) \in R^n$ is the system state, $f : R^n \times [t_0, \infty] \to R^n$ is a continuous vector-valued function, t_0 and $x_0 \in R^n$ denote the initial time and the initial state vector, respectively.

Definition 1 [46]: We say the solution of (1) is uniformly ultimately bounded (UUB) if there exists a compact set $\Omega \subset R^n$ such that for all $x(t_0) = x_0 \in \Omega$, there exist an $\varepsilon > 0$ and a number $T(\varepsilon, x_0)$ such that $||x(t)|| < \varepsilon$ for all $t \geq T(\varepsilon, x_0) + t_0$. In particular, if the compact set $\Omega = R^n$, then the solution of system (1) is GUUB.

Usually, the following result is used to analyze the convergence of the tracking error in the field of ANNC. Let function $V(t) \geq 0$ be a continuous and bounded function defined for $t \in [0, \infty)$. If $\dot{V}(t) \leq -\bar{\gamma} V(t) + \bar{\kappa}$ where $\bar{\gamma}$ and $\bar{\kappa}$ are positive constants, then $V(t) \leq \left[V(0) - \frac{\bar{\kappa}}{\bar{\gamma}}\right] e^{-\bar{\gamma} t} + \frac{\bar{\kappa}}{\bar{\gamma}}$. In particular, $\lim_{t \to \infty} V(t) \leq \frac{\bar{\kappa}}{\bar{\gamma}}$ which implies that $V(t)$ will converge to the neighborhood around zero with radius $\frac{\bar{\kappa}}{\bar{\gamma}}$.

2.2 RBFNNs approximation

As pointed out in [26], it has been proved in [2] and [47] that RBFNNs can be employed as a tool for modeling uncertain nonlinear functions appearing in the control systems owing to their linearly parameterized structure and good capabilities in function approximation. In this paper, an unknown continuous nonlinear function $h_{ij}(Z_i) : R^m \to R$ will be approximated over a compact set $\Omega_{Z_i} \subset R^m$ by an RBFNN, that is, the following relation holds

$$h_{ij}(Z_i) = S_{ij}(Z_i)^{\mathrm{T}} W_{ij} + \epsilon_{ij}(Z_i) \tag{2}$$

where the input vector [1] $Z_i \in \Omega_{Z_i} \subset R^m$, the optimal weight vector $W_{ij} \in R^{l_{ij}}$, the NN node number $l_{ij} > 1$, $\epsilon_{ij}(Z_i)$ is the NN inherent approximation error which is bounded over the compact set, i.e., $|\epsilon_{ij}(Z_i)| \leq \epsilon_{ij}$ where ϵ_{ij} is an unknown constant, and $S_{ij}(Z_i) = [s_{ij1}(Z_i), \ldots, s_{ijl_{ij}}(Z_i)]^{\mathrm{T}} : \Omega_{Z_i} \to R^{l_{ij}}$ is a known smooth vector function with $s_{ijq}(Z_i)$ being chosen as the commonly used Gaussian functions, which has the following form

$$s_{ijq}(Z_i) = \exp\left[\frac{-(Z_i - \mu_{ijq})^{\mathrm{T}}(Z_i - \mu_{ijq})}{\eta_i^2}\right], \quad q = 1, \ldots, l_{ij} \tag{3}$$

where μ_{ijq} is the center of the receptive field and $\eta_i > 0$ is the [1] of the Gaussian functions. The optimal weight vector $W_{ij} = [w_{ij1}, \ldots, w_{ijl_{ij}}]^{\mathrm{T}}$ is defined as

$$W_{ij} := \arg \min_{\hat{W}_{ij} \in R^{l_{ij}}} \left\{ \sup_{Z_i \in \Omega_{Z_i}} |h_{ij}(Z_i) - S_{ij}(Z_i)^{\mathrm{T}} \hat{W}_{ij}| \right\} \tag{4}$$

where $\hat{W}_{ij}$ is the estimate of W_{ij}.

2.3 Key definition and lemmas

Definition 2 *(nth-order smoothly switching function):* For all $z \in R^p$ and given constants $0 < r_1 < r_2$, the function $\mathbf{m}(z)$ is called an nth-order smoothly switching function with n being a finite positive integer, if it satisfies the following conditions:

(a) when $||z|| \leq r_1$, $\mathbf{m}(z) = 1$;
(b) when $||z|| \geq r_2 > r_1$, $\mathbf{m}(z) = 0$;
(c) $\mathbf{m}(z)$ is nth-order continuous differentiable.

In particular, for all $z_{ij} \in R, j = 1, \ldots, m$, the following switching function is constructed

$$\mathbf{m}(z_{ij}) = \begin{cases} 1, & |z_{ij}| \leq r_{ij1} \\ \cos^{n+1}\left(\frac{\pi}{2} \sin^{n+1}\left(\frac{\pi}{2} \frac{|z_{ij}|^2 - r_{ij1}^2}{r_{ij2}^2 - r_{ij1}^2}\right)\right), & \text{otherwise} \\ 0, & |z_{ij}| \geq r_{ij2} \end{cases}$$

with $r_{ij2} > r_{ij1} > 0$.

Let

$$M(Z_i) := \prod_{j=1}^{m} \mathbf{m}(z_{ij}) \tag{5}$$

with $Z_i = [z_{i1}, \ldots, z_{im}]^{\mathrm{T}} \in R^m$, which is the key to design the globally stable ABNNC.

Lemma 1 *The function $\mathbf{m}(z_{ij})$ is an n th-order smoothly switching function.*

Proof By using the definition of the derivative, this lemma can be verified directly. To save space, the detailed proof is omitted here. □

Lemma 2 [7]: *The following inequality holds for any $\epsilon > 0$ and for any $\eta \in R$*

$$0 \leq |\eta| - \eta \tanh\left(\frac{\eta}{\epsilon}\right) \leq \kappa \epsilon \tag{6}$$

where κ is a constant satisfying $\kappa = e^{-(\kappa+1)}$, i.e., $\kappa \approx 0.2785$.

3 Problem formulation and main result

3.1 System description and problem formulation

In this paper, consider the following uncertain MIMO nonlinear system which is assumed that it has unique analytical solution on the given interval

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i), \quad i = 1, 2, \ldots, n-1 \\ \dot{x}_n = g_n(\bar{x}_n)u + f_n(\bar{x}_n) \\ y = x_1 \end{cases} \tag{7}$$

where $x_i = [x_{i1}, \ldots, x_{ip}]^{\mathrm{T}} \in R^p, u = [u_1, \ldots, u_p]^{\mathrm{T}} \in R^p$, and $y = [y_1, \ldots, y_p]^{\mathrm{T}} \in R^p$ represent the measurable state, the control input, and the output of system (7), respectively; p is a positive integer; $\bar{x}_i$ is defined as $\bar{x}_i := [x_1^{\mathrm{T}}, \ldots, x_i^{\mathrm{T}}]^{\mathrm{T}} \in R^{ip}$; $f_i(\bar{x}_i) = [f_{i1}(\bar{x}_i), \ldots, f_{ip}(\bar{x}_i)]^{\mathrm{T}}$ and $g_i(\bar{x}_i) = \mathrm{diag}\{g_{i1}(\bar{x}_i), \ldots, g_{ip}(\bar{x}_i)\}$ with the unknown smooth functions $f_{ij} : R^{ip} \to R$ and $g_{ij} : R^{ip} \to R, i = 1, \ldots, n, j = 1, \ldots, p$.

It is worth stating that in many cases, since uncertain MIMO nonlinear systems are often derived from problems in physical world, existence, and uniqueness are often obvious for the physical reasons. Notwithstanding this, a mathematical statement about existence and uniqueness is worthwhile. Uniqueness would be of importance if, for instance, we wished to approximate the solution numerically. If two solutions passed through a point, then successive approximations could very well jump from one solution to the other-with misleading consequences.

Remark 1 The system (7) is in the canonical strict-feedback form. In the field of ANNC, the tracking/regulation problem of such system has been extensively studied, such as continuous-time systems [7, 9, 10, 25], discrete-time systems [12, 13, 39], and time-delay systems [21, 22].

The objective of this paper is to design a direct ANNC law $u(t)$ for system (7) such that

1. all the signals in the closed-loop MIMO system remain GUUB;
2. the system output y can track a known reference trajectory $y_r = [y_{r1}, \ldots, y_{rp}]^{\mathrm{T}}$, i.e.,

$$\lim_{t\to\infty} ||y - y_r|| \le \epsilon_0 \tag{8}$$

for any $\epsilon_0 > 0$.

Remark 2 Some ANNC schemes have been established for different uncertain MIMO nonlinear systems (e.g., see [30–37]). Nevertheless, all the existing ANNC schemes only can guarantee the closed-loop MIMO systems being SGUUB. To the best of our knowledge, until now still no globally stable ANNC approaches have been developed for MIMO nonlinear systems. In this paper, we attempt to design an ANNC such that the closed-loop MIMO system is GUUB.

To design the desired controller, the following assumptions on system functions are made.

Assumption 1 [26]: For $i = 1, \ldots, n$, suppose that there exist the known positive smooth functions $\varphi_i(\bar{x}_i)$ and the unknown constants $\varrho_i \ge 0$ such that

$$||f_i(\bar{x}_i)|| \le \varrho_i \varphi_i(\bar{x}_i), \quad \forall \bar{x}_i \in R^{ip}. \tag{9}$$

Remark 3 It should be emphasized that the above assumption is necessary to design the extra robust controller when we develop the globally stable ANNC scheme later. This assumption is very similar that made in [26], where $f_i(\bar{x}_i) \in R$ are assumed to be bounded by known functions. Here, the upper bounds are allowed to be unknown constants multiplied by known functions. Compared with the previous hypothesis, a weaker assumption is given in this paper.

Assumption 2 [38]: For $i = 1, \ldots, n, j = 1, \ldots, p$, the signs of $g_{ij}(\bar{x}_i)$ are known, and there exist the known positive smooth functions $\bar{g}_{ij}(\bar{x}_i), \underline{g}_{ij}(\bar{x}_i)$ and the positive constant $\underline{g}_0$ such that

$$\underline{g}_0 < \underline{g}_{ij}(\bar{x}_i) \le |g_{ij}(\bar{x}_i)| \le \bar{g}_{ij}(\bar{x}_i), \quad \forall \bar{x}_i \in R^{ip}. \tag{10}$$

Remark 4 Condition (10) implies that system functions $g_{ij}(\bar{x}_i)$ are strictly either positive or negative, which may limit the class of systems under investigation. In the past decades, lots of significant research results about ANNC still have been obtained under the similar assumptions in the existing literature (e.g., see [38–41]). Without loss of generality, here we assume that $g_{ij}(\bar{x}_i)$ are positive. Based on condition (10), for all $\bar{x}_i \in R^{ip}$, we can have

$$||g_i(\bar{x}_i)|| \le \bar{G}_i(\bar{x}_i) \text{ and } ||g_i^{-1}(\bar{x}_i)|| \le \bar{g}_i(\bar{x}_i)$$

where $\bar{G}_i(\bar{x}_i)$ and $\bar{g}_i(\bar{x}_i)$ are the known smooth positive functions.

Assumption 3 [19]: Suppose that the time derivatives of $g_i(\bar{x}_i)$ along the solutions of (7), denoted by $\dot{g}_i$, satisfy

$$||\dot{g}_i|| \le v_i \beta_i(\bar{x}_i), \quad \forall \bar{x}_i \in R^{ip} \tag{11}$$

where $\beta_i(\bar{x}_i)$ are known positive smooth functions and $v_i \ge 0$ are unknown constants.

Remark 5 Assumption 3 is similar with the conditions made in [16, 19], where $\dot{g}_i \in R$ are assumed to be bounded by known constants. Here, the upper bounds are allowed to be unknown constants multiplied by known functions. Compared with the existing hypothesis, a weaker assumption is given. The conditions (10) and (11) play the same important role as Assumption 1 in the globally stable ANNC scheme design. Moreover, from condition

(11) we can see that the affine term $g_n(\cdot)$ is independent of state x_n. By using this special structure property, the controller singularity problem is avoided completely without projection algorithm [38, 42].

Assumption 4 [38]: The reference signal y_r and its derivatives up to the nth order are continuous and bounded.

3.2 Globally stable ANNC design

In this subsection, based on Assumptions 1–4 made in the above subsection, we give the detailed design procedure of the globally stable ANNC law by using the backstepping technique. To this end, define the following n error variables

$$\begin{cases} z_1 = y - y_r \\ z_i = x_i - \alpha_{i-1}(\bar{x}_{i-1}, \bar{y}_r^{(i-1)}, \bar{\hat{\epsilon}}_{i-1}, \bar{\hat{\tau}}_{i-1}, \overline{\hat{W}}_{i-1}), \quad i = 2, \ldots, n \end{cases} \tag{12}$$

where $z_i = [z_{i1}, \ldots, z_{ip}]^{\mathrm{T}}$ and $\bar{y}_r^{(i-1)} = [y_r, \dot{y}_r, \ldots, y_r^{(i-1)}]^{\mathrm{T}}$; $\bar{\hat{\epsilon}}_{i-1} = [\hat{\epsilon}_1, \ldots, \hat{\epsilon}_{i-1}]^{\mathrm{T}}$, $\bar{\hat{\tau}}_{i-1} = [\hat{\tau}_1, \ldots, \hat{\tau}_{i-1}]^{\mathrm{T}}$, and $\overline{\hat{W}}_{i-1} = [\hat{W}_1^{\mathrm{T}}, \ldots, \hat{W}_{i-1}^{\mathrm{T}}]^{\mathrm{T}}$; the estimates of the unknown parameters ϵ_k, τ_k and the NN weight vectors W_k are denoted by $\hat{\epsilon}_k, \hat{\tau}_k$ and $\hat{W}_k$, $k = 1, \ldots, i-1$, respectively; α_{i-1} denote the virtual control variables which will be designed later step by step. Now, we proceed to the backstepping design procedure with n steps.

Step 1 Under the coordinate transformation (12), the time derivative of z_1 along the solution of the first subsystem of (7) is given by

$$\begin{aligned} \dot{z}_1 &= g_1(\bar{x}_1)x_2 + f_1(\bar{x}_1) - \dot{y}_r \\ &= g_1(\bar{x}_1)(\alpha_1 + z_2 + g_1^{-1}(\bar{x}_1)(f_1(\bar{x}_1) - \dot{y}_r)). \end{aligned} \tag{13}$$

Take a Lyapunov function as

$$V_1 = \frac{1}{2} z_1^{\mathrm{T}} g_1^{-1}(\bar{x}_1) z_1$$

whose time derivative along the solution of (13) is

$$\begin{aligned} \dot{V}_1 &= z_1^{\mathrm{T}} g_1^{-1}(\bar{x}_1)\dot{z}_1 \\ &\quad - \frac{1}{2} z_1^{\mathrm{T}} g_1^{-1}(\bar{x}_1)\dot{g}_1(\bar{x}_1) g_1^{-1}(\bar{x}_1) z_1 \\ &= z_1^{\mathrm{T}}[\alpha_1 + z_2 + g_1^{-1}(\bar{x}_1)(f_1(\bar{x}_1) - \dot{y}_r)] \\ &\quad - \frac{1}{2} z_1^{\mathrm{T}} g_1^{-1}(\bar{x}_1)\dot{g}_1(\bar{x}_1) g_1^{-1}(\bar{x}_1) z_1. \end{aligned} \tag{14}$$

Let

$$\begin{aligned} h_1(Z_1) &:= g_1^{-1}(\bar{x}_1)(f_1(\bar{x}_1) - \dot{y}_r) \\ &\quad - \frac{1}{2} g_1^{-1}(\bar{x}_1)\dot{g}_1(\bar{x}_1) g_1^{-1}(\bar{x}_1) z_1 \end{aligned} \tag{15}$$

where

$$Z_1 := [x_1^{\mathrm{T}}, y_r^{\mathrm{T}}, \dot{y}_r^{\mathrm{T}}]^{\mathrm{T}} \in \Omega_{Z_1} \subset R^{3p}$$

with a compact set Ω_{Z_1}. Thus, Eq. (14) can be rewritten as

$$\dot{V}_1 = z_1^{\mathrm{T}}[\alpha_1 + z_2 + h_1(Z_1)]. \tag{16}$$

We design the first virtual controller as

$$\alpha_1 = -k_1 z_1 + M(Z_1)\alpha_1^{an} + (1 - M(Z_1))\alpha_1^{r} \tag{17}$$

where $k_1 > 0$ is a design parameter, the function $M(\cdot)$ is defined as shown in (5), $\alpha_1^{an} = -S_1(Z_1)^{\mathrm{T}}\hat{W}_1 - \hat{\epsilon}_1 \Upsilon(\frac{z_1}{\varpi})$ and $\alpha_1^{r} = -\hat{\tau}_1 \gamma_1(Z_1)\Upsilon\left(\frac{\gamma_1(Z_1)z_1}{\varpi}\right)$ with the basis function $S_1(Z_1)$ and the function $\gamma_1(Z_1)$ to be defined in the following subsection, $\varpi > 0$ being a design parameter, $\Upsilon(\frac{z_1}{\varpi}) := [\tanh(\frac{z_{11}}{\varpi}), \ldots, \tanh(\frac{z_{1p}}{\varpi})]^{\mathrm{T}}$ and $\Upsilon(\frac{\gamma_1(Z_1)z_1}{\varpi}) := [\tanh(\frac{\gamma_1(Z_1)z_{11}}{\varpi}), \ldots, \tanh(\frac{\gamma_1(Z_1)z_{1p}}{\varpi})]^{\mathrm{T}}$.

Substituting (17) into (16) yields

$$\begin{aligned} \dot{V}_1 &= z_1^{\mathrm{T}}[-k_1 z_1 + M(Z_1)\alpha_1^{an} \\ &\quad + (1 - M(Z_1))\alpha_1^{r} + z_2 + h_1(Z_1)]. \end{aligned} \tag{18}$$

Step $i (2 \le i \le n-1)$: The derivative of $z_i = x_i - \alpha_{i-1}$ is given by

$$\begin{aligned} \dot{z}_i &= g_i(\bar{x}_i)(\alpha_i + z_{i+1}) + f_i(\bar{x}_i) - \dot{\alpha}_{i-1} \\ &= g_i(\bar{x}_i)(\alpha_i + z_{i+1}) + f_i(\bar{x}_i) \\ &\quad - \left[\sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\left(g_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j)\right) + \phi_{i-1}\right] \end{aligned} \tag{19}$$

where

$$\begin{aligned} \phi_{i-1} &= \frac{\partial \alpha_{i-1}}{\partial y_r}\dot{y}_r + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} \\ &\quad + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\epsilon}_j}\dot{\hat{\epsilon}}_j + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\tau}_j}\dot{\hat{\tau}}_j \\ &\quad + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{W}_j}\dot{\hat{W}}_j. \end{aligned}$$

Consider the following Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2} z_i^{\mathrm{T}} g_i^{-1}(\bar{x}_i) z_i$$

whose time derivative along the solution of (19) is

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i-1} + z_i^{\mathrm{T}} g_i^{-1}(\bar{x}_i)\dot{z}_i - \frac{1}{2} z_i^{\mathrm{T}} g_i^{-1}(\bar{x}_i)\dot{g}_i(\bar{x}_i) g_i^{-1}(\bar{x}_i) z_i \\ &= \dot{V}_{i-1} + z_i^{\mathrm{T}}\Big\{\alpha_i + z_{i+1} + g_i^{-1}(\bar{x}_i) \\ &\quad \left[f_i(\bar{x}_i) - \left(\sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\left(g_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j)\right) + \phi_{i-1}\right)\right\} \\ &\quad - \frac{1}{2} z_i^{\mathrm{T}} g_i^{-1}(\bar{x}_i)\dot{g}_i(\bar{x}_i) g_i^{-1}(\bar{x}_i) z_i. \end{aligned} \tag{20}$$

Let

$$h_i(Z_i) := g_i^{-1}(\bar{x}_i)\left[f_i(\bar{x}_i) - \left(\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left(g_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j)\right) + \phi_{i-1}\right)\right] - \frac{1}{2}g_i^{-1}(\bar{x}_i)\dot{g}_i(\bar{x}_i)g_i^{-1}(\bar{x}_i)z_i \quad (21)$$

where

$$Z_i := \left[x_1^{\mathrm{T}}, \ldots, x_i^{\mathrm{T}}, \alpha_{i-1}^{\mathrm{T}}, \left(\frac{\partial\alpha_{(i-1)1}}{\partial x_1}\right)^{\mathrm{T}}, \ldots, \left(\frac{\partial\alpha_{(i-1)1}}{\partial x_{i-1}}\right)^{\mathrm{T}}, \ldots, \left(\frac{\partial\alpha_{(i-1)p}}{\partial x_1}\right)^{\mathrm{T}}, \ldots, \left(\frac{\partial\alpha_{(i-1)p}}{\partial x_{i-1}}\right)^{\mathrm{T}}, \phi_{i-1}^{\mathrm{T}}\right]^{\mathrm{T}} \in \Omega_{Z_i} \subset R^{(i-1)p^2+(i+2)p}.$$

with a compact set Ω_{Z_i}. Then, equation (20) can be rewritten as

$$\dot{V}_i = \dot{V}_{i-1} + z_1^{\mathrm{T}}[\alpha_i + z_{i+1} + h_i(Z_i)]. \quad (22)$$

Design the ith virtual controller as

$$\alpha_i = -z_{i-1} - k_i z_i + M(Z_i)\alpha_i^{an} + (1 - M(Z_i))\alpha_i^r \quad (23)$$

where $k_i > 0$ is a design parameter, $\alpha_i^{an} = -S_i(Z_i)^{\mathrm{T}}\hat{W}_i - \hat{\epsilon}_i\Upsilon\left(\frac{z_i}{\varpi}\right)$ and $\alpha_i^r = -\hat{\tau}_i\gamma_i(Z_i)\Upsilon\left(\frac{\gamma_i(Z_i)z_i}{\varpi}\right)$ with the basis function $S_i(Z_i)$ and the function $\gamma_i(Z_i)$ to be defined in the following subsection, $\Upsilon(\frac{z_i}{\varpi}) := [\tanh(\frac{z_{i1}}{\varpi}), \ldots, \tanh(\frac{z_{ip}}{\varpi})]^{\mathrm{T}}$ and $\Upsilon(\frac{\gamma_i(Z_i)z_i}{\varpi}) := [\tanh(\frac{\gamma_i(Z_i)z_{i1}}{\varpi}), \ldots, \tanh(\frac{\gamma_i(Z_i)z_{ip}}{\varpi})]^{\mathrm{T}}$.

Substituting (23) into (22) yields

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + z_i^{\mathrm{T}}[-z_{i-1} - k_i z_i + M(Z_i)\alpha_i^{an} + (1 - M(Z_i))\alpha_i^r + z_{i+1} + h_i(Z_i)]\\ &= z_1^{\mathrm{T}}[-k_1 z_1 + M(Z_1)\alpha_1^{an} + (1 - M(Z_1))\alpha_1^r + z_2 + h_1(Z_1)]\\ &\quad + \sum_{q=2}^{i} z_q^{\mathrm{T}}[-z_{q-1} - k_q z_q + M(Z_q)\alpha_q^{an} + (1 - M(Z_q))\alpha_q^r + z_{q+1} + h_q(Z_q)]. \end{aligned} \quad (24)$$

Step n The derivative of $z_n = x_n - \alpha_{n-1}$ is

$$\begin{aligned}\dot{z}_n &= g_n(\bar{x}_n)u + f_n(\bar{x}_n) - \dot{\alpha}_{n-1}\\ &= g_n(\bar{x}_n)u + f_n(\bar{x}_n) - \left[\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left(g_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j)\right) + \phi_{n-1}\right]\end{aligned} \quad (25)$$

where

$$\phi_{n-1} = \frac{\partial\alpha_{n-1}}{\partial y_r}\dot{y}_r + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(j)}}y_r^{(j+1)} + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\epsilon}_j}\dot{\hat{\epsilon}}_j + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\tau}_j}\dot{\hat{\tau}}_j + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{W}_j}\dot{\hat{W}}_j.$$

Consider the following Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2}z_n^{\mathrm{T}}g_n^{-1}(\bar{x}_n)z_n.$$

The time derivative of V_n along the solution of (25) is given by

$$\dot{V}_n = \dot{V}_{n-1} + z_n^{\mathrm{T}}\left\{u + g_n^{-1}(\bar{x}_n)[f_n(\bar{x}_n) - \left(\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left(g_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j)\right) + \phi_{n-1}\right)\right\} - \frac{1}{2}z_n^{\mathrm{T}}g_n^{-1}(\bar{x}_n)\dot{g}_n(\bar{x}_n)g_n^{-1}(\bar{x}_n)z_n. \quad (26)$$

Let

$$h_n(Z_n) := g_n^{-1}(\bar{x}_n)\left[f_n(\bar{x}_n) - \left(\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left(g_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j)\right) + \phi_{n-1}\right)\right] - \frac{1}{2}g_n^{-1}(\bar{x}_n)\dot{g}_n(\bar{x}_n)g_n^{-1}(\bar{x}_n)z_n \quad (27)$$

where

$$Z_n := \left[x_1^{\mathrm{T}}, \ldots, x_n^{\mathrm{T}}, \alpha_{n-1}^{\mathrm{T}}, \left(\frac{\partial\alpha_{(n-1)1}}{\partial x_1}\right)^{\mathrm{T}}, \ldots, \left(\frac{\partial\alpha_{(n-1)1}}{\partial x_{n-1}}\right)^{\mathrm{T}}, \ldots, \left(\frac{\partial\alpha_{(n-1)p}}{\partial x_1}\right)^{\mathrm{T}}, \ldots, \left(\frac{\partial\alpha_{(n-1)p}}{\partial x_{n-1}}\right)^{\mathrm{T}}, \phi_{n-1}^{\mathrm{T}}\right]^{\mathrm{T}} \in \Omega_{Z_n} \subset R^{(n-1)p^2+(n+2)p}$$

with a compact set Ω_{Z_n}.

Design the actual controller as

$$u = -z_{n-1} - k_n z_n + M(Z_n)\alpha_n^{an} + (1 - M(Z_n))\alpha_n^r \quad (28)$$

where $k_n > 0$ is a design parameter, the function $M(\cdot)$ is defined as shown in (5), $\alpha_n^{an} = -S_n(Z_n)^{\mathrm{T}}\hat{W}_n - \hat{\epsilon}_n\Upsilon\left(\frac{z_n}{\varpi}\right)$ and $\alpha_n^r = -\hat{\tau}_n\gamma_n(Z_n)\Upsilon\left(\frac{\gamma_n(Z_n)z_n}{\varpi}\right)$ with the basis function $S_n(Z_n)$ and the function $\gamma_n(Z_n)$ to be defined in the following subsection, $\Upsilon(\frac{z_n}{\varpi}) := [\tanh(\frac{z_{n1}}{\varpi}), \ldots, \tanh(\frac{z_{np}}{\varpi})]^{\mathrm{T}}$ and $\Upsilon(\frac{\gamma_n(Z_n)z_n}{\varpi}) := [\tanh(\frac{\gamma_n(Z_n)z_{n1}}{\varpi}), \ldots, \tanh(\frac{\gamma_n(Z_n)z_{np}}{\varpi})]^{\mathrm{T}}$.

By substituting (28) into (26), we have

$$\dot{V}_n = \dot{V}_{n-1} + z_n^{\mathrm{T}}[-z_{n-1} - k_n z_n + M(Z_n)\alpha_n^{an} + (1 - M(Z_n)) \alpha_n^r + h_n(Z_n)] = z_1^{\mathrm{T}}[-k_1 z_1 + M(Z_1)\alpha_1^{an} + (1 - M(Z_1))\alpha_1^r + z_2 + h_1(Z_1)] + \sum_{q=2}^{n-1} z_q^{\mathrm{T}}[-z_{q-1} - k_q z_q + M(Z_q)\alpha_q^{an} + (1 - M(Z_q)) \alpha_q^r + z_{q+1} + h_q(Z_q)] + z_n^{\mathrm{T}}[-z_{n-1} - k_n z_n + M(Z_n)\alpha_n^{an} + (1 - M(Z_n))\alpha_n^r + h_n(Z_n)]. \tag{29}$$

Remark 6 The actual control (28) is different from all the existing results of uncertain MIMO nonlinear systems (e.g., see [38–42]). Here, the actual control input (28) includes two parts, a conventional adaptive neural controller α_n^{an} dominating the NNs approximation domain and an extra robust controller α_n^r to take charge of outside the NNs approximation domain. The switching terms $M(\cdot)$ appearing in the equations (17), (23), and (28) are to switch off the control α_i^{an} once the NNs inputs run out the NNs approximation domain, and the extra robust controllers α_i^r begin to work at the same time. Such a switching controller can guarantee the ultimate closed-loop MIMO system being GUUB.

3.3 Stability analysis and main results

Theorem 1 *Based on Assumptions 1–4, consider the closed-loop MIMO system consisting of the plant (7), the reference signal y_r, the virtual control laws (17), (23), the actual control law (28), and the adaptation laws (35). For the bounded initial states, the following results are obtained.*

1. *All the closed-loop signals remain GUUB.*
2. *The tracking error $z_1 = y - y_r$ uniformly converges to a small neighborhood around zero*

$$\Omega^* = \left\{ z_1 \middle| ||z_1|| \le \sqrt{\frac{2\delta^*}{k^* \lambda_{\min}(g_1^{*-1})}} \right\}.$$

Proof Consider the overall Lyapunov function as follows

$$V = V_n + \frac{1}{2}\sum_{i=1}^{n} \tilde{W}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{W}_i + \sum_{i=1}^{n} \frac{1}{2\rho_i} \tilde{\Theta}_i^{\mathrm{T}} \tilde{\Theta}_i \tag{30}$$

where $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$ are the adaptation matrices, ρ_i are the positive design parameters, $\tilde{W}_i = \hat{W}_i - W_i$ and $\tilde{\Theta}_i = \hat{\Theta}_i - \Theta_i$ with $\hat{W}_i, W_i, \hat{\Theta}_i$ and Θ_i to be defined later.

Considering (29), we can obtain the time derivative of V as

$$\dot{V} = \dot{V}_n + \sum_{i=1}^{n} \dot{\hat{W}}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{W}_i + \sum_{i=1}^{n} \frac{1}{\rho_i} \tilde{\Theta}_i^{\mathrm{T}} \dot{\hat{\Theta}}_i$$
$$= z_1^{\mathrm{T}}[-k_1 z_1 + M(Z_1)\alpha_1^{an} + (1 - M(Z_1))\alpha_1^r + z_2 + h_1(Z_1)] + \sum_{q=2}^{n-1} z_q^{\mathrm{T}}[-z_{q-1} - k_q z_q + M(Z_q)\alpha_q^{an} + (1 - M(Z_q))\alpha_q^r + z_{q+1} + h_q(Z_q)] + z_n^{\mathrm{T}}[-z_{n-1} - k_n z_n + M(Z_n)\alpha_n^{an} + (1 - M(Z_n))\alpha_n^r + h_n(Z_n)] + \sum_{i=1}^{n} \dot{\hat{W}}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{W}_i + \sum_{i=1}^{n} \frac{1}{\rho_i} \tilde{\Theta}_i^{\mathrm{T}} \dot{\hat{\Theta}}_i$$
$$= -\sum_{i=1}^{n} k_i z_i^{\mathrm{T}} z_i + \sum_{i=1}^{n} z_i^{\mathrm{T}} \left[M(Z_i)\alpha_i^{an} + (1 - M(Z_i))\alpha_i^r + h_i(Z_i) \right] + \sum_{i=1}^{n} \dot{\hat{W}}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{W}_i + \sum_{i=1}^{n} \frac{1}{\rho_i} \tilde{\Theta}_i^{\mathrm{T}} \dot{\hat{\Theta}}_i. \tag{31}$$

In fact, the functions $h_i(Z_i), i = 1, \ldots, n$ are unknown and cannot be directly used since they include some uncertain functions $f_i(\bar{x}_i), g_i(\bar{x}_i)$. Since it can be verified that the functions $h_i(Z_i)$ are continuous, we can employ RBFNNs to compensate the unknown functions $h_i(Z_i)$ over the compact sets $Z_i \in \Omega_{Z_i}$ as follows

$$h_i(Z_i) = S_i(Z_i)^{\mathrm{T}} W_i + \epsilon_i(Z_i) \tag{32}$$

where $S_i(Z_i) := \mathrm{diag}\{S_{i1}(Z_i), \ldots, S_{ip}(Z_i)\}$, $W_i := [W_{i1}^{\mathrm{T}}, \ldots, W_{ip}^{\mathrm{T}}]^{\mathrm{T}}$, and $\epsilon_i(Z_i) := [\epsilon_{i1}(Z_i), \ldots, \epsilon_{ip}(Z_i)]^{\mathrm{T}}$, respectively, denote the basis function matrices, the ideal weight vectors, and the inherent approximation error vectors with $S_{ij}(Z_i), W_{ij}$ and $\epsilon_{ij}(Z_i), j = 1, \ldots, p$ defined as shown in (2), and $||\epsilon_i(Z_i)|| \le \epsilon_i$ with unknown constants $\epsilon_i > 0$.

Meanwhile, based on Assumptions 1–3 and the detailed forms of the functions $h_i(Z_i)$ as shown in (15), (21), and (27), by computation we can determine that there exist the known positive smooth functions $\gamma_i(Z_i)$ and the unknown constants $\tau_i > 0$ which are dependent of the unknown parameters ϱ_i and v_i such that

$$||h_i(Z_i)|| \le \tau_i \gamma_i(Z_i). \tag{33}$$

Substituting (32) into (31) yields

$$
\begin{aligned}
\dot{V} =& -\sum_{i=1}^{n} k_i z_i^{\mathrm{T}} z_i + \sum_{i=1}^{n} z_i^{\mathrm{T}} [M(Z_i)(\alpha_i^{an} + h_i(Z_i)) + (1 - M(Z_i))(\alpha_i^{r} + h_i(Z_i))] \\
& + \sum_{i=1}^{n} \dot{\hat{W}}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{W}_i + \sum_{i=1}^{n} \frac{1}{\rho_i} \tilde{\Theta}_i^{\mathrm{T}} \dot{\hat{\Theta}}_i = -\sum_{i=1}^{n} k_i z_i^{\mathrm{T}} z_i + \sum_{i=1}^{n} z_i^{\mathrm{T}} \\
& \left[M(Z_i)\left(-S_i(Z_i)^{\mathrm{T}} \tilde{W}_i - (\epsilon_i + \tilde{\epsilon}_i) \Upsilon\left(\frac{z_i}{\varpi}\right) + \epsilon_i(Z_i) \right) \right. \\
& \left. + (1 - M(Z_i))(-(\tau_i + \tilde{\tau}_i)\gamma_i(Z_i) \Upsilon\left(\frac{\gamma_i(Z_i) z_i}{\varpi}\right) + h_i(Z_i)) \right] \\
& + \sum_{i=1}^{n} \dot{\hat{W}}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{W}_i + \sum_{i=1}^{n} \frac{1}{\rho_i} \tilde{\Theta}_i^{\mathrm{T}} \dot{\hat{\Theta}}_i \\
=& -\sum_{i=1}^{n} k_i z_i^{\mathrm{T}} z_i - \sum_{i=1}^{n} M(Z_i) z_i^{\mathrm{T}} S_i(Z_i)^{\mathrm{T}} \\
& \tilde{W}_i - \sum_{i=i}^{n} \tilde{\Theta}_i^{\mathrm{T}} \Phi_i + \sum_{i=i}^{n} \dot{\hat{W}}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{W}_i + \sum_{i=1}^{n} \frac{1}{\rho_i} \tilde{\Theta}_i^{\mathrm{T}} \dot{\hat{\Theta}}_i \\
& + \sum_{i=1}^{n} M(Z_i) z_i^{\mathrm{T}} \left[-\epsilon_i \Upsilon\left(\frac{z_i}{\varpi}\right) + \epsilon_i(Z_i) \right] + \sum_{i=1}^{n} (1 - M(Z_i)) z_i^{\mathrm{T}} \\
& \left[-\tau_i \gamma_i(Z_i) \Upsilon\left(\frac{\gamma_i(Z_i) z_i}{\varpi}\right) + h_i(Z_i) \right]
\end{aligned} \tag{34}
$$

where $\hat{\Theta} = [\hat{\epsilon}_i, \hat{\tau}_i]^{\mathrm{T}}$, $\Theta = [\epsilon_i, \tau_i]^{\mathrm{T}}$ and

$$
\Phi_i = \begin{bmatrix} M(Z_i) z_i^{\mathrm{T}} \Upsilon\left(\frac{z_i}{\varpi}\right) \\ (1 - M(Z_i)) z_i^{\mathrm{T}} \gamma_i(Z_i) \Upsilon\left(\frac{\gamma_i(Z_i) z_i}{\varpi}\right) \end{bmatrix}.
$$

Based on Eq. (34), design the parameter adaptive laws as

$$
\begin{cases} \dot{\hat{W}}_i = \Gamma_i (M(Z_i) S_i(Z_i) z_i - \sigma_i \hat{W}_i) \\ \dot{\hat{\Theta}}_i = \rho_i (\Phi_i - \sigma_i \hat{\Theta}_i) \end{cases} \tag{35}
$$

where σ_i are the positive design parameters.

By substituting (35) into (34), the derivative of V becomes

$$
\begin{aligned}
\dot{V} =& -\sum_{i=1}^{n} k_i z_i^{\mathrm{T}} z_i \underbrace{- \sum_{i=1}^{n} \sigma_i \tilde{W}_i^{\mathrm{T}} \hat{W}_i - \sum_{i=1}^{n} \sigma_i \tilde{\Theta}_i^{\mathrm{T}} \hat{\Theta}_i}_{\text{(I)}} \\
& + \underbrace{\sum_{i=1}^{n} M(Z_i) z_i^{\mathrm{T}} \left[-\epsilon_i \Upsilon\left(\frac{z_i}{\varpi}\right) + \epsilon_i(Z_i) \right]}_{\text{(II)}} \\
& + \underbrace{\sum_{i=1}^{n} (1 - M(Z_i)) z_i^{\mathrm{T}} \left[-\tau_i \gamma_i(Z_i) \Upsilon\left(\frac{\gamma_i(Z_i) z_i}{\varpi}\right) + h_i(Z_i) \right]}_{\text{(III)}}.
\end{aligned} \tag{36}
$$

By using Lemma 2 and noticing $0 \le M(Z_i) \le 1$, the following inequalities hold

$$
\begin{aligned}
\text{(I)}: & -\sum_{i=1}^{n} \sigma_i \tilde{W}_i^{\mathrm{T}} \hat{W}_i - \sum_{i=1}^{n} \sigma_i \tilde{\Theta}_i^{\mathrm{T}} \hat{\Theta}_i \\
& \le -\sum_{i=1}^{n} \frac{\sigma_i ||\tilde{W}_i||^2}{2} - \sum_{i=1}^{n} \frac{\sigma_i ||\tilde{\Theta}_i||^2}{2} \\
& \quad + \sum_{i=1}^{n} \left(\frac{\sigma_i ||W_i||^2}{2} + \frac{\sigma_i ||\Theta_i||^2}{2} \right)
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
\text{(II)}: & \sum_{i=1}^{n} M(Z_i) z_i^{\mathrm{T}} \left[-\epsilon_i \Upsilon\left(\frac{z_i}{\varpi}\right) + \epsilon_i(Z_i) \right] \\
& \le \sum_{i=1}^{n} M(Z_i) p \kappa \varpi \epsilon_i.
\end{aligned} \tag{38}
$$

Using inequality (33), the definition of function $\Upsilon(\cdot)$ and Lemma 2, we have

$$
\begin{aligned}
\text{(III)}: & \sum_{i=1}^{n} (1 - M(Z_i)) z_i^{\mathrm{T}} \\
& \left[-\tau_i \gamma_i(Z_i) \Upsilon\left(\frac{\gamma_i(Z_i) z_i}{\varpi}\right) + h_i(Z_i) \right] \\
& \le \sum_{i=1}^{n} (1 - M(Z_i)) \left[-\tau_i \sum_{q=1}^{p} \gamma_i(Z_i) z_{iq} \right. \\
& \quad \left. \tanh\left(\frac{\gamma_i(Z_i) z_{iq}}{\varpi}\right) + \sum_{q=1}^{p} |z_{iq}| \tau_i \gamma_i(Z_i) \right] \\
& \le \sum_{i=1}^{n} (1 - M(Z_i)) p \kappa \varpi \tau_i.
\end{aligned} \tag{39}
$$

Based on (37)–(39), Eq. (36) satisfies the following inequality

$$
\begin{aligned}
\dot{V} \le & -\sum_{i=1}^{n} k_i z_i^{\mathrm{T}} z_i - \sum_{i=1}^{n} \frac{\sigma_i ||\tilde{W}_i||^2}{2} \\
& - \sum_{i=1}^{n} \frac{\sigma_i ||\tilde{\Theta}_i||^2}{2} \\
& + \sum_{i=1}^{n} \left(\frac{\sigma_i ||W_i||^2}{2} + \frac{\sigma_i ||\Theta_i||^2}{2} \right. \\
& \left. + M(Z_i) p \kappa \varpi \epsilon_i + (1 - M(Z_i)) p \kappa \varpi \tau_i \right).
\end{aligned} \tag{40}
$$

Let

$$
\begin{aligned}
\delta^* = & \sum_{i=1}^{n} \left(\frac{\sigma_i ||W_i||^2}{2} + \frac{\sigma_i ||\Theta_i||^2}{2} + M(Z_i) \right. \\
& \left. p \kappa \varpi \epsilon_i + (1 - M(Z_i)) p \kappa \varpi \tau_i \right).
\end{aligned}
$$

If we choose k_i such that $k_i > \frac{k^*}{2\underline{g}_0}, i = 1, 2, \ldots, n$, where k^* is a positive constant, and choose σ_i and Γ_i such that $\sigma_i > \max\left\{k^* \lambda_{\max}(\Gamma_i^{-1}), \frac{k^*}{\rho_i}\right\}, i = 1, 2, \ldots, n$, then from (40) we have the following inequality

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{n} \frac{k^*}{2} z_i^{\mathrm{T}} g_i^{-1}(\bar{x}_i) z_i \quad -\sum_{i=1}^{n} \frac{k^*}{2} \tilde{W}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{W}_i \\ & -\sum_{i=1}^{n} \frac{k^*}{2\rho_i} \tilde{\Theta}_i^{\mathrm{T}} \tilde{\Theta}_i + \delta^* \leq -k^* \left[\sum_{i=1}^{n} \frac{1}{2} z_i^{\mathrm{T}} g_i^{-1}(\bar{x}_i) z_i \right. \\ & \left. + \sum_{i=1}^{n} \frac{\tilde{W}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{W}_i}{2} + \sum_{i=1}^{n} \frac{1}{2\rho_i} \tilde{\Theta}_i^{\mathrm{T}} \tilde{\Theta}_i \right] + \delta^* \leq -k^* V + \delta^*.\end{aligned} \tag{41}$$

From (41), we have $V(t) \leq (V(0) - \frac{\delta^*}{k^*})^{-k^* t} + \frac{\delta^*}{k^*}$, which implies that all the signals, including $z_i, \tilde{W}_i$ and $\tilde{\Theta}_i, i = 1, \ldots, n$, are uniformly bounded. Since W_i and Θ_i are bounded, it is easily seen that $\hat{W}_i$ and $\hat{\Theta}_i$ are bounded. Next, it can be seen that x_1 is bounded for $z_1 = x_1 - y_r$ and y_r being bounded. For $i = 2, \ldots, n$, from $x_i = z_i + \alpha_{i-1}$ and the definitions of virtual control inputs α_{i-1}, it can be shown that α_{i-1} and x_i are all bounded. Using (28), we can conclude that the actual control u is also bounded. Consequently, all the signals in the closed-loop system remain bounded. On the other hand, from inequality (41), it can be easily shown that

$$\begin{aligned}\frac{1}{2} z_1^{\mathrm{T}} g_1^{-1}(\bar{x}_1) z_1 \leq & \sum_{i=1}^{n} \frac{1}{2} z_i^{\mathrm{T}} g_i^{-1}(\bar{x}_i) z_i \\ & \leq V(t) \leq \left(V(0) - \frac{\delta^*}{k^*}\right)^{-k^* t} \\ & + \frac{\delta^*}{k^*}.\end{aligned} \tag{42}$$

Note that $g_1(\bar{x}_1)$ is a constant matrix, denoted by g_1^*, that is, $g_1(\bar{x}_1)$ is independent of x_1. Otherwise, it is in contradiction with condition (11) in Assumption 3. Therefore, we can get the following inequality

$$\frac{1}{2} \lambda_{\min}(g_1^{*-1}) z_1^2 \leq \left(V(0) - \frac{\delta^*}{k^*}\right)^{-k^* t} + \frac{\delta^*}{k^*}$$

which implies that the tracking error $z_1 = y - y_r$ will eventually converge to

$$\Omega^* = \left\{ z_1 \middle| ||z_1|| \leq \sqrt{\frac{2\delta^*}{k^* \lambda_{\min}(g_1^{*-1})}} \right\}.$$

□

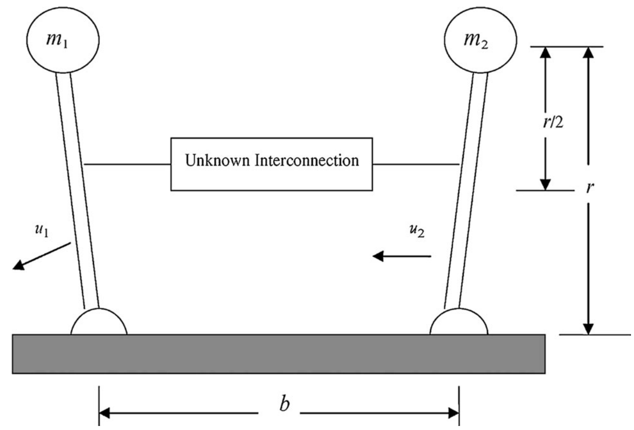


Fig. 1 Two inverted pendulums connected by unknown device [20]

4 Simulation example

In order to show the effectiveness of the control scheme proposed in this paper, we consider a double-inverted pendulum model [20] connected by an unknown device, which is shown in Fig. 1. For the purpose of simulation, the unknown device is specified as a spring. The system has the following form

$$\begin{cases} \dot{x}_1 = f_1(\bar{x}_1) + g_1(\bar{x}_1) x_2 \\ \dot{x}_2 = f_2(\bar{x}_2) + g_2(\bar{x}_2) u \\ y = x_1 \end{cases} \tag{43}$$

where the system states $x_1 = [x_{11}, x_{12}]^{\mathrm{T}}, x_2 = [x_{21}, x_{22}]^{\mathrm{T}}$, the systems outputs $y = [y_1, y_2]^{\mathrm{T}}$ and the control inputs $u = [u_1, u_2]^{\mathrm{T}}$,

$$f_1(\bar{x}_1) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad g_1(\bar{x}_1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$f_2(\bar{x}_2) = \begin{pmatrix} \left(\frac{m_1}{J_1} gr - \frac{kr^2}{4J_1}\right) \sin(x_{11}) + \frac{kr}{2J_1}(l - b) + \frac{kr^2}{4J_1} \sin(x_{12}) \\ \left(\frac{m_2}{J_2} gr - \frac{kr^2}{4J_2}\right) \sin(x_{12}) + \frac{kr}{2J_2}(l - b) + \frac{kr^2}{4J_2} \sin(x_{11}) \end{pmatrix},$$

$$g_2(\bar{x}_2) = \begin{pmatrix} \frac{1}{J_1} & 0 \\ 0 & \frac{1}{J_2} \end{pmatrix},$$

the end masses of pendulum are $m_1 = 2\,\mathrm{kg}$ and $m_2 = 2.5\,\mathrm{kg}$, the moments of inertia are $J_1 = 0.5\,\mathrm{kg} \cdot \mathrm{m}^2$ and $J_2 = 0.625\,\mathrm{kg} \cdot \mathrm{m}^2$, the constant of connecting spring is $k = 100\,\mathrm{N/m}$, the pendulum height is $r = 0.5\,\mathrm{m}$, the natural length of the spring is $l = 0.5\,\mathrm{m}$, and the gravitational acceleration is $g = 9.81\,\mathrm{m/s^2}$. The distance between the pendulum hinges is $b = 0.4\,\mathrm{m}$.

For the simulation, the known functions in Assumptions 1–3 are given as follows

$$\varphi_1(\bar{x}_1) = 1, \beta_1(\bar{x}_1) = 1,$$
$$\bar{G}_1(\bar{x}_1) = 2, \bar{g}_1(\bar{x}_1) = 1.5$$
$$\varphi_2(\bar{x}_2) = (441 + 152x_{11}^2 + 202x_{12}^2)^{\frac{1}{2}},$$
$$\beta_2(\bar{x}_2) = 1, \bar{G}_2(\bar{x}_2) = 4, \bar{g}_2(\bar{x}_2) = 1.$$

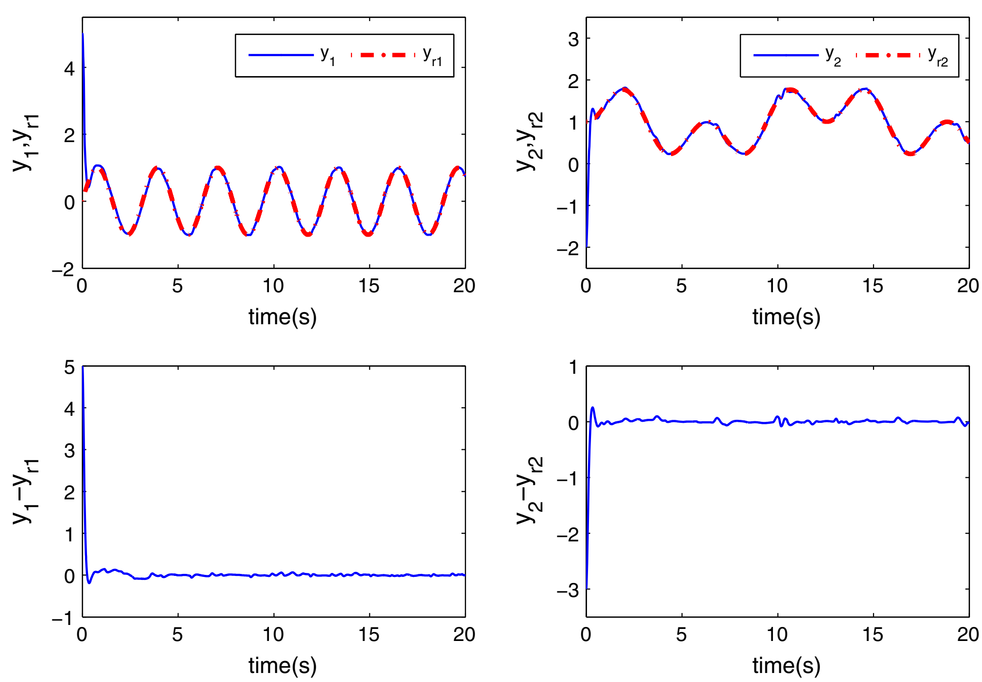
The reference signal is $y_r = [y_{r1}, y_{r2}]^{\mathrm{T}} = [\sin(2t), \sin(\frac{t}{2})\sin(t) + 1]^{\mathrm{T}}$. According to the design procedure proposed in Sect. 3, we can easily obtain the virtual control α_1 and the actual control $u(t)$ which are in the same form as (17) and (28) but different from where the functions $\gamma_1(Z_1)$ and $\gamma_2(Z_2)$ are given as follows

$$\begin{aligned}\gamma_1(Z_1) &= 1 + \frac{1}{2}(x_{11}^2 + x_{12}^2 + 0.2)^{1/2} \\ &\quad + \frac{1}{2}(y_{r1}^2 + y_{r2}^2 + 1)^{1/2} \\ &\quad + (\dot{y}_{r1}^2 + \dot{y}_{r2}^2 + 0.5)^{1/2}, \\ \gamma_2(Z_2) &= \varphi_2(\bar{x}_2) + \sqrt{2}\Bigg(\left(\frac{\partial\alpha_{11}}{\partial x_1}\right)^2 \\ &\quad + \left(\frac{\partial\alpha_{12}}{\partial x_1}\right)^2 + \bar{G}_1^2(\bar{x}_1) \\ &\quad + x_2^2 + \phi_1^2 + 1\Bigg)^{1/2} \\ &\quad + \frac{1}{2}[(x_2^2 + 0.1)^{1/2} \\ &\quad + (\alpha_1^2 + 0.1)^{1/2}].\end{aligned}$$

All simulations are run by the Matlab "ode45" method and the max step size is set to be 0.01. RBFNNs $S_1(Z_1)^{\mathrm{T}}\hat{W}_1$ and $S_2(Z_2)^{\mathrm{T}}\hat{W}_2$ are employed in this simulation, where $S_1(Z_1) = \mathrm{diag}\{S_{11}(Z_1), S_{12}(Z_1)\}$, $\hat{W}_1 = [\hat{W}_{11}^{\mathrm{T}}, \hat{W}_{12}^{\mathrm{T}}]^{\mathrm{T}}$, $S_2(Z_2) = \mathrm{diag}\{S_{21}(Z_2), S_{22}(Z_2)\}$, $\hat{W}_2 = [\hat{W}_{21}^{\mathrm{T}}, \hat{W}_{22}^{\mathrm{T}}]^{\mathrm{T}}$. Specifically, both NNs $S_{11}(Z_1)^{\mathrm{T}}\hat{W}_{11}$ and $S_{12}(Z_1)^{\mathrm{T}}\hat{W}_{12}$ contain 729 nodes (i.e., $l_{11} = l_{12} = 729$) with centers $\mu_{1jq}(j = 1, 2, q = 1, \ldots, l_{11})$ evenly spaced in $[-1, 1] \times [-1, 2] \times [-1, 1] \times [-1, 2] \times [-2, 2] \times [-1.5, 1.5]$ and width $\eta_1 = 0.34$; the other NNs $S_{21}(Z_2)^{\mathrm{T}}\hat{W}_{21}$ and $S_{22}(Z_2)^{\mathrm{T}}\hat{W}_{22}$ contain 59049 nodes (i.e., $l_{21} = l_{22} = 59049$) with centers $\mu_{2jq}(j = 1, 2, q = 1, \ldots, l_{21})$ evenly spaced in $[-1, 1] \times [-1, 2] \times [-1, 1] \times [-1, 3] \times [-3, 7] \times [-3, 7] \times [-20, 40] \times [-20, 10] \times [-20, 40] \times [-20, 10] \times [-20, 10] \times [-20, 10]$ and width $\eta_2 = 0.39$. The rest of design parameters used in the simulation are summarized as: $k_1 = k_2 = 10.0, \Gamma_1 = \Gamma_2 = \mathrm{diag}\{30\}, \rho_1 = 10^{-3}, \rho_2 = 1.5 \times 10^{-3}, \sigma_1 = \sigma_2 = 10^{-3}, r_{1j1} = 1.0, r_{1j2} = 2.0, j = 1, \ldots, 6, r_{2j1} = 1.0, r_{2j2} = 3.0, j = 1, \ldots, 12$ and $\varpi = 10$. The initial states are chosen as $[x_{11}(0), x_{12}(0), x_{21}(0), x_{22}(0)]^{\mathrm{T}} = [5, -2, 2, 3]^{\mathrm{T}}$ also outside the NNs approximation domain, $\hat{W}_i(0) = 0$ and $\hat{\Theta}_i(0) = 0, i = 1, 2$.

Figures 2, 3, 4, and 5 show the simulation results of adopting controller (45) to the double-inverted pendulum model for tracking reference signal y_r. From Fig. 2, we can see that fairly good tracking performance is also obtained. The boundedness of the system state x_2, the control input u, the adaptive parameters $\hat{W}_i$ and $\hat{\Theta}_i, i = 1, 2$ are shown in

Fig. 2 Trajectories of the outputs y_i, the reference signals y_{ri}, and the tracking errors $y_i - y_{ri}, i = 1, 2$



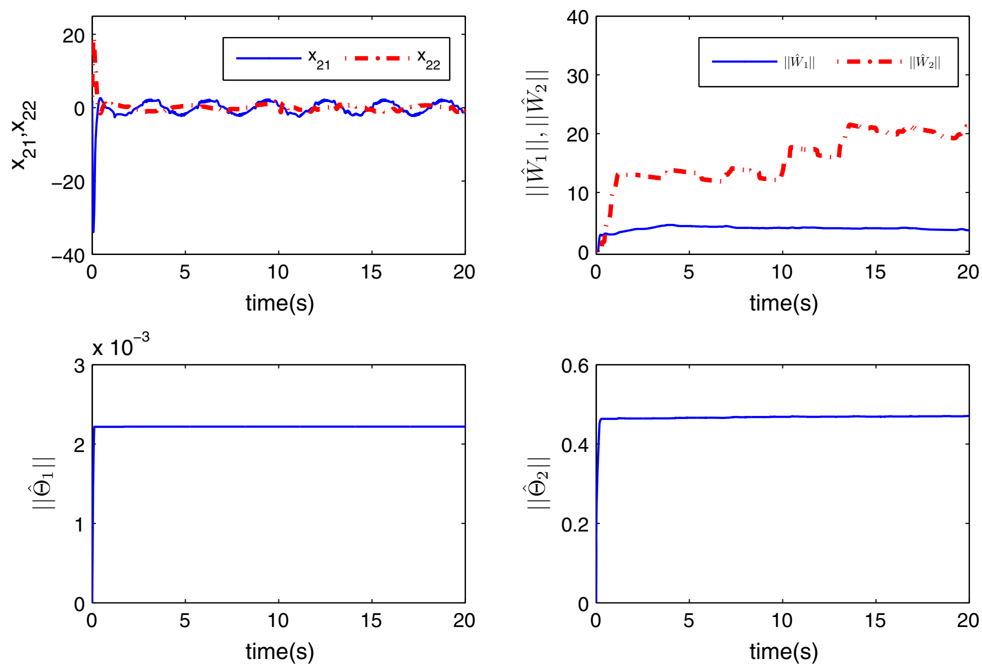


Fig. 3 Trajectories of the states x_{2i}, the norms $||\hat{W}_i||, ||\hat{\Theta}_i||, i = 1, 2$

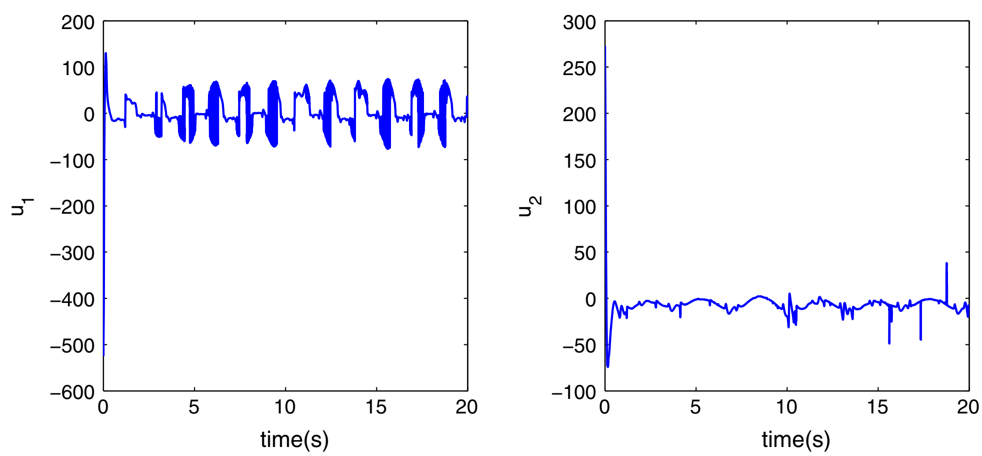


Fig. 4 Trajectories of the control inputs $u_i, i = 1, 2$

Figs. 3 and 4, respectively. The switching signals are depicted in Fig. 5.

To further show the advantage of the control scheme proposed in this paper, we now present a comparison experiment as following. For (17) and (28), let $M(Z_1) = M(Z_2) = 1$, and then they are reduced to the conventional controller developed in [38]. The simulation results are shown in Fig. 6, from which it can be seen that the tracking performance is very poor. This is because the conventional adaptive neural controller is valid under the assumption that the initial condition of the system must be within a small compact set (generally, such a compact is much less than the approximation domain of NNs). That is, the existing control schemes just can guarantee the semi-global stability of the closed-loop systems. However, our approach can obtain a global result since we no longer require the initial condition of the system is within a small compact set.

Remark 7 From Fig. 5, it can be seen that the function $M(\cdot)$ takes values between 0 and 1, which is different from the general switching function taking values 0 or 1. In fact, the function $M(\cdot)$ proposed in this paper has the derivatives up to the nth order, which guarantees the successful design of the ANNC by using the backstepping technique.

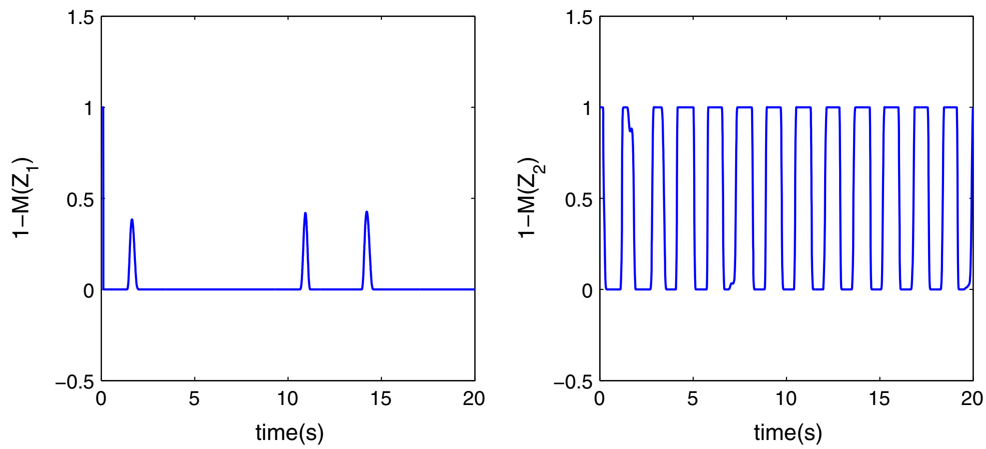


Fig. 5 Trajectories of the functions $1 - M(Z_i), i = 1, 2$

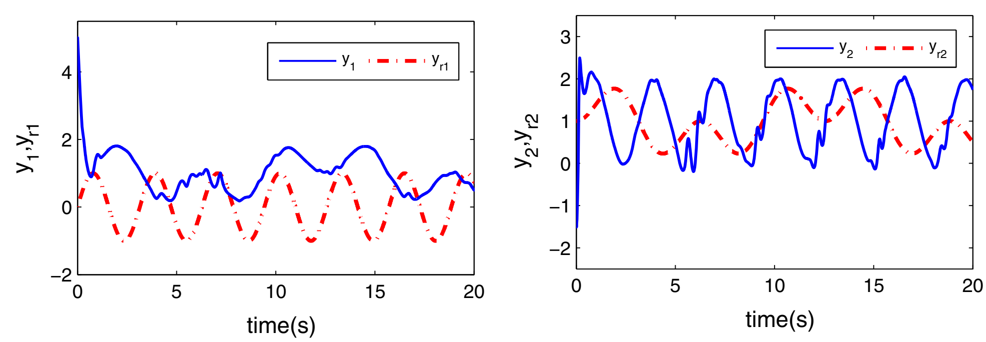


Fig. 6 Trajectories of the outputs y_i, the reference signals y_{ri} using (17) and (28) with $M(Z_i) = 1, i = 1, 2$

5 Conclusions

In this paper, it is the first time to develop a globally stable direct ANNC scheme for a class of uncertain MIMO nonlinear systems. By constructing a novel nth-order smoothly switching function, an appropriate switching controller is designed to ensure the closed-loop MIMO system being GUUB. In each backstepping design procedure, all the unknown parts are approximated by employing an RBFNN to reduce the number of adaptive parameters, so a simplified ANNC strategy is obtained and it is easy to implement in practice. Finally, it has been shown that all the signals of the closed-loop MIMO system are GUUB and a good tracking performance is obtained. It should be mentioned that this paper has used RBF neural network to model uncertain dynamical systems. Of course, it is well known that there are some modeling algorithms reported in the literature such as fuzzy algorithms [48, 50] and genetic algorithm [49], which have several advantages. For system (7), how to design the feasible control scheme based on these modeling algorithms is an interesting topic.

Acknowledgments This work is supported by National Natural Science Foundation of China (61174213, 61203074), Natural Science Foundation of Anhui Province (1608085MF144, 1608085QF131), and the Foundation of University Research and Innovation Platform Team for Intelligent Perception and Computing of Anhui Province.

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