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ORIGINAL ARTICLE

Solution of non-convex economic load dispatch problem using Grey Wolf Optimizer

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Abstract Grey Wolf Optimizer (GWO) is a recently developed meta-heuristic search algorithm inspired by grey wolves (Canis lupus), which simulate the social stratum and hunting mechanism of grey wolves in nature and based on three main steps of hunting: searching for prey, encircling prey and attacking prey. This paper presents the application of GWO algorithm for the solution of non-convex and dynamic economic load dispatch problem (ELDP) of electric power system. The performance of GWO is tested for ELDP of small-, medium- and large-scale power systems, and the results are verified by a comparative study with lambda iteration method, Particle Swarm Optimization algorithm, Genetic Algorithm, Biogeography-Based Optimization, Differential Evolution algorithm, pattern search algorithm, NN-EPSO, FEP, CEP, IFEP and MFEP. Comparative results show that the GWO algorithm is able to provide very competitive results compared to other well-known conventional, heuristics and meta-heuristics search algorithms.

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1 Introduction

In the recent power system networks, there are various generating resources like thermal, hydro, nuclear etc. Also, the load demand varies during a day and attains different peak values. Thus, it is required to decide which generating unit to turn on and at what time it is needed in the power system network and also the sequence in which the units must be shut down keeping in mind the cost-effectiveness of turning on and shutting down of respective units. The entire process of computing and making these decisions is known as unit commitment (UC). The unit which is decided or scheduled to be connected to the power system network, as and when required, is known to be committed unit. Unit commitment in power systems refers to the problem of determining the on/off states of generating units that minimize the operating cost for a given time horizon.

Electrical power plays a pivotal role in the modern world to satisfy various needs. It is therefore very important that the electrical power generated is transmitted and distributed efficiently in order to satisfy the power requirement. Electrical power is generated in several ways. The most significant crisis in the planning and operation of electric power generation system is the effective scheduling of all generators in a system to meet the required demand. The economic load dispatch (ELD) problem is the most important optimization problem in scheduling the generation among thermal generating units in power system.

Economic dispatch in electric power system refers to the short-term discernment of the optimal generation output of various electric utilities, to meet the system load demand, at the minimum possible cost, subject to various system and operating constraints viz. operational and transmission constraints. The economic load dispatch problem (ELDP) means that the electric utilities (i.e. generators) real and reactive power are allowed to vary within certain limits so as to meet a particular load demand within lowest fuel cost. The ultimate aim of the ELD problem is to minimize the operation cost of the power generation system, while supplying the required power demanded. In addition to this, the various operational constraints of the system should also be satisfied.

The problem of ELD is usually multimodal, discontinuous and highly nonlinear. Although the cost curve of thermal generating units is generally modelled as a smooth curve, the input–output characteristics are nonlinear by nature because of valve-point loading effects, Prohibited Operating Zones (POZ), ramp rate limits etc.

In recent years, various evolutionary, heuristic and meta-heuristics optimization algorithms have been developed simulating natural phenomena such as: Genetic Algorithm (GA) [1], Ant Colony Optimization (ACO) [2], Particle Swarm Optimization [3], Simulating Annealing (SA) [4], Gravitational Local Search (GLSA) [5], Big-Bang Big-Crunch (BBBC) [6], Gravitational Search Algorithm (GSA) [7], Curved Space Optimization (CSO) [8], Charged System Search (CSS) [9], Central Force Optimization (CFO) [10], Artificial Chemical Reaction Optimization Algorithm (ACROA) [11], Black Hole (BH) [12] algorithm, Ray Optimization Algorithm (ROA) [13], Small-World Optimization Algorithm (SWOA) [14], Galaxy-based Search Algorithm (GbSA) [15], Shuffled Frog Leaping Algorithm (SFLA) [16], Snake Algorithm [17], Biogeography-Based Optimization [18], Marriage in Honey Bees Optimization algorithm (MBO) [19], Artificial Fish-Swarm Algorithm (AFSA) [20], Termite Algorithm (TA) [21], Wasp Swarm Algorithm (WSA) [22], Monkey Search Algorithm (MSA) [23], Bee Collecting Pollen Algorithm (BCPA) [24], Cuckoo Search Algorithm (CSA) [25], Dolphin Partner Optimization (DPO) [26], Firefly Algorithm [27], Krill Herd (KH) algorithm [28], Fruit fly Optimization Algorithm (FOA) [29] and Distributed BBO [30]. Out of these heuristics evolutionary search algorithm, some of these are used to solve ELDP, Combined Economic Load Dispatch Problem (CELDP), Dynamic Economic Dispatch Problem (DEDP) and Combined Economic Emission Dispatch (CEED) and are reported in numerous literatures as: Evolutionary Programming [31], Particle Swarm Optimization [32], Genetic Algorithm [32, 33], Improved Genetic Algorithm [34], Adaptive PSO and Chaotic PSO [35], Cardinal Priority Ranking-based Decision-making [36], Gravitational Search Algorithm [37, 42, 45], Biogeography-based Optimization [38, 39, 44], Intelligent Water Drop Algorithm [40], Hybrid Harmony Search Algorithm [41], Firefly Algorithm [43], Cuckoo Search Algorithm [46, 54], Biogeography-based Optimization [44], Differential harmony Search [47], Hybrid Particle Swarm Optimization and Gravitational Search Algorithm [48], Differential Evolution [49], Modified Ant Colony Optimization [50], Modified Harmony Search [51], Hybrid GA-MGA [52] and Artificial Bee Colony [53]. Although no optimization algorithm can perform general enough to solve all optimizations problems, each optimization algorithm have their own advantages and disadvantages. The limitations of some of these well-known optimization algorithms are listed below.

The major limitations of the numerical techniques and dynamic programming method are the size or dimensions of the problem, large computational time and complexity in programming. The mixed integer programming methods for solving the ELDP fails when the participation of number of units increases because they require a large memory and suffer from great computational delay. Gradient descent method is distracted for non-differentiable search spaces.

The Lagrangian relaxation (LR) approach fails to obtain solution feasibility and solution quality of problems and becomes complex if the number of units is more. The Branch and Bound (BB) method employs a linear function to represent fuel cost, start-up cost and obtains a lower and upper bounds. The difficulty of this method is the exponential growth in the execution time for systems of a large practical size. An Expert System (ES) algorithm rectifies the complexity in calculations and saving in computation time. But it faces the problem if the new schedule is differing from schedule in database. The fuzzy theory method using fuzzy set solves the forecasted load schedules error, but it suffers from complexity.

The Hopfield neural network technique considers more constraints, but it may suffer from numerical convergence due to its training process. The Simulated Annealing (SA) and Tabu Search (TS) are powerful, general-purpose stochastic optimization technique, which can theoretically converge asymptotically to a global optimum solution with probability one. But it takes much time to reach the near-global minimum. Particle Swarm Optimization (PSO) has simple concept, easy implementation, relative robustness to control parameters and computational efficiency [55], although it has numerous advantages, it get trapped in a local minimum, when handling heavily constrained problems due to the limited local/global searching capabilities [56, 57]. Differential Evolution (DE) algorithm has the ability to find the true global minimum regardless of the initial parameters values and requires few control parameters. It has parallel processing nature and fast convergence as compared to conventional optimization algorithm. Although it does not always give an exact global optimum due to premature convergence and may require tremendously high computation time because of a large number of fitness evaluations, the Biogeography-Based Optimization (BBO) is an efficient algorithm for power system optimization, which does not take unnecessary computational time and is good for exploiting the solutions. The solutions obtained by BBO algorithm do not die at the end of each generation like the other optimization algorithm, but the convergence becomes slow for medium- and large-scale systems. Gravitational Search Algorithm has the advantages to explore better optimized results, but due to the cumulative effect of the fitness function on mass, masses get heavier and heavier over the course of iteration. This causes masses to remain in close proximity and neutralize the gravitational forces of each other in later iterations, preventing them from rapidly exploiting the optimum [55]. Therefore, increasing effect of the cost function on mass, masses get greater over the course of iteration and search process and convergence becomes slow. To overcome the limitation of GSA, Mirjalili [55] proposed an Adaptive gbest-Guided Gravitational Search Algorithm (AgGGSA), in which the best mass is archived and utilized to accelerate the exploitation phase, enriching the weakness of GSA. Grey Wolf Optimizer (GWO) is a recently developed powerful evolutionary algorithm proposed by Seyedali Mirjalili [57] and has the ability to converge to a better quality near-optimal solution and possesses better convergence characteristics than other prevailing techniques reported in the recent literatures. Also, GWO has a good balance between exploration and exploitation that result in high local optima avoidance. Ghazzai et al. [62, 63] applied GWO for cell planning problem for the fourth-generation (4G) LTE cellular networks. Muangkote et al. [64] proposed Improved Grey Wolf Optimizer for evaluated by adopting the IGWO to training q-Gaussian Radial Basis Functional-link nets (qRBFLNs) neural networks.

2 Economic load dispatch problem formulation

The scheduling of electric utilities along with the distribution of the generation power which must be planned to meet the load demand for a specific time period represents the unit commitment problem (UCP). ELDP refers the optimal generation schedule for the generation system to deliver the required load demand plus transmission loss with the optimal generation fuel cost. Noteworthy economical benefits can be achieved by searching a better solution to the ELDP. The economic dispatch problem is defined so as to optimize the total operational cost of an electric power system while meeting the total load demand plus transmission losses within utilities generating limits [56].

The overall objective of ELDP of electric power system is to plan the devoted (Committed) electric utilities outputs so as to congregate the load demand at optimal operating cost while satisfying all generating utilities constraints and various operational constraints of the electric utilities. The ELDP is a constrained optimization problem, and it can be mathematically expressed as follows [56]:

$$\min[FC(P_n)] = \sum_{n=1}^{\mathrm{NEU}} \left(a_n P_n^2 + b_n P_n + c_n\right) \quad \$/\mathrm{Hour} \tag{1}$$

subject to:

(i) The energy balance equation:

$$\sum_{n=1}^{\mathrm{NEU}} P_n = P_{\mathrm{Demand}} + P_{\mathrm{Loss}}. \tag{2}$$

(ii) The inequality constraints:

$$P_n^{\min} \le P_n \le P_n^{\max} \quad (n = 1, 2, 3, \ldots, \mathrm{NEU}). \tag{3}$$

where, a_n, b_n and c_n are cost coefficients. P_{Demand} is load demand. P_{Loss} is power transmission loss. NEU is the number of electric generating units. P_n is real power generation and will act as decision variable.

The most simple and approximate method of expressing power transmission loss, P_{Loss} as a function of generator powers is through George's Formula using B-coefficients and mathematically can be expressed as [56]:

$$P_{\mathrm{Loss}} = \sum_{n=1}^{\mathrm{NEU}} \sum_{m=1}^{\mathrm{NEU}} P_{g_n} B_{nm} P_{g_m} \ \mathrm{MW}. \tag{4}$$

where P_{g_n} and P_{g_m} are the real power generations at the nth and mth buses, respectively.

B_{nm} is the loss coefficients which are constant under certain assumed conditions and NEU is the number of electric generating units.

The constrained ELDP can be converted to unconstrained ELD problem using penalty of definite value, which can be mathematically expressed as:

$$\min[FC(P_n)] = \sum_{n=1}^{\mathrm{NEU}} F_n(P_n) + 1000 * \mathrm{abs}\left(\sum_{n=1}^{\mathrm{NEU}} P_n - P_{\mathrm{Demand}} - \sum_{n=1}^{\mathrm{NEU}} \sum_{m=1}^{\mathrm{NEU}} B_{nm} P_n P_m\right) \tag{5}$$

The Eq. (5) represents the unconstrained ELDP including penalty factor of $\sum_{n=1}^{\mathrm{NEU}} \sum_{m=1}^{\mathrm{NEU}} B_{nm} P_n P_m$.

The complete unconstrained ELDP having (NEU-1) variables can be represented as:

$$\min[FC(P_n)] = \sum_{n=1}^{NEU}(a_nP_n^2 + b_nP_n + c_n) + 1000 * \mathrm{abs}\left(\sum_{n=1}^{NEU} P_n - P_{\mathrm{Demand}} - \sum_{n=1}^{NEU}\sum_{m=1}^{NEU} B_{nm}P_nP_m\right) \quad (6)$$

3 Grey Wolf Optimizer (GWO)

Grey Wolf Optimizer is a recently developed powerful evolutionary algorithm proposed by Mirjalili [57], to solve non-convex engineering optimization problem. Grey wolf (Canis lupus) belongs to Canidae family. Grey wolves are referred as pinnacle predators, meaning that they are at the top of the victuals sequence. Grey wolves mostly prefer to live in a group. The group size is 5–12 on average. One of the most particular interests in grey wolves is that they have a very strict social dominant hierarchy as shown in Fig. 1.

The privileged wolves are both male and female, named as alpha (α) wolves. The alpha wolves are most responsible for making decisions about quiescent place, hunting, time to arouse and all other activities. The alpha's decisions are dictated to the group. On the other hand, some kind of democratic behaviour has also been pragmatic, in which alpha wolves obey the other wolves in the group. In gatherings, the entire group acknowledges the alpha wolves by holding their tails down. As order of alpha wolves are followed by other wolves, they are named as leading wolves [58]. The alpha wolves are only permitted to companion in the group. Fascinatingly, the alpha wolves are not necessarily the strongest associate of the group but the best in terms of supervision the group. This shows that the association and regulation of a group is much more important than its power.

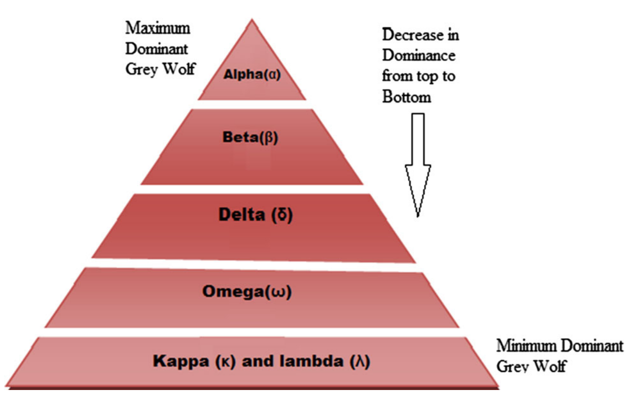


Fig. 1 Hierarchy levels of grey wolves

The next level in the chain of command of grey wolves is beta (β) wolves. The beta wolves are secondary wolves that help the alpha wolves in supervisory or other group actions. The beta wolves may be female or male and are possibly the best candidate to be the alpha in case any of the alpha wolf die or becomes aged. The beta wolf should reverence the alpha wolf, but orders the other wolves which are low in the hierarchy level. They play the responsibility of a consultant to the alpha wolves and discipliner for the group. The beta wolves support the alpha's wolves domination right through the group and gives response to the alpha wolves.

The lowest ranking grey wolves are delta (δ) wolves. The delta wolves theatre the character of scapegoat. Delta wolves forever have to put forward to other overriding wolves in the hierarchy chain. They are preceding wolves, which are permissible for scoff. It may appear that delta wolves are not much vital entity in the whole group, but whole group face internal warfare and tribulations in case of losing the delta wolves. This is due to frustration and venting of violence of all wolves by the delta(s). This assists gratifying the intact group and maintaining the ascendancy configuration.

In some cases, the delta wolves are also the governess (i.e. babysitters) in the group.

If a wolf is not an alpha, beta or delta, they are called inferior [or named as be omega (ω) wolf]. Omega wolves have to submit to alpha and beta wolves, but they direct the kappa (κ) wolves and lambda (λ) wolves of lowest hierarchy levels. Elders, scouts, hunters, caretakers and sentinels fit in this class.

Escort is accountable for inspection of restrictions of the province and caveat the group in case of danger. Sentinels guard and pledge the protection of the group. Elder wolves are veteran wolves, who second hand to be alpha or beta. Hunters assist the alphas and betas when hunting quarry and providing food for the group. At last, the caretakers are responsible for thoughtful for the ill, weak and injured wolves in the group.

Besides social hierarchy of wolves, group hunting is another interesting societal behaviour of grey wolves. According to Muro et al. [59], the main phases of grey wolf hunting are as follows:

- Tracking, chasing and approaching the prey
- Pursuing, encircling and harassing the prey until it stops moving
- Attack towards the prey

The steps for tracking, chasing and approaching towards the prey are show in Fig. 2a–c. The process of pursuing, encircling and harassing the prey is shown in Fig. 2d–f and attack towards the prey are shown in Fig. 2g–i.

Fig. 2 **a–c** Tracking, chasing and approaching towards the prey; **d–f** Pursuing, encircling and harassing the prey; **g–i** attack towards the prey

3.1 Mathematical formulation of social behaviour of grey wolves

In order to mathematically model the social governance of wolves when designing Grey Wold Optimizer (GWO), assume the fittest solution as the alpha (α). Consequently, the second and third best solutions are named beta (β) and delta (δ), respectively. The rest of the candidate solutions are assumed to be omega (ω), kappa (κ) and lambda (λ). In the GWO algorithm, the optimization (i.e. hunting) is guided by α, β and δ. The ω, κ and λ wolves trail these three wolves.

3.2 Encircling or trapping prey

As mentioned above, grey wolves encircle prey during the hunt. In order to mathematically model encircling behaviour, the following equations are proposed:

$$\vec{D} = \left| \vec{C} \cdot \vec{X}_{\text{Prey}}(t) - \vec{X}_{\text{GWolf}}(t) \right| \tag{7}$$

$$\vec{X}_{\text{GWolf}}(t+1) = \vec{X}_{\text{Prey}}(t) - \vec{A} \cdot \vec{D} \tag{8}$$

where t indicates the current iteration, $\vec{A}$ and $\vec{C}$ are coefficient vectors, $\vec{X}_{\text{Prey}}$ is the position vector of the prey, and $\vec{X}_{GWolf}$ indicates the position vector of a grey wolf.

The vectors $\overrightarrow{A}$ and $\overrightarrow{C}$ are calculated as follows:

$$\overrightarrow{A} = 2\overrightarrow{a} \cdot \overrightarrow{r_1} - \overrightarrow{a} \tag{9}$$

$$\overrightarrow{C} = 2 \cdot \overrightarrow{r_2} \tag{10}$$

where components of $\overrightarrow{a}$ are linearly decreased from 2 to 0 over the course of iterations, and $\vec{r}_1$ and $\vec{r}_2$ are random vectors between 0 and 1.

The effect of Eqs. (7) and (8) as 2D positions vector and possible neighbours are illustrated in Fig. 3a. According to Fig. 3a, a grey wolf in the position of (X, Y) can update its position according to the position of the prey (X^*, Y^*). Different places around the best agent can be reached with respect to the current position by adjusting the value of $\vec{A}$ and $\vec{C}$ vectors.

The possible updated positions of a grey wolf in three-dimensional space are shown in Fig. 3b.

The random positions vectors, which allow grey wolves to reach any position between the points, are shown in Fig. 4.

Therefore, a grey wolf can update its position inside the space around the prey in any random location by using Eqs. (7) and (8) (Fig. 5).

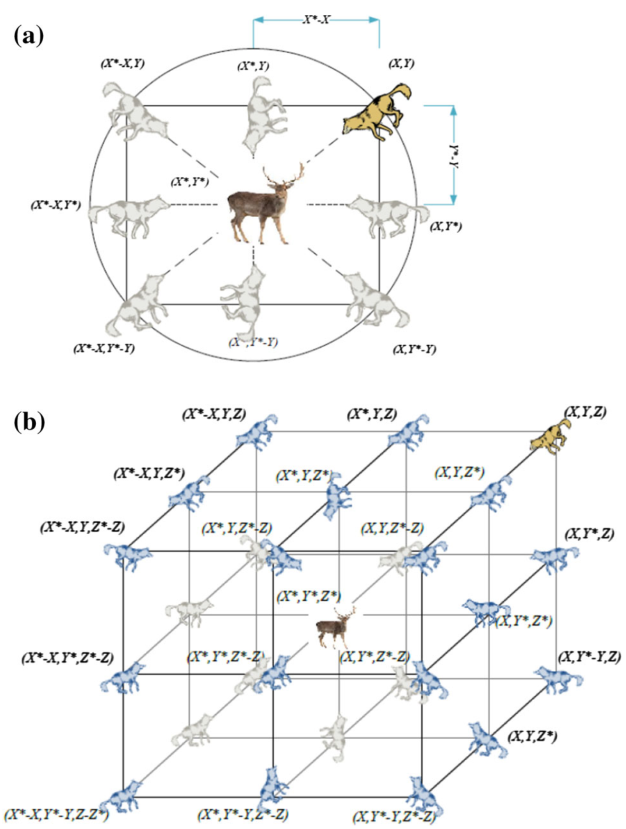


Fig. 3 **a** Two-dimensional position vectors and possible next location w.r.t. prey [57]. **b** Three-dimensional position vectors and possible next location w.r.t. prey [57]

3.3 Hunting of prey

Grey wolves have the ability to recognize the location of prey and enclose or trap them. The hunt is usually guided by the alpha. The beta and delta might also participate in hunting occasionally. However, in an abstract search space, we have no idea about the location of the optimum (prey). In order to mathematically simulate the hunting behaviour of grey wolves, we suppose that the alpha (best candidate solution) beta and delta have better knowledge about the potential location of prey. Therefore, we save the first three best solutions obtained so far and oblige the other search agents (including the delta, kappa and lambda) to update their positions according to the position of the best search agent. The score and positions of first three search agents (i.e. alpha, beta and delta) can be updated using the Eqs. (11), (12) and (13), respectively.

$$\vec{D}_{\text{Alpha}} = \left| \vec{C}_1 \cdot \vec{X}_{\text{Alpha}} - \vec{X} \right| \tag{11}$$

$$\vec{D}_{\text{Beta}} = \left| \vec{C}_2 \cdot \vec{X}_{\text{Beta}} - \vec{X} \right| \tag{12}$$

$$\vec{D}_{\text{Delta}} = \left| \vec{C}_3 \cdot \vec{X}_{\text{Delta}} - \vec{X} \right| \tag{13}$$

The position vector of prey with respect to alpha, beta and delta wolves can be calculated using the following mathematical formulation:

$$\vec{X}_1 = \vec{X}_{\text{Alpha}} - \vec{A}_1 \cdot (\vec{D}_{\text{Alpha}}) \tag{14}$$

$$\vec{X}_2 = \vec{X}_{\text{Beta}} - \vec{A}_2 \cdot (\vec{D}_{\text{Beta}}) \tag{15}$$

$$\vec{X}_3 = \vec{X}_{\text{Delta}} - \vec{A}_3 \cdot (\vec{D}_{\text{Delta}}) \tag{16}$$

The best position can be calculated taking average of alpha, beta and delta wolves as depicted below in Eq. (17)

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{17}$$

Figure 3a, b shows how a search agent updates its position according to alpha, beta and delta in a 2D and 3D search space, respectively. It can be observed that the final position would be in a random place within a circle, which is defined by the positions of alpha, beta and delta in the search space. In other words, alpha, beta and delta wolves estimate the position of the prey, and other wolves update their positions randomly around the prey.

3.4 Flow chart for economic load dispatch using GWO

Initialize a set of candidate for solution X. This solution comprises of the number of generations of the system that will be optimized, which resulted a minimum cost by

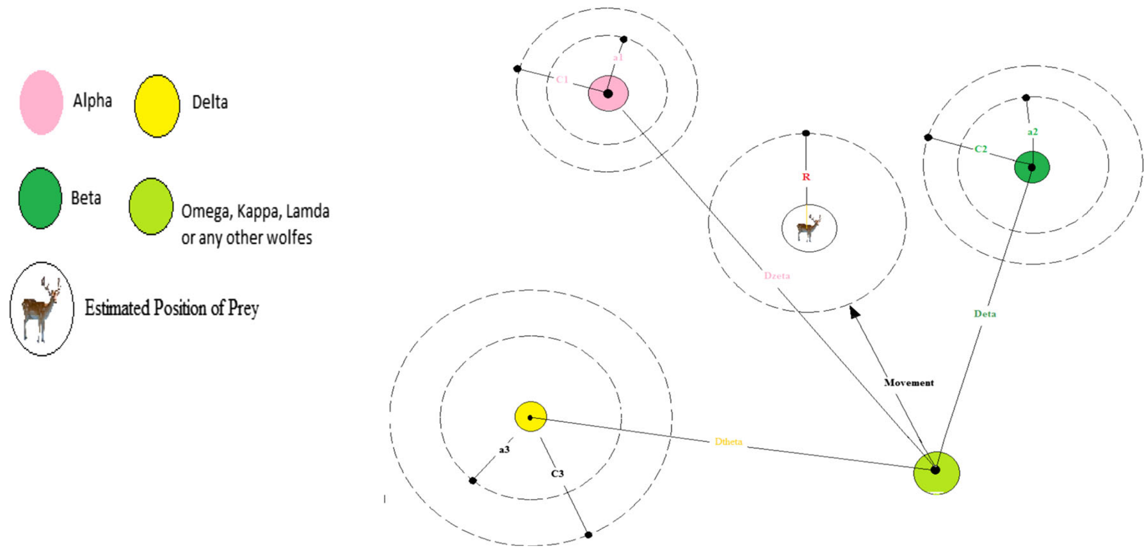
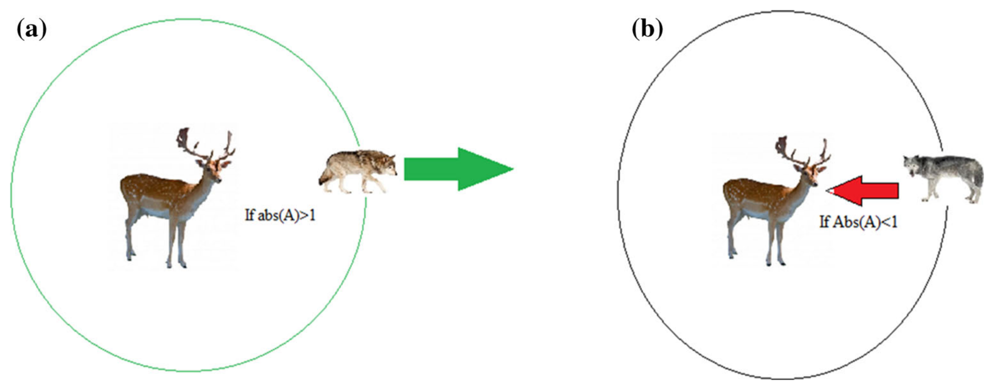


Fig. 4 Updating of position of alpha, beta and delta grey wolves in GWO

Fig. 5 **a** Searching for prey, **b** attacking for prey



fulfilling all the constraints. The variables of the optimal ELDP are expressed as follows:

$$X = \begin{bmatrix} x_1 & x_2 & x_3 & \ldots & x_{\mathrm{NEU}} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ x_{\mathrm{SA}} & \ldots & \ldots & \ldots & x_{\mathrm{SA}\times\mathrm{NEU}} \end{bmatrix}_{\mathrm{SA}\times\mathrm{NEU}} \tag{18}$$

where NEU is the number of generating units and SA is the number of search agents, which is generated randomly for initialization. Equation (6) was applied in the performance evaluation of the ELDP until the optimum cost is achieved. For inequality constraints, similar to any other techniques, when the solutions obtained for any iteration are out of boundaries, GWO chooses the boundaries values, while for equality constraint, when it is violated, the penalty factor of 1000 is implemented and embedded in the cost function as per Eq. (5). The algorithm will continue until the maximum iteration is met, and the optimum results are obtained. The flow chart of GWO algorithm is given below in Fig. 6.

4 Test systems, results and discussion

In order to show the effectiveness of the GWO algorithm for economic load dispatch problem, benchmark test system of small-, medium- and large-scale power systems having standard IEEE bus systems have been taken into consideration.

4.1 Test system-I: small-scale power system

For small-scale power plants, three different cases are taken into consideration:

Case-I The first test system consists of 3-generating units with a load demand of 150 MW [60]. Test data of 3-generating-unit system are taken from [60], loss coefficient matrices are used to calculate the corresponding transmission. The corresponding results are compared with lambda iteration method [60] and Particle Swarm Optimization

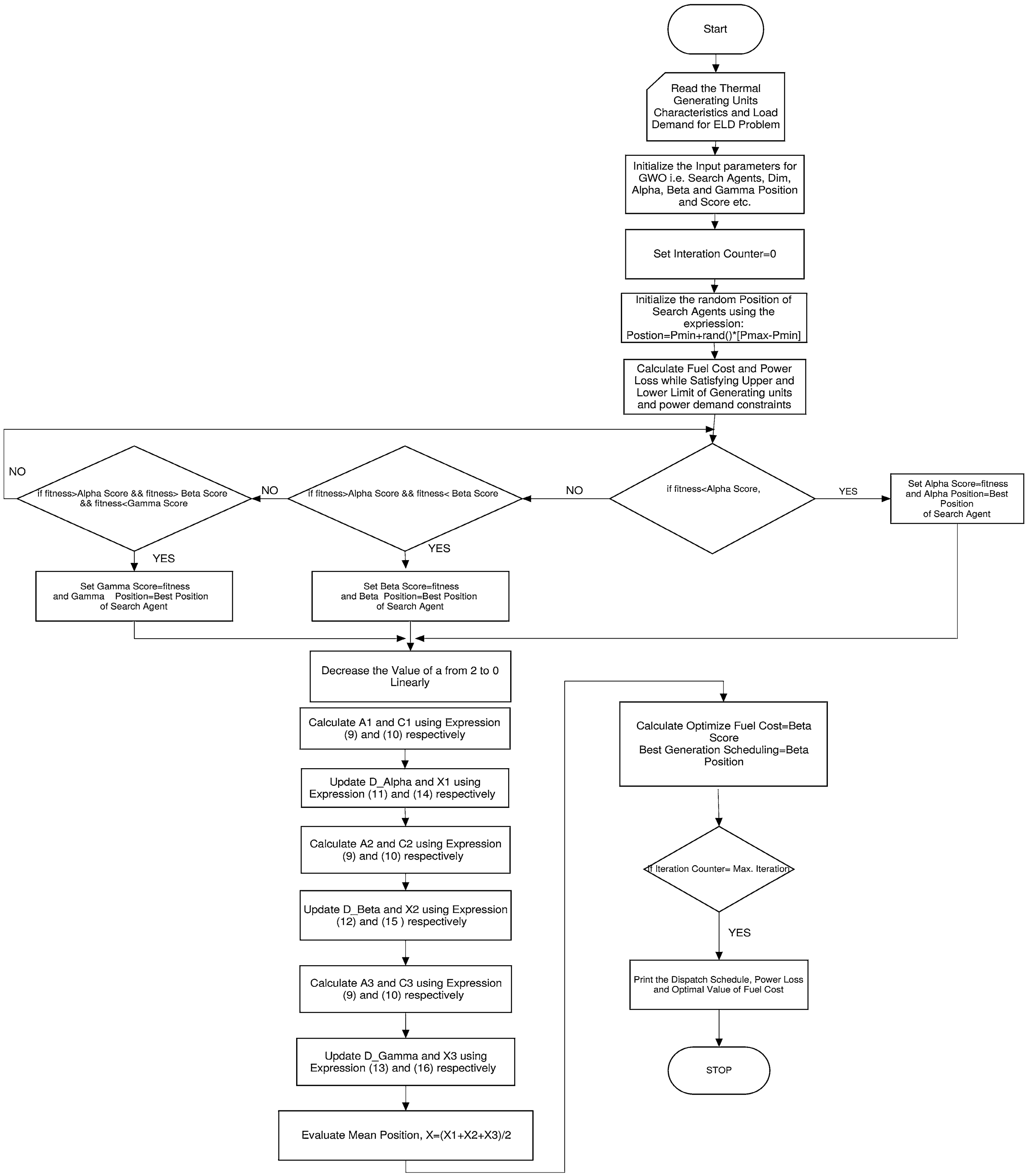


Fig. 6 Flow chart of GWO for ELD problem

(PSO) [60]. Table 1 shows that total fuel cost for 3-unit generating model for 150 MW load demand using GWO algorithm is 1597.4815 Rs./Hour and power loss is 2.344 MW, which is less than lambda iteration method and PSO.

Case-II The second test system also consisting of 3-generating-unit system [61] is tested for two different load demands of 850 and 1050 MW including transmission losses. The corresponding results are compared with lambda iteration method [61], Genetic Algorithm (GA) [61], Particle

Table 1 Economic load dispatch for 3-generating-unit system [case-I] (Load demand = 150 MW)

Method	Load demand (MW)	P_1 (MW)	P_2 (MW)	P_3 (MW)	Fuel cost (Rs./h)	P_{loss} (MW)	No. of iteration	Elapsed time (s)
Lambda iteration [60]	150	33.4401	64.0974	55.1011	1599.9	2.66	250	NA
PSO [60]	150	33.0858	64.4545	54.8325	1598.79	2.37	250	NA
GWO	150	30.4998	64.6208	54.8994	**1597.4815**	**2.3444**	250	**4.761541**

The bold results show the superiority of Grey Wolf Optimizer over other well known algorithms

Table 2 Economic load dispatch for 3-generating-unit system [case-II] (Load demand = 850 MW)

Method	Load demand (MW)	Generation scheduling			Fuel cost (Rs./h)	Best cost	Average cost	Worst cost
		U1	U2	U3				
Lambda Iteration	850	382.258	127.419	340.323	8575.68	NA	NA	NA
GA	850	382.255	127.418	340.3202	8575.64	NA	NA	NA
PSO	850	394.524	200	255.4756	8280.81	NA	NA	NA
ABC	850	300.266	149.733	400	8253.1	NA	NA	NA
GWO	850	**300.51**	**149.81**	**399.6777**	**8253.1053**	**8253.1053**	**8253.10558**	**8253.1061**

The bold results show the superiority of Grey Wolf Optimizer over other well known algorithms

Table 3 Economic load dispatch for 3-generating-unit system [case-II] (Load demand = 1050 MW)

Method	Load demand (MW)	Generation scheduling			Cost (Rs./h)	Best cost	Average cost	Worst cost
		U1	U2	U3				
Lambda iteration	1050	487.5	162.5	400	10212.459	NA	NA	NA
GA	1050	487.498	162.499	400	10212.44	NA	NA	NA
PSO	1050	492.699	157.3	400	10123.73	NA	NA	NA
ABC	1050	492.699	157.301	400	10123.73	NA	NA	NA
GWO	1050	492.847	157.393	399.7609	**10123.7196**	**10123.72**	**10123.7347**	**10123.7392**

The bold results show the superiority of Grey Wolf Optimizer over other well known algorithms

Swarm Optimization (PSO) [61] and Artificial Bee Colony (ABC) [61]. Tables 2 and 3 show the comparison of results with different methodologies, and it is found that optimal value of fuel cost obtained by GWO cost is much less that lambda iteration, GA, PSO and ABC. The convergence curve of test case-I and case-II is shown in Fig. 7a–d.

Case-III The third test case consists of 6-generating-unit system without valve-point loading [60]. The results of 6-generating-unit systems are tested for load demands of 600, 700, 800, 900 and 1000 MW and are shown in Table 4, and effectiveness of GWO for 6-generating-unit system is compared with lambda iteration method [60] and Particle Swarm Optimization (PSO) [60]. Corresponding analysis of results (Table 5) shows that GWO algorithm yields better fuel cost and power loss as compared to lambda iteration method and Particle Swarm Optimization algorithm. Also, the convergence of algorithm is much better than these algorithms.

4.2 Test system-II: medium-scale power system

Medium-scale power systems are tested for two different benchmark systems.

Case-I 13-Generating-unit system [65] considering valve-point effect for load demand of 1800 MW. The performance of GWO for 13-unit test system is compared with NN-EPSO [66] (Table 6), CEP [65], FEP [65], MFEP [65], IFEP [65] (Tables 7, 8), and it is found that the convergence of GWO is very fast as compared to NN-EPSO, CEP, FEP, MFEP and IFEP. Convergence curve for 13-generating-unit system is shown in Fig. 8a.

Case-II 20-Generating-unit system [67] without valve-point loading considering transmission losses is tested for convergence parameters, and it is found that optimization converges up to 193 iterations. The convergence curve for the same is depicted in Fig. 8b.

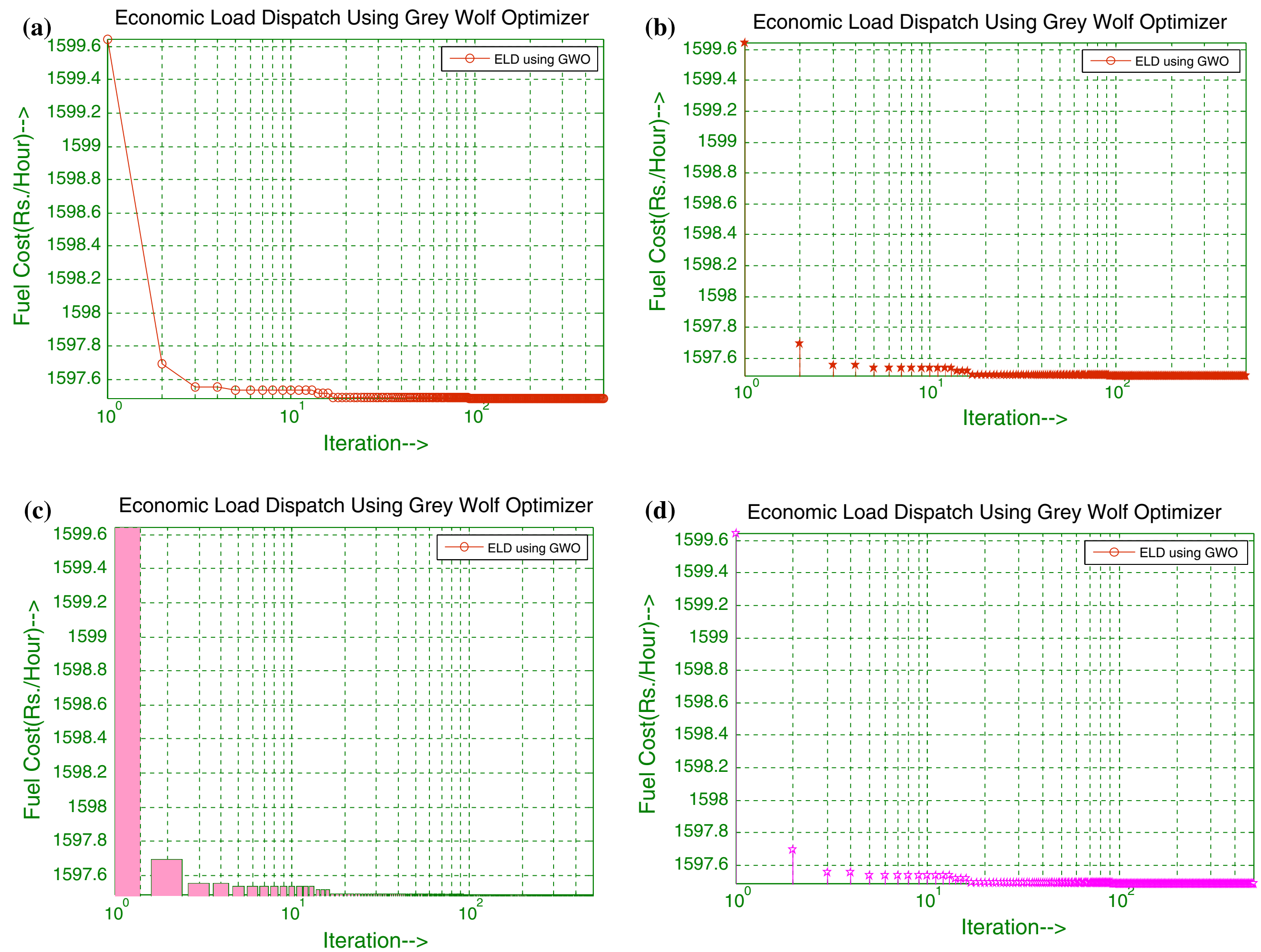


Fig. 7 Convergence of GWO algorithm for ELDP [3-generating-unit system (case-I and case-II)]

4.3 Test system-III: large-scale power system

Large-scale power systems are tested for three different benchmark systems.

Case-I 38-Generating-unit system [68] without valve-point loading is tested for load demand of 6000 MW and performance of proposed algorithm is compared with Biogeography-Based Optimization, pattern search method and λ-logic-based method [69]. Best, mean and average cost and time (in seconds) are depicted in Table 9, corresponding generation scheduling is shown in Fig. 9, and from comparative analysis, it has been found that performance of proposed method is much better than BBO, PS, DE and λ-logic-based method.

Case-II Korean power system consisting of 140-generating-unit system [70] without valve-point effect is tested

Table 4 Economic load dispatch for 6-generating units

Method	No. of iterations	Load demand (MW)	Generation scheduling						Fuel cost (Rs./h)	P_{Loss} (MW)	Elapsed time (s)
			P_1	P_2	P_3	P_4	P_5	P_6			
GWO	1000	600	23.7823	10.0026	95.6928	100.6695	203.1324	180.954	32,091.5107	14.2377	5.560456
	1000	700	28.2962	10.0027	118.8999	118.8304	230.7961	212.6049	36,908.451	19.4303	7.032372
	1000	800	32.0952	14.6235	141.4852	136.1825	257.8341	243.1175	41,892.3867	25.3379	7.035746
	1000	900	36.9656	21.196	163.7997	153.1386	284.1867	272.697	47,040.3513	31.9837	6.942486
	1000	1000	41.1915	27.88	186.4387	170.5973	310.6946	302.6778	52,355.7728	39.4798	6.960886

Table 5 Comparison of results for 6-generating-unit system

Load demand (MW)	Methods	P_1 (MW)	P_2 (MW)	P_3 (MW)	P_4 (MW)	P_5 (MW)	P_6 (MW)	Fuel cost (Rs./h)	P_{Loss}	Iteration time (s)
600	Lambda iteration	23.7909	10.22	95.25	10.12309	202.967	181.34	32132.29	14.7988	NA
	PSO	23.8602	10	95.6394	100.7081	202.8315	181.1978	32094.72	14.2373	NA
	GWO	23.7823	10.0026	95.6928	100.6695	203.1324	180.954	**32091.5107**	14.2377	**5.560456**
700	Lambda iteration	28.29	10.0901	118.9873	118	230.2372	213.9068	36912.32	19.5114	NA
	PSO	28.29	10	118.9583	118.6747	230.763	212.7449	36912.22	19.43	NA
	GWO	28.2962	10.0027	118.8999	118.8304	230.7961	212.6049	**36908.451**	**19.4303**	**7.032372**
800	Lambda iteration	32.9521	14.7126	141.5988	136.0345	258.1009	243.8011	41897.25	27.5	NA
	PSO	32.586	14.4839	141.5475	136.0435	257.6624	243.0073	41896.7	25.33	NA
	GWO	32.0952	14.6235	141.4852	136.1825	257.8341	243.1175	**41892.3867**	25.3379	**7.035746**
900	Lambda iteration	36.9889	22.1821	163.01	153.2168	284.1482	273.0581	47045.32	32.6131	NA
	PSO	36.848	21.0774	163.9304	153.263	284.1696	272.7301	47045.25	31.98	NA
	GWO	36.9656	21.196	163.7997	153.1386	284.1867	272.697	**47040.3513**	31.9837	**6.942486**
1000	Lambda iteration	40.3969	28.1002	187	171.2136	310.721	303.1006	52362.07	40.5323	NA
	PSO	41.1657	27.7786	186.5604	170.5795	310.8297	302.568	52361.65	39.4821	NA
	GWO	41.1915	27.88	186.4387	170.5973	310.6946	302.6778	**52355.7728**	**39.4798**	**6.960886**

The bold results show the superiority of Grey Wolf Optimizer over other well known algorithms

Table 6 Comparison of results for medium-scale power systems

Comparison of results for 13-generating-unit system [Load demand = 1800 MW]													
Method	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}
NN-EPSO	490	189	214	160	90	120	103	88	104	13	58	66	55
GWO	807.1247	144.869	297.9434	60	60	60	60	60	60.0362	40	40.0267	55	55

Table 7 Comparison of results for medium-scale power systems [13-unit benchmark system]

Comparison of results for 13-generating-unit system [Load demand = 1800 MW]													
Method	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}
NN-EPSO	490	189	214	160	90	120	103	88	104	13	58	66	55
GWO	807.1247	144.869	297.9434	60	60	60	60	60	60.0362	40	40.0267	55	55

NN-EPSO		Grey Wolf Optimizer (proposed method)	
Load demand (MW)	1800 MW	Load demand (MW)	1800 MW
Fuel cost ($)	18442.59	Fuel cost ($)	18051.11
Iteration time (s)	NA	Iteration time (s)	3.116071

Table 8 Comparison of results for medium-scale power systems

Comparison of convergence time for 13-generating-unit system			
Method	Best time (s)	Mean time (s)	Worst time (s)
CEP [65]	293.41	294.96	NA
FEP [65]	166.43	168.11	NA
MFEP [65]	315.98	317.12	NA
IFEP [65]	156.81	157.43	NA
GWO	**3.116071**	**3.182163**	**3.228003**

The bold results show the superiority of Grey Wolf Optimizer over other well known algorithms

for load demand of 49,342 MW, and it is found that elapsed time is 28,052.541250 s, which is very large. It had been observed that algorithm does not converge to optimal value up to 100,000 iterations. The convergence curve for 140-unit test system is shown in Fig. 10.

Case-III Third large-scale test system is tested for 520-generating-unit system. The test data for 520-unit system were obtained by adding the units of test systems of 140-units [70] three times-generating unit system, and it is found that system goes out of memory.

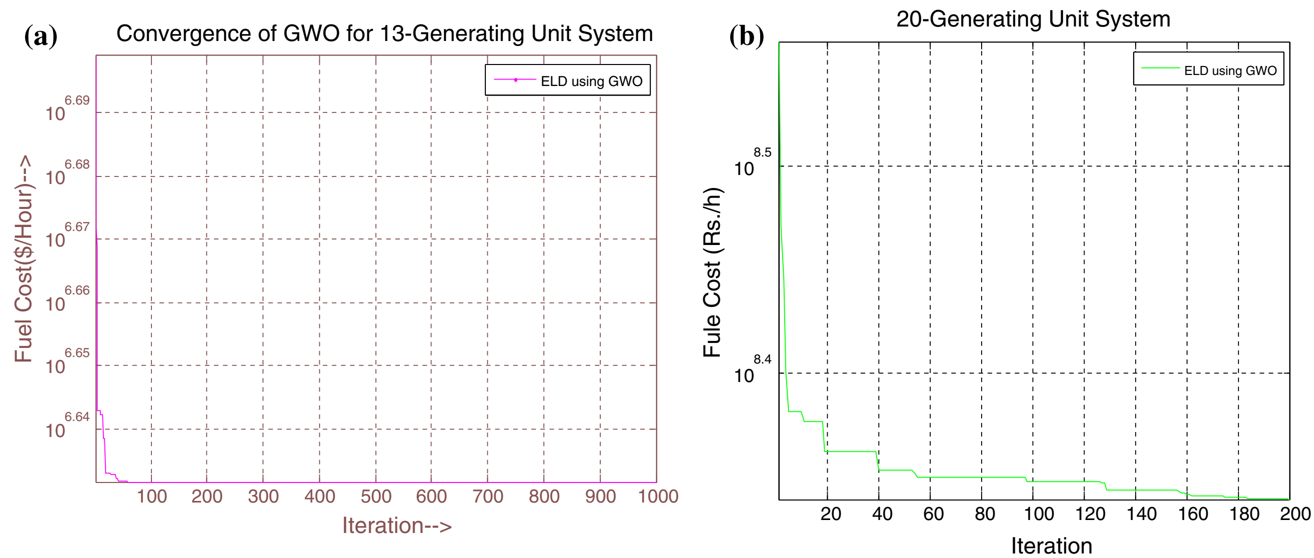


Fig. 8 Convergence of GWO for medium-scale power system (13 and 20-unit system)

Table 9 Comparative analysis of results for 38-generating-unit system

38-Generating-unit system characteristics and results								
Generating unit characteristics					Comparison of results			
a ($/MW^2)	*b* ($/MW)	*c* ($)	P_{min}	P_{max}	Biogeography-Based optimization (BBO)	λ-logic-based method	Pattern search (PS)	Grey Wolf Optimizer (GWO)
0.3133	796.9	64,782	220	550	550	426.6061	258.3397	429.7056
0.3133	796.9	64,782	220	550	550	426.6061	258.3397	416.2439
0.3127	795.5	64,670	200	500	500	429.6633	238.3397	408.4052
0.3127	795.5	64,670	200	500	500	429.6633	238.3397	412.4527
0.3127	795.5	64,670	200	500	375.6216	429.6633	238.3397	433.6422
0.3127	795.5	64,670	200	500	200	429.6633	238.3397	425.6522
0.3127	795.5	64,670	200	500	200	429.6633	238.3397	435.6207
0.3127	795.5	64,670	200	500	200	429.6633	238.3397	437.6536
0.7075	915.7	172,832	114	500	114	114	196.2345	115.2751
0.7075	915.7	172,832	114	500	114.6486	114	196.2345	116.883
0.7515	884.2	176,003	114	500	162.1622	119.7681	196.2345	130.7939
0.7083	884.2	173,028	114	500	114	127.0729	196.2345	153.2393
0.4211	1250.1	91,340	110	500	129.2432	110	196.2345	110
0.5145	1298.6	63,440	90	365	90	90	196.2345	90.028
0.5691	1298.6	65,468	82	365	153.2432	82	196.2345	82.0111
0.5691	1290.8	77,282	120	325	120	120	196.2345	120
2.5881	238.1	190,928	65	315	204.3243	159.5981	196.2345	157.1682
3.8734	1149.5	285,372	65	315	65	65	196.2345	65
3.6842	1269.1	271,676	65	315	65	65	196.2345	65.0326
0.4921	696.1	39,197	120	272	120	272	196.2345	271.9524
0.5728	660.2	45,576	120	272	182.4324	272	196.2345	271.959
0.3572	803.2	28,770	110	260	110	160	196.2345	259.81
0.9415	818.2	36,902	80	190	187.2973	130.6487	190	120.8832
52.123	33.5	105,510	10	150	27.027	10	150	12.3567
1.1421	805.4	22,233	60	125	125	113.3051	125	107.634
2.0275	707.1	30,953	55	110	110	88.0669	110	92.4117

Table 9 continued

38-Generating-unit system characteristics and results								
Generating unit characteristics					Comparison of results			
a ($/MW^2)	*b* ($/MW)	*c* ($)	P_{min}	P_{max}	Biogeography-Based optimization (BBO)	λ-logic-based method	Pattern search (PS)	Grey Wolf Optimizer (GWO)
3.0744	833.6	17,044	35	75	75	37.5051	75	39.6668
16.765	2188.7	81,079	20	70	70	20	70	20.005
26.355	1024.4	125,767	20	70	70	20	70	20.0014
30.575	837.1	121,915	20	70	70	20	70	20.0302
25.098	1305.2	120,780	20	70	70	20	70	20.013
33.722	716.6	104,441	20	60	60	20	60	20.007
23.915	1633.9	83,224	25	60	60	35	60	25.0032
32.562	969.6	111,281	18	60	60	18	60	18.008
18.362	2625.8	64,142	8	60	60	8	60	8.006
23.915	1633.9	103,519	25	60	60	25	60	25.002
8.482	694.7	13,547	20	38	38	21	38	22.4379
9.693	655.9	13,518	20	38	38	21	38	20.0048
Results obtained by Grey Wolf Optimizer								
Best, mean and worst fuel cost					Best, mean and worst time (s)			
Best fuel cost	9419270.188				Best time	9.457007		
Mean fuel cost	9419978.978				Mean time	9.668424333		
Worst fuel cost	9421100				Worst time	9.973747		

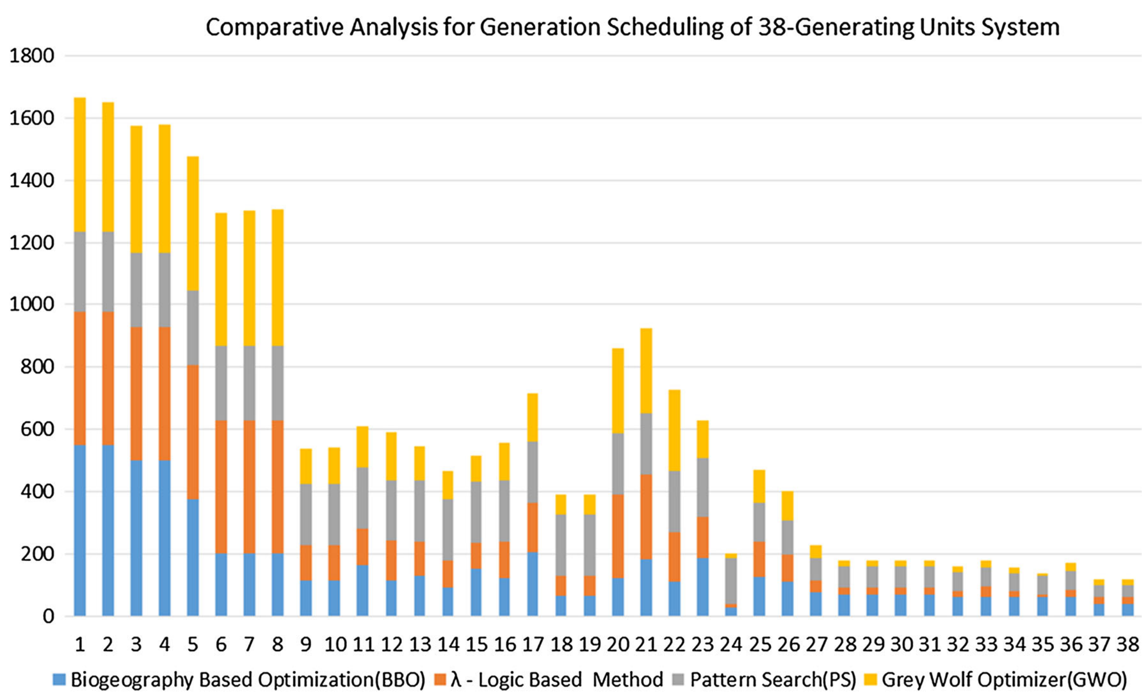


Fig. 9 Generation scheduling of 38-generating-unit system

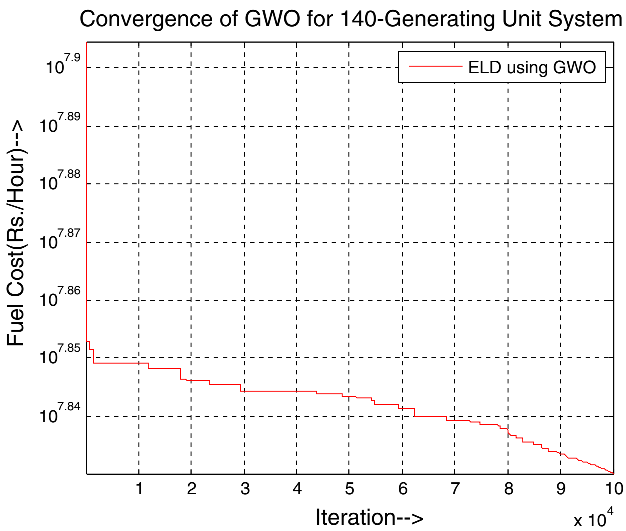


Fig. 10 Convergence of GWO for large-scale power system

5 Conclusions

In this research paper, application of GWO algorithm is presented for the solution of non-convex and dynamic ELDP of electric power system. Performance of GWO algorithm is tested for small-, medium- and large-scale power plants. The effectiveness of proposed GWO algorithm is tested with the standard IEEE bus system consisting of 3-, 6-, 13-, 20-, 38-, 140- and 520-generating-unit model.

The results obtained show that GWO has been successfully implemented to solve different ELD problems; moreover, GWO is able to provide very spirited results in terms of minimizing total fuel cost and lower transmission loss. Also, convergence of GWO is very fast as compared to lambda iteration method, Particle Swarm Optimization (PSO) algorithm, Genetic Algorithm (GA), Biogeography-Based Optimization (BBO), Differential Evolution (DE) algorithm, pattern search algorithm, NN-EPSO, FEP, CEP, IFEP and MFEP for small- and medium-scale power systems

Also, it has been observed that the GWO has the ability to converge to a better quality near-optimal solution and possesses better convergence characteristics than other widespread techniques reported in the recent literatures. It is also clear from the results obtained by different trials that the GWO shows a good balance between exploration and exploitation that result in high local optima avoidance. This superior capability is due to the adaptive value of A. It is because half of the iterations are devoted to exploration, $\vec{A}$ >1 and the rest to exploitation $\vec{A}$ <1. Thus, this algorithm may become very promising for solving some more complex power system optimization problems such as: economic load dispatch for quadratic and cubical cost function, Single and Multi-objective economic load dispatch including valve-point effect, Economic Load Dispatch incorporating wind Power, Economic Load Dispatch incorporating Solar Power, Hydro-Thermal and Wind-Thermal Scheduling of electric power system. Thermal Scheduling incorporating Smart Grids, Hydro-Thermal Scheduling incorporating Smart Grids, Single and Multi-Objective Unit Commitment Problem formulation, Multi-Objective and Multi-Area Unit Commitment Problem.

6 Future scope

Recently developed algorithms like Ant Lion Optimizer (ALO), Multi Verse Optimizer (MVO), Dragonfly Algorithm (DA), and Ions Motion Optimization algorithm (IMO) proposed by Seyedali Mirjalili can be applied for the solution of non-convex ELDP for improved performance.

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