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ORIGINAL ARTICLE

An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets

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Abstract In this paper, a novel outranking approach for multi-criteria decision-making (MCDM) problems is proposed to address situations where there is a set of numbers in the real unit interval and not just a specific number with a neutrosophic set. Firstly, the operations of interval neutrosophic sets and their related properties are introduced. Then some outranking relations for interval neutrosophic numbers (INNs) are defined based on ELECTRE IV, and the properties of the outranking relations are further discussed in detail. Additionally, based on the outranking relations of INNs, a ranking approach is developed in order to solve MCDM problems. Finally, two practical examples are provided to illustrate the practicality and effectiveness of the proposed approach. Moreover, a comparison analysis based on the same examples is also conducted.

Keywords Interval-valued neutrosophic sets · Multi-criteria decision-making · Outranking · ELECTRE IV

1 Introduction

In order to deal with different types of uncertainties, Zadeh [1] proposed his seminal theory of fuzzy sets (FSs) in 1965; since then, it has been applied successfully in various fields [2]. However, in certain cases it is hard to define the grade of membership of an FS with a specific value. Therefore, Turksen [3] introduced interval-valued fuzzy sets (IVFSs) in 1986. Subsequently, Atanassov [4, 5] proposed intuitionistic fuzzy sets (IFSs) to deal with the lack of knowledge of non-membership degrees. Moreover, Gau and Buehrer [6] defined vague sets, and according to Bustince, these vague sets and Atanassov's IFSs are mathematically equivalent objects [7]. To date, IFS has been widely applied in solving multi-criteria decision-making (MCDM) problems [8–10] in fields such as medical diagnosis [11], neural networks [12], color region extraction [13, 14] and market prediction [15]. IFSs were subsequently extended to interval-valued intuitionistic fuzzy sets (IVIFSs) [16], as well as to intuitionistic interval FSs with triangular intuitionistic fuzzy numbers [17]. In order to cope with these situations where people are hesitant in expressing their preference with regard to objects in a decision-making process, hesitant fuzzy sets (HFSs) were introduced by Torra [18] and Narukawa [19]. In addition, some further extensions have been proposed [20–22], and methods with intuitionistic fuzzy numbers from some new groups' decision-making analysis have been developed [23, 24].

Despite the fact that the theory of FSs has been developed and generalized, it could not deal with all types of uncertainties, such as indeterminate and inconsistent information, in real decision-making problems. For example [25], when an expert gives the opinion about a certain statement, he or she may say that the possibility that the statement is true is 0.5, the degree of false statement is 0.6, and the possibility that he or she is not sure is 0.2. A further example relates to the field of medicine. Sometimes it is difficult for a doctor to make a certain diagnosis when a patient is suffering from a disease; therefore, she/he will often give an analysis with a degree of truth and falsity, as well as indeterminacy, such as 60 % of "yes", 30 % of "no" and 20 % of "not sure". These issues are beyond the scope of the FSs and IFSs. Therefore, some new theories are required.

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For this purpose, Smarandache [26, 27] developed neutrosophic logic and neutrosophic sets (NSs) in 1995. The NS is a set where each element of the universe has a degree of truth, indeterminacy and falsity and which lies in $]0^{-}, 1^{+}[$, the nonstandard unit interval [28]. Clearly, this is the extension to the standard interval [0, 1] as in the IFS. Furthermore, the uncertainty present here, i.e., indeterminacy factor, is independent of truth and falsity values, while the incorporated uncertainty is dependent of the degrees of belongingness and non-belongingness in IFSs [29]. Additionally, with regard to the aforementioned example about expert statement, it can be expressed as $x(0.5, 0.2, 0.6)$ by NSs.

To date, some extensions to NS have been proposed. For example, a single-valued neutrosophic set (SVNS), which is a variation of an NS [25], was introduced to facilitate its practical use. With regard to SVNS, Majumdar and Samant [29] proposed a measure for the entropy, while Ye [30] suggested methods for solving MCDM problems using a cross-entropy and correlation coefficient [31]. Moreover, Liu and Wang [32] proposed a single-valued neutrosophic normalized weighted Bonferroni mean operator. Additionally, MCDM methods were suggested for simplified neutrosophic sets (SNSs) [31, 33]. However, as the degree of truth, falsity and indeterminacy about a certain statement could not be defined precisely in real situations; an interval neutrosophic set (INS) was required, which is similar to an IVIFS. Wang et al. [34] proposed the concept of an INS and developed its set-theoretical operators. Furthermore, the operations of an INS were discussed in [35, 36], and its correlation coefficients have also been considered [37], as well the cosine similarity measure of INS was proposed [38]. Additionally, on the basis of INSs, MCDM methods using aggregation operators were proposed in [39]. Moreover, in order to utilize the advantages of INSs and interval-valued HFSs, an interval neutrosophic hesitant fuzzy set (INHFS) was proposed and its novel aggregation operators were developed [40]. In order to deal with MCDM problems in a neutrosophic linguistic environment, Broumi and Smarandache [41] introduced a neutrosophic trapezoid linguistic weighted geometric aggregation operator and an extended TOPSIS method was developed [42]. Somewhat differently, Tian et al. [43] proposed a simplified neutrosophic linguistic Bonferroni mean operator and a simplified neutrosophic linguistic normalized weighted Bonferroni mean operator to resolve MCDM problems where the evaluation information takes the form of simplified neutrosophic linguistic numbers. At the same time, multi-valued neutrosophic sets (MVNSs) were introduced [44], based on which the power aggregation operators for MVNNs were defined [45]. On the basis of soft set theory [46–48], neutrosophic soft set and its operations were defined [49, 50], and interval-valued neutrosophic soft sets with their relations have been discussed [51–55].

The methods for solving MCDM problems using INSs that are outlined above are all function models in nature. In a function method, the priority of the alternatives will be obtained being aggregated by given weights according to aggregation operators. The order of the alternatives has properties as transitivity, completeness and reflexivity. In these function models, it is assumed that the criteria are independent and subject to compensation, which causes information loss and ignores the actual will of decision-makers. For example, for an alternative A, even if it is worse than alternative B with regard to a criterion, it may be compensated by other criteria and the final ranking maybe $A > B$. Clearly, this fails to meet some practical requirements; therefore, another method, namely the relation model, is needed to deal with these problems. The relation model adopts an outranking relation [56], which proceeds by pair-wise comparison of alternatives for each criterion in order to rank and classify alternatives in terms of their priority according to the criteria. The ELECTRE and PROMETHREE methods are the main representatives in this field; moreover, in an outranking relation, the hypothesis is weaker and more related to actual circumstance [57].

Since they were originally developed by Benayoun, Roy and Sussmann in 1966 [58], the ELECTRE methods have been successfully and widely used in many actual decision-making problems, from agriculture [59] to medical science [60], from finance [61] to economics [62] and from project selection [63] to communication and transportation [63, 64]. Theoretical research into ELECTRE has been intensive, and different versions of this method have been proposed for dealing with different decision-making situations; these include ELECTRE I, ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE TRI-C and ELECTRE IS [58, 65–67]. Furthermore, some approaches based on ELECTRE have been developed to solve MCDM problems, including those where the evaluation values are IFSs, SNSs [57], interval type-2 FSs [68], IVIFSs [69] and HFSs [70, 71]. Moreover, this approach has been extended to solve MCDM problems where the assessments of the alternative are the form of linguistic information [72, 73].

Most of the above methods are needed to define the relative importance of the coefficients of criteria. However, people are often not willing or able to provide information about the specific role (i.e., importance) that is associated with each criterion in the aggregation procedure [58]. In other words, people do not know how to assign a value to those coefficients, and this does not mean that each criterion has the same weight. However, ELECTRE IV is highly appropriate for cases like this [74]. For this reason, some outranking relations of interval neutrosophic numbers (INNs) based on ELECTRE IV are developed in this paper, and the related properties of INNs are discussed. Furthermore, an outranking approach for MCDM problems using INNs is proposed.

The rest of paper is organized as follows. Section 2 briefly introduces the concepts and operations of NSs, SNSs and INSs. Subsequently in Sect. 3, some outranking relations of INNs that are based on ELECTRE IV are defined; moreover, some properties are also discussed. In Sect. 4, an outranking approach for MCDM problems with INNs is developed. Then in Sect. 5, two examples are presented to illustrate the proposed methods and a comparative analysis and discussion are provided. Finally, in Sect. 6 conclusions are drawn.

2 Preliminaries

In this section, some basic concepts and definitions, including definitions of NSs, SNSs and INSs, are introduced. The operations of INNs are also provided as they will be utilized in the rest of the paper.

2.1 NS, SNS and INS

Definition 1 [34] Let X be a set with a generic element in X denoted by x. A NS A in X is described using a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, that is, $T_A(x): X \to]0^-, 1^+[$, $I_A(x): X \to]0^-, 1^+[$, and $F_A(x): X \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup\, T_A(x) + \sup\, I_A(x) + \sup\, F_A(x) \leq 3^+$.

Definition 2 [34] A NS A is contained in the other NS B, denoted as $A \subseteq B$, if and only if $\inf\, T_A(x) \leq \inf\, T_B(x)$, $\sup\, T_A(x) \leq \sup\, T_B(x)$, $\inf\, I_A(x) \leq \inf\, I_B(x)$, $\sup\, I_A(x) \leq \sup\, I_B(x)$, $\inf\, F_A(x) \leq \inf\, F_B(x)$, and $\sup\, F_A(x) \leq \sup\, F_B(x)$ for $x \in X$.

Since it is difficult to apply NSs to practical problems, Ye [31] reduced the NSs of nonstandard intervals into a kind of SNSs of standard intervals, which preserved the operations of the NSs.

Definition 3 [31] A NS A in X is characterized by $T_A(x)$, $I_A(x)$ and $F_A(x)$, which are singleton subintervals/subsets in the real standard [0, 1], that is $T_A(x): X \to [0, 1]$, $I_A(x): X \to [0, 1]$, and $F_A(x): X \to [0, 1]$. Then, a simplification of A is denoted by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$$

which is called a SNS and is a subclass of NSs.

Definition 4 [34] An INS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. For each point x in X, it is that $T_A(x) = [\inf\, T_A(x), \sup\, T_A(x)]$, $I_A(x) = [\inf\, I_A(x), \sup\, I_A(x)]$, $F_A(x) = [\inf\, F_A(x), \sup\, F_A(x)] \subseteq [0, 1]$ and $0 \leq \sup\, T_A(x) + \sup\, I_A(x) + \sup\, F_A(x) \leq 3$, $x \in X$. Only the subunitary interval of [0, 1] is considered, and it is a subclass of a NS. Therefore, all INSs are clearly NSs.

2.2 Operations for INNs

Definition 5 [39] Let two INNs be $a = \langle[\inf\, T_a(x), \sup\, T_a(x)], [\inf\, I_a(x), \sup\, I_a(x)], [\inf\, F_a(x), \sup\, F_a(x)]\rangle$, and $b = \langle[\inf\, T_b(x), \sup\, T_b(x)], [\inf\, I_b(x), \sup\, I_b(x)], [\inf\, F_b(x), \sup\, F_b(x)]\rangle$, and $\lambda > 0$. The operations for the INNs are defined below.

$$(1)\ \lambda a = \left\langle \left[1-(1-\inf T_a(x))^{\lambda},\ 1-(1-\sup T_a(x))^{\lambda}\right], \left[(\inf I_a(x))^{\lambda},\ (\sup I_a(x))^{\lambda}\right], \left[(\inf F_a(x))^{\lambda},\ (\sup F_a(x))^{\lambda}\right] \right\rangle;$$

$$(2)\ a^{\lambda} = \left\langle \left[(\inf\ T_a(x))^{\lambda},\ (\sup\ T_a(x))\right]^{\lambda}, \left[1-(1-\inf\ I_a(x))^{\lambda},\ 1-(1-\sup\ I_a(x))^{\lambda}\right], \left[1-(1-\inf\ F_a(x))^{\lambda},\ 1-(1-\sup\ F_a(x))^{\lambda}\right] \right\rangle;$$

$$(3)\ a+b = \langle[\inf T_a(x)+\inf T_b(x)-\inf T_a(x)\cdot\inf T_b(x), \sup T_a(x)+\sup T_b(x)-\sup T_a(x)\cdot\sup T_b(x)], [\inf T_a(x)\cdot\inf I_b(x), \sup I_a(x)\cdot\sup I_b(x)], [\inf F_a(x)\cdot\inf F_b(x), \sup F_a(x)\cdot\sup F_b(x)]\rangle;$$

$$(4)\ a\cdot b = \langle[\inf T_a(x)\cdot\inf T_b(x), \sup T_a(x)\cdot\sup T_b(x)], [\inf T_a(x)+\inf I_b(x)-\inf T_a(x)\cdot\inf I_b(x), \sup I_a(x)+\sup I_b(x)-\sup I_a(x)\cdot\sup I_b(x)], [\inf F_a(x)+\inf F_b(x)-\inf F_a(x)\cdot\inf F_b(x), \sup F_a(x)+\sup F_b(x)-\sup F_a(x)\cdot\sup F_b(x)]\rangle.$$

Theorem 1 [39] *Let a, b and c be three INNs and* $a = \langle[\inf\, T_a(x), \sup\, T_a(x)], [\inf\, I_a(x), \sup\, I_a(x)], [\inf\, F_a(x), \sup\, F_a(x)]\rangle$, $b = \langle[\inf\, T_b(x), \sup\, T_b(x)], [\inf\, I_b(x), \sup\, I_b(x)], [\inf\, F_b(x), \sup\, F_b(x)]\rangle$ and $c = \langle[\inf\, T_c(x), \sup\, T_c(x)], [\inf\, I_c(x), \sup\, I_c(x)], [\inf\, F_c(x), \sup\, F_c(x)]\rangle$. *Then the following equations are true.*

(1) $a + b = b + a$,
(2) $a \cdot b = b \cdot a$,
(3) $\lambda(a + b) = \lambda a + \lambda b,\ \lambda > 0$,
(4) $(a \cdot b)^{\lambda} = a^{\lambda} + b^{\lambda},\ \lambda > 0$,
(5) $\lambda_1 a + \lambda_2 a = (\lambda_1 + \lambda_2)a,\ \lambda_1 > 0,\ \lambda_2 > 0$,
(6) $a^{\lambda_1} \cdot a^{\lambda_2} = a^{(\lambda_1+\lambda_2)},\ \lambda_1 > 0,\ \lambda_2 > 0$,
(7) $(a + b) + c = a + (b + c)$,
(8) $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Example 1 If two INNs are $a = \langle[0.7, 0.8], [0.0, 0.1], [0.1, 0.2]\rangle$, and $b = \langle[0.4, 0.5], [0.2, 0.3], [0.3, 0.4]\rangle$ and $\lambda = 2$, then:

(1) $\lambda a = \langle[0.91, 0.96], [0, 0.01], [0.01, 0.04]\rangle$;
(2) $a^{\lambda} = \langle[0.49, 0.64], [0, 0.19], [0.19, 0.36]\rangle$;
(3) $a + b = \langle[0.82, 0.90], [0, 0.05], [0.03, 0.08]\rangle$ and
(4) $a \cdot b = \langle[0.28, 0.40], [0.20, 0.37], [0.37, 0.52]\rangle$.

3 Outranking relations of INNs

A criterion with both preference and indifference thresholds is called a pseudo-criterion. Roy [75, 76] provided a definition of the pseudo-criterion, in which it is a function g_j associated with two threshold functions, $q_j(\cdot)$ and $p_j(\cdot)$, which satisfies the following condition: for all ordered pairs of actions, $(a, a') \in S \times S$, $g_j(a) \geq g_j(a')$, $g_j(a) + p_j(g_j(a'))$ and $g_j(a) + q_j(g_j(a'))$ are non-decreasing monotone functions of $g_j(a')$, and $p_j(g_j(a')) \geq q_j(g_j(a')) \geq 0$ for all $a \in S$. Here, $q_j(g_j(a'))$ is the greatest performance difference, for which the situation of indifference holds on to criterion g_j between two actions a and a', and $p_j(g_j(a'))$ is the smallest performance difference, for which the situation of preference occurs on criterion g_j between a and a'. Based on the definition, the pseudo-criterion between two INNs can be defined as follows.

Definition 6 Given two INNs a and b, where $a = \langle[\inf T_a, \sup T_a], [\inf I_a, \sup I_a], [\inf F_a, \sup F_a]\rangle$, and $b = \langle[\inf T_b, \sup T_b], [\inf I_b, \sup I_b], [\inf F_b, \sup F_b]\rangle$. Let $g(a) = (\inf T_a + \sup T_a) - (\inf I_a + \sup I_a) - (\inf F_a + \sup F_a)$, $g(b) = (\inf T_b + \sup T_b) - (\inf I_b + \sup I_b) - (\inf F_b + \sup F_b)$, $g(T_{ab}) = (\inf T_a + \sup T_a) - (\inf T_b + \sup T_b)$, $g(I_{ab}) = (\inf I_a + \sup I_a) - (\inf I_b + \sup I_b)$, and $g(F_{ab}) = (\inf F_b + \sup F_b) - (\inf F_a + \sup F_a)$. Assume that $g(a, b) = g(a) - g(b) = g(T_{ab}) + g(I_{ab}) + g(F_{ab})$. Let p and q represent the preference threshold and indifference threshold, respectively. The following relations exist between the two INNs:

(1) If $p < g(a, b)$, then a is strongly preferred to b, denoted by $P(a, b)$ or $a >_S b$.
(2) If $q < g(a, b) \leq p$, then a is weakly preferred to b, denoted by $Q(a, b)$ or $a >_W b$.
(3) If $-q < g(a, b) \leq q$, then a is indifferent to b, denoted by $I(a, b)$, or represented by $a \sim_I b$.

Clearly, there are three binary relations between two INNs a and b. They are a strong dominance relation, weak dominance relation and indifference relation. If a is strongly preferred to b, it means that a strongly dominates b, whereas if a is weakly preferred to b, it means that a weakly dominates b.

Example 2 Assume preference threshold $p = 0.4$, and the indifference threshold $q = 0.2$.

(1) If $a = \langle[0.7, 0.8], [0.0, 0.1], [0.1, 0.2]\rangle$ and $b = \langle[0.4, 0.5], [0.2, 0.3], [0.3, 0.4]\rangle$ are two INNs, then $g(a, b) = 1.4 > p$. Therefore, a and b satisfy $P(a, b)$, that is, a is strictly preferred to b.
(2) If $a = \langle[0.6, 0.7], [0.2, 0.4], [0.3, 0.4]\rangle$ and $b = \langle[0.4, 0.5], [0.2, 0.3], [0.3, 0.4]\rangle$ are two INNs, then $q < g(a, b) = 0.3 \leq p$. Therefore, a and b satisfy $Q(a, b)$, that is, a is weakly preferred to b.
(3) If $a = \langle[0.6, 0.7], [0.3, 0.4], [0.4, 0.5]\rangle$ and $b = \langle[0.4, 0.5], [0.3, 0.4], [0.2, 0.3]\rangle$ are two INNs, then $-q < g(a, b) = 0 < q$. Hence, a is indifferent to b, that is, $I(a, b)$.

Property 1 *Let a, b and c be INNs, if $a >_S b$ and $b >_S c$, then $a >_S c$.*

Proof According to Definition 6, if $a >_S b$ then $g(a, b) > p$, that is, $g(a) - g(b) > p$. Similarly, if $b >_S c$ then $g(b) - g(c) > p$. Therefore, $g(a) - g(b) + g(b) - g(c) > 2p$. Thus, $g(a) - g(c) > p$. Based on Definition 6, $a >_S c$ is achieved and, therefore, the property is proven.

Property 2 *Let a, b and c be three INNs, then the following results can be achieved.*

(1) *The strong dominance relations are categorized into*:
  (1a) *Irreflexivity* $\forall a \in$ INNs, $a \not>_S a$;
  (1b) *Asymmetry* $\forall a, b \in$ INNs, $a >_S b \Rightarrow b \not>_S a$;
  (1c) *Transitivity* $\forall a, b, c \in$ INNs, $a >_S b,\ b >_S c \Rightarrow a >_S c$.
(2) *The weak dominance relations are categorized into*:
  (2a) *Irreflexivity* $\forall a \in$ INNs, $a \not>_W a$;
  (2b) *Asymmetry* $\forall a, b \in$ INNs, $a >_W b \Rightarrow b \not>_W a$;
  (2c) *Non-transitivity* $\exists a, b, c \in$ INNs, $a >_W b,\ b >_W c \not\Rightarrow a >_W c$.
(3) *The indifference relations are categorized into*:
  (3a) *Reflexivity* $\forall a \in$ INNs, $a \sim_I a$;
  (3b) *Symmetry* $\forall a, b \in$ INNs, $a \sim_I b \Rightarrow b \sim_I a$;

(3c) *Non-transitivity* $\exists a, b, c \in$ INNs, $a \sim_I b$, $b \sim_I c \nRightarrow a \sim_I c$.

$g(a) - g(a) = 0 < q < p$, $a \not>_S a$, $a \not>_W a$, $a \sim_I a$ holds according to Definition 6. Thus, (1a), (2a) and (3a) are true. Similarly, if $a >_S b$, then $g(a) - g(b) > p \Rightarrow g(b) - g(a) < -p$. Therefore, $b \not>_S a$ and (1b) is true, as are (2b) and (3b). According to Property 1 and Property 2, (1c) is true. However, (2c) and (3c) need to be proven.

Example 3 (2c) and (3c) which are provided in Property 2 can be exemplified as follows.

(1) If $a >_W b$, $b >_W c$, assume $g(a) - g(b) = 0.25$, $g(b) - g(c) = 0.25$ and $q = 0.2, p = 0.4$, then $g(a) - g(c) = 0.5 > p$. According to Definition 6, $a >_S b$ in this case. Therefore, (2c) is achieved.

(2) If $a \sim_I b$, then $-q \le g(a, b) \le q$, that is, $-q \le g(a) - g(b) \le q$. If $b \sim_I c$, then $-q \le g(b) - g(c) \le q$. Assume $q = 0.2, p = 0.4$, $g(a) - g(b) = 0.2$ and $g(b) - g(c) = 0.1$, then $g(a) - g(c) = 0.3$. In this case, $a >_W c$ according to Definition 6; therefore, $a \sim_I b$, $b \sim_I c \Rightarrow a \sim_I c$ is obtained, and (3c) is realized.

Definition 7 Let alternatives A and B be collections of INNs, $A = \{a_1, a_2, a_3, \ldots, a_m\}$, and $B = \{b_1, b_2, b_3, \ldots, b_m\}$, a_i, $b_j \in$ INNs $(i, j = 1, 2, \ldots, m)$. In order to make a pair-wise comparison of the alternatives A and B, the following notations are defined:

Let $n_P(A, B)$ represent the number of the criteria for which $P(a_i, b_j)$.
Let $n_Q(A, B)$ represent the number of the criteria for which $Q(a_i, b_j)$.
Let $n_I(A, B)$ represent the number of the criteria for which $I(a_i, b_j) \wedge (g(a_i, b_j) > 0)$.

Given two alternatives A and B, the following statements provide insights into the outranking relation [65]:

(1) *A outranks B*, denoted by AOB, which means that the analyst has enough reasons to ascertain that in the eyes of the decision-maker, A is at least as good as B.

(2) *A does not outrank B*, which means that the arguments in favor of the proposition "A is at least as good as B" are judged insufficient (this does not imply that there are arguments in favor of "B is at least as good as A").

The binary relations $\succ$ (preference), I (indifference) and R (incomparability) between two alternatives A and B may arise as follows:

$AIB \Leftrightarrow AOB$ and BOA;
$A \succ B \Leftrightarrow AOB$ and not BOA;
$A \prec B \Leftrightarrow$ not AOB and BOA; and
$ARB \Leftrightarrow$ not AOB and not BOA.

Furthermore, the preference relation consists of two situations. They are a strong preference and weak preference, denoted by $A \succ_S B$ and $A \succ_W B$ respectively.

Based on [77], the rules of dominance were established as follows.

Definition 8 Assume m is the total number of criteria. Then:

(1) $A \succ_S B$: (a) for no criterion B is strictly preferred to A; (b) if the number of criteria for which B is weakly preferred in relation to A is inferior or equal to the number of criteria for which A is strictly preferred to B; and (c) and if the number of criteria, for which the performance of B is better than that of A, is strictly inferior to the number of criteria for which the performance of A is better than that of alternative B.

$$A \succ_S B \Leftrightarrow \begin{cases} n_P(B,A) = 0 \text{ and } n_Q(B,A) \le n_P(A,B) \text{ and} \\ n_Q(B,A) + n_I(B,A) < n_I(A,B) + n_Q(A,B) + n_P(A,B). \end{cases}$$

(2) $A \succ_W B$: (a) if under no criteria B is strictly preferred to A; (b) if the number of criteria, for which the performance of B is superior to that of A, is strictly inferior to the number of criteria for which the performance of A is superior to that of alternative B; (c) if the additional condition required for the relation $A \succ_S B$ to hold is not verified; and (d) if there is a single criterion for which B is strictly preferred to A, then A is strictly preferred to B in at least half of the criteria.

$$A \succ_W B \Leftrightarrow \begin{cases} n_P(B,A) = 0 \quad \text{and} \quad n_Q(B,A) + n_I(B,A) < n_I(A,B) + n_Q(A,B) + n_P(A,B) \quad \text{and} \quad \text{not } A \succ_S B \\ \text{or} \\ n_P(B,A) = 1 \quad \text{and} \quad n_P(A,B) \ge m/2. \end{cases}$$

By this definition, the outranking relation between each pair-wise can be achieved.

Example 4 Assume that the preference threshold is $p = 0.4$ and the indifference threshold is $q = 0.2$. For alternatives A_1 and A_2, three criteria are applied for evaluation purposes. The performance of each criterion with regard to every alternative is as follows:

$$A_1 = [A_{11} \quad A_{12} \quad A_{13}] = \begin{array}{l}[\langle[0.4,0.5],\ [0.2,0.3],\ [0.3,0.4]\rangle] \\ [\langle[0.4,0.6],\ [0.1,0.3],\ [0.2,0.4]\rangle] \\ [\langle[0.7,0.9],\ [0.2,0.3],\ [0.4,0.5]\rangle]\end{array}$$

$$A_2 = [A_{21} \quad A_{22} \quad A_{23}] = \begin{array}{l}[\langle[0.6,0.7],\ [0.1,0.2],\ [0.2,0.3]\rangle] \\ [\langle[0.6,0.7],\ [0.1,0.2],\ [0.2,0.3]\rangle] \\ [\langle[0.3,0.6],\ [0.3,0.5],\ [0.8,0.9]\rangle]\end{array}$$

According to Definition 6, $g(A_{11}, A_{21}) = -0.8$, $g(A_{21}, A_{11}) = 0.8 > p$, $g(A_{22}, A_{12}) = 0.5 > p$ and $g(A_{13}, B_{23}) = 1.8 > p$. Therefore, $P(A_{21}, A_{11})$, $P(A_{22}, A_{12})$ and $P(A_{13}, A_{23})$. That is, the relationship between alternatives A_1 and A_2 is that with regard to the former two criteria, A_2 is strictly preferred to A_1, while A_1 is strictly preferred to A_2 with regard to the third criterion. For the pair (A_1, A_2), $n_P(A_1, A_2) = 1$, $n_Q(A_1, A_2) = 0$, $n_I(A_1, A_2) = 0$, $n_P(A_2, A_1) = 2$, $n_Q(A_2, A_1) = 0$ and $n_I(A_2, A_1) = 0$. According to Definition 8, $A_2 \succ_W A_1$, which means that A_2 is weakly dominated by A_1.

For ease of presentation, let $n_s(A_i)$ represent the number that has strong dominant relation with the alternative A_i, for which $A_i \succ_S A_k$ between the pair (A_i, A_k) $(k = 1, 2, \ldots, n)$. Similarly, $n_w(A_i)$ represents the number for which $A_i \succ_W A_k$, for $k = 1, 2, \ldots, n$.

Definition 9 The rules of the outranking relation between the pair (A_i, A_k) are constructed as follows.

(1) If $n_s(A_i) > n_s(A_j)$, then it implies A_i outranking A_j, denoted $A_i \succ A_j$.

(2) If $n_s(A_i) = n_s(A_j)$, and $n_w(A_i) > n_w(A_j)$, then $A_i \succ A_j$.

(3) If $n_s(A_i) = n_s(A_j)$, $n_w(A_i) = n_w(A_j)$ and $A_i \succ_S A_j$, then $A_i \succ A_j$.

Subsequently, the sequence with a partial or complete order of alternatives is established.

4 An outranking approach for MCDM with INSs

In this section, an outranking method for MCDM problems where the outranking rules of INSs are applied is proposed.

Assume that the MCDM ranking/selection problem with INSs consists of n alternatives $A = \{A_1, A_2, \ldots, A_n\}$ and each alternative is evaluated by m criteria $C = \{c_1, c_2, \ldots, c_m\}$. The evaluation of every alternative according to each criterion is transformed into the interval neutrosophic decision matrix $R = (A_{ij})_{n \times m}$, where $A_{ij} = \langle T_{A_{ij}}, I_{A_{ij}}, F_{A_{ij}} \rangle$ is a criterion value, denoted by INN, where $T_{A_{ij}}$ indicates the truth-membership function that the alternative A_i satisfies the criterion c_j; $I_{A_{ij}}$ indicates the indeterminacy-membership function that the alternative A_i satisfies the criterion c_j; and $F_{A_{ij}}$ indicates the falsity-membership function that the alternative A_i satisfies the criterion c_j. Let the preference threshold for each criterion $P = \{p_1, p_2, \ldots, p_m\}$ and the indifference threshold $Q = \{q_1, q_2, \ldots, q_m\}$.

In the following steps, a procedure to rank and select the most desirable alternative(s) is outlined.

Step 1 Calculate the performance matrix G.

The performance value of every alternative A_i on each criterion c_j is denoted as $g(A_i)_j$. Using Definition 6, the performance matrix G can be calculated:

$$G = \begin{bmatrix} g(A_1)_1 & g(A_1)_2 & \cdots & g(A_1)_m \\ g(A_2)_1 & g(A_2)_2 & \cdots & g(A_2)_m \\ \cdots & \cdots & \cdots & \cdots \\ g(A_n)_1 & g(A_n)_2 & \cdots & g(A_n)_m \end{bmatrix}$$

Step 2 Calculate the difference matrix D.

The difference value $g(A_i, A_k)_j$ demonstrates the performance difference between two alternatives A_i and A_k on each criterion c_j. According to the score function value of each alternative with regard to every criterion, $g(A_i, A_k)_j = g(A_i)_j - g(A_i)_j$ $(i, k = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$, and thus, the difference matrix can be constructed as the following:

$$D = \begin{bmatrix} g(A_1, A_1)_1 & g(A_1, A_1)_2 & \cdots & g(A_1, A_1)_m \\ g(A_1, A_2)_1 & g(A_1, A_2)_2 & \cdots & g(A_1, A_2)_m \\ \cdots & \cdots & \cdots & \cdots \\ g(A_n, A_n)_1 & g(A_n, A_n)_2 & \cdots & g(A_n, A_n)_m \end{bmatrix}$$

Step 3 Obtain the binary relationship between two the INNs with regard to each criterion for all alternatives.

Clearly, the difference value $g(A_i, A_k)_j$ must be compared with the preference threshold p_j and indifference threshold q_j to determine the relationships. According to Definition 6, if $g(A_i, A_k)_j > p_j$, then $A_{ij} >_S A_{kj}$. Moreover, if $q_j < g(A_i, A_k)_j \le p_j$, then $A_{ij} >_W A_{kj}$ and if $-q_j \le g(A_i, A_k)_j \le q_j$, then $A_{ij} >_I A_{kj}$.

Step 4 Count the number of outranking relations.

Using Definition 7, count the number of outranking relations $n_P(A_i, A_k)$, $n_Q(A_i, A_k)$ and $n_I(A_i, A_k)$ for the pair-wise (A_i, A_k) regarding all criteria. Clearly, any pair-wise (A_i, A_k), $n_P(A_i, A_k)$ represents the number when $A_{ij} >_S A_{kj}$ for $j = 1, 2, \ldots, m$. Similarly, $n_Q(A_i, A_k)$ represents the number when $A_{ij} >_W A_{kj}$ and $n_I(A_i, A_k)$ represents the number when $A_{ij} \sim_I A_{kj}$ and $g(A_i, A_k)_j > 0$ for $j = 1, 2, \ldots, m$.

Step 5 Determine the outranking relations between the pair-wise (A_i, A_k) according to Definition 8.
Step 6 Count $n_s(A_i)$ and $n_w(A_i)$ for alternative A_i.
Step 7 Select the alternative or alternatives with the most outranking relations according Definition 9.

Using Definition 9, select the alternative or alternatives with the largest $n_s(A_i)$. If two or more have the maximum of $n_s(A_i)$, then compare $n_w(A_i)$. Thus, one or more alternatives with the most outranking relations are distilled.

Step 8 Repeat Step 4–Step 7 for the remaining alternatives until the remainder is null.
Step 9 Rank the alternatives.

The complete or partial order for all alternatives is subsequently established.

5 An illustrative example

In this section, to demonstrate the application of the proposed decision-making method and its effectiveness, two examples of MCDM problems with alternatives are provided.

5.1 Two examples of outranking approach for MCDM with INSs

Example 5 A decision-making problem adapted from Refs. [39, 78] will be used. There is a panel with four possible alternatives to invest money: A_1, A_2, A_3 and A_4. The investment company must take a decision according to the following three criteria: c_1, c_2 and c_3. The preference threshold $P = \{0.2, 0.2, 0.2\}$ and the indifference threshold $Q = \{0.1, 0.1, 0.1\}$. The four possible alternatives are to be evaluated under the above three criteria, and the evaluation values are to be transformed into INNs, as shown in the following interval neutrosophic decision matrix D:

$$D = \begin{bmatrix} \langle [0.4, 0.5], [0.2, 0.3], [0.3, 0.4] \rangle & \langle [0.4, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle & \langle [0.7, 0.9], [0.2, 0.3], [0.4, 0.5] \rangle \\ \langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3] \rangle & \langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3] \rangle & \langle [0.3, 0.6], [0.3, 0.5], [0.8, 0.9] \rangle \\ \langle [0.3, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle & \langle [0.5, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle & \langle [0.4, 0.5], [0.2, 0.4], [0.7, 0.9] \rangle \\ \langle [0.7, 0.8], [0.0, 0.1], [0.1, 0.2] \rangle & \langle [0.6, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle & \langle [0.6, 0.7], [0.3, 0.4], [0.8, 0.9] \rangle \end{bmatrix}$$

Step 1 Calculate the performance matrix G:

$$G = \begin{bmatrix} -0.3 & 0 & 0.2 \\ 0.5 & 0.5 & -1.6 \\ -0.3 & -0.1 & -1.3 \\ 1.1 & 0.6 & -1.1 \end{bmatrix}$$

Step 2 Calculate the difference matrix D.
The difference value $g(A_i, A_k)_j$ between the two alternatives A_i and A_k on each criterion c_j satisfies $g(A_i, A_k)_j = g(A_i)_j - g(A_i)_j$. Subsequently, the difference matrix can be drawn up as shown in Table 1.
Step 3 Obtain the binary relationship between the two INNs on each criterion for all alternatives.
Compare $g(i, k)_j$ with p_j and q_j, and then, the results can be seen in Table 2.
Step 4 Compute the number of every outranking relation of each alternative with the other alternatives for all the criteria, as shown in Table 3.
Step 5 Determine the outranking relations according to Definition 8, as shown in Table 4.
Step 6 Count $n_s(A_i)$ and of every alternative A_i as shown in Table 5.
Step 7 Select the alternative or alternatives with the most outranking relations according to Definition 9.

It is clear that A_4 is the best alternative.

Step 8 Repeat Step 4 to Step 7 for the remaining alternatives until the remainder is null.

The procedure is continuously repeated, and subsequently A_1, A_2 and A_3 are distilled.

Step 9 Rank the alternatives.

Based on the above steps, the final order $A_4 \succ A_1 \succ A_2 \succ A_3$ can be ascertained.

Example 6 The following scenario is adapted from references [20, 79]. Recently, an overseas investment department decided to select a pool of alternatives from several foreign countries based on preliminary surveys. After a thorough investigation, five projects (alternatives) were taken into consideration, i.e., $\{A_1, A_2, A_3, A_4, A_5\}$. There were many factors that affected the investment decision, but four factors were eventually considered based on the experience of the personnel concerned, including c_1: resources (such as the suitability of the minerals and their exploration potential); c_2: infrastructure (such as railway and highway facilities); and c_3: the development environment (such as development vitality and the stability, politics and policy of the region).

The decision-makers, including experts and executive managers, were gathered to determine the information needed to make a decision. The evaluation value ranges from 0 to 1, and each decision-maker can use an interval value to express his/her opinion; in other words, the three interval values denote to what degree the project x_i matches the criterion c_j, as well as the degrees of indeterminacy and falsity. Each decision-maker gave his/her own evaluations based on the surveys of the five projects as well as his/her knowledge and experience. Then all the evaluations given were assembled. Consequently, following a heated discussion, they came to a consensus on the final evaluations which are expressed by INNs as shown in Table 6.

Table 1 The performance difference of projects on each criterion

	c_1	c_2	c_3
(A_1, A_1)	0	0	0
(A_1, A_2)	−0.8	−0.5	1.8
(A_1, A_3)	0	0.1	1.5
(A_1, A_4)	−1.4	−0.6	1.3
(A_2, A_1)	0.8	0.5	−1.8
(A_2, A_2)	0	0	0
(A_2, A_3)	0.8	0.6	−0.3
(A_2, A_4)	−0.6	−0.1	−0.5
(A_3, A_1)	0	−0.1	−1.5
(A_3, A_2)	−0.8	−0.6	0.3
(A_3, A_3)	0	0	0
(A_3, A_4)	−1.4	−0.7	−0.2
(A_4, A_1)	1.4	0.6	−1.3
(A_4, A_2)	0.6	0.1	0.5
(A_4, A_3)	1.4	0.7	0.2
(A_4, A_4)	0	0	0

Table 2 The relationship between each pair-wise on the criteria

	c_1	c_2	c_3
(A_1, A_1)	$\sim_I$	$\sim_I$	$\sim_I$
(A_1, A_2)	$<_S$	$<_S$	$>_S$
(A_1, A_3)	$\sim_I$	$\sim_I$	$>_S$
(A_1, A_4)	$<_S$	$<_S$	$>_S$
(A_2, A_1)	$>_S$	$>_S$	$<_S$
(A_2, A_2)	$>_S$	$\sim_I$	$\sim_I$
(A_2, A_3)	$>_S$	$>_S$	$<_S$
(A_2, A_4)	$<_S$	$\sim_I$	$<_S$
(A_3, A_1)	$\sim_I$	$\sim_I$	$<_S$
(A_3, A_2)	$<_S$	$<_S$	$>_S$
(A_3, A_3)	$\sim_I$	$\sim_I$	$\sim_I$
(A_3, A_4)	$<_S$	$<_S$	$<_W$
(A_4, A_1)	$>_S$	$>_S$	$<_S$
(A_4, A_2)	$>_S$	$\sim_I$	$>_S$
(A_4, A_3)	$>_S$	$>_S$	$>_W$
(A_4, A_4)	$\sim_I$	$\sim_I$	$\sim_I$

Table 3 The relation numbers between pair-wise for all the criteria

	$n_P(A_i, A_k)$	$n_Q(A_i, A_k)$	$n_I(A_i, A_k)$
(A_1, A_1)	0	0	0
(A_1, A_2)	1	0	0
(A_1, A_3)	1	0	1
(A_1, A_4)	1	0	0
(A_2, A_1)	2	0	0
(A_2, A_2)	0	0	0
(A_2, A_3)	2	0	0
(A_2, A_4)	0	0	0
(A_3, A_1)	0	0	0
(A_3, A_2)	1	0	0
(A_3, A_3)	0	0	0
(A_3, A_4)	0	0	0
(A_4, A_1)	2	0	0
(A_4, A_2)	2	0	1
(A_4, A_3)	2	1	0
(A_4, A_4)	0	0	0

Table 4 The outranking relation matrix of the projects on the criteria

	A_1	A_2	A_3	A_4
A_1	/		$\succ_S$	
A_2	$\succ_W$	/	$\succ_W$	
A_3			/	
A_4	$\succ_S$	$\succ_S$	$\succ_S$	/

Table 5 The relation numbers between each project and the others

	$n_s(A_i)$	$n_w(A_i)$
A_1	1	0
A_2	0	2
A_3	0	0
A_4	3	0

The preference threshold $P = \{0.2, 0.2, 0.2\}$ and the indifference threshold $Q = \{0.1, 0.1, 0.1\}$.

Step 1 Calculate the performance matrix G:

$$G = \begin{bmatrix} 0.8 & 0.5 & -0.4 \\ 0.8 & 0.6 & -0.3 \\ 0.9 & 1.0 & -1.6 \\ 0.6 & 0.7 & 0.1 \\ 0.6 & 0.4 & 0 \end{bmatrix}$$

Table 6 The performance matrix of the projects on the criteria

	c_1	c_2	c_3
A_1	$\langle[0.7, 0.9], [0.2, 0.3], [0.1, 0.2]\rangle$	$\langle[0.4, 0.8], [0.1, 0.2], [0.1, 0.3]\rangle$	$\langle[0.3, 0.4], [0.3, 0.4], [0.2, 0.2]\rangle$
A_2	$\langle[0.6, 0.7], [0.1, 0.2], [0.1, 0.1]\rangle$	$\langle[0.7, 0.9], [0.2, 0.3], [0.2, 0.3]\rangle$	$\langle[0.3, 0.4], [0.3, 0.4], [0.1, 0.2]\rangle$
A_3	$\langle[0.8, 0.9], [0.2, 0.3], [0.1, 0.2]\rangle$	$\langle[0.6, 0.9], [0.1, 0.1], [0.1, 0.2]\rangle$	$\langle[0.2, 0.4], [0.6, 0.9], [0.3, 0.4]\rangle$
A_4	$\langle[0.5, 0.8], [0.2, 0.2], [0.1, 0.2]\rangle$	$\langle[0.8, 0.9], [0.2, 0.4], [0.1, 0.3]\rangle$	$\langle[0.5, 0.6], [0.3, 0.4], [0.1, 0.2]\rangle$
A_5	$\langle[0.6, 0.8], [0.2, 0.3], [0.1, 0.2]\rangle$	$\langle[0.5, 0.8], [0.2, 0.3], [0.2, 0.2]\rangle$	$\langle[0.4, 0.6], [0.2, 0.4], [0.1, 0.3]\rangle$

Step 2 Calculate the difference value $g(A_i, A_k)_j$ as shown in Table 7.

Step 3 Obtain the binary relationship between two the INNs on each criterion for all alternatives.

Compare $g(i, k)_j$ with p_j and q_j, and then, the results can be seen in Table 8.

Step 4 Compute the number of every outranking relation for each alternative with the other alternatives for all the criteria, as shown in Table 9.

Step 5 Determine the outranking relations according to Definition 8, as shown in Table 10.

Step 6 Count $n_s(A_i)$ and $n_w(A_i)$ of every alternative A_i as shown in Table 11.

Step 7 Select the alternative or alternatives with the most outranking relations according to Definition 9.

It is clear that A_4 is the best alternative.

Step 8 Repeat Step 4–Step 7 for the remaining alternatives until the remainder is null.

The procedure is continuously repeated, and subsequently, A_3, A_2 and A_1 as well as A_5 are distilled. It should be noted that there are two alternatives that were distilled in the last repetition, as A_1 and A_5 are either indistinguishable or incomparable.

Step 9 Rank the alternatives.

The final order is $A_4 \succ A_3 \succ A_2 \succ A_1, A_5$.

Table 7 The difference matrix of the projects on each criterion

	c_1	c_2	c_3
(A_1, A_1)	0	0	0
(A_1, A_2)	0	−0.1	−0.1
(A_1, A_3)	−0.1	−0.5	1.2
(A_1, A_4)	0.2	−0.2	−0.5
(A_1, A_5)	0.2	0.1	−0.4
(A_2, A_1)	0	0.1	0.1
(A_2, A_2)	0	0	0
(A_2, A_3)	−0.1	−0.4	1.3
(A_2, A_4)	0.2	−0.1	−0.4
(A_2, A_5)	0.2	0.2	−0.3
(A_3, A_1)	0.1	0.5	−1.2
(A_3, A_2)	0.1	0.4	−1.3
(A_3, A_3)	0	0	0
(A_3, A_4)	0.3	0.3	−1.7
(A_3, A_5)	0.3	0.6	−1.6
(A_4, A_1)	−0.2	0.2	0.5
(A_4, A_2)	−0.2	0.1	0.4
(A_4, A_3)	−0.3	−0.3	1.7
(A_4, A_4)	0	0	0
(A_4, A_5)	0	0.3	0.1
(A_5, A_1)	−0.2	−0.1	0.4
(A_5, A_2)	−0.2	−0.2	0.3
(A_5, A_3)	−0.3	−0.6	1.6
(A_5, A_4)	0	−0.3	−0.1
(A_5, A_5)	0	0	0

5.2 A comparison analysis and discussion

To validate the feasibility of the proposed decision-making method, a comparative study with other methods was conducted. The comparison analysis includes two cases: One is a comparison with the existing methods, which use interval value neutrosophic information, that were outlined in [39, 80]; in the other, the proposed method is compared to the methods that use simplified neutrosophic information, which were introduced in [30, 31, 57, 81].

Case 1 The proposed approach is compared with some of the methods that use interval neutrosophic information.

With regard to the method in [80], firstly the similarity measures were calculated and used to determine the final ranking order of all the alternatives, and then, two aggregation operators were developed to aggregate the interval neutrosophic information [39]. To solve the MCDM problem in Example 5, the results from using different methods are shown in Table 12.

According to the results presented in Table 12, if methods 1 and 2 in [80] are used, then the best alternatives are A_4 and A_2, respectively, and the worst one is A_1. By using methods 3 and 4 in [39], the best ones are A_4 and A_1, respectively, and the worst one is A_3. Finally, with regard to the proposed method in this paper, the best one is A_4 and the worst one is A_3. In the similarity measures in [80], the distances between INSs are first calculated and any difference is then amplified in the results using criteria weights, which cause a distortion in the similarity between

Table 8 The relationship between each pair-wise on the criteria

	c_1	c_2	c_3
(A_1, A_1)	$\sim_I$	$\sim_I$	$\sim_I$
(A_1, A_2)	$\sim_I$	$\sim_I$	$\sim_I$
(A_1, A_3)	$\sim_I$	$\prec_S$	$\succ_S$
(A_1, A_4)	$\succ_W$	$\prec_W$	$\prec_S$
(A_1, A_5)	$\succ_W$	$\sim_I$	$\prec_S$
(A_2, A_1)	$\sim_I$	$\sim_I$	$\sim_I$
(A_2, A_2)	$\sim_I$	$\sim_I$	$\sim_I$
(A_2, A_3)	$\sim_I$	$\prec_S$	$\succ_S$
(A_2, A_4)	$\succ_W$	$\sim_I$	$\prec_S$
(A_2, A_5)	$\succ_W$	$\succ_W$	$\prec_S$
(A_3, A_1)	$\sim_I$	$\succ_S$	$\prec_S$
(A_3, A_2)	$\sim_I$	$\succ_S$	$\prec_S$
(A_3, A_3)	$\sim_I$	$\sim_I$	$\sim_I$
(A_3, A_4)	$\succ_S$	$\succ_S$	$\prec_S$
(A_3, A_5)	$\succ_S$	$\succ_S$	$\prec_S$
(A_4, A_1)	$\prec_W$	$\succ_W$	$\succ_S$
(A_4, A_2)	$\prec_W$	$\sim_I$	$\succ_S$
(A_4, A_3)	$\prec_S$	$\prec_S$	$\succ_S$
(A_4, A_4)	$\sim_I$	$\sim_I$	$\sim_I$
(A_4, A_5)	$\sim_I$	$\succ_S$	$\sim_I$
(A_5, A_1)	$\prec_W$	$\sim_I$	$\succ_S$
(A_5, A_2)	$\prec_W$	$\prec_W$	$\succ_S$
(A_5, A_3)	$\prec_S$	$\prec_S$	$\succ_S$
(A_5, A_4)	$\sim_I$	$\prec_S$	$\sim_I$
(A_5, A_5)	$\sim_I$	$\sim_I$	$\sim_I$

an alternative and the ideal alternative. While in [39], the degree of distortion was reduced by the INS aggregation operators. Furthermore, different aggregation operators lead to different rankings, because the operators emphasize decision-makers' judgments differently. Method 3 in [39] uses an interval neutrosophic number weighted averaging (INNWA) operator, whereas method 4 in [39] uses an interval neutrosophic number weighted geometric (INNWG) operator. The INNWA operator is based on an arithmetic average and emphasizes a group's major points, while the INNWG operator emphasizes an individual's major points. That is the reason why the results gained by method 3 and method 4 in [39] are different. Additionally, the four orders in [39] and [80] are obtained when the weight vector of the criteria is given by $W = (0.35, 0.25, 0.4)$. By comparison, the proposed method in this paper does not consider the relative importance coefficients of the criteria. However, the ranking gained by the proposed method is the same as that of the INNWA operator, which emphasizes a group's major points. Therefore, even without the criteria weights, the proposed method is effective.

Case 2 The proposed approach is compared with some methods that use simplified neutrosophic information. The comparison results can be found in Table 13.

Table 9 The relation numbers between each pair-wise for all the criteria

	$n_P(A_i, A_k)$	$n_Q(A_i, A_k)$	$n_I(A_i, A_k)$
(A_1, A_1)	0	0	0
(A_1, A_2)	0	0	0
(A_1, A_3)	1	0	0
(A_1, A_4)	0	1	0
(A_1, A_5)	0	1	1
(A_2, A_1)	0	0	2
(A_2, A_2)	0	0	0
(A_2, A_3)	1	0	0
(A_2, A_4)	0	1	0
(A_2, A_5)	0	2	0
(A_3, A_1)	1	0	1
(A_3, A_2)	1	0	1
(A_3, A_3)	0	0	0
(A_3, A_4)	2	0	0
(A_3, A_5)	2	0	0
(A_4, A_1)	1	1	0
(A_4, A_2)	1	0	1
(A_4, A_3)	1	0	0
(A_4, A_4)	0	0	3
(A_4, A_5)	1	0	1
(A_5, A_1)	1	0	0
(A_5, A_2)	1	0	0
(A_5, A_3)	1	0	0
(A_5, A_4)	0	0	0
(A_5, A_5)	0	0	0

Table 10 The outranking relation matrix of the projects on criteria

	A_1	A_2	A_3	A_4	A_5
A_1	/				
A_2	$\succ_W$	/			
A_3			/	$\succ_S$	$\succ_S$
A_4	$\succ_S$	$\succ_S$		/	$\succ_S$
A_5					/

Table 11 The relation numbers between each project and the others

	$n_s(A_i)$	$n_w(A_i)$
A_1	0	0
A_2	0	1
A_3	2	0
A_4	3	0
A_5	0	0

As shown in Table 13, the difference between the results is the sequence of A_4 and A_2. Ref. [30] ranks the alternatives that correspond to the cross-entropy values between the ideal alternative and another alternative to select the

Table 12 The results of different methods with INSs

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Method 1 in [80]	$A_4 \succ A_2 \succ A_3 \succ A_1$	A_4	A_1
Method 2 in [80]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
Method 3 in [39]	$A_4 \succ A_1 \succ A_2 \succ A_3$	A_4	A_3
Method 4 in [39]	$A_1 \succ A_4 \succ A_2 \succ A_3$	A_1	A_3
The proposed method	$A_4 \succ A_1 \succ A_2 \succ A_3$	A_4	A_3

most desirable one. Refs. [31, 81] employ correlation coefficients and the aggregation operator, respectively. It can be seen that the result of the proposed method is the same as those in [31, 81], whereas it differs from that in [57], which utilized a method based on ELECTRE. There are particular reasons for the differences between the final rankings of the compared methods and the proposed approach. Firstly, due to the use of criteria weights, the aggregation values, correlation coefficients and cross-entropy measures of SNSs were utilized by Ye [30, 31, 81], and this led to differences in the final results. Secondly, the outranking relations somehow retain the inherent thoughts of decision-makers, which is a prerequisite of guaranteeing the accuracy of the final outcome. Conversely, the evaluation information of decision-makers is compensated when using operators and measures. Furthermore, the difference between the result in [57] and the proposed method derives from the relative importance coefficients of the criteria used in that approach, as the proposed method does not account for the importance of the criteria.

From the comparison analysis presented above, it can been seen that the proposed approach is more flexible and reliable in managing MCDM problems in an interval neutrosophic environment than the compared methods, which shows that the outranking approach developed in this paper has certain advantages. Firstly, it is simple and can also be used to solve problems with the preference information that is expressed by INSs and SNSs. Secondly, as it is not compensatory with regard to all the criteria, the approach takes the importance of every decision-maker fully into account. This means that it can avoid losing and distorting the preference information provided which makes the final results better correspond with real life decision-making problems. However, the proposed method has a limitation in that it always needs a large amount of computation; fortunately, this can be easily reduced by the assistance of programming software, such as MATLAB.

6 Conclusion

As a new branch of NSs, INSs can be applied in addressing problems with uncertain, imprecise, incomplete and inconsistent information that exist in real scientific and engineering applications. This paper has proposed an approach for solving MCDM problems using INSs and developed some aggregation operators. However, it has been found that in certain circumstances, the importance of each criterion is not ascertained. Consequently, an outranking approach for solving MCDM problems with INSs was established based on the ELECTRE IV. Therefore, some outranking relations for INNs were defined, and the properties relating to the outranking relations were discussed in detail. Furthermore, two illustrative examples that demonstrated the application of the proposed decision-making method and a comparison analysis were provided.

The contribution of this study is that the proposed approach is simple and convenient with regard to computing, and effective in decreasing the loss of evaluative information. The validity and feasibility of the proposed approach have been verified through illustrative examples and a comparison analysis, and the results of the latter demonstrated that the method proposed can provide more reliable and precise outcomes than other methods. Therefore, this approach has great potential for managing MCDM problems in an interval neutrosophic environment, in which the criterion values that relate to the alternatives are evaluated by INNs and the criterion weights are unknown. However, it should be noted that it does not mean that each criterion has the same weight. The limitation of

Table 13 The results of different methods with SNSs

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Method in [30]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
Method in [81]	$A_4 \succ A_2 \succ A_3 \succ A_1$	A_4	A_1
Method in [31]	$A_4 \succ A_2 \succ A_3 \succ A_1$	A_4	A_1
Method in [57]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
The proposed method	$A_4 \succ A_2 \succ A_3 \succ A_1$	A_4	A_1

this approach is that it needs a large amount of computation which does not affect its application. Furthermore, the outranking approach with INSs, which is based on ELECTRE IV, can be used to enlarge their application with regard to MCDM problems.

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