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ORIGINAL ARTICLE

Prioritized aggregation operators of trapezoidal intuitionistic fuzzy sets and their application to multicriteria decision-making

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Abstract A trapezoidal intuitionistic fuzzy set, some operational laws, score and accuracy functions for trapezoidal intuitionistic fuzzy values are presented in this paper. Then, the trapezoidal intuitionistic fuzzy prioritized weighted averaging (TIFPWA) operator and trapezoidal intuitionistic fuzzy prioritized weighted geometric (TIFPWG) operator are proposed to aggregate the trapezoidal intuitionistic fuzzy information. Furthermore, a multicriteria decision-making method based on the TIFPWA and TIFPWG operators and the score and accuracy functions of trapezoidal intuitionistic fuzzy values is established to deal with the multicriteria decision-making problem in which the criteria are in different priority level. Finally, a practical example about software selection for considering various prioritized relationships between the criteria of decision-making is given to demonstrate its practicality and effectiveness.

Keywords Trapezoidal intuitionistic fuzzy set · Score function · Accuracy function · Trapezoidal intuitionistic fuzzy prioritized weighted averaging (TIFPWA) operator · Trapezoidal intuitionistic fuzzy prioritized weighted geometric (TIFPWG) operator · Multicriteria decision-making

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1 Introduction

Atanassov [1] introduced the concept of intuitionistic fuzzy set, which is a generalization of the concept of fuzzy set [2]. The intuitionistic fuzzy set has received more and more attention since its appearance [3]. Later, Liu and Yuan [4] introduced the concept of a triangular intuitionistic fuzzy set. The fundamental characteristic of the triangular intuitionistic fuzzy set is that the values of its membership function and nonmembership function are triangular fuzzy numbers rather than exact numbers. Then Wang [5] proposed the triangular intuitionistic fuzzy weighted geometric (TIFWG) operator, triangular intuitionistic fuzzy ordered weighted geometric (TIFOWG) operator and triangular intuitionistic fuzzy hybrid geometric (TIFHG) operator and developed an approach based on the TIFWG and the TIFHG operators to deal with multiple attribute group decision-making problems with triangular intuitionistic fuzzy information. Wang [6] proposed the fuzzy number intuitionistic fuzzy weighted averaging (FIFWA) operator, fuzzy number intuitionistic fuzzy ordered weighted averaging (FIFOWA) operator and fuzzy number intuitionistic fuzzy hybrid aggregation (FIFHA) operator and developed an approach based on the FIFHA operator to deal with multiple attribute decision-making problems with triangular intuitionistic fuzzy information. Furthermore, Wei et al. [7] developed an induced triangular intuitionistic fuzzy ordered weighted geometric (I-TIFOWG) operator and applied the I-TIFOWG operator to group decision-making problems with triangular intuitionistic fuzzy information. Hence, we can see that fuzzy number intuitionistic fuzzy set is a very useful tool to deal with uncertainty. More and more multicriteria decision methods [8, 9] have been applied under triangular intuitionistic fuzzy environment. However, these mentioned decision-making methods only deal with decision-making problems with triangular intuitionistic fuzzy sets and may generate distorted conclusions in some multicriteria decision-making problems due to the lack of considering various relationships between the criteria of decision-making in some cases. As is known to all, the trapezoidal fuzzy information is a generalization of triangular fuzzy information. Furthermore, the trapezoidal fuzzy number is a typical fuzzy number in practical applications, and the triangular fuzzy number is a special case of the trapezoidal fuzzy number. Therefore, the extension of triangular intuitionistic fuzzy sets to trapezoidal intuitionistic fuzzy sets and trapezoidal intuitionistic fuzzy multicriteria decision-making problems considering the prioritization among the criteria shall be worth paying attention to the research topic. To do it, the purposes of the paper are to introduce a trapezoidal intuitionistic fuzzy set, some operational laws, score and accuracy functions for trapezoidal intuitionistic fuzzy values and to propose the trapezoidal intuitionistic fuzzy prioritized weighted averaging (TIFPWA) operator and trapezoidal intuitionistic fuzzy prioritized weighted geometric (TIFPWG) operator. Then, the TIFPWA and TIFPWG operators are applied to multicriteria decision-making problems. The remainder of this paper is organized as follows. Section 2 introduces some basic concepts related to intuitionistic fuzzy sets, triangular intuitionistic fuzzy sets, some operational laws of triangular intuitionistic fuzzy values, and the prioritized average operator proposed by Yager [10]. By a generalization of a triangular intuitionistic fuzzy set, a trapezoidal intuitionistic fuzzy set, some operational laws of trapezoidal intuitionistic fuzzy values, and the score function and accuracy function of trapezoidal intuitionistic fuzzy values are introduced in Sect. 3. In Sect. 4, another score function of a trapezoidal intuitionistic fuzzy value is defined, and then the TIFPWA operator and TIFPWG operator are proposed to aggregate the trapezoidal intuitionistic fuzzy information. Section 5 develops a multicriteria decision-making method with trapezoidal intuitionistic fuzzy information based on the TIFPWA operator and TIFPWG operator and the score function and accuracy function of trapezoidal intuitionistic fuzzy values. In Sect. 6, an illustrative example shows the implementation process and effectiveness of the proposed method. Conclusions and some remarks are given in Sect. 7.

2 Preliminaries

In this section, we briefly describe some basic concepts related to intuitionistic fuzzy sets and fuzzy number intuitionistic fuzzy sets, some basic operational laws of fuzzy number intuitionistic fuzzy values, and the prioritized averaging operator which was originally introduced by Yager [10].

Atanassov [1] extended a fuzzy set [2] to an intuitionistic fuzzy set and gave the following definition.

Definition 1 [1]. An intuitionistic fuzzy set A in the universe of discourse X is defined as

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\},$$

where $\mu_A(x)$: $X \to [0, 1]$ and $v_A(x)$: $X \to [0, 1]$ are, respectively, the membership degree and nonmembership degree of the element x to the set A under the condition $0 \le \mu_A(x) + v_A(x) \le 1$ for $x \in X$. Then $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$, $x \in X$, is called Atanassov's intuitionistic index or a hesitancy degree of the element x in the set A. It is obvious that $0 \le \pi_A(x) \le 1$, $x \in X$.

By a generalization of an intuitionistic fuzzy set, Liu and Yuan [4] introduced the concept of a fuzzy number intuitionistic fuzzy set, in which the fundamental characteristic of the fuzzy number intuitionistic fuzzy set is that the values of membership function and nonmembership function are triangular fuzzy numbers and gave the following definition.

Definition 2 [4]. Let X be a universe of discourse, a triangular intuitionistic fuzzy set $\tilde{A}$ in X is defined as the following form:

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle | x \in X\},$$

where $\mu_{\tilde{A}}(x) \subset [0,1]$ and $v_{\tilde{A}}(x) \subset [0,1]$ are two triangular fuzzy numbers $\mu_{\tilde{A}}(x) = (\mu^1_{\tilde{A}}(x), \mu^2_{\tilde{A}}(x), \mu^3_{\tilde{A}}(x)) : X \to [0,1]$ and $v_{\tilde{A}}(x) = (v^1_{\tilde{A}}(x), v^2_{\tilde{A}}(x), v^3_{\tilde{A}}(x)) : X \to [0,1]$ with the condition $0 \le \mu^3_{\tilde{A}}(x) + v^3_{\tilde{A}}(x) \le 1$, $x \in X$.

For convenience, let $\mu_{\tilde{A}}(x) = (a, b, c)$ and $v_{\tilde{A}}(x) = (l, m, p)$, thus a triangular intuitionistic fuzzy value is denoted by $\tilde{a} = \langle (a, b, c), (l, m, p)\rangle$.

Then, Liu and Yuan [4] presented some operational laws of two fuzzy number intuitionistic fuzzy values.

Definition 3 [4]. Let $\tilde{a}_1 = \langle (a_1, b_1, c_1), (l_1, m_1, p_1)\rangle$ and $\tilde{a}_2 = \langle (a_2, b_2, c_2), (l_2, m_2, p_2)\rangle$ be two triangular intuitionistic fuzzy values. Then, there are the following operational laws:

$$(1)\ \tilde{a}_1 + \tilde{a}_2 = \langle (a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2, c_1 + c_2 - c_1c_2), (l_1l_2, m_1m_2, p_1p_2)\rangle;$$

$$(2)\ \tilde{a}_1 \times \tilde{a}_2 = \langle (a_1a_2, b_1b_2, c_1c_2), (l_1 + l_2 - l_1l_2, m_1 + m_2 - m_1m_2, p_1 + p_2 - p_1p_2)\rangle;$$

$$(3)\ \lambda\tilde{a}_1 = \left\langle \left(1 - (1 - a_1)^\lambda, 1 - (1 - b_1)^\lambda, 1 - (1 - c_1)^\lambda\right), \times \left(l_1^\lambda, m_1^\lambda, p_1^\lambda\right) \right\rangle, \lambda \ge 0;$$

(4) $\tilde{a}_1^\lambda = \left\langle \left(a_1^\lambda, b_1^\lambda, c_1^\lambda\right), \left(1-(1-l_1)^\lambda, 1-(1-m_1)^\lambda,\right.\right.$
$\left.\left. 1-(1-p_1)^\lambda\right)\right\rangle, \lambda \geq 0.$

Furthermore, Wang [5, 6] proposed the score function and accuracy function of a triangular intuitionistic fuzzy value as follows.

Definition 4 [5, 6]. Let $\tilde{a} = \langle (a,b,c),(l,m,p)\rangle$ be a triangular intuitionistic fuzzy value, then a score function s of a triangular intuitionistic fuzzy value can be represented as follows:

$$s(\tilde{a}) = \frac{a+2b+c}{4} - \frac{l+2m+p}{4}, \quad s(\tilde{a}) \in [-1,1], \tag{1}$$

where the larger the value of $s(\tilde{a})$, the larger the triangular intuitionistic fuzzy value $\tilde{a}$.

Definition 5 [5, 6]. Let $\tilde{a} = \langle (a,b,c),(l,m,p)\rangle$ be a triangular intuitionistic fuzzy value, an accuracy function h of a triangular intuitionistic fuzzy value can be represented as follows:

$$h(\tilde{a}) = \frac{a+2b+c}{4} + \frac{l+2m+p}{4}, \quad h(\tilde{a}) \in [0,1], \tag{2}$$

where the larger the value of $h(\tilde{a})$, the more the degree of accuracy of the triangular intuitionistic fuzzy value $\tilde{a}$.

As presented above, the score function s and the accuracy function h are, respectively, defined as the difference and the sum of the membership function $\mu_{\tilde{A}}(x)$ and the nonmembership function $v_{\tilde{A}}(x)$. Based on the score function s and the accuracy function h, Wang [5, 6] gave an order relation between two triangular intuitionistic fuzzy values, which is defined as follows.

Definition 6 [5, 6]. Let $\tilde{a}_1 = \langle (a_1,b_1,c_1),(l_1,m_1,p_1)\rangle$and $\tilde{a}_2 = \langle (a_2,b_2,c_2),(l_2,m_2,p_2)\rangle$ be two triangular intuitionistic fuzzy values. Thus, $s(\tilde{a}_1)$ and $s(\tilde{a}_2)$ are the scores of $\tilde{a}_1$ and $\tilde{a}_2$, respectively, and $h(\tilde{a}_1)$ and $h(\tilde{a}_2)$ are the accuracy degrees of $\tilde{a}_1$ and $\tilde{a}_2$, respectively. Then, the order relation between two triangular intuitionistic fuzzy values is in the following:

(1) If $s(\tilde{a}_1) > s(\tilde{a}_2)$, then $\tilde{a}_1 > \tilde{a}_2$;
(2) If $s(\tilde{a}_1) = s(\tilde{a}_2)$, and
  (a) if $h(\tilde{a}_1) = h(\tilde{a}_2)$, then $\tilde{a}_1 = \tilde{a}_2$;
  (b) if $h(\tilde{a}_1) > h(\tilde{a}_2)$, then $\tilde{a}_1 > \tilde{a}_2$.

Yager [10] originally introduced the prioritized average operator and defined as follows:

Definition 7 [10]. Let $G = \{G_1, G_2, \ldots, G_n\}$ be a collection of criteria and there is a prioritization between the criteria expressed by the linear ordering $G_1 > G_2 > \ldots > G_n$, which indicates criterion G_j has a higher priority than G_k if $j > k$. The value $G_j(x)$ is the performance of any alternative x under criterion G_j and satisfies $G_j(x) \in [0, 1]$. If

$$\mathrm{PA}(G_i(x)) = \sum_{j=1}^{n} w_j G_j(x), \tag{3}$$

where $w_j = T_j / \sum_{j=1}^{n} T_j$, $T_j = \prod_{k=1}^{j-1} G_k(x)$ $(j = 2, 3, \ldots, n)$, and $T_1 = 1$. Then PA is called the prioritized averaging operator.

3 Trapezoidal intuitionistic fuzzy sets

In this section, we extend triangular intuitionistic fuzzy sets to trapezoidal intuitionistic fuzzy sets, which are preferred in practice, then present the score function and accuracy function of trapezoidal intuitionistic fuzzy values.

As a generalization of a triangular intuitionistic fuzzy set, a trapezoidal intuitionistic fuzzy set is presented as the follows.

Definition 8 Let X be a universe of discourse, a trapezoidal intuitionistic fuzzy set $\tilde{A}$ in X is defined as the following form:

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle | x \in X\},$$

where $\mu_{\tilde{A}}(x) \subset [0,1]$ and $v_{\tilde{A}}(x) \subset [0,1]$ are two trapezoidal fuzzy numbers $\mu_{\tilde{A}}(x) = (\mu_{\tilde{A}}^1(x), \mu_{\tilde{A}}^2(x), \mu_{\tilde{A}}^3(x), \mu_{\tilde{A}}^4(x)) : X \to [0,1]$ and $v_{\tilde{A}}(x) = (v_{\tilde{A}}^1(x), v_{\tilde{A}}^2(x), v_{\tilde{A}}^3(x), v_{\tilde{A}}^4(x)) : X \to [0,1]$ with the condition $0 \leq \mu_{\tilde{A}}^4(x) + v_{\tilde{A}}^4(x) \leq 1$, $x \in X$.

For convenience, let $\mu_{\tilde{A}}(x) = (a,b,c,d)$ and $v_{\tilde{A}}(x) = (l,m,n,p)$, thus a trapezoidal intuitionistic fuzzy value is denoted by $\tilde{a} = \langle (a,b,c,d),(l,m,n,p)\rangle$.

If $b = c$ and $m = n$ hold in a trapezoidal intuitionistic fuzzy value $\tilde{a}$, it is reduced to the triangular intuitionistic fuzzy value, which is considered as a special case of the trapezoidal intuitionistic fuzzy value.

Definition 9 Let $\tilde{a}_1 = \langle (a_1,b_1,c_1,d_1),(l_1,m_1,n_1,p_1)\rangle$ and $\tilde{a}_2 = \langle (a_2,b_2,c_2,d_2),(l_2,m_2,n_2,p_2)\rangle$ be two trapezoidal intuitionistic fuzzy values. Then, there are the following operational laws:

(1) $\tilde{a}_1 + \tilde{a}_2 = \langle (a_1+a_2-a_1a_2, b_1+b_2-b_1b_2, c_1+c_2$
$-c_1c_2, d_1+d_2-d_1d_2),(l_1l_2, m_1m_2, n_1n_2, p_1p_2)\rangle;$

(2) $\tilde{a}_1 \times \tilde{a}_2 = \langle (a_1a_2, b_1b_2, c_1c_2, d_1d_2),(l_1+l_2-l_1l_2, m_1$
$+m_2-m_1m_2,, n_1+n_2-n_1n_2, p_1+p_2-p_1p_2)\rangle;$

(3) $\lambda\tilde{a}_1 = \left\langle \left(1-(1-a_1)^\lambda, 1-(1-b_1)^\lambda, 1-(1-c_1)^\lambda,\right.\right.$
$\left.\left. 1-(1-d_1)^\lambda\right), \left(l_1^\lambda, m_1^\lambda, n_1^\lambda, p_1^\lambda\right)\right\rangle, \quad \lambda > 0;$

(4) $\tilde{a}_1^{\lambda} = \left\langle (a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}, d_1^{\lambda}), \left(1-(1-l_1)^{\lambda}, 1-(1-m_1)^{\lambda}, 1-(1-n_1)^{\lambda}, 1-(1-p_1)^{\lambda}\right)\right\rangle, \quad \lambda \geq 0.$

Definition 10 Let $\tilde{a} = \langle (a,b,c,d),(l,m,n,p)\rangle$ be a trapezoidal intuitionistic fuzzy value, then a score function S of a trapezoidal intuitionistic fuzzy value can be presented as follows:

$$S(\tilde{a}) = \frac{a+b+c+d}{4} - \frac{l+m+n+p}{4}, \quad S(\tilde{a}) \in [-1,1], \tag{4}$$

where the larger the value of $S(\tilde{a})$, the larger the trapezoidal intuitionistic fuzzy value $\tilde{a}$. Especially, when $b = c$ and $m = n$ hold in a trapezoidal intuitionistic fuzzy value $\tilde{a}$, it is reduced to the score function of the triangular intuitionistic fuzzy value, which is considered as a special case.

Definition 11 Let $\tilde{a} = \langle (a,b,c,d),(l,m,n,p)\rangle$ be a trapezoidal intuitionistic fuzzy value, an accuracy function H of a trapezoidal intuitionistic fuzzy value can be presented as follows:

$$H(\tilde{a}) = \frac{a+b+c+d}{4} + \frac{l+m+n+p}{4}, \quad H(\tilde{a}) \in [0,1], \tag{5}$$

where the larger the value of $H(\tilde{a})$, the more the degree of accuracy of the trapezoidal intuitionistic fuzzy value $\tilde{a}$. Especially, when $b = c$ and $m = n$ hold in a trapezoidal intuitionistic fuzzy value $\tilde{a}$, it is reduced to the accuracy function of the triangular intuitionistic fuzzy value, which is considered as a special case.

4 Prioritized aggregation operators of trapezoidal intuitionistic fuzzy sets

It should note that the score function $S(\tilde{a})$ is between -1 and 1 in Sect. 3. In order to facilitate the following study, we introduce another score function defined as follows:

$$S'(\tilde{a}) = \frac{4+a+b+c+d}{8} - \frac{l+m+n+p}{8}, \quad S'(\tilde{a}) \in [0,1]. \tag{6}$$

Since $0 \leq a, b, c, d, l, m, n, p \leq 1$, we can easily obtain $0 \leq S'(\tilde{a}) \leq 1$ and derive that the order relation between two trapezoidal intuitionistic fuzzy values is also valid using the new defined score function in Definition 6.

Based on the prioritized average operators [10] and the operational lows of trapezoidal intuitionistic fuzzy values, we propose the TIFPWA operator and TIFPWG operator in the section.

4.1 Trapezoidal intuitionistic fuzzy prioritized weighted averaging (TIFPWA) operator

The prioritized average operator [10] has usually been used in situations where the input arguments are the exact values. We shall extend the prioritized average operator to accommodate the situations where the input arguments are trapezoidal intuitionistic fuzzy information. Based on Definition 7, we give the following definition of the TIFPWA operator.

Definition 12 Let $\tilde{a}_j = \langle (a_j,b_j,c_j,d_j),(l_j,m_j,n_j,p_j)\rangle$ ($j = 1, 2, \ldots, n$) be a collection of trapezoidal intuitionistic fuzzy values. Then, a TIFPWA operator is defined as follows:

$$\begin{aligned}\mathrm{TIFPWA}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n) &= \frac{T_1}{\sum_{j=1}^n T_j}\tilde{a}_1 \oplus \frac{T_2}{\sum_{j=1}^n T_j}\tilde{a}_2 \oplus \cdots \oplus \frac{T_n}{\sum_{j=1}^n T_j}\tilde{a}_n \\ &= \mathop{\oplus}_{j=1}^{n}\left(\frac{T_j}{\sum_{j=1}^n T_j}\tilde{a}_j\right),\end{aligned} \tag{7}$$

where $T_j = \prod_{k=1}^{j-1} S'(\tilde{a}_k)$ ($j = 2, 3,\ldots, n$), $T_1 = 1$ and $S'(\tilde{a}_k)$ is the score of a trapezoidal intuitionistic fuzzy value a_k.

Based on the operational laws of trapezoidal intuitionistic fuzzy values described in Sect. 3, we can derive the following Theorem 1.

Theorem 1 *Let $\tilde{a}_j = \langle (a_j,b_j,c_j,d_j),(l_j,m_j,n_j,p_j)\rangle$ ($j = 1, 2, \ldots, n$) be a collection of trapezoidal intuitionistic fuzzy values. Then, their aggregated value using the TIFPWA operator is also a trapezoidal intuitionistic fuzzy value and*

$$\begin{aligned}&\mathrm{TIFPWA}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n) \\ &= \frac{T_1}{\sum_{j=1}^n T_j}\tilde{a}_1 \oplus \frac{T_2}{\sum_{j=1}^n T_j}\tilde{a}_2 \oplus \cdots \oplus \frac{T_n}{\sum_{j=1}^n T_j}\tilde{a}_n = \mathop{\oplus}_{j=1}^{n}\left(\frac{T_1}{\sum_{j=1}^n T_j}\tilde{a}_j\right) \\ &= \left\langle \left(1-\prod_{j=1}^n (1-a_j)^{T_j/\sum_{j=1}^n T_j}, 1-\prod_{j=1}^n (1-b_j)^{T_j/\sum_{j=1}^n T_j}, \right.\right. \\ &\quad \left. 1-\prod_{j=1}^n (1-c_j)^{T_j/\sum_{j=1}^n T_j}, 1-\prod_{j=1}^n (1-d_j)^{T_j/\sum_{j=1}^n T_j}\right), \\ &\quad \left.\left(\prod_{j=1}^n l_j^{T_j/\sum_{j=1}^n T_j}, \prod_{j=1}^n m_j^{T_j/\sum_{j=1}^n T_j}, \prod_{j=1}^n n_j^{T_j/\sum_{j=1}^n T_j}, \prod_{j=1}^n p_j^{T_j/\sum_{j=1}^n T_j}\right)\right\rangle\end{aligned} \tag{8}$$

where $T_j = \prod_{k=1}^{j-1} S'(\tilde{a}_k)$ ($j = 2, 3,\ldots, n$), $T_1 = 1$ and $S'(\tilde{a}_k)$ is the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k$.

It is obvious that there are the following properties (P1–P3):

P1 Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j), (l_j, m_j, n_j, p_j) \rangle$ ($j = 1, 2, \ldots, n$) be a collection of trapezoidal intuitionistic fuzzy values. Then $T_j = \prod_{k=1}^{j-1} S'(\tilde{a}_k)$ ($j = 2, 3, \ldots, n$), $T_1 = 1$ and $S'(\tilde{a}_k)$ is the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k$. If each $\tilde{a}_j$ ($j = 1, 2, \ldots, n$) is equal, i.e., $\tilde{a}_j = \tilde{a}$ for $j = 1, 2, \ldots, n$, thus

$$\mathrm{TIFPWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a}. \tag{9}$$

P2 Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j), (l_j, m_j, n_j, p_j) \rangle$ ($j = 1, 2, \ldots, n$) be a collection of trapezoidal intuitionistic fuzzy values. Then $T_j = \prod_{k=1}^{j-1} S'(\tilde{a}_k)$ ($j = 2, 3, \ldots, n$), $T_1 = 1$ and $S'(\tilde{a}_k)$ is the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k$. Thus, let

$$\tilde{a}^- = \left\langle \left(\min_j a_j, \min_j b_j, \min_j c_j, \min_j d_j \right), \left(\max_j l_j, \max_j m_j, \max_j n_j, \max_j p_j \right) \right\rangle,$$
$$\tilde{a}^+ = \left\langle \left(\max_j a_j, \max_j b_j, \max_j c_j, \max_j d_j \right), \left(\min_j l_j, \min_j m_j, \min_j n_j, \min_j p_j \right) \right\rangle$$

Then

$$\tilde{a}^- \le \mathrm{TIFPWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \le \tilde{a}^+. \tag{10}$$

P3 Let $\tilde{a}_j$ ($j = 1, 2, \ldots, n$) and $\tilde{a}_j^*$ ($j = 1, 2, \ldots, n$) be two collections of trapezoidal intuitionistic fuzzy values. Then $T_j = \prod_{k=1}^{j-1} S'(\tilde{a}_k)$, $T_j^* = \prod_{k=1}^{j-1} S'(\tilde{a}_k^*)$ ($j = 2, 3, \ldots, n$), $T_1 = T_1^* = 1$ and $S'(\tilde{a}_k)$ is the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k$, $S'(\tilde{a}_k^*)$ the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k^*$. If $\tilde{a}_j \le \tilde{a}_j^*$ for $j = 1, 2, \ldots, n$, then

$$\mathrm{TIFPWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \le \mathrm{TIFPWA}\left(\tilde{a}_1^*, \tilde{a}_2^*, \ldots, \tilde{a}_n^*\right). \tag{11}$$

4.2 Trapezoidal intuitionistic fuzzy prioritized weighted geometric (TIFPWG) operator

Based on the TIFPWA operator and the geometric mean, here, we define a TIFPWG operator.

Definition 13 Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j), (l_j, m_j, n_j, p_j) \rangle$ ($j = 1, 2, \ldots, n$) be a collection of trapezoidal intuitionistic fuzzy values. Then, a TIFPWG operator is defined as follows:

$$\mathrm{TIFPWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a}_1^{T_1/\sum_{j=1}^n T_j} \otimes \tilde{a}_2^{T_2/\sum_{j=1}^n T_j} \otimes \cdots \otimes \tilde{a}_n^{T_n/\sum_{j=1}^n T_j} = \bigotimes_{j=1}^n \tilde{a}_j^{T_j/\sum_{j=1}^n T_j}, \tag{12}$$

where $T_j = \prod_{k=1}^{j-1} S'(\tilde{a}_k)$ ($j = 2, 3, \ldots, n$), $T_1 = 1$ and $S'(\tilde{a}_k)$ is the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k$.

Based on the operational laws of trapezoidal intuitionistic fuzzy values described in Sect. 3, we can derive the Theorem 2.

Theorem 2 *Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j), (l_j, m_j, n_j, p_j) \rangle$ ($j = 1, 2, \ldots, n$) be a collection of trapezoidal intuitionistic fuzzy values, then their aggregated value using the TIFPWG operator is also a trapezoidal intuitionistic fuzzy value and*

$$\begin{aligned}
&\mathrm{TIFPWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \\
&= \tilde{a}_1^{T_1/\sum_{j=1}^n T_j} \otimes \tilde{a}_2^{T_2/\sum_{j=1}^n T_j} \otimes \cdots \otimes \tilde{a}_n^{T_n/\sum_{j=1}^n T_j} = \bigotimes_{j=1}^n \tilde{a}_j^{T_j/\sum_{j=1}^n T_j} \\
&= \left\langle \left(\prod_{j=1}^n a_j^{T_j/\sum_{j=1}^n T_j}, \prod_{j=1}^n b_j^{T_j/\sum_{j=1}^n T_j}, \prod_{j=1}^n c_j^{T_j/\sum_{j=1}^n T_j}, \prod_{j=1}^n d_j^{T_j/\sum_{j=1}^n T_j} \right), \right. \\
&\quad \left(1 - \prod_{j=1}^n (1 - l_j)^{T_j/\sum_{j=1}^n T_j}, 1 - \prod_{j=1}^n (1 - m_j)^{T_j/\sum_{j=1}^n T_j}, \right. \\
&\quad \left. \left. 1 - \prod_{j=1}^n (1 - n_j)^{T_j/\sum_{j=1}^n T_j}, 1 - \prod_{j=1}^n (1 - p_j)^{T_j/\sum_{j=1}^n T_j} \right) \right\rangle
\end{aligned} \tag{13}$$

where $T_j = \prod_{k=1}^{j-1} S'(\tilde{a}_k)$ ($j = 2, 3, \ldots, n$), $T_1 = 1$ and $S'(\tilde{a}_k)$ is the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k$.

It is obvious that the TIFPWG operator has the following properties (P1–P3):

P1 Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j), (l_j, m_j, n_j, p_j) \rangle$ ($j = 1, 2, \ldots, n$) be a collection of trapezoidal intuitionistic fuzzy values. Then $T_j = \prod_{k=1}^{j-1} S'(\tilde{a}_k)$ ($j = 2, 3, \ldots, n$), $T_1 = 1$ and $S'(\tilde{a}_k)$is the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k$. If each $\tilde{a}_j$ ($j = 1, 2, \ldots, n$) is equal, i.e., $\tilde{a}_j = \tilde{a}$ for $j = 1, 2, \ldots, n$, thus

$$\mathrm{TIFPWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a}. \tag{14}$$

P2 Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j), (l_j, m_j, n_j, p_j) \rangle$($j = 1, 2, \ldots, n$) be a collection of trapezoidal intuitionistic fuzzy values. Then $T_j = \prod_{k=1}^{j-1} S'(\tilde{a}_k)$ ($j = 2, 3, \ldots, n$), $T_1 = 1$ and $S'(\tilde{a}_k)$ is the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k$. Thus, let

$$\tilde{a}^- = \left\langle \left(\min_j a_j, \min_j b_j, \min_j c_j, \min_j d_j \right), \left(\max_j l_j, \max_j m_j, \max_j n_j, \max_j p_j \right) \right\rangle,$$
$$\tilde{a}^+ = \left\langle \left(\max_j a_j, \max_j b_j, \max_j c_j, \max_j d_j \right), \left(\min_j l_j, \min_j m_j, \min_j n_j, \min_j p_j \right) \right\rangle.$$

Then

$$\tilde{a}^- \leq \text{TIFPWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \tilde{a}^+. \quad (15)$$

P3 Let $\tilde{a}_j$ ($j = 1, 2, \ldots, n$) and $\tilde{a}_j^*$ ($j = 1, 2, \ldots, n$) be two collections of trapezoidal intuitionistic fuzzy values. Then $T_j = \prod_{k=1}^{j-1} S'(\tilde{a}_k)$, $T_j^* = \prod_{k=1}^{j-1} S(\tilde{a}_k^*)$ ($j = 2, 3, \ldots, n$), $T_1 = T_1^* = 1$ and $S'(\tilde{a}_k)$is the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k$, $S'(\tilde{a}_k^*)$ is the score of a trapezoidal intuitionistic fuzzy value $\tilde{a}_k^*$. If $\tilde{a}_j \leq \tilde{a}_j^*$ for $j = 1, 2, \ldots, n$, then

$$\text{TIFPWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \text{TIFPWG}\left(\tilde{a}_1^*, \tilde{a}_2^*, \ldots, \tilde{a}_n^*\right). \quad (16)$$

5 Decision-making method with trapezoidal intuitionistic fuzzy information

This section develops an approach based on the TIFPWA and TIFPWG operators and the score function and accuracy function to deal with a trapezoidal intuitionistic fuzzy multicriteria decision-making problem, in which the criteria are in different priority level.

For a multicriteria decision-making problem with trapezoidal intuitionistic fuzzy information, Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives and let $G = \{G_1, G_2, \ldots, G_n\}$ be a collection of criteria, then there is a prioritization between the criteria expressed by the linear ordering $G_1 > G_2 > \ldots > G_n$ which indicates criterion G_j has a higher priority than G_s if $j < s$. Suppose that $\tilde{R} = (\tilde{r}_{ij})_{m\times n} = \left(\left\langle (a_{ij}, b_{ij}, c_{ij}, d_{ij}), (l_{ij}, m_{ij}, n_{ij}, p_{ij})\right\rangle\right)_{m\times n}$ is a trapezoidal intuitionistic fuzzy decision matrix, where $(a_{ij}, b_{ij}, c_{ij}, d_{ij}) \subset [0, 1]$ indicates the degree that the alternative A_i satisfies the criterion G_j and $(l_{ij}, m_{ij}, n_{ij}, p_{ij}) \subset [0, 1]$ indicates the degree that the alternative A_i does not satisfy the criterion G_j with $0 \leq d_{ij} + p_{ij} \leq 1$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

In the following, we apply the TIFPWA and TIFPWG operators and the score function and accuracy function to a multicriteria decision-making problem with trapezoidal intuitionistic fuzzy information, which can be described as the following procedures:

Step 1 Calculate the values of T_{ij} ($i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$) by utilizing $T_{ij} = \prod_{k=1}^{j-1} S'(\tilde{r}_{ik})$ ($i = 1, 2, \ldots, m$; $j = 2, 3, \ldots, n$) and $T_{i1} = 1$ for $i = 1, 2, \ldots, m$.

Step 2 Utilize the decision information of the given decision matrix $\tilde{R}$ and the TIFPWA and TIFPWG operators $\tilde{r}_i = \langle (a_i, b_i, c_i, d_i), (l_i, m_i, n_i, p_i)\rangle = \text{TIFPWA}(\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in})$ and $\tilde{r}_i = \langle (a_i, b_i, c_i, d_i), (l_i, m_i, n_i, p_i)\rangle = \text{TIFPWG}(\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in})$ ($i = 1, 2, \ldots, m$) to obtain the collective overall trapezoidal intuitionistic fuzzy values of $\tilde{r}_i$ ($i = 1, 2, \ldots, m$) for each alternative A_i ($i = 1, 2, \ldots, m$).

Step 3 Calculate the score $S'(\tilde{r}_i)$ ($i = 1, 2, \ldots, m$) of the collective overall trapezoidal intuitionistic fuzzy values of $\tilde{r}_i$ ($i = 1, 2, \ldots, m$) to rank all the alternatives of A_i ($i = 1, 2, \ldots, m$) and to select the best one(s) (if there is no difference between two scores $S'(\tilde{r}_i)$ and $S'(\tilde{r}_j)$, then we need to calculate the accuracy degrees $H(\tilde{r}_i)$ and $H(\tilde{r}_j)$ of the collective overall trapezoidal intuitionistic fuzzy values, respectively, and then rank the alternatives A_i and A_j in accordance with the accuracy degrees $H(\tilde{r}_i)$ and $H(\tilde{r}_j)$).

Step 4 Rank all the alternatives of A_i ($i = 1, 2, \ldots, m$) and select the best one(s) according to $S'(\tilde{r}_i)$ and $H(\tilde{r}_i)$ ($i = 1, 2, \ldots, m$).

Step 5 End.

6 Illustrative example

In this section, an illustrative example of a software selection problem adapted from Wang [9] for a multicriteria decision-making problem of alternatives is used as the demonstration of the application of the proposed multicriteria decision-making method in a realistic scenario, as well as the implementation process and effectiveness of the proposed method.

We consider a software selection problem for a multicriteria decision-making problem in which five candidate software systems are given as the set of five alternatives $A = (A_1, A_2, A_3, A_4, A_5)$ and the investment company must take a decision according to the following four criteria: (1) G_1: the contribution to organization performance; (2) G_2: the effort to transform from current system; (3) G_3: the costs of hardware/software investment; (4) G_4: the outsourcing software developer reliability. Then, we suppose that the prioritization relationship between the four criteria is $G_1 > G_2 > G_3 > G_4$. The investment company must take a decision according to the above four criteria to be evaluated by the decision maker or expert under the trapezoidal intuitionistic fuzzy environment, as listed in the following trapezoidal intuitionistic fuzzy decision matrix:

$$\tilde{R} = \begin{bmatrix} \langle(0.4,0.5,0.6,0.7),(0.1,0.1,0.1,0.1)\rangle & \langle(0.0,0.1,0.2,0.3),(0.2,0.3,0.4,0.5)\rangle & \langle(0.3,0.4,0.5,0.6),(0.1,0.1,0.1,0.1)\rangle & \langle(0.3,0.4,0.5,0.6),(0.1,0.2,0.3,0.4)\rangle \\ \langle(0.3,0.4,0.5,0.5),(0.0,0.1,0.1,0.1)\rangle & \langle(0.2,0.3,0.4,0.5),(0.0,0.1,0.2,0.3)\rangle & \langle(0.0,0.1,0.1,0.2),(0.5,0.6,0.7,0.8)\rangle & \langle(0.3,0.4,0.5,0.5),(0.0,0.1,0.1,0.2)\rangle \\ \langle(0.1,0.1,0.1,0.1),(0.6,0.7,0.8,0.9)\rangle & \langle(0.0,0.1,0.1,0.2),(0.3,0.4,0.5,0.6)\rangle & \langle(0.2,0.3,0.4,0.5),(0.1,0.2,0.2,0.3)\rangle & \langle(0.1,0.2,0.3,0.4),(0.3,0.4,0.5,0.6)\rangle \\ \langle(0.7,0.7,0.7,0.7),(0.1,0.1,0.1,0.1)\rangle & \langle(0.4,0.5,0.6,0.7),(0.0,0.1,0.2,0.2)\rangle & \langle(0.2,0.3,0.4,0.5),(0.1,0.2,0.3,0.3)\rangle & \langle(0.1,0.2,0.3,0.4),(0.4,0.5,0.6,0.6)\rangle \\ \langle(0.0,0.1,0.2,0.2),(0.5,0.6,0.7,0.8)\rangle & \langle(0.4,0.4,0.4,0.4),(0.0,0.1,0.2,0.3)\rangle & \langle(0.6,0.7,0.7,0.8),(0.0,0.1,0.1,0.2)\rangle & \langle(0.1,0.2,0.3,0.3),(0.2,0.3,0.4,0.5)\rangle \end{bmatrix}$$

Hence, we utilize the developed method to obtain the most desirable software system (s) as follows:

Step 1 Utilizing $T_{ij} = \prod_{k=1}^{j-1} S'(\tilde{r}_{ik})$ ($i = 1, 2, 3, 4, 5$; $j = 2, 3, 4$) and $T_{i1} = 1$ for $i = 1, 2, 3, 4, 5$, we can calculate the values of T_{ij} ($i = 1, 2, 3, 4, 5$ and $j = 2, 3, 4$) as follows:

$$T = \begin{bmatrix} 1.0000 & 0.7250 & 0.2900 & 0.1957 \\ 1.0000 & 0.6750 & 0.4050 & 0.0911 \\ 1.0000 & 0.1750 & 0.0569 & 0.0327 \\ 1.0000 & 0.8000 & 0.5700 & 0.3206 \\ 1.0000 & 0.2375 & 0.1484 & 0.1187 \end{bmatrix}.$$

Step 2 Utilizing the TIFPWA operator, we can obtain the collective overall trapezoidal intuitionistic fuzzy values of $\tilde{r}_i$ ($i = 1, 2, 3, 4, 5$) for a software system A_i ($i = 1, 2, 3, 4, 5$) as follows:

$$\tilde{r}_1 = \langle(0.2661, 0.3689, 0.4727, 0.5781), (0.1255, 0.1524, 0.1737, 0.1917)\rangle,$$
$$\tilde{r}_2 = \langle(0.2201, 0.3211, 0.4095, 0.4542), (0.0000, 0.1397, 0.1783, 0.2135)\rangle,$$
$$\tilde{r}_3 = \langle(0.0916, 0.1128, 0.1220, 0.1466), (0.4940, 0.6035, 0.6958, 0.8014)\rangle,$$
$$\tilde{r}_4 = \langle(0.4827, 0.5303, 0.5813, 0.6369), (0.0000, 0.1403, 0.1920, 0.1920)\rangle,$$
$$\tilde{r}_5 = \langle(0.1642, 0.2495, 0.3133, 0.3402), (0.0000, 0.3588, 0.4536, 0.5759)\rangle.$$

Utilizing the TIFPWG operator, we can also obtain the collective overall trapezoidal intuitionistic fuzzy values of $\tilde{r}_i$ ($i = 1, 2, 3, 4, 5$) for a software system A_i ($i = 1, 2, 3, 4, 5$):

$$\tilde{r}_1 = \langle(0.0000, 0.2808, 0.4021, 0.5125), (0.1341, 0.1798, 0.2294, 0.2840)\rangle,$$
$$\tilde{r}_2 = \langle(0.0000, 0.2824, 0.3455, 0.4214), (0.1213, 0.2263, 0.2931, 0.3744)\rangle,$$
$$\tilde{r}_3 = \langle(0.0000, 0.1070, 0.1095, 0.1226), (0.5452, 0.6487, 0.7526, 0.8629)\rangle,$$
$$\tilde{r}_4 = \langle(0.3605, 0.4559, 0.5369, 0.6098), (0.1152, 0.1816, 0.2519, 0.2519)\rangle,$$
$$\tilde{r}_5 = \langle(0.0000, 0.1593, 0.2607, 0.2641), (0.3801, 0.4853, 0.5877, 0.6996)\rangle.$$

Step 3 Calculate the score values of $S'(\tilde{r}_i)$ ($i = 1, 2, 3, 4, 5$) for the collective overall trapezoidal intuitionistic fuzzy values of $\tilde{r}_i$ ($i = 1, 2, 3, 4, 5$), which are shown in Table 1.

Step 4 From Table 1, we can rank all the software systems of A_i ($i = 1, 2, 3, 4, 5$) according to the score values of $S'(\tilde{r}_i)$ ($i = 1, 2, 3, 4, 5$) for the collective overall trapezoidal intuitionistic fuzzy values of $\tilde{r}_i$ ($i = 1, 2, 3, 4, 5$), which are shown in Table 2. Note that "$\succ$" means "preferred to". We can see two kinds of ranking orders of the alternatives are the same and the most desirable software system is the alternative A_4.

Compared with some relevant papers [5–9] which proposed triangular intuitionistic fuzzy decision-making approaches, the decision information used in [5–9] is triangular intuitionistic fuzzy sets, whereas the decision information in this paper is trapezoid intuitionistic fuzzy sets. As mentioned above, the trapezoid intuitionistic fuzzy set is a further generalization of a triangular intuitionistic fuzzy set. So the decision-making method proposed in this paper is more typical in applications. Furthermore, the

Table 1 The score values for the alternatives by utilizing the TIFPWA and TIFPWG operators

Alternative A_i	Score value (TIFPWA)	Score value (TIFPWG)
A_1	0.6303	0.5460
A_2	0.6092	0.5043
A_3	0.2348	0.1912
A_4	0.7134	0.6453
A_5	0.4599	0.3164

Table 2 Ranking orders of the alternatives

Aggregation operator	Ranking order
TIFPWA	$A_4 \succ A_1 \succ A_2 \succ A_5 \succ A_3$
TIFPWG	$A_4 \succ A_1 \succ A_2 \succ A_5 \succ A_3$

decision-making approach proposed in this paper can be used to solve not only decision-making problems with trapezoid intuitionistic fuzzy information but also decision-making problems with triangular intuitionistic fuzzy information, whereas the method in [5–9] is only suitable for decision-making problems with triangular intuitionistic fuzzy information. On the other hand, the decision-making models in [5–9] may generate distorted conclusions in some multicriteria decision-making problems due to the lack of considering various relationships between the criteria of decision-making, whereas the decision-making models proposed in this paper provide various prioritized relationships between the criteria of decision-making. Therefore, the decision-making method proposed in the paper is suitable for solving multicriteria decision-making problems with trapezoidal intuitionistic fuzzy prioritized criteria and is more reasonable in some cases.

7 Conclusion

This paper introduced a trapezoidal intuitionistic fuzzy set and its score and accuracy functions. Then, the TIFPWA operator and TIFPWG operator were presented to aggregate the trapezoidal intuitionistic fuzzy information. Furthermore, based on the TIFPWA and TIFPWG operators and the score function and accuracy function, we have developed the trapezoidal intuitionistic fuzzy multicriteria decision-making approach, in which the criteria are in different priority level. The TIFPWA and TIFPWG operators are utilized to aggregate the trapezoidal intuitionistic fuzzy information corresponding to each alternative and get the collective overall values of the alternatives, then we can rank the alternatives according to the values of the score and accuracy functions and select the most desirable one(s). Finally, an illustrative example of software selection for considering various prioritized relationships between the criteria of decision-making was given to demonstrate its practicality and effectiveness.

Because distorted conclusions will be generated in some multicriteria decision-making problems due to the lack of considering various relationships between the criteria of decision-making, the advantage of the proposed methods is more suitable for solving multicriteria decision-making problems with trapezoidal intuitionistic fuzzy prioritized criteria. The future work is to improve the aggregated algorithm and to apply the proposed method to some other practical decision-making problems, such as supply chain management and water resource schedule.

References

1. Atanassov K (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20:87–96
2. Zadeh LA (1965) Fuzzy sets. Inf Control 8(5):338–356
3. Chen TY (2011) Experimental analysis of score functions in multi-criteria decision-making with intuitionistic fuzzy sets. J Chin Inst of Ind Eng 28(3):206–225
4. Liu F, Yuan XH (2007) Fuzzy number intuitionistic fuzzy set. Fuzzy Syst Math 21(1):88–91
5. Wang XF (2008) Fuzzy number intuitionistic fuzzy geometric aggregation operators and their application to decision making. Control Decis 23(6):607–612
6. Wang XF (2008) Fuzzy number intuitionistic fuzzy arithmetic aggregation operators. Int J Fuzzy Syst 10(2):104–111
7. Wei GW, Zhao XF, Lin R (2010) Some induced aggregating operators with fuzzy number intuitionistic fuzzy information and their applications to group decision making. Int J Comput Intell Syst 3(1):84–95
8. Qiu JX (2011) Evaluating model of mechanical automation with fuzzy number intuitionistic fuzzy information. Int J Adv Comput Technol 3(6):42–47
9. Wang Y (2012) An approach to software selection with triangular intuitionistic fuzzy information. Int J Adv Comput Technol 4(2):284–290
10. Yager RR (2008) Prioritized aggregation operators. Int J Approx Reason 48:263–274