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ORIGINAL ARTICLE

A developed distance method for ranking generalized fuzzy numbers

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Abstract Fuzzy logic is one of the effective tools to handle uncertainty and vagueness in engineering and mathematics. One major part of fuzzy logic is ranking fuzzy numbers. In many fuzzy program systems, ranking fuzzy numbers has a remarkable role in decision making and data analysis. Despite the fact that a variety of methods exists for ranking fuzzy numbers, no one can rank fuzzy numbers perfectly in all cases and situations. In this paper, a new method for ranking fuzzy numbers based on the left and right using distance method and α-cut has been presented. To achieve this, a fuzzy distance measure between two generalized fuzzy numbers is proposed. The new measure is expanded with the help of the fuzzy ambiguity measure. The calculation of this method is derived from generalized trapezoidal fuzzy numbers and distance method concepts. Furthermore, a comparison of generalized fuzzy numbers between the proposed method and other resembled methods is provided.

Keywords Fuzzy number · Distance method · Ranking

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1 Introduction

Making the best decision under surrounding circumstance has always been a major challenge for scientists and researchers across the world. To achieve decision makers' aims, a variety of well-known methods including fuzzy set theory, SWOT analysis and analytical hierarchal process (AHP) have been developed in previous researches such as [29, 30, 33]. Fuzzy set theory has been applied to multifarious scopes which demands to control ambiguous and unreliable values. Fuzzy numbers are a specific division of fuzzy sets and can be regarded as an influential expansion of ordinary numbers [17]. Various studies have dealt with ranking fuzzy numbers. They used this concept in a variety of researches. In one of the recent ones, Sepehriar et al. in [29] used this concept as an efficient way in regard to supplier selection. In the first steps for ranking fuzzy numbers, Jain [10, 11] recommended a method, using the concept of maximizing set for ranking fuzzy numbers. His technique showed that the decision maker only considers the right side membership function. Regular way for developing ranking fuzzy numbers was suggested in [18]. After that, Dubios and Parde [19] used maximizing sets for ranking fuzzy numbers. One year later, Baldwin and Guil [20] pointed out that these two methods have some unsettling drawbacks. Subsequently, Adamo [21] used the concept of preference function α-preference rule. The concept of preference function was introduced by Chang [7]. Moreover, Yager [22, 23] proposed four indices which may be employed for ranking fuzzy quantities between 0 and 1. Bortlan and Degani [24] measured and reconsidered some of these ranking methods. Chen and Hwang [25] comprehensively reviewed the existing approaches and indicated some unreasonable conditions that arise among them, and more recently, some ranking techniques have presented in [26, 27] to define an improved fuzzy distance measure. After that, in [28, 30], new approaches proposed for measuring fuzzy distance. In this paper, a new method for the distance between two fuzzy numbers has been proposed. To achieve this, a new criterion has been used through α-cut concept. This paper includes 5 Sections. In Sect. 2, some basic definitions in regard to fuzzy numbers are reviewed. In Sect. 3, a new method for ordering fuzzy numbers is proposed. Section 4 encompasses numerical examples, and the final section includes conclusions.

2 Preliminaries

The basic definitions and concepts of the fuzzy set theory are given as follows from [31, 32].

Definition 2.1: Let X be a universe set. A fuzzy set C of X is defined by a membership function $\mu_C(x): R \rightarrow [0,1]$, where $\mu_C(x), \forall x \in X$ indicates the degree of x in C.

Definition 2.2: A trapezoidal fuzzy number C is a fuzzy number with a membership function μ_C which can be denoted as a quartet (c_1, c_2, c_3, c_4). In these equations, if $c_2 = c_3$, C becomes a triangular fuzzy number.

$$\mu_C(x) = \begin{cases} \frac{x-c_1}{c_2-c_1}, & c_1 \le x \le c_2 \\ 1, & c_2 \le x \le c_3 \\ \frac{c_4-x}{c_4-c_3}, & c_3 \le x \le c_4 \\ 0, & \text{OW} \end{cases}$$

Definition 2.3: Throughout the paper, we assume that $X = R$.

Definition 2.4: An extended fuzzy number C is described as any fuzzy subset of the universe set X with membership function μ_C as follows; where c_1, c_2, c_3, c_4 are real numbers. μ_C (x) is a continuous mapping from X to a closed interval [0, 1].

(a) μ_C $(x) = 0$, for all $x \in (\infty, c_1)$.
(b) μ_C is strictly increasing on $[c_1, c_2]$.
(c) μ_C $(x) = 1$, for all $x \in [c_2, c_3]$.
(d) μ_C is strictly decreasing on $[c_3, c_4]$.
(e) μ_C $(x) = 0$ for all $x \in [c_4, +\infty)$.

Definition 2.5: The α-cut of a fuzzy number C, where $0 < \alpha \le 1$ is a set defined as $C_\alpha = \inf\{x \in R | \mu_C(x) \ge \alpha\}$, According to the definition, every α-cut of a fuzzy number is a closed interval. Hence, we have $C_\alpha = \left[C_\alpha^-, C_\alpha^+\right]$, where;

$$C_\alpha^- = \inf\{x \in R | \mu_C(x) \ge \alpha\},$$
$$C_\alpha^+ = \sup\ \{x \in R | \mu_c(x) \ge \alpha\}.$$

A set of all fuzzy numbers on real line is denoted by $F(R)$, and in this article, we have: $C_\alpha = \{x \in R | \mu_C(x) = 1\}$.

Definition 2.6: The $D_{p,q}$-distance, indexed by parameters $1 \le p \le +\infty, 0 \le q \le 1$, between two fuzzy numbers A and C is a nonnegative function on $F(R) \times F(R)$ as following:

$$D_{p,q}(A,C) = \begin{cases} \left[(1-q)\int_0^1 \left|A_\alpha^- - C_\alpha^-\right|^p d\alpha + q\int_0^1 \left|A_\alpha^+ - C_\alpha^+\right|^p d\alpha\right]^{1/p}, \text{if } p < \infty, \\ (1-q)\sup_{0 \le \alpha \le 1}\left(\left|A_\alpha^- - C_\alpha^-\right|^p\right) + q\inf_{0 \le \alpha \le 1}\left|A_\alpha^+ - C_\alpha^+\right|^p), \text{if } p = \infty. \end{cases}$$

In this paper, we suppose $p = 2$, $q = \frac{1}{2}$. Therefore,

$$\left[D_{2,1/2}(A,C)\right]^2 = \frac{1}{2}\left(\int_0^1 \left|A_\alpha^- - C_\alpha^-\right|^2 d\alpha + \int_0^1 \left|A_\alpha^+ - C_\alpha^+\right|^2 d\alpha\right).$$

Definition 2.7: Let $A = (a_1, a_2, a_3, a_4)$ and $C = (c_1, c_2, c_3, c_4)$ be two trapezoidal fuzzy numbers, and

$$A_\alpha = \left[A_\alpha^-, A_\alpha^+\right] = [(1-\alpha)a_1 + \alpha a_2, \alpha a_3 + (1-\alpha)a_4],$$
$$C_\alpha = \left[C_\alpha^-, C_\alpha^+\right] = [(1-\alpha)c_1 + \alpha c_2, \alpha c_3 + (1-\alpha)c_4].$$

Define $D_{2,1/2}$-distance for two trapezoidal fuzzy numbers A and C on $F(R)$ as follows:

$$\left[D_{2,1/2}(A,C)\right]^2 = \frac{1}{6}\left[\sum_{i=1}^4 (c_i - a_i)^2 + \sum_{i \in \{1,3\}} (c_i - a_i)(c_{i+1} - a_{i+1})\right].$$

Let $A = (a_1, a_2, a_3, a_4)$, $C = (c_1, c_2, c_3, c_4)$ be two trapezoidal fuzzy numbers and $x \in R$. Define:

$$x \ge 0, xA = (xa_1, xa_2, xa_3, xa_4),$$
$$x < 0, xA = (xa_4, xa_3, xa_2, xa_1),$$

$A + C = (a_1 + c_1, a_2 + c_2, a_3 + c_3, a_4 + c_4)$.

3 Proposed approach

We define minimum crisp value $\mu_{F_{min}}^{(x)}$ and a maximum crisp value $\mu_{F_{max}}^{(x)}$ to be the benchmark, where their characteristic functions $\mu_{F_{min}}^{(x)}$ and $\mu_{F_{max}}^{(x)}$ are as follow:

$$\mu_{F_{\min}}^{(x)} = \begin{cases} 1, x = F_{\min}, \\ 0, x \ne F_{\min}. \end{cases}, \quad \mu_{F_{\max}}^{(x)} = \begin{cases} 1, x = F_{\max}, \\ 0, x \ne F_{\max}. \end{cases}$$

Now assume fuzzy numbers A_i, $i = 1, \ldots, n$ on $F(R)$ are given; then, the minimum crisp value $F_{\min}$ and the maximum crisp value $F_{\max}$ are defined as:

$$F_{\min} = \inf \bigcup_{i=1}^{n} \sup(A_i), i = 1, \ldots, n$$

$$F_{\max} = \sup \bigcup_{i=1}^{n} \sup(A_i), i = 1, \ldots, n.$$

Let $A_i, A_j \in F(R)(\text{if } i \neq j)$ be two arbitrary fuzzy numbers. Define the rank of A_i, A_j by $D_{2,\frac{1}{2}}$ on $F(R)$ as:

$$\begin{cases} A_i > A_j \leftrightarrow D_{2,\frac{1}{2}}(A_i, F_{\min}) > D_{2,\frac{1}{2}}(A_j, F_{\min}) \\ A_i \sim A_j \leftrightarrow D_{2,\frac{1}{2}}(A_i, F_{\min}) = D_{2,\frac{1}{2}}(A_j, F_{\min}) \\ A_i < A_j \leftrightarrow D_{2,\frac{1}{2}}(A_i, F_{\min}) < D_{2,\frac{1}{2}}(A_j, F_{\min}) \end{cases}$$

Besides, we formulate orders $\geq$, $\leq$ as $A_i \geq A_j \leftrightarrow A_i > A_j \text{ or } A_i \sim A_j, A_i \leq A_j \leftrightarrow A_i < A_j or A_i \sim A_j$.

Moreover, we use $F_{\max}$, $F_{\min}$ indexes and for two arbitrary fuzzy numbers$A_i, A_j \in F(R)\ \{if i \neq j\}$, we define A_i, A_j; $D_{2,\frac{1}{2}}$ on $F(R)$ as:

$$\begin{cases} A_i > A_j \leftrightarrow D_{2,\frac{1}{2}}(A_i, F_{\max}) < D_{2,\frac{1}{2}}(A_j, F_{\max}), \\ A_i \sim A_j \leftrightarrow D_{2,\frac{1}{2}}(A_i, F_{\max}) = D_{2,\frac{1}{2}}(A_j, F_{\max}), \\ A_i < A_j \leftrightarrow D_{2,\frac{1}{2}}(A_i, F_{\max}) > D_{2,\frac{1}{2}}(A_j, F_{\max}). \end{cases}$$

To shed light on the mentioned approach, the following example is given.

Example 3.1: Consider the following trapezoidal numbers as follows: $A_1 = (2, 4, 4, 6)$, $A_2 = (3, 5, 5, 6)$, $A3 = (3, 4, 5, 7)$.

Their membership functions and inverse functions are shown in the Table 1. The fuzzy numbers and the minimum and maximum crisp values are illustrated in Fig. 1.

Subsequently, we have:

$$F_{\min} = \inf \bigcup_{i=1}^{3} \sup(A_{i,;1 \leq i \leq 3}) = 2,$$

$$F_{\max} = \inf \bigcup_{i=1}^{3} \sup(A_{i,;1 \leq i \leq 3}) = 7,$$

$$g_{\min}^{(x)} = \begin{cases} g_{\min}^{-(x)} = 2, \\ g_{\min}^{+(x)} = 2. \end{cases}, \quad g_{\max}^{(x)} = \begin{cases} g_{\max}^{-(x)} = 7, \\ g_{\max}^{+(x)} = 7. \end{cases}$$

Therefore, we can get that the $D_{2,\frac{1}{2}}$-distances between minimum crisp value and fuzzy numbers are 2.30, 2.71, 3.05, respectively. Thus, $A_3 > A_2 > A_1$.

Now we verify that the reasonable axioms are valid for the proposed approach.

Let S be a set of fuzzy quantities which the $D_{2,\frac{1}{2}}$-distance method is applicable, and Γ, μ be finite subset of S. When E has a higher rank than F when $D_{2,\frac{1}{2}}$-distance applied to the fuzzy quantities in Γ, we have: $E > F$ on Γ. E–F on μ and $E \geq F$ on Γ are accordingly concluded.

The following axioms show reasonable properties of the ordering approach $D_{2,\frac{1}{2}}$-distance.

1. For $(E, F) \in \Gamma^2$, $E \geq F$, $F \geq E$ on Γ, we have E–F on Γ.
2. Let $(E, F) \in (\Gamma \cap \mu)^2$. We obtain ranking order $E \geq F$ on μ if and only if $E \geq F$ on Γ.
3. For $(E, F) \in \Gamma^2$, $\inf \sup(E) > \sup \sup(F)$, we have $E \geq F$ on Γ.

3.1. For $(E, F) \in \Gamma^2$, $\inf \sup(E) > \sup \sup(F)$, we have $E > F$ on Γ.

4. For $(E, F, G) \in \Gamma^3, E \geq F \geq G$ on Γ, we have $E \geq G$ on Γ.
5. Let $E, F, E + G, F + G$ be elements of S. If $E \geq F$ on $\{E, F\}$, then $E + G \geq F + G$ on $\{E + G, E + F\}$.

5.1. Let $E, F, E + G, F + G$ be elements of S and $G = \emptyset$. If $E > F \text{on} \{E, F\}$, then $E + G \geq F + G$ on $\{E + G, E + F\}$.

Table 1 Fuzzy numbers A_1, A_2, A_3 and their membership and inverse functions

Fuzzy numbers	Membership function	Inverse function
A_1	$\mu_{A_1}(x) = \begin{cases} x-2, & 2 \leq x \leq 4 \\ \frac{6-x}{2}, & 4 \leq x \leq 6 \\ 0, & \text{OW} \end{cases}$	$g_{A_1}^{(x)} = \begin{cases} A_1^{-(x)} = x+2, \\ A_1^{+(x)} = 6-2x \end{cases}$
A_2	$\mu_{A_2}(x) = \begin{cases} x-3, & 3 \leq x \leq 5 \\ 6-x, & 5 \leq x \leq 6 \\ 0, & \text{OW} \end{cases}$	$g_{A_2}^{(x)} = \begin{cases} A_2^{-(x)} = x+3, \\ A_2^{+(x)} = 6-x \end{cases}$
A_3	$\mu_{A_3}(x) = \begin{cases} x-3, & 3 \leq x \leq 4 \\ 1, & 4 \leq x \leq 5 \\ \frac{7-x}{2}, & 5 \leq x \leq 7 \\ 0, & \text{OW} \end{cases}$	$g_{A_3}^{(x)} = \begin{cases} g_3^{-(x)} = x+3, \\ g_3^{+(x)} = 7-2x \end{cases}$

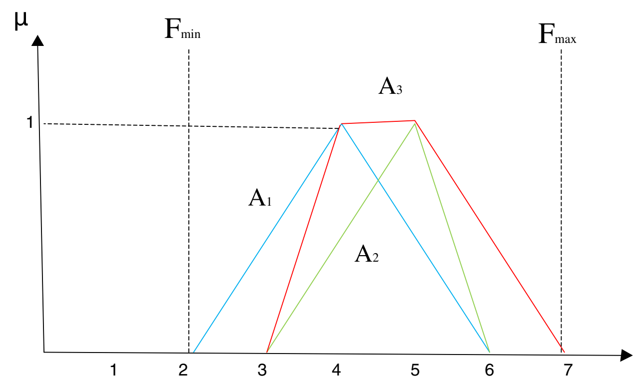

Fig. 1 Fuzzy numbers A_1, A_2, A_3, $F_{\max}$, $F_{\min}$

4 Numerical examples

In this section, the proposed method is compared with some similar examples taken from [1–15].

Example 4.1: Consider three sets of fuzzy numbers:

Set1: $A = (5, 6, 6, 10)$, $B = (5, 8, 8, 10)$, $C = (5, 9, 9, 10)$.

Set2: $A = (4, 5, 8, 9)$, $B = (4, 8, 8, 10)$, $C = (6, 8, 8, 10)$.

Set3: $A = (4, 6, 6, 7)$, $B = (4, 6, 7, 9)$, $C = (4, 5, 6, 9)$.

A comparison with other methods has been gathered in Table 2.

Table 2 A comparison between ranking methods

Authors	Fuzzy number	Set1	Set2	Set3
Proposed method		$A < B < C$	$A < B < C$	$A < C < B$
Mag [2]	A	6.21	6.58	6.04
	B	8.11	7.31	7.54
	C	9.33	8.14	6.58
Results		$A < B < C$	$A < B < C$	$A < C < B$
Sign distance method $p = 1$ [1]	A	13.1	12.78	11.42
	B	15.4	14.65	13.4
	C	17.3	15.57	12.3
Results		$A < B < C$	$A < B < C$	$A < C < B$
Sign distance method $p = 2$ [1]	A	10.1	9.88	8.1
	B	11.78	11.1	10.98
	C	12.6	11.88	9.24
Results		$A < B < C$	$A < B < C$	$A < B < C$
Asady and Zendehman [3]	A	7.11	7.01	6.51
	B	8.31	8.21	8.01
	C	10.27	8.35	6.5
Results		$A < B < C$	$A < B < C$	$A < C < B$

Example 4.2: Consider these fuzzy numbers $A_1 = (0.1, 0.2, 0.3)$, $A_2 = (0.3, 0.6, 0.5)$, $A_3 = (0.6, 0.7, 0.8)$, $B_1 = (0.01, 0.01, 0.1)$, $B_2 = (0.5, 0.6, 0.7)$, $B_3 = (0.9, 1, 1)$, $C_1 = (0.36, 0.46, 1)$, $C_2 = (0.16, 0.7, 0.8)$, $D_1 = (0.01, 0.1, 0.5, 1)$, $D_2 = (0.5, 0.7, 0.7)$, $E_1 = (0.3, 0.5, 0.7)$, $E_2 = (0.7, 0.5, 0.2)$, $E_3 = (0.3, 0.5, 0.7, 0.9)$. We rank them with some ranking methods according to Table 3.

5 Conclusions

Even though distance method is one of the commonly used methods in ranking fuzzy numbers, most of these methods do not give a satisfactory result as it is indiscriminative. In this paper, a new method for ranking fuzzy numbers based on the left and right using distance method and α-cut has been presented. Moreover, some examples are given to illustrate that the proposed approach has the distinct characteristics. It is obvious that the new method gives an intuitively discriminate result than the existing methods. Additionally, the proposed approach can provide decision makers with a new alternative to rank fuzzy numbers. It enriches the theories and methods for ranking fuzzy numbers.

Table 3 The results of ranking methods

Proposed method	$A_1 < A_2 < A_3$	$B_1 < B_2 < B_3$	$C_1 < C_2$	$D_1 < D_2$	$E_1 < E_2 < E_3$
Yager [15]	$A_1 < A_2 < A_3$	$B_1 < B_2 < B_3$	$C_1 > C_2$	$D_1 < D_2$	$E_1 < E_2 < E_3$
Kerre [14]	$A_1 \sim A_2 < A_3$	$B_2 < B_1 < B_3$	$C_1 > C_2$	$D_1 > D_2$	$E_1 < E_2 < E_3$
Chang [7]	$A_1 < A_2 < A_3$	$B_1 < B_2 < B_3$	$C_1 < C_2$	$D_1 > D_2$	$E_1 < E_2 < E_3$
Bass and Kwakernaak [5]	$A_1 \sim A_2 < A_3$	$B_1 \sim B_2 < B_3$	$C_2 < C_1$	$D_1 < D_2$	$E_1 < E_2 < E_3$
Adamo [4] $\alpha = 0.9$	$A_1 < A_2 < A_3$	$B_1 < B_2 < B_3$	$C_1 < C_2$	$D_1 < D_2$	$E_1 > E_2 > E_3$
Adamo [4] $\alpha = 0.5$	$A_1 < A_2 < A_3$	$B_1 < B_2 < B_3$	$C_1 > C_2$	$D_1 > D_2$	$E_1 > E_2 > E_3$
Baldwin and Guild (I.P) [6]	$A_1 \sim A_2 < A_3$	$B_1 \sim B_2 < B_3$	$C_1 > C_2$	$D_1 < D_2$	$E_1 > E_2 > E_3$
Baldwin and Guild [6] (g.)	$A_1 \sim A_2 < A_3$	$B_1 \sim B_2 < B_3$	$C_1 > C_2$	$D_1 < D_2$	$E_1 > E_2 > E_3$
Baldwin and Guild (R.a.) [6]	$A_1 \sim A_2 < A_3$	$B_1 \sim B_2 < B_3$	$C_1 > C_2$	$D_1 < D_2$	$E_1 > E_2 > E_3$
Jain [10, 11]	$A_1 < A_2 < A_3$	$B_1 < B_2 < B_3$	$C_1 < C_2$	$D_1 > D_2$	$E_1 > E_2 > E_3$
$K = 1$					
Duboa-Prad [8] (PD)	$B_1 \sim B_2 < B_3$	$B_1 \sim B_2 < B_3$	$C_1 < C_2$	$D_1 < D_2$	$E_1 > E_2 > E_3$
Kim and Park [12] $K = 0.5$	$A_1 < A_2 < A_3$	$B_1 < B_2 < B_3$	$C_1 < C_2$	$D_1 < D_2$	$E_1 > E_2 > E_3$
Fortemps and Roubens [9]	$A_1 < A_2 < A_3$	$B_1 < B_2 < B_3$	$C_1 > C_2$	$D_1 < D_2$	$E_1 > E_2 > E_3$
Lio and Wang [13] $\lambda = 0.5$	$A_1 < A_2 < A_3$	$B_1 < B_2 < B_3$	$C_1 \sim C_2$	$D_1 < D_2$	$E_1 > E_2 > E_3$

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