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ORIGINAL ARTICLE

# Design of $H_\infty$ control of neural networks with time-varying delays

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**Abstract** This paper deals with the $H_\infty$ control problem of neural networks with time-varying delays. The system under consideration is subject to time-varying delays and various activation functions. Based on constructing some suitable Lyapunov–Krasovskii functionals, we establish new sufficient conditions for $H_\infty$ control for two cases of time-varying delays: (1) the delays are differentiable and have an upper bound of the delay-derivatives and (2) the delays are bounded but not necessary to be differentiable. The derived conditions are formulated in terms of linear matrix inequalities, which allow simultaneous computation of two bounds that characterize the exponential stability rate of the solution. Numerical examples are given to illustrate the effectiveness of our results.

**Key words** Neural networks · $H_\infty$ control · Stabilization · Time-delay systems · Lyapunov function · Linear matrix inequalities

## 1 Introduction

In the area of control, signal processing, pattern recognition and image processing, delayed neural networks have many useful applications. Some of these applications require that the equilibrium points of the designed network be stable. In both biological and artificial neural systems, time delays due to integration and communication are ubiquitous and often become a source of instability. The time delays in electronic neural networks are usually time-varying, and sometimes vary violently with respect to time due to the finite switching speed of amplifiers and faults in the electrical circuitry. Therefore, stability and control of delayed neural networks is a very important issue, and various stability criteria have been reported in the literature (see, for example, [1–6]). In conducting a periodicity or stability analysis of a neural network, the conditions to be imposed on the neural network are determined by the characteristics of various activation functions as well as network parameters. When neural networks are designed for problem solving, it is desirable for their activation functions to be general. To facilitate the design of neural networks, it is important that the neural networks with various activation functions and time-varying delays are studied. On the other hand, the problem of $H_\infty$ control of dynamical time-delay systems are of practical and theoretical interest due to their useful applications in image processing, especially in classification of patterns, associative memories and other areas (see, for example, [7–14]). For the $H_\infty$ control problem, appropriate methods for linear time-delay systems usually make use of the Lyapunov functional approach, whereby the $H_\infty$ conditions are obtained via solving either matrix inequalities or algebraic Riccati-type equations [15–17]. Regarding $H_\infty$ control of neural networks, the papers [18–20] proposed a state feedback $H_\infty$ control law for the asymptotic stabilization of neural networks with constant time delays. To the best of our knowledge, the $H_\infty$ control problem of neural networks with time-varying delays has not been fully studied yet, which are important in both theories and applications. This motivates our research.

In this paper, we investigate the $H_\infty$ control with exponential stability for neural networks with time-varying

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delays. The novel features here are that the time-varying delay is present in the observation output with various activation functions, and the controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. Based on constructing a set of augmented Lyapunov–Krasovskii functionals, a $H_\infty$ controller is designed to achieve exponential stabilization of the neural networks for two cases of time-varying delays: the delays are differentiable and have an upper bound of the delay-derivatives and the delays are bounded but not necessary to be differentiable. The conditions are obtained in terms of LMIs, which allow simultaneous computation of two bounds that characterize the exponential stability rate of the solution and can be easily determined by utilizing MATLABs LMI Control Toolbox.

This paper is organized as follows. Section 2 presents notations, definitions and auxiliary propositions required for the proof of the main results. $H_\infty$ control design for delayed neural networks for two cases of time-varying delays: (1) the delays are differentiable and have an upper bound of the delay-derivatives and (2) the delays are bounded but not necessary to be differentiable, and numerical examples of the results are presented in Sects. 3 and 4, respectively.

## 2 Preliminaries

The following notations will be used throughout this paper. $R^+$ denotes the set of all real non-negative numbers; $R^n$ denotes the $n$-dimensional space with the scalar product $\langle\cdot,\cdot\rangle$ and the vector norm $\|\cdot\|$; $R^{n\times r}$ denotes the space of all matrices of $(n \times r)$-dimension. Matrix $A$ is symmetric if $A = A^T$, where $A^T$ denotes the transpose of $A$. $I_n$ denotes the identity matrix in $R^n$; $\lambda(A)$ denotes the set of all eigenvalues of $A$; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $C([a,b], R^n)$ denotes the set of all $R^n$-valued continuous functions on $[a,b]$; $L_2([0,\infty], R^r)$ denotes the set of all square-integrable $R^r$-valued functions on $[0,\infty]$. Matrix $A$ is semi-positive definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$ for all $x \in R^n$; $A$ is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A \geq B$ means $A - B \geq 0$. Let us denote $x_t := \{x(t+s), s \in [-h, 0]\}$ the segment of the trajectory $x(t)$ with the norm $\|x_t\| = \sup_{t\in[-h,0]} \|x(t+s)\|$.

Consider the following delayed neural networks with control input and observation output

$$
\begin{aligned}
\dot{x}(t) &= -Ax(t) + W_0 f(x(t)) + W_1 g(x(t-\tau_1(t))) \\
&\quad + Bu(t) + B_1 w(t), \\
z(t) &= Cx(t) + W_2 h(x(t-\tau_2(t))) + Du(t), \\
x(t) &= \phi(t), \quad t \in [-\tau, 0],
\end{aligned}
\tag{2.1}
$$

where $\tau = \max\{\tau_1, \tau_2\}, x(t) \in R^n$ is the state vector of the neural networks; $u(t) \in L_2([0,s), R^m), s > 0, m \leq n$, is the control input; $w(t) \in L_2([0,\infty), R^r), r \leq n$, is the uncertain input of the neural networks; $z(t) \in R^l, l \leq n$, is the observation output; $n$ is the number of neurals; $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T$, $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T$, $h(x(t)) = [h_1(x_1(t)), h_2(x_2(t)), \ldots, h_n(x_n(t))]^T$ are the neural activation functions; the diagonal matrix $A = \mathrm{diag}(a_1, a_2, \ldots, a_n), a_i > 0$, represents the self-feedback term; the matrices $W_0, W_1 \in R^{n\times n}, W_2 \in R^{l\times n}$ denote, respectively, the connection weights; $B \in R^{n\times m}, D \in R^{l\times m}$ denote the control input matrices; $B_1 \in R^{n\times r}$ denotes the uncertain/perturbation input matrix; $C \in R^{l\times n}$ denotes the observation output matrix; the initial functions $\phi(t) \in C([-\tau,0], R^n)$ with the uniform norm $||\phi|| = \max_{t\in[-\tau,0]} \|\phi(t)\|$; the time-varying delay functions $\tau_1(t)$, $\tau_2(t)$ satisfy either (H1) or (H2):

$$
\begin{aligned}
&(\mathrm{H1}) \quad 0 \leq \tau_1(t) \leq \tau_1, \quad \dot{\tau}_1(t) \leq \delta_1 < 1, \quad \forall t \in R^+; \\
&\qquad\quad\;\; 0 \leq \tau_2(t) \leq \tau_2, \quad \dot{\tau}_2(t) \leq \delta_2 < 1, \quad \forall t \in R^+, \\
&(\mathrm{H2}) \quad 0 \leq \tau_1(t) \leq \tau_1, \quad 0 \leq \tau_2(t) \leq \tau_2, \quad \forall t \in R^+.
\end{aligned}
$$

In this paper, we consider various activation functions $f(x)$, $g(x)$, $h(x)$, $f(0) = h(0) = g(0) = 0$, which are globally Lipschitzian with the Lipschitz constants $\xi_i > 0$, $\sigma_i, > 0$, $\eta_i > 0$ such that

$$
\begin{aligned}
&|f_i(x_1) - f_i(x_2)| \leq \xi_i |x_1 - x_2|, \quad i = 1,2,\ldots,n, \quad \forall x_1, x_2 \in R \\
&|g_i(x_1) - g_i(x_2)| \leq \sigma_i |x_1 - x_2|, \quad i = 1,2,\ldots,n, \quad \forall x_1, x_2 \in R \\
&|h_i(x_1) - h_i(x_2)| \leq \eta_i |x_1 - x_2|, \quad i = 1,2,\ldots,n, \quad \forall x_1, x_2 \in R.
\end{aligned}
\tag{2.2}
$$

**Definition 2.1** Given $\beta > 0$. The system (2.1), where $w(t) = 0$, is $\beta$-stabilizable if there is a feedback control law $u(t) = Kx(t)$ such that every solution $x(t, \phi)$ of the closed-loop system

$$
\begin{aligned}
\dot{x}(t) &= -(A - BK)x(t) + W_0 f(x(t)) + W_1 g(x(t-\tau_1(t))), \\
x(t) &= \phi(t), \quad t \in [-\tau, 0],
\end{aligned}
\tag{2.3}
$$

satisfies

$$
\exists N > 0: \quad \|x(t,\phi)\| \leq N\|\phi\| e^{-\beta t}, \quad \forall t \in R^+.
$$

**Definition 2.2** Let the numbers $\beta > 0$, $\gamma > 0$ be given. The $H_\infty$ control problem for system (2.1) has a solution if there exists a memoryless state feedback controller $u(t) = Kx(t)$ satisfying the following two requirements:

1. The system (2.1) is $\beta$-stabilizable.
2. There is a number $c_0 > 0$ such that

$$
\sup \frac{\int_0^\infty \|z(t)\|^2 \mathrm{d}t}{c_0\|\phi\|^2 + \int_0^\infty \|w(t)\|^2 \mathrm{d}t} \leq \gamma,
\tag{2.4}
$$

where the supremum is taken over all $\phi(t) \in C([-\tau,0], R^n)$ and the nonzero uncertainty $w(t) \in$

$L_2([0,\infty), R^r)$. In this case, we say that the feedback $H_\infty$ control $u(t) = Kx(t)$ exponentially stabilizes the system.

The following lemmas are essential for the proofs in the subsequent section.

**Proposition 2.1** *Let P, Q be matrices of appropriate dimensions and Q is symmetric positive definite. Then*

$$2\langle Py, x\rangle - \langle Qy, y\rangle \le \langle PQ^{-1}P^T x, x\rangle, \quad \forall (x, y).$$

The proof of the above proposition is easily derived from completing the square:

$$0 \le \langle Q(y - Q^{-1}P^T x), y - Q^{-1}P^T x\rangle.$$

**Proposition 2.2** (Schur complement lemma [21]) *Given constant symmetric matrices X, Y and Z, where Y > 0. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Yx \end{pmatrix} < 0.$$

**Proposition 2.3** (Razumikhin stability theorem [22]) *Consider the time-delay system $\dot{x}(t) = f(t, x_t), x(t) = \phi(t)$, $t \in [-h, 0]$. Assume that $u, v, w : R^+ \to R^+$ are nondecreasing, and u(s), v(s) are positive for $s \ge 0$, v(0) = u(0) = 0, and q > 1. If there is a function $V(t, x) : R^+ \times R^n \to R^+$ such that*

1. $u(\|x\|) \le V(t, x) \le v(\|x\|),\ t \in R^+, x \in R^n$
2. $\dot{V}(t, x(t)) \le -w(\|x(t)\|)$ if $V(t+s, x(t+s)) \le qV(t, x(t)), \forall s \in [-h, 0], t \in R^+,$

*then the zero solution of system is asymptotically stable.*

## 3 Main results

As in [9, 11] we assume that

$$D^T[C \quad W_2] = 0, \quad D^T D = I_m. \tag{3.1}$$

Let us denote

$$\mu_1 = (1-\delta_1)^{-1},\ \mu_2 = (1-\delta_2)^{-1},$$
$$W_2^I = W_2 W_2^T + I_l,\ w = \lambda_{\max}(W_2^I),$$
$$T_1(A,P) = -0.5(A^T P + PA) + \beta P + C^T W_2^I C + \mu_2 w e^{2\beta\tau_2} HH,$$
$$T_2(A,P) = -0.5(A^T P + PA) + \beta P + \mu_1 GG + 2FD_1 + FD_2 F,$$
$$F = \mathrm{diag}(\xi_1, \xi_2, \ldots, \xi_n),\ G = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n),$$
$$H = \mathrm{diag}(\eta_1, \eta_2, \ldots, \eta_n),$$
$$\alpha_1 = \lambda_{\min}(P),\ \sigma^2 = \max\{\sigma_i^2, i = 1, 2\ldots, n\},$$
$$\eta^2 = \max\{\eta_i^2, i = 1, 2\ldots, n\},$$
$$\alpha_2 = \lambda_{\max}(P) + \sigma^2\mu_1\tau_1 + \lambda_{\max}(W_2^I)\eta^2\mu_2\tau_2 e^{2\beta\tau_2},\ N = \sqrt{\frac{\alpha_2}{\alpha_1}}.$$

**Theorem 3.1** *Assume the condition (H1). Given $\beta > 0$, the $H_\infty$ control of system (2.1) has a solution if there exist a symmetric positive definite matrix $P \in R^{n\times n}$, and two diagonal positive definite matrices $D_i \in R^{n\times n}, i = 1, 2$, such that the following LMIs hold:*

$$\begin{pmatrix} T_1(A,P) & PB & PW_1 & PB_1 \\ B^T P & -\frac{4}{5} I_n & 0 & 0 \\ W_1^T P & 0 & -e^{-2\beta\tau_1} I_n & 0 \\ B_1^T P & 0 & 0 & -\gamma I_n \end{pmatrix} < 0, \tag{3.2}$$

$$\begin{pmatrix} T_2(A,P) & PW_0 + D_1 \\ W_0^T P + D_1 & -D_2 \end{pmatrix} < 0. \tag{3.3}$$

*The feedback $H_\infty$ control law is defined by*

$$u(t) = \frac{1}{2} B^T P x(t), \quad t \in R^+. \tag{3.4}$$

*Proof* Consider the following time-varying Lyapunov–Krasovskii functional for the closed-loop system (2.3):

$$V(t, x_t) = V_1 + V_2 + V_3$$

where

$$V_1(t, x_t) = \langle Px(t), x(t)\rangle$$
$$V_2(t, x_t) = \mu_1 \int_{t-\tau_1(t)}^{t} e^{2\beta(s-t)} \langle g(x(s)), g(x(s))\rangle \mathrm{d}s$$
$$V_3(t, x_t) = \mu_2 e^{2\beta\tau_2} \int_{t-\tau_2(t)}^{t} e^{2\beta(s-t)} \langle W_2^I h(x(s)), h(x(s))\rangle \mathrm{d}s.$$

It is easy to verify that

$$\alpha_1 ||x(t)||^2 \le V(t, x_t) \le \alpha_2 ||x_t||^2, \quad t \in R^+. \tag{3.5}$$

Taking the time derivative of $V(\cdot)$ in $t$ along the solution we obtain

$$\begin{aligned}
\dot{V}(t, x_t) &= 2\langle P\dot{x}(t), x(t)\rangle + \mu_1 \langle g(x(t)), g(x(t))\rangle \\
&\quad - \mu_1 e^{-2\beta\tau_1}(1 - \dot{\tau}_1(t))\langle g(x(t-\tau_1(t))), g(x(t-\tau_1(t)))\rangle \\
&\quad - 2\beta V_2(t, x_t) + \mu_2 e^{2\beta\tau_2}\langle W_2^I h(x(t)), h(x(t))\rangle \\
&\quad - \mu_2(1 - \dot{\tau}_2(t))\langle W_2^I h(x(t-\tau_2(t))), h(x(t-\tau_2(t)))\rangle \\
&\quad - 2\beta V_3(t, x_t) \\
&\le \langle -2PAx(t), x(t)\rangle + 2\langle PW_0 f(x(t), x(t)\rangle \\
&\quad + 2\langle PW_1 g(x(t-\tau_1(t)), x(t)\rangle \\
&\quad + \langle PBB^T Px(t), x(t)\rangle + 2\langle PB_1 w(t), x(t)\rangle \\
&\quad + \mu_1\langle g(x(t)), g(x(t))\rangle + \mu_2 e^{2\beta\tau_2}\langle W_2^I h(x(t)), h(x(t))\rangle \\
&\quad - e^{-2\beta\tau_1}\langle g(x(t-\tau_1(t)), g(x(t-\tau_1(t)))\rangle - 2\beta V_2(t, x_t) \\
&\quad - \langle W_2^I h(x(t-\tau_2(t)), h(x(t-\tau_2(t)))\rangle - 2\beta V_3(t, x_t).
\end{aligned}$$

Using Proposition 2.1 for the estimation

$$\begin{aligned}
&2\langle PW_1 g(\cdot), x(t)\rangle - e^{-2\beta\tau_1}\langle g(\cdot), g(\cdot)\rangle \\
&\quad \le e^{2\beta\tau_1}\langle PW_1 W_1^T Px(t), x(t)\rangle,
\end{aligned}$$

we have

$$\begin{aligned}\dot V(t,x_t)+2\beta V(t,x_t)&\le\langle -2PAx(t),x(t)\rangle+2\beta\langle Px(t),x(t)\rangle\\&+\langle PBB^TPx(t),x(t)\rangle+2\langle PB_1w(t),x(t)\rangle\\&+e^{2\beta\tau_1}\langle PW_1W_1^TPx(t),x(t)\rangle+2\langle PW_0f(x(t),x(t)\rangle\\&+\mu_1\langle g(x(t)),g(x(t))\rangle+\mu_2e^{2\beta\tau_2}\langle W_2^Ih(x(t)),h(x(t))\rangle\\&-\langle W_2^Ih(x(t-\tau_2(t)),h(x(t-\tau_2(t))\rangle.\end{aligned}\tag{3.6}$$

By adding and substituting into the right-hand side of the inequality (3.6) four items

$$\langle C^TW_2^ICx(t),x(t)\rangle,\quad \frac{1}{\gamma}\langle PB_1B_1^TPx(t),x(t)\rangle,$$
$$2\langle D_1f(x(t)),x(t)\rangle,\quad \langle D_2f(x(t)),f(x(t))\rangle,$$

and using the condition (2.2) and the diagonal matrices $G>0$, $H>0$, $F>0$ for the following estimations

$$\begin{aligned}-\langle D_1f(x(t)),x(t)\rangle&\le\langle FD_1x(t),x(t)\rangle,\\\langle D_2f(x(t)),f(x(t))\rangle&\le\langle FD_2Fx(t),x(t)\rangle,\\\langle g(x(t)),g(x(t))\rangle&\le\langle GGx(t),x(t)\rangle,\\\langle W_2^Ih(x(t)),h(x(t))\rangle&\le w\langle HHx(t),x(t)\rangle,\end{aligned}$$

we have

$$\begin{aligned}\dot V(t,x_t)+2\beta V(t,x_t)&\le\langle(-2A^TP+2\beta P+C^TW_2^IC+2FD_1\\&+\mu_1GG+FD_2F+\mu_2we^{2\beta\tau_2}HH)x(t),x(t)\rangle\\&+\frac{1}{\gamma}\langle PB_1B_1^TPx(t),x(t)\rangle+e^{2\beta\tau_1}\langle PW_1W_1^TPx(t),x(t)\rangle\\&+2\langle D_1f(x(t)),x(t)\rangle-\langle D_2f(x(t)),f(x(t))\rangle\\&+2\langle PB_1w(t),x(t)\rangle\\&+\langle PBB^TPx(t),x(t)\rangle+2\langle PB_1w(t),x(t)\rangle\\&+2\langle PW_0f(x(t),x(t)\rangle\\&-\langle C^TW_2^ICx(t),x(t)\rangle-\langle W_2^Ih(x(t-\tau_2(t)),h(x(t-\tau_2(t))\rangle\\&-\frac{1}{\gamma}\langle PB_1B_1^TPx(t),x(t)\rangle.\end{aligned}$$

Then, using the Schur complement lemma and Proposition 2.2, we obtain

$$\begin{aligned}\dot V(t,x_t)+2\beta V(t,x_t)&\le\langle(-2A^TP+2\beta P+C^TW_2^IC)x(t),x(t)\rangle\\&+\frac{5}{4}\langle PBB^TPx(t),x(t)\rangle+2\langle PB_1w(t),x(t)\rangle\\&+\langle(\mu_2we^{2\beta\tau_2}HH+2FD_1+\mu_1GG+FD_2F)x(t),x(t)\rangle\\&+2\langle(PW_0+D_1)f(x(t)),x(t)\rangle-\langle D_2f(x(t)),f(x(t))\rangle\\&+\frac{1}{\gamma}\langle PB_1B_1^TPx(t),x(t)\rangle+e^{2\beta\tau_1}\langle PW_1W_1^TPx(t),x(t)\rangle\\&+2\langle PB_1w(t),x(t)\rangle-\frac{1}{4}\langle PBB^TPx(t),x(t)\rangle\\&-\langle C^TW_2^ICx(t),x(t)\rangle-\frac{1}{\gamma}\langle PB_1B_1^TPx(t),x(t)\rangle\\&-\langle W_2^Ih(x(t-\tau_2(t)),h(x(t-\tau_2(t))\rangle.\end{aligned}$$

Therefore, we obtain

$$\begin{aligned}\dot V(t,x_t)+2\beta V(t,x_t)&\le\langle Nx(t),x(t)\rangle+\langle\mathcal{M}z(t),z(t)\rangle\\&+2\langle PB_1w(t),x(t)\rangle-\langle C^TW_2^ICx(t),x(t)\rangle\\&-\langle W_2^Ih(x(t-\tau_2(t)),h(x(t-\tau_2(t))\rangle\\&-\frac{1}{4}\langle PBB^TPx(t),x(t)\rangle-\frac{1}{\gamma}\langle PB_1B_1^TPx(t),x(t)\rangle.\end{aligned}\tag{3.7}$$

where $z(t) = [x(t), f(x(t))]$ and

$$N=T_1(A,P)+e^{2\beta\tau_1}PW_1W_1^TP+\frac{5}{4}PBB^TP+\frac{1}{\gamma}PB_1B_1^TP,$$
$$\mathcal{M}=\begin{pmatrix}T_2(A,P)&PW_0+D_1\\W_0^TP+D_1&-D_2\end{pmatrix}.$$

Letting $w(t)=0$, and noting that

$$\frac{1}{4}\langle PBB^TPx(t),x(t)\rangle\ge0,\quad\langle C^TW_2^ICx(t),x(t)\rangle\ge0,$$
$$\langle W_2^Ih(x(t-\tau_2),h(x(t-\tau_2)\rangle\ge0,\quad\langle PB_1B_1^TPx(t),x(t)\rangle\ge0,$$

and that $N<0$ is, by Schur complement lemma, equivalent to $\mathcal{N}<0$, where

$$\mathcal{N}=\begin{pmatrix}T_1(A,P)&PB&PW_1&PB_1\\B^TP&-\frac{4}{5}I_n&0&0\\W_1^TP&0&-e^{-2\beta\tau_1}I_n&0\\B_1^TP&0&0&-\gamma I_n\end{pmatrix},$$

we obtain from (3.2), (3.3) that

$$\dot V(t,x_t)+2\beta V(t,x_t)\le0.\tag{3.8}$$

Therefore, from differential inequality (3.8), it follows that

$$V(t,x_t)\le V(0,x_0)e^{-2\beta t},\quad\forall t\ge0.$$

Using the condition (3.5), we have

$$\|x(t,\phi)\|\le N\|\phi\|e^{-\beta t},\quad\forall t\ge0.$$

To complete the proof of the theorem, it remains to show the $\gamma$-optimal level condition (2.4). For this, we consider the relation

$$\begin{aligned}\int_0^s[\|z(t)\|^2-\gamma\|w(t)\|^2]\mathrm{d}t&=\int_0^s\left[\|z(t)\|^2-\gamma\|w(t)\|^2+\dot V(t,x_t)\right]\mathrm{d}t\\&-\int_0^s\dot V(t,x_t)\mathrm{d}t.\end{aligned}$$

Since $V(t, x_t)\ge0$, $t\ge0$, we have

$$-\int_0^s \dot{V}(t,x_t)\mathrm{d}t = V(0,x_0) - V(s,x_s) \le V(0,x_0), \quad \forall s \ge 0,$$

and hence

$$\int_0^s [\|z(t)\|^2 - \gamma\|w(t)\|^2]\mathrm{d}t \le \int_0^s \Big[\|z(t)\|^2 - \gamma\|w(t)\|^2 + \dot{V}(t,x_t)\Big]\mathrm{d}t + V(0,x_0). \tag{3.9}$$

Observe that the value of $\|z(t)\|^2$ is defined due to (2.1) and (3.1) as

$$\begin{aligned}\|z(t)\|^2 &= \langle C^T Cx(t), x(t)\rangle + 2\langle C^T W_2 h(x(t-\tau_2(t))), x(t)\rangle \\ &\quad + \langle W_2^T W_2 h(x(t-\tau_2(t))), h(x(t-\tau_2(t)))\rangle \\ &\quad + \frac{1}{4}\langle PBB^T Px(t), x(t)\rangle\end{aligned}$$

Using Proposition 2.1, we have

$$2\langle C^T W_2 h(x(t-\tau_2(t))), x(t)\rangle \le \langle C^T W_2 W_2^T Cx(t), x(t)\rangle + \|h(x(t-\tau_2(t)))\|^2,$$

then

$$\begin{aligned}\|z(t)\|^2 &\le \langle C^T W_2^I Cx(t), x(t)\rangle \\ &\quad + \langle W_2^I h(x(t-\tau_2(t))), h(x(t-\tau_2(t)))\rangle \\ &\quad + \frac{1}{4}\langle PBB^T Px(t), x(t)\rangle\end{aligned} \tag{3.10}$$

Submitting the estimation of $\dot{V}(t,x_t)$ and $\|z(t)\|^2$ respectively defined from (3.7) and (3.10) into (3.9), we obtain

$$\begin{aligned}&\int_0^s [||z(t)||^2 - \gamma||w(t)||^2]\mathrm{d}t \\ &\le \int_0^s \Big[-\frac{1}{\gamma}\langle PB_1B_1^T Pw(t),\, x(t)\rangle \\ &\quad + 2\langle PB_1 w(t),\, x(t)\rangle - \gamma||w(t)||^2]\mathrm{d}t + V(0,x_0).\end{aligned}$$

Applying Proposition 2.1 for the estimation

$$\langle PB_1 w(t), x(t)\rangle - \gamma\|w(t)\|^2 \le \frac{1}{\gamma}\langle PB_1B_1^T Pw(t), x(t)\rangle,$$

we have

$$\int_0^s [\|z(t)\|^2 - \gamma\|w(t)\|^2]\mathrm{d}t \le V(0,x_0) \le \alpha_2\|\phi\|^2,$$

equivalently,

$$\int_0^s \|z(t)\|^2\mathrm{d}t \le \gamma\int_0^s \|w(t)\|^2\mathrm{d}t + \alpha_2\|\phi\|^2.$$

Letting $s \to \infty$, and setting $c_0 = \frac{\alpha_2}{\gamma} > 0$, we obtain that

$$\frac{\int_0^\infty \|z(t)\|^2\mathrm{d}t}{c_0\|\phi\|^2 + \int_0^\infty \|w(t)\|^2\mathrm{d}t} \le \gamma,$$

for all nonzero $w(t) \in L_2([0,\infty), R^r)$, $\phi(t) \in C([-h, 0], R^n)$. This completes the proof of the theorem.

*Remark 3.1* Theorem 3.1 provides sufficient conditions for solving the $H_\infty$ control problem of the Hopfield delayed neural network (2.1) in terms of LMIs, which allows for an arbitrary prescribed stability degree $\beta$. The LMI feasibility will depend on parameters of the system under consideration as well as some upper limits for the Lipschitz constants and the time delays. The feasibility of the LMIs (3.2)–(3.3) can be tested by the reliable and efficient MATLABs LMI Control Toolbox [23].

In the sequel, the $H_\infty$ control problem for the system (2.1) will be solved further with no restriction on the derivative of the time-varying delay function. For this, we set

$$\begin{aligned}&\tau = \max\{\tau_1,\tau_2\}, \quad W_2^I = W_2W_2^T + I_l, \\ &T_1(A,P) = -0.5[A^T(P+e^{-\tau}I_n) + (P+e^{-\tau}I_n)A] \\ &\qquad + \sigma^2(P+e^{-\tau}I) + C^TW_2^IC, \\ &T_2(A,P) = -0.5[A^T(P+e^{-\tau}I_n) + (P+e^{-\tau}I_n)A] \\ &\qquad + \eta^2we^{\tau}(P+e^{-\tau}I) + 2FD_1 + FD_2F, \\ &F = \mathrm{diag}(\xi_1,\xi_2,\ldots,\xi_n), \quad p = \lambda_{\max}(P), \quad w = \lambda_{\max}(W_2^I), \\ &\sigma^2 = \max\{\sigma_i^2, i = 1,2\ldots,n\}, \quad \eta^2 = \max\{\eta_i^2, i = 1,2\ldots,n\}.\end{aligned}$$

**Theorem 3.2** *Assume the condition (H2). The $H_\infty$ control of system (2.1) has a solution if there exist a symmetric positive definite matrix $P$ and two diagonal positive definite matrices $D_i \in R^{n\times n}, i = 1,2$, such that the following LMIs hold:*

$$\begin{pmatrix} T_1(A,P) & PB+e^{-\tau}B & PW+e^{-\tau}W_1 & PB_1+e^{-\tau}B_1 \\ B^TP+e^{-\tau}B^T & -\frac{4}{5}I_n & 0 & 0 \\ W_1^TP+e^{-\tau}W_1^T & 0 & -e^{-\tau}I_n & 0 \\ B_1^TP+e^{-\tau}B_1^T & 0 & 0 & -\gamma I_n \end{pmatrix} < 0, \tag{3.11}$$

$$\begin{pmatrix} T_2(A,P) & PW_0 + e^{-\tau}W_0 + D_1 \\ W_0^TP + e^{-\tau}W_0^T + D_1 & -D_2 \end{pmatrix} < 0. \tag{3.12}$$

*The feedback $H_\infty$ control law is defined by*

$$u(t) = \frac{1}{2}B^T(P+e^{-\tau}I_n)x(t), \quad t \ge 0. \tag{3.13}$$

*Proof* Let us set $P_\tau = P + e^{-\tau}\ I_n$. We consider the following Lyapunov–Krasovskii functional

$$V(x(t)) = \langle P_\tau x(t), x(t)\rangle, \tag{3.14}$$

Taking the time derivative of $V(\cdot)$ in $t$ along the solution and using the feedback control (3.13), we obtain

$$\begin{aligned}\dot V(x(t)) &= 2\langle P_\tau \dot x(t), x(t)\rangle\\ &= -2\langle P_\tau Ax(t), x(t)\rangle + \langle P_\tau BB^T P_\tau x(t), x(t)\rangle\\ &\quad + 2\langle P_\tau W_0 f(x(t)), x(t)\rangle + 2\langle P_\tau W_1 g(x(t-\tau_1(t))), x(t)\rangle\\ &\quad + 2\langle P_\tau B_1 w(t), x(t)\rangle.\end{aligned}$$

Using Proposition 2.1, we have

$$\begin{aligned}2\langle P_\tau W_1 g(x(t-\tau_1(t))), x(t)\rangle &\le e^\tau \langle P_\tau W_1 W_1^T P_\tau x(t), x(t)\rangle\\ &\quad + e^{-\tau}\|g(x(t-\tau_1(t))\|^2.\end{aligned}$$

Then, we have

$$\begin{aligned}\dot V(x(t)) &= -2\langle P_\tau Ax(t), x(t)\rangle + \langle P_\tau BB^T P_\tau x(t), x(t)\rangle\\ &\quad + e^\tau\langle P_\tau W_1 W_1^T P_\tau x(t), x(t)\rangle + e^{-\tau}\|g(x(t-\tau_1(t)))\|^2\\ &\quad + 2\langle P_\tau W_0 f(x(t)), x(t)\rangle + 2\langle P_\tau B_1 w(t), x(t)\rangle.\end{aligned} \tag{3.15}$$

By adding and substituting into the right-hand side of the inequality (3.15) five items

$$\langle C^T W_2^I Cx(t), x(t)\rangle,\ \frac{1}{\gamma}\langle PB_1B_1^T Px(t), x(t)\rangle,\ 2\langle D_1 f(x(t)), x(t)\rangle,$$
$$\langle D_2 f(x(t)), f(x(t))\rangle,\ \langle W_2^I h(x(t-\tau_2(t))),\ h(x(t-\tau_2(t)))\rangle,$$

and using the condition (2.2) and the diagonal matrices $D_1 > 0$, $D_2 > 0$, $H > 0$, $F > 0$ for the following estimations

$$\begin{aligned}&-\langle D_1 f(x(t)), x(t)\rangle \le \langle FD_1 x(t), x(t)\rangle,\\ &\langle D_2 f(x(t)), f(x(t))\rangle \le \langle FD_2 Fx(t), x(t)\rangle,\\ &\langle g(x(t-\tau_1(t))), g(x(t-\tau_1(t)))\rangle\\ &\quad \le \sigma^2\langle x(t-\tau_1(t)), x(t-\tau_1(t))\rangle,\\ &\langle W_2^I h(x(t-\tau_2(t))), h(x(t-\tau_2)))\rangle\\ &\quad \le \eta^2 w\langle x(t-\tau_2(t)), x(t-\tau_2(t)))\rangle,\end{aligned}$$

we have

$$\begin{aligned}\dot V(x(t)) &\le \left\langle\left(-2P_\tau A + \frac{5}{4}P_\tau BB^T P_\tau + \frac{1}{\gamma}P_\tau B_1 B_1^T P_\tau\right)x(t), x(t)\right\rangle\\ &\quad + \langle (C^T W_2^I C + 2FD_1 + FD_2 D)x(t), x(t)\rangle\\ &\quad + 2\langle D_1 f(x(t)), x(t)\rangle + 2\langle P_\tau W_0 f(x(t)), x(t)\rangle\\ &\quad - \langle D_2 f(x(t)), f(x(t))\rangle\\ &\quad + e^\tau\langle P_\tau W_1 W_1^T P_\tau x(t), x(t)\rangle + e^{-\tau}\sigma^2\|x(t-\tau_1(t))\|^2\\ &\quad + \eta^2 w\langle x(t-\tau_2(t)), x(t-\tau_2(t))\rangle - \frac{1}{\gamma}\langle P_\tau B_1 B_1^T P_\tau x(t), x(t)\rangle\\ &\quad - \frac{1}{4}\langle P_\tau B_1 B_1^T P_\tau x(t), x(t)\rangle - \langle C^T W_2^I Cx(t), x(t)\rangle\\ &\quad - \langle W_2^I(h(x(t-\tau_2(t))), h(x(t-\tau_2(t)))\rangle + 2\langle P_\tau B_1 w(t), x(t)\rangle.\end{aligned} \tag{3.16}$$

In the light of the Razumikhin theorem, Proposition 2.3, we assume that for any $\epsilon > 0$, such that

$$V(t+s, x(t+s)) < (1+\epsilon)V(t, x(t)), \quad \forall s \in [-2h, 0],$$

and using the condition (3.14), we have

$$\begin{aligned}e^{-\tau}\|x(t-\tau_1(t)\|^2 &\le V(t-\tau_1(t), x(t-\tau_1(t)))\\ &\le (1+\epsilon)V(t, x(t)) = (1+\epsilon)\langle P_\tau x(t), x(t)\rangle,\\ e^{-\tau}\|x(t-\tau_2(t)\|^2 &\le V(t-\tau_2(t), x(t-\tau_2(t)))\\ &\le (1+\epsilon)V(t, x(t)) = (1+\epsilon)\langle P_\tau x(t), x(t)\rangle.\end{aligned}$$

Therefore, from (3.16) it follows that

$$\begin{aligned}\dot V(x(t)) &\le \left\langle\left(-2P_\tau A + \frac{5}{4}P_\tau BB^T P_\tau + \frac{1}{\gamma}P_\tau B_1 B_1^T P_\tau\right)x(t), x(t)\right\rangle\\ &\quad + \langle (C^T W_2^I C + 2FD_1 + FD_2 D)x(t), x(t)\rangle\\ &\quad + (\sigma^2 + \eta^2 we^\tau)(1+\epsilon)\langle P_\tau x(t), x(t)\rangle\\ &\quad + e^\tau\langle P_\tau W_1 W_1^T P_\tau x(t), x(t)\rangle\\ &\quad + 2\langle D_1 f(x(t)), x(t)\rangle + 2\langle P_\tau W_0 f(x(t)), x(t)\rangle\\ &\quad - \langle D_2 f(x(t)), f(x(t))\rangle\\ &\quad - \frac{1}{\gamma}\langle P_\tau B_1 B_1^T P_\tau x(t), x(t)\rangle + 2\langle P_\tau B_1 w(t), x(t)\rangle\\ &\quad - \langle C^T W_2^I Cx(t), x(t)\rangle - \frac{1}{4}\langle P_\tau B_1 B_1^T P_\tau x(t), x(t)\rangle\\ &\quad - \langle W_2^I(h(x(t-\tau_2(t))), h(x(t-\tau_2(t)))\rangle.\end{aligned} \tag{3.17}$$

Now letting $\epsilon \to 0^+$, and $w(t) = 0$ in (3.17), we obtain

$$\begin{aligned}\dot V(x(t)) &\le \langle Nx(t), x(t)\rangle + \langle \mathcal{M}z(t), z(t)\rangle\\ &\quad - \langle C^T W_2^I Cx(t), x(t)\rangle\\ &\quad - \langle W_2^I h(x(t-\tau_2(t)), h(x(t-\tau_2(t))\rangle\\ &\quad - \frac{1}{4}\langle P_\tau BB^T P_\tau x(t), x(t)\rangle\\ &\quad - \frac{1}{\gamma}\langle P_\tau B_1 B_1^T P_\tau x(t), x(t)\rangle,\end{aligned} \tag{3.18}$$

where $z(t) = [x(t), f(x(t))]$ and

$$N = T_1(A, P_\tau) + e^\tau P_\tau W_1 W_1^T P_\tau + \frac{5}{4}P_\tau BB^T P_\tau + \frac{1}{\gamma}P_\tau B_1 B_1^T P_\tau,$$

$$\mathcal{M} = \begin{pmatrix} T_2(A, P_\tau) & P_\tau W_0 + D_1\\ W_0^T P_\tau + D_1 & -D_2\end{pmatrix}.$$

Note that $N < 0$ is, by Schur complement lemma, equivalent to $\mathcal{N} < 0$, where

$$\mathcal{N} = \begin{pmatrix} T_1(A, P_\tau) & P_\tau B & P_\tau W_1 & P_\tau B_1\\ B^T P_\tau & -\frac{4}{5}I_n & 0 & 0\\ W_1^T P_\tau & 0 & -e^{-\tau}I_n & 0\\ B_1^T P_\tau & 0 & 0 & -\gamma I_n\end{pmatrix}.$$

Since

$\langle C^T W_2^I C x(t), x(t)\rangle \geq 0,$
$\langle W_2^I h(x(t-\tau_2(t)), h(x(t-\tau_2(t))\rangle \geq 0,$
$\langle P_\tau B B^T P_\tau x(t), x(t)\rangle \geq 0, \langle P_\tau B_1 B_1^T P_\tau x(t), x(t)\rangle \geq 0,$

the conditions (3.18) gives

$$\dot{V}(x(t)) \leq \langle N x(t), x(t)\rangle + \langle \mathcal{M} z(t), z(t)\rangle, \quad \forall t \geq 0,$$

and hence taking the conditions (3.11), (3.12) into account, there is $\alpha > 0$ such that

$$\dot{V}(x(t)) \leq -\alpha \|x(t)\|^2, \quad \forall t \geq 0.$$

Hence, the zero solution of the closed-loop system, by using the Razuminkhin-type stability theorem, Proposition 2.3, is asymptotically stable. The exponential estimation of the solution, as in the proof of Theorem 3.1, follows from the differential inequality

$$\dot{V}(t, x(t)) \leq -\frac{\alpha}{p+e^{-\tau}} V(t, x(t)), \quad t \geq 0,$$

and hence

$$\|x(t,\phi)\| \leq \sqrt{\frac{p+e^{-\tau}}{\lambda_{\min}(P)}} \|\phi\| e^{-\frac{\alpha}{2(p+e^{-\tau})}t}, \quad \forall t \geq 0.$$

The condition (2.4) is proved by the same arguments used in Theorem 3.1. This completes the proof of the theorem.

*Remark 3.2* Note that by using the Razumikhin stability theorem, only knowledge of the upper bound of the time-delay function is required in condition (H2) and no additional information of the delay is necessary, which is of particular interest for many practical processes. However, unlike the LMI conditions obtained in Theorem 3.1 that allow for an arbitrary prescribed stability degree $\beta$, the exponential rate of the system (2.1) obtained in Theorem 3.2 depends on the solution $P$ of the LMIs (3.11) and (3.12) as well as on the time delay.

## 4 Numerical examples

*Example 4.1* Consider the system (2.1) with the delay function $\tau_1(t) = \sin^2(0.25t), \tau_2(t) = \cos^2(0.4t)$ and

$$A = \begin{pmatrix} 28 & 0 \\ 0 & 29 \end{pmatrix}, B = \begin{pmatrix} 0.3 \\ 0.1 \end{pmatrix}, B_1 = \begin{pmatrix} 0.3 & 0 \\ -0.9 & 0 \end{pmatrix},$$
$$W_0 = \begin{pmatrix} -8 & 0 \\ -9 & 0 \end{pmatrix}, W_1 = \begin{pmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{pmatrix}, W_2 = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix},$$
$$C = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, D = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$
$$\xi_1 = 1.2, \xi_2 = 1.3, \sigma_1 = 1.3, \sigma_2 = 0.9, \eta_1 = 0.7, \eta_2 = 1.3.$$

Given $\beta = 0.5$, $\delta_1 = 0.5$, $\delta_2 = 0.8$, $\gamma = 100$, by using LMI toolbox of MATLAB, we have both the LMI (3.2), (3.3) feasible with

$$P = \begin{pmatrix} 4.4236 & -0.0313 \\ -0.0313 & 2.9222 \end{pmatrix},$$
$$D_1 = \begin{pmatrix} 22.5359 & 0 \\ 0 & 4.9215 \end{pmatrix}, D_2 = \begin{pmatrix} 30.8554 & 0 \\ 0 & 19.3603 \end{pmatrix}.$$

The feedback $H_\infty$ control is defined by (3.4) as

$$u(t) = 0.6620 x_1(t) + 0.1414 x_2(t), \quad t \geq 0,$$

and the solution $x(t, \phi)$ satisfies

$$\|x(t,\phi)\| \leq 5.3830 e^{-0.5t} \|\phi\|, \quad \forall t \geq 0.$$

Figure 1 shows the trajectories of solutions $x_1(t)$ and $x_2(t)$ of the closed-loop system (2.1) with the initial condition $\phi(t) = (1, 0.2), t \in [1, 0]$.

*Example 4.2* Consider the system (2.1) with the time-delay functions

$$\begin{cases} \tau_1(t) = 2\sin^2 t, & \text{if } t \in I = [2k\pi, (2k+1)\pi], \quad k = 0, 1, 2, \ldots, \\ \tau_1(t) = 0 & \text{if } t \in R^+ \setminus I, \end{cases}$$

$$\begin{cases} \tau_2(t) = \beta(t), & \text{if } t \in [0, 1] \\ \tau_2(t) = \beta(t-k), & \text{if } t \in [k, k+1], k = 1, 2, \ldots, \end{cases}$$

where $\beta(t) = t, t \in [0, 0.5]; = -t+1, t \in (0.5, 1]$.

$$A = \begin{pmatrix} 120 & 0 \\ 0 & 180 \end{pmatrix}, \quad B = \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.3 & 0 \\ -0.9 & 0 \end{pmatrix},$$
$$W_0 = \begin{pmatrix} -2 & 0 \\ -3 & 0 \end{pmatrix}, \quad W_1 = \begin{pmatrix} -0.12 & 0 \\ -0.13 & -0.014 \end{pmatrix},$$
$$W_2 = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 \\ 3 & 4 \end{pmatrix}, \quad D = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$
$$\xi_1 = 1.1 \quad \xi_2 = 1.3, \quad \sigma_1 = 1.2, \quad \sigma_2 = 0.7,$$
$$\eta_1 = 0.7, \quad \eta_2 = 1.3.$$

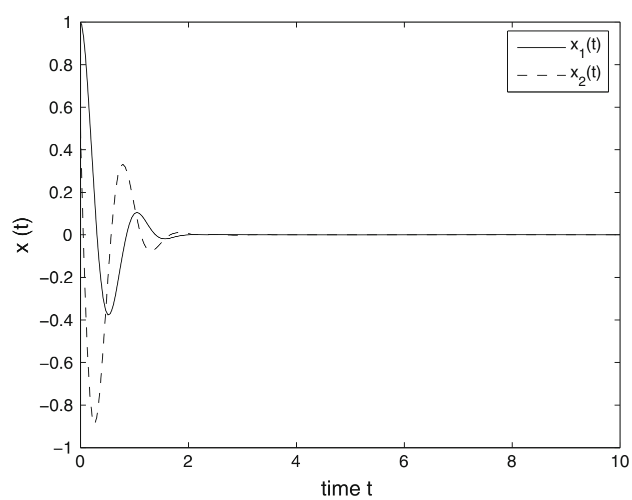


**Fig. 1** The trajectories of $x_1(t)$, and $x_2(t)$ of (2.1)

It is worth noting that the delay functions $\tau_1(t)$,$\tau_2(t)$ are bounded $\tau_1 = 2$, $\tau_2 = 1$, but non-differentiable, and therefore, the methods used in [11–14] are not applicable to this system. Given $\beta = 0.5$, $\gamma = 1000$, by using LMI toolbox of MATLAB, we have both the LMI (3.11), (3.12) feasible with

$$P = \begin{pmatrix} 5.2819 & 0.0525 \\ 0.0525 & 3.4978 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 26.0699 & 0 \\ 0 & 35.2669 \end{pmatrix},$$
$$D_2 = \begin{pmatrix} 111.9514 & 0 \\ 0 & 129.8264 \end{pmatrix}.$$

The feedback $H_\infty$ control is defined by (3.13) as

$$u(t) = 12.1419x_1(t) + 10.1327x_2(t), \quad t \geq 0.$$

Figure 2 shows the trajectories of solutions $x_1(t)$ and $x_2(t)$ of the closed-loop system (2.1) with the initial condition $\phi(t) = (-1, 0.6), t \in [-2, 0]$.

## 5 Conclusion

The $H_\infty$ control problem with exponential stability for neural networks with time-varying delays has been studied. Based on constructing a set of augmented Lyapunov–Krasovskii functionals, new sufficient conditions for $H_\infty$ control have been established for two cases of time-varying delays: the delays are differentiable and have an upper bound of the delay-derivatives; and the delays are bounded but not necessary to be differentiable. The derived conditions are formulated in terms of LMIs. Upon the feasibility of the LMIs, all the control parameters can be easily computed and the design of a $H_\infty$ controller can be accomplished.

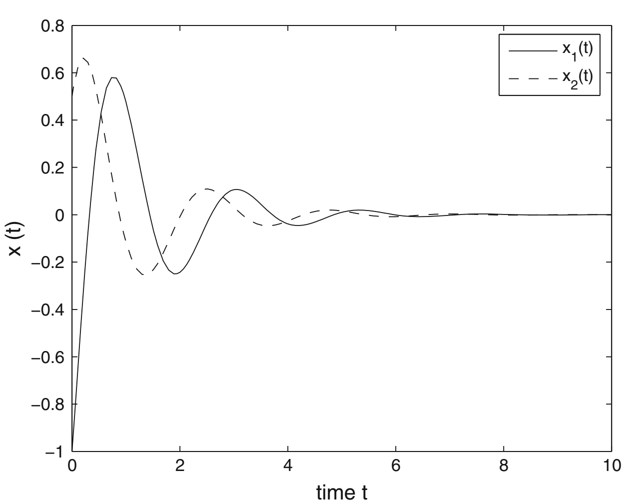


**Fig. 2** The trajectories of $x_1(t)$, and $x_2(t)$ of (2.1)

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