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Structure and stability analysis of a Takagi–Sugeno fuzzy PI controller with application to tissue hyperthermia therapy

Y. Ding, H. Ying, S. Shao

Abstract In this paper, we first reveal the analytical structure of a simple Takagi–Sugeno (TS) fuzzy PI controller relative to the linear PI controller. The fuzzy controller consists of two linear input fuzzy sets, four TS fuzzy rules with linear consequent, Zadeh fuzzy logic AND and the centroid defuzzifier. We prove that the fuzzy controller is actually a nonlinear PI controller with the gains changing with process output. Utilizing the well-known small Gain Theorem in control theory, we then derive sufficient conditions for global stability of the fuzzy control systems involving the TS fuzzy PI controller. Finally, as an application demonstration, we apply the fuzzy PI controller to control issue temperature, in computer simulation, during hyperthermia therapy. The relationship between heat energy and tissue temperature is represented by a linear time-varying model with a time delay. The sufficient conditions for global stability are used to design a stable fuzzy control system. Our simulation results show that the fuzzy PI control system achieves satisfactory temperature control performance. The control system is robust and stable even when the model parameters are changed suddenly and significantly.

1 Introduction

Analytical analysis of structures of fuzzy controllers with respect to conventional control theory is regarded as an important way to advance fuzzy control techniques, since many important but difficult issues in fuzzy control techniques, such as stability, analysis, design and robustness, can be analytically investigated by utilizing powerful conventional control theory. The analytical analysis of Mamdani-type fuzzy controllers has been relatively well conducted [1, 4, 6, 14–17]. The Mamdani-type fuzzy PI, PD and PID controllers are actually nonlinear PI, PD, and PID controllers with variable gains, respectively [14, 16, 9].

Another major type of fuzzy controllers, namely Takagi–Sugeno (TS) fuzzy controllers, was developed in 1985 [10] and has often been used. In most cases, TS fuzzy controllers are treated and used a block-box controllers. Recently, we began to explore the exact and analytical relationship between TS fuzzy, PI, PD and PID controllers and their linear counterparts [18–21]. Also, only a few stability results on TS fuzzy control systems are available [11–13].

In this paper, we first establish the relationship between a simple TS fuzzy PI controller and the linear PI controller. The structure of the simple TS fuzzy PI controller is analytically derived. Then we employ the Small Gain Theorem to analyze the bounded-input–bounded-output stability.

(BIBO) of the TS fuzzy PI control system involving nonlinear processes. Finally, the BIBO stability conditions that we have developed are employed to design a stable TS fuzzy control system for tissue temperature control during hyperthermia. Computer simulation shows that the performance is satisfactory.

The approach used in this study is different from the existing ones, i.e., those employed in [11–13] where the Lyapunov methods are utilized for asymptotic stability analysis and design of some TS fuzzy control systems without knowing the analytical structure of the TS fuzzy controllers involved. Our approach is to derive the analytical structure of the TS fuzzy controllers first and then apply the Small Gain Theorem to the derived structure to establish system BIBO stability conditions for analysis and design purposes. Because our stability work is directly based on the controller structure, the stability results have the potential to be less conservative than the results in existence. On the other hand, however, BIBO stability, while useful, is less informative than asymptotic stability.

2 Configuration of the TS fuzzy PI controller

The simple TS fuzzy PI controller consists of two inputs and one output. The input variables are error and change of error (rate, for short) of process output with respect to output setpoint. They are denoted as follows:

$$e(nT) = \mathrm{SP}(nT) - y(nT),$$

$$r(nT) = e(nT) - e(nT - T),$$

where n is a positive integer, T is the sampling period and $\mathrm{SP}(nT)$ the setpoint. We denote $e(nT)$, $r(nT)$ and $y(nT)$ as

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error, rate and process output, respectively. Each of the two input variables is fuzzified by two input fuzzy sets, named "positive" and "negative", whose membership functions are defined over $(-\infty, \infty)$, as shown in Fig. 1. In $[-L, L]$, where L is a design parameter, the membership functions are:

$$\mu_P(e) = \frac{L + e(nT)}{2L} \quad \text{for positive error,}$$

$$\mu_N(e) = \frac{L - e(nT)}{2L} \quad \text{for negative error,}$$

$$\mu_P(r) = \frac{L + r(nT)}{2L} \quad \text{for positive rate,}$$

$$\mu_N(r) = \frac{L - r(nT)}{2L} \quad \text{for negative rate,}$$

where

$$\mu_P(e) + \mu_N(e) = 1 \quad \text{and} \quad \mu_P(r) + \mu_N(r) = 1.$$

Outside $[-L, L]$, the memberships are either zero or one.

The fuzzy PI controller uses following four fuzzy control rules:

r_1. IF $e(nT)$ is positive AND $r(nT)$ is positive THEN $\Delta u_1(nT) = a_1 e(nT) + b_1 r(nT)$.

r_2. IF $e(nT)$ is positive AND $r(nT)$ is negative THEN $\Delta u_2(nT) = a_2 e(nT) + b_2 r(nT)$.

r_3. IF $e(nT)$ is negative AND $r(nT)$ is positive THEN $\Delta u_3(nT) = a_3 e(nT) + b_3 r(nT)$.

r_4. IF $e(nT)$ is negative AND $r(nT)$ is negative THEN $\Delta u_4(nT) = a_4 e(nT) + b_4 r(nT)$.

where $\Delta u_i(nT)$ $(i = 1, 2, 3, 4)$ is the contribution of rule r_i to the change of the fuzzy controller output. In the rule consequent, a_i and b_i are eight design parameters. In the rules, Zadeh fuzzy logic AND is used and the resulting memberships for the four rules are:

$$\mu_{r_1} = \min(\mu_P(e), \mu_P(r)) \quad \text{for} \Delta u_1(nT),$$

$$\mu_{r_2} = \min(\mu_P(e), \mu_N(r)) \quad \text{for} \Delta u_2(nT),$$

$$\mu_{r_3} = \min(\mu_N(e), \mu_P(r)) \quad \text{for} \Delta u_3(nT),$$

$$\mu_{r_4} = \min(\mu_N(e), \mu_N(r)) \quad \text{for} \Delta u_4(nT), \tag{1}$$

The widely used centroid defuzzifer is employed to calculate the output change of the fuzzy PI controller:

$$\Delta u(nT) = \frac{\sum_{i=1}^{4} \Delta u_i \mu_{r_i}}{\sum_{i=1}^{4} \mu_{r_i}}. \tag{2}$$

The new output of the fuzzy controller at $nT + T$ is

$$u(nT + T) = u(nT) + \Delta u(nT).$$

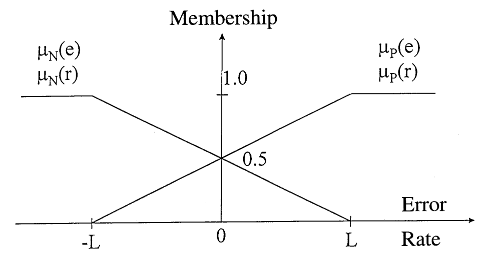

Fig. 1. The input membership functions for the TS fuzzy PI controller

3
Structural analysis of the TS fuzzy PI controller

In this section, we derive analytically the structure of the TS fuzzy PI controller. From (2), we obtain

$$\Delta u(nT) = \frac{\sum_{i=1}^{4} \Delta u_i \mu_{r_i}}{\sum_{i=1}^{4} \mu_{r_i}} = \sum_{i=1}^{4} \frac{\mu_{r_i}}{\sum_{i=1}^{4} \mu_{r_i}} (a_i e(nT) + b_i r(nT))$$

$$= \sum_{i=1}^{4} (K_I^i(e, r) e(nT) + K_P^i(e, r) r(nT)), \tag{3}$$

where

$$K_P^i(e, r) = \frac{\mu_{r_i} b_i}{\sum_{i=1}^{4} \mu_{r_i}} \quad \text{and} \quad K_I^i(e, r) = \frac{\mu_{r_i} a_i}{\sum_{i=1}^{4} \mu_{r_i}} \quad (i = 1\text{–}4).$$

Recall that the linear PI controller in incremental form is

$$\Delta u(nT) = \bar{K}_i e(nT) + \bar{K}_P r(nT), \tag{4}$$

where $\bar{K}_P$ and $\bar{K}_i$ are proportional-gain and integral-gain, respectively. Comparing (3) with (4), one sees that the output of the TS fuzzy controller is the sum of the outputs of four nonlinear PI controllers, each of which has variable proportional-gain $K_P^i(e, r)$ and integral gain $K_I^i(e, r)$ that change with $e(nT)$ and $r(nT)$. We can also rewrite (3) as follows:

$$\Delta u(nT) = K_I(e, r) e(nT) + K_P(e, r) r(nT), \tag{5}$$

where

$$K_P(e, r) = \sum_{i=1}^{4} K_P^i(e, r) \quad \text{and} \quad K_I(e, r) = \sum_{i=1}^{4} K_I^i(e, r).$$

This is to say the TS fuzzy controller is a nonlinear PI controller with variable proportional-gain, $K_P(e, r)$, and integral-gain, $K_I(e, r)$, being the sum of the corresponding gains of the four above-mentioned nonlinear PI controllers. We call $K_P(e, r)$ and $K_I(e, r)$ dynamic proportional-gain and integral-gain, [14] respectively, because they change with $e(nT)$ and $r(nT)$.

In the analysis so far, we have related the structure of the TS fuzzy PI controller to that of the linear PI controller. Since the only differences between them are the gains, we now derive the analytical expressions for $K_P(e, r)$ and $K_I(e, r)$. First, we need to divide the error-rate input space into 20 different input combinations (ICs) as shown in Fig. 2. These divisions are necessary because they will result in, in each of the 20 ICs, a unique inequality between $e(nT)$ and $r(nT)$ when each of the

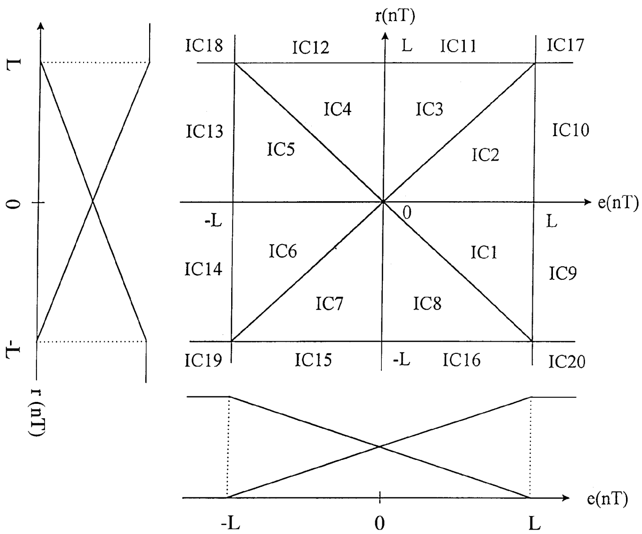

Fig. 2. Division of input space for analytically deriving structure of the TS fuzzy PI controller

four fuzzy rules is evaluated by Zadeh fuzzy logic AND [14]. After applying defuzzification algorithm (2) to each of the 20 resulting memberships, we obtain the expressions for $K_P(e, r)$ and $K_I(e, r)$ as shown in Table 1. From the table, one sees the following:

(1) When $e(nT)$ and $r(nT)$ are in IC1–IC8, $K_P(e, r)$ and $K_I(e, r)$ are determined by all a_i and b_i as well as $e(nT)$ and $r(nT)$. More specifically, $K_P(e, r)$ is determined by b_1–b_4, $e(nT)$ and $r(nT)$, and $K_I(e, r)$ by a_1–a_4, $e(nT)$ and $r(nT)$. Take IC1 as an example, for $K_P(e, r)$, the weight of b_3 and b_4 decreases while the weight of b_1 and b_2 increases with increase of $e(nT)$, and at the same time, the weight of b_1 in $K_P(e, r)$ decreases and the weight of b_2 in $K_P(e, r)$ increases with the increase of $r(nT)$. Also in IC1, for $K_I(e, r)$, with the increase of $e(nT)$, the contribution of a_3 and a_4 to $K_I(e, r)$ decreases whereas the contribution of a_1 and a_2 to $K_I(e, r)$ increases, and meanwhile the weight of a_1 in $K_I(e, r)$ decreases and the weight of a_2 in $K_I(e, r)$ increases with the increase of $r(nT)$.

(2) When $e(nT)$ and $r(nT)$ are in IC9 to IC16, $K_P(e, r)$ is determined by b_i in two of the four fuzzy rules, $e(nT)$ and $r(nT)$ whereas $K_I(e, r)$ is determined by a_i in the same two fuzzy rules, $e(nT)$ and $r(nT)$. For instance, in IC9, $K_P(e, r)$ is determined by b_1, b_2, $e(nT)$ and $r(nT)$ and $K_I(e, r)$ by a_1, a_2, $e(nT)$ and $r(nT)$. The weight of b_1 in $K_P(e, r)$ increases but the weight of b_2 in $K_P(e, r)$ decreases with increase of $r(nT)$. The weight of a_1 in $K_I(e, r)$ increases and the weight of a_2 in $K_I(e, r)$ decreases with the increase of $r(nT)$.

(3) When $e(nT)$ and $r(nT)$ are in IC17 to IC20, the TS fuzzy PI controller becomes a linear PI controller. The proportional-gain is b_i and integral-gain is a_i (i depends on IC number).

(4) In $[-L, L] \times [-L, L]$, when $e(nT) = 0$, $r(nT) = 0$, the TS fuzzy PI controller becomes a linear PI controller with proportional-gain $(b_1 + b_2 + b_3 + b_4)/4$ and integral-gain $(a_1 + a_2 + a_3 + a_4)/4$.

(5) The TS fuzzy PI controller may switch from one control algorithm in one IC to another one in another IC, depending on the change of $e(nT)$ and $r(nT)$. But the controller output is always continuous and smooth on the boundaries of any adjacent ICs involved.

In order to visualize how the variable gains, $K_P(e, r)$ and $K_I(e, r)$, change with $e(nT)$ and $r(nT)$, we provide three-dimensional plots of $K_P(e, r)$ in Figs. 3–5, where three different combinations of the parameters are used. As we pointed out above, $K_P(e, r)$ is determined only by b_i and L, and is independent of a_i. The parameter values for Figs. 3, 4 and 5 are, respectively, $b_1 = 6$, $b_2 = 1$, $b_3 = 4$ and $b_4 = 2.5$, $b_1 = 2.5$, $b_2 = 1$, $b_3 = 6$ and $b_4 = 4$, and $b_1 = 1$, $b_2 = 6$, $b_3 = 4$ and $b_4 = 2.5$. For all the three sets of the parameter values, L is always 1. The figures demonstrate that, by using different parameter values, one can obtain different characteristics of the gain variation. The fuzzy controller designer needs to choose proper parameter values to generate the desired gain variation for his/her particular application. The variation of $K_I(e, r)$ with respect to $e(nT)$ and $r(nT)$, which is parameterized by a_i and L, can also be shown in a similar manner.

The above structure results for the TS fuzzy PI controller can easily be extended to the TS fuzzy PD controller with similar configuration. Note that the linear PD controller in position form is

$$u(nT) = \bar{K}_p e(nT) + \bar{K}_d r(nT),$$

where $\bar{K}_p$ and $\bar{K}_d$ are the proportional-gain and derivative-gain, respectively. Hence, if the output of the four fuzzy rules is controller output, instead of incremental output, the above analysis will show that the TS fuzzy controller is a nonlinear PD controller with variable proportional-gain $K'_P(e, r)$ and derivative-gain $K'_D(e, r)$:

$$u(nT) = K'_P(e, r)e(nT) + K'_D(e, r)r(nT).$$

Here, expression for $K'_P(e, r)$ are those for $K_I(e, r)$ whereas expressions for $K'_D(e, r)$ are those for $K_P(e, r)$ in Table 1. Summarizing the above, we have:

Theorem 1 The simple TS fuzzy PI (or PD) controller is a nonlinear PI (or PD) controller with variable proportional-gain and integral-gain (or derivative-gain).

4
BIBO stability analysis

In this section, we analyze the BIBO stability of the TS fuzzy control system by using the Small Gain Theorem [5, 7], which is a generally applicable tool in nonlinear control theory. We used the theorem before to derive sufficient stability conditions for the Mamdani-type fuzzy control systems [2].

Assuming a TS fuzzy control system consisting of a nonlinear process, denoted by P, controlled by the TS fuzzy PI controller, the process output is $P(u(nT))$. We treat the TS fuzzy controller as a nonlinear operator mapping input $e(nT)$ to output $\Delta u(nT)$ and designate this operator as C, then $\Delta u(nT) = C(e(nT))$. Since C is different in different ICs, we

Table 1 Integral-gain $K_I(e, r)$ and proportional-gain $K_P(e, r)$ of the simple TS fuzzy PI controller when $e(nT)$ and $r(nT)$ are in the 20 different ICs shown in Fig. 2

IC#	$K_I(e, r)$	$K_P(e, r)$
IC1	$\frac{(L-\lvert r(nT)\rvert)a_1+(L+\lvert r(nT)\rvert)a_2+(L-\lvert e(nT)\rvert)(a_3+a_4)}{2(2L-\lvert e(nT)\rvert)}$	$\frac{(L-\lvert r(nT)\rvert)b_1+(L+\lvert r(nT)\rvert)b_2+(L-\lvert e(nT)\rvert)(b_3+b_4)}{2(2L-\lvert e(nT)\rvert)}$
IC2	$\frac{(L+\lvert r(nT)\rvert)a_1+(L-\lvert r(nT)\rvert)a_2+(L-\lvert e(nT)\rvert)(a_3+a_4)}{2(2L-\lvert e(nT)\rvert)}$	$\frac{(L+\lvert r(nT)\rvert)b_1+(L-\lvert r(nT)\rvert)b_2+(L-\lvert e(nT)\rvert)(b_3+b_4)}{2(2L-\lvert e(nT)\rvert)}$
IC3	$\frac{(L+\lvert e(nT)\rvert)a_1+(L-\lvert e(nT)\rvert)a_3+(L-\lvert r(nT)\rvert)(a_2+a_4)}{2(2L-\lvert r(nT)\rvert)}$	$\frac{(L+\lvert e(nT)\rvert)b_1+(L-\lvert e(nT)\rvert)b_3+(L-\lvert r(nT)\rvert)(b_2+b_4)}{2(2L-\lvert r(nT)\rvert)}$
IC4	$\frac{(L-\lvert e(nT)\rvert)a_1+(L+\lvert e(nT)\rvert)a_3+(L-\lvert r(nT)\rvert)(a_2+a_4)}{2(2L-\lvert r(nT)\rvert)}$	$\frac{(L-\lvert e(nT)\rvert)b_1+(L+\lvert e(nT)\rvert)b_3+(L-\lvert r(nT)\rvert)(b_2+b_4)}{2(2L-\lvert r(nT)\rvert)}$
IC5	$\frac{(L+\lvert r(nT)\rvert)a_3+(L-\lvert r(nT)\rvert)a_4+(L-\lvert e(nT)\rvert)(a_1+a_2)}{2(2L-\lvert e(nT)\rvert)}$	$\frac{(L+\lvert r(nT)\rvert)b_3+(L-\lvert r(nT)\rvert)b_4+(L-\lvert e(nT)\rvert)(b_1+b_2)}{2(2L-\lvert e(nT)\rvert)}$
IC6	$\frac{(L-\lvert r(nT)\rvert)a_3+(L+\lvert r(nT)\rvert)a_4+(L-\lvert e(nT)\rvert)(a_1+a_2)}{2(2L-\lvert e(nT)\rvert)}$	$\frac{(L-\lvert r(nT)\rvert)b_3+(L+\lvert r(nT)\rvert)b_4+(L-\lvert e(nT)\rvert)(b_1+b_2)}{2(2L-\lvert e(nT)\rvert)}$
IC7	$\frac{(L-\lvert e(nT)\rvert)a_2+(L+\lvert e(nT)\rvert)a_4+(L-\lvert r(nT)\rvert)(a_1+a_3)}{2(2L-\lvert r(nT)\rvert)}$	$\frac{(L-\lvert e(nT)\rvert)b_2+(L+\lvert e(nT)\rvert)b_4+(L-\lvert r(nT)\rvert)(b_1+b_3)}{2(2L-\lvert r(nT)\rvert)}$
IC8	$\frac{(L+\lvert e(nT)\rvert)a_2+(L-\lvert e(nT)\rvert)a_4+(L-\lvert r(nT)\rvert)(a_1+a_3)}{2(2L-\lvert r(nT)\rvert)}$	$\frac{(L+\lvert e(nT)\rvert)b_2+(L-\lvert e(nT)\rvert)b_4+(L-\lvert r(nT)\rvert)(b_1+b_3)}{2(2L-\lvert r(nT)\rvert)}$
IC9	$\frac{(L-\lvert r(nT)\rvert)a_1+(L+\lvert r(nT)\rvert)a_2}{2L}$	$\frac{(L-\lvert r(nT)\rvert)b_1+(L+\lvert r(nT)\rvert)b_2}{2L}$
IC10	$\frac{(L+\lvert r(nT)\rvert)a_1+(L-\lvert r(nT)\rvert)a_2}{2L}$	$\frac{(L+\lvert r(nT)\rvert)b_1+(L-\lvert r(nT)\rvert)b_2}{2L}$
IC11	$\frac{(L+\lvert e(nT)\rvert)a_1+(L-\lvert e(nT)\rvert)a_3}{2L}$	$\frac{(L+\lvert e(nT)\rvert)b_1+(L-\lvert e(nT)\rvert)b_3}{2L}$
IC12	$\frac{(L-\lvert e(nT)\rvert)a_1+(L+\lvert e(nT)\rvert)a_3}{2L}$	$\frac{(L-\lvert e(nT)\rvert)b_1+(L+\lvert e(nT)\rvert)b_3}{2L}$
IC13	$\frac{(L+\lvert r(nT)\rvert)a_3+(L-\lvert r(nT)\rvert)a_4}{2L}$	$\frac{(L+\lvert r(nT)\rvert)b_3+(L-\lvert r(nT)\rvert)b_4}{2L}$
IC14	$\frac{(L-\lvert r(nT)\rvert)a_3+(L+\lvert r(nT)\rvert)a_4}{2L}$	$\frac{(L-\lvert r(nT)\rvert)b_3+(L+\lvert r(nT)\rvert)b_4}{2L}$
IC15	$\frac{(L-\lvert e(nT)\rvert)a_2+(L+\lvert e(nT)\rvert)a_4}{2L}$	$\frac{(L-\lvert e(nT)\rvert)b_2+(L+\lvert e(nT)\rvert)b_4}{2L}$
IC16	$\frac{(L+\lvert e(nT)\rvert)a_2+(L-\lvert e(nT)\rvert)a_4}{2L}$	$\frac{(L+\lvert e(nT)\rvert)b_2+(L-\lvert e(nT)\rvert)b_4}{2L}$
IC17	a_1	b_1
IC18	a_3	b_3
IC19	a_4	b_4
IC20	a_2	b_2

need to discuss stability for every IC. When $e(nT)$ and $r(nT)$ are in the region IC1,

$$
\begin{aligned}
\|C(e(nT))\| &= \|K_I(e, r)e(nT)+K_P(e, r)r(nT)\| \\
&= \|K_I(e, r)e(nT)+K_P(e, r)(e(nT)-e(nT-T))\| \\
&\leqslant (K_I(e, r)+K_P(e, r))|e(nT)|+K_P(e, r)|e(nT-T)| \\
&\leqslant (K_I(e, r)+K_P(e, r))|e(nT)|+K_P(e, r)L.
\end{aligned}
$$

Since

$$
\begin{aligned}
&K_I(e, r)+K_P(e, r) \\
&=\frac{(L-|r(nT)|)a_1+(L+|r(nT)|)a_2+(L-|e(nT)|)(a_3+a_4)}{2(2L-|e(nT)|)} \\
&+\frac{(L-|r(nT)|)b_1+(L+|r(nT)|)b_2+(L-|e(nT)|)(b_3+b_4)}{2(2L-|e(nT)|)} \\
&\leqslant K_I^{\max}+K_P^{\max},
\end{aligned}
$$

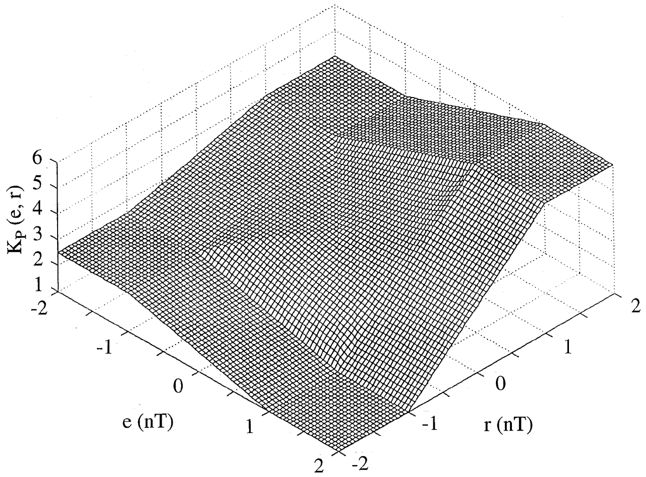

Fig. 3. A three-dimensional plot of proportional-gain $K_P(e, r)$, of the TS fuzzy controller changing with $e(nT)$ and $r(nT)$. The design parameters are chosen as follows: $b_1 = 6$, $b_2 = 1$, $b_3 = 4$, $b_4 = 2.5$ and $L = 1$

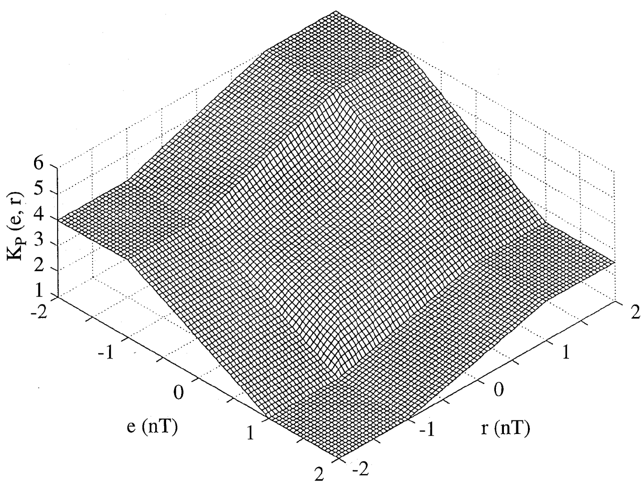

Fig. 4. A three-dimensional plot of proportional-gain $K_P(e, r)$, of the TS fuzzy controller changing with $e(nT)$ and $r(nT)$. The design parameters are chosen as follows: $b_1 = 2.5$ $b_2 = 1$, $b_3 = 6$, $b_4 = 4$ and $L = 1$

where

$$K_I^{max} = \frac{a_1 + 2a_2 + a_3 + a_4}{2} \quad \text{and} \quad K_P^{max} = \frac{b_1 + 2b_2 + b_3 + b_4}{2}$$

Therefore, in IC1,
$\|C(e(nT))\| \leqslant (K_I^{max} + K_P^{max})|e(nT)| + K_P^{max}L.$

Also, in IC1,

$$\|P(u(nT))\| \leqslant \|P\|\,|u(nT)|,$$

where

$$\|P\| := \sup_{u_1 \neq u_2, n \geqslant 0} \frac{|P(u_1(nT)) - P(u_2(nT))|}{|u_1(nT) - u_2(nT)|}$$

is the operator norm of a given P, which is the gain of the given nonlinear process over a set of admissible fuzzy control signals that have any meaningful function norms. Using the above

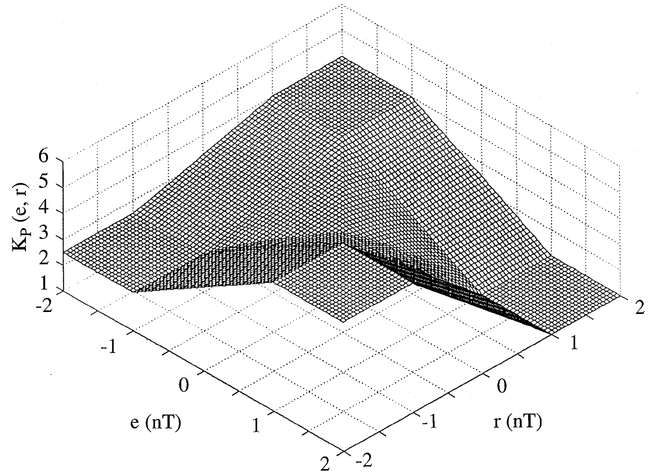

Fig. 5. A three-dimensional plot of proportional-gain $K_P(e, r)$, of the TS fuzzy controller changing with $e(nT)$ and $r(nT)$. The design parameters are chosen as follows: $b_1 = 1$, $b_2 = 6$, $b_3 = 4$, $b_4 = 2.5$ and $L = 1$

inequalities and the Small Gain Theorem, we obtain the following sufficient conditions for the BIBO stability of the nonlinear TS fuzzy PI control systems in IC1:

(1) $\|P\| < \infty$, and
(2) $(K_I^{max} + K_P^{max})\,\|P\| < 1$.

Actually, these two conditions are applicable to all the ICs and the expressions for K_I^{max} and K_P^{max}, which are different in different ICs, are shown in Table 2. We now state this result in the form of theorem as follows:

Theorem 2 The sufficient conditions for the nonlinear TS fuzzy PI control systems to be BIBO stable are: (1) the given nonlinear process has a bounded norm (gain) (i.e., $\|P\| < \infty$); and (2) the design parameters of the TS fuzzy PI controller satisfy

$$(K_I^{max} + K_P^{max})\,\|P\| < 1, \tag{6}$$

where K_I^{max} and K_P^{max} are given in Table 2.

5
Application to tissue temperature control in hyperthermia

Hyperthermia is an effective thermal therapy for destroying diseased tissue such as cancers. During hyperthermia, diseased tissue is killed by maintaining tissue temperature at 43–46°C for 30–60 min. High-performance temperature control during hyperthermia is difficult because hyperthermia is a dynamic, time-varying process involving (1) varying blood perfusion rates, both in time and in space, (2) variation in physical and physiological properties of the tissue under the treatment, and (3) time delay due to heat transfer in the tissue. Clinical studies have found several reasons responsible for failure of some hyperthermia therapies, one of which is the absence of tightly controlled temperature distributions in the treated tissue. It is clinically desirable that temperature in treated tissue rises quickly to a preferred level, e.g. 43°C, and stays at that level throughout the treatment to ensure the killing of the diseased tissue volume. To protect the surrounding normal tissues as well as avoid tissue carbonization, temperature in the treated

Table 2 K_I^{max} and K_P^{max} needed to use BIBO sufficient condition in (6) for the 20 different ICs shown in Fig. 2

IC#	K_I^{max}	K_P^{max}
IC1 & IC8	$\frac{a_1+2a_2+a_3+a_4}{2}$	$\frac{b_1+2b_2+b_3+b_4}{2}$
IC2 & IC3	$\frac{2a_1+a_2+a_3+a_4}{2}$	$\frac{2b_1+b_2+b_3+b_4}{2}$
IC4 & IC5	$\frac{a_1+a_2+2a_3+a_4}{2}$	$\frac{b_1+b_2+2b_3+b_4}{2}$
IC6 & IC7	$\frac{a_1+a_2+a_3+2a_4}{2}$	$\frac{b_1+b_2+b_3+2b_4}{2}$
IC9	$\frac{a_1+2a_2}{2}$	$\frac{b_1+2b_2}{2}$
IC10	$\frac{2a_1+a_2}{2}$	$\frac{2b_1+b_2}{2}$
IC11	$\frac{2a_1+a_3}{2}$	$\frac{2b_1+b_3}{2}$
IC12	$\frac{a_1+2a_3}{2}$	$\frac{b_1+2b_3}{2}$
IC13	$\frac{2a_3+a_4}{2}$	$\frac{2b_3+b_4}{2}$
IC14	$\frac{a_3+2a_4}{2}$	$\frac{b_3+2b_4}{2}$
IC15	$\frac{a_2+2a_4}{2}$	$\frac{b_2+2b_4}{2}$
IC16	$\frac{2a_2+a_4}{2}$	$\frac{2b_2+b_4}{2}$
IC17	a_1	b_1
IC18	a_3	b_3
IC19	a_4	b_4
IC20	a_2	b_2

tissue should fluctuate only slightly, e.g., 1°C, around the preferred temperature level.

The best mathematical model relating heating energy to tissue temperature is a bio-heat transfer equation that is a complicated three-dimensional partial differential equation with no closed-form analytical solution. Numeric solution takes prohibitively long time. For control simulation study, a linear first-order model with a time delay may be used, which approximates the bio-heat partial differential equation reasonably well [3]. The model is

$$P(s)=\frac{K}{TS+1}e^{-\tau s}, \tag{7}$$

where K, which is constant gain of the model, is in the range 0.12–24.6°C/W and T, which is fixed time constant, is in the range 43–2570 s. The value of time delay τ varies from patient to patient (typically 10–70 s) and, for typical patients, it is 45 s. The previous study [3] indicates that the parameter values for typical patients are: $K=1.1$, $T=250$ and $\tau=45$ and we use these values as nominal values in the following fuzzy control system design. We emphasize that in medical applications, control system stability needs to be rigorously guaranteed before the system can clinically be implemented. Therefore, our above stability conditions provide an effective means to design a stable fuzzy control system for hyperthermia.

We choose the sampling period to be 1 s. We represent the delay term e^{-45s} in the model by a fourth-order Pade approximation [8] and convert the continuous-time model to a discrete-time one as follows:

$$P(z)=\frac{0.0027z^4-0.0121z^3+0.0203z^2-0.0152z+0.0043}{z^5-4.5619z^4+8.3324z^3-7.6173z^2+3.4855z-0.6386}.$$

By using the Maximum Modulus Theorem in complex analysis, we find

$$\|P\|=\sup_{|z|=1}|P(z)|=0.0079.$$

To design a BIBO stable fuzzy PI control system for the model, we select such values of the eight design parameters in the four rule consequent of the TS fuzzy PI controller that the fuzzy control system will be stable according to our stability conditions (i.e., Theorem 2 and Table 2). We find that $a_1=0.005$, $a_2=0.007$, $a_3=0.004$, $a_4=0.006$, $b_1=2$, $b_2=1.62$, $b_3=1.4$, $b_4=2$ and $L=1$ not only make the system stable (see Table 3) but also achieve good temperature performance. With these values, we plot three dimensionally how proportional-gain $K_P(e, r)$ and integral-gain $K_I(e, r)$ of the TS fuzzy PI controller change with $e(nT)$ and $r(nT)$ in Figs. 6 and 7, respectively.

Computer simulation shows the designed TS fuzzy PI control system performs well. The fuzzy controller can quickly (i.e., in 300 s) drive tissue temperature to the desired level without overshoot and maintain the temperature at that level afterwards (see temperature curve for the period between 0 and 799 s in Fig. 8). To further test the robustness and stability of the designed fuzzy control system, we increase K and T suddenly by 20% (to $K=1.32$ and $T=300$) at time 800 s and then drop them suddenly back to their original values ($K=1.1$ and $T=250$) at time 1500 s (see temperature curve for the

Table 3 The values of K_I^{max} and K_P^{max} for our designed fuzzy temperature controller, which are used in sufficient stability condition in (6) for the different ICs shown in Fig. 2

IC#	K_I^{max}	K_P^{max}	$(K_I^{max}+K_P^{max})\|P\|$ where $\|P\|=0.0079$
IC1 & IC8	0.0145	4.320	0.0343
IC2 & IC3	0.0135	4.510	0.0358
IC4 & IC5	0.0130	4.210	0.0335
IC6 & IC7	0.0140	4.510	0.0358
IC9	0.0095	2.620	0.0208
IC10	0.0085	2.810	0.0223
IC11	0.0070	2.700	0.0214
IC12	0.0065	2.400	0.0191
IC13	0.0070	2.400	0.0191
IC14	0.0080	2.700	0.0215
IC15	0.0095	2.810	0.0223
IC16	0.0100	2.620	0.0208
IC17	0.0050	2.000	0.0159
IC18	0.0040	1.400	0.0111
IC19	0.0060	2.000	0.0159
IC20	0.0070	1.620	0.0129

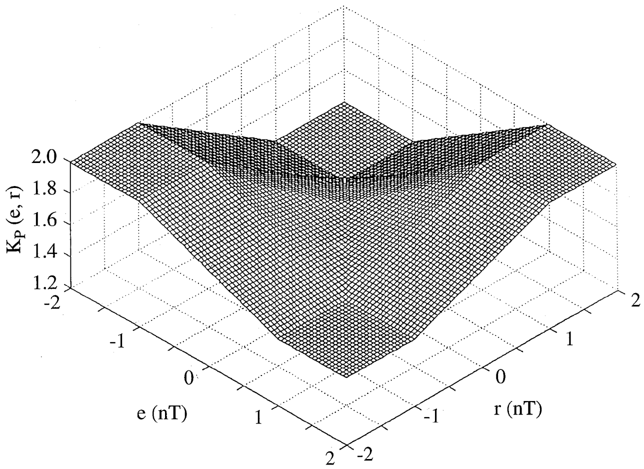

Fig. 6. A three-dimensional plot of proportional-gain $K_P(e, r)$, of the TS fuzzy controller changing with $e(nT)$ and $r(nT)$. The design parameters are chosen so as to result in a stable TS fuzzy tissue temperature control system: $a_1 = 0.005$, $a_2 = 0.007$, $a_3 = 0.004$, $a_4 = 0.006$, $b_1 = 2$, $b_2 = 1.62$, $b_3 = 1.4$, $b_4 = 2$ and $L = 1$

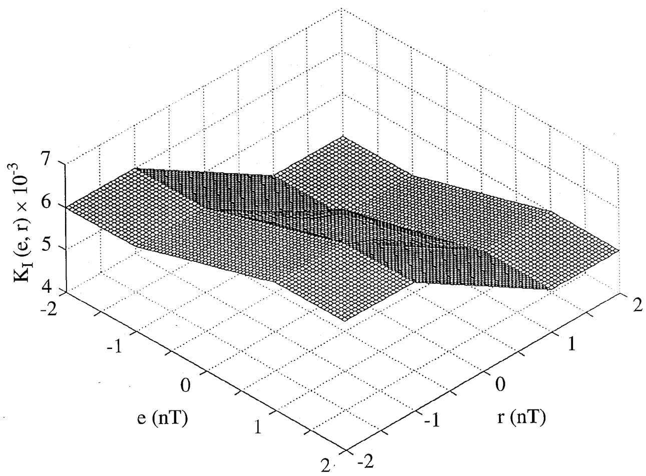

Fig. 7. A three-dimensional plot of proportional-gain $K_I(e, r)$, of the TS fuzzy controller changing with $e(nT)$ and $r(nT)$. The design parameters are chosen so as to result in a stable TS fuzzy tissue temperature control system: $a_1 = 0.005$, $a_2 = 0.007$, $a_3 = 0.004$, $a_4 = 0.006$, $b_1 = 2$, $b_2 = 1.62$, $b_3 = 1.4$, $b_4 = 2$ and $L = 1$

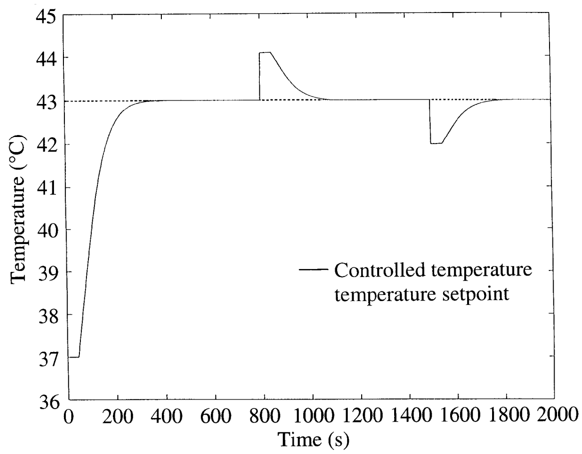

Fig. 8. For the time period of 0–799 s, the nominal model parameters ($K = 1.1$, $T = 250$ and $\tau = 45$) are used to simulate the designed TS fuzzy control system. At time 800 s, gain and time constant of the model suddenly increased by 20% and then suddenly returned to their original values at time 1500 s

period after 800 s in Fig. 8). The maximal temperature error due to the parameter changes is only 1.1°C and the fuzzy controller eliminates the error quickly. These simulation results show that our designed fuzzy control system is robust and stable even in the face of sudden and significant changes in the process parameters Indeed, our theoretical calculation using Theorem 2 proves that after the changes in the parameter values, the fuzzy control system is stable (note that $\|P\| = 0.1194$ after the 20% increase in the parameter values from the nominal values).

6 Conclusions

We have analytically proved that the simple TS fuzzy PI controller is a nonlinear PI controller with variable proportional-gain and integral-gain changing with process output. The explicit expressions for the gains are derived. Based on these analytical results, we have used the Small Gain Theorem to establish sufficient conditions for BIBO stability of the fuzzy control system involving the simple TS fuzzy PI controller. The stability conditions are used to design a stable fuzzy control system for control of tissue temperature in hyperthermia. Computer simulation and theoretical analysis show that the designed TS fuzzy PI control system is robust and stable even when there are sudden and significant changes in the model parameter values.

Based on the methodology developed in this paper, analytical structures of other more complex TS fuzzy controllers and their corresponding BIBO stability can be analyzed.

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