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A fuzzy PID controller for nonlinear and uncertain systems

J. H. Kim, S. J. Oh

Abstract In order to control systems that contain nonlinearities or uncertainties, control strategies must deal with the effects of these. Since most control methods based on mathematical models have been mainly focused on stability robustness against nonlinearities or uncertainties, they are limited in their ability to improve the transient responses. In this paper, a nonlinear fuzzy PID control method is suggested, which can stably improve the transient responses of systems disturbed by nonlinearities or unknown mathematical characteristics. Although the derivation of the control law is based on the design procedure for general fuzzy logic controllers, the resultant control algorithm has analytical form with time varying PID gains rather than linguistic form. This means the implementation of the proposed method can be easily and effectively applied to real-time control situations. Control simulations are carried out to evaluate the transient performance of the suggested method through example systems, by comparing its responses with those of the nonlinear fuzzy PI control method developed in [9].

Key words nonlinear and uncertain systems, fuzzy logic controller, fuzzy PID control, time varying PID gains

1 Introduction

As industry has developed, the demand has increased for control system design to accomplish more accurate and faster control by improving a transient response. In order to satisfy this requirement an effect of modelling error or uncertainties must be considered when choosing and developing a control theory.

During the past several years, fuzzy control has emerged as one of the most active and important application branches of fuzzy theory since the first realisation of the fuzzy controller using Zadeh's fuzzy logic by Mamdani [1] in 1974. Since the number of literature on fuzzy control and application in industrial processes has been growing rapidly, it is not simple to make a comprehensive survey, so that the references [2, 3] are cited for survey purpose.

A fuzzy logic controller (FLC) is based on fuzzy logic which is much closer in spirit to human thinking and natural language than the traditional logical system. From this perspective, the essential part of the FLC is a set of linguistic control rules related by the dual concepts of fuzzy implication and the compositional rules of inference. Thus, in essence, the FLC provides an algorithm which converts the linguistic control strategy based on expert knowledge into an automatic control strategy. In this regard, the methodology of the FLC appears particularly useful in cases where processes are too complex for analysis by conventional control techniques, or where the available sources of information are inexact or uncertain. In order to overcome the difficulty and complexity in generating fuzzy control rules, Sugeno et al. [4–6] tried to develop a method which provides a systematic design procedure of an FLC and assures the stability of it within fuzzy control theory. Another trend also appeared in the direction of designing an FLC systematically and assuring stability with the aid of conventional control theories, such as sliding mode control [7] and PI control [8–10].

In this paper, a nonlinear fuzzy PID control law is derived in order to generate a simple design procedure and to improve transient responses for nonlinear uncertain systems, by developing the fuzzy PI control law suggested by Ying et al. [8, 9]. Although the derivation is developed based on the design procedure of the general FLC, the resultant control law has an analytical PID controller form rather than linguistic form. The stability of the nonlinear fuzzy PID control system is entirely dependent upon the linear PID controller which is naturally generated in developing the nonlinear fuzzy PID control law.

2 Derivation of a nonlinear fuzzy PID control law

Most of the popular fuzzy controllers developed so far take two inputs, such as error and rate of change of error (rate for short) about a setpoint. However, the nonlinear fuzzy PID controller proposed in this paper has an additional input named accelerated rate of change of error (acc for short) to improve the transient response of nonlinear uncertain systems. The configuration of the FLC suggested is shown in Fig. 1.

With these three inputs the structure of the FLC is composed of two independent parallel fuzzy control blocks, each of which contains the corresponding fuzzy control rules and a defuzzifier. The incremental output of the FLC is formed by algebraically adding the outputs of the two fuzzy control blocks.

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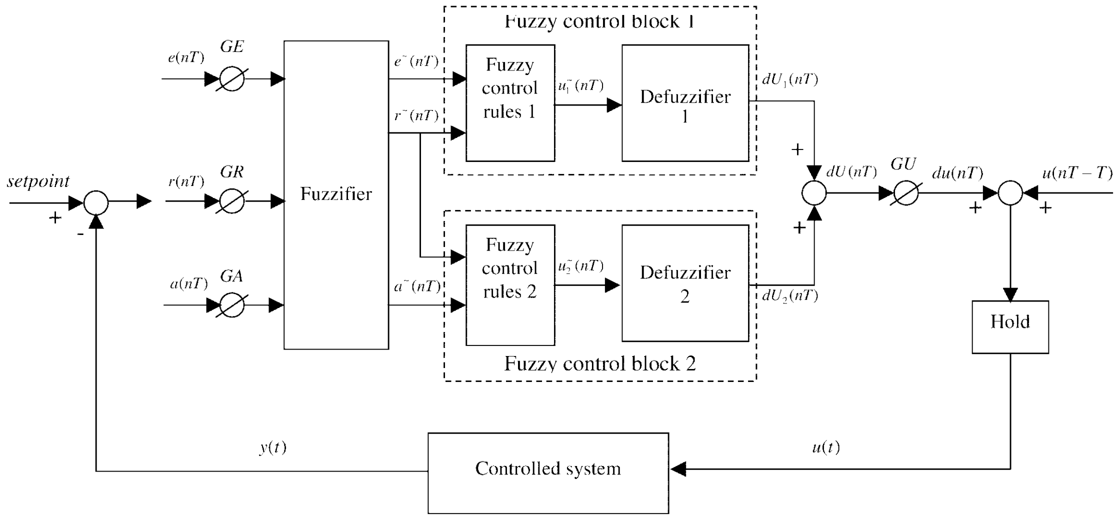


Fig. 1. The structure of FLC suggested in this paper

The following notations are employed.

$$e(t) = \text{setpoint} - y(t), \quad e(nT) = \text{sample}\ [e(t)]$$

$$e^{\sim}(nT) = F(e^*), \quad e^* = GE \times e(nT)$$

$$r(nT) = [e(nT) - e(nT - T)]/T$$

$$r^{\sim}(nT) = F(r^*), \quad r^* = GR \times r(nT)$$

$$a(nT) = [r(nT) - r(nT - T)]/T$$
$$= [e(nT) - 2e(nT - T) + e(nT - 2T)]/T^2$$

$$a^{\sim}(nT) = F(a^*), \quad a^* = GA \times a(nT)$$

$$u(nT) = \mathrm{d}u(nT) + u(nT - T),$$
$$\mathrm{d}u(nT) = GU \times \mathrm{d}U(nT)$$

$$\mathrm{d}U(nT) = \mathrm{d}U_1(nT) + \mathrm{d}U_2(nT)$$

where n is positive integer and T is the sampling period. $y(nT)$, $e(nT)$, $r(nT)$ and $a(nT)$ denote process output, error, rate and acc at sampling time nT, respectively. GE (gain for error) is the input scaler for error, GR (gain for rate) the input scaler for rate, GA (gain for acc) the input scaler for acc and GU (gain for controller output) the output scaler of the FLC. $F(\cdot)$ describes the fuzzification of the scaled input signals. $\mathrm{d}U(nT)$ denotes the incremental output of the FLC at sampling time $nT \cdot \mathrm{d}U_i(nT)$ $(i = 1, 2)$ designate the incremental output of the fuzzy control block i from the defuzzification of the fuzzy set 'output i' $u_i^{\sim}(nT)$ at sampling time nT. Thus the FLC includes the following components:

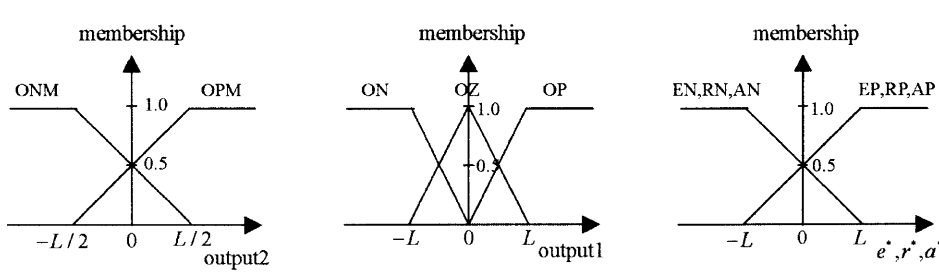
(1) Input scalers GE, GR, GA and output scaler GU
(2) Fuzzification algorithms for scaled error e^*, scaled rate r^*, scaled acc a^* and output of each control block
(3) Fuzzy control rules for each control block
(4) Fuzzy decision-making logic to evaluate the fuzzy control rules for each control block
(5) A defuzzification algorithm to obtain the crisp output of each control block for the control of process.

2.1
Fuzzification algorithms

The fuzzification algorithms for scaled inputs are shown in Fig. 2a.

The fuzzy set 'error' has two members EP (error_positive) and EN (error negative); the fuzzy set 'rate' has two members RP (rate positive) and RN (rate negative); the fuzzy set 'acc' also has two members AP (acc positive) and AN (acc negative). The fuzzy set 'output1' has three members OP (output_positive), OZ (output_zero) and ON (output_negative) as shown in Fig. 2b for the fuzzification of the incremental output of fuzzy control block 1. The fuzzy set 'output2' has two members OPM (output_positive_middle) and ONM (output_negative_middle) as shown in Fig. 2c for the fuzzification of the incremental output of fuzzy control block 2.

Although the grades of membership function of the output members may be decided from the fuzzy control



a Incremental output of fuzzy control block 2

b Incremental output of fuzzy control block 1

c Inputs e^*, r^* and a^* of FLC

Fig. 2a–c. Fuzzification algorithms for inputs and outputs of FLC rules, the definition of fuzzy set 'output1' and 'output2' are necessary for the fuzzification and fuzzy control rules. It should be noted that the fuzzification algorithm of the fuzzy set 'output2' is different from that of the 'output1'. Because the fuzzy control block 2 has a characteristic to compensate the behaviour of the fuzzy control block 1.

2.2 Fuzzy control rules and fuzzy logic for evaluating the fuzzy control rules

Fuzzy control rules must be made based on expert experience and control engineering knowledge, or based on the operator's control action. In this paper, fuzzy control rules were made based on expert experience and control engineering knowledge, and each control rule set comprises four fuzzy control rules for each fuzzy control block. For the fuzzy control block 1, four control rules are given as:

$(\mathrm{R1})_1$: IF error = EP and rate = RP
THEN output = OP

$(\mathrm{R2})_1$: IF error = EP and rate = RN
THEN output = OZ

$(\mathrm{R3})_1$: IF error = EN and rate = RP
THEN output = OZ

$(\mathrm{R4})_1$: IF error = EN and rate = RN
THEN output = ON

For the fuzzy control block 2, four control rules, different from those of the fuzzy control block 1, are given as:

$(\mathrm{R1})_2$: IF rate = RP and acc = AP
THEN output = OPM

$(\mathrm{R2})_2$: IF rate = RP and acc = AN
THEN output = ONM

$(\mathrm{R3})_2$: IF rate = RN and acc = AP
THEN output = OPM

$(\mathrm{R4})_2$: IF rate = RN and acc = AN
THEN output = ONM

A fuzzy control logic is developed to evaluate each fuzzy control rule. The fuzzy logics considered in this work are logic of Zadeh and Lukasiewicz. In evaluating the control rules, it is proper to use the Zadeh AND logic to evaluate the individual control rule, but the Lukasiewicz OR logic to evaluate the implied OR between control rule $(\mathrm{R2})_1$ and $(\mathrm{R3})_1$ in control block 1. The control rules $(\mathrm{R1})_1$–$(\mathrm{R4})_1$ and $(\mathrm{R1})_2$–$(\mathrm{R4})_2$ all employ the Zadeh AND of two conditions in the antecedents, such as one on the scaled error and the other on the scaled rate. Since the Zadeh AND is the minimum of two values, two different conditions arise for each rule in the fuzzy control blocks, namely, one when the scaled error is less than the scaled rate and one when the scaled rate is less than the scaled error in control block 1. In a similar manner, two conditions also arise between scaled rate and scaled acc in control block 2. The eight different combinations of scaled error and scaled rate constituting inputs to the control rules are shown graphically in Fig. 3 for control block 1. For control block 2, the eight different combinations of scaled rate and scaled acc are shown in Fig. 4.

These combinations of inputs must be considered when the fuzzy control rules are evaluated. The results of evaluating the fuzzy control rules $(\mathrm{R1})_1$–$(\mathrm{R4})_1$ when scaled error and rate are within the interval $[-L, L]$, are given in Table 1. In Table 1, μ_{EP} and μ_{EN} (μ_{RP} and μ_{RN}) mean the membership values of EP and EN (RP and RN) in the fuzzy set 'error' (rate). For example, when the values of scaled error e^* and rate r^* are given, let the membership values obtained by using the fuzzification algorithm shown in Fig. 2(a), be given as μ_{EP} and μ_{RP}. Then, say in rule $(\mathrm{R1})_1$, the membership value associated with the member, ON, of the fuzzy set 'output1' is the $\mathrm{Min}(\mu_{\mathrm{EP}}, \mu_{\mathrm{RP}})$. In this way, the membership values are given as listed in Table 1.

Notice that

$$\mu_{\mathrm{EP}} = [r^* + L]/2L = [GE \times e(nT) + L]/2L \tag{1}$$

$$\mu_{\mathrm{EN}} = [L - GE \times e(nT)]/2L \tag{2}$$

$$\mu_{\mathrm{RP}} = [GR \times r(nT) + L]/2L \tag{3}$$

$$\mu_{\mathrm{RN}} = [L - GR \times r(nT)]/2L \tag{4}$$

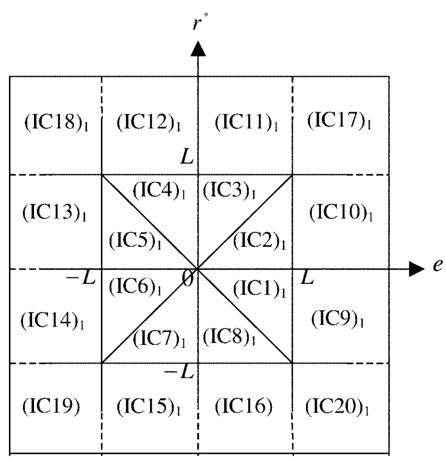


Fig. 3. Possible input combinations of e^* and r^* for the control block 1

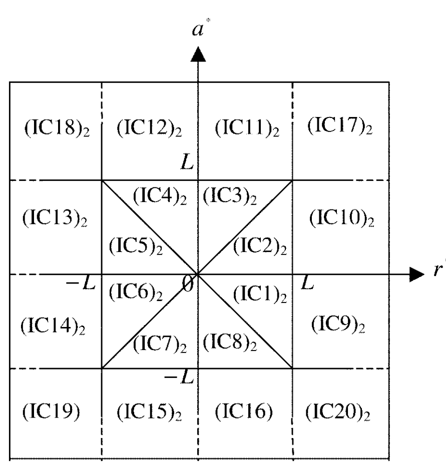


Fig. 4. Possible input combinations of r^* and a^* for the control block 2

Table 1. Results of evaluating the fuzzy control rules for all combinations of inputs when e^* and r^* are within the range $[-L, L]$

Input combination of e^* and r^*	Membership obtained by evaluating fuzzy control rules			
	$(R1)_1$	$(R2)_1$	$(R3)_1$	$(R4)_1$
$(IC1)_1$	μ_{RP}	μ_{RN}	μ_{EN}	μ_{EN}
$(IC2)_1$	μ_{RP}	μ_{RN}	μ_{EN}	μ_{EN}
$(IC3)_1$	μ_{EP}	μ_{RN}	μ_{EN}	μ_{RN}
$(IC4)_1$	μ_{EP}	μ_{RN}	μ_{EN}	μ_{RN}
$(IC5)_1$	μ_{EP}	μ_{EP}	μ_{RP}	μ_{RN}
$(IC6)_1$	μ_{EP}	μ_{EP}	μ_{RP}	μ_{RN}
$(IC7)_1$	μ_{RP}	μ_{EP}	μ_{RP}	μ_{EN}
$(IC8)_1$	μ_{RP}	μ_{EP}	μ_{RP}	μ_{EN}

Table 2. Results of evaluating the fuzzy control rules for all combinations of inputs when r^* and a^* are within the range $[-L, L]$

Input combination of r^* and a^*	Membership obtained by evaluating fuzzy control rules			
	$(R1)_2$	$(R2)_2$	$(R3)_2$	$(R4)_2$
$(IC1)_2$	μ_{AP}	μ_{AN}	μ_{RN}	μ_{RN}
$(IC2)_2$	μ_{AP}	μ_{AN}	μ_{RN}	μ_{RN}
$(IC3)_2$	μ_{RP}	μ_{AN}	μ_{RN}	μ_{AN}
$(IC4)_2$	μ_{RP}	μ_{AN}	μ_{RN}	μ_{AN}
$(IC5)_2$	μ_{RP}	μ_{RP}	μ_{AP}	μ_{AN}
$(IC6)_2$	μ_{RP}	μ_{RP}	μ_{AP}	μ_{AN}
$(IC7)_2$	μ_{AP}	μ_{RP}	μ_{AP}	μ_{RN}
$(IC8)_2$	μ_{AP}	μ_{RP}	μ_{AP}	μ_{RN}

$$\mu_{EP} + \mu_{EN} = 1 \tag{5}$$

$$\mu_{RP} + \mu_{RN} = 1 \tag{6}$$

Also, be aware that in this case the Lukasiewicz OR deduces to the sum of the grades of membership being Ord, since this sum can never be greater than unity for the fuzzy controller under study.

In the same manner, Table 2 shows the results of evaluating the fuzzy control rules for all combinations of inputs when the scaled rate r^* and acc a^* are within the interval $[-L, L]$, for the case of Fig. 4.

Notice that

$$\mu_{RP} = [GR \times r(nT) + L]/2L \tag{7}$$

$$\mu_{RN} = [L - GR \times r(nT)]/2L \tag{8}$$

$$\mu_{AP} = [GA \times a(nT) + L]/2L \tag{9}$$

$$\mu_{AN} = [L - GA \times a(nT)]/2L \tag{10}$$

$$\mu_{RP} + \mu_{RN} = 1 \tag{11}$$

$$\mu_{AP} + \mu_{AN} = 1 \tag{12}$$

2.3 Defuzzification algorithm

In this work, the center of area method is used as the defuzzification algorithm, which amounts to a normalisation of the grades of membership of the members of the fuzzy set being defuzzified to a sum of one. Thus the defuzzified output of a fuzzy set is defined as

$$dU = \frac{\sum(\text{membership of member}) \times (\text{value of member})}{\sum(\text{memberships})} \tag{13}$$

The value used in the defuzzification algorithm, is the value for the members of the fuzzy set, which are chosen as those values for which the grade of membership is maximal. Therefore, these values for the control block 1 are L for the fuzzy member OP, 0 for the fuzzy member OZ and $-L$ for the fuzzy member ON as shown in Fig. 2(b). The values used in the defuzzication algorithm for control block 2 are $L/2$ for the fuzzy member OPM and $-L/2$ for the fuzzy member ONM.

When the defuzzification algorithm given as Eq. (13) is applied to Table 1 and the Lukasiewicz OR logic is used for the membership of the member OZ of the fuzzy set 'output1', the incremental output of fuzzy control block1 at sampling time nT, $dU_1(nT)$, can be described by the following equations.

If $GR \times |r(nT)| \le GE \times |e(nT)| \le L$,

$$dU_1(nT) = \frac{0.5 \times L}{2L - GE \times |e(nT)|} \times [GE \times e(nT) + GR \times r(nT)] \tag{14}$$

If $GE \times |e(nT)| \le GR \times |r(nT)| \le L$,

$$dU_1(nT) = \frac{0.5 \times L}{2L - GR \times |r(nT)|} \times [GE \times e(nT) + GR \times r(nT)] \tag{15}$$

These results can be observed by careful examination of Fig. 3 and Table 1.

If scaled error and/or scaled rate are not within the range $[-L, L]$ of the fuzzification algorithm shown in Fig. 2(a), the incremental output of fuzzy control block 1 is listed in Table 3.

In a similar way, when the defuzzification algorithm is applied to Table 2, the incremental output of fuzzy control block 2 at sampling time nT, $dU_2(nT)$, can be given by the following two equations.

If $GA \times |a(nT)| \le GR \times |r(nT)| \le L$,

$$dU_2(nT) = \frac{0.25 \times L}{2L - GR \times |r(nT)|}[GA \times a(nT)] \tag{16}$$

Table 3. The incremental output of the fuzzy control block 1 when e^* and/or r^* are not within the range $[-L, L]$ of the fuzzification algorithm

Input combinations as shown in Fig. 3	Incremental output of the fuzzy control block 1, $dU_1(nT)$
$(IC9)_1$, $(IC10)_1$	$[GR \times r(nT) + L]/2$
$(IC11)_1$, $(IC12)_1$	$[GE \times e(nT) + L]/2$
$(IC13)_1$, $(IC14)_1$	$[GR \times r(nT) - L]/2$
$(IC15)_1$, $(IC16)_1$	$[GE \times e(nT) - L]/2$
$(IC17)_1$	L
$(IC18)_1$, $(IC20)_1$	0
$(IC19)_1$	$-L$

If $GR \times |r(nT)| \leq GA \times |a(nT)| \leq L$,

$$\mathrm{d}U_2(nT) = \frac{0.25 \times L}{2L - GA \times |a(nT)|}[GA \times a(nT)] \quad (17)$$

If scaled rate and/or scaled acc are not within the range $[-L, L]$ of the fuzzification algorithm, the incremental output of fuzzy control block 2 is listed in Table 4.

Consequently, the overall incremental output of the FLC, $\mathrm{d}U(nT)$, can be obtained by adding incremental output $\mathrm{d}U_1(nT)$ from fuzzy control block 1 and incremental output $\mathrm{d}U_2(nT)$ from fuzzy control block 2.

$$\mathrm{d}U(nT) = \mathrm{d}U_1(nT) + \mathrm{d}U_2(nT) \quad (18)$$

Then the crisp value of incremental output, $\mathrm{d}u(nT)$, can be obtained by multiplying $\mathrm{d}U(nT)$ by output scaler GU.

$$\mathrm{d}u(nT) = GU \times \mathrm{d}U(nT) \quad (19)$$

Thus far, the process through which the incremental output can be obtained using the FLC structure suggested in Fig. 1, has been developed.

Conclusively, the incremental output of the FLC can be divided into four different forms according to the following conditions:

(1) If $GR \times |r(nT)| \leq GE \times |e(nT)| \leq L$ and $GA \times |a(nT)| \leq GR \times |r(nT)| \leq L$,

$$\mathrm{d}u(nT) = \frac{0.5 \times L \times GU}{2L - GE \times |e(nT)|}[GE \times e(nT) + GR \times r(nT)] + \frac{0.25 \times L \times GU}{2L - GR \times |r(nT)|} \times [GA \times a(nT)] \quad (20)$$

(2) If $GR \times |r(nT)| \leq GE \times |e(nT)| \leq L$ and $GR \times |r(nT)| \leq GA \times |a(nT)| \leq L$,

$$\mathrm{d}u(nT) = \frac{0.5 \times L \times GU}{2L - GE \times |e(nT)|}[GE \times e(nT) + GR \times r(nT)] + \frac{0.25 \times L \times GU}{2L - GA \times |a(nT)|} \times [GA \times a(nT)] \quad (21)$$

(3) If $GE \times |e(nT)| \leq GR \times |r(nT)| \leq L$ and $GA \times |a(nT)| \leq GR \times |r(nT)| \leq L$,

$$\mathrm{d}u(nT) = \frac{0.5 \times L \times GU}{2L - GR \times |r(nT)|}[GE \times e(nT) + GR \times r(nT)] + \frac{0.25 \times L \times GU}{2L - GR \times |r(nT)|} \times [GA \times a(nT)] \quad (22)$$

(4) If $GE \times |e(nT)| \leq GR \times |r(nT)| \leq L$ and $GR \times |r(nT)| \leq GA \times |a(nT)| \leq L$,

$$\mathrm{d}u(nT) = \frac{0.5 \times L \times GU}{2L - GR \times |r(nT)|}[GE \times e(nT) + GR \times r(nT)] + \frac{0.25 \times L \times GU}{2L - GA \times |a(nT)|} \times [GA \times a(nT)] \quad (23)$$

If scaled error, rate and/or acc are not within the interval $[-L, L]$ the incremental output of the FLC is obtained from the combinations of incremental outputs for the fuzzy control blocks given in Tables 3 and 4.

Here, if we carefully observe Eq. (20), then we can find an important fact described as below.

$$\mathrm{d}u(nT) = \frac{0.5 \times L \times GU \times GE}{2L - GE \times |e(nT)|} \times e(nT) + \frac{0.5 \times L \times GU \times GR}{2L - GE \times |e(nT)|} \times r(nT) + \frac{0.25 \times L \times GU \times GA}{2L - GR \times |r(nT)|} \times a(nT) \quad (24)$$

Let

$$K_i = \frac{0.5 \times L \times GU \times GE}{2L - GE \times |e(nT)|}, \quad K_p = \frac{0.5 \times L \times GU \times GR}{2L - GE \times |e(nT)|}, \quad K_d = \frac{0.5 \times L \times GU \times GA}{2L - GR \times |r(nT)|} \quad (25)$$

Then the following equation can be written and the fuzzy controller in this work can be considered as a PID type controller with gains K_p, K_i and K_d which are changed nonlinearly according to the error, rate and acc. That is,

$$\mathrm{d}u(nT) = K_i\, e(nT) + K_p\, r(nT) + K_d\, a(nT) \quad (26)$$

This nonlinear type PID controller may be named a nonlinear fuzzy PID controller, where K_p is defined a proportional gain, K_i an integral gain and K_d a derivative gain. In a similar fashion, K_p, K_i and K_d can also be solved for Eqs. (21)–(23).

Let us define the constant proportional gain K_p^*, integral gain K_i^* and derivative gain K_d^* when error, rate and acc are zero in Eq. (25).

$$K_p^* = \frac{GU \times GR}{4}, \quad K_i^* = \frac{GU \times GE}{4}, \quad K_d^* = \frac{GU \times GA}{8} \quad (27)$$

There are infinitely many combinations of GE, GR, GA and GU so that Eq. (27) may hold true. Once GE, GR and GA

Table 4. The incremental output of the fuzzy control block 2 when r^* and/or a^* are not within the range $[-L, L]$ of the fuzzification algorithm

Input combinations as shown in Fig. 4	Incremental output of the fuzzy control block 2, $\mathrm{d}U_2(nT)$
$(IC9)_2$, $(IC10)_2$, $(IC13)_2$, $(IC14)_2$	$0.5 \times GA \times a(nT)$
$(IC11)_2$, $(IC12)_2$, $(IC17)_2$, $(IC18)_2$	$0.5 \times L$
$(IC15)_2$, $(IC16)_2$, $(IC19)_2$, $(IC20)_2$	$-0.5 \times L$

are selected, GU can be uniquely determined to satisfy Eq. (27).

A design procedure for the suggested nonlinear fuzzy PID controller is as follows.

Step 1 Input scalers GE, GR and GA for error, rate and acc, respectively, are properly selected from the input/output data of a controlled process.

Step 2 Constant proportional gain K_p^* is selected so that it may satisfy a rising time requirement in a controller design specification.

Step 3 Output scaler GU is decided under K_p^* and then constant integral gain K_i^* and derivative gain K_d^* are decided from Eq. (27).

Step 4 The linear PID control parameters obtained from step 2 and 3 should be tuned in order for the controlled process to be stable and to exhibit better transient behaviour.

Step 5 When constant PID gains are tuned properly, a nonlinear fuzzy PID control law results from substituting constant PID gains into Eqs. (20)–(23).

In conclusion, the nonlinear fuzzy PID control method can be readily applied to nonlinear and/or uncertain systems only if constant proportional gain K_p^* is selected from the input/output data so that K_p^* may satisfy a rising time requirement, regardless of imperfect model information and nonlinearities. Unfortunately, there is no mathematical expression to sufficiently prove the stability of the nonlinear fuzzy PID control system even though fuzzy control rules are designed in a stable manner. By the way, the derivation procedure of the FLC makes the linear PID control law given as Eq. (27). And also the PID gains K_p, K_i and K_d of the nonlinear PID are adjusted around the linear PID gains in the direction that the plant output has a better performance and is more stable than that of the linear PID. Therefore, the stability is fully dependent upon the linear PID control system. That is, the nonlinear PID control system is always stable as long as the linear PID control system is stable.

3 Computer simulations

In order to assure the effectiveness of the nonlinear fuzzy PID controller, computer simulations were executed for the following example plants.

(1) Plant open-loop transfer function $10/(s(s+1))$

This plant model is an illustration of a stable underdamped system. It was used to test whether the nonlinear fuzzy PID controller can comprise and operate correctly or not, even when model information cannot be used at all. The results were given in Fig. 5.

As is shown, the nonlinear fuzzy PID control system exhibits a good unit step response with nearly zero overshoot, faster rising time and more satisfactory settling time than that of the given plant and the nonlinear fuzzy PI control system in [9]. In this respect, it is verified that the nonlinear fuzzy PID controller can be designed only using input/output information about the given controlled plant.

(2) Plant described by nonlinear differential equation $\ddot{y} + \dot{y} = 0.5y^2 + 2u$

This model is an illustration of nonlinear plants diverged slowly. This was used to assure that the nonlinear fuzzy PID has a nonlinear characteristic and is stable controller. The results were shown in Fig. 6. As is shown, the nonlinear fuzzy PID exhibits a good transient response and a stable control action despite of the divergent controlled process.

While, the nonlinear fuzzy PI does not diverge but exhibits a damped fluctuating transient response. From the simulation results, that the fuzzy PID controller turns out a nonlinear controller and exhibits a good stable transient and steady state performance without regard to a point that a given plant is linear or nonlinear.

(3) Plant open-loop transfer function $(e^{-0.2s})/(s(s+1))$

This is a plant model of time delay or nonminimum phase systems. The simulation results were given in Fig. 7. As was expected, the nonlinear fuzzy PID exhibits better performance than that of the nonlinear fuzzy PI.

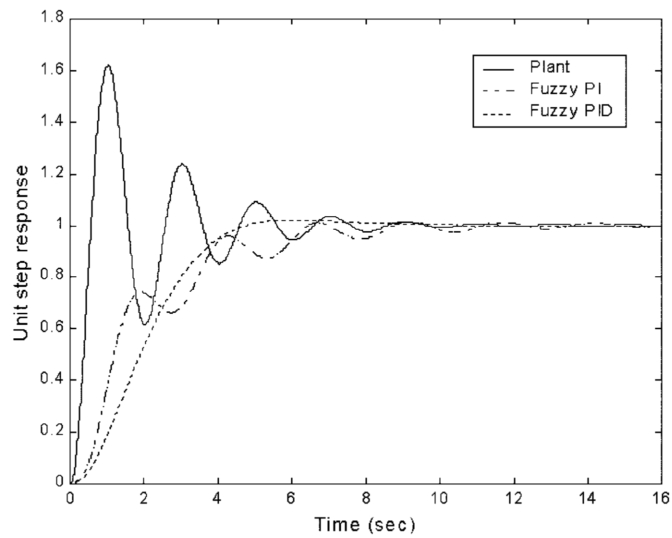


Fig. 5. Comparison of unit step responses for simulation (1)

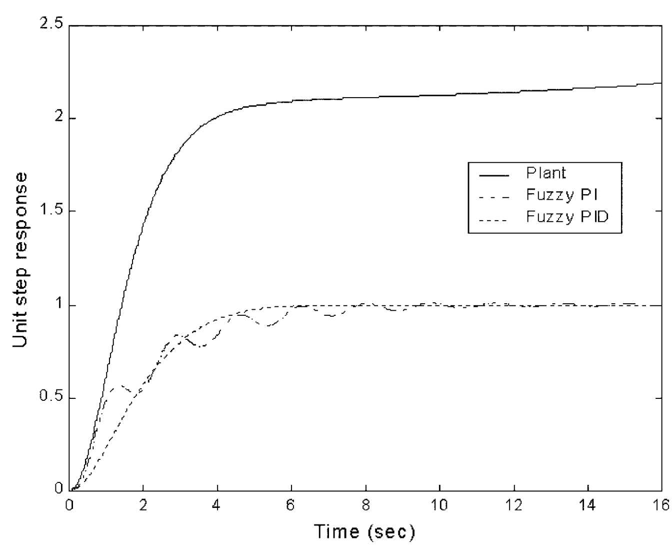


Fig. 6. Comparison of unit step responses for simulation (2)

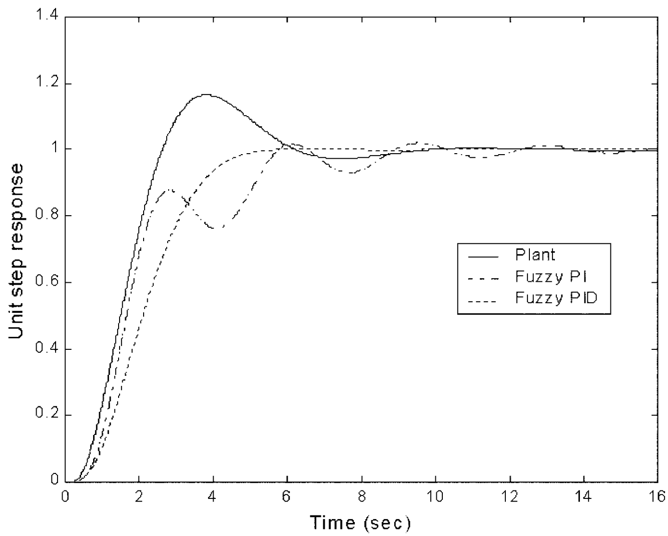


Fig. 7. Comparison of unit step responses for simulation (3)

By the way, in the design of the nonlinear fuzzy PID controller, it was also known that the combination of GE, GR and GA based on the given input/output relation and GU based on the proportional gain K_p^* must be selected carefully, especially proportional gain used to decide GU, against the possibility of divergence. According to the simulation experience, when the controlled process is stable minimum phase system, the selection of K_p^* may be allowed to be the value slightly greater than unity and then the performance may almost be identical regardless of variant values of K_p^*. When the controlled process is nonlinear and/or nonminimum phase system, K_p^* must be selected as the value smaller than unity in order not to generate excessive control input, and it must be tuned step by step with small incremental values to obtain stable desired output.

4
Conclusion

In this paper, a nonlinear PID control algorithm was derived in order to control nonlinear and/or uncertain systems. The nonlinear fuzzy PID controller has the characteristic of a nonlinear controller with time varying PID gains. While it is easy to design for the linear or nonlinear time invariant systems, it is more or less hard to design and requires careful tuning of controller parameters for nonminimum phase systems. The most important advantage of the nonlinear fuzzy PID controller is that it is possible to design a control system whose plant dynamics is not known, by only using the input/output information. Also, a linear PID controller can naturally be derived under the design procedure of the nonlinear fuzzy PID controller, although plant dynamics is not known.

The usefulness and effectiveness were verified through the computer simulations for example systems. Since the resultant control law has an analytical form and the number of fuzzy control rules is rather small, controller designers can expect an effective implementation of a control system in real time without computational burden.

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