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FUZZY SYSTEMS AND THEIR MATHEMATICS

# A novel combination rule for conflict management in data fusion

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## Abstract

How to handle conflict in Dempster-Shafer evidence theory is an open issue. Many approaches have been proposed to solve this problem. The existing approaches can be divided into two kinds. The first is to improve the combination rule, and the second is to modify the data model. A typical method to improve combination rule is to assign the conflict to the total ignorance set $\Theta$. However, it does not make full use of conflict information. A novel combination rule is proposed in this paper, which assigns the conflicting mass to the power set (ACTP). Compared with modifying data model, the advantage of the proposed method is the sequential fusion, which greatly decrease computational complexity. To demonstrate the efficacy of the proposed method, some numerical examples are given. Due to the less information loss, the proposed method is better than other methods in terms of identifying the correct evidence, the speed of convergence and computational complexity.

**Keywords** Dempster-Shafer evidence theory · Conflict management · Power set · Combination rule · Target recognition

## 1 Introduction

Data fusion has always been a hot topic of research. Many theories proposed to address it, such as the Dempster-Shafer evidence theory (Xiao 2020; Fei and Wang 2022), information fusion in quantum theory (Xiao and Pedrycz 2022; Deng et al. 2023), entropy-based approaches (Pan and Gao 2023; Yang et al. 2022; Zhou et al. 2021), possibility theory (Solaiman and Bossé 2019; Zhou et al. 2022).

Among these methods, the Dempster-Shafer evidence theory has gotten a lot of attention. Compared to probability theory, it requires fewer conditions and can handle uncertain information such as fuzzy, missing, and contradictory information. Therefore it is widely used in decision-making (Xiao 2019; Song et al. 2019; Liao et al. 2020), risk analysis (Chen and Deng 2022; Liang et al. 2021), uncertainty measurements (Moral-García and Abellán 2021; Meng et al. 2020), classification system (Li et al. 2023a), pattern classification (Zhun-Ga Liu et al. 2019; Huang and Xiao 2023; Xiao et al. 2022a), information fusion (Xiao 2022; Xinyang et al. 2022; Xiao et al. 2022b), group decision making settings (Li et al. 2023b; Zhang and Li 2022). Dempster's combination rule is a crucial method for multi-source combination. But when fusing the highly conflicting evidences, counter-intuitive conditions often occur.

Zadeh (1979) presented an example to illustrate the drawback of the combination rule, that has caused widespread concern. Dealing with contradictory evidence has been extensively researched. The solutions can be divided into two kinds: improving the combination rules and modifying the data model. Yager (1987), Dubois and Prade (1992), Smets (1990) and Lefevre et al. (2002) proposed other combination rules. In addition, Murphy (2000) presented an improved combination rule, which evenly distributes the basic probabilities of the collected evidence. Then, a weighted average in terms of the similarity of evidences was presented for conflict management (Zhang and Deng 2019). Furthermore, several recent works were reported in this area. Liu et al. proposed a new classifier fusion method (Liu et al. 2020). A geometric approach to the theory of evidence was proposed by Cuzzolin (2008). Abellan et al. presented a new hybrid rule and analyzed the mathematical properties Abellán et al. (2021). A method of dealing with high conflicts from the perspective of the network was proposed by Xiong et al. (2021). Deng et

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al. proposed a method can handle conflicting evidence and produce a fused result that reflects the degree of agreement among the source (Deng et al. 2021). In various fields, conflict management remains a widely discussed and relevant topic.

Yager (1987) pointed out that the normalization of the combination rule is the main reason for counter-intuitive results. To address this issue, the conflict is assigned to the total ignorance set $\Theta$ without the normalization step. Yager's method partially mitigates the counter-intuitive situation of Dempster's combination rule. However, there are two shortcomings. One is that the convergence rate is relatively slow, and the other is that the uncertainty degree is relatively high. The main reason is that almost all conflicts are not utilized. If all of the conflict is assigned to $\Theta$, it would result in the loss of some useful information. In fact, the conflict information contains certain information that could be utilized.

To overcome the problem discussed above, a novel conflict allocation method is proposed. The main contribution is to assign the conflicting mass to the power set (Song and Deng 2021), rather than the total ignorance set. The proposed method offers significant advantages, mainly due to its capability of integrating high-conflict evidence while minimizing the loss of information and accurately identifying the target. Compared to other methods, it shows superior convergence performance and reduces computational complexity. Furthermore, the proposed method supports sequential fusion, making it appropriate for high real-time update systems.

This paper is structured as follows. In Sect. 2, a brief overview of the concepts of Dempster-Shafer evidence theory and a discussion of Yager's rule. In Sect. 3, a novel method that assigns the conflicting mass to the power set is proposed. The framework and pseudo code of the proposed method are also provided. In Sect. 4, some numerical examples are presented to compare it with other commonly methods. Concluded in Sect. 5.

## 2 Preliminaries

In this section, brief introductions to previous knowledge are presented.

### 2.1 Dempster-Shafer evidence theory

Dempster-Shafer evidence theory was first proposed by Dempster (2008) and then expanded by Shafer (1976). It has been researched ever since (Deng 2020; Song et al. 2018; Deng and Jiang 2020), and some applications are being studied in fault diagnosis (Gong et al. 2018) and human reliability analysis (Gao et al. 2021).

#### 2.1.1 Frame of discernment

Let $\Theta$ be a fixed set of n exclusive and exhaustive elements, called the framework of discernment (FOD), defined as follows (Shafer 1976):

$$\Theta = \{\theta_1, \theta_2, ..., \theta_n\}. \tag{1}$$

The power set of $\Theta$ is denoted as $2^\Theta$, and has $2^n$ elements, $2^\Theta$ is indicated by:

$$\begin{aligned} 2^\Theta = \{&\emptyset, \{\theta_1\}, \{\theta_2\}, ..., \{\theta_n\}, \{\theta_1, \theta_2\}, ..., \\ &\{\theta_1, \theta_n\}, ..., \{\theta_1, ..., \theta_n\}, ..., \Theta\}. \end{aligned} \tag{2}$$

It is different from power sets, as a new type of set called random permutation set have been proposed (Deng 2022). It has received a lot of attention (Zhou et al. 2023; Chen et al. 2023).

#### 2.1.2 Mass function

For $\Theta$, the basic probability assignment (BPA), also known as the mass function, can be defined as follows (Shafer 1976):

$$m : 2^\Theta \rightarrow [0, 1], \tag{3}$$

constrained conditions as follows:

$$\begin{cases} \sum\limits_{A \in 2^\Theta} m(A) = 1, \\ m(\emptyset) = 0. \end{cases} \tag{4}$$

If $A \in 2^\Theta$ and $A \neq \emptyset$, $m(A)$ represents the degree of belief in the supporting evidence $A$. The larger $m(A)$, the stronger the evidence that proves. There are some studies about mass function (Xiao 2021; Han et al. 2016; Chen et al. 2021), which has been used in many fields. Related references include (Deng and Jiang 2022; Chang et al. 2021).

#### 2.1.3 Dempster's combination rule

Given two BPAs $m_1$ and $m_2$, their combination $m_1 \bigoplus m_2$ can be mathematically defined as follows (Shafer 1976):

$$m(A) = \begin{cases} \frac{1}{1-k} \sum\limits_{B \bigcap C = A} m_1(B) m_2(C) & A \neq \emptyset, \\ 0 & A = \emptyset, \end{cases} \tag{5}$$

where $A, B, C \in 2^\Theta$, $k$ is a normalization factor,

$$k = \sum_{B \bigcap C = \emptyset} m_1(B) m_2(C). \tag{6}$$

The conflict coefficient $k$ represents the degree of conflict between two BPAs. When $k = 0$ indicates that $m_1$ and $m_2$ are consistent with each other, while $k = 1$ means that $m_1$ and $m_2$ are completely contradictory. In other words, the two pieces of evidence strongly support different hypotheses that are incompatible with each other. However, it should be noted that when combining highly contradictory evidence using Dempster's combination rule, the resulting conclusion may be counter-intuitive (Zadeh 1979).

### 2.2 Yager's rule

Yager proposed a rule to overcome the counter-intuitive problem of the Dempster's combination rule, which can be described as (Yager 1987):

$$m_Y(A) = \begin{cases} \sum\limits_{B \bigcap C = A} m_1(B) m_2(C) & A \neq \emptyset, A \subset \Theta, \\ 0 & A = \emptyset, \\ \sum\limits_{B \bigcap C = \Theta} m_1(B) m_2(C) + k & A = \Theta, \end{cases} \tag{7}$$

where $A, B, C \in 2^{\Theta}$, $k$ is conflicting mass,

$$k = \sum_{B \bigcap C = \emptyset} m_1(B) m_2(C).$$

Yager's rule proposes to allocate conflicting mass $k$ to the total ignorance set $\Theta$. However, it remains to be discussed whether this approach to managing conflicts is reasonable or not (Yang and Dong-Ling 2013).

### 2.3 Murphy's rule

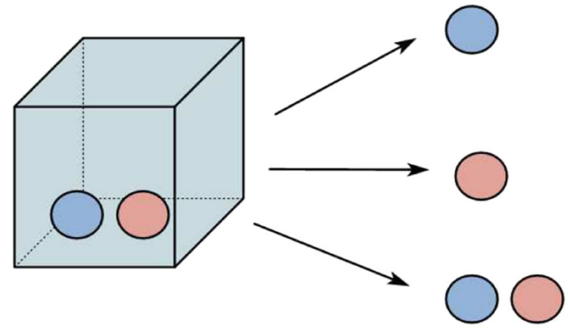
Suppose there are n BPAs $m_1, m_2, ..., m_n$ on $\Theta$, Murphy's rule is expressed as (Murphy 2000),

$$m_{AM}(A) = \frac{1}{n} \sum_{i=1}^{n} m_i(A). \tag{8}$$

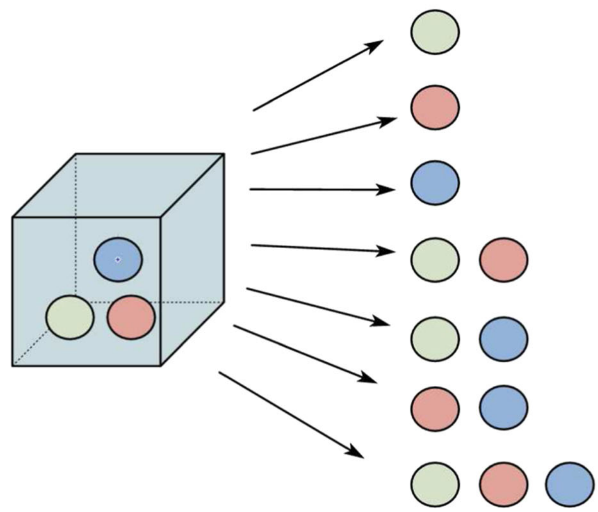
Then use Eq. (5) and Eq. (6) combine $m_{AM(A)}$ n - 1 times to get $m_M(A)$. In Murphy's rule, all BPAs are assigned to the equal weight without taking into account the difference or similarity among different BPAs.

## 3 The proposed method

A reasonable method for conflict management named ACTP is proposed in this section. The corresponding framework is shown in Sect. 3.1. The proposed algorithm is illustrated in Sect. 3.2. A simple example is provided in Sect. 3.3.



(a) 2 balls in the box



(b) 3 balls in the box

**Fig. 1** The situations of taking balls out of the box. **a** When there are two balls in the box, there are three possible outcomes when taking balls out of the box. **b** When there are three balls in the box, there are seven possible outcomes when taking balls out of the box

### 3.1 Assign conflict to the power set (ACTP)

***Example 1*** Suppose $\Theta = \{A, B\}$, two BPAs as follows,

$$m_1 : m_1(\{A\}) = 1, m_1(\{B\}) = 0,$$
$$m_2 : m_2(\{A\}) = 0, m_2(\{B\}) = 1.$$

As shown in Example 1, it can be seen that $m_1$ and $m_2$ are completely opposite. If we apply Yager's rule, $m_Y(\{A\}) = m_Y(\{B\}) = 0$, $m_Y(\{\Theta\}) = 1$. Obviously, after the conflict is given to $\Theta$, the result is unreasonable.

Suppose there are two balls of different colors in an opaque box. If we take the ball or balls out of the box, then we can get three ($2^2 - 1 = 3$) situations with the same possibility as shown in Fig. 1a. If there are three balls of different colors, then there are seven ($2^3 - 1 = 7$) situations with the same possibility as shown in Fig. 1b. The main idea of the proposed method is assigning conflict to all situations with the same

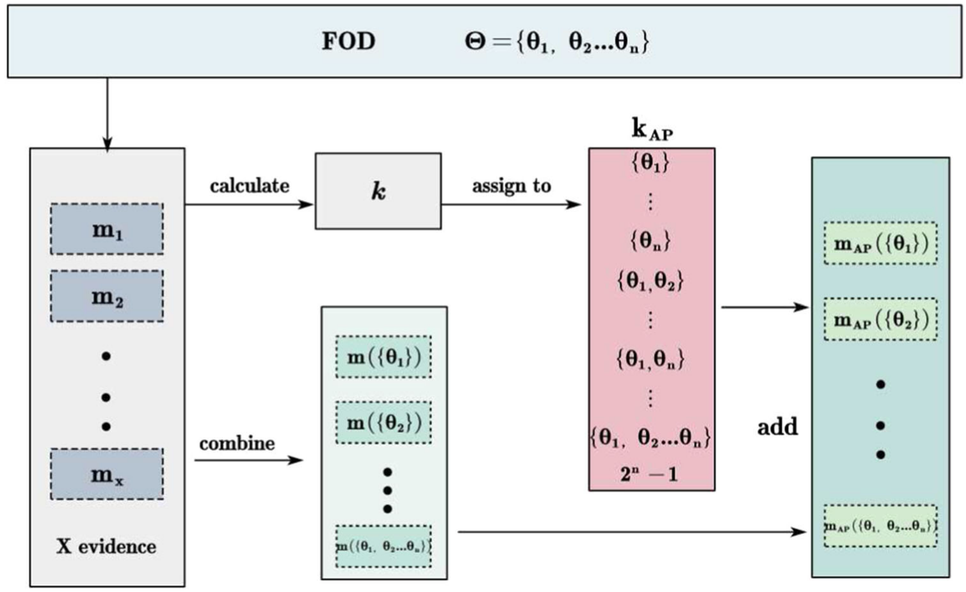



**Fig. 2** The framework of ACTP

**Table 1** The results of different methods for Example 3

| | $\{A\}$ | $\{B\}$ | $\{A, B\}$[a] | $\Theta$[1] |
|---|---|---|---|---|
| Dempster Shafer (1976) | 0.6923 | 0.3077 | 0 | \ |
| Yager (1987) | 0.3600 | 0.1600 | 0 | 0.4800 |
| Proposed method | 0.5200 | 0.3200 | 0.1600 | \ |

[a] In Table 1, $m(\{A, B\}) \neq m(\Theta)$. $m(\Theta)$ is the total ignorance set, which means all the mass functions in $2^\Theta$ are totally unknown, including $m(\{A, B\})$, $m(\{A\})$ and $m(\{B\})$

possibility, that is, all elements in the power set except empty set. By doing this, useful information will not be lost and will be distributed reasonably.

A novel combination rule for conflict management in data fusion which assigns conflict to the power set (ACTP) is proposed. The framework of ACTP is described as Fig. 2. In FOD $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$, assume $m_1, m_2...m_x$ are $x$ pieces of evidence. The power set $2^\Theta$ contains $2^n$ elements. Conflicting mass $k$ is assigned to all elements in the power set except empty set. In this way, $k$ is divided into $2^n - 1$ parts. We can then utilize the proposed rule to combine the evidence and derive the final combined result.

## 3.2 The proposed algorithm

Let $\Theta$ be a fixed set, which has n mutually exclusive and exhaustive elements. The power set of $\Theta$ is $2^\Theta$, in which the number of elements is $2^n$. The formula is given as:

$$\mathbf{k}_{AP} = \frac{k}{2^n - 1}, \tag{9}$$

where $k$ is defined in Eq. (6) and $n$ represents each subset of the power set $2^\Theta$ except empty set. $\mathbf{k}_{AP}$ is used to redistribute the conflicting mass, which divides $k$ into $2^n - 1$.

For example, given a FOD with 4 elements,

$$\mathbf{k}_{AP} = \frac{k}{2^4 - 1} = \frac{k}{15}.$$

As shown in the example, conflict is divided into 15 on average.

Given two BPAs $m_1$ and $m_2$. For the combination of $m_1 \bigoplus m_2$, the resulting mass denoted $m_{AP}$ is given as:

$$m_{AP}(A) = \begin{cases} \sum\limits_{B \bigcap C = A} m_1(B)m_2(C) + \mathbf{k}_{AP}, & A \neq \emptyset, \\ 0, & A = \emptyset, \end{cases} \tag{10}$$

$A$ denotes all subsets in $\Theta$ excluding the empty set $\emptyset$, and $m_{AP}(A)$ represents the belief value that support the proposition $A$.

## 3.3 A simple example

A simple example to show the calculation of ACTP.

***Example 2*** Suppose $\Theta = \{A, B, C\}$, two BPAs as follows,

$$m_1 : m_1(\{A\}) = 0.6, m_1(\{C\}) = 0.2, m_1(\{A, B\}) = 0.2,$$
$$m_2 : m_2(\{B\}) = 0.7, m_2(\{A, B, C\}) = 0.3.$$

As shown in Example 2, it can be seen that the value of $m_1$ supports the object A and $m_2$ support the object $B$, while $m_1(\{A\}) = 0.6$ and $m_2(\{B\}) = 0.7$. They also have multiple

**Table 2** The results of different methods for Example 4

| | $\{A\}$ | $\{B\}$ | $\{C\}$ | $\{A, B\}$ | $\{A, C\}$ | $\{B, C\}$ | $\{A, B, C\}^1$ | $\Theta^a$ |
|---|---|---|---|---|---|---|---|---|
| Dempster's rule Shafer (1976) | 0 | 1 | 0 | 0 | 0 | 0 | 0 | \ |
| Yager's rule Yager (1987) | 0 | 0.0001 | 0 | 0 | 0 | 0 | 0 | 0.9999 |
| Proposed method | 0.1428 | 0.1429 | 0.1428 | 0.1428 | 0.1428 | 0.1428 | 0.1428 | \ |

[a] In Table 2, $m(\{\Theta\})$ including $m(\{A, B, C\})$, $m(\{A, B\})$, $m(\{A, C\})$, $m(\{B, C\})$, $m(\{A\})$, $m(\{B)$ and $m(\{C\})$. So $m(\{A, B, C\}) \neq m(\Theta)$

**Table 3** Five BPAs in Example 5

| BPA | $\{A\}$ | $\{B\}$ | $\{A, B\}$ | $\{A, C\}$ | $\{B, C\}$ |
|---|---|---|---|---|---|
| $m_1$ | 0.5 | 0 | 0.3 | 0 | 0.2 |
| $m_2$ | 0 | 0.8 | 0 | 0 | 0.2 |
| $m_3$ | 0.6 | 0.2 | 0 | 0.2 | 0 |
| $m_4$ | 0.7 | 0.1 | 0 | 0.2 | 0 |
| $m_5$ | 0.8 | 0 | 0 | 0.1 | 0.1 |

objects, while $m_1(\{A, B\}) = 0.2$ and $m_2(\{A, B, C\}) = 0.3$. The calculation processes are given as follows:

First calculate the conflicting mass $k$ according to Eq. (6),

$$k = m_1(\{A\}) \times m_2(\{B\}) + m_1(\{C\}) \times m_2(\{B\})$$
$$= 0.6 \times 0.7 + 0.2 \times 0.7 = 0.56.$$

Then, get the $\mathbf{k}_{AP}$ according to Eq. (9),

$$\mathbf{k}_{AP} = \frac{k}{2^n - 1} = \frac{0.56}{2^3 - 1} = 0.08.$$

Finally, using the combination rules based on Eq. (10).

$$m_{AP}(\{A\}) = m_1(\{A\}) \times m_2(\{A, B, C\}) + \mathbf{k}_{AP}$$
$$= 0.18 + 0.08 = 0.26,$$
$$m_{AP}(\{B\}) = m_1(\{A, B\}) \times m_2(\{B\}) + \mathbf{k}_{AP}$$
$$= 0.14 + 0.08 = 0.22,$$
$$m_{AP}(\{C\}) = m_1(\{C\}) \times m_2(\{A, B, C\}) + \mathbf{k}_{AP}$$
$$= 0.06 + 0.08 = 0.14,$$
$$m_{AP}(\{A, B\}) = m_1(\{A, B\}) \times m_2(\{A, B, C\}) + \mathbf{k}_{AP}$$
$$= 0.06 + 0.08 = 0.14,$$
$$m_{AP}(\{A, B, C\}) = m_{AP}(\{A, C\}) = m_{AP}(\{B, C\}) = \mathbf{k}_{AP} = 0.08$$

It can be seen from the result that $m_{AP}(A)$ is the largest, so object $A$ is the target.

## 4 Examples and discussions

In this part, some examples are given to demonstrate the benefits of the proposed method.

***Example 3*** Suppose $\Theta = \{A, B\}$, and the following are the two BPAs,

$$m_1 : m_1(\{A\}) = 0.6, m_1(\{B\}) = 0.4,$$
$$m_2 : m_2(\{A\}) = 0.6, m_2(\{B\}) = 0.4.$$

As shown in Example 3, $m_1$ and $m_2$ has the same value, while $m_1(\{A\}) = 0.6$ and $m_2(\{A\}) = 0.6$, $m_1(\{B\}) = 0.4$ and $m_2(\{B\}) = 0.4$. The results shown in Table 1.

As shown in Table 1, it can be observed that all methods correctly identify $A$. ACTP is found to be as effective as the other methods when dealing with non-conflicting evidence.

***Example 4*** Suppose $\Theta = \{A, B, C\}$, and the following are the two BPAs (Zadeh 1979),

$$m_1 : m_1(\{A\}) = 0.99, m_1(\{B\}) = 0.01,$$
$$m_2 : m_2(\{B\}) = 0.01, m_2(\{C\}) = 0.99.$$

As demonstrated in Example 4, two sources are in high conflict, where source 1 strongly supports $A$ and source 2 supports $C$. The resulting combination is presented in Table 2.

Table 2 shows that when using Dempster's rules, the result is $m(\{B\}) = 1$. This is counter-intuitive. Using Yager's rule, the result is $m_Y(\{B\}) = 0.0001$ and $m_Y(\Theta) = 0.9999$, indicating that B only has minimal support and most of the conflict is assigned to the total ignorance set. Hence, both Dempster's and Yager's rules are deemed unreasonable.

According to ACTP, $m_{AP}(B)$ is not significantly greater than other mass functions, indicating that $B$ does not have more support than others. This result is evidently more reasonable, as $A$, $B$, and $C$ have approximate probabilities.

***Example 5*** Suppose $\Theta = \{A, B, C\}$, and there are five BPAs as shown in Table 3 (Guo and Li 2011),

In Example 5, we presented the outcomes of the other five common methods of combining evidence and compared them with our proposed method. The experimental results are illustrated in Tables 4, 5 and Fig. 3. Some discussions are detailed as follows.

(1) Identify the correct evidence

As shown in Table 4, Dempster's rule produces counterintuitive results. Although the following evidence supports $A$,

**Table 4** Results of different methods for Example 5

| | | Time = 1 ($m_{12}$) | Time = 2 ($m_{123}$) | Time = 3 ($m_{1234}$) | Time = 4 ($m_{12345}$) |
|---|---|---|---|---|---|
| Dempster Shafer (1976) | $\{B\}$ | 0.9200 | 0.9259 | 0.8621 | 0.7576 |
| | $\{C\}$ | 0 | 0.0741 | 0.1379 | 0.2424 |
| | $\{BC\}$ | 0.0800 | 0 | 0 | 0 |
| Yager (1987) | $\{B\}$ | 0.4600 | 0.1000 | 0.0100 | 0.0010 |
| | $\{C\}$ | 0 | 0.0080 | 0.0016 | 0.0003 |
| | $\{BC\}$ | 0.0400 | 0 | 0 | 0 |
| | $\{\Theta\}$ | 0.5000 | 0.8920 | 0.9884 | 0.9987 |
| Murphy (2000) | $\{A\}$ | 0.1964 | 0.4450 | 0.7820 | 0.9464 |
| | $\{B\}$ | 0.7143 | 0.5284 | 0.2096 | 0.0515 |
| | $\{C\}$ | 0 | 0.0158 | 0.0072 | 0.0019 |
| | $\{AB\}$ | 0.0321 | 0.0029 | 0.0002 | 0 |
| | $\{AC\}$ | 0 | 0.0009 | 0.0005 | 0.0001 |
| | $\{BC\}$ | 0.0572 | 0.0070 | 0.0005 | 0.0001 |
| Li et al. (2001) | $\{A\}$ | 0.1250 | 0.3271 | 0.4448 | 0.5193 |
| | $\{B\}$ | 0.6600 | 0.3973 | 0.2818 | 0.2207 |
| | $\{C\}$ | 0 | 0.0080 | 0.0016 | 0.0003 |
| | $\{AB\}$ | 0.0750 | 0.0892 | 0.0741 | 0.0599 |
| | $\{AC\}$ | 0 | 0.0595 | 0.0988 | 0.0999 |
| | $\{BC\}$ | 0.1400 | 0.1189 | 0.0988 | 0.0999 |
| Liang et al. (2008) | $\{A\}$ | 0.1250 | 0.4970 | 0.06289 | 0.6865 |
| | $\{B\}$ | 0.6600 | 0.2985 | 0.1900 | 0.1438 |
| | $\{C\}$ | 0 | 0.0215 | 0.0068 | 0.0017 |
| | $\{AB\}$ | 0.0750 | 0.0631 | 0.0471 | 0.0387 |
| | $\{AC\}$ | 0 | 0.0385 | 0.0644 | 0.0648 |
| | $\{BC\}$ | 0.1400 | 0.0814 | 0.0628 | 0.0645 |
| Proposed method | $\{A\}$ | 0.0714 | 0.2824 | 0.5222 | 0.6858 |
| | $\{B\}$ | 0.5314 | 0.2396 | 0.1070 | 0.0557 |
| | $\{C\}$ | 0.0714 | 0.1191 | 0.0986 | 0.0744 |
| | $\{AB\}$ | 0.0714 | 0.0825 | 0.0583 | 0.0391 |
| | $\{AC\}$ | 0.0714 | 0.1111 | 0.0970 | 0.0547 |
| | $\{BC\}$ | 0.1114 | 0.0825 | 0.0583 | 0.0508 |
| | $\{ABC\}$ | 0.0714 | 0.0825 | 0.0583 | 0.0391 |

**Table 5** Results of different methods for Example 5

| | Time=1 | Time=2 | Time=3 | Time=4 |
|---|---|---|---|---|
| Dempster Shafer (1976) | B | B | B | B |
| Yager (1987) | B | B | B | B |
| Murphy (2000) | B | B | A | A |
| Li et al. (2001) | B | B | A | A |
| Liang et al. (2008) | B | A | A | A |
| Proposed method | B | A | A | A |

the Dempster's rule supports $C(C = 0.8578)$, but completely against $A(A = 0)$. No matter how much more evidence there is in support of A, Yager's rule supports $C(C = 0.00129)$ and is totally against $A(A = 0)$. The value of the total ignorance set $\Theta$ is increasing, which means increasing uncertainty in the system. Other methods and ACTP can correctly identify $A$.

(2) The speed of convergence

According to Table 5, Murphy and Li et al. identify the correct target until the 4th evidence appears. In comparison,

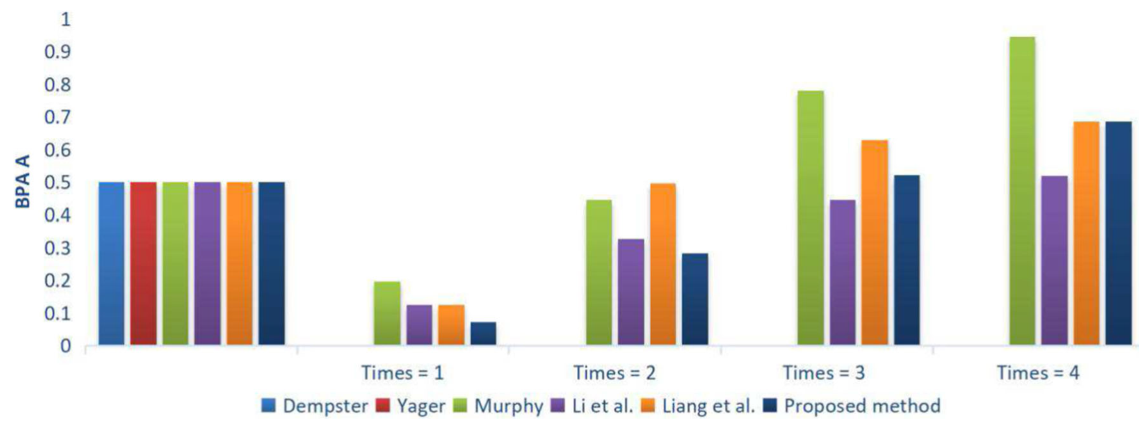



**Fig. 3** Mass function value of target A

**Table 6** Five sources of information

| BPA | $\{F_1\}$ | $\{F_2\}$ | $\{F_3\}$ |
|---|---|---|---|
| $m_1$ | 0.5 | 0.2 | 0.3 |
| $m_2$ | 0 | 0.9 | 0.1 |
| $m_3$ | 0.55 | 0.1 | 0.35 |
| $m_4$ | 0.55 | 0.1 | 0.35 |
| $m_5$ | 0.55 | 0.1 | 0.35 |

**Table 7** Fused results of ACTP

| | $m(F_1)$ | $m(F_2)$ | $m(F_3)$ |
|---|---|---|---|
| ACTP | 0.3923 | 0.1020 | 0.2270 |

the convergence speed of the proposed method is better, since identifying correct target A only by two times combined.

(3) Computational complexity

In Fig. 3, Murphy's method achieves a much higher value than other methods. If there are n evidences in the system, Murphy's method efficiently combines conflicting evidence by simple averaging. Then use the Dempster's method to combine n-1 times. The proposed method is a sequential fusion method. When one piece of evidence comes, integrate it immediately. In comparison, the proposed method has lower computational complexity. For example, given a system with 10000 evidence, Murphy's method needs 9999 times Dempster's combination process. While the proposed method just need 1 time combination process.

In conclusion, the proposed method is better than other methods in terms of identify the correct evidence, the speed of convergence and computational complexity.

**Table 8** Results of different methods

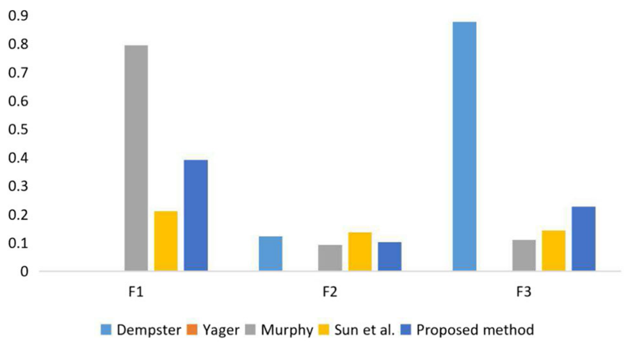
| | $m(F_1)$ | $m(F_2)$ | $m(F_3)$ |
|---|---|---|---|
| Dempster Shafer (1976) | 0 | 0.1228 | 0.8772 |
| Yager (1987) | 0 | 0.00018 | 0.00129 |
| Murphy (2000) | 0.7958 | 0.0932 | 0.1110 |
| Quan Sun and Ye (2000) | 0.2110 | 0.1380 | 0.1440 |
| ACTP | 0.3923 | 0.1020 | 0.2270 |

## 5 Applicaiton

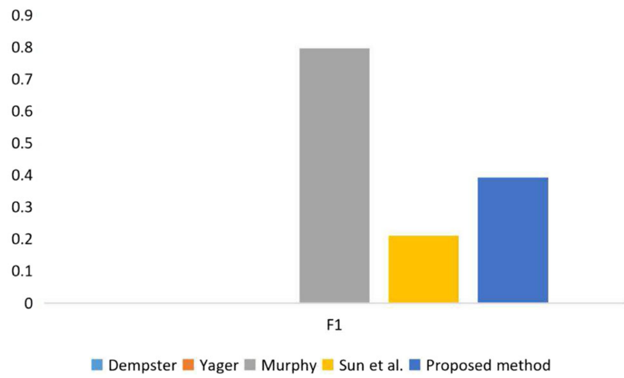
In this section, ACTP is applied in a real-time update system to illustrate its practicability and efficiency. The result is also discussed as well. The data in Li et al. (2020) is used for.

The problem is described as follows. Assuming a radar system is detecting targets from the air. $\Theta = \{F_1, F_2, F_3\}$, which means there are three targets. There is a group of five sources of information, $m_1$ to $m_5$, providing evidence of the target's existence. The detection results are shown in Table 6, where each target corresponds to a BPA $m_i$, $i \in \{1, 2, 3, 4, 5\}$. For example, $m_1(F_1) = 0.5$ indicates that there is a 50% chance of the target object $F_1$ existing within the scanning range of the radar. By using the rule of ACTP in Eq. (10), these BPAs can be combined to obtain the belief function of the existence of the target object, which provides support for further decision-making. The results are shown in Table 7. It can be seen that BPA of $F_1$ is the highest one, which means $F_1$ is the target.

To demonstrate the effectiveness ACTP, we also compared the data with other methods. The results are shown in Table 8 and Fig. 4. Obviously, facing conflicting sources of information $m_2$, ACTP can effectively identify the target $F_1$, which is consistent with Murphy's method and sun et al.'s method. Additionally, Murphy's method has the highest belief degree ($F_1 = 0.7958$) which is higher than Sun et al.'s method ($F_1 = 0.2110$) and ACTP ($F_1 = 0.3923$).




(a) BPAs for different objectives




(b) BPAs for $F_1$

**Fig. 4** Mass function value

**Table 9** Two other sources of information

| BPA | $\{F_1\}$ | $\{F_2\}$ | $\{F_3\}$ |
|---|---|---|---|
| $m_6$ | 0.4 | 0.25 | 0.35 |
| $m_7$ | 0.05 | 0.15 | 0.8 |

In the real-time system, new information will always be updated. Assuming the radar system has detected two additional sources, as indicated in Table 9. The results of the fusion using different methods are presented in Table 10. Obviously, as shown in Table 10, in Murphy's method, the result of sequential fusion is different from the result of simultaneous synthesis. The weighted average method is not suitable for real-time systems. On the contrary, our proposed methods ACTP is simple, fast and accurate. Therefore, this method is particularly suitable for real-time update systems that continuously generate new data.

Our method is not only applicable for target recognition, but it also holds an advantage in group decision-making settings (Dong et al. 2022; Yang et al. 2023). In the process of group decision-making, to achieve consensus, ACTP can be utilized to calculate the evidence trustworthiness among decision-makers and determine the final consensus. In software risk assessment (Chen and Deng 2023), ACTP can also be used to combine different risk assessment values and ultimately form a risk ranking.

**Table 10** Result of the comparison

| | | $\{F_1\}$ | $\{F_2\}$ | $\{F_3\}$ |
|---|---|---|---|---|
| Murphy (2000) | m'[a] | 0.4153 | 0.1644 | 0.4203 |
| | m''[b] | 0.3714 | 0.2571 | 0.3714 |
| Proposed | m | 0.2777 | 0.1737 | 0.2450 |

[a] m' is the result of averaging $m$, $m_6$ and $m_7$, and then use Dempster's method combine 2 times

[b] m'' is the result of averaging $m_1 ... m_7$, and then use Dempster's method combine 6 times

## 6 Conclusion

In this paper, a novel conflict management based on assigning conflict to the power set has been proposed. In the case of high conflict, the performance of fusing evidence can be improved when using ACTP. The experimental results illustrate that the evidence combination rule we proposed is better than other methods. The following is a list of the advantages,

(1) ACTP can identify the target correctly in different alternatives.
(2) Compared with the existing combined method, ACTP has the best convergence performance.
(3) The important point is that ACTP can achieve sequential fusion, which has lower computational complexity. Therefore, it is suitable for high real-time update systems.

In the future, based on ACTP, we can do more research.

(a) According to the main ideas of conflict handling proposed in this paper, more conflict management methods can be designed. (b) Besides target recognition, this new combination rule has the potential application in practical engineering base on sensor data fusion, such as fault diagnosis and risk analysis.

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**Availability of data and materials** All data and materials generated or analysed during this study are included in this article.

**Code availability** The code of the current study is available from the corresponding author on reasonable request.

## Declarations

**Conflict of interest** Author Xingyuan Chen declares that she had no conflict of interest. Author Yong Deng declares that he has no conflict of interest.

**Ethical statement** This article does not contain any studies with human participants or animals performed by any of the authors.

**Consent to participate** Informed consent was obtained from all individual participants included in the study.

**Consent for publication** The participant has consented to the submission of the case report to the journal.

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