Soft Computing (2023) 27:14095–14126
https://doi.org/10.1007/s00500-023-07928-0

APPLICATION OF SOFT COMPUTING

Swarm-based hybrid optimization algorithms: an exhaustive analysis and its applications to electricity load and price forecasting

Rahul Kottath[1,3] · **Priyanka Singh**[2] · **Anirban Bhowmick**[3]

Accepted: 11 February 2023 / Published online: 12 March 2023

Abstract

In this work, we intend to propose multiple hybrid algorithms with the idea of giving a choice to the particles of a swarm to update their position for the next generation. To implement this concept, Cuckoo Search Algorithm (CSA), Grey Wolf Optimization (GWO), Harris Hawks Optimization (HHO), and Whale Optimization Algorithm (WOA) have been utilized. Exhaustive possible combinations of these algorithms are developed and benchmarked against the base algorithms. These hybrid algorithms have been validated on twenty-four well-known unimodal and multimodal benchmarks functions, and detailed analysis with varying dimensions and population size is discussed for the same. Further, the efficacy of these algorithms has been tested on short-term electricity load and price forecasting applications. For this purpose, the algorithms have been combined with Artificial Neural Networks (ANNs) to evaluate their performance on the ISO New Pool England dataset. The results demonstrate that hybrid optimization algorithms perform superior to their base algorithms in most test cases. Furthermore, the results show that the performance of CSA-GWO is significantly better than other algorithms.

Keywords Artificial neural network · Cuckoo search algorithm · Grey wolf optimization · Harris hawks optimization · Whale optimization algorithm

1 Introduction

Optimization has always been a popular area of research in various fields of science and technology. In real-world problems, resources, time, and money are always limited, necessitating the need for different optimization algorithms. According to the "No-free-lunch" theorem, there is no single algorithm available that works well in all applications. Hence, an optimization algorithm with improved performance is always needed. Optimization algorithms systematically find the solution for a particular problem with or without constraints. Traditional mathematical optimization methods such as linear programming and the Newton–Raphson method face difficulty in solving complex problems due to discontinuity, higher dimension, computation of derivatives, etc. Many nature-inspired algorithms are proposed in the literature to mitigate these issues.

Nature-inspired algorithms (NIA) are meta-heuristic algorithms inspired by some of the natural processes around us. NIAs can be classified into various categories: swarm based, evolution based, human based, bio based, math based, and physics based. Particle Swarm Optimization (PSO) (Dhalwar et al. 2016), Ant Lion Optimization (ALO) (Mafarja and Mirjalili 2019), Ant Colony Optimization (Kumar et al. 2020), Grey Wolf Optimization (GWO) (Faris et al. 2018), Cuckoo Search Algorithm (CSA) (Miao et al. 2021), Salps Swarm Optimization (SSA) (Bairathi and Gopalani 2021), Harris Hawks Optimization (HHO) (Fan et al. 2020), Whale Optimization Algorithm (WOA) (Mafarja and Mirjalili 2017), Follow The Leader (FTL) (Singh and Kottath 2022a; Singh et al. 2022) are a few examples of the swarm-based optimization algorithms proposed over the period. Evolution-based optimization techniques include Differential Evolution (DE)

✉ Priyanka Singh
priyankamnnit925@gmail.com; priyanka.si@srmap.edu.in

Rahul Kottath
rahulkottath@gmail.com

Anirban Bhowmick
anirban.bhowmick@vitbhopal.ac.in

1 Digital Tower, Bentley Systems India Private Limited, Pune, India

2 Department of Computer Science and Engineering, SRM University-AP, Amaravati, AP 522502, India

3 School of Electrical and Electronics Engineering, VIT Bhopal University, Bhopal, MP 466114, India

(Deng et al. 2021), Genetic Algorithm (GA) (Wang et al. 2020), Evolutionary Programming (EP) (Hong et al. 2018), Evolutionary Strategies (ES) (Cofnas 2018), etc. Influencer Buddy Optimization (IBO) (Kottath and Singh 2023), Teaching Learning-Based Optimization (TLBO) (Peng et al. 2019), Culture Algorithm (CA) (Chen et al. 2020b), Corona Virus Herd Immunity Optimization (CHIO) (Al-Betar et al. 2020), Forensic-Based Investigation Optimization (FBIO) (Kuyu and Vatansever 2021) are well-known algorithms under the category of human-based optimization algorithms. Bio-based optimization algorithms include Biogeography Based Optimization (BBO) (Chen et al. 2019), Virus Colony Search (VCS) (Li et al. 2016), Satin Bowerbird Optimizer (SBO) (Moosavi and Bardsiri 2017), Earthworm Optimization Algorithm (EOA) (Wang et al. 2018), etc., whereas Hill Climbing (HC) (El Yafrani and Ahiod 2018) and Sine Cosine Algorithm (SCA) (Li and Wang 2020) come under math-based optimization. Atom Search Optimization (ASO) (Zhao et al. 2019), Electromagnetic Field Optimization (EFO) (Abedinpourshotorban et al. 2016), Multi-verse Optimizer (MVO) (Benmessahel et al. 2018), Simulated Annealing (SA) (Fathollahi-Fard et al. 2019) are some of the famous examples of physics-based optimization algorithms.

Swarm intelligence has received wide popularity due to its performance in solving various optimization problems. Particle Swarm Optimization is the pioneering work in the area of swarm intelligence and has motivated the evolution of various swarm-based algorithms. The communication among the swarm brings the exploitation property, whereas the randomness added with different operators brings the exploration property within the search space. A balance between these two properties is essential for an optimization algorithm to find an optimal solution. Many of the proposed meta-heuristic algorithms have the ability to avoid local optima, but they suffer from problems like parameter selection, premature convergence, etc. In the past few years, many hybrid optimization algorithms have been proposed to alleviate these issues (Ahmadian et al. 2021; Jakubik et al. 2021; Chong et al. 2021). The hybrid algorithm improves information exchange within candidates and diversity within the population, thus enhancing the ability to solve complex engineering problems (Singh and Kottath 2021). Wang et al. proposed a hybrid algorithm CS-PSO by combining Cuckoo Search with PSO, which improves the algorithm's ability to avoid local optima (Wang et al. 2011). Zhang et al. proposed a hybrid optimization technique by combining PSO along with Tabu Search for flexible job-shop scheduling problems (Zhang et al. 2009). Yang et al. proposed a hybrid model based on the Fruit Fly optimization algorithm and neural network to optimize the network parameters and predict underwater acoustic signal (Yang et al. 2018). Singh and Dwivedi integrated ANN and FTL algorithms to learn the weight parameters of network architecture and showed the efficiency of the proposed model over short-term electricity load forecasting problem (Singh and Dwivedi 2018). Further, the author extended their work and proposed a hybrid model based on ANN and optimization algorithm using Controlled Gaussian Mutation (CGM) for electricity demand prediction (Singh et al. 2019).

The literature shows that hybrid algorithms are capable of generating better results than traditional ones. Based on a similar idea, we have combined multiple algorithms in a single stack to find an optimal solution. In this work, a novel method of combining swarm-based optimization techniques has been proposed. The motivation behind this work comes from multiple model systems where the individual particles of an algorithm update their positions by selecting the best available solutions from several optimization algorithms. The novelty of this work lies in developing a framework by combining multiple algorithms which can be extended to any swarm-based approach. To verify the proposed framework, we have utilized four well-known optimization algorithms: CSA, GWO, HHO, and WOA, as the base algorithms. The selection of these optimization algorithms has been solely made on the basis of their wide acceptability and implementation to solve a huge range of complex problems. Also, the number of algorithms is chosen four to limit the number of possible combinations of algorithms for the analysis of our proposed model. The set of all possible combinations of these algorithms, namely CSA-GWO, CSA-HHO, CSA-WOA, GWO-HHO, GWO-WOA, and HHO-WOA, has been evaluated in this work. As the algorithms are executed in parallel, the order of algorithms mentioned in the name does not affect their performance. The proposed hybrid algorithms are tested on twenty-four standard unimodal and multimodal benchmark functions. Furthermore, the performance of hybrid algorithms is evaluated on electricity load and price forecasting problems. Also, the Friedman test has been performed to show the significance of generated results. The major contributions of our work are as follows:

- A novel approach of combining two different optimization algorithms in a single framework has been proposed.
- Exhaustive combinations of CSA, GWO, HHO, and WOA have been evaluated.
- The proposed algorithms are validated on twenty-four unimodal and multimodal benchmark functions with varying dimensions and population sizes.
- Further, they have been combined with ANN to further show their performance on electricity load and price forecasting problems.

The rest of the paper is organized as follows. Section 2 gives background details of Artificial Neural Networks and different optimization algorithms. Section 3 describes the proposed novel approach of combining CSA, GWO, HHO, and WOA with a detailed flow diagram. Experimental results with a detailed analysis are presented in Sect. 4. Section 5 discusses the results and compiles the interpretations of the results. Finally, Sect. 6 concludes the work by showing some light on the future scope.

2 Related works

In this section, we provide the background details of artificial neural networks and the mathematical representation of different optimization algorithms utilized in this work. The detailed theory about these optimization algorithms can be read from their base papers as per the references.

2.1 Artificial Neural Network

An Artificial Neural Network models the biological neural network of our brain to learn the mapping between input and output neurons. ANN architectures have evolved a lot over the period, and the multi-layer perceptron (MLP) is one of the most widely used networks. This consists of an input layer, one or more hidden layers, and an output layer connected through network weights. ANN has been one of the major choices for time series prediction applications such as prediction of rainfall, water demand, electricity load (Singh and Dwivedi 2019), etc.

For a two-layered feed forward network containing n input neurons h hidden neurons and m output neurons, the output Y_k can be obtained as:

$$Y_k = \sum_{j=1}^{n} \left(W_{k,j} * \frac{1}{1 + exp\left(-\sum_{i=1}^{n} W_{j,i} X_i + b_j\right)} + b_k \right) \tag{1}$$

where b_j represents the bias term in the hidden layer and b_k represents the bias terms in the output layer. Machine learning problems can be widely classified as regression and classification problems. Neural networks and their variants have been widely used for these applications. Selection of hyper-parameters such as the number of neurons in each layer, number of layers in the network, choice of activation function, etc. has always been challenging, and hence, the trial-and-error method is used for the same (Hamzaçebi 2008).

2.2 Optimization algorithms

Several optimization algorithms have been proposed in the literature. Algorithms, such as PSO, CSA, HHO, GWO, and WOA, show similar behavior of updating the individual candidate for the next generation. As mentioned earlier, a combination of two or more algorithms may improve the accuracy of the given problem and overcome the drawback of the existing algorithm. This section discusses the basics of CSA, GWO, HHO, and WOA optimization algorithms.

2.2.1 Cuckoo Search Algorithm (CSA)

The Cuckoo Search Algorithm is inspired by the aggressive reproduction strategy of female cuckoo birds. Cuckoo birds lay their eggs in communal nests, and the host birds raise them (Gandomi et al. 2013). Cuckoos use some strategies to increase the hatching probability of their eggs and reduce the probability of abandonment of eggs by the host birds. The cuckoo search algorithm works on three basic rules:

- Each cuckoo lays one egg at a time and dumps their egg in a randomly chosen nest among available host nests
- Best nests with good eggs will be sent for next generation
- p_a is a probability that an egg laid by a cuckoo can be discovered by the host bird, and the number of host nests is fixed. In this situation, a host can either throw the egg away or leave the nest and build a new nest. Basu and Chowdhury (2013).

A cuckoo uses Levy flight distribution to create a new nest based on the previous best nests. The new nest is calculated as follows:

$$X_{\text{next}} = X_c + \alpha * r_1 * \text{step}\left(X_c - X_{\text{cbest}}\right) \tag{2}$$

where step is calculated using Mantegna's algorithm and $\alpha > 0$ (Yang 2010). Apart from basic CSA, numerous variants have been published by researchers and practitioners to improve the performance of existing versions. Chen and Kunjie proposed a hybrid meta-heuristic algorithm by combining biogeography-based optimization (BBO) and CSA to identify the photo-voltaic model parameters. As CSA is good at global exploration while BBO favors local exploitation, it thus brings a good combination of exploration and exploitation (Chen and Yu 2019).

2.2.2 Grey Wolf Optimizer (GWO)

Grey Wolf Optimizer is a population-based algorithm that imitates the leadership hierarchy and group hunting behavior of wolves (Nadimi-Shahraki et al. 2021). In this algorithm, three categories of leader wolves, namely α, β, and δ have been considered the best solutions (Faris et al. 2018). These three groups of wolves lead the remaining wolves, termed as ω wolves, toward good search space to find the global solution. The hunting strategy followed by wolves can be described in three main steps:

- Encircling Prey:

$$X_{next} = X_{prey,c} - A.|C.X_{prey,c} - X_c| \tag{3}$$
$$C = 2.r_2 \tag{4}$$
$$A = 2.a.r_1 - a, \qquad a = 2\left(1 - \frac{ite}{Maxite}\right) \tag{5}$$

where r_1 and r_2 are random numbers. ite is current iteration; Maxite is maximum number of iterations. X_c is current position, and X_{next} is next position of the wolf. $X_{\text{prey},c}$ is position of prey in current iteration.

$$X_{\text{next}} = \begin{cases} X_r - r_1|X_r - 2.r_2.X_c| & p \geq 0.5 \\ (X_{\text{prey}} - X_{\text{avg}}) - r_3\,(lb + r_4\,(ub - lb)) & p < 0.5 \end{cases} \tag{8}$$

where X_{next} and X_c are the position of hawks in the next iteration and current iteration, respectively. X_{prey} is the best position of prey; X_{avg} is the average location of hawks in current iteration. p, r_1, r_2, r_3, and r_4 are the random numbers between 0 and 1. The energy of prey decreases during its escaping behavior, which can be modeled as follows:

$$E = E_0\left(2 - \frac{2t}{T}\right) \tag{9}$$
$$X_{\text{next}} = \begin{cases} X_{\text{prey}} - X_c - E|2(1-r_5).X_{\text{prey}} - X_c| & r_5 \geq 0.5 \text{ and } |E| \geq 0.5 \;\; Prey\ has\ energy\ to\ escape\ (Soft\ besiege) \\ X_{\text{prey}} - E|X_{\text{prey}} - X_c| & r_5 \geq 0.5 \text{ and } |E| < 0.5 \;\; Prey\ is\ extremely\ tired\ (Hard\ besiege) \end{cases} \tag{10}$$

- Hunting:

$$\begin{aligned} X_{1,c} &= X_\alpha - A_1.|C_1 X_\alpha - X_c| \\ X_{2,c} &= X_\beta - A_1.|C_2 X_\beta - X_c| \\ X_{3,c} &= X_\delta - A_1.|C_3 X_\delta - X_c| \end{aligned} \tag{6}$$

where X_α, X_β, and X_δ are the best solutions of current iteration. C_1, C_2 and C_3 are calculated using Equ. 4. Wolves belonging to ω should be update their position using:

$$X_c = \frac{X_{1,c} + X_{2,c} + X_{3,c}}{3} \tag{7}$$

- Attacking the prey: the grey wolves finish the hunt by attacking the prey when it stops moving.

Due to simplicity and few control parameters, the GWO algorithm has been applied in different areas to solve optimization problems such as the economic load dispatch problem (Nithiyananthan and Ramachandran 2013), feature selection (Kiziloz and Deniz 2021), scheduling problem (Chen et al. 2021), and recommendation system (Katarya and Verma 2018).

2.2.3 Harris Hawks Optimizer (HHO)

Harris Hawks Optimizer is a meta-heuristic algorithm that imitates the cooperative behavior and prey-catching manner of Harris hawks (Chen et al. 2020a). The HHO algorithm is expressed between two phases: exploration and exploitation phase.

Soft besiege with progressive rapid dives:

$$Y = \left\{ \begin{matrix} X_{\text{prey}} - E|2(1-r_5).X_{\text{prey}} - X_c| \\ Z = Y + S \times \text{LF}(d) \end{matrix} \right\} r_5 < 0.5 \text{ and } |E| \geq 0.5 \tag{11}$$

where LF is Levy flight function; S is random vector of size d. The final position of hawks can be mathematically updated using the following:

$$X_{next} = \begin{cases} Y \; if \; F(Y) < F(X_c)) \\ Z \; if \; F(Z) < F(X_c)) \end{cases} \tag{12}$$

Hard besiege with progressive rapid dives:

$$Y = \left\{ \begin{matrix} X_{\text{prey}} - E|2(1-r_5).X_{\text{prey}} - X_{\text{avg}}| \\ Z = Y + S \times \text{LF}(d) \end{matrix} \right\} r_5 < 0.5 \text{ and } |E| < 0.5 \tag{13}$$

The final strategy to update the position of hawks can be mathematically represented as:

$$X_{next} = \begin{cases} Y \; if \; F(Y) < F(X_c)) \\ Z \; if \; F(Z) < F(X_c)) \end{cases} \tag{14}$$

2.2.4 Whale Optimization Algorithm (WOA)

The whale optimization algorithm is a population-based optimization algorithm that mimics the bubble-net feeding behavior of humpback whales while foraging (Aljarah et al. 2018). The bubble net created by humpback whales helps trap the prey and makes it easier for the whale to hunt closer to the surface (Mafarja and Mirjalili 2017). The algorithm depicts the exploitation phase by encircling the prey and creating a bubble net and the exploration phase by randomly searching for prey. The exploration and exploitation phase can be represented as follows:

- Exploitation Phase: the whale moves around prey by shrinking encircling mechanism and upward spiral-shaped path.

$$X_{\text{next}} = \begin{cases} X_{\text{best}} - A.|C.X_{\text{best}} - X_c| & \text{if } r_2 < 0.5 \\ |X_{\text{best}} - X_c|.e^{bl}.cos(2\pi l) + X_{\text{best}} & \text{if } r_2 \geq 0.5 \end{cases} \tag{15}$$

  where $A = 2.a.r_1 - a$, $C = 2.r_1$, a is linearly decreasing from 2 to 0, b defines spiral shape, l lies between -1 and 1, and r_1 and r_2 random numbers between 0 and 1.
- Exploration phase: a random position $X_r and$ is used to update the position of whales. A is a vector with random values between 1 and -1 that force a solution to move away from the best solution.

$$X_{\text{next}} = X_r - A.|C.X_r - X_c| \tag{16}$$

3 Proposed methodology

This section details the proposed hybrid optimization algorithms and their approach to combining various algorithms. This section also discusses the method to develop the hybrid model by combining hybrid optimization algorithms with ANN.

3.1 Proposed hybrid algorithm

A hybrid algorithm is one in which aspects of multiple algorithms are combined in a single framework. Exploration and exploitation are the two primary ingredients of an optimization algorithm. The algorithms proposed in the literature are intended to balance these two properties. One of the significant issues with optimization algorithms is that they fail to generalize the performance on multiple problems. The convergence rate and accuracy of the CSA algorithm have been considered as its main limitation (Cuong-Le et al. 2021). The single search strategy in GWO makes it insufficient for various optimization problems (Faris et al. 2018). Major limitations of the HHO algorithm include the problem of solutions diversity and the problem of local optima (Elgamal et al. 2020). GWO suffers majorly due to a limited degree of exploration (Subramanian et al. 2020). This paper evaluates an exhaustive combination of CSA, GWO, HHO, and WOA. Overall, six different hybrid algorithms, namely CSA-GWO, CSA-HHO, CSA-WOA, GWO-HHO, GWO-WOA, and HHO-WOA, are developed with a possible combination of two. These combinations ensure that the positive aspects of the algorithms will be retained, and the performance can be ensured in multiple optimization problems.

In this work, two algorithms are chosen from the base algorithms to create their hybrid algorithm. After the initialization step, each particle changes its position using updated equations from the selected algorithms. In each iteration, the particle selects the best position generated from these updated positions. The essence of both algorithms is incorporated in a single stack through this modification. Each particle gets an option to update its position based on the best available options, allowing each particle to explore the search space in the best possible way. This process of particle updating its position goes on until the termination condition is met. Finally, the best solution is selected from the obtained solutions. A generalized flow diagram of the proposed hybrid algorithm is shown in Fig. 1. The updated equations of CSA, GWO, HHO, and WOA are given in Eq. (2), Eqs (3)–(7), Eqs (8)–(14), and Eqs (15)–(16), respectively.

3.2 Proposed hybrid model

Proposed hybrid algorithms (CSA-GWO, CSA-HHO, CSA-WOA, GWO-HHO, GWO-WOA, and HHO-WOA) have been applied to the time series prediction problem by replacing the gradient descent algorithm of ANN with the proposed hybrid optimization algorithms. This combination of ANN and hybrid algorithms has been tested on real-life prediction problems to analyze their performance. For better clarification, the framework of the proposed hybrid algorithm, ANN-CSA-GWO, is summarized in Table 1.

4 Experiment and results

In this section, we validate our proposed hybrid algorithms on benchmark functions and test them on two different time series prediction problems to verify their efficacy and performance. All the simulations have been performed on MATLAB R2020b software and Python 3.8 on a Windows 10, 64-bit machine with Intel(R) Core(TM) i5 CPU 760 @ 2.80 GHz.

4.1 Parameter settings

To show the effectiveness of the proposed hybrid algorithms, they have been compared with CSA, GWO, HHO, and WOA algorithms. The parameters of CSA (Shehab et al. 2017), GWO (Kohli and Arora 2018), HHO (Heidari et al. 2019), and WOA (Mirjalili and Lewis 2016) algorithm have been chosen from their respective base papers, and the same parameters have been used in hybrid algorithms as shown in Table 2. During the validation phase, algorithms were tested

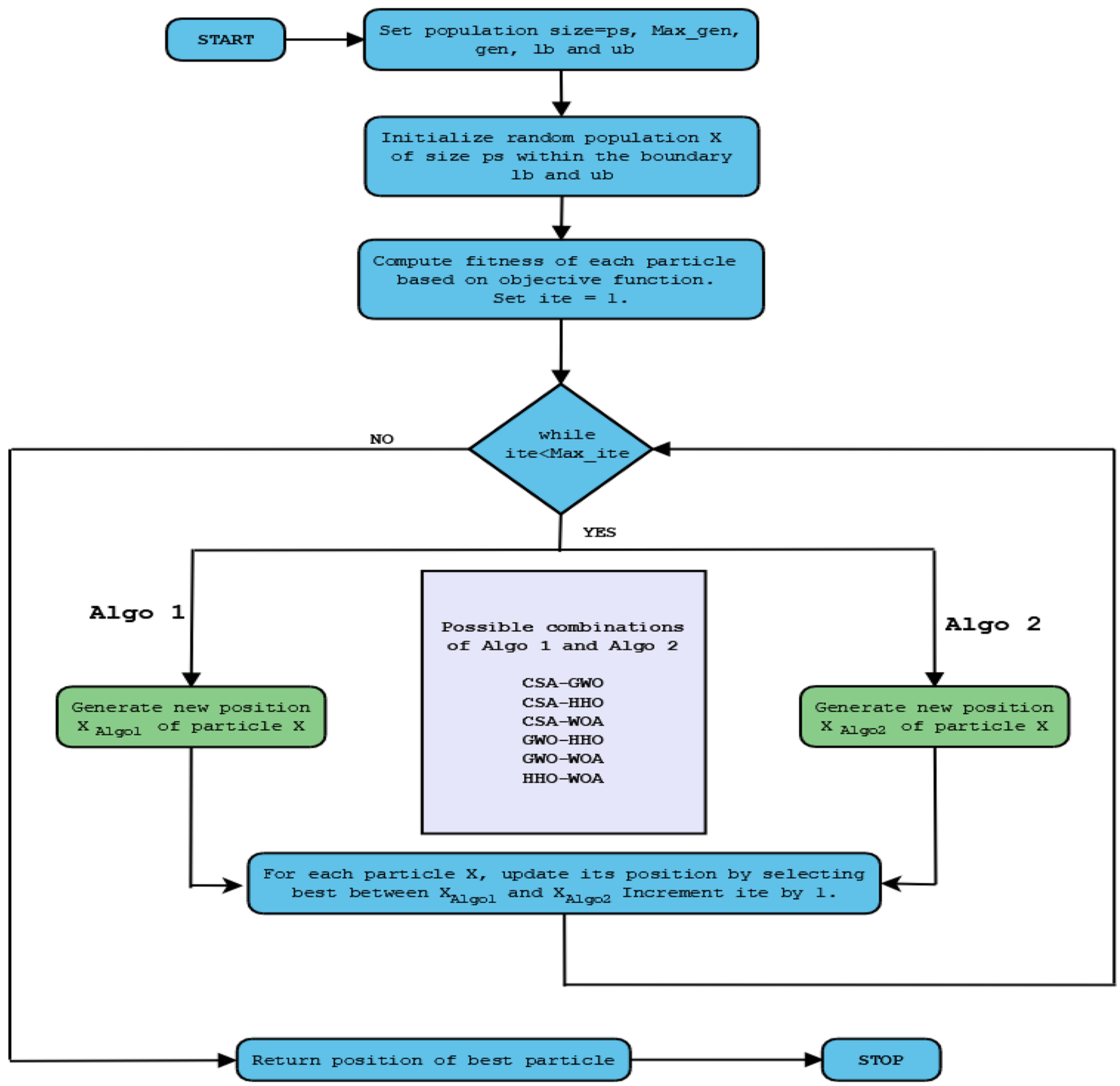

Fig. 1 Flowchart of proposed model

for 30 epochs with 100 particles and 500 iterations over 2, 5, 10, and 20 dimensions for unimodal and multimodal functions. Furthermore, in the testing phase, these algorithms have been integrated with ANN for short-term electricity load and price forecasting problems, which is executed for 2000 iterations with 100 population sizes and 20 hidden neurons.

4.2 Validation on benchmark functions

The performance evaluation of the proposed models has been carried out on twenty-four standard benchmark functions. These functions are categorized into two classes: unimodal (F1–F12) and multimodal (F13–F24). The mathematical equations for these functions and their *Domain* are given in Table 13 (Appendix). Unimodal functions have only one global solution, whereas multimodal functions have multiple local solutions with one global solution. Thus, unimodal functions can be utilized to validate the exploitation strategy of the optimization algorithm, whereas multimodal functions can be used to test both exploration and exploitation.

4.2.1 Influence on dimension

Tables 3 and 4 show the results generated by different optimization algorithms. All the mentioned algorithms have been implemented in four dimensions (2, 5, 10, 20) to analyze their performance with an increasing number of input variables. In Table 3, for the F1 function, GWO, CSA-HHO, CSA-WOA, GWO-HHO, GWO, WOA, and HHO-WOA obtained global values in 2D and HHO-WOA in 5D. In F5 function, CSA-GWO, GWO-WOA, and HHO-WOA generated optimal results in 2D, whereas CSA-GWO and GWO-WOA

Table 1 Pseudocode of the proposed hybrid model

Step 1 Set population size pop, maximum generation Max_{ite}, ite, g_{best}, and hidden neurons N. Set initial parameters of CSA and GWO algorithm as given in Table 1. Initialize random weight vectors (population), X_{current} of size pop and dimension, D within lower and upper bound.

$$\text{Weights of dimension}(D) = m * n + n + n + o \begin{cases} m = \text{Number of input neurons} \\ n = \text{Number of hidden neurons} \\ o = \text{Number of output neurons} \end{cases}$$

Step 2 Evaluate the fitness of current weight vectors, X_{current}.

Step 3 For i^{th} weight vector in X_{current}, find a new position using of CSA and GWO update equation.

$X^i_{\text{CSA}} = X^i_{\text{current}} + \alpha * r_1 * step\left(X^i_{\text{current}} - X_{\text{cbest}}\right)$

$X^i_{GWO} = X_{\text{prey},c} - A.|C.X_{\text{prey},c} - X^i_{\text{current}}|$

A detailed description of these equations is given in Sect. 2.

Step 4 Evaluate the fitness of both X_{CSA} and X_{GWO}

Step 5 For i^{th} weight vector, update its position by selecting best from X^i_{CSA} and X^i_{GWO} based on the fitness values.

Step 6 Update g_{best} vector.

Step 7 Increment ite by one.

Step 8 If ite reaches Max_{ite} then terminate otherwise go to Step-2.

In the end, the optimal weight solution is achieved, which is further used to test the data.

Table 2 Experimental parameter values

S No.	Models	Parameters	Value
1	CSA	P_a is a probability that egg laid by a cuckoo can be discovered by the host bird	0.25
		β for Mantegna's algorithm to find step size	1.5
2	GWO	a is decreasing step parameter	[2,0]
		r_1 and r_2 are random numbers	[0,1]
3	HHO	β for Mantegna's algorithm to find step size	1.5
		$p, r_{1,2}, r_{3,4}$ and r_5 are random numbers	[0,1]
		E_0 initial energy	[−1,1]
4	WOA	a linearly decreased from 2 to 0	[2,0]
		l is a random number	[−1,1]
		p is a random number	[0,1]
		a is decreasing step parameter	[2,0]

in 5D. Similar results are obtained from CSA-WOA for F7 and F12 benchmark functions. CSA-HHO and HHO-WOA obtained the best value for F8, F9, and F11 unimodal benchmark functions in all dimensions. Also, HHO and WOA algorithm obtained their optimal values for F9 and F11 functions in 2, 5, 10, and 20 dimensions. Table 4 shows that HHO, CSA-GWO, CSA-WOA, and HHO-WOA generated optimal values for F13, F15, F19, and F23 benchmark functions in all dimensions. GWO, WOA, and CSA-HHO obtained the best values for F13 function in both 2D and 5D. Also, CSA-HHO generated the best function values for F15, F19, and F23 multimodal benchmark functions. For a better understanding, the best values obtained for unimodal and multimodal functions are highlighted in Table 3 and Table 4.

Tables 3 and 4 show that CSA-GWO and HHO-WOA generated superior results for almost all the benchmark functions in different dimensions. In contrast, the performance of CSA-HHO, GWO-HHO, and GWO-WOA is inferior to their base algorithms under given circumstances. Results generated by CSA-HHO are significantly closer to CSA-GWO and better than their respective base algorithm in almost all dimensions. Figure 2 shows the convergence curve of different algorithms over benchmark functions. The convergence plot shows that HHO-GWO and CSA-GWO have good convergence compared to other algorithms. On the other hand, the performance of CSA algorithm has been inferior in all dimensions for both unimodal and multimodal functions, which are visualized from both Tables 3 and 4.

Table 3 Comparison of optimal results obtained from different algorithms over unimodal functions

FID	Dim	Fmin	CSA	GWO	HHO	WOA	CSA-GWO	CSA-HHO	CSA-WOA	GWO-HHO	GWO-WOA	HHO-WOA
F1	2	0	1.03E-24	**0**	1.3E-124	5.2E-186	1.2E-245	**0**	**0**	**0**	**0**	**0**
	5	0	4.52E-14	2.3E-154	2E-105	2.7E-109	1.8E-217	1.2E-156	5E-290	5.4E-202	1.5E-138	**0**
	10	0	1.71E-07	1.38E-89	1.7E-111	1E-102	8.2E-206	2.95E-90	1.4E-241	1.8E-126	1.4E-115	**1.7E-304**
	20	0	0.005408	2.71E-56	3.3E-109	1.8E-100	1.5E-207	5.74E-56	2.6E-204	1.06E-94	1.8E-110	**7.7E-293**
F2	2	0	2.47E-12	3.2E-171	4.74E-66	2E-107	8E-129	4.2E-178	4.4E-238	**5E-265**	7.6E-184	3.5E-203
	5	0	5.26E-06	1.83E-82	3.55E-57	1.77E-63	4.6E-112	1.68E-85	1.5E-148	3.2E-118	5.49E-82	**2.8E-170**
	10	0	0.020927	1.71E-49	5.08E-57	1.18E-59	1.5E-106	1.71E-50	2.8E-119	3.79E-78	1.62E-67	**4.9E-162**
	20	0	3.758263	3.06E-31	3.54E-58	1.18E-56	2.2E-103	2.05E-31	3.5E-109	1.8E-59	7.02E-65	**5.3E-157**
F3	2	0	1.64E-21	1.3E-260	1.4E-110	2.11E-64	2.1E-218	2.8E-282	**0**	1.8E-289	4.4E-193	5.7E-306
	5	0	8.96E-10	3.23E-85	5.9E-102	2.75E-07	1.7E-169	4.61E-93	1.8E-213	2.97E-82	6.25E-36	**2.9E-263**
	10	0	0.101506	1.66E-41	2.8E-100	2.611021	4.5E-153	9.79E-42	3.7E-141	1.08E-40	2.67E-09	**7E-245**
	20	0	460.1265	5.87E-19	2.88E-95	2613.053	4.9E-132	1.27E-15	4.74E-95	4.07E-18	0.209449	**3.2E-237**
F4	2	0	1.62E-10	2.5E-163	1E-62	2.93E-20	1.2E-124	9.9E-181	**6.4E-232**	1.8E-203	9.1E-125	2E-172
	5	0	0.000256	1.31E-58	3.91E-54	2.51E-08	3E-104	1.77E-59	1.6E-138	3.8E-59	2.04E-23	**1.1E-144**
	10	0	0.403132	1.22E-28	2.47E-54	0.418819	4.4E-103	3.25E-28	3.6E-112	1.9E-24	0.091263	**8E-139**
	20	0	9.318119	4.07E-15	2.18E-53	5.868642	2.1E-102	1.13E-13	1E-107	1.13E-09	7.196903	**2.3E-133**
F5	2	0	3.33E-22	4.73E-09	4.6E-10	1.55E-10	**0**	2.78E-28	1.02E-28	2.15E-15	**0**	**0**
	5	0	1.58E-11	2.94E-07	1.05E-06	1.03E-07	**0**	6.4E-17	2.01E-13	1.37E-11	**0**	1.39E-12
	10	0	8.43E-05	1.84E-06	4.08E-06	1.14E-05	8.21E-28	6.3E-10	1.37E-09	9.07E-10	**1.4E-29**	2.61E-09
	20	0	2.035969	0.050718	1.01E-05	0.00066	**9.63E-13**	3.58E-06	6.87E-07	0.082598	1.85E-11	1.04E-06
F6	2	0	5.39E-15	4.66E-08	7.41E-09	8.94E-09	**3.7E-32**	3.34E-18	1.36E-10	4.44E-10	3.7E-32	1.31E-14
	5	0	1.16E-06	0.1871	0.008433	0.133334	**9.86E-32**	0.088884	3.97E-11	0.066667	0.022222	4.63E-14
	10	0	0.183959	0.666668	0.188071	0.622547	**1.87E-20**	0.644445	1.92E-06	0.666941	0.577778	7.34E-08
	20	0	2.15048	0.666671	0.240195	0.666692	**4.63E-08**	0.666667	0.049676	0.667022	0.666667	0.001184
F7	2	0	2.58E-24	**0**	9E-124	3.9E-180	6.6E-252	**0**	**0**	**0**	**0**	**0**
	5	0	8.57E-14	4.7E-154	4.1E-112	1.4E-109	1.1E-218	1.2E-156	1.6E-289	8.1E-204	1.4E-140	**0**
	10	0	2.01E-07	5.1E-90	1.2E-112	7E-103	2E-209	2.3E-90	1.5E-237	3.2E-124	2.4E-114	**7.4E-300**
	20	0	0.00477	9E-56	1.8E-108	2.2E-98	2.4E-205	2.1E-56	3.3E-214	1.31E-95	3.5E-112	**6.5E-299**

Table 3 continued

FID	Dim	Fmin	CSA	GWO	HHO	WOA	CSA-GWO	CSA-HHO	CSA-WOA	GWO-HHO	GWO-WOA	HHO-WOA
F8	2	0	1.05E-30	**0**	6.4E-137	1.5E-193	1E-271	**0**	**0**	**0**	**0**	**0**
	5	0	2.04E-23	2.1E-226	6.8E-131	9.8E-152	1.8E-265	1E-230	**0**	2.6E-272	1.8E-202	**0**
	10	0	2.4E-16	2.8E-180	2.6E-133	1.3E-148	8.5E-265	7.3E-178	**0**	7E-193	5.1E-184	**0**
	20	0	7.85E-11	1.3E-153	1.4E-134	2.2E-149	1.1E-258	1.2E-146	**0**	2.7E-151	5.2E-182	**0**
F9	2	0	1.3E-111	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	5	0	4.61E-54	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	10	0	6.03E-21	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	20	0	8.6E-05	7.6E-187	**0**	**0**	**0**	4.3E-177	**0**	1.3E-170	6.4E-265	**0**
F10	2	0	7.06E-24	**0**	5.7E-128	3.8E-176	5.4E-254	**0**	**0**	**0**	**0**	**0**
	5	0	4.23E-13	3.2E-149	4.1E-107	6.1E-111	3E-218	7E-153	8.9E-289	2.4E-198	3.8E-139	**0**
	10	0	4.29E-06	9.25E-89	7E-111	1E-101	1.2E-205	1.28E-88	2.6E-237	4.9E-123	3.9E-113	**1.4E-302**
	20	0	0.166844	2.85E-54	2.4E-103	7.5E-99	9.5E-205	8.27E-55	3.1E-217	3.49E-92	4.3E-110	**1.1E-297**
F11	2	−1	**−1**	**−1**	**−1**	**−1**	**−1**	**−1**	**−1**	**−1**	**−1**	**−1**
	5	−1	−0.18262	0.93253	**−1**	**−1**	**−1**	0.186216	**−1**	**−1**	**−1**	**−1**
	10	−1	0	0.998349	**−1**	**−1**	**−1**	0.998338	**−1**	−0.93339	−0.93339	**−1**
	20	−1	0	0.997165	**−1**	**−1**	**−1**	0.99668	**−1**	**−1**	**−1**	**−1**
F12	2	0	1.95E-23	**0**	2.1E-120	5.9E-127	9.8E-243	**0**	**0**	**0**	4E-264	**0**
	5	0	7.93E-12	4.4E-115	2.4E-106	1.15E-28	1.5E-186	1.1E-119	4.2E-243	9.2E-118	3.35E-50	**1.2E-273**
	10	0	0.003752	2.18E-57	5.1E-99	0.064291	3.4E-145	1.77E-56	6.2E-147	1.24E-55	9.85E-14	**2.5E-247**
	20	0	25.11315	8.14E-28	3.21E-90	198.5244	2.6E-101	3.58E-24	1.73E-63	4.91E-28	0.004645	**2.5E-218**

Bold values indicate the best value obtained for the particular function

Table 4 Comparison of optimal results obtained from different algorithms over multimodal functions

FID	Dim	Fmin	CSA	GWO	HHO	WOA	CSA-GWO	CSA-HHO	CSA-WOA	GWO-HHO	GWO-WOA	HHO-WOA
F13	2	0	3.38E-14	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	5	0	0.155055	**0**	**0**	**0**	**0**	**0**	**0**	0.733295	0.165827	**0**
	10	0	11.51068	0.245335	**0**	4.74E-16	**0**	3.2413	**0**	5.852009	2.785884	**0**
	20	0	55.82305	0.520274	**0**	9.47E-16	**0**	17.60318	**0**	26.44648	21.59057	**0**
F14	2	0	6.6E-10	8.88E-16	**8.88E-16**	2.07E-15	**8.88E-16**	**8.88E-16**	**8.88E-16**	**8.88E-16**	**8.88E-16**	**8.88E-16**
	5	0	0.003773	2.19E-15	**8.88E-16**	3.49E-15	**8.88E-16**	**8.88E-16**	**8.88E-16**	2.07E-15	1.72E-15	**8.88E-16**
	10	0	2.330277	5.03E-15	**8.88E-16**	4.09E-15	**8.88E-16**	4.32E-15	**8.88E-16**	4.56E-15	3.02E-15	**8.88E-16**
	20	0	9.031059	1.27E-14	**8.88E-16**	3.97E-15	**8.88E-16**	5.39E-15	**8.88E-16**	4.8E-15	3.85E-15	**8.88E-16**
F15	2	0	1.43E-07	0.003205	**0**	0.002712	**0**	**0**	**0**	0.002712	0.000247	**0**
	5	0	0.02858	0.01585	**0**	0.044677	**0**	0.003315	**0**	0.030788	0.004189	**0**
	10	0	0.063862	0.023547	**0**	0.049652	**0**	0.014184	**0**	0.067763	0.053719	**0**
	20	0	0.97006	0.003287	**0**	0.00498	**0**	0.002217	**0**	0.002889	0.001725	**0**
F16	2	0	6.85E-20	1.58E-08	1.01E-11	4.43E-09	**2.36E-31**	2.51E-27	**2.42E-31**	3.21E-14	**2.36E-31**	**2.36E-31**
	5	0	5.46E-08	1.55E-07	9.8E-07	1.35E-06	**9.43E-32**	2.86E-17	1.5E-11	1.7E-10	**9.44E-32**	6.18E-10
	10	0	0.058975	3.69E-07	1.51E-06	2.45E-05	**7.79E-28**	9.41E-11	9.14E-09	0.001093	2.45E-26	7.56E-08
	20	0	2.136613	0.005935	1.28E-06	0.001651	**4.32E-13**	4.27E-07	3.22E-07	0.29694	0.109052	4.56E-07
F17	2	0	2.64E-20	1.31E-08	1.7E-10	8.45E-09	**1.35E-32**	7.73E-27	**1.35E-32**	4.28E-14	**1.35E-32**	**1.35E-32**
	5	0	6.92E-10	4.02E-07	1.67E-06	2.27E-06	**1.35E-32**	4.73E-15	5.47E-11	3.68E-10	0.000366	2.39E-09
	10	0	0.001897	1.93E-06	5.54E-06	0.000456	**1.83E-26**	7.19E-09	4.99E-08	0.006432	0.002532	3.71E-07
	20	0	2.307969	0.016554	6.08E-06	0.006921	**2.25E-12**	6.72E-06	4.19E-06	0.147054	0.015045	3.66E-06
F18	2	0	3.16E-08	2.14E-05	0.000239	0.003499	**2.42E-11**	5.79E-10	1.13E-05	1.87E-05	**2.18E-11**	0.000207
	5	0	0.006158	0.000117	0.001475	0.119196	3.32E-05	**1.64E-05**	0.000192	0.007101	5.74E-05	0.002257
	10	0	0.088128	0.025437	0.003466	0.203238	0.000487	**9.14E-05**	0.001162	0.082284	0.009721	0.005666
	20	0	0.309467	0.121482	0.00708	0.276572	0.001573	**0.000732**	0.007339	0.201379	0.073897	0.007112
F19	2	0.9	**0.9**	0.903334	**0.9**	0.923333	**0.9**	**0.9**	**0.9**	0.913333	**0.9**	**0.9**
	5	0.9	1.000028	0.977703	**0.9**	0.98008	**0.9**	0.996965	**0.9**	0.98112	0.983333	**0.9**
	10	0.9	1.002189	1.028186	**0.9**	0.972454	**0.9**	1.001751	**0.9**	1.003387	0.973333	**0.9**
	20	0.9	1.062238	1.215668	**0.9**	0.98271	**0.9**	1.04705	**0.9**	1.055491	0.96	**0.9**

Table 4 continued

FID	Dim	Fmin	CSA	GWO	HHO	WOA	CSA-GWO	CSA-HHO	CSA-WOA	GWO-HHO	GWO-WOA	HHO-WOA
F20	2	0	4.84E-05	0.003329	3.96E-64	0.049937	1.7E-121	1.3E-160	**0**	**0**	3.5E-41	**0**
	5	0	0.099873	0.099873	3.84E-56	0.086557	1.2E-103	0.096544	5.5E-141	0.089886	0.089886	**2.7E-153**
	10	0	0.48922	0.099873	6.26E-56	0.123215	5.1E-99	0.099873	1.4E-112	0.103207	0.106544	**8.6E-150**
	20	0	2.160052	0.113544	3.36E-54	0.126548	3.38E-94	0.10322	2.4E-105	0.14654	0.169873	**1.8E-140**
F21	2	0	1.35E-06	1.9E-27	6.75E-24	1.01E-05	3.7E-106	1.2E-132	**1.2E-198**	8.3E-128	7.42E-96	1.8E-147
	5	0	0.000276	2.33E-25	8.72E-32	3.51E-05	2.3E-105	3.8E-91	**3.6E-189**	3.1E-105	3.43E-62	9.5E-153
	10	0	0.006399	6.01E-45	8.81E-16	0.000488	9.2E-108	1.47E-79	**9.2E-193**	1.62E-87	2.12E-54	5.1E-149
	20	0	0.457634	4.25E-40	5.89E-24	0.000142	2.6E-108	1.2E-67	**1.2E-188**	1.57E-71	2.09E-47	2.2E-156
F22	2	0	2.69E-09	4.2E-175	1.51E-66	0.01099	1.2E-130	3.4E-192	3.9E-236	**2.4E-266**	1.9E-178	2.7E-202
	5	0	0.041786	0.04178	9.78E-59	0.033857	3.96E-87	0.04178	4.3E-110	0.04178	0.037602	**6.4E-145**
	10	0	0.001259	0.00077	0.00051	0.000673	0.000434	0.001041	0.00051	0.000703	0.000592	**0.000396**
	20	0	2.41E-07	1.04E-06	**5.15E-08**	5.49E-08	**5.15E-08**	2.15E-07	**5.15E-08**	5.94E-08	5.56E-08	**5.19E-08**
F23	2	−1	−0.99999	**−1**	**−1**	−0.9	**−1**	**−1**	**−1**	**−1**	**−1**	**−1**
	5	−1	−0.11585	1.97E-06	**−1**	−0.16667	**−1**	−0.62108	**−1**	−0.13332	−0.1	**−1**
	10	−1	1.74E-07	5.19E-09	**−1**	−0.1	**−1**	8.66E-08	**−1**	2.89E-06	5.73E-34	**−1**
	20	−1	6.79E-10	1.82E-12	**−1**	−0.13333	**-1**	7.02E-10	**−1**	6.35E-10	3.29E-18	**-1**
F24	2	0	3.05E-23	6.37E-09	6.67E-12	2.08E-10	**1.5E-32**	3.77E-28	4.32E-28	3.81E-15	**1.5E-32**	1.43E-30
	5	0	1.42E-09	4.85E-07	5.6E-07	9.76E-07	**1.5E-32**	1.06E-16	5.8E-11	0.005969	0.002984	1.73E-12
	10	0	0.127908	0.051163	1.26E-06	0.011996	**2.26E-28**	2.69E-09	1.86E-07	0.10552	0.047748	1.21E-08
	20	0	6.740223	0.344794	4.04E-06	0.025692	**8.5E-13**	1.04E-05	4.27E-06	0.925722	0.504388	2.09E-06

Bold values indicate the best value obtained for the particular function

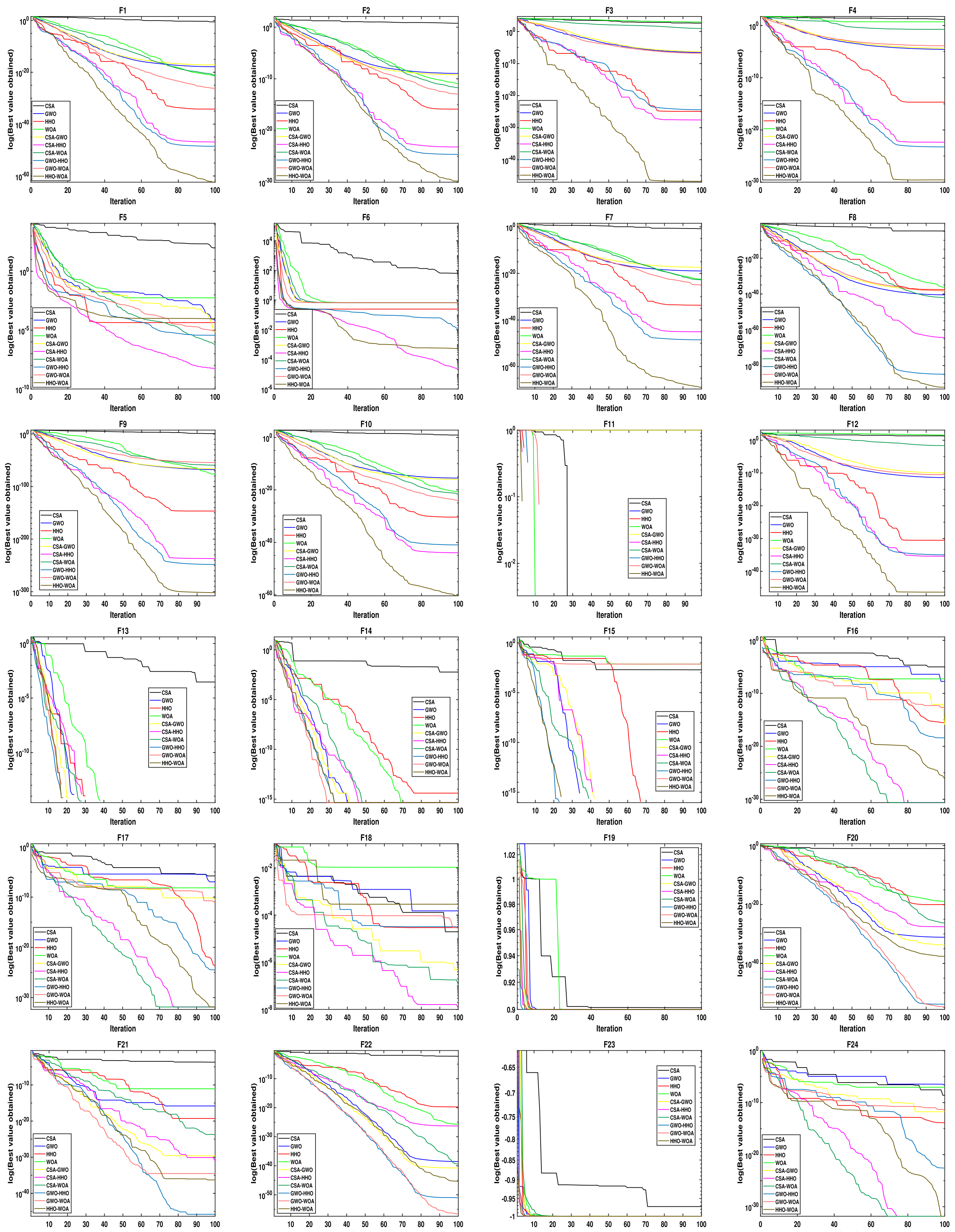

Fig. 2 Convergence plot of unimodal and multimodal benchmark functions

Table 5 Effect of population size on F1 to F6 unimodal functions

FID	Algorithms	Population = 25		Population = 50		Population = 75		Population = 100		Population = 125		Population = 150	
		Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F1	CSA	1.4E-08	8.1E-09	9.9E-08	4.5E-08	1.8E-07	7.9E-08	2E-07	9.5E-08	2.2E-07	8.4E-08	2.6E-07	1.4E-07
	GWO	3.8E-55	1.2E-54	1.2E-72	3.9E-72	4.2E-83	1.3E-82	9E-90	1.8E-89	4.3E-95	1.5E-94	5E-101	1E-100
	HHO	2.2E-97	1.2E-96	1E-104	8E-104	3E-106	1E-105	1E-110	5E-110	1E-115	4E-115	7E-112	4E-111
	WOA	2.8E-75	1.3E-74	2.1E-90	7E-90	4.9E-97	2.7E-96	3E-103	1E-102	6E-105	2E-104	2E-109	1E-108
	CSA-GWO	2E-190	0	7E-202	0	8E-209	0	7E-213	0	7E-211	0	3E-215	0
	CSA-HHO	1.6E-57	3.8E-57	1.7E-73	4.6E-73	4.3E-83	1.1E-82	3.3E-89	1.3E-88	6.8E-95	3.3E-94	2.9E-98	1.6E-97
	CSA-WOA	3E-198	0	2E-214	0	2E-222	0	5E-236	0	5E-238	0	9E-245	0
	GWO-HHO	7.9E-76	3.2E-75	4E-100	2E-99	1E-115	5E-115	2E-126	5E-126	2E-132	1E-131	1E-138	7E-138
	GWO-WOA	6.8E-81	2E-80	3E-98	1.4E-97	1E-110	6E-110	1E-118	3E-118	1E-120	5E-120	1E-126	3E-126
	HHO-WOA	**6E-253**	**0**	**2E-285**	**0**	**3E-297**	**0**	**4E-305**	**0**	**0**	**0**	**0**	**0**
F2	CSA	0.00347	0.00113	0.01069	0.00325	0.0173	0.00512	0.01965	0.00401	0.02276	0.00413	0.02311	0.0046
	GWO	2.2E-30	5.1E-30	4.7E-40	6.2E-40	1E-45	3E-45	1.9E-49	4.5E-49	5.3E-53	7.9E-53	1.5E-55	2.1E-55
	HHO	1.7E-51	5.9E-51	6E-55	2.9E-54	6E-55	2.3E-54	4.2E-57	2E-56	2.3E-58	7.8E-58	6.7E-59	3.6E-58
	WOA	8.4E-51	4.5E-50	9E-56	4.8E-55	1.7E-58	7.6E-58	4.6E-59	2.2E-58	6E-62	2E-61	3.4E-61	1.3E-60
	CSA-GWO	2.2E-97	9.8E-97	3E-102	7E-102	3E-102	2E-101	8E-105	4E-104	1E-108	6E-108	3E-109	1E-108
	CSA-HHO	2.9E-32	4E-32	8E-41	1.7E-40	2.8E-46	7.2E-46	2.4E-50	4.6E-50	2.6E-53	4.5E-53	1.2E-55	3.4E-55
	CSA-WOA	1E-99	4E-99	8E-106	4E-105	2E-112	9E-112	7E-123	3E-122	4E-121	2E-120	1E-125	8E-125
	GWO-HHO	1.7E-54	5.8E-54	2.5E-66	4.6E-66	5.8E-73	1.9E-72	6.6E-77	1.9E-76	1.9E-80	6.7E-80	1.2E-83	4.5E-83
	GWO-WOA	1.3E-52	6.4E-52	2.7E-62	8.6E-62	4.7E-65	2.3E-64	1.3E-68	2.8E-68	1.1E-70	3.3E-70	9.4E-70	5.2E-69
	HHO-WOA	**2E-136**	**9E-136**	**5E-151**	**3E-150**	**5E-159**	**2E-158**	**1E-165**	**0**	**3E-167**	**0**	**2E-167**	**0**
F3	CSA	0.03794	0.02595	0.07468	0.02734	0.09731	0.05307	0.09686	0.03013	0.10648	0.04754	0.10624	0.03736
	GWO	1.6E-22	6.3E-22	1.6E-31	5.7E-31	8E-37	2.5E-36	1.5E-41	5.3E-41	1E-45	3.5E-45	5.9E-48	1.9E-47
	HHO	5.4E-82	3E-81	2.8E-92	1.4E-91	2.5E-93	1.4E-92	2E-99	6E-99	2E-103	8E-103	2E-102	1E-101
	WOA	374.073	717.691	43.899	69.2829	14.9292	24.3934	1.92778	4.02285	1.68281	2.41093	1.55444	3.43958
	CSA-GWO	1E-117	6E-117	8E-140	3E-139	5E-138	2E-137	5E-147	3E-146	1E-153	5E-153	5E-161	3E-160
	CSA-HHO	2.9E-23	9.9E-23	3.9E-32	1E-31	2.3E-37	6.7E-37	2.6E-40	1.4E-39	1.5E-44	7E-44	3.1E-47	1.2E-46
	CSA-WOA	8E-108	4E-107	1E-122	6E-122	3E-133	2E-132	8E-139	5E-138	8E-149	3E-148	2E-143	9E-143
	GWO-HHO	1.2E-21	5.6E-21	4.7E-32	1.1E-31	2.2E-36	7.2E-36	7.5E-40	3E-39	4.7E-44	1.2E-43	6.7E-46	3.1E-45
	GWO-WOA	0.74909	1.49903	9.5E-05	0.00028	2.8E-07	7.9E-07	8.6E-09	2.6E-08	2.2E-11	6.6E-11	1.9E-13	3.6E-13
	HHO-WOA	**7E-204**	**0**	**2E-227**	**0**	**1E-234**	**0**	**3E-250**	**0**	**5E-243**	**0**	**3E-254**	**0**

Table 5 continued

FID	Algorithms	Population = 25		Population = 50		Population = 75		Population = 100		Population = 125		Population = 150	
		Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F4	CSA	0.09973	0.0305	0.2509	0.05288	0.35041	0.06229	0.40308	0.08212	0.42942	0.06146	0.44354	0.0892
	GWO	3.3E-17	6.3E-17	1.6E-22	2.3E-22	2E-25	6.5E-25	2.1E-28	4.3E-28	2.4E-30	9.5E-30	3.8E-32	1.3E-31
	HHO	3.3E-47	1.8E-46	3.1E-52	1.1E-51	3.2E-52	1.7E-51	4.1E-54	1.7E-53	2.5E-55	1.3E-54	5E-55	2.6E-54
	WOA	6.42112	10.7776	2.99733	8.8931	0.72492	2.5233	0.24001	0.81947	0.00315	0.01142	0.00398	0.01768
	CSA-GWO	4.1E-88	2.3E-87	1E-99	4E-99	5E-102	1E-101	1E-103	5E-103	4E-103	2E-102	2E-106	7E-106
	CSA-HHO	1.4E-16	2.6E-16	1.6E-22	2.7E-22	4.6E-26	1.1E-25	1.7E-28	3E-28	3.4E-30	1.3E-29	1.8E-32	4.3E-32
	CSA-WOA	7.8E-94	4.3E-93	3E-101	1E-100	1E-108	5E-108	5E-111	3E-110	7E-115	4E-114	3E-117	2E-116
	GWO-HHO	1.8E-13	7.1E-13	2.4E-18	6.4E-18	6.9E-22	1.1E-21	4.6E-25	9.7E-25	2.2E-26	3.6E-26	3.1E-28	7.5E-28
	GWO-WOA	7.57656	6.56775	2.811	3.73476	0.30281	1.18363	0.08776	0.42891	0.01116	0.04234	0.00011	0.00048
	HHO-WOA	**1E-118**	**5E-118**	**4E-132**	**2E-131**	**1E-137**	**5E-137**	**5E-139**	**3E-138**	**3E-142**	**2E-141**	**1E-141**	**7E-141**
F5	CSA	5.4E-06	5.2E-06	2.9E-05	1.6E-05	5.6E-05	2.6E-05	7.8E-05	3.3E-05	9.1E-05	3.4E-05	8.5E-05	3.7E-05
	GWO	0.00835	0.04572	2.7E-06	9.6E-07	2.2E-06	7.5E-07	2E-06	4.3E-07	0.00836	0.04578	1.5E-06	3.8E-07
	HHO	1E-04	0.00015	1.5E-05	2.5E-05	8.4E-06	1.1E-05	3.6E-06	4.6E-06	1.6E-06	1.9E-06	2.7E-06	3.4E-06
	WOA	0.0086	0.03377	0.00014	0.00012	3E-05	2.1E-05	1.1E-05	9.3E-06	4.9E-06	4.2E-06	2.5E-06	2.1E-06
	CSA-GWO	**1.5E-11**	**3.2E-11**	**5.8E-20**	**1.3E-19**	1.2E-23	3.5E-23	1.7E-27	7.3E-27	3.3E-30	1.2E-29	1E-31	1.9E-31
	CSA-HHO	6.7E-11	6.1E-11	3.2E-10	1.8E-10	4.1E-10	3.3E-10	3.9E-10	2E-10	4.8E-10	2.1E-10	4.8E-10	3.2E-10
	CSA-WOA	1.5E-06	8.4E-07	1.5E-07	1.4E-07	1E-08	1.2E-08	1.7E-09	3.3E-09	2.5E-10	3.2E-10	6.7E-11	8.6E-11
	GWO-HHO	0.02467	0.0753	0.00826	0.04522	0.00836	0.04577	1.3E-09	1.2E-09	6.6E-10	4.3E-10	2.6E-10	1.5E-10
	GWO-WOA	3.4E-11	8.1E-11	2E-19	6.9E-19	**9.2E-25**	**3.4E-24**	**1.6E-30**	**6.3E-30**	**1.7E-32**	**3.4E-32**	8.2E-33	**1.3E-32**
	HHO-WOA	5.3E-05	4.6E-05	6.4E-07	4.8E-07	4.4E-08	4.5E-08	5.5E-09	7.3E-09	7.6E-10	8.5E-10	1.7E-10	2.6E-10
F6	CSA	0.52737	0.17466	0.32251	0.22259	0.25213	0.17107	0.13084	0.1035	0.12033	0.07825	0.12692	0.07963
	GWO	0.66667	8E-06	0.66667	1.8E-05	0.66668	2.5E-05	0.66667	3.5E-05	0.66668	5.5E-05	0.66667	4E-06
	HHO	0.22622	0.03997	0.20033	0.05828	0.19009	0.0542	0.16114	0.0683	0.15468	0.05812	0.15036	0.05863
	WOA	0.64217	0.127	0.64835	0.10121	0.64453	0.12162	0.55586	0.25238	0.57794	0.23017	0.55638	0.25289
	CSA-GWO	**1E-07**	**2.1E-07**	**1.8E-13**	**4.3E-13**	**1.5E-17**	**5.7E-17**	**1.6E-20**	**4.4E-20**	8.4E-23	**4.5E-22**	**1.2E-26**	**4.7E-26**
	CSA-HHO	0.66667	6.1E-07	0.66667	3.3E-07	0.66633	0.00187	0.65864	0.04398	0.64444	0.12171	0.66667	7.6E-08
	CSA-WOA	0.0019	0.00208	0.00013	0.00018	1.1E-05	1.7E-05	2.3E-06	3.6E-06	6.5E-07	7.3E-07	1.4E-07	1.6E-07
	GWO-HHO	0.66676	0.00035	0.6669	0.00122	0.64454	0.12173	0.66729	0.00194	0.6668	0.00058	0.64449	0.12172
	GWO-WOA	0.66667	1.2E-08	0.57778	0.2305	0.66667	5E-17	0.64444	0.12172	0.64444	0.12172	0.62222	0.16914
	HHO-WOA	0.00921	0.01439	5.4E-05	8.7E-05	2.4E-06	3.6E-06	8.1E-08	1.1E-07	1.4E-08	1.7E-08	4.4E-09	6.9E-09

Bold values indicate the best value obtained for the particular function

Table 6 Effect of population size on F7 to F12 unimodal functions

FID	Algorithms	Population = 25		Population = 50		Population = 75		Population = 100		Population = 125		Population = 150	
		Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F7	CSA	1.7E-08	1.1E-08	7.5E-08	3.1E-08	1.5E-07	7.3E-08	2.2E-07	1.1E-07	2.5E-07	1E-07	2.6E-07	6.8E-08
	GWO	3.1E-54	9.2E-54	2.3E-73	4.4E-73	8.4E-83	3.2E-82	3.1E-90	8.6E-90	4.1E-96	2E-95	2E-100	8E-100
	HHO	4E-99	1.8E-98	5E-104	2E-103	7E-108	4E-107	2E-108	7E-108	5E-113	2E-112	4E-114	2E-113
	WOA	1.1E-76	3.7E-76	2.6E-88	1.4E-87	1E-99	4E-99	7E-105	3E-104	2E-106	5E-106	1E-107	6E-107
	CSA-GWO	9E-188	0	1E-201	0	5E-207	0	7E-209	0	8E-216	0	4E-215	0
	CSA-HHO	5.3E-55	2.9E-54	1.2E-73	3.6E-73	1.6E-83	5.5E-83	2.5E-90	7.8E-90	4.3E-97	1.2E-96	7E-101	4E-100
	CSA-WOA	9E-197	0	5E-217	0	6E-228	0	1E-239	0	5E-240	0	4E-238	0
	GWO-HHO	1.2E-75	6.1E-75	7E-102	3E-101	3E-113	1E-112	3E-126	2E-125	3E-132	1E-131	2E-135	8E-135
	GWO-WOA	1.4E-81	7.1E-81	9E-101	3E-100	1E-110	7E-110	3E-117	1E-116	2E-120	7E-120	1E-124	5E-124
	HHO-WOA	**9E-253**	**0**	**3E-284**	**0**	**1E-294**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
F8	CSA	1.4E-18	4.8E-18	5.1E-17	5.5E-17	1.3E-16	1E-16	3.8E-16	2.5E-16	5.3E-16	4.6E-16	5.5E-16	3.9E-16
	GWO	8E-111	4E-110	2E-146	9E-146	4E-167	0	2E-180	0	1E-190	0	1E-197	0
	HHO	1E-120	7E-120	2E-130	5E-130	3E-132	9E-132	5E-131	3E-130	3E-136	2E-135	3E-131	2E-130
	WOA	2.2E-97	1.2E-96	1E-122	5E-122	4E-137	2E-136	4E-148	2E-147	4E-151	2E-150	3E-160	1E-159
	CSA-GWO	2E-244	0	2E-263	0	1E-260	0	3E-261	0	1E-265	0	5E-263	0
	CSA-HHO	6E-114	3E-113	6E-144	3E-143	2E-166	0	1E-179	0	3E-190	0	4E-197	0
	CSA-WOA	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	GWO-HHO	6E-122	3E-121	2E-154	9E-154	1E-178	0	9E-191	0	2E-207	0	1E-218	0
	GWO-WOA	3E-112	1E-111	2E-147	1E-146	1E-172	0	8E-190	0	1E-198	0	3E-205	0
	HHO-WOA	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
F9	CSA	6.1E-26	1.6E-25	9.6E-23	3.2E-22	1.6E-21	3.6E-21	1.1E-20	2E-20	9.4E-21	1.4E-20	3.1E-20	6.2E-20
	GWO	6E-194	0	7E-248	0	9E-293	0	**0**	**0**	**0**	**0**	**0**	**0**
	HHO	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	WOA	4E-169	0	2E-256	0	5E-287	0	**0**	**0**	**0**	**0**	**0**	**0**
	CSA-GWO	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	CSA-HHO	2E-194	0	4E-254	0	4E-292	0	**0**	**0**	**0**	**0**	**0**	**0**
	CSA-WOA	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	GWO-HHO	5E-187	0	2E-245	0	5E-287	0	**0**	**0**	**0**	**0**	**0**	**0**
	GWO-WOA	6E-104	3E-103	5E-196	0	2E-265	0	1E-303	0	**0**	**0**	**0**	**0**
	HHO-WOA	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

Table 6 continued

FID	Algorithms	Population = 25		Population = 50		Population = 75		Population = 100		Population = 125		Population = 150	
		Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F10	CSA	2.5E-07	1.8E-07	1.5E-06	7.4E-07	2.8E-06	1.3E-06	3.3E-06	1.8E-06	4E-06	1.7E-06	4.6E-06	2.2E-06
	GWO	8.4E-54	2.5E-53	1.6E-71	3E-71	3.5E-81	1.5E-80	3.1E-88	1.3E-87	4.2E-95	1.1E-94	2E-99	4E-99
	HHO	2.7E-98	1.3E-97	1E-102	7E-102	1E-106	7E-106	3E-109	1E-108	5E-107	3E-106	6E-113	2E-112
	WOA	8.6E-71	4.2E-70	1.7E-89	7.4E-89	1E-95	5.5E-95	2E-101	6E-101	4E-102	2E-101	4E-106	2E-105
	CSA-GWO	1E-182	0	2E-201	0	3E-204	0	6E-209	0	2E-211	0	3E-216	0
	CSA-HHO	3.4E-55	1.1E-54	3.4E-72	1.1E-71	3.6E-82	1.1E-81	2.7E-89	7.3E-89	7.6E-95	1.9E-94	3E-100	9E-100
	CSA-WOA	3E-188	0	2E-218	0	1E-218	0	5E-240	0	9E-240	0	1E-243	0
	GWO-HHO	8.7E-75	3.9E-74	1E-100	7E-100	1E-113	6E-113	8E-123	4E-122	4E-132	1E-131	2E-138	5E-138
	GWO-WOA	1.4E-77	5.8E-77	5E-100	2E-99	7E-108	3E-107	2E-115	1E-114	7E-120	4E-119	1E-123	7E-123
	HHO-WOA	**5E-250**	**0**	**3E-280**	**0**	**4E-292**	**0**	**1E-304**	**0**	**0**	**0**	**0**	**0**
F11	**CSA**	0	0	0	0	0	0	0	0	0	0	0	0
	GWO	0.99843	0.0001	0.99838	7.1E-05	0.99837	6.2E-05	0.99835	4.2E-05	0.99834	6.8E-16	0.99834	6.8E-16
	HHO	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**
	WOA	−1	9.2E-17	−0.9334	0.36485	−1	7.1E-17	−0.9334	0.36485	−1	0	−1	5.8E-17
	CSA-GWO	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**
	CSA-HHO	0.99834	6.8E-16	0.99834	6.8E-16	0.99834	6.8E-16	0.99834	6.8E-16	0.98973	0.04713	0.99834	6.8E-16
	CSA-WOA	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**
	GWO-HHO	−1	9.2E-17	−0.9334	0.36485	**−1**	**0**	−1	4.1E-17	**−1**	**0**	−1	4.1E-17
	GWO-WOA	−1	4.1E-17	−0.8002	0.60975	−0.9334	0.36485	**−1**	**0**	**−1**	**0**	**−1**	**0**
	HHO-WOA	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**
F12	CSA	0.00128	0.00081	0.00259	0.00145	0.0039	0.00173	0.00354	0.00202	0.00435	0.00181	0.00385	0.00143
	GWO	2.3E-30	7.3E-30	5.9E-44	1.5E-43	4.4E-51	1.9E-50	3.2E-58	9E-58	5.8E-63	1.4E-62	1.8E-66	6.8E-66
	HHO	9.3E-78	4.6E-77	6.6E-87	3.5E-86	2.3E-98	8.6E-98	1.2E-95	6.6E-95	1.3E-98	6.9E-98	6E-100	3E-99
	WOA	21.8904	14.3078	3.44949	5.69751	0.67032	1.40104	0.07568	0.16315	0.01774	0.06162	0.0035	0.00566
	CSA-GWO	5.2E-98	2.9E-97	5E-120	3E-119	1E-140	6E-140	5E-136	3E-135	2E-150	1E-149	8E-154	4E-153
	CSA-HHO	1.9E-32	5.2E-32	1E-43	3.9E-43	1.2E-50	4.8E-50	2.6E-56	1E-55	1.5E-60	3.6E-60	1.4E-63	7.3E-63
	CSA-WOA	1E-108	6E-108	2E-130	1E-129	4E-142	2E-141	5E-141	2E-140	3E-156	2E-155	5E-151	3E-150
	GWO-HHO	7.1E-32	2.8E-31	1.3E-43	4.5E-43	3.8E-51	1.8E-50	4.5E-57	1.7E-56	3.1E-62	7.6E-62	4.2E-65	2E-64
	GWO-WOA	0.0004	0.00089	1.9E-08	5.3E-08	2.4E-11	6.6E-11	4.8E-13	1.4E-12	4.3E-16	9.4E-16	5.6E-17	8.1E-17
	HHO-WOA	**9E-193**	**0**	**1E-218**	**0**	**9E-233**	**0**	**1E-242**	**0**	**4E-252**	**0**	**4E-252**	**0**

Bold values indicate the best value obtained for the particular function

Table 7 Effect of population size on F13 to F18 multimodal functions

FID	Algorithms	Population = 25		Population = 50		Population = 75		Population = 100		Population = 125		Population = 150	
		Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F13	CSA	11.5621	2.75724	12.0439	3.14976	10.6823	2.50641	10.8712	2.19173	10.8821	1.87128	10.3702	2.09112
	GWO	0.55149	1.37834	1.0826	2.10681	0.20891	1.14424	0.17292	0.9471	**0**	**0**	**0**	**0**
	HHO	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	WOA	2.9992	8.30064	9.5E-16	3.6E-15	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	CSA-GWO	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	CSA-HHO	4.51538	1.81044	3.89952	1.35242	3.31761	1.28902	2.79949	1.09885	2.91944	1.24835	2.40934	1.00555
	CSA-WOA	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	GWO-HHO	9.78478	9.6914	6.50886	8.88624	5.9228	10.5159	4.32642	6.49192	6.07776	10.0437	4.88473	7.74396
	GWO-WOA	6.89837	9.00188	7.76067	7.40605	4.80896	7.33045	4.27832	6.50682	2.52056	4.27535	2.88538	5.22153
	HHO-WOA	**0**	**0**	0	**0**	**0**	**0**	**0**	0	**0**	**0**	**0**	**0**
F14	CSA	0.31572	0.32959	1.40244	0.67012	2.12107	0.49072	2.3761	0.66966	2.42696	0.50895	2.55015	0.52701
	GWO	8.1E-15	2E-15	5.4E-15	1.6E-15	5.3E-15	1.5E-15	4.8E-15	1.1E-15	4.4E-15	0	4.4E-15	0
	HHO	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**
	WOA	4.2E-15	3.2E-15	4.1E-15	2.4E-15	3.8E-15	2.5E-15	3.8E-15	2.5E-15	4.2E-15	2.6E-15	3.8E-15	2.1E-15
	CSA-GWO	1E-15	6.5E-16	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**
	CSA-HHO	4.3E-15	6.5E-16	4.4E-15	0	4.4E-15	0	4.4E-15	0	4.4E-15	0	4.3E-15	6.5E-16
	CSA-WOA	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**
	GWO-HHO	5.9E-15	2.4E-15	5.4E-15	1.9E-15	4.4E-15	1.3E-15	4.7E-15	9E-16	4.7E-15	9E-16	4.6E-15	6.5E-16
	GWO-WOA	4E-15	1.2E-15	3.7E-15	2E-15	3.5E-15	1.6E-15	3.4E-15	1.9E-15	2.5E-15	1.8E-15	2.5E-15	1.8E-15
	HHO-WOA	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**	**8.9E-16**	**0**
F15	CSA	0.07751	0.02238	0.07559	0.01763	0.06682	0.01198	0.06417	0.01442	0.06267	0.01483	0.06347	0.01437
	GWO	0.01819	0.03039	0.02422	0.02602	0.03108	0.04325	0.01447	0.01921	0.01727	0.02157	0.0219	0.03119
	HHO	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	WOA	0.08968	0.1851	0.04769	0.1066	0.0422	0.09328	0.04927	0.08228	0.06242	0.14343	0.02563	0.068
	CSA-GWO	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	CSA-HHO	0.02665	0.01664	0.02168	0.01427	0.01825	0.01176	0.01874	0.01147	0.01293	0.01311	0.01315	0.01387
	CSA-WOA	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	GWO-HHO	0.08017	0.08265	0.05736	0.07373	0.09383	0.11624	0.1074	0.1281	0.06109	0.07879	0.05506	0.07952
	GWO-WOA	0.05994	0.07898	0.07891	0.10575	0.08573	0.13018	0.08014	0.09555	0.05684	0.10794	0.05357	0.08146
	HHO-WOA	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

Table 7 continued

FID	Algorithms	Population = 25		Population = 50		Population = 75		Population = 100		Population = 125		Population = 150	
		Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F16	CSA	0.00686	0.01069	0.02993	0.02965	0.0357	0.02224	0.05913	0.05784	0.04997	0.02983	0.05212	0.0219
	GWO	0.00887	0.01367	0.00133	0.00507	5.1E-07	2E-07	3.5E-07	1.6E-07	0.00066	0.00361	3E-07	8.7E-08
	HHO	8.2E-05	0.00018	5.9E-06	8.5E-06	3.3E-06	4.7E-06	1E-06	9.4E-07	1.1E-06	1.3E-06	1.2E-06	1.3E-06
	WOA	0.12266	0.61008	0.00321	0.0122	0.00057	0.00207	0.00011	0.00033	4.7E-05	0.00017	4.3E-06	3.4E-06
	CSA-GWO	**1.1E-11**	**5.7E-11**	**1.1E-18**	**5.1E-18**	**6.7E-25**	**2E-24**	**3.1E-28**	**1.3E-27**	1.2E-30	**3.3E-30**	**1.7E-31**	**4.6E-31**
	CSA-HHO	1.4E-11	9.7E-12	4.9E-11	4E-11	6.3E-11	4E-11	7.5E-11	3.7E-11	8.9E-11	5.2E-11	8.9E-11	4.8E-11
	CSA-WOA	2.3E-06	1.1E-06	2.4E-07	1.4E-07	4.6E-08	2.4E-08	1.3E-08	1.4E-08	2.7E-09	2.4E-09	9.2E-10	9.3E-10
	GWO-HHO	0.07593	0.33474	0.00664	0.01144	0.00315	0.00717	0.00327	0.00743	0.00133	0.00505	0.0003	0.00166
	GWO-WOA	0.54875	1.96868	1.95161	7.12892	2.2E-13	1.2E-12	0.20811	1.13986	2.1E-21	1.2E-20	1.9E-31	7.5E-31
	HHO-WOA	4.8E-05	5.1E-05	1.9E-06	1.8E-06	2.2E-07	2E-07	4.7E-08	6.4E-08	1.2E-08	1.1E-08	3.7E-09	4E-09
F17	CSA	7.1E-05	5.1E-05	0.0007	0.00047	0.0013	0.00052	0.00176	0.00067	0.002	0.00107	0.00247	0.00107
	GWO	0.01884	0.03879	3.8E-06	1.6E-06	0.00333	0.01825	1.9E-06	7.9E-07	1.8E-06	5.7E-07	1.5E-06	5.6E-07
	HHO	0.00014	0.00021	4E-05	5.6E-05	1.9E-05	3.9E-05	3.9E-06	3.3E-06	3.4E-06	6.2E-06	4.1E-06	6.2E-06
	WOA	0.06276	0.07534	0.01046	0.02547	0.00146	0.00338	0.00044	0.002	0.0008	0.00278	2.7E-05	3.8E-05
	CSA-GWO	**2.5E-11**	**1.1E-10**	**4.7E-18**	**2E-17**	**5.8E-23**	**2.9E-22**	**1.1E-27**	**2.6E-27**	2.3E-30	**5.6E-30**	**1.6E-31**	**3.1E-31**
	CSA-HHO	1E-09	1.1E-09	4.8E-09	4.3E-09	5.4E-09	4.6E-09	7.5E-09	6.4E-09	7E-09	3.3E-09	8.5E-09	4.6E-09
	CSA-WOA	9.3E-06	5E-06	1.4E-06	7.4E-07	2.3E-07	1.6E-07	8.9E-08	7.3E-08	1.5E-08	1.3E-08	5.1E-09	5.1E-09
	GWO-HHO	0.04384	0.07211	0.02275	0.03908	0.00993	0.02916	0.00587	0.01955	0.00682	0.02454	0.00696	0.02369
	GWO-WOA	0.0149	0.02537	0.01746	0.0372	0.00618	0.01925	0.0011	0.00335	0.00293	0.00862	0.0018	0.00494
	HHO-WOA	0.00054	0.00203	4.8E-06	3.2E-06	1.7E-06	1.4E-06	2.6E-07	2.3E-07	6.6E-08	6.2E-08	2.5E-08	2E-08
F18	CSA	0.12326	0.03337	0.09755	0.02608	0.09309	0.03107	0.08248	0.02685	0.07332	0.01765	0.07736	0.02344
	GWO	0.10766	0.0701	0.07334	0.05714	0.041	0.03692	0.02561	0.03628	0.02464	0.03121	0.01763	0.02419
	HHO	0.0078	0.00837	0.00775	0.00969	0.00369	0.00349	0.00308	0.00338	0.00281	0.00347	0.00183	0.00235
	WOA	0.25205	0.0628	0.22236	0.05011	0.21515	0.04937	0.21179	0.04281	0.21293	0.05485	0.20121	0.05683
	CSA-GWO	0.00387	0.00422	0.00206	0.00188	0.00088	0.00072	0.00037	0.00025	0.00034	0.00018	0.00025	0.00016
	CSA-HHO	**0.00069**	**0.00042**	**0.00025**	**0.00013**	**0.00011**	**5.2E-05**	**8.1E-05**	**3.6E-05**	7.2E-05	**3.9E-05**	**6.8E-05**	**2.9E-05**
	CSA-WOA	0.00664	0.01018	0.00196	0.00124	0.00144	0.00101	0.00133	0.00127	0.00086	0.00061	0.00085	0.00048
	GWO-HHO	0.13605	0.05728	0.12074	0.0676	0.07452	0.05828	0.0768	0.05898	0.06546	0.05019	0.05389	0.03985
	GWO-WOA	0.0688	0.0438	0.03564	0.01825	0.01544	0.01054	0.01032	0.0065	0.006	0.00494	0.00619	0.00822
	HHO-WOA	0.01296	0.01473	0.00457	0.00727	0.00476	0.00813	0.00317	0.00456	0.00249	0.00269	0.00228	0.003

Bold values indicate the best value obtained for the particular function

Table 8 Effect of population size on F19 to F24 multimodal functions

FID	Algorithms	Population = 25		Population = 50		Population = 75		Population = 100		Population = 125		Population = 150	
		Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F19	CSA	1.00351	0.00117	1.00283	0.00093	1.00233	0.00082	1.00219	0.00053	1.00218	0.00051	1.00192	0.00041
	GWO	1.17673	0.28897	1.06543	0.15857	1.09173	0.20309	1.06893	0.21765	1.12502	0.26762	1.07699	0.18943
	HHO	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**
	WOA	1.02816	0.07765	0.98201	0.05579	0.99098	0.04185	0.98697	0.03966	0.97875	0.04424	0.9854	0.03891
	CSA-GWO	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**
	CSA-HHO	1.00305	0.00119	1.00224	0.00066	1.002	0.00072	1.00161	0.0005	1.00173	0.00044	1.00155	0.00041
	CSA-WOA	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**
	GWO-HHO	1.0567	0.06608	1.01901	0.05601	1.01944	0.04759	0.98867	0.04784	1.02127	0.02411	0.99289	0.06126
	GWO-WOA	0.98	0.04068	0.97667	0.04302	0.97	0.04661	0.98667	0.03457	0.98667	0.03457	0.99	0.03051
	HHO-WOA	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**	**0.9**	**4.5E-16**
F20	**CSA**	0.35611	0.06853	0.44069	0.07362	0.43778	0.08328	0.4611	0.06453	0.47736	0.064	0.44855	0.06677
	GWO	0.10321	0.01826	0.09987	5.1E-10	0.09987	3.5E-09	0.09987	7.5E-11	0.09987	1.1E-10	0.09987	6.9E-11
	HHO	1.5E-48	7.8E-48	3.6E-52	1.9E-51	3.6E-53	1.9E-52	1.6E-55	6E-55	3.3E-56	1.3E-55	1.8E-55	9.2E-55
	WOA	0.09323	0.05204	0.10656	0.0583	0.1099	0.08844	0.12322	0.05681	0.10322	0.04136	0.11322	0.06286
	CSA-GWO	2.7E-84	1.5E-83	8.3E-96	2.9E-95	6E-99	2.1E-98	1.8E-98	9.7E-98	8E-104	3E-103	8E-106	2E-105
	CSA-HHO	0.09987	1.1E-11	0.09987	1.6E-12	0.09987	1.6E-12	0.09987	5.1E-13	0.09987	2.3E-13	0.09987	3.9E-13
	CSA-WOA	7.5E-90	4.1E-89	2E-104	1E-103	1E-110	6E-110	1E-106	6E-106	9E-119	5E-118	7E-120	4E-119
	GWO-HHO	0.11987	0.04068	0.10654	0.02537	0.10321	0.01826	0.10321	0.01826	0.09987	4.4E-17	0.09987	2.3E-17
	GWO-WOA	0.13654	0.06149	0.13321	0.05467	0.11987	0.04068	0.12987	0.0535	0.13321	0.04795	0.12321	0.04302
	HHO-WOA	**2E-123**	**1E-122**	**1E-136**	**6E-136**	**3E-141**	**2E-140**	**1E-141**	**6E-141**	**1E-151**	**4E-151**	**2E-153**	**7E-153**
F21	CSA	0.00307	0.00292	0.00565	0.00531	0.00706	0.00461	0.00693	0.00526	0.00838	0.00552	0.00789	0.00447
	GWO	2.9E-23	1.6E-22	4.6E-27	2.5E-26	6.1E-42	3.3E-41	2.5E-43	1.4E-42	4.1E-50	2.3E-49	2.5E-51	1.4E-50
	HHO	1.8E-24	9.6E-24	4.6E-11	2.5E-10	2.5E-13	1.3E-12	9.1E-20	5E-19	3.1E-13	1.3E-12	2.5E-26	1.2E-25
	WOA	0.03606	0.18459	0.00422	0.02309	0.00016	0.00068	2E-05	7.4E-05	4E-05	0.0002	0.00013	0.0007
	CSA-GWO	2.5E-96	1.2E-95	3E-103	1E-102	2E-104	1E-103	6E-105	3E-104	4E-109	2E-108	2E-109	9E-109
	CSA-HHO	5.3E-53	2E-52	4.5E-66	1.8E-65	2E-74	5.7E-74	1.7E-78	8.9E-78	1.4E-83	7.2E-83	2.1E-84	1.2E-83
	CSA-WOA	**1E-159**	**5E-159**	**3E-168**	**0**	**1E-181**	**0**	**5E-193**	**0**	**9E-199**	**0**	**9E-191**	**0**
	GWO-HHO	1.2E-52	6.4E-52	4.7E-69	2.3E-68	4.5E-71	2.2E-70	9.3E-78	5.1E-77	5.5E-85	2.8E-84	1.2E-91	6.5E-91
	GWO-WOA	4.2E-35	1.7E-34	1.3E-44	4.1E-44	1.1E-50	5E-50	1.6E-53	9E-53	1.7E-60	6.5E-60	1.4E-62	7.6E-62
	HHO-WOA	5E-118	3E-117	5E-126	3E-125	2E-113	9E-113	5E-150	3E-149	6E-152	3E-151	3E-142	1E-141

Table 8 continued

FID	Algorithms	Population = 25		Population = 50		Population = 75		Population = 100		Population = 125		Population = 150	
		Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F22	CSA	0.00144	0.00018	0.0013	0.0002	0.00126	0.00017	0.00123	0.0001	0.00121	0.00015	0.00117	0.00013
	GWO	0.0012	0.0009	0.00106	0.00075	0.00091	0.00074	0.00107	0.00077	0.00075	0.0003	0.00094	0.00067
	HHO	0.00057	2E-06	0.00047	0.00021	0.00053	0.00014	0.00049	0.0002	0.00043	0.00024	0.00038	0.00027
	WOA	0.00074	0.00027	0.00071	0.0003	0.00061	9.3E-05	0.00061	0.00015	0.00065	0.00016	0.00066	0.0003
	CSA-GWO	**0.00047**	**0.00021**	0.00053	0.00014	**0.00045**	**0.00023**	0.00045	0.00023	**0.00034**	**0.00028**	**0.00032**	**0.00029**
	CSA-HHO	0.00129	0.0002	0.00119	0.00018	0.00111	0.0002	0.00101	0.00018	0.00104	0.00015	0.001	0.00015
	CSA-WOA	0.00059	0.00021	0.00059	0.00015	0.00057	2.7E-05	0.0006	0.00015	0.00062	0.00031	0.00053	0.00014
	GWO-HHO	0.00078	0.00036	0.00075	0.0004	0.00095	0.00048	0.00084	0.00041	0.00068	0.00021	0.00068	0.00019
	GWO-WOA	0.00066	0.00018	0.00062	0.0001	0.00066	0.0002	0.00057	0.00013	0.00057	0.00013	0.00055	0.00011
	HHO-WOA	0.00059	0.00013	**0.00046**	**0.00023**	0.00048	0.00022	**0.00043**	**0.00024**	0.00036	0.00028	0.00044	0.00025
F23	CSA	3E-07	1.9E-07	2.4E-07	1.4E-07	1.8E-07	6.7E-08	1.8E-07	8.6E-08	1.6E-07	7.8E-08	1.3E-07	6.3E-08
	GWO	2.5E-07	1.3E-06	2.2E-07	1.1E-06	6.2E-08	3.1E-07	1.5E-07	7.8E-07	1.4E-07	7.5E-07	4.6E-09	1.4E-09
	HHO	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**
	WOA	3.4E-05	7.8E-05	−0.0333	0.18258	−0.1	0.30513	1.4E-06	3.8E-06	−0.1	0.30513	−0.1333	0.34575
	CSA-GWO	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**
	CSA-HHO	2E-07	2.5E-07	1.6E-07	1E-07	1.3E-07	8.1E-08	9.7E-08	8.3E-08	9.7E-08	4.6E-08	8.2E-08	7.2E-08
	CSA-WOA	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**
	GWO-HHO	5E-06	1E-05	5E-06	8.6E-06	2.1E-06	5.1E-06	5.7E-06	1.5E-05	3.2E-06	5.3E-06	2.4E-06	4.3E-06
	GWO-WOA	−0.0333	0.18257	6.4E-21	2.6E-20	−0.0333	0.18257	−0.0333	0.18257	3.6E-33	1.2E-32	4.1E-35	3.5E-35
	HHO-WOA	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**	**−1**	**0**
F24	CSA	0.00499	0.00804	0.07487	0.061	0.11852	0.05379	0.14525	0.06371	0.13036	0.05672	0.16275	0.06941
	GWO	0.15726	0.09171	0.08741	0.08728	0.07524	0.07146	0.05429	0.07355	0.03029	0.04356	0.0332	0.04438
	HHO	2.3E-05	2.5E-05	7.8E-06	1E-05	3.3E-06	4.5E-06	2E-06	2.5E-06	2.1E-06	2.5E-06	9.4E-07	8.2E-07
	WOA	0.08379	0.10817	0.02457	0.047	0.00013	0.00011	0.01199	0.03094	0.006	0.02272	0.00598	0.02271
	CSA-GWO	**9.5E-12**	**2.9E-11**	**9E-21**	**2.8E-20**	**3.3E-24**	**1.2E-23**	**5.5E-28**	**2.5E-27**	**1.8E-29**	**5E-29**	**1.1E-30**	**2.6E-30**
	CSA-HHO	0.00298	0.01635	9.1E-10	7.5E-10	1.6E-09	1.1E-09	2.3E-09	1.6E-09	2.8E-09	1.7E-09	2.7E-09	1.6E-09
	CSA-WOA	2.3E-05	2.1E-05	3E-06	3.7E-06	1.1E-06	2.3E-06	1.3E-07	2E-07	4.6E-08	6.5E-08	1.5E-08	1.8E-08
	GWO-HHO	0.20046	0.10229	0.1322	0.08138	0.13249	0.10268	0.0946	0.08413	0.08743	0.07336	0.1115	0.09696
	GWO-WOA	0.22605	0.51281	0.07461	0.09998	0.0567	0.0724	0.05372	0.08008	0.03283	0.06848	0.05372	0.06482
	HHO-WOA	3.3E-05	3.3E-05	2.8E-06	6.5E-06	1.9E-07	2.7E-07	2.5E-08	3.6E-08	2.9E-09	6.5E-09	6.6E-10	1.2E-09

Bold values indicate the best value obtained for the particular function

4.2.2 Influence on population size

This section describes the effect of population size on the performance of the hybrid algorithms. These algorithms and their base algorithms are tested on 25, 50, 75, 100, 125, and 150 populations for 500 iterations. The simulation is performed over 30 epochs to analyze the stability of the obtained results. For this purpose, the dimensionality of the function has been fixed to 10 for analyzing the effect of varying populations on different algorithms. The mean and standard deviation of the generated results for all the benchmark functions are given in Tables 5, 6, 7, and 8. The best value obtained for each population size is highlighted in these tables for a better interpretation. From the tables, it can be observed that there are some cases in which multiple algorithms are performing equally well; hence, they are highlighted. Importance has been given to mean value to select the best. If the mean value is equal, then the standard deviation has been considered to select the best algorithm.

From Table 5, CSA-GWO, CSA-WOA, and HHO-WOA obtained zero standard deviation for F1 function over 25, 50, 75, 100, 125, and 150 population sizes, and HHO-WOA obtained global minima for 125 and 150 population sizes. HHO-WOA found zero std for F2 and F3 functions for 100, 125, and 150 populations. In Table 6, CSA-GWO, CSA-WOA, and HHO-WOA obtained zero std for F7, F8, F9, and F10 in different population sizes. HHO-WOA obtained minimum fitness value for all varying populations in F1, F2, F3, and F4 while CSA-GWO in F5 and F6 unimodal functions among compared algorithms. From Table 6, in F7 function, HHO-WOA generated the best average and zero std value, while CSA-GWO and CSA-WOA obtained zero std values with varying population sizes in a fixed dimension. CSA-WOA and HHO-WOA found global optima, i.e., zero average and std value in the F8 function. HHO, CSA-WOA, CSA-WOA, and HHO-WOA achieved global optima for the F9 and F11 functions and HHO-WOA with its best average value in the F10 and F12 functions.

Tables 7 and 8 show the mean and std of the compared algorithm on the F13–F24 benchmark functions over varying population size with fixed dimension. In F13 and F15, HHO, CSA-GWO, CSA-WOA, and HHO-WOA obtained their global optima and zero std values, whereas better solutions in F14 function for different populations. Also, CSA-GWO obtained the best mean solution for F16 and F17 and CSA-HHO in the F18 function. As shown in Table 8, HHO, CSA-GWO, CSA-WOA, and HHO-WOA found global optima for F19 and F23 over varying population sizes. In addition, HHO-WOA performed superior for F20 and F21 functions, while CSA-GWO for F22 and F24 functions.

From Tables 5, 6, 7, and 8, for population size 25, F1 F2, F3, F4, F7, F9, F10, F11, F12, F13, F15, F19, F20, F21, F23, and F24, the hybrid algorithms CSA-GWO, CSA-WOA, and HHO-WOA show better convergence, and among these HHO-WOA generates superior results. F14 and F22 functions do not show any proper pattern with increasing population size. It can be observed from Tables 5, 6, 7, and 8 that with the increase in the population, the performance of algorithms improves for all functions except F14 and F22. For small population sizes, CSA-GWO and CSA-HHO exhibit better performance than other algorithms in the case of F16 and F18. It can be observed that the performance of GWO-WOA on F16 improves when the population size is greater than 100. For function F24, with increasing population size, CSA-GWO significantly outperformed other algorithms. It is evident from Tables 5, 6, 7, and 8 that with the increase in population size, the performance of hybrid algorithms is comparable to or better than their base algorithms. From the results, it can be noticed that the performance of the algorithms saturates after 100 population. Therefore, for other experimental purposes, a 100 population size has been considered to evaluate the performance of hybrid algorithms.

4.3 Test on real-world problem

To demonstrate the effectiveness of the proposed hybrid algorithms, they are tested on short-term electricity load and price forecasting problems. This section discusses the results generated by hybrid algorithms when integrated with ANN and compares their results with base algorithms.

Electricity load and price forecasting are two important but crucial tasks in the deregulated power market. Error in prediction leads to substantial economic losses; therefore, an accurate model is required to meet future demand and mitigate the gap between supply and demand. Unfortunately, nonlinear and random behavior in load and price data makes forecasting difficult and unreliable. In the past few decades, various models have been proposed to improve the accuracy of electricity load and price forecasting problems (Kottath and Singh 2022; Singh and Kottath 2022b; Singh and Dwivedi 2022).

4.3.1 Problem statement

Short-term electricity load and price forecasting have become important issues in the power market. Therefore, developing an efficient and accurate forecasting model is necessary. ANN is one of the widely accepted models for time series prediction applications. The great learning capability, robustness, huge generalization ability, and high fault tolerance are a few characteristics of the ANN model (Singh and Dwivedi 2018). However, ANN carries certain limitations, such as the selection of an appropriate training algorithm, network architecture, choosing appropriate numbers of hidden neurons and hidden layers, etc. (Singh et al. 2019). One solution to these problems can be a hybrid model that uses an optimization algorithm to train the neural network. We have proposed hybrid models based on a similar concept by using the mentioned hybrid algorithms for training vanilla ANN.

4.3.2 Data description

The ISO New England electricity load dataset and ISO New England electricity price dataset have been used to test the performance of hybrid models. In the case of load forecasting, the models have been trained on hourly data from 2004 to 2007 and tested on out-of-sample data from 2008 to 2009. The purpose of this experiment is to forecast a day-ahead load on an hourly interval. The load data incorporated in the training dataset consist of eight input parameters (L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8) where L_1: *previous 24-hr. average load*, L_2: *24-hr lagged load*, L_3: *168-hr lagged load*, L_4: *dry bulb temperature* in °C, L_5: *dew point temperature* in °C, L_6: *hr. of the day*, L_7: *day of the week*, and L_8: *holiday/weekend indicator* (England 2009).

The electricity price data for 2004 to 2007 have been used to train the model for price forecasting, whereas out-of-sample data for 2008 are used to test the model. The dataset consists of fourteen input variables (P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}), where initial eight parameters $P_1 - P_8$ are same as $L_1 - L_8$ taken in load dataset, P_9: *system load*, P_{10}: *previous 24-hr. average price*, P_{11}: *24-hr lagged price*, P_{12}: *168-hr lagged price*, P_{13}: *24-hr lagged natural gas price*, and P_{14}: *168-hr average lagged natural gas price* (England 2009). Before training and testing the model, data are pre-processed using the MIN-MAX normalization technique to reduce the training time.

4.3.3 Evaluation metrics

We have used seven performance evaluation metrics to critically analyze hybrid models over forecasting results. These evaluation metrics are calculated between predicted and actual values. Mathematical equations of the metrics used are given below:

- Average error (AE)

$$\mathrm{AE} = \frac{1}{N}\sum_{j=1}^{n} Y_j - Y_j' \tag{17}$$

- Mean absolute error (MAE)

$$\mathrm{MAE} = \frac{1}{N}\sum_{j=1}^{n} |Y_j - Y_j'| \tag{18}$$

- Normalized mean squared error (NMSE)

$$\mathrm{NMSE} = \frac{1}{\Delta^2 N}\sum_{j=1}^{n}(Y_j - Y_j')^2 \tag{19}$$

- Root of mean squared error (RMSE)

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{j=1}^{n}(Y_j - Y_j')^2} \tag{20}$$

- Mean absolute percent error (MAPE)

$$\mathrm{MAPE} = \frac{1}{N}\sum_{j=1}^{n}\frac{|\ Y_j - Y_j'\ |}{Y_j} * 100 \tag{21}$$

- Directional Change (DC)

$$\mathrm{DC} = \frac{100}{N-1}\sum_{j=1}^{N-1} a_t,\ a_t = \left\{\begin{matrix} 0, \ \text{otherwise} \\ 1,\ if(Y_{j+1}' - Y_j)(Y_{j+1} - Y_j)\ >\ 0 \end{matrix}\right\} \tag{22}$$

- Pearson's correlation coefficient (r)

$$r = \frac{\sum_{j=1}^{n}(Y_j - \bar{Y})(Y_j' - \bar{Y}')}{\sqrt{\sum_{j=1}^{n}(Y_j - \bar{Y})^2 \sum_{j=1}^{n}(Y_j' - \bar{Y}')^2}} \tag{23}$$

where Y_j is actual price/ load value of day j, Y_j' is predicted price/ load value of day j, $\bar{Y}$ is mean of actual price/ load value, $\bar{Y}'$ is mean of predicted price/ load value and, N is number of elements in training data.

4.3.4 Result: electricity load forecasting

The ANN model has been trained with 20 hidden neurons for both electricity load and price forecasting. During the training phase, the neural network updates its weight to optimize the objective function, i.e., mean squared error. Figure 3 shows the convergence graph of ANN-CSA, ANN-GWO, ANN-HHO, ANN-WOA, ANN-CSA-GWO, ANN-CSA-HHO, ANN-CSA-WOA, ANN-GWO-HHO, ANN-GWO-WOA, ANN-HHO-WOA hybrid models during the training phase of neural network. From the figure, we can deduce that ANN-GWO-HHO generates minimum MSE at the end of the termination condition. It can be noted that hybrid algorithms converge earlier than standalone optimization algorithms when combined with the neural network. For example, ANN-CSA-GWO started with minimal convergence value and gave

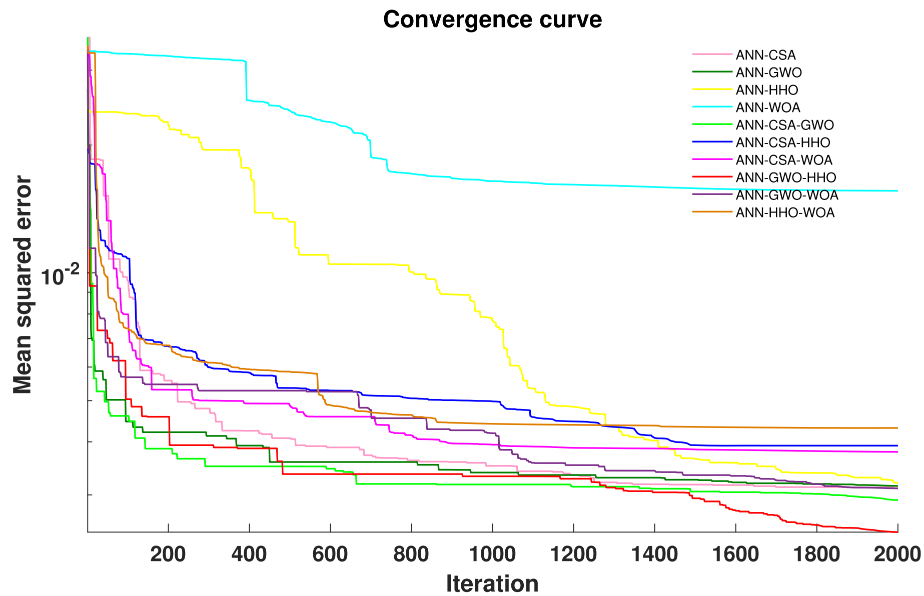

Fig. 3 Convergence plot of different algorithm for electricity load forecasting

tough competition to ANN-GWO-HHO till 1250 iterations, but later on, after 1500 iterations, the ANN-GWO-HHO converged rapidly. The graph also shows that ANN-GWO generated maximum training error. Overall we can generalize that the training error of hybrid models with combined algorithms generates less error than others.

A detailed analysis of results generated by different algorithms based on different error metrics is shown in Table 9. The forecasting results for the years 2008 and 2009 are shown in the table. Different evaluation metrics such as AE, MAE, MAPE, RMSE, NMSE, r, and DC have been used to compare the performance of different hybrid models ANN-CSA, ANN-GWO, ANN-HHO, ANN-WOA, ANN-CSA-GWO, ANN-CSA-HHO, ANN-CSA-WOA, ANN-GWO-HHO, ANN-GWO-WOA, and ANN-HHO-WOA. The table shows that the performance of the hybrid models ANN-CSA-GWO and ANN-WOA-HHO is superior to standalone and other combination algorithms. The MSE values show that the ANN-WOA-HHO model generates the least value, whereas ANN-CSA-GWO gives the second best. The hybrid model ANN-WOA failed to converge compared to other prediction models by generating maximum MAE and MAPE values of 1708.092 MWh and 12.28%, respectively, which proves its inefficacy over the electricity load forecasting problem. However, hybrid models based on WOA, such as ANN-CSA-WOA, ANN-GWO-WOA, and ANN-HHO-WOA, perform superior to the ANN-WOA algorithm. The MAPE metric shows the least value of 4.372268% for ANN-CSA-GWO, whereas the ANN-WOA with the maximum value of 12.28163 MWh. The RMSE metric has a similar response to MAE, generating the least value for ANN-WOA-HHO. The last row of the table depicts the Friedman test statistic generated by all the hybrid models for the electricity load forecast. The Friedman values show that ANN-CSA-GWO and ANN-WOA-HHO produced almost similar values. Overall, Table 9 depicts that the CSA-GWO and WOA-HHO hybrid algorithms add better learning ability to ANN than other algorithms.

Table 10 shows the MAE and RMSE values of ANN-CSA, ANN-GWO, ANN-HHO, ANN-WOA, ANN-CSA-GWO, ANN-CSA-HHO, ANN-CSA-WOA, ANN-GWO-HHO, ANN-GWO-WOA, ANN-HHO-WOA hybrid algorithms monthly. The table shows that for the month of October and November, ANN-WOA-HHO generated a minimum MAE value of 430.2016 MWh and 576.8031 MWh, and RMSE value of 553.6002 MWh and 742.1469 MWh, respectively. Based on the results, we conclude that ANN-GWO-WOA is superior among other hybrid models discussed, whereas ANN-CSA-GWO ranked second.

Figure 4 shows the bar graph of different hybrid algorithms discussed in terms of MAE and RMSE over days for 2008. From the graph, it can be noted that all the hybrid algorithms generated maximum MAE and RMSE on Monday. ANN-GWO-HHO generated minimum MAE and RMSE on Tuesday, Wednesday, Thursday, Friday, and Monday. On Saturday and Sunday, ANN-GWO-HHO generated results closer to minimal values. It is visible from the graph that ANN-WOA performed severely in terms of MAE and RMSE for all the days.

4.3.5 Result: electricity price forecasting

To deeply analyze the performance of the hybrid algorithms, they are integrated with ANN and applied to solve the electricity price forecasting problem. Figure 5 shows the plot between MSE and the increasing number of iter-

Table 9 Performance metrics of different algorithms on load forecasting

Algorithm	AE	MAE	MAPE	RMSE	NMSE	r	DC	*F*-Value
ANN-CSA	−123.407	714.7331	4.979028	921.154	0.106288	0.946676	66.45386	171
ANN-GWO	7.139642	664.9789	4.502328	882.9531	0.097656	0.950179	68.57436	64
ANN-HHO	3.109397	680.5928	4.617914	898.11	0.101037	0.948198	67.71362	158
ANN-WOA	−226.256	1708.092	12.28163	2215.132	0.61464	0.626071	55.1217	237
ANN-CSA	−7.7416	643.0749	4.372268	858.3883	0.092297	0.952756	70.41555	**47**
ANN-CSA-HHO	−140.889	806.186	5.586227	1030.684	0.133068	0.932746	71.21359	158
ANN-CSA-WOA	−49.4397	715.2502	4.805343	960.095	0.115465	0.940832	67.55401	169
ANN-GWO-HHO	−58.4613	631.6132	4.417615	818.9399	0.084009	0.957306	72.13133	49
ANN-GWO-WOA	−7.09361	657.0277	4.445229	882.1714	0.097483	0.950079	68.92778	98
ANN-HHO-WOA	−70.7691	791.7542	5.398814	1048.828	0.137794	0.929013	66.11184	224

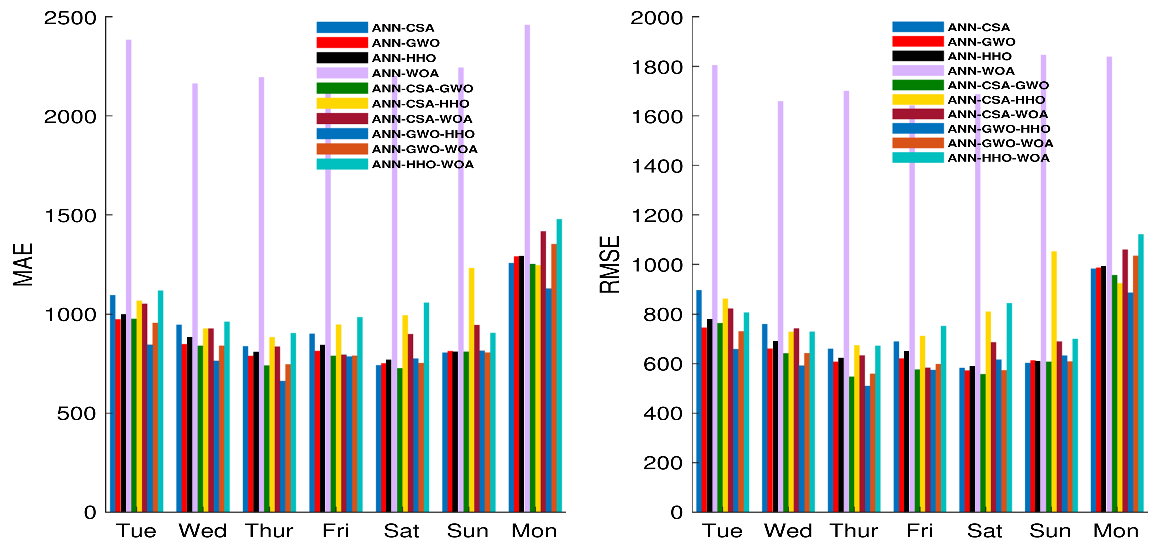

Fig. 4 Graph comparison of MAE and RMSE on a daily basis for electricity load forecasting

ations. The figure plots the reducing convergence values of MSE generated by ANN-CSA, ANN-GWO, ANN-HHO, ANN-WOA, ANN-CSA-GWO, ANN-CSA-HHO, ANN-CSA-WOA, ANN-GWO-HHO, ANN-GWO-WOA, and ANN-HHO-WOA models. From the figure, we can see that ANN-WOA generated maximum MSE and didn't converge well. It can be noted that ANN-CSA-GWO showed good convergence while ANN-GWO, ANN-CSA-HHO, and ANN-GWO-HHO showed similar patterns in terms of MSE. The figure reveals that most hybrid models generate less training error than others.

The test results of hybrid models are shown in Table 11. Different evaluation metrics such as AE, MAE, MAPE, RMSE, NMSE, r, and DC have been used to compare the performance of different hybrid models. The table shows that the performance of ANN-CSA-GWO and ANN-WOA-HHO are superior to single and other hybrid algorithms. The MSE and MAPE values generated by ANN-CSA-GWO and ANN-WOA-HHO are the best and second best compared to other algorithms. The ANN-WOA model failed to converge by generating maximum MAE and MAPE values of 15.97964 \$/MWh and 20.70291%, respectively, which proves its inefficacy over the electricity price forecasting problem. However, hybrid models based on WOA, such as ANN-CSA-WOA, ANN-GWO-WOA, and HHO-WOA, perform superior to the base ANN-WOA model. The table shows that ANN-CSA-GWO generates the least MAPE value of 7.77011%. The last row of the table depicts the Friedman test statistic for the electricity price forecast, and values show that ANN-CSA-GWO got the least value of 51. These results show that hybrid algorithms perform better than individual algorithms when combined with the ANN model.

Table 12 shows the MAE and RMSE values generated by ANN-CSA, ANN-GWO, ANN-HHO, ANN-WOA, ANN-CSA-GWO, ANN-CSA-HHO, ANN-CSA-WOA, ANN-GWO-HHO, ANN-GWO-WOA, ANN-HHO-WOA models on a monthly basis. The table reveals that ANN-GWO produced a minimum MAE value of 10.25275 \$/MWh, 7.372667

Table 10 MAE and RMSE results of different algorithms on a monthly basis for the year 2008 (Load)

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
MAE												
ANN-CSA	731.339	596.744	712.672	709.779	630.72	981.271	945.24	786.972	773.912	513.011	637.931	831.966
ANN-GWO	698.849	550.571	583.097	535.385	507.798	994.819	966.458	778.041	738.961	459.249	592.437	809.667
ANN-HHO	712.5	557.731	621.905	586.355	551.045	990.07	966.991	786.522	768.028	469.74	599.716	822.458
ANN-WOA	1456.64	1450.83	1452.47	1408.44	1455.89	2410.8	2817.84	2066.62	1870.83	1390.6	1496.02	1601.43
ANN-CSA-GWO	691.171	544.618	555.395	510.262	**488.003**	953.958	914.702	766.366	710.515	**430.202**	**576.803**	800.4
ANN-CSA-HHO	802.959	668.884	727.631	869.41	841.893	997.215	1028.79	802.626	837.396	662.597	688.275	931.459
ANN-CSA-WOA	745.355	597.707	644.225	599.465	528.231	1118.48	1093.75	820.024	796.266	449.883	630.617	882.674
ANN-GWO-HHO	**614.632**	**538.335**	**555.933**	**506.978**	526.321	**857.805**	**832.398**	**740.175**	706.26	465.351	606.493	**708.935**
ANN-GWO-WOA	702.491	554.083	588.857	526.918	496.882	972.303	925.006	774.875	737.789	445.41	582.248	807.75
ANN-HHO-WOA	781.671	679.873	687.909	602.938	613.711	1131.88	1230.84	905.596	882.676	549.461	732.73	835.948
RMSE												
ANN-CSA	906.893	757.351	861.874	854.735	800.662	1331.8	1198.2	1046.84	992.383	650.968	793.036	1026.98
ANN-GWO	893.206	721.23	739.257	691.511	696.033	1358.25	1186.89	1019.22	951.692	595.031	764.12	1001.64
ANN-HHO	910.758	733.679	780.383	747.789	736.135	1340.5	1199.66	1038.26	978.316	605.612	771.789	1014.16
ANN-WOA	1906.8	1915.32	1922.01	1811.69	1776.02	3064.7	3519.68	2431.75	2301.53	1758.29	1905.76	2032.24
ANN-CSA-GWO	886.821	716.893	708.289	651.319	667.736	1310.47	1161.88	1024.61	930.377	**553.6**	**742.147**	997.403
ANN-CSA-HHO	1010.47	838.452	915.772	1011.7	1026.15	1336.36	1307.92	1078.48	1063.06	829.511	886.421	1142.89
ANN-CSA-WOA	925.929	786.629	796.422	744.958	698.505	1571.82	1366.03	1086.71	1023.33	599.083	816.897	1098.42
ANN-GWO-HHO	**799.852**	**684.491**	**708.526**	**643.572**	**677.197**	**1148.17**	**1062.61**	**962.297**	**901.101**	593.112	772.459	**880.934**
ANN-GWO-WOA	901.18	728.579	746.776	680.86	693.829	1337.09	1179.84	1043.16	955.868	583.082	763.157	1007.21
ANN-HHO-WOA	991.554	889.545	879.999	773.435	793.91	1555.91	1555.99	1159.26	1127.53	749.37	936.738	1079.56

Bold values indicate the best value obtained for the particular function

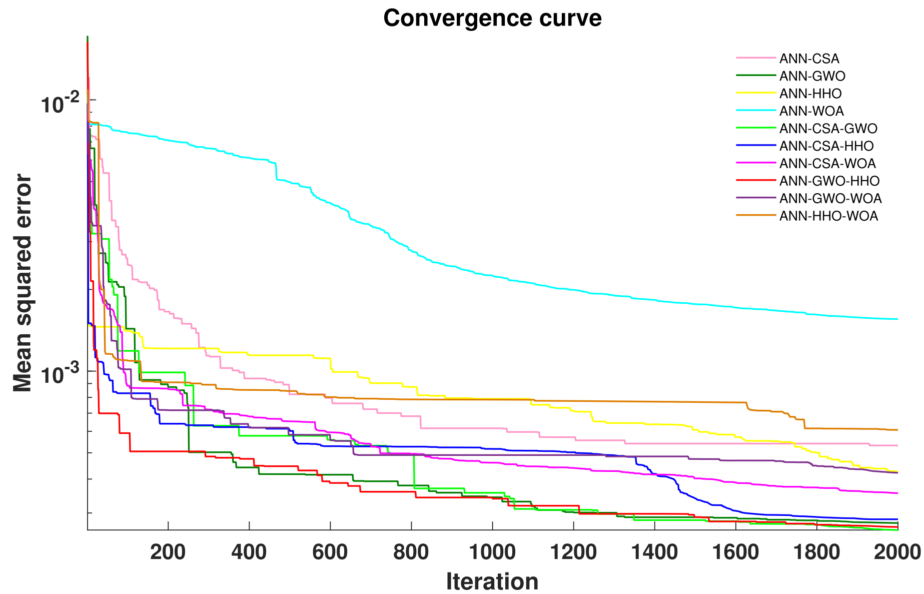

Fig. 5 Convergence plot of different algorithms for electricity price forecasting

Table 11 Performance metrics of different algorithms on price forecasting

Algorithm	AE	MAE	MAPE	RMSE	NMSE	r	DC	*F*-value
ANN-CSA	2.56683	9.75351	11.73066	14.83025	0.29384	0.84936	62.84868	185
ANN-GWO	0.47793	6.50169	7.87161	10.89445	0.15857	0.91999	67.14107	62
ANN-HHO	3.05388	8.24178	9.72379	13.19292	0.23254	0.89376	61.83536	160
ANN-WOA	−3.48734	15.97964	20.70291	21.67580	0.62773	0.69113	50.34726	237
ANN-CSA-GWO	−0.09636	6.33643	7.77011	10.27028	0.14092	0.92758	66.46932	**51**
ANN-CSA-HHO	0.39581	6.54366	7.94166	10.82737	0.15663	0.92085	67.01583	113
ANN-CSA-WOA	1.54633	7.73620	9.34303	12.34787	0.20371	0.89930	62.05169	188
ANN-GWO-HHO	0.30783	6.38974	7.80168	10.56421	0.14911	0.92418	66.91336	59
ANN-GWO-WOA	2.70054	8.18251	9.59493	13.52100	0.24425	0.89145	63.20164	107
ANN-HHO-WOA	4.25953	10.20551	11.87356	16.16019	0.34891	0.84005	56.16532	213

Bold values indicate the best value obtained for the particular function

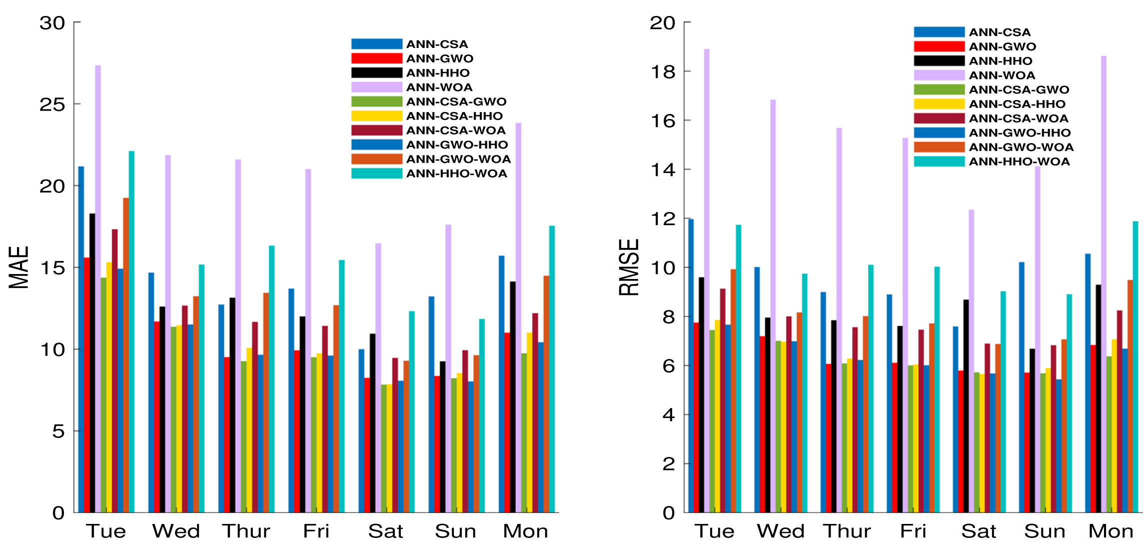

Fig. 6 Graph comparison of MAE and RMSE on a daily basis for electricity load forecasting

Table 12 MAE and RMSE results of different algorithms on monthly basis for the year 2008 (Price)

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
MAE												
ANN-CSA	11.6413	9.23053	7.87786	8.27402	11.894	16.2985	19.53	6.96564	6.07899	5.57691	5.88465	7.84812
ANN-GWO	**10.2527**	7.12326	5.86001	5.44849	**7.37267**	11.6878	11.0832	4.02861	3.89604	**3.09508**	3.3305	4.81871
ANN-HHO	12.7311	8.20171	7.50034	8.44648	11.6604	13.4337	11.7291	4.74123	4.80608	4.80492	4.63517	6.18115
ANN-WOA	18.4219	13.8326	14.014	11.7899	16.9983	26.3327	25.5449	13.9304	11.2871	10.7768	11.4498	16.9543
ANN-CSA-GWO	10.3475	6.95951	5.79273	**5.0805**	7.5109	**11.3942**	**9.741**	4.11152	3.92777	3.19221	**3.25043**	**4.69155**
ANN-CSA-HHO	10.9517	7.24507	5.93907	5.17111	7.46769	11.6295	10.3062	4.19531	3.99317	3.25799	3.29058	4.95754
ANN-CSA-WOA	11.2028	8.11648	6.49928	7.18219	9.87565	13.7488	13.2618	4.32461	4.06388	4.8072	4.38585	5.29783
ANN-GWO-HHO	10.3592	**6.88197**	**5.56974**	5.21513	7.45522	11.4757	10.5921	**3.90951**	**3.77023**	3.10359	3.29886	4.97168
ANN-GWO-WOA	12.1339	8.53178	6.81559	8.09664	11.807	13.2999	13.3487	4.62567	4.10051	4.39791	4.49244	6.37829
ANN-HHO-WOA	14.789	9.7278	9.10474	9.65748	13.7115	17.0006	17.0861	5.4035	5.73877	6.32211	5.84705	7.9516
RMSE												
ANN-CSA	14.753	11.6834	10.0416	10.5451	16.0198	27.4294	26.8145	8.91004	8.18478	6.97552	7.52455	9.82245
ANN-GWO	**13.6138**	9.40462	7.73413	6.93306	11.4615	22.811	16.1993	5.10928	5.16506	**3.93193**	4.44764	6.55171
ANN-HHO	17.2003	10.4484	9.46698	10.8112	17.8568	24.781	18.0103	5.90317	6.21672	6.06508	6.14146	8.43409
ANN-WOA	24.6626	17.2712	17.2051	14.6591	21.6127	36.7572	31.9141	17.4174	14.6374	13.0796	14.4252	21.7928
ANN-CSA-GWO	13.6139	9.14128	7.52514	**6.32975**	**10.9136**	**21.5084**	**13.8865**	5.16528	5.08106	4.02647	**4.3966**	**6.31737**
ANN-CSA-HHO	14.4987	9.40551	7.664	6.76449	11.8878	22.318	15.0919	5.21035	5.18614	4.10737	4.43727	6.84981
ANN-CSA-WOA	14.5199	10.2	8.47925	9.35245	14.906	24.7882	18.543	5.47384	5.23656	6.0101	5.72869	7.04844
ANN-GWO-HHO	13.5492	**9.06807**	**7.39073**	6.69045	11.4554	22.1383	15.018	**4.9581**	**4.97645**	3.95142	4.41833	6.65962
ANN-GWO-WOA	16.5985	10.6877	8.78734	10.5707	18.1029	25.7022	20.6427	5.99088	5.27001	5.53485	5.89	8.34717
ANN-HHO-WOA	19.9029	12.0772	11.2979	12.3221	20.8248	29.7912	25.8674	6.86529	7.1904	7.62351	7.3628	10.2962

Bold values indicate the best value obtained for the particular function

$/MWh, and 3.095078 $/MWh for Jan, May, and Oct, whereas minimum RMSE value of 13.61383 $/MWh, and 3.931929 $/MWh for Jan and Oct. ANN-CSA-GWO generated minimum MAE for Apr, Jun, July, Nov and Dec, whereas minimum RMSE for Apr, May, June, July, Nov and Dec. The table shows that the ANN-WOA-HHO model performed well in terms of MAE and RMSE for the remaining months. From the results, we can conclude that ANN-CSA-GWO gives superior results for most of the months, and ANN-WOA-HHO generated similar results while ANN-WOA generated the maximum error. Figure 6 shows the bar graph between MAE, RMSE, and days of the week. The graph shows the comparison of MAE and RMSE generated by different hybrid algorithms daily. The graph shows that MAE and RMSE values on Tuesday and Monday are higher than on the other days. The graph shows that ANN-CSA-GWO and ANN-GWO-HHO generated minimum MAE and RMSE values, whereas ANN-CSA, ANN-WOA, and ANN-HHO-WOA generated maximum MAE and RMSE for all the days. ANN-CSA-GWO outranked all stated algorithms on all weekdays except Sundays.

5 Discussions

In this section, we discuss the valuable outcome of this work. This section gives a brief interpretation of the obtained results from this work given in the previous section.

From the literature, the performance of an algorithm can be improved by combining it with other appropriate algorithms. Based on this, CSA, GWO, HHO, and WOA algorithms are chosen and combined to form several new algorithms combinations. Tables 5, 6, 7, 8, and 9 show the impact of the increasing population on different hybrid algorithms. The outcome reveals that the performance of the algorithms saturates after a population size of 100.

According to the "No-Free-Lunch" theorem, there is no algorithm that can perform well for all applications. Hence, this method of combining multiple algorithms can reduce the performance degradation of algorithms when the application changes. The key takeaways from this work can be listed as follows:

- A novel method of combining multiple algorithms in a single stack has been discussed and devised in this paper
- Four different optimization algorithms: CSA, GWO, HHO, and WOA, have been utilized to create exhaustive combinations of two algorithms.
- The hybrid algorithms are tested on twenty-four unimodal and multimodal benchmark functions to evaluate their performance along with the base algorithms.
- Two real-world time series prediction problems (electricity load and electricity price) have been used to further test the performance of the hybrid algorithms by integrating them with the ANN.
- The evaluation results show that ANN-CSA-GWO and ANN-GWO-HHO outranked other hybrid models.
- The metrics show that the performance of WOA has been significantly improved when combined with other algorithms. Hence, it can be utilized to improve the performance of the algorithms in different applications.
- The Friedman values 47 and 49 for electricity load and price show the improved performance of ANN-CSA-GWO on these applications.

The main advantage of the proposed algorithm is that every particle can update the position based on multiple options available. This allows us to combine the positives of multiple algorithms in the same stack. In optimization, some algorithms perform well on particular problems but poorly on others. Through this hybrid method, we can combine the algorithms to make them more generalized. The proposed algorithm gives a method of creating hybrid algorithms without degrading their performance. Though this provides a better option for us to combine algorithms, the selection of the algorithms is one of the challenging tasks here. It is important to have complementary algorithms to be combined in order to get a cumulative performance. If the algorithms selected are not complementary, we may end up having similar performance with increased computation. This can be considered one of the challenges of the proposed algorithm.

6 Conclusions and future work

Hybrid optimization techniques combine multiple optimization algorithms in a single framework. This work discusses a novel method of combining different optimization algorithms. The algorithms CSA, GWO, HHO, and WOA are chosen as the base for making the hybrid algorithms. Exhaustive combinations of these algorithms, namely CSA-GWO, CSA-HHO, CSA-WOA, GWO-HHO, GWO-WOA, and HHO-WOA, are discussed in detail. The performance of these algorithms is tested in twenty-four well-known unimodal and multimodal benchmark functions. Rigorous analysis has been performed by varying the dimensions and population size for these functions. CSA-GWO performed well in almost all the benchmark functions, and CSA-WOA and HHO-WOA generated competitive results. It can be interpreted that the hybrid algorithm performance is superior to the base algorithms when the individual algorithms perform well. To test the algorithm's efficacy, they have been applied to short-term electricity load and price forecast problems. The hybrid algorithms are combined with ANN to learn the network parameters during training. The results indicate that the performance of ANN-CSA-GWO is superior in all the test cases. The remaining hybrid algorithms are competitive enough but not in the problem under discussion. The reason for superior performance is that hybridization ensures better exploration in the search space, which helps the algorithms converge faster. In the future, more algorithms can be tested for making hybrid algorithms and can also be tested in different classification and regression problems.

Author Contributions RK took part in conceptualization, methodology, data curation, manuscript writing, programming, review and editing, validation and testing, result and discussions. PS involved in conceptualization, methodology, data curation, programming, manuscript writing, review and editing, validation and testing, result and discussions. AB took part in data curation, reviewing, software, validation, result and discussions.

Funding For this study, no funding has been granted from any of the funding agencies.

Data Availability Enquiries about data availability should be directed to the authors.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent As this study does not include any participants, informed consent is not applicable.

Appendix A

In Appendix, the mathematical expressions of twenty-four standard benchmark functions utilized in this work are given Table 13.

Table 13 Benchmark Functions

FID	Equation	Domain
F1	$f(\mathbf{x}) = f(x_1, ..., x_n) = \sum_{i=1}^{n} x_i^2$	$[-5.12,5.12]$
F2	$f(\mathbf{x}) = f(x_1, ..., x_n) = \sum_{i=1}^{n} \lvert x_i\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$	$[-100,100]$
F3	$f(\mathbf{x}) = f(x_1, ..., x_n) = \sum_{i=1}^{d} \sum_{j=1}^{i} x_j^2$	$[-65, 65]$
F4	$f(\mathbf{x}) = f(x_1, ..., x_n) = \max_{i=1,...,n} \lvert x_i\rvert$	$[-100,100]$
F5	$f(\mathbf{x}) = f(x_1, ..., x_n) = \sum_{i=1}^{d} ([x_i + 0.5])^2$	[-100, 100]
F6	$f(\mathbf{x}) = f(x_1, ..., x_n) = (x_1 - 1)^2 + \sum_{i=2}^{d} i(2x_i^2 - x_{i-1})^2$	$[-10,10]$
F7	$f(\mathbf{x}) = f(x_1, ..., x_n) = \sum_{i=1}^{n-1} (x_i^2)^{(x_{i+1}^2+1)} + (x_{i+1}^2)^{(x_i^2+1)}$	$[-1,4]$
F8	$f(\mathbf{x}) = f(x_1, ..., x_n) = \sum_{i=1}^{n} \lvert x_i\rvert^{i+1}$	$[-1,1]$
F9	$f(\mathbf{x}) = f(x_1, ..., x_n) = \sum_{i=1}^{n} x_i^{10}$	$[-10,10]$
F10	$f(\mathbf{x}) = f(x_1, ..., x_n) = \sum_{i=1}^{n} i x_i^2$	$[-10,10]$
F11	$f(\mathbf{x}) = f(x_1, ..., x_n) = exp(-\sum_{i=1}^{n} (x_i/\beta)^{2m}) - 2exp(-\sum_{i=1}^{n} x_i^2) \prod_{i=1}^{n} cos^2(x_i)$	$[-2\pi,2\pi]$
F12	$f(\mathbf{x}) = f(x_1, ..., x_n) = \sum_{i=1}^{n} x_i^2 + (\sum_{i=1}^{n} 0.5ix_i)^2 + (\sum_{i=1}^{n} 0.5ix_i)^4$	$[-5,10]$
F13	$f(x) = f(x_1, ..., x_n) = 10n + \sum_{i=1}^{n} (x_i^2 - 10cos(2\pi x_i))$	$[-5.12,5.12]$
F14	$f(\mathbf{x}) = f(x_1, ..., x_n) = -a.exp(-b\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - exp(\frac{1}{n}\sum_{i=1}^{n} cos(cx_i)) + a + exp(1)$	$[-32,32]$
F15	$f(\mathbf{x}) = f(x_1, ..., x_n) = 1 + \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n} cos(\frac{x_i}{\sqrt{i}})$	$[-600,600]$
F16	$f(\mathbf{x}) = f(x_1, ..., x_n) = \frac{\pi}{n}\left\{10\,sin(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 \left[1 + 10sin^2(\pi y_{i+1})\right] + (y_n - 1)^2\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{x_i+1}{4} u(x_i, a, k, m) = \begin{cases} k(x_i - 1)^m > a \\ 0 - a < x_i < a \\ k(-x_i - a)^m\ x_i < -a \end{cases}$	$[-50,50]$
F17	$f(\mathbf{x}) = f(x_1, ..., x_n) = 0.1\left\{sin^2(3\pi x_1) + \sum_{i=1}^{n} (x_i - 1)^2[1 + sin^2(3\pi x_i + 1)] + (x_n - 1)^2[1 + sin^2(2\pi x_n)]\right\} +$ $\sum_{i=1}^{n} u(x_i, 5, 100, 4)$	$[-50,50]$
F18	$f(\mathbf{x}) = f(x_1, ..., x_n) = \left[\left(\lvert\lvert\mathbf{x}\rvert\rvert^2 - n\right)^2\right]^{\alpha} + \frac{1}{n}\left(\frac{1}{2}\lvert\lvert\mathbf{x}\rvert\rvert^2 + \sum_{i=1}^{n} x_i\right) + \frac{1}{2}$	$[-2,2]$
F19	$f(\mathbf{x}) = f(x_1 ... x_n) = 1 + \sum_{i=1}^{n} sin^2(x_i) - 0.1e^{(\sum_{i=1}^{n} x_i^2)}$	$[-10,10]$
F20	$f(\mathbf{x}) = f(x_1, ..., x_n) = 1 - cos(2\pi\sqrt{\sum_{i=1}^{D} x_i^2}) + 0.1\sqrt{\sum_{i=1}^{D} x_i^2}$	$[-100,100]$
F21	$f(\mathbf{x}) = f(x_1, ..., x_n) = \sum_{i=1}^{n} \epsilon_i \lvert x_i\rvert^i$	$[-5,5]$
F22	$f(\mathbf{x}) = f(x_1, ..., x_n) = (\sum_{i=1}^{n} \lvert x_i\rvert)exp(-\sum_{i=1}^{n} sin(x_i^2))$	$[-2\pi,2\pi]$
F23	$f(\mathbf{x}) = f(x_1, ..., x_n) = \left(\sum_{i=1}^{n} sin^2(x_i) - e^{-\sum_{i=1}^{n} x_i^2}\right) e^{-\sum_{i=1}^{n} sin^2\sqrt{\lvert x_i\rvert}}$	$[-10,10]$
F24	$f(\mathbf{x}) = f(x_1, ..., x_n) = sin^2 + \sum_{i=1}^{d-1} (w_i - 1)^2 \left[1 + 10sin^2(\pi w_i + 1)\right] + (w_d - 1)^2[1 + sin^2(2\pi w_d)]$	$[-10,10]$

References

Abedinpourshotorban H, Shamsuddin SM, Beheshti Z, Jawawi DN (2016) Electromagnetic field optimization: a physics-inspired metaheuristic optimization algorithm. Swarm and Evolutionary Computation 26:8–22

Ahmadian MM, Salehipour A, Cheng T (2021) A meta-heuristic to solve the just-in-time job-shop scheduling problem. European Journal of Operational Research 288(1):14–29

Al-Betar M.A, Alyasseri Z.A.A, Awadallah M.A, Doush I.A (2020). Coronavirus herd immunity optimizer (chio). Neural Computing and Applications 33, 5011–5042

Aljarah I, Faris H, Mirjalili S (2018) Optimizing connection weights in neural networks using the whale optimization algorithm. Soft Computing 22:11–15

Bairathi D, Gopalani D (2021) An improved salp swarm algorithm for complex multi-modal problems. Soft Computing 25(15):10441–10465

Basu, M. , & Chowdhury, A. (2013). Cuckoo search algorithm for economic dispatch. Energy**60**99–108

Benmessahel I, Xie K, Chellal M (2018) A new evolutionary neural networks based on intrusion detection systems using multiverse optimization. Applied Intelligence 48(8):2315–2327

Chen H, Heidari AA, Chen H, Wang M, Pan Z, Gandomi A H (2020) Multi-population differential evolution-assisted harris hawks optimization: Framework and case studies. Future Generation Computer Systems 111:175–198

Chen R , Yang B, Li S, Wang S, Cheng Q (2021). An effective multi-population grey wolf optimizer based on reinforcement learning for flow shop scheduling problem with multi-machine collaboration. Computers & Industrial Engineering, 162, 107738

Chen W, Panahi M, Khosravi K, Pourghasemi H R, Rezaie F, Parvinnezhad D (2019)Spatial prediction of groundwater potentiality using ANFIS ensembled with teaching-learning-based and biogeography-based optimization. Journal of Hydrology, 572, 435–448

Chen X, Yu K (2019) Hybridizing cuckoo search algorithm with biogeography-based optimization for estimating photovoltaic model parameters. Solar Energy 180:192–206

Chen X, Zhang L, Zhang Z (2020) An integrated model for maintenance policies and production scheduling based on immune-culture algorithm. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 234(5):651–663

Chong HY, Yap HJ, Tan SC, Yap KS, Wong SY (2021) Advances of metaheuristic algorithms in training neural networks for industrial applications. Soft Computing 25(16):11209–11233

Cofnas N (2018) Judaism as a group evolutionary strategy. Human Nature 29(2):134–156

Cuong-Le T, Minh, H- L., Khatir, S., Wahab, M.A., Tran, M.T., Mirjalili, S. (2021) A novel version of cuckoo search algorithm for solving optimization problems. Expert Syst Appl 186:115669

Deng W, Xu J, Song Y, Zhao H (2021) Differential evolution algorithm with wavelet basis function and optimal mutation strategy for complex optimization problem. Applied Soft Computing 100:106724

Dhalwar S, Kottath R, Kumar V, Raj A.N.J, Poddar S (2016). Adaptive parameter based particle swarm optimisation for accelerometer calibration. In: 2016 IEEE 1st international conference on power electronics, intelligent control and energy systems (ICPEICES) (pp 1–5)

Elgamal Z.M, Yasin N.B.M, Tubishat M, Alswaitti M, Mirjalili S (2020). An improved harris hawks optimization algorithm with simulated annealing for feature selection in the medical field. IEEE Access, **8**, 186638–186652

El Yafrani M, Ahiod B (2018) Information Sciences. Efficiently solving the traveling thief problem using hill climbing and simulated annealing 432:231–244

England I.N (2009). - Reliable electricity. Competitive prices. Clean-energy transition. Reliable electricity. Competitive prices. clean-energy transition. https://www.iso-ne.com/

Fan Q, Chen Z, Xia Z (2020) A novel quasi-reflected harris hawks optimization algorithm for global optimization problems. Soft Computing 24(19):14825–14843

Faris H, Aljarah I, Al-Betar MA, Mirjalili S (2018) Grey wolf optimizer: a review of recent variants and applications. Neural computing and applications 30(2):413–435

Fathollahi-Fard AM, Govindan K, Hajiaghaei-Keshteli M, Ahmadi A (2019) A green home health care supply chain: New modified simulated annealing algorithms. Journal of Cleaner Production 240:118200

Gandomi AH, Yang, X- S., Alavi, A.H. (2013) Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems. Eng comput 29:117–135

Hamzaçebi C (2008) Improving artificial neural networks' performance in seasonal time series forecasting. Information Sciences 17823:4550–4559

Heidari AA, Mirjalili S, Faris H, Aljarah I, Mafarja M, Chen H (2019) Harris hawks optimization: Algorithm and applications. Future generation computer systems 97:849–872

Hong L, Drake JH, Woodward JR, Özcan E (2018) A hyper-heuristic approach to automated generation of mutation operators for evolutionary programming. Applied Soft Computing 62:162–175

Jakubik J, Binding A, Feuerriegel S (2021) Directed particle swarm optimization with gaussian-process-based function forecasting. European Journal of Operational Research 295:1157–169

Katarya R, Verma OP (2018) Recommender system with grey wolf optimizer and fcm. Neural Computing and Applications 30(5):1679–1687

Kiziloz H E, Deniz A (2021) An evolutionary parallel multiobjective feature selection framework. Computers & Industrial Engineering 159:107481

Kohli M, Arora S (2018) Chaotic grey wolf optimization algorithm for constrained optimization problems. Journal of computational design and engineering 5(4):458–472

Kottath R , Singh P. (2022). A meta-heuristic learning approach for short-term price forecasting. In: Soft computing: theories and applications. Springer, (pp 147–156)

Kottath R, Singh P (2023) Influencer buddy optimization: Algorithm and its application to electricity load and price forecasting problem. Energy 263:125641

Kumar R, Singh R, Ashfaq H (2020) Stability enhancement of multi-machine power systems using ant colony optimization-based static synchronous compensator. Computers & Electrical Engineering 83:106589

Kuyu Y.Ç, Vatansever F (2021). Modified forensic-based investigation algorithm for global optimization. Engineering with Computers, 38(4), 3197–3218

Li MD, Zhao H, Weng XW, Han T (2016) A novel nature-inspired algorithm for optimization: Virus colony search. Advances in Engineering Software 92:65–88

Li N, Wang L (2020) Bare-bones based sine cosine algorithm for global optimization. Journal of Computational Science 47:101219

Mafarja, M.M. , & Mirjalili, S. (2017). Hybrid whale optimization algorithm with simulated annealing for feature selection. Neurocomputing**260**(21)302–312

Mafarja MM, Mirjalili S (2019) Hybrid binary ant lion optimizer with rough set and approximate entropy reducts for feature selection. Soft Computing 23(15):6249–6265

Miao C, Chen G, Yan C, Wu Y (2021) Path planning optimization of indoor mobile robot based on adaptive ant colony algorithm. Computers & Industrial Engineering 156:107230

Mirjalili S, Lewis A (2016) The whale optimization algorithm. Advances in engineering software 95:51–67

Moosavi SHS, Bardsiri VK (2017) Satin bowerbird optimizer: A new optimization algorithm to optimize anfis for software development effort estimation. Engineering Applications of Artificial Intelligence 60:1–15

Nadimi-Shahraki MH, Taghian S, Mirjalili S (2021) An improved grey wolf optimizer for solving engineering problems. Expert Systems with Applications 166:113917

Nithiyananthan K, Ramachandran V (2013) Versioning-based service-oriented model for multi-area power system online economic load dispatch. Computers & Electrical Engineering 39(2):433–440

Peng ZK, Zhang SX, Zheng SY, Long YL (2019) Collective information-based teaching-learning-based optimization for global optimization. Soft Computing 23(22):11851–11866

Shehab M, Khader AT, Al-Betar MA (2017) A survey on applications and variants of the cuckoo search algorithm. Applied Soft Computing 61:1041–1059

Singh P, Dwivedi P (2018) Integration of new evolutionary approach with artificial neural network for solving short term load forecast problem. Applied energy 217:537–549

Singh P, Dwivedi P (2019) A novel hybrid model based on neural network and multi-objective optimization for effective load forecast. Energy 182:606–622

Singh P, Dwivedi P (2022). Very short-term load forecasting with deep learning neural network in Delhi, India. In: Soft computing: theories and applications, (pp 125–134). Springer

Singh, P. , Dwivedi, P. , Kant, V. (2019). A hybrid method based on neural network and improved environmental adaptation method using controlled gaussian mutation with real parameter for short-term load forecasting. Energy**174**(2)460–477

Singh P, Kottath R (2021) An ensemble approach to meta-heuristic algorithms: Comparative analysis and its applications. Computers & Industrial Engineering 162:107739

Singh P, Kottath R (2022) Chaos follow the leader algorithm: Application to data classification. Journal of Computational Science 65:101886

Singh P, Kottath R (2022) Influencer-defaulter mutation-based optimization algorithms for predicting electricity prices. Utilities Policy 79:101444

Singh P, Kottath R, Tejani G.G (2022). Ameliorated follow the leader: algorithm and application to truss design problem. In: Structures (Vol. 42, pp 181–204)

Subramanian P, Sahayaraj JM, Senthilkumar S, Alex DS (2020) A hybrid grey wolf and crow search optimization algorithm-based optimal cluster head selection scheme for wireless sensor networks. Wireless Personal Communications 113(2):905–925

Wang F, Luo L, He X- s, Wang Y (2011). Hybrid optimization algorithm of pso and cuckoo search. In: 2011 2nd international conference on artificial intelligence, management science and electronic commerce (AIMSEC), (pp 1172–1175)

Wang GG, Deb S, Coelho LDS (2018) Earthworm optimisation algorithm: a bio-inspired metaheuristic algorithm for global optimisation problems. Int J Bio-Inspir Comput 12:11–22

Wang Y, Zhang Z, Zhang LY, Feng J, Gao J, Lei P (2020) A genetic algorithm for constructing bijective substitution boxes with high nonlinearity. Information Sciences 523:152–166

Yang H, Wang S, Li G, Mao T (2018). A new hybrid model based on fruit fly optimization algorithm and wavelet neural network and its application to underwater acoustic signal prediction. Mathematical problems in engineering, 2018(3), 1–8

Yang X- S (2010). Nature-inspired metaheuristic algorithms. Luniver press

Zhang G, Shao X, Li P, Gao L (2009) An effective hybrid particle swarm optimization algorithm for multi-objective flexible job-shop scheduling problem. Computers & Industrial Engineering 56(4):1309–1318

Zhao W, Wang L, Zhang Z (2019) Atom search optimization and its application to solve a hydrogeologic parameter estimation problem. Knowledge-Based Systems 163:283–304