



Group decision-making based on 2-tuple linguistic T -spherical fuzzy COPRAS method

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Abstract

Data mining is a thoroughly advanced method that evaluates and makes more sense of a variety in electronic commerce (e-commerce)-related knowledge, discovering useful ideas, predicting user actions, and assisting enterprises selection in modifying competitive strategy, minimizing cost, and attaining the finest results. Data mining has already become more popular in recent years. In this research paper, we propose a multi-attribute group decision-making (MAGDM) method under T -spherical fuzzy environment for selecting an optimal data mining strategy which is an important part of modern decision-making research. The information aggregation operators play an important role in solving MAGDM problems. Some point aggregation operators based on the 2-tuple linguistic T -spherical fuzzy numbers, including 2-tuple linguistic T -spherical fuzzy point weighted averaging (2TLT-SFPWA) operator, 2-tuple linguistic T -spherical fuzzy point weighted geometric (2TLT-SFPWG) operator, 2-tuple linguistic T -spherical fuzzy generalized point weighted averaging (2TLT-SFGPWA) operator and 2-tuple linguistic T -spherical fuzzy generalized point weighted geometric (2TLT-SFGPWG) operator, are proposed which competently capture all the aspects of human opinions expressible in terms of yes, no, cessation and denial with no limitation. The proposed aggregation operators are valid and have some basic properties which are keenly analyzed. Furthermore, the complex proportional assessment (COPRAS) method is developed on the basis of 2-tuple linguistic T -spherical fuzzy point aggregation operators. Finally, a numerical example is illustrated for demonstrating the effectiveness of the proposed work along with comparative analysis which verifies the reliability and efficacy of its outcomes. In the end, we conclude some results from the numerical analysis, i.e., to balance the long-term development of e-commerce, data mining can mine massive amounts of data which boosts the growth of e-commerce in future.

Keywords 2-Tuple linguistic T -spherical fuzzy set · MAGDM · Point weighted averaging operator · Point weighted geometric operator · COPRAS method

1 Introduction

Multiple attribute decision-making (MADM) is the technique for selecting the optimal alternative from a finite collection of alternatives based on numerous, typically conflicting attributes (or called criteria). MAGDM is fascinating and challenging everyday problem that is important to decision theory. However, how to successfully obtain attribute information in the face of uncertainty and diversity in the decision-making environment is the main vexing challenge of MAGDM. Thus, Zadeh (1965) introduced the concept of fuzzy set (FS) to deal with real-life problems involving variability or inaccuracy. Atanassov (1986) extended the idea of FS and proposed the intuitionistic fuzzy set (IFS) which consists of membership degree (MD) and non-membership

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degree (NMD) to describe the unreliability of an event with a condition that the sum of the MD and NMD should not exceed 1. Yager (2014) strengthened this concept by introducing the notion of Pythagorean fuzzy set (PyFS) in which the range for assigning values to MD and NMD was increased. Yager (2016) also proposed a generalized orthopair FS commonly known as q -ROFS. Wang et al. (2019b) developed some aggregation operators for fusing q -ROF information. They extended the Muirhead mean to q -ROF environment and proposed a family of q -ROF Muirhead mean operators. Darko and Liang (2020) proposed some q -ROF Hamacher aggregation operators, i.e., the q -ROF Hamacher average operator, the weighted q -ROF Hamacher average operator, the q -ROF Hamacher Maclaurin symmetric mean operator and the weighted q -ROF Hamacher Maclaurin symmetric mean operator along with their special properties. Xing et al. (2019) developed a novel category of point weighted aggregation operators to aggregate q -ROF information. These recommended operators can divide q -ROFN membership and non-membership based on different criteria.

Somehow, the pairs of IFS, PyFS and q -ROFS deal with inaccuracy that occurs in real life but these duplets discuss only two dimensions of human opinion, i.e., like and dislike, although a human opinion also involves certain extent of abstinence and rejection. Cuong and Kreinovich (2014) suggested that the pair, representing an IFS or its generalized form, represents the human opinion only in terms of MD and NMD, where the abstinence degree (AD) and the refusal degree (RD) are kind of ignored, leading to the loss of knowledge. For this reason, they proposed the idea of a picture fuzzy set (PFS) in the form of triplets using MD, AD and NMD with a limitation that their sum is not greater than 1. Kahraman and Gündođdu (2018) originally presented the idea of spherical fuzzy sets (SFs). In 2019, F.K. Gündođdu and C. Kahraman, (2019) developed a MAGDM technique, namely spherical fuzzy TOPSIS (SF-TOPSIS) method, to tackle the complex decision-making problems. Later, Kahraman et al. (2019) applied the SF-TOPSIS to choose the most appropriate site for a hospital. Mahmood et al. (2019) presented the T -spherical fuzzy set (T -SFS) with the basic operations and aggregation operators which increased the range for assigning MD, Ashraf and Abdullah (2019); Ashraf et al. (2020) put forward the decision-making strategies based on spherical fuzzy aggregation operators. Akram et al. (2020b) developed novel spherical fuzzy prioritized weighted aggregation operators to solve MAGDM problems. Ullah et al. (2020) introduced Hamacher aggregation operators based on T -spherical fuzzy numbers and discussed their basic properties. They proposed the idea of T -SF Hamacher-weighted averaging and T -SF Hamacher-weighted geometric aggregation operators to incorporate four aspects of human opinion including yes, no, abstinence and refusal with no limitations. Ashraf et al. (2019) introduced spherical fuzzy t -norms and

t -conorms, developed spherical fuzzy negator and some classifications of spherical fuzzy t -norms and spherical fuzzy t -conorms to aggregate the SF information, and examined the spaces of spherical fuzzy membership grades and picture fuzzy membership grades graphically. Zeng et al. (2020) proposed new operational laws with T -SF information for Einstein geometric interaction operators and Einstein averaging interactive aggregation operators. Some fundamental characteristics and benefits of proposed aggregation operators are also discussed. Liu et al. (2020) proposed the linguistic T -SFNs, the linguistic T -spherical fuzzy weighted averaging operator, and extended multi-attributive border approximation area comparison method in the linguistic spherical fuzzy environment. Further, Akram et al. (2020a, 2021a, b, c), Akram et al. (2021), Naz et al. (2022a, b, c), Liu et al. (2022) and Zahid et al. (2022) introduced several decision-making methods under generalized fuzzy scenario.

Processing manner of 2-tuple linguistic information can effectively avoid the distortion and loss of information. Herrera and Martinez (2000a) proposed the representation model of fuzzy 2-tuple linguistic term to handle linguistic decision-making problems. In last few decades, various 2-tuple linguistic aggregation operators and decision methods have been proposed to combine the individual preference information into a collective one, such as the generalized 2-tuple linguistic Pythagorean fuzzy Heronian mean operator, 2-tuple linguistic Pythagorean fuzzy geometric Heronian mean operator and its weighted form (Deng et al. 2019). Ju et al. (2020) developed the q -ROF 2-tuple linguistic weighted averaging operator and the q -ROF 2-tuple linguistic weighted geometric operator. Furthermore, they proposed the q -ROF 2-tuple linguistic Muirhead mean operator and the q -ROF 2-tuple linguistic dual Muirhead mean operator on the basis of Muirhead mean operator and dual Muirhead mean operator. Wang et al. (2019a) proposed a concept of interval-valued q -ROF 2-tuple linguistic sets and discussed its properties. They developed some weighted averaging and geometric aggregation operators for

interval-valued q -ROF 2-tuple linguistic sets and characterized their desirable properties. On the basis of Archimedean t -norm and t -conorm, Mo and Huang (2020) developed the dual hesitant fuzzy geometric Heronian mean operator and dual hesitant fuzzy geometric weighted Heronian mean operator. Some properties and special cases are also discussed. Rong et al. (2020a) introduced the complex q -ROF 2-tuple linguistic Maclaurin symmetric mean operator, and the complex q -ROF 2-tuple linguistic dual Maclaurin symmetric mean operator along with several attractive characteristics of the developed operators. Rong et al. (2020b) proposed the hesitant fuzzy linguistic Hamy mean operator, hesitant fuzzy linguistic dual Hamy mean operator, and its weighted form along with their properties. Even though the MADM methods provide numerically significant and consis-

tent rankings in a particular situation, the optimal solution can be presented in a different manner. The decision-makers' task of determining the optimal conclusion in a scenario might be complicated by the random selection of the most favorable alternative. Based on the outcome, it is highly challenging to determine the capacity or capability strengths of more than 100 MADM approaches. This issue is still regarded as a riddle due to the lack of accepted standards in the literature for analyzing MADM approaches under uncertainty.

The COPRAS method, developed by Zavadskas et al. (1994), is the most valuable and suitable method to solve complex MAGDM problems. Based on the benefit and cost criteria, it gives information accurately on comparing with other existing approaches. The most crucial advantage of COPRAS method is that it portrays the ratio to the worst and the best results simultaneously. In this regard, COPRAS method was extended under several models of FS theory. To evaluate the pulmonary disease, Zheng et al. (2018) developed the hesitant fuzzy linguistic COPRAS approach. Mishra et al. (2019) introduced extended COPRAS approach to evaluate the multiple criteria decision-making problems based on hesitant FSs. Buyukozkan and Gocer (2019) presented an integrated model associated with AHP-COPRAS to evaluate the digital supply chain partner selection within Pythagorean fuzzy environment. Dorfeshan and Mousavi (2019) developed a new integrated TOPSIS-COPRAS method to determine the critical path of projects. Over the past few decades, considerable research has been done on information aggregation. The aggregation operator is a method that can combine a specified number of inputs into a single output. Yager (1988) developed a class of generalized ordered weighted average aggregation operators based on the ordered weighted average operator. Xu and Yager (2006) also developed some geometric aggregation operators. It must be noted that these aggregation operators are based on original information, therefore the uncertainty of the aggregated arguments cannot be minimized. However, in some situations, it is essential to reduce information ambiguity.

The motivations of this article are as follows:

- (1) The T -SFS has been proved to be an efficient tool in expressing DMs evaluation values in MAGDM procedure. However, the existing theories about T -SFS fail to depict uncertain information through the 2-tuple linguistic representation model. The 2-tuple linguistic model has stronger capability to describe linguistic information and it also can avoid information distortion loss in dealing with linguistic decision issues. The 2TLT-SFSs are more powerful than 2TLIFSs, 2TLPyFSs, and 2TL q -ROFs. It is more efficient to use 2TLT-SFSs to express evaluation values of the DMs in the MAGDM process.
- (2) Some effective information processing and evaluation methods need to be developed in order to aggregate

the information. So in this article, we give point aggregation operators for 2TLT-SFS, which can reduce the degree of uncertainty of the information, and propose 2TLT-SFPWA operator, 2TLT-SFPWG operator, 2TLT-SFGPWA operator and 2TLT-SFGPWG operator, as effective tools to aggregate the 2TLT-SFNs. The point weighted aggregation operator is more appropriate in considering the redistribution between the different number of attributes by the variable vector and in reducing the impact of the inappropriate evaluation values by measuring the degree of hesitance between any two individuals.

- (3) In some recent publications, COPRAS method has received much attention from researchers but there is currently no progress in extending COPRAS to the 2TLT-SF domain. So, we develop a novel 2TLT-SF-COPRAS method by extending the COPRAS method into 2TLT-SFN.

In considering the aforementioned justifications and arguments, the novelty of this article is demonstrated in the following three ways:

- (1) To aggregate 2TLT-SFNs, we develop four new aggregation operators based on point operations, such as 2TLT-SFPWA operator, 2TLT-SFPWG operator, 2TLT-SFGPWA operator and 2TLT-SFGPWG operator. Desirable properties of proposed operators are also examined.
- (2) We design a novel 2TLT-SF-COPRAS method based on the 2TLT-SFPWA and 2TLT-SFPWG operators. Specifically, we utilize the 2TLT-SFPWA operator and 2TLT-SFPWG operator to fuse the evaluation preferences of the DMs. Then, with the integration of the 2TLT-SFPWA operator (2TLT-SFPWG operator) and the 2TLT-SF-COPRAS method, we appraise the alternatives.
- (3) Finally, we test the applicability of our proposed 2TLT-SF-COPRAS method by solving a problem to select the best data mining task.

In order to accomplish this, we organize the rest of our paper as follows. In Sect. 2, we review fundamental concepts associated with T -spherical fuzzy set (T -SFS), 2-tuple linguistic terms and 2TLT-SFS. In Sect. 3, depending on point operators on 2TLT-SFNs, we develop 2TLT-SFPWA operator, 2TLT-SFPWG operator, 2TLT-SFGPWA operator and 2TLT-SFGPWG operator and also discuss their properties. In Sect. 4, we give a novel technique of COPRAS method for solving MAGDM with 2TLT-SF information. In Sect. 5, a practical illustration for selecting the optimal user through data mining tasks is addressed and comparative analysis by

comparing numerical results with the existing methodologies is demonstrated. Finally, Sect. 6 summarizes the paper.

2 Preliminaries

In this section, some correlative basic concepts of T -SFS, 2-tuple linguistic terms and 2TLT-SFS are recapped to facilitate the next sections.

Mahmood et al. (2019) defined the T -spherical fuzzy set as an extension of q -ROFS and SFS as follows:

Definition 1 (Mahmood et al. 2019) For any universal set L , a T -SFS is of the form

$$T = \{ \langle l, a_T(l), b_T(l), c_T(l) \rangle \mid l \in L \}, \tag{1}$$

where $a_T, b_T, c_T : L \rightarrow [0, 1]$ represent the MD, abstinence degree (AD) and NMD, respectively, with the condition $0 \leq a_T^q(l) + b_T^q(l) + c_T^q(l) \leq 1$ for positive number $q \geq 1$, and $r(l) = \sqrt[q]{1 - (a_T^q(l) + b_T^q(l) + c_T^q(l))}$ is known as the degree of refusal of l in T . To express information conveniently, the triplet (a, b, c) is known as a T -spherical fuzzy number (T -SFN).

Definition 2 (Herrera and Martinez 2000b) Let $S = \{s_i \mid i = 0, \dots, k\}$ be a linguistic term set and $\beta \in [0, k]$ is a number value representing the aggregation result of linguistic symbolic. Then, the function Δ used to obtain the 2-tuple linguistic information equivalent to β is defined as

$$\begin{aligned} \Delta : [0, k] &\rightarrow S \times [-0.5, 0.5), \\ \Delta(\beta) &= \begin{cases} s_i, & i = \text{round}(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5), \end{cases} \end{aligned} \tag{2}$$

where $\text{round}(\cdot)$ is the usual round operation, s_i has the closest index label to β , and α is the value of the symbolic translation.

Definition 3 (Herrera and Martinez 2000b) Let $S = \{s_i \mid i = 0, \dots, k\}$ be a linguistic term set and (s_i, α) be a 2-tuple linguistic information, then there exists an inverse function Δ^{-1} that restores the 2-tuple linguistic information to its equivalent numerical value $\beta \in [0, k]$, where

$$\begin{aligned} \Delta^{-1} : S \times [-0.5, 0.5) &\rightarrow [0, k], \\ \Delta^{-1}(s_i, \alpha) &= i + \alpha = \beta. \end{aligned} \tag{3}$$

Inspired by the ideas of 2-tuple linguistic term and T -SFS, Akram et al. (2022) developed the new concept of 2TLT-SFS. The mathematical representation of 2TLT-SFS is described as follows:

Definition 4 (Akram et al. 2022) Let L be a universal set. A 2TLT-SFS \mathfrak{T} in L is defined as

$$\mathfrak{T} = \{ \langle l, ((s_a(l), \zeta(l)), (s_b(l), \psi(l)), (s_c(l), \varphi(l))) \mid l \in L \}, \tag{4}$$

where $(s_a(l), \zeta(l))$, $(s_b(l), \psi(l))$, and $(s_c(l), \varphi(l))$ represent the MD, abstinence degree, and NMD, respectively, with the conditions $s_a(l), s_b(l), s_c(l) \in \mathfrak{T}$, $\zeta(l), \psi(l), \varphi(l) \in [-0.5, 0.5)$, $0 \leq \Delta^{-1}(s_a(l), \zeta(l)) \leq k$, $0 \leq \Delta^{-1}(s_b(l), \psi(l)) \leq k$, $0 \leq \Delta^{-1}(s_c(l), \varphi(l)) \leq k$ and $0 \leq (\Delta^{-1}(s_a(l), \zeta(l)))^q + (\Delta^{-1}(s_b(l), \psi(l)))^q + (\Delta^{-1}(s_c(l), \varphi(l)))^q \leq k^q$. For convenience, we say $\gamma = ((s_a, \zeta), (s_b, \psi), (s_c, \varphi))$, a 2TLT-SFN, where $0 \leq \Delta^{-1}(s_a, \zeta) \leq k$, $0 \leq \Delta^{-1}(s_b, \psi) \leq k$, $0 \leq \Delta^{-1}(s_c, \varphi) \leq k$ and $0 \leq (\Delta^{-1}(s_a, \zeta))^q + (\Delta^{-1}(s_b, \psi))^q + (\Delta^{-1}(s_c, \varphi))^q \leq k^q$.

In order to compare any two 2TLT-SFNs, their score and accuracy functions are defined as follows:

Definition 5 (Akram et al. 2022) Let $\gamma = ((s_a, \zeta), (s_b, \psi), (s_c, \varphi))$ be a 2TLT-SFN. Then, the score function \mathcal{S} of a 2TLT-SFN γ can be represented as

$$\begin{aligned} \mathcal{S}(\gamma) &= \Delta \left\{ \frac{k}{2} \left(1 + \left(\frac{\Delta^{-1}(s_a, \zeta)}{k} \right)^q - \left(\frac{\Delta^{-1}(s_c, \varphi)}{k} \right)^q \right) \right\}, \\ \mathcal{S}(\gamma) &\in [0, k], \end{aligned} \tag{5}$$

and its accuracy function \mathcal{H} is defined as

$$\begin{aligned} \mathcal{H}(\gamma) &= \Delta \left\{ k \left(\left(\frac{\Delta^{-1}(s_a, \zeta)}{k} \right)^q + \left(\frac{\Delta^{-1}(s_c, \varphi)}{k} \right)^q \right) \right\}, \\ \mathcal{H}(\gamma) &\in [0, k]. \end{aligned} \tag{6}$$

Definition 6 (Akram et al. 2022) Let $\gamma_1 = ((s_{a_1}, \zeta_1), (s_{b_1}, \psi_1), (s_{c_1}, \varphi_1))$ and $\gamma_2 = ((s_{a_2}, \zeta_2), (s_{b_2}, \psi_2), (s_{c_2}, \varphi_2))$ be two 2TLT-SFNs; then, these two 2TLT-SFNs can be compared according to the following rules:

- (1) If $\mathcal{S}(\gamma_1) > \mathcal{S}(\gamma_2)$, then $\gamma_1 \succ \gamma_2$;
- (2) If $\mathcal{S}(\gamma_1) < \mathcal{S}(\gamma_2)$, then $\gamma_1 \prec \gamma_2$;
- (3) If $\mathcal{S}(\gamma_1) = \mathcal{S}(\gamma_2)$, then

- If $\mathcal{H}(\gamma_1) > \mathcal{H}(\gamma_2)$, then $\gamma_1 \succ \gamma_2$;
- If $\mathcal{H}(\gamma_1) < \mathcal{H}(\gamma_2)$, then $\gamma_1 \prec \gamma_2$;
- If $\mathcal{H}(\gamma_1) = \mathcal{H}(\gamma_2)$, then $\gamma_1 \sim \gamma_2$.

Definition 7 (Akram et al. 2022) Let $\gamma = ((s_a, \zeta), (s_b, \psi), (s_c, \varphi))$, $\gamma_1 = ((s_{a_1}, \zeta_1), (s_{b_1}, \psi_1), (s_{c_1}, \varphi_1))$ and $\gamma_2 = ((s_{a_2}, \zeta_2), (s_{b_2}, \psi_2), (s_{c_2}, \varphi_2))$ be three 2TLT-SFNs, $q \geq 1$, then

1.

$$\begin{aligned} &\gamma_1 \oplus \gamma_2 \\ &= \left(\Delta \left(k^q \sqrt[1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_1}, \zeta_1)}{k} \right)^q \right) \left(1 - \left(\frac{\Delta^{-1}(s_{a_2}, \zeta_2)}{k} \right)^q \right) \right] \right), \\ &\quad \Delta \left(k \left(\frac{\Delta^{-1}(s_{b_1}, \psi_1)}{k} \right) \left(\frac{\Delta^{-1}(s_{b_2}, \psi_2)}{k} \right) \right), \\ &\quad \Delta \left(k \left(\frac{\Delta^{-1}(s_{c_1}, \varphi_1)}{k} \right) \left(\frac{\Delta^{-1}(s_{c_2}, \varphi_2)}{k} \right) \right) \end{aligned} \Bigg);$$

2.

$$\gamma_1 \otimes \gamma_2 = \left(\Delta \left(k \sqrt[k]{1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_1}, \varsigma_1)}{k} \right)^q \right) \left(1 - \left(\frac{\Delta^{-1}(s_{a_2}, \varsigma_2)}{k} \right)^q \right)} \right), \right. \\ \left. \Delta \left(k \sqrt[k]{1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_1}, \psi_1)}{k} \right)^q \right) \left(1 - \left(\frac{\Delta^{-1}(s_{b_2}, \psi_2)}{k} \right)^q \right)} \right), \right. \\ \left. \Delta \left(k \sqrt[k]{1 - \left(1 - \left(\frac{\Delta^{-1}(s_{c_1}, \varphi_1)}{k} \right)^q \right) \left(1 - \left(\frac{\Delta^{-1}(s_{c_2}, \varphi_2)}{k} \right)^q \right)} \right) \right)$$

3.

$$\lambda \gamma = \left(\Delta \left(k \sqrt[k]{1 - \left(1 - \left(\frac{\Delta^{-1}(s_a, \varsigma)}{k} \right)^q \right)^\lambda} \right), \right. \\ \left. \Delta \left(k \left(\frac{\Delta^{-1}(s_b, \psi)}{k} \right)^\lambda \right), \right. \\ \left. \Delta \left(k \left(\frac{\Delta^{-1}(s_c, \varphi)}{k} \right)^\lambda \right) \right), \lambda > 0;$$

4.

$$\gamma^\lambda = \left(\Delta \left(k \left(\frac{\Delta^{-1}(s_a, \varsigma)}{k} \right)^\lambda \right), \right. \\ \left. \Delta \left(k \sqrt[k]{1 - \left(1 - \left(\frac{\Delta^{-1}(s_b, \psi)}{k} \right)^q \right)^\lambda} \right), \right. \\ \left. \Delta \left(k \sqrt[k]{1 - \left(1 - \left(\frac{\Delta^{-1}(s_c, \varphi)}{k} \right)^q \right)^\lambda} \right) \right), \lambda > 0.$$

3 2TLT-SF point aggregation operators

In this section, we present a new class of aggregation operators to aggregate 2TLT-SF information such as the 2TLT-SFPWA operator, 2TLT-SFPWG operator, 2TLT-SFGPWA operator and 2TLT-SFGPWG operator. Moreover, we discuss some properties of the proposed operators.

3.1 2TLT-SFPWA operators

Definition 8 Let Ω be the set of all 2TLT-SFNs, and $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs and $\xi_i, \zeta_i \in [0, 1]$. Then, the series of 2TLT-SFPWA operators: $\Omega^m \rightarrow \Omega$ is defined, if

- (1) $2TLT - SFPWAD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \omega_1 D_{\xi_1}^n(\gamma_1) \oplus \omega_2 D_{\xi_2}^n(\gamma_2) \oplus \dots \oplus \omega_m D_{\xi_m}^n(\gamma_m);$
- (2) $2TLT - SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \omega_1 F_{\xi_1, \zeta_1}^n(\gamma_1) \oplus \omega_2 F_{\xi_2, \zeta_2}^n(\gamma_2) \oplus \dots \oplus \omega_m F_{\xi_m, \zeta_m}^n(\gamma_m);$
- (3) $2TLT - SFPWAG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \omega_1 G_{\xi_1, \zeta_1}^n(\gamma_1) \oplus \omega_2 G_{\xi_2, \zeta_2}^n(\gamma_2) \oplus \dots \oplus \omega_m G_{\xi_m, \zeta_m}^n(\gamma_m);$
- (4) $2TLT - SFPWAH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \omega_1 H_{\xi_1, \zeta_1}^n(\gamma_1) \oplus \omega_2 H_{\xi_2, \zeta_2}^n(\gamma_2) \oplus \dots \oplus \omega_m H_{\xi_m, \zeta_m}^n(\gamma_m);$

$$(5) 2TLT - SFPWAJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \omega_1 J_{\xi_1, \zeta_1}^n(\gamma_1) \oplus \omega_2 J_{\xi_2, \zeta_2}^n(\gamma_2) \oplus \dots \oplus \omega_m J_{\xi_m, \zeta_m}^n(\gamma_m),$$

where as $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ is the weight vector of $(\gamma_1, \gamma_2, \dots, \gamma_m)$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1$.

By the defined operational laws in Sect. 2, we can get the following Theorem.

Theorem 1 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs, and $\xi_i, \zeta_i \in [0, 1]$, and $\xi_i + \zeta_i \leq 1$, so the aggregated value by the series of 2TLT-SFPWA operators is also 2TLT-SFNs and

(1) $2TLT - SFPWAD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$\left(\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)^q + \xi_i \pi_{\gamma_i}^q \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q + (1 - \xi_i) \pi_{\gamma_i}^q \right)^{\omega_i} \right) \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q + (1 - \xi_i) \pi_{\gamma_i}^q \right)^{\omega_i} \right) \right) \right)_q$$

(2) $2TLT - SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$\left(\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)^q F_{\xi_i, \zeta_i}^n(\gamma_i) \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} F_{\xi_i, \zeta_i}^n(\gamma_i) \right) \right) \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} F_{\xi_i, \zeta_i}^n(\gamma_i) \right) \right) \right) \right)_q$$

where

$$\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ = \left(\left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)^q + \xi_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}} \right) \\ \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ = \left(\left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q + \zeta_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}} \right) \\ \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ = \left(\left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q + \zeta_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}} \right).$$

(3) $2TLT-SFPWAG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \xi_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \zeta_i^n \right)^{\frac{\omega_i}{q}} \right) \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \zeta_i^n \right)^{\frac{\omega_i}{q}} \right) \right) \right)_q$$

(4) $2TLT-SFPWAH_{\xi, \zeta}^m(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \xi_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} H_{\xi_i, \zeta_i}^n(\gamma_i) \right) \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} H_{\xi_i, \zeta_i}^n(\gamma_i) \right) \right) \right)_q$$

where

$$\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{H_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q_{\gamma_i} \right. \\ \left. + \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q_{\gamma_i} \right) (1 - (1 - \zeta_i)^n) \right. \\ \left. - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q_{\gamma_i} \xi_i \left(\sum_{t=0}^{n-1} \xi_i^{n-1-t} (1 - \zeta_i)^t \right) \right)^{\frac{1}{q}}, \\ \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{H_{\xi_i, \zeta_i}^n(\gamma_i)} \\ = \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q_{\gamma_i} + \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q_{\gamma_i} \right) (1 - (1 - \zeta_i)^n) \right. \\ \left. - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q_{\gamma_i} \xi_i \left(\sum_{t=0}^{n-1} \xi_i^{n-1-t} (1 - \zeta_i)^t \right) \right)^{\frac{1}{q}}$$

(5) $2TLT-SFPWAJ_{\xi, \zeta}^m(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^{\omega_i} J_{\xi_i, \zeta_i}^n(\gamma_i) \right) \right)^{\frac{1}{q}} \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \xi_i^n \right) \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \xi_i^n \right) \right) \right)_q$$

where

$$\left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)_{J_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q_{\gamma_i} \right. \\ \left. + \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q_{\gamma_i} \right) (1 - (1 - \xi_i)^n) \right. \\ \left. - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q_{\gamma_i} \xi_i \left(\sum_{t=0}^{n-1} \zeta_i^{n-1-t} (1 - \xi_i)^t \right) \right)^{\frac{1}{q}}.$$

Proof We will prove Eq. 1 by the use of mathematical induction on m , and the others will prove to be equivalent. As

$$0 \leq \frac{\Delta^{-1}(s_{a_i}, s_i)}{k}, \\ \frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k}, \frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \leq 1, \\ q \geq 10 \leq \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q + \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \\ + \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \leq 1,$$

and

$$\left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ = \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q_{(\gamma_i)} \\ + \xi_i \pi_{(\gamma_i)}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i}, \\ \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ = \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q_{(\gamma_i)} \\ + \zeta_i \pi_{(\gamma_i)}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i}, \\ \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ = \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q_{(\gamma_i)} \\ + \zeta_i \pi_{(\gamma_i)}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i}$$

When $m = 2$, from the operational laws, we have

$$\begin{aligned}
 & 2TLT - SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2) = \omega_1 \gamma_1 \oplus \omega_2 \gamma_2 \\
 & = \left(\begin{array}{l} \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_1, \zeta_1}^n(\gamma_1) \right)^{\omega_1} \right)^{\frac{1}{q}}, \\ \Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_1} \right) F_{\xi_1, \zeta_1}^n(\gamma_1) \right), \\ \Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_1} \right) F_{\xi_1, \zeta_1}^n(\gamma_1) \right) \oplus \\ \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_2, \zeta_2}^n(\gamma_2) \right)^{\omega_2} \right)^{\frac{1}{q}}, \\ \Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_2} \right) F_{\xi_2, \zeta_2}^n(\gamma_2) \right), \\ \Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_2} \right) F_{\xi_2, \zeta_2}^n(\gamma_2) \right) \end{array} \right)_q \\
 & = \left(\begin{array}{l} \left(\Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_1, \zeta_1}^n(\gamma_1) \right)^{\omega_1} \right) + 1 \right. \\ \left. - \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_2, \zeta_2}^n(\gamma_2) \right)^{\omega_2} \right. \\ \left. - \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_1, \zeta_1}^n(\gamma_1) \right)^{\omega_1} \right. \\ \left. \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_2, \zeta_2}^n(\gamma_2) \right)^{\omega_2} \right)^{\frac{1}{q}} \right) \\ \Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_1} \right) F_{\xi_1, \zeta_1}^n(\gamma_1) \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_2} F_{\xi_2, \zeta_2}^n(\gamma_2) \right), \\ \Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_1} \right) F_{\xi_1, \zeta_1}^n(\gamma_1) \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_2} F_{\xi_2, \zeta_2}^n(\gamma_2) \right) \end{array} \right)_q \\
 & = \left(\begin{array}{l} \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_1, \zeta_1}^n(\gamma_1) \right)^{\omega_1} \right. \\ \left. \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_2, \zeta_2}^n(\gamma_2) \right)^{\omega_2} \right)^{\frac{1}{q}}, \\ \Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_1} \right) F_{\xi_1, \zeta_1}^n(\gamma_1) \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_2} F_{\xi_2, \zeta_2}^n(\gamma_2) \right), \\ \Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_1} \right) F_{\xi_1, \zeta_1}^n(\gamma_1) \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_2} F_{\xi_2, \zeta_2}^n(\gamma_2) \right) \end{array} \right)_q
 \end{aligned}$$

$$\left(\begin{array}{l} \Delta \left(k \left(1 - \prod_{i=1}^2 \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_i, \zeta_i}^n(\gamma_i) \right)^{\omega_i} \right)^{\frac{1}{q}}, \\ \Delta \left(k \left(\prod_{i=1}^2 \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} \right) F_{\xi_i, \zeta_i}^n(\gamma_i) \right), \\ \Delta \left(k \left(\prod_{i=1}^2 \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} \right) F_{\xi_i, \zeta_i}^n(\gamma_i) \right) \end{array} \right)_q$$

Consequently, the result is true for $m = 2$. If Eq. 1 holds for $m = t$, i.e.,

$$\begin{aligned}
 & 2TLT - SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_t) \\
 & = \left(\begin{array}{l} \Delta \left(k \left(1 - \prod_{i=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_i, \zeta_i}^n(\gamma_i) \right)^{\omega_i} \right)^{\frac{1}{q}}, \\ \Delta \left(k \left(\prod_{i=1}^t \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} \right) F_{\xi_i, \zeta_i}^n(\gamma_i) \right), \\ \Delta \left(k \left(\prod_{i=1}^t \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} \right) F_{\xi_i, \zeta_i}^n(\gamma_i) \right) \end{array} \right)_q
 \end{aligned}$$

Next, when $m = t + 1$, then from the operational laws, we have

$$\begin{aligned}
 & 2TLT - SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_{t+1}) \\
 & = 2TLT - SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_t) \oplus \omega_{t+1} F_{\xi, \zeta}^n(\gamma_{t+1}) \\
 & = \left(\begin{array}{l} \Delta \left(k \left(1 - \prod_{i=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_i, \zeta_i}^n(\gamma_i) \right)^{\omega_i} \right)^{\frac{1}{q}}, \\ \Delta \left(k \left(\prod_{i=1}^t \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} \right) F_{\xi_i, \zeta_i}^n(\gamma_i) \right), \\ \Delta \left(k \left(\prod_{i=1}^t \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} \right) F_{\xi_i, \zeta_i}^n(\gamma_i) \right) \oplus \\ \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) F_{\xi_{t+1}, \zeta_{t+1}}^n(\gamma_{t+1}) \right)^{\omega_{t+1}} \right)^{\frac{1}{q}}, \\ \Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} \right) F_{\xi_{t+1}, \zeta_{t+1}}^n(\gamma_{t+1}) \right), \\ \Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} \right) F_{\xi_{t+1}, \zeta_{t+1}}^n(\gamma_{t+1}) \right) \end{array} \right)_q
 \end{aligned}$$

$$\begin{aligned}
 & \left(\begin{array}{c} \Delta \left(k \left(1 - \prod_{i=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^{\omega_i} \right) \\ + 1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right)_{F_{\xi_{t+1}, \zeta_{t+1}}^n(\gamma_{t+1})}^{\omega_{t+1}} \\ - \left(1 - \prod_{i=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^{\omega_i} \right) \\ \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right)_{F_{\xi_{t+1}, \zeta_{t+1}}^n(\gamma_{t+1})}^{\omega_{t+1}} \right) \end{array} \right)^{\frac{1}{q}} \\
 & \Delta \left(k \left(\prod_{i=1}^t \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_{t+1}, \zeta_{t+1}}^n(\gamma_{t+1})}^{\omega_{t+1}} \right) \\
 & \Delta \left(k \left(\prod_{i=1}^t \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_{t+1}, \zeta_{t+1}}^n(\gamma_{t+1})}^{\omega_{t+1}} \right) \\
 & \left(\begin{array}{c} \Delta \left(k \left(1 - \prod_{i=1}^{t+1} \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right) \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^{\omega_i} \right)^{\frac{1}{q}} \\ \Delta \left(k \left(\prod_{i=1}^{t+1} \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right) \\ \Delta \left(k \left(\prod_{i=1}^{t+1} \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right) \end{array} \right)^{\frac{1}{q}}
 \end{aligned}$$

Thus, Eq. 1 holds for $m = t + 1$. Hence, Eq. 1 holds for all m . \square

Theorem 2 Let $\gamma_i = ((s_{a_i}, \zeta_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs with the weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1, \alpha > 0$, then

- (1) $2TLT - SFPWAD_{\xi}^n(\alpha\gamma_1, \alpha\gamma_2, \dots, \alpha\gamma_m) = \alpha 2TLT - SFPWAD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (2) $2TLT - SFPWAF_{\xi, \zeta}^n(\alpha\gamma_1, \alpha\gamma_2, \dots, \alpha\gamma_m) = \alpha 2TLT - SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (3) $2TLT - SFPWAG_{\xi, \zeta}^n(\alpha\gamma_1, \alpha\gamma_2, \dots, \alpha\gamma_m) = \alpha 2TLT - SFPWAG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (4) $2TLT - SFPWAH_{\xi, \zeta}^n(\alpha\gamma_1, \alpha\gamma_2, \dots, \alpha\gamma_m) = \alpha 2TLT - SFPWAH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (5) $2TLT - SFPWAJ_{\xi, \zeta}^n(\alpha\gamma_1, \alpha\gamma_2, \dots, \alpha\gamma_m) = \alpha 2TLT - SFPWAJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$.

Proof We will prove Eq. 2 holds for all m , and the others can be obtained corresponding. From the operational law in Sect. 2, we have

$$\alpha\gamma_i = \left(\begin{array}{c} \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right)^\alpha \right) \right)^{\frac{1}{q}} \\ \Delta \left(k \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^\alpha \right)_i \\ \Delta \left(k \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^\alpha \right)_i \end{array} \right)_q$$

and

$$\begin{aligned}
 & 2TLT - SFPWAF_{\xi, \zeta}^n(\alpha\gamma_1, \alpha\gamma_2, \dots, \alpha\gamma_m) \\
 & \left(\begin{array}{c} \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^{\alpha\omega_i} \right) \right)^{\frac{1}{q}} \\ \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\alpha\omega_i} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right) \\ \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\alpha\omega_i} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right) \end{array} \right)_q
 \end{aligned}$$

and hence

$$\begin{aligned}
 & \alpha 2TLT - SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \\
 & \left(\begin{array}{c} \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^{\omega_i} \right) \right)^{\frac{1}{q}} \\ \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right) \\ \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right) \end{array} \right)_q \\
 & = \alpha \left(\begin{array}{c} \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^{\omega_i} \right) \right)^{\frac{1}{q}} \\ \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right) \\ \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right) \end{array} \right)_q \\
 & = 2TLT - SFPWAF_{\xi, \zeta}^n(\alpha\gamma_1, \alpha\gamma_2, \dots, \alpha\gamma_m)
 \end{aligned}$$

\square

Theorem 3 Let $\gamma_i = ((s_{a_i}, \zeta_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q$ and $\gamma'_i = ((s'_{a_i}, \zeta'_i), (s'_{b_i}, \psi'_i), (s'_{c_i}, \varphi'_i))_q (i = 1, 2, \dots, m)$ be two collections of 2TLT-SFNs, and their weight vector is $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1$, then

- (1) $2TLT - SFPWAD_{\xi}^n(\gamma_1 \oplus \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) = 2TLT - SFPWAD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \oplus 2TLT - SFPWAD_{\xi}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (2) $2TLT - SFPWAF_{\xi, \zeta}^n(\gamma_1 \oplus \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) = 2TLT - SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \oplus 2TLT - SFPWAF_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (3) $2TLT - SFPWAG_{\xi, \zeta}^n(\gamma_1 \oplus \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) = 2TLT - SFPWAG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \oplus 2TLT - SFPWAG_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;

- (4) $2TLT-SFPWAH_{\xi, \zeta}^n(\gamma_1 \oplus \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) = 2TLT-SFPWAH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \oplus 2TLT-SFPWAH_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (5) $2TLT-SFPWAJ_{\xi, \zeta}^n(\gamma_1 \oplus \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) = 2TLT-SFPWAJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \oplus 2TLT-SFPWAJ_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$.

Proof We will prove Eq. 2 holds for all m , and the others can be proved corresponding. From the operational laws, we

$$\begin{aligned} \gamma_i \oplus \gamma'_i &= \left(\Delta \left(k \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{\gamma_i} + \left(\frac{\Delta^{-1}(s'_{a_i}, \varsigma'_i)}{k} \right)_{\gamma'_i} - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{\gamma_i} \left(\frac{\Delta^{-1}(s'_{a_i}, \varsigma'_i)}{k} \right)_{\gamma'_i} \right)^{\frac{1}{q}} \right), \right. \\ &\quad \Delta \left(k \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{\gamma_i} \left(\frac{\Delta^{-1}(s'_{b_i}, \psi'_i)}{k} \right)_{\gamma'_i} \right), \\ &\quad \left. \Delta \left(k \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{\gamma_i} \left(\frac{\Delta^{-1}(s'_{c_i}, \varphi'_i)}{k} \right)_{\gamma'_i} \right) \right)_q \\ 2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1 \oplus \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) &= \left(\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right)^q - \left(\frac{\Delta^{-1}(s'_{a_i}, \varsigma'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} + \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right)^{\frac{1}{q}} \right), \right. \\ &\quad \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s'_{b_i}, \psi'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right) \right), \\ &\quad \left. \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s'_{c_i}, \varphi'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right) \right) \right)_q \\ &= \left(\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right)^q \right. \right. \right. \\ &\quad \left. \left. \left(1 - \left(\frac{\Delta^{-1}(s'_{a_i}, \varsigma'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right)^{q \omega_i} \right) \right), \right. \\ &\quad \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s'_{b_i}, \psi'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right) \right), \\ &\quad \left. \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s'_{c_i}, \varphi'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right) \right) \right)_q \\ &= \left(\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right)^q \right. \right. \right. \\ &\quad \left. \left. \left(1 - \left(\frac{\Delta^{-1}(s'_{a_i}, \varsigma'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right)^{q \omega_i} \right) \right), \right. \\ &\quad \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s'_{b_i}, \psi'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right) \right), \\ &\quad \left. \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s'_{c_i}, \varphi'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right) \right) \right)_q \end{aligned}$$

And

$$\begin{aligned} &2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \oplus 2TLT-SFPWAF_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m) \\ &= \left(\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right)^q + 1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s'_{a_i}, \varsigma'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right)^q - \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right)^q \right) \right. \right. \\ &\quad \left. \left. \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s'_{a_i}, \varsigma'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right)^q \right) \right) \right), \right. \\ &\quad \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s'_{b_i}, \psi'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right) \right), \\ &\quad \left. \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s'_{c_i}, \varphi'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right) \right) \right)_q \\ &= \left(\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \right)^q \right. \right. \right. \\ &\quad \left. \left. \left(1 - \left(\frac{\Delta^{-1}(s'_{a_i}, \varsigma'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right)^{q \omega_i} \right) \right), \right. \\ &\quad \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s'_{b_i}, \psi'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right) \right), \\ &\quad \left. \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \left(\frac{\Delta^{-1}(s'_{c_i}, \varphi'_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma'_i)} \right) \right) \right)_q \\ &= 2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1 \oplus \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m). \end{aligned}$$

□

Moreover, the series of 2TLT-SFPWA operator have the following properties.

Theorem 4 (Idempotency) *If $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ are equal, that is, $\gamma_i = \gamma = ((s_a, \varsigma), (s_b, \psi), (s_c, \varphi))$ then*

- (1) $2TLT-SFPWAD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = D_{\xi}^n$;
- (2) $2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = F_{\xi, \zeta}^n$;
- (3) $2TLT-SFPWAG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = G_{\xi, \zeta}^n$;
- (4) $2TLT-SFPWAH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = H_{\xi, \zeta}^n$;
- (5) $2TLT-SFPWAJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = J_{\xi, \zeta}^n$.

Theorem 5 (Monotonicity) *Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q$ and $\gamma'_i = ((s'_{a_i}, \varsigma'_i), (s'_{b_i}, \psi'_i), (s'_{c_i}, \varphi'_i))_q (i = 1, 2, \dots, m)$ be two collections of 2TLT-SFNs, if $\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)$*

$$\begin{aligned} &\geq \left(\frac{\Delta^{-1}(s'_{a_i}, \varsigma'_i)}{k} \right), \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right) \geq \left(\frac{\Delta^{-1}(s'_{b_i}, \psi'_i)}{k} \right) \text{ and} \\ &\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right) \\ &\geq \left(\frac{\Delta^{-1}(s'_{c_i}, \varphi'_i)}{k} \right) \text{ holds for all } j, \text{ then} \end{aligned}$$

- (1) $2TLT-SFPWAD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq 2TLT-SFPWAD_{\xi}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (2) $2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq 2TLT-SFPWAF_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (3) $2TLT-SFPWAG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq 2TLT-SFPWAG_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (4) $2TLT-SFPWAH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq 2TLT-SFPWAH_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (5) $2TLT-SFPWAJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq 2TLT-SFPWAJ_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$.

Theorem 6 (Boundedness) Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs, then

$$\begin{aligned} (1) \quad &d_{D_{\xi}^n}^- \leq 2TLT-SFPWAD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq d_{D_{\xi}^n}^+; \\ (2) \quad &d_{F_{\xi, \zeta}^n}^- \leq 2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq d_{F_{\xi, \zeta}^n}^+; \\ (3) \quad &d_{G_{\xi, \zeta}^n}^- \leq 2TLT-SFPWAG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq d_{G_{\xi, \zeta}^n}^+; \\ (4) \quad &d_{H_{\xi, \zeta}^n}^- \leq 2TLT-SFPWAH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq d_{H_{\xi, \zeta}^n}^+; \\ (5) \quad &d_{J_{\xi, \zeta}^n}^- \leq 2TLT-SFPWAJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq d_{J_{\xi, \zeta}^n}^+ \end{aligned}$$

where $d_{\Delta}^+ = \left(\max_i \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{\Delta} \right) \right)$, $\min_i \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{\Delta} \right)$, $\min_i \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{\Delta} \right)$ and $d_{\Delta}^- = \left(\min_i \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{\Delta} \right) \right)$, $\max_i \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{\Delta} \right)$, $\max_i \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{\Delta} \right)$, Δ denotes $D_{\xi}^n, F_{\xi, \zeta}^n, G_{\xi, \zeta}^n, H_{\xi, \zeta}^n, J_{\xi, \zeta}^n$.

We have the special cases, by giving different values to the parameters n, ω, q .

Theorem 7 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ a collection of 2TLT-SFNs, $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ are the weight of $(\gamma_1, \gamma_2, \dots, \gamma_m)$, with $\omega_i \in [0, 1], q \geq 1$ and $\sum_{i=1}^m \omega_i = 1$, then

- (1) If $q = 1$, then the series of 2TLT-SFPWA operators will all be reduced to the series of 2TLFPWA operators.
- (2) If $q = 2$, then the series of 2TLT-SFPWA operators will all be reduced to the series of 2TLSFPWA operators.
- (3) If $n = 0$, then the series of 2TLT-SFPWA operators will all be reduced to the 2TLT-SFWA operator.

- (4) If $n = 0, q = 1$, then the series of 2TLT-SFPWA operators will all be reduced to a 2TLFPWA operator.
- (5) If $n = 0, q = 2$, then the series of 2TLT-SFPWA operators will all be reduced to 2TLSFWA operator.
- (6) If $n = 0, \omega = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})^T$, then the series of 2TLT-SFPWA operators will all be reduced to a 2TLT-SFA operator.
- (7) If $n = 0, \omega = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})^T, q = 1$, then the series of 2TLT-SFPWA operators will all be reduced to a 2TLFPWA operator.
- (8) If $n = 0, \omega = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})^T, q = 2$, then the series of 2TLT-SFPWA operators will all be reduced to a 2TLSFA operator.

3.2 2TLT-SFPWG operators

Definition 9 Let Ω be the set of all 2TLT-SFNs, and $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs and $\xi_i, \zeta_i \in [0, 1]$. Then, the series of 2TL T-SFPWG operators: $\Omega^m \rightarrow \Omega$ will define, if

- (1) $2TLT - SFPWGD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (D_{\xi_1}^n(\gamma_1))^{\omega_1} \otimes (D_{\xi_2}^n(\gamma_2))^{\omega_2} \otimes \dots \otimes (D_{\xi_m}^n(\gamma_m))^{\omega_m}$;
- (2) $2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (F_{\xi_1, \zeta_1}^n(\gamma_1))^{\omega_1} \otimes (F_{\xi_2, \zeta_2}^n(\gamma_2))^{\omega_2} \otimes \dots \otimes (F_{\xi_m, \zeta_m}^n(\gamma_m))^{\omega_m}$;
- (3) $2TLT - SFPWGG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (G_{\xi_1, \zeta_1}^n(\gamma_1))^{\omega_1} \otimes (G_{\xi_2, \zeta_2}^n(\gamma_2))^{\omega_2} \otimes \dots \otimes (G_{\xi_m, \zeta_m}^n(\gamma_m))^{\omega_m}$;
- (4) $2TLT - SFPWGH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (H_{\xi_1, \zeta_1}^n(\gamma_1))^{\omega_1} \otimes (H_{\xi_2, \zeta_2}^n(\gamma_2))^{\omega_2} \otimes \dots \otimes (H_{\xi_m, \zeta_m}^n(\gamma_m))^{\omega_m}$;
- (5) $2TLT - SFPWJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (J_{\xi_1, \zeta_1}^n(\gamma_1))^{\omega_1} \otimes (J_{\xi_2, \zeta_2}^n(\gamma_2))^{\omega_2} \otimes \dots \otimes (J_{\xi_m, \zeta_m}^n(\gamma_m))^{\omega_m}$

where $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ is the weight vector of $(\gamma_1, \gamma_2, \dots, \gamma_m)$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1$.

From the operational laws defined in Definition 7, we get the following theorem.

Theorem 8 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs, taking $\xi_i, \zeta_i \in [0, 1]$, and $\xi_i + \zeta_i \leq 1$, then the aggregated value from the series of 2TL T-SFPWG operators is also 2TLT-SFNs and

$$(1) \quad 2TLT - SFPWGD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \left(\Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q + (1 - \xi_i) \pi_{(\gamma_i)}^q \right)^{\frac{\omega_i}{q}} \right) \right), \right. \\ \left. \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q + \xi_i \pi_{(\gamma_i)}^q \right) \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \right. \\ \left. \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{(\gamma_i)}^q + \xi_i \pi_{(\gamma_i)}^q \right) \right)^{\omega_i} \right)^{\frac{1}{q}} \right) \right)_q$$

(2) $2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{l} \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)^{\omega_i} \right) \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}, \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \right)^{\omega_i} \right)^{\frac{1}{q}} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}, \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \right)^{\omega_i} \right)^{\frac{1}{q}} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \end{array} \right)_q$$

where

$$\begin{aligned} & \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ &= \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q + \xi_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}} \\ & \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ &= \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q + \zeta_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}} \\ & \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ &= \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{(\gamma_i)}^q + \zeta_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}} \end{aligned}$$

(3) $2TLT - SFPWGG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{l} \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)^q \xi_i^n \right)^{\frac{\omega_i}{q}} \right) \right), \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \zeta_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \zeta_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right) \end{array} \right)_q$$

(4) $2TLT - SFPWGH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{l} \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)^{\omega_i} \right) \right)_{H_{\xi_i, \zeta_i}^n(\gamma_i)}, \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \xi_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right)_{(\gamma_i)}, \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \xi_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right)_{(\gamma_i)} \end{array} \right)_q$$

where

$$\begin{aligned} & \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{H_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q \right. \\ & \left. + \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q \right) (1 - (1 - \xi_i)^n) \right. \\ & \left. - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q \xi_i \left(\sum_{t=0}^{n-1} \zeta_i^{n-1-t} (1 - \xi_i)^t \right) \right)^{\frac{1}{q}} \end{aligned}$$

(5) $2TLT - SFPWJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{l} \Delta \left(k \left(\prod_{i=1}^m \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q \right) \right), \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{\omega_i} \zeta_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{\omega_i} \zeta_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right) \end{array} \right)_q$$

where

$$\begin{aligned} & \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{J_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q \right. \\ & \left. + \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q \right) (1 - (1 - \zeta_i)^n) \right. \\ & \left. - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q \zeta_i \left(\sum_{t=0}^{n-1} \xi_i^{n-1-t} (1 - \zeta_i)^t \right) \right)^{\frac{1}{q}} \end{aligned}$$

Same as the 2TLT-SFPWA, we have the following properties of 2TL T-SFPWG. To save space, proofs of those properties are omitted.

Theorem 9 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs with the weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1$, then

- (1) $2TLT - SFPWGD_{\xi}^n(\gamma_1^\alpha, \gamma_2^\alpha, \dots, \gamma_m^\alpha) = (2TLT - SFPWGD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m))^\alpha$;
- (2) $2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1^\alpha, \gamma_2^\alpha, \dots, \gamma_m^\alpha) = (2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m))^\alpha$;
- (3) $2TLT - SFPWGG_{\xi, \zeta}^n(\gamma_1^\alpha, \gamma_2^\alpha, \dots, \gamma_m^\alpha) = (2TLT - SFPWGG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m))^\alpha$;
- (4) $2TLT - SFPWGH_{\xi, \zeta}^n(\gamma_1^\alpha, \gamma_2^\alpha, \dots, \gamma_m^\alpha) = (2TLT - SFPWGH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m))^\alpha$;

$$(5) \quad 2TLT - SFPWJ_{\xi, \zeta}^n(\gamma_1^\alpha, \gamma_2^\alpha, \dots, \gamma_m^\alpha) = (2TLT - SFPWJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m))^\alpha.$$

Theorem 10 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q$ and $\gamma'_i = ((s'_{a_i}, \varsigma'_i), (s'_{b_i}, \psi'_i), (s'_{c_i}, \varphi'_i))_q (i = 1, 2, \dots, m)$ be two collections of 2TLT-SFNs, then

- (1) $2TLT - SFPWGD_{\xi}^n(\gamma_1 \otimes \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) = 2TLT - SFPWGD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \otimes 2TLT - SFPWGD_{\xi}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (2) $2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1 \otimes \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) = 2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \otimes 2TLT - SFPWGF_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (3) $2TLT - SFPWGG_{\xi, \zeta}^n(\gamma_1 \otimes \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) = 2TLT - SFPWGG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \otimes 2TLT - SFPWGG_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (4) $2TLT - SFPWGH_{\xi, \zeta}^n(\gamma_1 \otimes \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) = 2TLT - SFPWGH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \otimes 2TLT - SFPWGH_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$;
- (5) $2TLT - SFPWJ_{\xi, \zeta}^n(\gamma_1 \otimes \gamma'_1, \gamma_2 \oplus \gamma'_2, \dots, \gamma_m \oplus \gamma'_m) = 2TLT - SFPWJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \otimes 2TLT - SFPWJ_{\xi, \zeta}^n(\gamma'_1, \gamma'_2, \dots, \gamma'_m)$.

The 2TL T-SFPWG operators series have properties same like the 2TLT-SFPWA operators such as idempotency, monotonicity and boundedness under some defined conditions, which are omitted in order to save space. We get the special cases through giving different values to the parameters n, ω, q .

Theorem 11 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ a collection of 2TLT-SFNs, $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ is the weight of $(\gamma_1, \gamma_2, \dots, \gamma_m)$, with $\omega_i \in [0, 1], q \geq 1$ and $\sum_{i=1}^m \omega_i = 1$, then

- (1) If $q = 1$, then the series of 2TL T-SFPWG operators will all be reduced to the series of 2TLFPWG operators.
- (2) If $q = 2$, then the series of 2TL T-SFPWG operators will all be reduced to the series of 2TLSFPWG operators.
- (3) If $n = 0$, then the series of 2TL T-SFPWG operators will all be reduced to the 2TLT-SFWG operator.
- (4) If $n = 0, q = 1$, then the series of 2TL T-SFPWG operators will all be reduced to 2TLFPWG operator.
- (5) If $n = 0, q = 2$, then the series of 2TL T-SFPWG operators will all be reduced to 2TLSFWG operator.

(6) If $n = 0, \omega = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})^T, q = 1$, then the series of 2TL T-SFPWG operators will all be reduced to a 2TLT-SFG operator.

(7) If $n = 0, \omega = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})^T, q = 1$, then the series of 2TL T-SFPWG operators will all be reduced to a 2TLFPWG operator.

(8) If $n = 0, \omega = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})^T, q = 2$, then the series of 2TL T-SFPWG operators will all be reduced to a 2TLSFG operator.

3.3 2TLT-SFGPWA operators

Definition 10 Let Ω be the set of all 2TLT-SFNs, and $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs and $\xi_i, \zeta_i \in [0, 1], \lambda > 0$. Then, the series of 2TLT-SFGPWA operators: $\Omega^m \rightarrow \Omega$ define, if

- (1) $2TLT - SFGPWAD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (\omega_1 D_{\xi_1}^n(\gamma_1))^{\frac{1}{\lambda}} \oplus (\omega_2 D_{\xi_2}^n(\gamma_2))^{\frac{1}{\lambda}} \oplus \dots \oplus (\omega_m D_{\xi_m}^n(\gamma_m))^{\frac{1}{\lambda}}$;
- (2) $2TLT - SFGPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (\omega_1 F_{\xi_1, \zeta_1}^n(\gamma_1))^{\frac{1}{\lambda}} \oplus (\omega_2 F_{\xi_2, \zeta_2}^n(\gamma_2))^{\frac{1}{\lambda}} \oplus \dots \oplus (\omega_m F_{\xi_m, \zeta_m}^n(\gamma_m))^{\frac{1}{\lambda}}$;
- (3) $2TLT - SFGPWAG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (\omega_1 G_{\xi_1, \zeta_1}^n(\gamma_1))^{\frac{1}{\lambda}} \oplus (\omega_2 G_{\xi_2, \zeta_2}^n(\gamma_2))^{\frac{1}{\lambda}} \oplus \dots \oplus (\omega_m G_{\xi_m, \zeta_m}^n(\gamma_m))^{\frac{1}{\lambda}}$;
- (4) $2TLT - SFGPWAH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (\omega_1 H_{\xi_1, \zeta_1}^n(\gamma_1))^{\frac{1}{\lambda}} \oplus (\omega_2 H_{\xi_2, \zeta_2}^n(\gamma_2))^{\frac{1}{\lambda}} \oplus \dots \oplus (\omega_m H_{\xi_m, \zeta_m}^n(\gamma_m))^{\frac{1}{\lambda}}$;
- (5) $2TLT - SFGPWAJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (\omega_1 J_{\xi_1, \zeta_1}^n(\gamma_1))^{\frac{1}{\lambda}} \oplus (\omega_2 J_{\xi_2, \zeta_2}^n(\gamma_2))^{\frac{1}{\lambda}} \oplus \dots \oplus (\omega_m J_{\xi_m, \zeta_m}^n(\gamma_m))^{\frac{1}{\lambda}}$

where $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ is the weight vector of $(\gamma_1, \gamma_2, \dots, \gamma_m)$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1$. From operational laws defined in Sect. 2, we get the following Theorem.

Theorem 12 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs, taking $\xi_i, \zeta_i \in [0, 1]$, and $\xi_i + \zeta_i \leq 1$, then the aggregated value by the series of 2TLT-SFGPWA operators is also 2TLT-SFNs and

(1) $2TLT - SFGPWAD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{l} \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q + \xi_i \pi_{(\gamma_i)}^q \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q - (1 - \xi_i) \pi_{\gamma_i}^q \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right) \right), \\ \Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{(\gamma_i)}^q - (1 - \xi_i) \pi_{\gamma_i}^q \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right) \right) \end{array} \right)_q$$

(2) $2TLT - SFGPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{l} \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^{q\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right), \\ \Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^q \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^q \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right) \end{array} \right)_q$$

where

$$\begin{aligned} & \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ &= \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{\gamma_i}^q + \xi_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}}, \\ & \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ &= \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{\gamma_i}^q + \zeta_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}}, \\ & \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} \\ &= \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{\gamma_i}^q + \zeta_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}} \end{aligned}$$

(3) $2TLT - SFGPWAG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{l} \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^{q\lambda} \xi_i^{n\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right), \\ \Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \zeta_i^n \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \zeta_i^n \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right) \end{array} \right)_q$$

(4) $2TLT - SFGPWAH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{l} \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^{q\lambda} \xi_i^{n\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right), \\ \Delta \left(k \left(\prod_{i=1}^m \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q H_{\xi_i, \zeta_i}^n(\gamma_i) \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right) \right), \\ \Delta \left(k \left(\prod_{i=1}^m \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q H_{\xi_i, \zeta_i}^n(\gamma_i) \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right) \right) \end{array} \right)_q,$$

where $\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{H_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q + \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \right) (1 - (1 - \zeta_i)^n) - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q_{(\gamma_i)} \zeta_i \left(\sum_{t=0}^{n-1} \xi_i^{n-1-t} (1 - \zeta_i)^t \right) \right)^{\frac{1}{q}}$

$\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{H_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q + \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \right) (1 - (1 - \zeta_i)^n) - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q_{(\gamma_i)} \zeta_i \left(\sum_{t=0}^{n-1} \xi_i^{n-1-t} (1 - \zeta_i)^t \right) \right)^{\frac{1}{q}}$

(5) $2TLT - SFGPWA J_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{c} \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q J_{\xi_i, \zeta_i}^n(\gamma_i) \xi_i^{n\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right), \\ \Delta \left(k \left(\prod_{i=1}^m \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^{q\lambda} \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(\prod_{i=1}^m \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^{q\lambda} \right)^{\lambda} \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right) \end{array} \right)_q$$

where

$$\begin{aligned} & \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)_{J_{\xi_i, \zeta_i}^n(\gamma_i)} \\ &= \left(\left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)_{(\gamma_i)}^q \right. \\ &+ \left. \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)_{(\gamma_i)}^q \right) (1 - (1 - \xi_i)^n) \right. \\ &\left. - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q \xi_i \left(\sum_{t=0}^{n-1} \zeta_i^{n-1-t} (1 - \xi_i)^t \right) \right)^{\frac{1}{q}}. \end{aligned}$$

$$\omega_2 \left(F_{\xi_2, \zeta_2}^n(\gamma_2) \right)^\lambda = \left(\begin{array}{c} \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^{q\lambda} \right)^{F_{\xi_2, \zeta_2}^n(\gamma_2)} \right)^{\omega_1} \right)^{\frac{1}{q}}, \\ \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \right)^{F_{\xi_2, \zeta_2}^n(\gamma_2)} \right)^{\lambda} \right)^{\frac{\omega_1}{q}}, \\ \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \right)^{F_{\xi_2, \zeta_2}^n(\gamma_2)} \right)^{\lambda} \right)^{\frac{\omega_1}{q}} \end{array} \right)_q$$

Proof We will prove Eq. 2 holds for all m , and similarly, we can prove the others. We will prove Eq. 2 based on the mathematical induction on m . When $m = 2$, we have

$$\omega_1 \left(F_{\xi_1, \zeta_1}^n(\gamma_1) \right)^\lambda = \left(\begin{array}{c} \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^{q\lambda} \right)^{F_{\xi_1, \zeta_1}^n(\gamma_1)} \right)^{\omega_1} \right)^{\frac{1}{q}}, \\ \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \right)^{F_{\xi_1, \zeta_1}^n(\gamma_1)} \right)^{\lambda} \right)^{\frac{\omega_1}{q}}, \\ \Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \right)^{F_{\xi_1, \zeta_1}^n(\gamma_1)} \right)^{\lambda} \right)^{\frac{\omega_1}{q}} \end{array} \right)_q$$

$$\begin{aligned} & \omega_1 \gamma_1^\lambda \oplus \omega_2 \gamma_2^\lambda \\ & \left(\Delta \left(k \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^{q\lambda} \right)_{F_{\xi_1, \xi_1}^n(\gamma_1)} \right) \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^{q\lambda} \right)_{F_{\xi_2, \xi_2}^n(\gamma_2)} \right) \right)^{\frac{1}{q}} \right), \\ & = \left(\Delta \left(k \left(\left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \right)_{F_{\xi_1, \xi_1}^n(\gamma_1)} \right)^\lambda \right)^{\frac{\omega_1}{q}} \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \right)_{F_{\xi_2, \xi_2}^n(\gamma_2)} \right)^\lambda \right)^{\frac{\omega_2}{q}} \right), \\ & \left(\Delta \left(k \left(\left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \right)_{F_{\xi_1, \xi_1}^n(\gamma_1)} \right)^\lambda \right)^{\frac{\omega_1}{q}} \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \right)_{F_{\xi_2, \xi_2}^n(\gamma_2)} \right)^\lambda \right)^{\frac{\omega_2}{q}} \right) \right)_q \end{aligned}$$

$$\begin{aligned} & 2TLT\text{-SFGPWA} F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = (\omega_1 \gamma_1^\lambda \oplus \omega_2 \gamma_2^\lambda)^{\frac{1}{\lambda}} \\ & \left(\Delta \left(k \left(1 - \prod_{i=1}^2 \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^{q\lambda} \right)_{F_{\xi_i, \xi_i}^n(\gamma_i)} \right) \right)^{\frac{1}{q\lambda}} \right), \\ & = \left(\Delta \left(k \left(1 - \left(1 - \prod_{i=1}^2 \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \right)_{F_{\xi_i, \xi_i}^n(\gamma_i)} \right)^\lambda \right) \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right), \\ & \left(\Delta \left(k \left(1 - \left(1 - \prod_{i=1}^2 \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \right)_{F_{\xi_i, \xi_i}^n(\gamma_i)} \right)^\lambda \right) \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right) \right)_q \end{aligned}$$

So, result is hold for $m = 2$. If Eq. 2 holds for $m = t$, that is

$$\begin{aligned} & 2TLT\text{-SFGPWA} F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_t) \\ & \left(\Delta \left(k \left(1 - \prod_{i=1}^t \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^{q\lambda} \right)_{F_{\xi_i, \xi_i}^n(\gamma_i)} \right) \right)^{\frac{1}{q\lambda}} \right), \\ & = \left(\Delta \left(k \left(1 - \left(1 - \prod_{i=1}^t \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \right)_{F_{\xi_i, \xi_i}^n(\gamma_i)} \right)^\lambda \right) \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right), \\ & \left(\Delta \left(k \left(1 - \left(1 - \prod_{i=1}^t \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \right)_{F_{\xi_i, \xi_i}^n(\gamma_i)} \right)^\lambda \right) \right)^{\omega_i} \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right) \right)_q \end{aligned}$$

$$(1) \ 2TLT - SFGPWGD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$$

$$= \left(\Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q - (1 - \xi_i) \pi_{\gamma_i}^q \right)^\lambda \right)^{\omega_i} \right) \right)^{\frac{1}{\lambda}} \right), \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q + \zeta_i \pi_{\gamma_i}^q \right)^\lambda \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{(\gamma_i)}^q + \zeta_i \pi_{\gamma_i}^q \right)^\lambda \right)^{\omega_i} \right)^{\frac{1}{q}} \right) \right)_q$$

Then, when $m = t + 1$, we have $2TLT - SFGPWA F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_{t+1}) = 2TLT - SFGPWA F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_t)$. Thus, Eq. 2 is true for $m = t + 1$. Hence, Eq. 2 holds for all m . \square

The series of $2TLT - SFGPWA$ operator have same properties to $2TLT - SFPWA$ operators such as idempotency, monotonicity and boundedness under some defined conditions, which are excluded in order to save space.

3.4 2TLT-SFGPWG operators

Definition 11 Let Ω be the set of all $2TLT - SFNs$, and $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of $2TLT - SFNs$ and $\xi_i, \zeta_i \in [0, 1], \lambda > 0$. Then, the series of $2TLT - SFGPWG$ operators: $\Omega^m \rightarrow \Omega$ define, if

- (1) $2TLT - SFGPWGD_{\xi}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \frac{1}{\lambda} ((\lambda D_{\xi_1}^n(\gamma_1))^{\omega_1} \otimes (\lambda D_{\xi_2}^n(\gamma_2))^{\omega_2} \otimes \dots \otimes (\lambda D_{\xi_m}^n(\gamma_m))^{\omega_m})$;
- (2) $2TLT - SFGPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \frac{1}{\lambda} ((\lambda F_{\xi_1, \zeta_1}^n(\gamma_1))^{\omega_1} \otimes (\lambda F_{\xi_2, \zeta_2}^n(\gamma_2))^{\omega_2} \otimes \dots \otimes (\lambda F_{\xi_m, \zeta_m}^n(\gamma_m))^{\omega_m})$;
- (3) $2TLT - SFGPWGG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \frac{1}{\lambda} ((\lambda G_{\xi_1, \zeta_1}^n(\gamma_1))^{\omega_1} \otimes (\lambda G_{\xi_2, \zeta_2}^n(\gamma_2))^{\omega_2} \otimes \dots \otimes (\lambda G_{\xi_m, \zeta_m}^n(\gamma_m))^{\omega_m})$;
- (4) $2TLT - SFGPWGH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \frac{1}{\lambda} ((\lambda H_{\xi_1, \zeta_1}^n(\gamma_1))^{\omega_1} \otimes (\lambda H_{\xi_2, \zeta_2}^n(\gamma_2))^{\omega_2} \otimes \dots \otimes (\lambda H_{\xi_m, \zeta_m}^n(\gamma_m))^{\omega_m})$;
- (5) $2TLT - SFGPWGJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \frac{1}{\lambda} ((\lambda J_{\xi_1, \zeta_1}^n(\gamma_1))^{\omega_1} \otimes (\lambda J_{\xi_2, \zeta_2}^n(\gamma_2))^{\omega_2} \otimes \dots \otimes (\lambda J_{\xi_m, \zeta_m}^n(\gamma_m))^{\omega_m})$

where $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ is the weight vector of $(\gamma_1, \gamma_2, \dots, \gamma_m)$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1$.

Theorem 13 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \varphi_i))_q (i = 1, 2, \dots, m)$ be a collection of $2TLT - SFNs$, taking $\xi_i, \zeta_i \in [0, 1]$, and $\xi_i + \zeta_i \leq 1$, then the aggregated value by the series of $2TLT - SFGPWG$ operators are also $2TLT - SFNs$ and

$$(2) \ 2TLT - SFGPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$$

$$= \left(\Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^q \right)^\lambda \right)^{\omega_i} \right) \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}}, \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^{q\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right), \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)}^{q\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right) \right)_q$$

where $\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q + \xi_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}}$, $\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q + \zeta_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}}$, $\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{F_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{(\gamma_i)}^q + \zeta_i \pi_{\gamma_i}^q \frac{1 - (1 - \xi_i - \zeta_i)^n}{\xi_i + \zeta_i} \right)^{\frac{1}{q}}$.

$$(3) \ 2TLT - SFGPWGG_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$$

$$= \left(\begin{array}{l} \Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)_{(\gamma_i)} \xi_i^n \right)^\lambda \right) \omega_i \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right) \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)} \zeta_i^{n\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right) \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{(\gamma_i)} \zeta_i^{n\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right) \end{array} \right)_q$$

(4) $2TLT-SFGPWGH_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{l} \Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)_{\gamma_i} \right)^{\lambda} \right) \omega_i \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right) \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{H_{\xi_i, \zeta_i}^n(\gamma_i)} \zeta_i^{n\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right) \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{H_{\xi_i, \zeta_i}^n(\gamma_i)} \zeta_i^{n\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right) \end{array} \right)_q$$

where $\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{H_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{\gamma_i}^q + \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{\gamma_i} \right)^q (1 - (1 - \zeta_i)^n) - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)_{\gamma_i}^q \zeta_i (\sum_{t=0}^{n-1} \xi_i^{n-1-t} (1 - \zeta_i)^t) \right)^{\frac{1}{q}}$

$\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{H_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{\gamma_i}^q + \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{\gamma_i} \right)^q (1 - (1 - \zeta_i)^n) - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)_{\gamma_i}^q \zeta_i (\sum_{t=0}^{n-1} \xi_i^{n-1-t} (1 - \zeta_i)^t) \right)^{\frac{1}{q}}$

(5) $2TLT - SFGPWGJ_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$

$$= \left(\begin{array}{l} \Delta \left(k \left(1 - \left(1 - \prod_{i=1}^m \left(1 - \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)_{J_{\xi_i, \zeta_i}^n(\gamma_i)} \right)^\lambda \right) \omega_i \right)^{\frac{1}{\lambda}} \right)^{\frac{1}{q}} \right) \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)} \zeta_i^{n\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right) \\ \Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)_{(\gamma_i)} \zeta_i^{n\lambda} \right)^{\omega_i} \right)^{\frac{1}{q\lambda}} \right) \end{array} \right)_q$$

where $\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k}\right)_{J_{\xi_i, \zeta_i}^n(\gamma_i)} = \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k}\right)_{(\gamma_i)}\right)^q + \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k}\right)_{(\gamma_i)}\right)^q \left(1 - (1 - \xi_i)^n\right) - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k}\right)_{(\gamma_i)}^q \xi_i \left(\sum_{t=0}^{n-1} \zeta_i^{n-1-t} (1 - \xi_i)^t\right)^{\frac{1}{q}}$.

Theorem 14 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \phi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs with the weight vector is $\omega = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})^T$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1, q \geq 1$, then the operation of complement on aggregation operator is as follows:

- (1) $\left[2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1^C, \gamma_2^C, \dots, \gamma_m^C)\right]^C = 2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (2) $\left[2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1^C, \gamma_2^C, \dots, \gamma_m^C)\right]^C = 2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (3) $\left[2TLT-SFGPWAF_{\xi, \zeta}^n(\gamma_1^C, \gamma_2^C, \dots, \gamma_m^C)\right]^C = 2TLT-SFGPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (4) $\left[2TLT-SFGPWGF_{\xi, \zeta}^n(\gamma_1^C, \gamma_2^C, \dots, \gamma_m^C)\right]^C = 2TLT-SFGPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$.

Theorem 15 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \phi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs with their weight vector is $\omega = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})^T$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1, q \geq 1, \lambda > 0$, then

- (1) $\lim_{n \rightarrow \infty} 2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = 2TLT-SFPWAD_{\xi+\zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (2) $\lim_{n \rightarrow \infty} 2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = 2TLT - SFPWGD_{\xi+\zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (3) $\lim_{n \rightarrow \infty} 2TLT-SFGPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = 2TLT-SFGPWAD_{\xi+\zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (4) $\lim_{n \rightarrow \infty} 2TLT-SFGPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = 2TLT-SFGPWGD_{\xi+\zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$.

To save space, proof of the theorem is omitted.

Theorem 16 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \phi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs, with the weight vector is $\omega = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})^T$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^m \omega_i = 1, \lambda > 0, q \geq 1$. If $\xi_i =$

$$\frac{\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k}\right)_{(\gamma_i)}^q}{\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k}\right)_{(\gamma_i)}^q + \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k}\right)_{(\gamma_i)}^q}, \zeta_i = \frac{\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k}\right)_{(\gamma_i)}^q}{\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k}\right)_{(\gamma_i)}^q + \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k}\right)_{(\gamma_i)}^q}, \text{ then}$$

- (1) $2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = 2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (2) $2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = 2TLT - SFPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (3) $2TLT-SFGPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = 2TLT-SFGPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (4) $2TLT-SFGPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = 2TLT-SFGPWGF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$.

Proof We proof Eq. 1 holds, and other can be proved correspondingly. From the Theorem 1, we get

$$2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \left[\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q - \xi_i \pi_{\gamma_i}^q \frac{1-(1-\xi_i-\zeta_i)^n}{\xi_i+\zeta_i} \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q + \zeta_i \pi_{\gamma_i}^q \frac{1-(1-\xi_i-\zeta_i)^n}{\xi_i+\zeta_i} \right)^{\omega_i} \right) \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{c_i}, \phi_i)}{k} \right)_{(\gamma_i)}^q + \zeta_i \pi_{\gamma_i}^q \frac{1-(1-\xi_i-\zeta_i)^n}{\xi_i+\zeta_i} \right)^{\omega_i} \right) \right) \right]_q$$

Since $\xi_i = \frac{\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k}\right)_{(\gamma_i)}^q}{\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k}\right)_{(\gamma_i)}^q + \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k}\right)_{(\gamma_i)}^q}, \zeta_i = \frac{\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k}\right)_{(\gamma_i)}^q}{\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k}\right)_{(\gamma_i)}^q + \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k}\right)_{(\gamma_i)}^q}$, we have

$$2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) = \left[\Delta \left(k \left(1 - \prod_{i=1}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)_{(\gamma_i)}^q - \frac{\xi_i \pi_{\gamma_i}^q}{\xi_i+\zeta_i} \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)_{(\gamma_i)}^q + \frac{\zeta_i \pi_{\gamma_i}^q}{\xi_i+\zeta_i} \right)^{\omega_i} \right) \right), \right. \\ \left. \Delta \left(k \left(\prod_{i=1}^m \left(\left(\frac{\Delta^{-1}(s_{c_i}, \phi_i)}{k} \right)_{(\gamma_i)}^q + \frac{\zeta_i \pi_{\gamma_i}^q}{\xi_i+\zeta_i} \right)^{\omega_i} \right) \right) \right]_q = 2TLT-SFPWAF_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$$

□

Further, we discuss the relationships and the differences between 2TLT-SFPWA, 2TLT-SFPWG, 2TLT-SFGPWA and 2TLT-SFGPWG operators. In case when $\lambda = 1$, and the 2TLT-SFGPWG operator is reduced to the 2TLT-SFPWG operator in Definition 9. Contrastingly, 2TLT-SFPWA is the arithmetic aggregation operator. So, 2TLT-SFPWG operator behaved its geometric form. Similarly, 2TLT-SFGPWG operator is geometric form of 2TLT-SFGPWA operator. Since $\prod_{i=1}^m x_i^{\lambda_i} \leq \sum_{i=1}^m \lambda_i x_i$ when $x_i > 0, \lambda_i > 0, \sum_{i=1}^m \lambda_i = 1$, the relationship between the aggregated values obtained by the 2TLT-SFPWA, 2TLT-SFPWG, 2TLT-SFGPWA and 2TLT-SFGPWG operators is shown as follows:

Theorem 17 Let $\gamma_i = ((s_{a_i}, \varsigma_i), (s_{b_i}, \psi_i), (s_{c_i}, \phi_i))_q (i = 1, 2, \dots, m)$ be a collection of 2TLT-SFNs, then

- (1) $2TLT-SFPWG F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq 2TLT-SFPWA F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (2) $2TLT-SFGPWG F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq 2TLT-SFGPWA F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (3) $2TLT-SFPWG F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq 2TLT-SFGPWA F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$;
- (4) $2TLT-SFGPWG F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m) \leq 2TLT-SFPWA F_{\xi, \zeta}^n(\gamma_1, \gamma_2, \dots, \gamma_m)$.

From the Theorem 17, we get that the values obtained from the 2TLT-SFPWG operators are not bigger than ones obtained by the 2TLT-SFPWA and 2TLT-SFGPWA operators. Same as the values obtained from GPFPCG operator are not bigger than the ones obtained by the 2TLT-SFPWA and 2TLT-SFGPWA operators for any $\lambda_i > 0$. Thus, DMs can select the any of four different operators according to the preference and actual needs.

4 An extended COPRAS method for MAGDM within 2TLT-SF environment

The COPRAS method is a reliable approach to solve MAGDM problems. This section introduces a new extended COPRAS method based on the series of 2TLT-SFPWA and 2TLT-SFPWG operators to build the ranking procedure.

To solve the group decision-making problem, we choose a set of m alternatives $\{A_1, A_2, \dots, A_m\}$ and a set of n attributes $\{Q_1, Q_2, \dots, Q_n\}$. Let $(w_1, w_2, \dots, w_n)^T$ be the weighting vector of attributes satisfying $0 \leq w_j \leq 1$ and $\sum_{j=1}^n w_j = 1$. Suppose $\{e_1, e_2, \dots, e_g\}$ is the collection of DMs having weights $(\varpi_1, \varpi_2, \dots, \varpi_g)^T$, satisfying $0 \leq \varpi_l \leq 1$ and $\sum_{l=1}^g \varpi_l = 1$. Suppose that the l th expert has evaluated the alternative A_i under Q_j and provided the decision matrices with 2TLT-SFNs, $R^l = (r_{ij}^l)_{m \times n}$ ($l =$

$1, 2, \dots, g$), where $r_{ij}^l = ((s_{a_{ij}}^l, \varsigma_{ij}^l), (s_{b_{ij}}^l, \psi_{ij}^l), (s_{c_{ij}}^l, \phi_{ij}^l))$, $0 \leq \Delta^{-1}(s_{a_{ij}}^l, \varsigma_{ij}^l) \leq k, 0 \leq \Delta^{-1}(s_{b_{ij}}^l, \psi_{ij}^l) \leq k, 0 \leq \Delta^{-1}(s_{c_{ij}}^l, \phi_{ij}^l) \leq k$ and $(\Delta^{-1}(s_{a_{ij}}^l, \varsigma_{ij}^l))^q + (\Delta^{-1}(s_{b_{ij}}^l, \psi_{ij}^l))^q + (\Delta^{-1}(s_{c_{ij}}^l, \phi_{ij}^l))^q \leq k^q, i = 1, 2, \dots, m, j = 1, 2, \dots, n$.

The 2TLT-SF-COPRAS method is constructed to get the best choice. The computation steps are described as follows:

Step 1. In this step, we integrate the individual decision matrices into an aggregated decision matrix $R = (r_{ij})_{m \times n}$ by using (3) of Theorem 1 ((3) of Theorem 8).

$$r_{ij} = 2TLT-SFPWAG_{\xi, \zeta}^n(r_{ij}^1, r_{ij}^2, \dots, r_{ij}^g). \tag{7}$$

$$\hat{r}_{ij} = 2TLT-SFPWGG_{\xi, \zeta}^n(r_{ij}^1, r_{ij}^2, \dots, r_{ij}^g). \tag{8}$$

We will develop an extended COPRAS method based on the 2TLT-SFPWAG $_{\xi, \zeta}^n$ (2TLT-SFPWGG $_{\xi, \zeta}^n$), named 2TLT-SFPWAG $_{\xi, \zeta}^n$ -COPRAS (2TLT-SFPWGG $_{\xi, \zeta}^n$ -COPRAS) method, to tackle the information in the aggregated decision matrix.

Step 2. Utilizing the aggregated decision matrix, we sum the values of benefit attributes. The sum of the values of the benefit attributes by using 2TLT-SFPWAG $_{\xi, \zeta}^n$ operator can be computed as follows:

$$\mathcal{P}_i = \left(\begin{array}{c} \Delta \left(k \left(1 - \left(\prod_{i=1; i \in N_B}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)^q \xi_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right) \right), \\ \Delta \left(k \left(\prod_{i=1; i \in N_B}^m \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \zeta_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(\prod_{i=1; i \in N_B}^m \left(\left(\frac{\Delta^{-1}(s_{c_i}, \phi_i)}{k} \right)^q \zeta_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right) \end{array} \right)_q \tag{9}$$

The sum of the values of the benefit attributes by using 2TLT-SFPWGG $_{\xi, \zeta}^n$ operator can be computed as follows:

$$\hat{\mathcal{P}}_i = \left(\begin{array}{c} \Delta \left(k \left(\prod_{i=1; i \in N_B}^m \left(\left(\frac{\Delta^{-1}(s_{a_i}, \varsigma_i)}{k} \right)^q \xi_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(1 - \prod_{i=1; i \in N_B}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \zeta_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(1 - \prod_{i=1; i \in N_B}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \phi_i)}{k} \right)^q \zeta_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right) \end{array} \right)_q \tag{10}$$

And then find the score function of \mathcal{P}_i and $\hat{\mathcal{P}}_i$.

Step 3. Similarly, utilizing aggregated decision matrix, we sum the values of cost attributes. The sum of the

values of the cost attributes by using 2TLT – SFPWAG $_{\xi, \zeta}^n$ operator can be computed as follows:

$$\mathcal{R}_i = \left(\begin{array}{l} \Delta \left(k \left(1 - \left(\prod_{i=1; i \in N_C}^m \left(1 - \left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \xi_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right) \right), \\ \Delta \left(k \left(\prod_{i=1; i \in N_C}^m \left(\left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \zeta_i^n \right)^{\frac{\omega_i}{q}} \right) \right), \\ \Delta \left(k \left(\prod_{i=1; i \in N_C}^m \left(\left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \zeta_i^n \right)^{\frac{\omega_i}{q}} \right) \right) \end{array} \right)^q \tag{11}$$

The sum of the values of the cost attributes by using 2TLT – SFPWGG $_{\xi, \zeta}^n$ operator can be computed as follows:

$$\hat{\mathcal{R}}_i = \left(\begin{array}{l} \Delta \left(k \left(\prod_{i=1; i \in N_C}^m \left(\left(\frac{\Delta^{-1}(s_{a_i}, s_i)}{k} \right)^q \xi_i^n \right)^{\frac{\omega_i}{q}} \right) \right), \\ \Delta \left(k \left(1 - \prod_{i=1; i \in N_C}^m \left(1 - \left(\frac{\Delta^{-1}(s_{b_i}, \psi_i)}{k} \right)^q \zeta_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right), \\ \Delta \left(k \left(1 - \prod_{i=1; i \in N_C}^m \left(1 - \left(\frac{\Delta^{-1}(s_{c_i}, \varphi_i)}{k} \right)^q \zeta_i^n \right)^{\omega_i} \right)^{\frac{1}{q}} \right) \end{array} \right)^q \tag{12}$$

And, also find the score function of \mathcal{R}_i and $\hat{\mathcal{R}}_i$.

Step 4. We determine each alternative $A_i (i = 1, 2, \dots, m)$'s relative significance $\mathcal{Q}_i (\hat{\mathcal{Q}}_i)$. Obviously, the more important the alternative is, the larger the value of $\mathcal{Q}_i (\hat{\mathcal{Q}}_i)$. The relative significance $\mathcal{Q}_i (\hat{\mathcal{Q}}_i)$ can be calculated as

$$\begin{aligned} \mathcal{Q}_i &= \mathcal{S}(\mathcal{P}_i) + \frac{\mathcal{S}(\mathcal{R}_{min}) \sum_{i=1}^m \mathcal{S}(\mathcal{R}_i)}{\mathcal{S}(\mathcal{R}_i) \sum_{i=1}^m \frac{\mathcal{S}(\mathcal{R}_{min})}{\mathcal{S}(\mathcal{R}_i)}} \\ &= \mathcal{S}(\mathcal{P}_i) + \frac{\sum_{i=1}^m \mathcal{S}(\mathcal{R}_i)}{\mathcal{S}(\mathcal{R}_i) \sum_{i=1}^m \frac{1}{\mathcal{S}(\mathcal{R}_i)}}. \end{aligned} \tag{13}$$

$$\begin{aligned} \hat{\mathcal{Q}}_i &= \mathcal{S}(\hat{\mathcal{P}}_i) + \frac{\mathcal{S}(\hat{\mathcal{R}}_{min}) \sum_{i=1}^m \mathcal{S}(\hat{\mathcal{R}}_i)}{\mathcal{S}(\hat{\mathcal{R}}_i) \sum_{i=1}^m \frac{\mathcal{S}(\hat{\mathcal{R}}_{min})}{\mathcal{S}(\hat{\mathcal{R}}_i)}} \\ &= \mathcal{S}(\hat{\mathcal{P}}_i) + \frac{\sum_{i=1}^m \mathcal{S}(\hat{\mathcal{R}}_i)}{\mathcal{S}(\hat{\mathcal{R}}_i) \sum_{i=1}^m \frac{1}{\mathcal{S}(\hat{\mathcal{R}}_i)}}. \end{aligned} \tag{14}$$

The $\mathcal{Q}_i (\hat{\mathcal{Q}}_i)$ from Eq. (13) (Eq. (14)) reflects the satisfaction measure of each alternative.

Step 5. Based on the $\mathcal{Q}_i (\hat{\mathcal{Q}}_i)$, the maximal relative significance value $\mathcal{K} (\hat{\mathcal{K}})$ can be determined. Utilize Eq. (15) (Eq. (16)) to identify the value of $\mathcal{K} (\hat{\mathcal{K}})$.

$$\mathcal{K} = \max_{1 \leq i \leq m} \mathcal{Q}_i. \tag{15}$$

$$\hat{\mathcal{K}} = \max_{1 \leq i \leq m} \hat{\mathcal{Q}}_i. \tag{16}$$

Thus, the alternative(s) with the associated maximal relative significance value is selected among the possible alternatives.

Step 6. Moreover, calculate the utility degree $\mathcal{U}_i (\hat{\mathcal{U}}_i)$ of each alternative utilizing the relative significance $\mathcal{Q}_i (\hat{\mathcal{Q}}_i)$. The $\mathcal{U}_i (\hat{\mathcal{U}}_i)$ can be determined by using Eq. (17) (Eq. (18)).

$$\mathcal{U}_i = \left(\frac{\mathcal{Q}_i}{\mathcal{K}} \right) \times 100\%. \tag{17}$$

$$\hat{\mathcal{U}}_i = \left(\frac{\hat{\mathcal{Q}}_i}{\hat{\mathcal{K}}} \right) \times 100\%. \tag{18}$$

Step 7. Rank the alternatives in the descending order of $\mathcal{U}_i (\hat{\mathcal{U}}_i)$. Hence, the bigger the value $\mathcal{U}_i (\hat{\mathcal{U}}_i)$, the higher is the rank of the alternative $A_i (i = 1, 2, \dots, m)$.

5 The problem enumerate and validity analysis

In this section, we will provide a problem enumerate and a validity analysis to verify our proposed methods.

5.1 The problem enumerate

Data mining integrated the concept and technique of databases, artificial intelligence, machine learning, statistics, and other areas. It is the methodology of searching into content's obscured regularities from unique perspectives for categorizing it into valuable data. This information is collected and assembled in precise aspects such as data warehouses, efficient analysis, and data mining algorithms, which help in decision-making as well as other data requirements and finally reduce costs and increase earnings. It is one of the most helpful tool for researchers, businesspeople, and users which enables them to extract the required information from massive data sets. The terminology

Knowledge discovery in database also refers to data mining. The methods concerned with knowledge discovery are data cleansing, data unification, data selection, data modification, data mining, trend interpretation, and data representation. Companies employ such techniques to retrieve particular knowledge from gigantic databases in order to deal with business challenges. It mainly transforms unprocessed data into insightful knowledge. It is related to data science carried out by an individual in a particular context, with a particular set of data and with a specific goal. Various services, such as text mining, web mining, audio and video mining, picture data mining, and social media mining,

are all part of this process. Simple or advanced software is used to perform these tasks. Data mining can be outsourced to have the task done quickly and cheaply. E-commerce is rapidly gaining ground in business world. If the changeover is performed appropriately, it opens the door for successful productivity, digital programs, reduced transaction costs, and enhanced client relations. Data mining technology is becoming more and more dominant in e-commerce. It has grown into a specialized area of basic and practical computer science study, especially when it comes to e-commerce. It has rich academic value and therefore will effectively fix numerous practical problems when implemented in e-commerce. When data mining innovation and e-commerce are merged, enterprises will be more likely to recognize one’s product offerings, make smarter choices, achieve a competitive edge, and have a wide variety of application opportunities. It will end up making e-commerce sites more productive and offer more advantages to companies. Web data mining for e-commerce can expose a plethora of information that is not visible to enterprises boost sales which enhances enterprise customer relationships, enhance site effectiveness, boost system performance, and grab a growing amount of customers. In this research, we will study a variety of the most advanced techniques and strategies utilized in the disciplines of e-commerce and data mining. It is obvious that e-commerce has raised and tends to generate latest research challenges in a very diverse wide range of data domains and data mining tasks. Here, the COPRAS method is utilized to evaluate the best data mining task for further research process. In this research article, we choose five different data mining tasks as a case study to show which one is more precise and better than others. A group of three DMs (users) $\{e_1, e_2, e_3\}$ with weighting vector $(0.24, 0.42, 0.34)^T$ were invited to evaluate the five data mining tasks namely

- (1) Classification analysis (\mathcal{CA});
- (2) Regression analysis (\mathcal{RA});
- (3) Sequence analysis (\mathcal{SA});
- (4) Prediction analysis (\mathcal{PA});
- (5) Association analysis (\mathcal{AA}).

After careful analysis, these four attributes are preferred for the evaluation: (1) Optimize enterprise resources (\mathcal{OER}); (2) Manage customer data (\mathcal{MCD}); (3) Assess business credit (\mathcal{ABC}); (4) Determine the abnormal events (\mathcal{DAE}) with the corresponding weight vector $(0.13, 0.15, 0.41, 0.31)^T$. Here, \mathcal{OER} and \mathcal{ABC} are benefit attributes, while \mathcal{MCD} and \mathcal{DAE} are cost attributes. Then, the individual evaluation matrices provided by three DMs (users) e_1, e_2 and e_3 , respectively, are expressed in the form of 2TLT-SF information with $n = 1, k = 8, q = 4, \xi = 0.4$ and $\zeta = 0.5$. Generally, n and q can take any values between zero to infinity. However, in some special situations, if n takes the value of 0, then the

$2TLT - SFPWAG_{\xi, \zeta}^n$ is reduced to the $2TLT - SFPWA$ operator and thus cannot reduce the uncertainty of data. Hence, from the application point of view, we generally advise DMs to take the values of $n = 1, q = 4$, which can not only effectively control the uncertainty of 2TLT-SFNs, but also simplify the process of calculation. Decision matrices $R^l = (r_{ij}^l)_{m \times n}$ given by the DMs $e_l (l = 1, 2, 3)$ are shown in Tables 1, 2 and 3.

5.2 Decision analysis with 2TLT – SFPWAG $_{\xi, \zeta}^n$ -COPRAS method

In this subsection, we utilize the proposed method to select the best data mining task. The detailed decision-making procedure from the extended 2TLT – SFPWAG $_{\xi, \zeta}^n$ -COPRAS method is described as follows:

- Step 1. We integrate the individual decision matrices given by three DMs into an aggregated decision matrix by using 2TLT – SFPWAG $_{\xi, \zeta}^n$ operator according to Eq. (7). The aggregated decision matrix $R = (r_{ij})_{m \times n}$ is given in Table 4.
- Step 2. Utilizing the aggregated decision matrix, we sum the collective values of the benefit attributes from Eq. (9) and obtain the score function of $\mathcal{P}_i (i = 1, 2, \dots, 5)$ for each alternative as shown in Table 5.
- Step 3. Similarly, utilizing the aggregated decision matrix, we sum the collective values of the cost attributes from Eq. (11) and obtain the score function of $\mathcal{R}_i (i = 1, 2, \dots, 5)$ for each alternative as shown in Table 6.
- Step 4. For each alternative, we compute the relative significance \mathcal{Q}_i from Eq. (13).

$$\begin{aligned} \mathcal{Q}_1 &= 3.8167 \\ &+ \frac{3.7744 + 3.9760 + 3.6511 + 3.9580 + 3.7713}{3.7744 \left(\frac{1}{3.7744} + \frac{1}{3.9760} + \frac{1}{3.6511} + \frac{1}{3.9580} + \frac{1}{3.7713} \right)} \\ &= 7.6913, \end{aligned}$$

$$\begin{aligned} \mathcal{Q}_2 &= 3.9715 \\ &+ \frac{3.7744 + 3.9760 + 3.6511 + 3.9580 + 3.7713}{3.9760 \left(\frac{1}{3.7744} + \frac{1}{3.9760} + \frac{1}{3.6511} + \frac{1}{3.9580} + \frac{1}{3.7713} \right)} \\ &= 7.6496, \end{aligned}$$

$$\begin{aligned} \mathcal{Q}_3 &= 3.8220 \\ &+ \frac{3.7744 + 3.9760 + 3.6511 + 3.9580 + 3.7713}{3.6511 \left(\frac{1}{3.7744} + \frac{1}{3.9760} + \frac{1}{3.6511} + \frac{1}{3.9580} + \frac{1}{3.7713} \right)} \\ &= 7.8274, \end{aligned}$$

$$\begin{aligned} \mathcal{Q}_4 &= 3.9864 \\ &+ \frac{3.7744 + 3.9760 + 3.6511 + 3.9580 + 3.7713}{3.9580 \left(\frac{1}{3.7744} + \frac{1}{3.9760} + \frac{1}{3.6511} + \frac{1}{3.9580} + \frac{1}{3.7713} \right)} \\ &= 7.6813, \end{aligned}$$

$$\mathcal{Q}_5 = 3.7827$$

Table 1 2-Tuple linguistic T -spherical fuzzy decision matrix given by e_1

	\mathcal{OER}	\mathcal{MCD}	\mathcal{ABC}	\mathcal{DAE}
\mathcal{CA}	$((s_2, 0), (s_4, 0), (s_2, 0))$	$((s_3, 0), (s_2, 0), (s_6, 0))$	$((s_4, 0), (s_4, 0), (s_5, 0))$	$((s_3, 0), (s_6, 0), (s_6, 0))$
\mathcal{RA}	$((s_5, 0), (s_2, 0), (s_2, 0))$	$((s_6, 0), (s_4, 0), (s_2, 0))$	$((s_3, 0), (s_3, 0), (s_5, 0))$	$((s_4, 0), (s_1, 0), (s_2, 0))$
\mathcal{SA}	$((s_6, 0), (s_3, 0), (s_4, 0))$	$((s_4, 0), (s_4, 0), (s_3, 0))$	$((s_5, 0), (s_4, 0), (s_3, 0))$	$((s_3, 0), (s_5, 0), (s_5, 0))$
\mathcal{PA}	$((s_5, 0), (s_5, 0), (s_2, 0))$	$((s_4, 0), (s_3, 0), (s_1, 0))$	$((s_6, 0), (s_4, 0), (s_2, 0))$	$((s_7, 0), (s_2, 0), (s_2, 0))$
\mathcal{AA}	$((s_7, 0), (s_3, 0), (s_2, 0))$	$((s_5, 0), (s_4, 0), (s_3, 0))$	$((s_6, 0), (s_3, 0), (s_3, 0))$	$((s_6, 0), (s_2, 0), (s_3, 0))$

Table 2 2-Tuple linguistic T -spherical fuzzy decision matrix given by e_2

	\mathcal{OER}	\mathcal{MCD}	\mathcal{ABC}	\mathcal{DAE}
\mathcal{CA}	$((s_3, 0), (s_4, 0), (s_2, 0))$	$((s_6, 0), (s_3, 0), (s_2, 0))$	$((s_3, 0), (s_5, 0), (s_2, 0))$	$((s_7, 0), (s_3, 0), (s_1, 0))$
\mathcal{RA}	$((s_5, 0), (s_3, 0), (s_4, 0))$	$((s_4, 0), (s_3, 0), (s_2, 0))$	$((s_7, 0), (s_2, 0), (s_1, 0))$	$((s_5, 0), (s_6, 0), (s_1, 0))$
\mathcal{SA}	$((s_6, 0), (s_2, 0), (s_1, 0))$	$((s_7, 0), (s_1, 0), (s_3, 0))$	$((s_6, 0), (s_3, 0), (s_4, 0))$	$((s_6, 0), (s_2, 0), (s_3, 0))$
\mathcal{PA}	$((s_7, 0), (s_2, 0), (s_3, 0))$	$((s_5, 0), (s_2, 0), (s_4, 0))$	$((s_4, 0), (s_5, 0), (s_2, 0))$	$((s_4, 0), (s_6, 0), (s_1, 0))$
\mathcal{AA}	$((s_4, 0), (s_5, 0), (s_3, 0))$	$((s_6, 0), (s_4, 0), (s_3, 0))$	$((s_5, 0), (s_4, 0), (s_3, 0))$	$((s_3, 0), (s_6, 0), (s_2, 0))$

Table 3 2-Tuple linguistic T -spherical fuzzy decision matrix given by e_3

	\mathcal{OER}	\mathcal{MCD}	\mathcal{ABC}	\mathcal{DAE}
\mathcal{CA}	$((s_6, 0), (s_1, 0), (s_2, 0))$	$((s_4, 0), (s_6, 0), (s_2, 0))$	$((s_2, 0), (s_3, 0), (s_4, 0))$	$((s_3, 0), (s_4, 0), (s_6, 0))$
\mathcal{RA}	$((s_4, 0), (s_5, 0), (s_2, 0))$	$((s_3, 0), (s_5, 0), (s_3, 0))$	$((s_5, 0), (s_2, 0), (s_3, 0))$	$((s_7, 0), (s_3, 0), (s_1, 0))$
\mathcal{SA}	$((s_3, 0), (s_7, 0), (s_1, 0))$	$((s_2, 0), (s_7, 0), (s_3, 0))$	$((s_4, 0), (s_2, 0), (s_5, 0))$	$((s_2, 0), (s_7, 0), (s_3, 0))$
\mathcal{PA}	$((s_6, 0), (s_4, 0), (s_3, 0))$	$((s_7, 0), (s_3, 0), (s_1, 0))$	$((s_2, 0), (s_6, 0), (s_1, 0))$	$((s_3, 0), (s_4, 0), (s_2, 0))$
\mathcal{AA}	$((s_5, 0), (s_3, 0), (s_4, 0))$	$((s_2, 0), (s_3, 0), (s_6, 0))$	$((s_6, 0), (s_1, 0), (s_5, 0))$	$((s_5, 0), (s_4, 0), (s_2, 0))$

$$\begin{aligned}
 &+ \frac{3.7744 + 3.9760 + 3.6511 + 3.9580 + 3.7713}{3.7713 \left(\frac{1}{3.7744} + \frac{1}{3.9760} + \frac{1}{3.6511} + \frac{1}{3.9580} + \frac{1}{3.7713} \right)} \\
 &= 7.6605.
 \end{aligned}$$

Step 5. Based on the \mathcal{Q}_i , we identify the maximum relative significance \mathcal{K} utilizing Eq. (15). The value of \mathcal{K} is given as

$$\mathcal{K} = 7.8274.$$

Step 6. Now, we calculate the utility degree \mathcal{U}_i of each alternative by using Eq. (17) as follows:

$$\begin{aligned}
 \mathcal{U}_1 &= \left(\frac{7.6913}{7.8274} \right) \times 100\% = 98.26\%, \\
 \mathcal{U}_2 &= \left(\frac{7.6496}{7.8274} \right) \times 100\% = 97.72\%, \\
 \mathcal{U}_3 &= \left(\frac{7.8274}{7.8274} \right) \times 100\% = 100\%, \\
 \mathcal{U}_4 &= \left(\frac{7.6813}{7.8274} \right) \times 100\% = 98.13\%, \\
 \mathcal{U}_5 &= \left(\frac{7.6605}{7.8274} \right) \times 100\% = 97.86\%.
 \end{aligned}$$

Step 7. Based on the utility degree $\mathcal{U}_i (i = 1, 2, \dots, 5)$, we rank the alternatives in the descending order as

$$\mathcal{SA} > \mathcal{CA} > \mathcal{PA} > \mathcal{AA} > \mathcal{RA}$$

. So, \mathcal{SA} is the best choice.

5.3 Decision analysis with 2TLT – SFPWGG $_{\xi, \zeta}^n$ -COPRAS method

In this subsection, we utilize the proposed method to select the best data mining task. The detailed decision-making procedure from the extended 2TLT – SFPWGG $_{\xi, \zeta}^n$ -COPRAS method is described as follows:

Step 1. We integrate the individual decision matrices given by three DMs into an aggregated decision matrix by using 2TLT – SFPWGG $_{\xi, \zeta}^n$ operator according to Eq. (8). The aggregated decision matrix $\hat{R} = (\hat{r}_{ij})_{m \times n}$ is given in Table 7.

Step 2. Utilizing the aggregated decision matrix, we sum the collective values of the benefit attributes from

Table 4 Aggregated decision matrix by 2TLT – SFPWAG $_{\xi, \zeta}^n$ operator

	\mathcal{OER}	\mathcal{MCD}
\mathcal{CA}	$((s_4, -0.2457), (s_2, 0.0994), (s_2, -0.3182))$	$((s_4, 0.0414), (s_3, -0.1030), (s_2, 0.1892))$
\mathcal{RA}	$((s_4, -0.2373), (s_3, -0.2771), (s_2, 0.2501))$	$((s_4, -0.3200), (s_3, 0.2157), (s_2, -0.0696))$
\mathcal{SA}	$((s_4, 0.3560), (s_3, -0.1620), (s_1, 0.1728))$	$((s_5, -0.3716), (s_2, 0.2728), (s_3, -0.4773))$
\mathcal{PA}	$((s_5, 0.0529), (s_3, -0.3477), (s_2, 0.2888))$	$((s_5, -0.3306), (s_2, 0.1277), (s_2, -0.4948))$
\mathcal{AA}	$((s_4, 0.4022), (s_3, 0.1264), (s_3, -0.4760))$	$((s_4, 0.1137), (s_3, 0.0502), (s_3, 0.1931))$
	\mathcal{ABC}	\mathcal{DAE}
\mathcal{CA}	$((s_3, -0.4769), (s_3, 0.3499), (s_3, -0.3477))$	$((s_5, -0.3901), (s_3, 0.2854), (s_2, 0.3772))$
\mathcal{RA}	$((s_5, -0.2266), (s_2, -0.1463), (s_2, -0.2023))$	$((s_5, -0.3306), (s_3, -0.4071), (s_1, -0.0069))$
\mathcal{SA}	$((s_4, 0.2196), (s_2, 0.3549), (s_3, 0.3866))$	$((s_4, -0.0817), (s_3, 0.2082), (s_3, -0.1483))$
\mathcal{PA}	$((s_4, -0.3633), (s_4, 0.2401), (s_1, 0.3287))$	$((s_4, 0.1800), (s_3, 0.3769), (s_1, 0.2570))$
\mathcal{AA}	$((s_4, 0.4944), (s_2, -0.0406), (s_3, 0.0012))$	$((s_4, -0.1150), (s_3, 0.3769), (s_2, -0.1463))$

Table 5 Sum up values and score functions of benefit type attributes by 2TLT – SFPWAG $_{\xi, \zeta}^n$ operator

Alternative	Sum up values (\mathcal{P}_i)	Scores ($\mathcal{S}(\mathcal{P}_i)$)
\mathcal{CA}	$((s_2, 0.0322), (s_4, 0.2847), (s_4, -0.2172))$	3.8167
\mathcal{RA}	$((s_3, 0.1362), (s_3, 0.4772), (s_3, 0.3497))$	3.9715
\mathcal{SA}	$((s_3, -0.0947), (s_4, -0.1436), (s_4, -0.0099))$	3.8220
\mathcal{PA}	$((s_3, -0.1775), (s_5, -0.1351), (s_3, -0.0342))$	3.9864
\mathcal{AA}	$((s_3, 0.0562), (s_4, -0.3783), (s_4, 0.1951))$	3.7827

Table 6 Sum up values and score functions of cost type attributes by 2TLT – SFPWAG $_{\xi, \zeta}^n$ operator

Alternative	Sum up values (\mathcal{R}_i)	Scores ($\mathcal{S}(\mathcal{R}_i)$)
\mathcal{CA}	$((s_3, -0.0786), (s_5, -0.1862), (s_4, 0.1752))$	3.7744
\mathcal{RA}	$((s_3, -0.0988), (s_5, -0.4562), (s_3, 0.1258))$	3.9760
\mathcal{SA}	$((s_3, -0.2483), (s_5, -0.3924), (s_5, -0.4876))$	3.6511
\mathcal{PA}	$((s_3, -0.1381), (s_5, -0.3647), (s_3, 0.2395))$	3.9580
\mathcal{AA}	$((s_3, -0.3994), (s_5, -0.1074), (s_4, 0.0905))$	3.7713

Eq. (10) and obtain the score function of $\hat{\mathcal{P}}_i (i = 1, 2, \dots, 5)$ for each alternative as shown in Table 8.

Step 3. Similarly, utilizing the aggregated decision matrix, we sum the collective values of the cost attributes from Eq. (12) and obtain the score function of $\hat{\mathcal{R}}_i (i = 1, 2, \dots, 5)$ for each alternative as shown in Table 9.

Step 4. For each alternative, we compute the relative significance $\hat{\mathcal{Q}}_i$ from Eq. (14).

$$\begin{aligned} \hat{\mathcal{Q}}_1 &= 4.1462 \\ &+ \frac{4.4836 + 4.6820 + 4.3876 + 4.5919 + 4.4880}{4.4836 \left(\frac{1}{4.4836} + \frac{1}{4.6820} + \frac{1}{4.3876} + \frac{1}{4.5919} + \frac{1}{4.4880} \right)} \\ &= 8.7140, \\ \hat{\mathcal{Q}}_2 &= 4.5120 \\ &+ \frac{4.4836 + 4.6820 + 4.3876 + 4.5919 + 4.4880}{4.6820 \left(\frac{1}{4.4836} + \frac{1}{4.6820} + \frac{1}{4.3876} + \frac{1}{4.5919} + \frac{1}{4.4880} \right)} \\ &= 8.8862, \end{aligned}$$

$$\begin{aligned} \hat{\mathcal{Q}}_3 &= 4.4881 \\ &+ \frac{4.4836 + 4.6820 + 4.3876 + 4.5919 + 4.4880}{4.3876 \left(\frac{1}{4.4836} + \frac{1}{4.6820} + \frac{1}{4.3876} + \frac{1}{4.5919} + \frac{1}{4.4880} \right)} \\ &= 9.1558, \\ \hat{\mathcal{Q}}_4 &= 4.3278 \\ &+ \frac{4.4836 + 4.6820 + 4.3876 + 4.5919 + 4.4880}{4.5919 \left(\frac{1}{4.4836} + \frac{1}{4.6820} + \frac{1}{4.3876} + \frac{1}{4.5919} + \frac{1}{4.4880} \right)} \\ &= 8.7879, \\ \hat{\mathcal{Q}}_5 &= 4.6053 \\ &+ \frac{4.4836 + 4.6820 + 4.3876 + 4.5919 + 4.4880}{4.4880 \left(\frac{1}{4.4836} + \frac{1}{4.6820} + \frac{1}{4.3876} + \frac{1}{4.5919} + \frac{1}{4.4880} \right)} \\ &= 9.1686. \end{aligned}$$

Step 5. Based on the $\hat{\mathcal{Q}}_i$, we identify the maximum relative significance $\hat{\mathcal{K}}$ utilizing Eq. (16). The value of $\hat{\mathcal{K}}$ is given as

$$\hat{\mathcal{K}} = 9.1686.$$

Table 7 Aggregated decision matrix by 2TLT – SFPWGG $_{\xi, \zeta}^n$ operator

	OER	MCD
CA	$((s_3, -0.2602), (s_3, 0.0373), (s_2, -0.3182))$	$((s_4, -0.4800), (s_4, -0.0203), (s_4, -0.3802))$
RA	$((s_4, -0.3142), (s_3, 0.3604), (s_3, -0.2304))$	$((s_3, 0.1795), (s_4, -0.4671), (s_2, 0.0900))$
SA	$((s_4, -0.2302), (s_5, -0.3506), (s_2, 0.3685))$	$((s_3, 0.1791), (s_5, -0.3109), (s_3, -0.4773))$
PA	$((s_5, -0.1272), (s_3, 0.3349), (s_2, 0.3917))$	$((s_4, 0.2258), (s_2, 0.2770), (s_3, -0.2824))$
AA	$((s_4, -0.0749), (s_4, -0.4600), (s_3, -0.1855))$	$((s_3, 0.1437), (s_3, 0.1501), (s_4, 0.0086))$
	ABC	DAE
CA	$((s_2, 0.2272), (s_4, -0.3494), (s_3, 0.3349))$	$((s_3, 0.4055), (s_4, -0.1303), (s_4, 0.4426))$
RA	$((s_4, 0.0515), (s_2, -0.0055), (s_3, 0.0889))$	$((s_4, 0.2258), (s_4, 0.1575), (s_1, 0.2317))$
SA	$((s_4, -0.0208), (s_3, -0.3314), (s_4, -0.3826))$	$((s_3, -0.2190), (s_5, -0.2081), (s_3, 0.2201))$
PA	$((s_3, -0.2300), (s_4, 0.4381), (s_2, -0.4720))$	$((s_3, 0.2993), (s_4, 0.2572), (s_1, 0.4841))$
AA	$((s_4, 0.4199), (s_3, -0.1710), (s_3, 0.4093))$	$((s_3, 0.3521), (s_4, 0.2572), (s_2, -0.0055))$

Table 8 Sum up values and score functions of benefit type attributes by 2TLT – SFPWGG $_{\xi, \zeta}^n$ operator

Alternative	Sum up values (\mathcal{P}_i)	Scores ($S(\mathcal{P}_i)$)
CA	$((s_4, -0.3593), (s_3, -0.4520), (s_2, 0.2578))$	4.1462
RA	$((s_5, -0.1642), (s_2, -0.1551), (s_2, 0.1790))$	4.5120
SA	$((s_5, -0.1857), (s_3, -0.4620), (s_2, 0.4722))$	4.4881
PA	$((s_4, 0.2907), (s_3, 0.0681), (s_1, 0.3424))$	4.3278
AA	$((s_5, 0.0526), (s_2, 0.1999), (s_2, 0.3765))$	4.6053

Table 9 Sum up values and score functions of cost type attributes by 2TLT – SFPWGG $_{\xi, \zeta}^n$ operator

Alternative	Sum up values (\mathcal{R}_i)	Scores ($S(\mathcal{R}_i)$)
CA	$((s_5, -0.1150), (s_3, -0.2893), (s_3, -0.0651))$	4.4836
RA	$((s_5, 0.1439), (s_3, -0.2338), (s_1, 0.1565))$	4.6820
SA	$((s_5, -0.4819), (s_3, 0.3101), (s_2, 0.1087))$	4.3876
PA	$((s_5, -0.0283), (s_3, -0.2939), (s_1, 0.4843))$	4.5919
AA	$((s_5, -0.2206), (s_3, -0.2289), (s_2, 0.1675))$	4.4880

Table 10 Relative significance and ranking results according to the parameter q by 2TLT – SFPWAG $_{\xi, \zeta}^n$ -COPRAS method

Parameter	Q_1	Q_2	Q_3	Q_4	Q_5	Ranking
$q = 1$	5.9012	5.8853	6.1764	6.0028	5.9572	$SA > PA > AA > CA > RA$
$q = 2$	6.8913	6.8459	7.1538	6.9435	6.8983	$SA > PA > AA > CA > RA$
$q = 3$	7.4166	6.8565	7.6132	7.4261	7.3943	$SA > PA > CA > AA > RA$
$q = 4$	7.6913	7.6496	7.8274	7.6813	7.6605	$SA > CA > PA > AA > RA$
$q = 5$	7.8348	7.8048	7.9262	7.8200	7.8063	$SA > CA > PA > AA > RA$

Table 11 Relative significance and ranking results according to the parameter q by 2TLT – SFPWGG $_{\xi, \zeta}^n$ -COPRAS method

Parameter	Q_1	Q_2	Q_3	Q_4	Q_5	Ranking
$q = 1$	10.4765	10.4493	10.8019	10.5482	10.7530	$SA > AA > PA > CA > RA$
$q = 2$	9.6963	9.8394	10.1901	9.8014	10.1709	$SA > AA > RA > PA > CA$
$q = 3$	9.1051	9.2962	9.6100	9.2019	9.6118	$AA > SA > RA > PA > CA$
$q = 4$	8.7140	8.8862	9.1558	8.7879	9.1686	$AA > SA > RA > PA > CA$
$q = 5$	8.4632	8.5963	8.8201	8.5139	8.8379	$AA > SA > RA > PA > CA$

Step 6. Now, we calculate the utility degree \hat{U}_i of each alternative by using Eq. (18) as follows:

$$\hat{U}_1 = \left(\frac{8.7140}{9.1686} \right) \times 100\% = 95.04\%,$$

$$\hat{U}_2 = \left(\frac{8.8862}{9.1686} \right) \times 100\% = 96.92\%,$$

$$\hat{U}_3 = \left(\frac{9.1558}{9.1686} \right) \times 100\% = 99.86\%,$$

$$\hat{U}_4 = \left(\frac{8.7879}{9.1686} \right) \times 100\% = 95.84\%,$$

$$\hat{U}_5 = \left(\frac{9.1686}{9.1686} \right) \times 100\% = 100\%.$$

Step 7. Based on the utility degree $\hat{U}_i (i = 1, 2, \dots, 5)$, we rank the alternatives in the descending order as $AA > SA > RA > PA > CA$. So, AA is the best choice.

5.4 Analyzing the influence of parameters on ranking results

In this subsection, we examine the influence of parameters on the ranking results of the proposed method. In this approach, n and q are two parameters, different parameter values show different risk preference attitudes of DMs in solving MAGDM problems. The proposed $2TLT - SFPWAG_{\xi, \zeta}^n - COPRAS$ ($2TLT - SFPWGG_{\xi, \zeta}^n - COPRAS$) is a technique that enables DMs to expand their decision evaluation space based on the parameters q and n . The main characteristic of our proposed operator(s) is not only the effective control of the degree of uncertainty of $2TLT - SFS$ but also the efficient and powerful model of practical decision-making problems for additional parameters.

5.4.1 Influence of the parameter q

To show the influence of parameter on ranking results, we utilize different values of parameter q to rank the alternatives by the proposed $2TLT - SFPWAG_{\xi, \zeta}^n - COPRAS$ and $2TLT - SFPWGG_{\xi, \zeta}^n - COPRAS$ methods. It can be seen in Tables 10 and 11 and Figs. 1 and 2, there is difference in the ranking results when parameter changes. The parameter q is very important and has major impact on the decision results. Influence of the parameter q on the relative significance and ranking results is explained by taking the fixed value of parameter $n (n = 1)$.

5.4.2 Influence of the parameter n

In practical MAGDM problems, the value of n can be seen as the DM's attitude parameter toward optimism and pes-

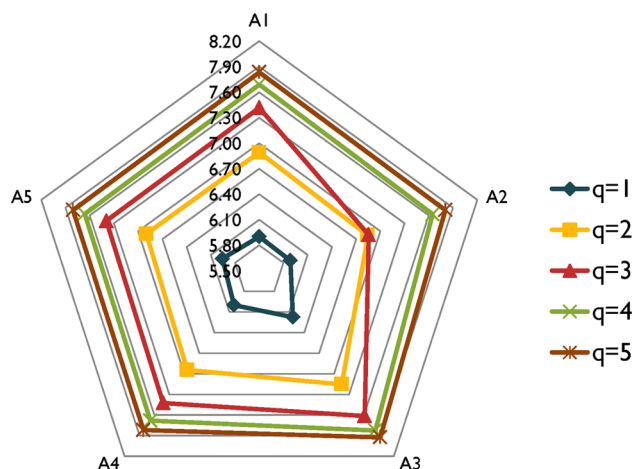


Fig. 1 Ranking results $A_i (i = 1, 2, 3, 4, 5)$ when $n = 1$ and $q = 1, 2, 3, 4, 5$ based on the $2TLT - SFPWAG_{\xi, \zeta}^n - COPRAS$ method

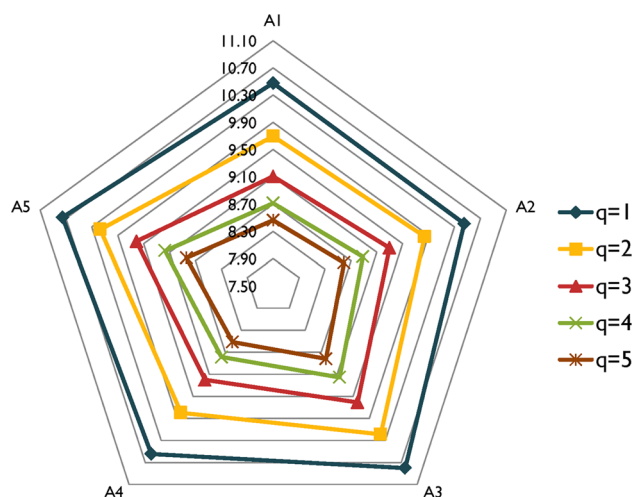


Fig. 2 Ranking results $A_i (i = 1, 2, 3, 4, 5)$ when $n = 1$ and $q = 1, 2, 3, 4, 5$ based on the $2TLT - SFPWGG_{\xi, \zeta}^n - COPRAS$ method

simism. If DMs are cautious about their decisions, then n should be given higher priority, and when decisions are uncertain, lower value of n is taken by DMs. By assigning different values to parameter n , we analyze the influence of the parameter n on relative significance and ranking results by taking fixed value of $q (q = 4)$. We can see in Tables 12 and 13 and Figs. 3 and 4, different ranking results are derived by different values of n in the $2TLT - SFPWAG_{\xi, \zeta}^n - COPRAS$ and $2TLT - SFPWGG_{\xi, \zeta}^n - COPRAS$ methods, which also demonstrates the elasticity of our proposed methods.

5.5 Validity analysis of the developed method

For further illustration of the effectiveness and advantages of the proposed method in this paper, we compare it with the other methods. Detailed evaluation results gained by using different MAGDM methods are given in Table 14, in

Table 12 Relative significance and ranking results according to the parameter n by $2TLT - SFPWAG_{\xi, \zeta}^n$ -COPRAS method

Parameter	Q_1	Q_2	Q_3	Q_4	Q_5	Ranking
$n = 1$	7.6913	7.6496	7.8274	7.6813	7.6605	$SA > CA > PA > AA > RA$
$n = 2$	7.7618	7.7010	7.8385	7.7319	7.7119	$SA > CA > PA > AA > RA$
$n = 3$	7.8237	7.7664	7.8710	7.7916	7.7778	$SA > CA > PA > AA > RA$
$n = 4$	7.8722	7.8247	7.9035	7.8437	7.8358	$SA > CA > PA > AA > RA$
$n = 5$	7.9083	7.8710	7.9301	7.8849	7.8812	$SA > CA > PA > AA > RA$

Table 13 Relative significance and ranking results according to the parameter n by $2TLT - SFPWGG_{\xi, \zeta}^n$ -COPRAS method

Parameter	Q_1	Q_2	Q_3	Q_4	Q_5	Ranking
$n = 1$	8.7140	8.8862	9.1558	8.7879	9.1686	$AA > SA > RA > PA > CA$
$n = 2$	8.4608	8.5667	8.7440	8.5091	8.7467	$AA > SA > RA > PA > CA$
$n = 3$	8.2978	8.3618	8.4781	8.3284	8.4765	$SA > AA > RA > PA > CA$
$n = 4$	8.1924	8.2307	8.3071	8.2116	8.3041	$SA > AA > PA > AA > CA$
$n = 5$	8.1245	8.1470	8.1972	8.1363	8.1939	$SA > AA > RA > PA > CA$

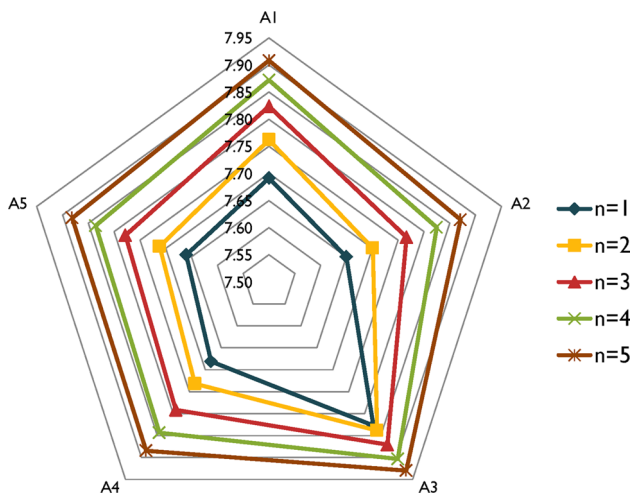


Fig. 3 Ranking results $A_i (i = 1, 2, 3, 4, 5)$ when $q = 4$ and $n = 1, 2, 3, 4, 5$ based on the $2TLT - SFPWAG_{\xi, \zeta}^n$ -COPRAS method

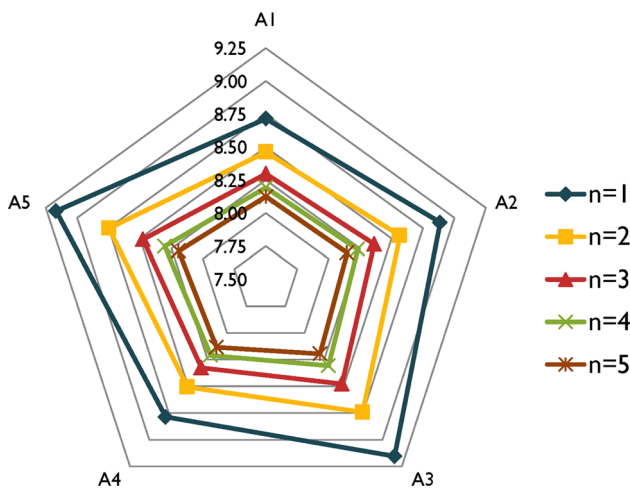


Fig. 4 Ranking results $A_i (i = 1, 2, 3, 4, 5)$ when $q = 4$ and $n = 1, 2, 3, 4, 5$ based on the $2TLT - SFPWGG_{\xi, \zeta}^n$ -COPRAS method

which we used linguistic picture fuzzy point weighted average COPRAS (LPFPWA-COPRAS), linguistic picture fuzzy point weighted geometric COPRAS (LPFPWG-COPRAS), linguistic spherical fuzzy point weighted average COPRAS (LSFPWA-COPRAS), linguistic spherical fuzzy point weighted geometric COPRAS (LSFPWG-COPRAS), 2-tuple linguistic T -spherical fuzzy point weighted average COPRAS ($2TLT - SFPWAG_{\xi, \zeta}^n$ -COPRAS), and 2-tuple linguistic T -spherical fuzzy point weighted geometric COPRAS ($2TLT - SFPWGG_{\xi, \zeta}^n$ -COPRAS) methods.

To explain the effectiveness of the proposed method in modeling fuzzy information, we present the characteristics of the proposed operators. We can see that the ranking results of the above methods are slightly different; however, the best alternative is SA or AA . This verifies that the developed $2TLT - SFPWAG_{\xi, \zeta}^n$ -COPRAS and $2TLT - SFPWGG_{\xi, \zeta}^n$ -COPRAS methods are reasonable and useful for MAGDM problems with $2TLT$ -SFNs and provide the powerful information in MAGDM.

- (1) From the aspect of data articulation, it is obvious that the valuable feature of our techniques is that they provide a diverse range of applications since they not only tackle problems that the existing methods can but also handle a few more circumstances that they are incapable to handle. The method proposed in this work is a very efficient tool for solving aggregated decision-making problems in the dynamic decision-making today's environment.
- (2) Some commonly utilized weighted aggregation operators, including $2TLT$ -SFWA and $2TLT$ -SFWG, are special examples of the proposed operators when $n = 0$. As a result, compared to all of those presented in the latest research, our approaches are more comprehensive and versatile. Furthermore, based on the parameters, the proposed operators can include almost all of the geomet-

Table 14 Comparison of decision results by utilizing different methods

Methods	Parameters	Relative significance of alternatives	Ranking
LPPWA-COPRAS	$q = 1, n = 1$	5.9012 5.8853 6.1764 6.0028 5.9572	$S_A \succ P_A \succ AA \succ CA \succ RA$
LPPWG-COPRAS	$q = 1, n = 1$	10.4765 10.4493 10.8019 10.5482 10.7530	$S_A \succ AA \succ P_A \succ CA \succ RA$
LSPPWA-COPRAS	$q = 2, n = 1$	6.8913 6.8459 7.1538 6.9435 6.8983	$S_A \succ P_A \succ AA \succ CA \succ RA$
LSPPWG-COPRAS	$q = 2, n = 1$	9.6963 9.8394 10.1901 9.8014 10.1709	$S_A \succ AA \succ RA \succ P_A \succ CA$
2TLT – SFPWAG $_{\xi, \zeta}^n$ -COPRAS	$q = 4, n = 1$	7.6913 7.6496 7.8274 7.6813 7.6605	$S_A \succ CA \succ P_A \succ AA \succ RA$
2TLT – SFPWGG $_{\xi, \zeta}^n$ -COPRAS	$q = 4, n = 1$	8.7140 8.8862 9.1558 8.7879 9.1686	$AA \succ SA \succ RA \succ P_A \succ CA$

ric and arithmetic aggregation operators for 2TLIFNs, 2TLPyFNs, and 2TLq-ROFNs. As a result, the proposed method is more efficient and general.

- (3) To reflect either positive or negative perspectives, DMs can employ the appropriate point weighted aggregation operators from the perspective of actual needs. In addition to the other operators, our techniques also take into consideration other aspects that indicate the DM’s interests, enabling them to choose different aspects in accordance with their risk preferences.

5.6 Advantages of the proposed work

Different aggregation operators have different functions, and the DMs can choose suitable aggregation operators according to the real decision-making environment. Our extended method therefore has some benefits and superiorities when compared to other methods. The merits of our developed approach are summarized as follows:

- Proposed methods are preferable to others because they effectively handle the redistribution of the multi-input arguments. Moreover, they have monotonicity with respect to the parameter n and can effect the risk attitude of the DMs. Thus, we can conclude that the proposed methods are significantly strong and have more comprehensive applications. The 2TLT-SFS can tackle practical problems both quantitatively and qualitatively. Therefore, proposed approach is clear and has less loss of data.
- 2TLT – SFPWAG $_{\xi, \zeta}^n$ -COPRAS and 2TLT – SFPWGG $_{\xi, \zeta}^n$ -COPRAS methods can provide more versatile and robust information fusion and make it more feasible to tackle MAGDM problems. From the point view of aggregation operators, we note that the existing methods are only based on the original information, and thus cannot control the uncertainty degree, while our method can redistribute the MD, AD and NMD in 2TLT-SFNs according to different principles and thus can get more intensive and objective information from the original 2TLT-SFS.
- Proposed methods consider both the benefit attributes and the cost attributes which provide more precise information compared with simply handling benefit attributes or cost attributes. Also, it increases the success rate of given data and the precision of decision results as well.

6 Conclusions

In this article, we covered some commonly used data mining tasks as well as some applications of these tasks toward e-commerce utilizing the 2TLT-SFS which increases the range for assigning MD, AD and NMD. The 2-tuple linguistic term can best describe the information given by the

DMs. Some novel 2TLT-SF point weighted aggregation operators, such as 2TLT-SFPWA operator, 2TLT-SFPWG operator, 2TLT-SFGPWA operator and 2TLT-SFGPWG operator, have been put forward for aggregating 2TLT-SFNs. As COPRAS method interprets the ratio to the worst and the best results, therefore, an extended 2TLT-SF point weighted COPRAS method for MAGDM under the 2TLT-SF environment was proposed. Subsequently, a novel approach to MAGDM based on 2TLT – SFPWAG $_{\xi, \zeta}^n$ -COPRAS method and 2TLT – SFPWGG $_{\xi, \zeta}^n$ -COPRAS method has been presented under 2TLT-SF circumstances. Further, a numerical instance related to the data mining tasks toward e-commerce has been presented to demonstrate the validity of the set forward concepts in MAGDM. Comparison analysis has been conducted, and the superiorities have been illustrated. As we have observed, q and n parameters of the aggregation operator also influence the rankings of the alternative. In addition to accommodating 2TLT-SFNs, the 2TLT-SFPWA and 2TLT-SFPWG operators have been distinguished from other operators by including the redistribution phenomenon among their attributes. In future, by utilizing point operators along with different applications of data mining such as artificial intelligence, science and engineering, automation, dynamic pricing, transportation, loan payment prediction, targeted marketing, financial crimes detection and many more will be extended to (1) 2-tuple linguistic complex interval-valued T -spherical fuzzy sets; (2) 2-tuple linguistic complex simplified interval-valued T -spherical fuzzy sets; and (3) 2-tuple linguistic complex hesitant T -spherical fuzzy sets. Although e-commerce is an ideal application, exploratory research is still required to enhance the marketing strategies of data mining, therefore, it is expected to be as famous and trendy as other relevant technologies in future study of decision-making.

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Declarations

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References

- Akram M, Khan A, Borumand Saeid A (2020a) Complex pythagorean Dombi fuzzy operators using aggregation operators and their decision-making. *Expert Syst* 38(2):e12626
- Akram M, Alsulami S, Khan A, Karaaslan F (2020b) Multi-criteria group decision-making using spherical fuzzy prioritized weighted aggregation operators. *Int J Comput Intell Syst* 13(1):1429–1446
- Akram M, Naz S, Edalatpanah SA, Mehreen R (2021a) Group decision-making framework under linguistic q -rung orthopair fuzzy Einstein models. *Soft Comput* 25(15):10309–10334
- Akram M, Sattar A, Peng X (2021b) A new decision making model using complex intuitionistic fuzzy Hamacher aggregation operators. *Soft Comput* 25(10):7059–7086
- Akram M, Kahraman C, Zahid K (2021c) Group decision-making based on complex spherical fuzzy VIKOR approach. *Knowl Based Syst* 216:106793
- Akram M, Naz S, Feng F, Shafiq A (2022) Assessment of hydropower plants in Pakistan: Muirhead mean-based 2-tuple linguistic T -spherical fuzzy model combining SWARA with COPRAS. *Arab J Sci Eng*. <https://doi.org/10.1007/s13369-022-07081-0>
- Akram M, Kahraman C, Zahid K (2021) Extension of TOPSIS model to the decision making under complex spherical fuzzy information. *Soft Comput* 25(16):10771–10795
- Ashraf S, Abdullah S (2019) Spherical aggregation operators and their application in multiattribute group decision-making. *Int J Intell Syst* 34(3):493–523
- Ashraf S, Abdullah S, Aslam M, Qiyas M, Kutbi MA (2019) Spherical fuzzy sets and its representation of spherical fuzzy t -norms and t -conorms. *J Intell Fuzzy Syst* 36(6):6089–6102
- Ashraf S, Abdullah S, Mahmood T (2020) Spherical fuzzy Dombi aggregation operators and their application in group decision making problems. *J Ambient Intell Humaniz Comput* 11(7):2731–2749
- Atanassov KT (1986) Intuitionistic fuzzy sets. *Fuzzy Sets Syst* 20(1):87–96
- Buyukozkan G, Gocer F (2019) A novel approach integrating AHP and COPRAS under Pythagorean fuzzy sets for digital supply chain partner selection. *IEEE Trans Eng Manage* 68(5):1486–1503
- Cuong BC, Kreinovich V (2014) Picture fuzzy sets. *J Comput Sci Cybern* 30(4):409–420
- Darko AP, Liang D (2020) Some q -rung orthopair fuzzy Hamacher aggregation operators and their application to multiple attribute group decision making with modified EDAS method. *Eng Appl Artif Intell* 87:103259
- Deng X, Wang J, Wei G (2019) Some 2-tuple linguistic Pythagorean Heronian mean operators and their application to multiple attribute decision-making. *J Exp Theor Artif Intell* 31(4):555–574
- Dorfeshan Y, Mousavi SM (2019) A group TOPSIS-COPRAS methodology with Pythagorean fuzzy sets considering weights of experts for project critical path problem. *J Intell Fuzzy Syst* 36(2):1375–1387
- Gündođdu FK, Kahraman C (2019) Spherical fuzzy sets and spherical fuzzy TOPSIS method. *J Intell Fuzzy Syst* 36(1):337–352
- Herrera F, Martinez L (2000a) A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Trans Fuzzy Syst* 8(6):746–752
- Herrera F, Martinez L (2000b) An approach for combining linguistic and numerical information based on the 2-tuple fuzzy linguistic representation model in decision-making. *Int J Uncertain Fuzz Knowl Based Syst* 8(05):539–562
- Ju Y, Wang A, Ma J, Gao H, Santibanez Gonzalez ED (2020) Some q -rung orthopair fuzzy 2-tuple linguistic Muirhead mean aggregation operators and their applications to multiple-attribute group decision making. *Int J Intell Syst* 35(1):184–213

- Kahraman C, Gündođdu FK, (2018) From 1D to 3D membership: spherical fuzzy sets, BOS, SOR. Polish Operational and Systems Research Society, September 24th–26th 2018. Palais Staszic, Warsaw, Poland
- Kahraman C, Gündođdu FK, Onar SC, Oztaysi B (2019) Hospital location selection using spherical fuzzy TOPSIS. In: In 11th Conference of the European society for fuzzy logic and technology (EUSFLAT 2019), Atlantis Press
- Liu P, Zhu B, Wang P, Shen M (2020) An approach based on linguistic spherical fuzzy sets for public evaluation of shared bicycles in China. *Eng Appl Artif Intell* 87:103295
- Liu P, Naz S, Akram M, Muzammal M (2022) Group decision-making analysis based on linguistic q -rung orthopair fuzzy generalized point weighted aggregation operators. *Int J Mach Learn Cybern* 13:883–906
- Mahmood T, Ullah K, Khan Q, Jan N (2019) An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Comput Appl* 31(11):7041–7053
- Mishra AR, Rani P, Pardasani KR (2019) Multiple-criteria decision-making for service quality selection based on Shapley COPRAS method under hesitant fuzzy sets. *Granul Comput* 4(3):435–449
- Mo J, Huang HL (2020) Archimedean geometric Heronian mean aggregation operators based on dual hesitant fuzzy set and their application to multiple attribute decision making. *Soft Comput* 24(19):14721–14733
- Naz S, Akram M, Muhiuddin G, Shafiq A (2022a) Modified EDAS method for MAGDM based on MSM operators with 2-tuple linguistic-spherical fuzzy sets. *Math Probl Eng* 2022:1–34
- Naz S, Akram M, Saeid AB, Saadat A (2022b) Models for MAGDM with dual hesitant q -rung orthopair fuzzy 2-tuple linguistic MSM operators and their application to COVID-19 pandemic. *Expert Syst* 39(8):e13005
- Naz S, Akram M, Al-Shamiri MA, Khalaf MM, Yousaf G (2022c) A new MAGDM method with 2-tuple linguistic bipolar fuzzy Heronian mean operators. *Math Biosci Eng* 19:3843–3878
- Rong Y, Liu Y, Pei Z (2020a) Complex q -rung orthopair fuzzy 2-tuple linguistic Maclaurin symmetric mean operators and its application to emergency program selection. *Int J Intell Syst* 35(11):1749–1790
- Rong Y, Pei Z, Liu Y (2020b) Hesitant fuzzy linguistic Hamy mean aggregation operators and their application to linguistic multiple attribute decision-making. *Math Probl Eng* 2020:1–22
- Ullah K, Mahmood T, Garg H (2020) Evaluation of the performance of search and rescue robots using T -spherical fuzzy Hamacher aggregation operators. *Int J Fuzzy Syst* 22(2):570–582
- Wang L, Garg H, Li N (2019a) Interval-valued q -rung orthopair 2-tuple linguistic aggregation operators and their applications to decision making process. *IEEE Access* 7:131962–131977
- Wang J, Zhang R, Zhu X, Zhou Z, Shang X, Li W (2019b) Some q -rung orthopair fuzzy Muirhead means with their application to multi-attribute group decision making. *J Intell Fuzzy Syst* 36(2):1599–1614
- Xing Y, Zhang R, Zhou Z, Wang J (2019) Some q -rung orthopair fuzzy point weighted aggregation operators for multi-attribute decision making. *Soft Comput* 23(22):11627–11649
- Xu ZS, Yager RR (2006) Some geometric aggregation operators based on intuitionistic fuzzy sets. *Int J Intell Syst* 35:417–433
- Yager RR (1988) On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Trans Syst Man Cybern* 18:183–190
- Yager RR (2014) Pythagorean membership grades in multi-criteria decision-making. *IEEE Trans Fuzzy Syst* 22:958–965
- Yager RR (2016) Generalized orthopair fuzzy sets. *IEEE Trans Fuzzy Syst* 26(5):1222–1230
- Zadeh LA (1965) Fuzzy sets. *Inf Control* 8(3):338–353
- Zahid K, Akram M, Kahraman C (2022) A new ELECTRE-based method for group decision-making with complex spherical fuzzy information. *Knowl Based Syst* 243:108525
- Zavadskas EK, Kaklauskas A, Sarka V (1994) The new method of multicriteria complex proportional assessment of projects. *Technol Econ Dev Econ* 1(3):131–139
- Zeng S, Munir M, Mahmood T, Naeem M (2020) Some T -Spherical fuzzy Einstein interactive aggregation operators and their application to selection of photovoltaic cells. *Math Probl Eng* 2020:1–16
- Zheng Y, Xu Z, He Y, Liao H (2018) Severity assessment of chronic obstructive pulmonary disease based on hesitant fuzzy linguistic COPRAS method. *Appl Soft Comput* 69:60–71

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