



Approximation operators and accuracy measures of rough sets from an infra-topology view

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Abstract

In this work, our target point is to focus on rough approximation operators generated from infra-topology spaces and examine their features. First, we show how infra-topology spaces are constructed from N_j -neighborhood systems under an arbitrary relation. Then, we exploit these infra-topology spaces to form new rough set models and scrutinize their master characterizations. The main advantages of these models are to preserve all properties of Pawlak approximation operators and produce accuracy values higher than those given in several methods published in the literature. One of the unique characterizations of the current approach is that all the approximation operators and accuracy measures produced by the current approach are identical under a symmetric relation. Finally, we present two medical applications of the current methods regarding Dengue fever and COVID-19 pandemic. Some debates regarding the pros and cons of the followed technique are given as well as some upcoming work are proposed.

Keywords N_j -neighborhood · Infra-topology · Infra-upper and infra-lower approximations · Accuracy and roughness measures · Dengue fever · COVID-19 pandemic

1 Introduction

To deal with the imperfect knowledge problems, which is a crucial issue for computer scientists, there are several techniques to manipulate them; one of the most important of them is rough set theory (Pawlak 1982). Its methodology relies on the categorizations of objects using equivalence relations. Approximation operators and accuracy measures are two major principles in this theory; they offer some information about the structure and size of boundary region.

To expand the applications of rough set theory, the strict stipulation of an equivalence relation has been replaced by different types of binary relations. This starts, in 1996, by Yao (1996, 1998); he defined right and left neighborhoods under

an arbitrary relation. Then, a lot of researchers exploited right and left neighborhoods to introduce various kinds of neighborhood systems which are induced from an arbitrary relation, or specific relations such as similarity (Abo-Tabl 2013), tolerance (Skowron and Stepaniuk 1996), quasiorder (Zhang et al. 2009) and dominance (Zhang et al. 2016). These neighborhood systems were aimed to extend the positive region which is considered an equivalent target for reducing the negative region. With the desire to improve the approximation operators and increase accuracy values of rough subsets, it was studied new types of neighborhood systems such as containment neighborhoods (Al-shami 2021a), M_j -neighborhoods (Al-shami 2022), subset neighborhoods (Al-shami and Ciucci 2022), E_j -neighborhoods (Al-shami et al. 2021), right maximal neighborhoods (Dai et al. 2018), core neighborhoods (Mareay 2016) and remote neighborhood (Sun et al. 2019). Abu-Donia (2008) initiated new rough set models with respect to a finite family of arbitrary relations.

The similarity behaviors of some topological and rough set concepts motivated many researchers to discuss rough set theory and its applications from topological viewpoint. Skowron (1988) and citesps41 were the first who explored the interaction among topological and rough set thoughts. Afterward, many interesting contributions to this direction have

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been made. An interesting method of creating a topological space from N_j -neighborhoods under arbitrary relation was provided by Lashin et al. (2005). The idea of this method is based on considering N_j -neighborhoods of each point as a subbase for a topology. The solution of missing attribute values problem was studied by Salama (2010). In Amer et al. (2017), the authors capitalized generalized open sets to define some lower and upper approximations. Järvinen and Radeleczki (2020) introduced the optimistic lower and upper approximations with respect to two equivalence relations. Many studies were contacted to explore and develop rough set notions depending on topological ideas such as Abo-Tabl (2014), Al-shami (2021b, 2022), Hosny (2018), Kondo and Dudek (2006), Salama (2020a) and Singh and Tiwari (2020). Moreover, it was applied rough set concepts induced from some generalizations of topology such as minimal structure (Azzam et al. 2020; El-Sharkasy 2021) and bitopology (Salama 2020b) to cope with some medical problems.

According to the contributions mentioned above, we see that the topological models of rough sets need to be studied. In line with this matter, this article proposes new types of rough set models inspired by an abstract structure called “infra-topology space.” The main motivations to display these models are: first, to relax some conditions of rough set models induced from topology, which make us in a position to dispense with some conditions that limit applications; second, to keep Pawlak properties of approximation operators that are evaporated in some previous approaches induced from topological structures such as Al-shami (2021b) and Amer et al. (2017); and finally, the desire of introducing a new model with better approximation operators and higher accuracy measures than some previous methods such as Abd El-Monsef et al. (2014), Allam et al. (2005, 2006) and Yao (1996, 1998).

The rest of this article is organized as follows. Section 2 contains the main principles and results of topology and rough sets we need to understand this manuscript as well as it shows the developments and motivations to present the new concepts. In Sect. 3, we present a method to generate infra-topology spaces from the different types of N_j -neighborhoods under an arbitrary relation. We capitalize from this method to initiate new rough set models and elaborate their main features in Sect. 4. In Sect. 5, we examine the performance of our method to analyze the data of dengue fever disease. Also, we design an algorithm to show how this method is applied to classify the risk of infection with COVID-19. In Sect. 6, we investigate the advantages of the followed technique, which highlights its importance as well as mention its limitation. Ultimately, in Sect. 7, we outline the main contributions and give some thoughts that can be applied to expand the scope of this manuscript.

2 Preliminaries

Some definitions and findings concerning infra-topology and rough sets that are necessary to understand this manuscript will be displayed in this section. The motivations and historical development of these concepts take into account in the presentation.

2.1 Rough approximations and neighborhood systems

Through this manuscript, the ordered pair (W, R) is called an approximation space, where W is a nonempty finite set and R is an arbitrary relation on W . If R is an equivalence relation, then we call (W, R) a Pawlak approximation space.

The first point of this branch of research begins from the following principle.

Definition 1 (Pawlak 1982) Let (W, R) be a Pawlak approximation space. Each subset A of W is associated with two sets according to the set of equivalences classes W/R as follows.

$$\underline{R}(A) = \cup\{U \in W/R : U \subseteq A\}, \text{ and}$$

$$\overline{R}(A) = \cup\{U \in W/R : U \cap A \neq \emptyset\}$$

The sets $\underline{R}(A)$ and $\overline{R}(A)$ are known as lower and upper approximations of A , respectively.

These approximations have some essential properties listed in the following result. In fact, these properties are considered as an indicator of the similarity of rough systems defined later to the original system of Pawlak.

Proposition 1 (Pawlak 1982) *The following statements describe the behaviors of the subsets $A, B \subseteq W$ and some operations (intersection, union, complement) under the lower and upper approximations generated from the set of equivalences classes W/R .*

(L1) $\underline{R}(A) \subseteq A$	(U1) $A \subseteq \overline{R}(A)$
(L2) $\underline{R}(\emptyset) = \emptyset$	(U2) $\overline{R}(\emptyset) = \emptyset$
(L3) $\underline{R}(W) = W$	(U3) $\overline{R}(W) = W$
(L4) If $A \subseteq B$, then $\underline{R}(A) \subseteq \underline{R}(B)$	(U4) If $A \subseteq B$, then $\overline{R}(A) \subseteq \overline{R}(B)$
(L5) $\underline{R}(A \cap B) = \underline{R}(A) \cap \underline{R}(B)$	(U5) $\overline{R}(A \cap B) \subseteq \overline{R}(A) \cap \overline{R}(B)$
(L6) $\underline{R}(A) \cup \underline{R}(B) \subseteq \underline{R}(A \cup B)$	(U6) $\overline{R}(A \cup B) = \overline{R}(A) \cup \overline{R}(B)$
(L7) $\underline{R}(A^c) = (\overline{R}(A))^c$	(U7) $\overline{R}(A^c) = (\underline{R}(A))^c$
(L8) $\underline{R}(\underline{R}(A)) = \underline{R}(A)$	(U8) $\overline{R}(\overline{R}(A)) = \overline{R}(A)$
(L9) $\underline{R}((\underline{R}(A))^c) = (\overline{R}(A))^c$	(U9) $\overline{R}((\overline{R}(A))^c) = (\underline{R}(A))^c$
(L10) If $A \in W/R$, then $\underline{R}(A) = A$	(U10) If $A \in W/R$, then $\overline{R}(A) = A$

To determine the knowledge obtained from a subset of data and give some insights into its structure, it was proposed

dividing it into three regions using approximation operators.

Definition 2 (Pawlak 1982) The positive, boundary and negative regions of a subset A of a Pawlak approximation space (W, R) are, respectively, defined as follows.

$$\begin{aligned} POS(A) &= \underline{R}(A), \\ B(A) &= \overline{R}(A) \setminus \underline{R}(A), \\ NEG(A) &= W \setminus \overline{R}(A). \end{aligned}$$

To capture the degree of completeness and incompleteness of knowledge obtained from a subset, the following measures were introduced.

Definition 3 (Pawlak 1982) The accuracy and roughness measures of a subset A of a Pawlak approximation space (W, R) are, respectively, given as follows.

$$\begin{aligned} M(A) &= \frac{|\underline{R}(A)|}{|\overline{R}(A)|}, \text{ where } A \neq \emptyset. \\ E(A) &= 1 - M(A). \end{aligned}$$

To overcome some obstacles caused by the equivalence relation, Yao (1996, 1998) replaced the equivalences classes by right and left neighborhoods which were defined as follows.

Definition 4 (Yao 1996, 1998) Let (W, R) be an approximation space. (Herein, R is not necessary an equivalence relation.) The right neighborhood and left neighborhood of $a \in W$, respectively, denoted by $N_r(a)$ and $N_l(a)$, are defined as follows.

$$\begin{aligned} N_r(a) &= \{b \in W : (a, b) \in R\}, \text{ and} \\ N_l(a) &= \{b \in W : (b, a) \in R\}. \end{aligned}$$

According to right and left neighborhoods, Yao defined the right (left) lower approximations and right (left) upper approximations as follows.

Definition 5 (Yao 1996, 1998) Let (W, R) be an approximation space. The j -lower and j -upper approximations of $A \subseteq W$ are given by the following formulations, where $j \in \{r, l\}$.

$$\begin{aligned} \underline{R}_j(A) &= \bigcup \{a \in W : N_k(a) \subseteq A\}, \text{ and} \\ \overline{R}_j(A) &= \bigcup \{a \in W : N_k(a) \cap A \neq \emptyset\}. \end{aligned}$$

Remark 1 Despite the new approximations expand the scope of applications, some properties of Pawlak approximation space, given in Proposition 1, are evaporated such as the lower approximation of empty set is itself and the upper approximation of the universal set is itself.

Afterward, the researchers interested in rough set theory endeavored to increase the accuracy measures and improve the approximations of rough subsets. These efforts produced several types of neighborhood systems listed in the following.

Definition 6 (Abd El-Monsef et al. 2014; Allam et al. 2005, 2006) Let (W, R) be an approximation space. The j -neighborhoods of $a \in W$, denoted by $N_j(a)$, are defined for each $j \in \{\langle r \rangle, \langle l \rangle, i, u, \langle i \rangle, \langle u \rangle\}$ as follows, where $\langle r \rangle, \langle l \rangle, i, u, \langle i \rangle, \langle u \rangle$, respectively, means a minimal right neighborhood, minimal left neighborhood, intersection neighborhood, union neighborhood, minimal intersection neighborhood and minimal union neighborhood.

- (i)
$$N_{\langle r \rangle}(a) = \begin{cases} \bigcap_{a \in N_r(b)} N_r(b) & : \text{ there exists } N_r(b) \text{ including } a \\ \emptyset & : \text{ Otherwise} \end{cases}$$
- (ii)
$$N_{\langle l \rangle}(a) = \begin{cases} \bigcap_{a \in N_l(b)} N_l(b) & : \text{ there exists } N_l(b) \text{ including } a \\ \emptyset & : \text{ Otherwise} \end{cases}$$
- (iii) $N_i(a) = N_r(a) \cap N_l(a).$
- (iv) $N_u(a) = N_r(a) \cup N_l(a).$
- (v) $N_{\langle i \rangle}(a) = N_{\langle r \rangle}(a) \cap N_{\langle l \rangle}(a).$
- (vi) $N_{\langle u \rangle}(a) = N_{\langle r \rangle}(a) \cup N_{\langle l \rangle}(a).$

Following a similar technique for the given in Definition 5, the above kinds of neighborhood systems were applied to formulate new approximations. On the other hand, to remove a shortcoming caused by using Pawlak accuracy measures when R is not equivalent, a slight modification was done to the definition of accuracy measures of rough subsets as illustrated in the following.

Definition 7 (Abd El-Monsef et al. 2014; Allam et al. 2005, 2006; Yao 1996, 1998) The accuracy measure of a subset A of an approximation space (W, R) is defined for each j as follows.

$$M_j(A) = \frac{|\underline{R}_j(A) \cap A|}{|\overline{R}_j(A) \cup A|}, \text{ where } A \neq \emptyset.$$

In the light of this line of research, some authors exploited some operations between j -neighborhood systems to define novel rough neighborhoods such as E_j -neighborhoods, C_j -neighborhoods, S_j -neighborhoods and M_j -neighborhoods. They were applied to handle some diseases and epidemics.

The following result will be helpful to interpret some relations of accuracy and roughness measures.

Proposition 2 (Al-shami and Ciucci 2022) *Let (W, R_1) and (W, R_2) be approximation spaces such that $R_1 \subseteq R_2$. Then $N_{1j}(a) \subseteq N_{2j}(a)$ for every $a \in W$ and $j \in \{l, r, u, i\}$.*

Definition 8 (see, Al-shami 2022) We call the approximations spaces (W, R_1) and (W, R_2) have the property of monotonicity accuracy (resp. monotonicity roughness) if $M_{R_1}(A) \geq M_{R_2}(A)$ (resp., $E_{R_1}(A) \leq E_{R_2}(A)$) whenever $R_1 \subseteq R_2$.

2.2 Topology and their related rough set concepts

We call a subfamily τ of the power set of a nonempty set W a topology on W if it is closed under arbitrary union and finite intersection. According to the famous set-theoretic result reports that the empty intersection (resp. empty union) of a family of subsets is the universal set (resp. empty set), we obtain the empty and universal sets are always members of a topology.

Definition 9 (see, Abd El-Monsef et al. 2014) A subset A of a topological space (W, τ_j) is said to be a j -open (resp. j -closed) provided that it belongs to τ_j (resp., its complement is a member of τ_j). We define the interior points of a set A , denoted by $Int(A)$, as the union of all open subsets of this set, and we define the closure points of a set, denoted by $Cl(A)$, as the intersection of all closed supersets of this set.

Skowron (1988) and Wiweger (1989) noted that the equivalences classes form a base for a specified type of topology (known as a quasidiscrete topology), which means there is a similarity between the behaviors of some topological and rough set concepts, for example, interior topological operator and lower approximation, and closure topological operator and upper approximation. This pioneering work paved to conducting deep investigations concerning rough set concepts from a topological standpoint. Later on, it was made use of N_j -neighborhood systems to establish new sorts of rough approximations inspired by topological structures. One of the followed manners to do that is demonstrated by the next interesting result.

Theorem 1 (Abd El-Monsef et al. 2014) *Let (W, R) be an approximation space. Then, for each j a family $\tau_j = \{U \subseteq W : N_j(a) \subseteq U \text{ for each } a \in U\}$ represents a topology on W .*

Starting from the above result, the approximations were defined topologically as follows.

Definition 10 (Abd El-Monsef et al. 2014) The j -lower and j -upper approximations of a set A induced from a topological space (W, τ_j) are given as follows.

$$\underline{T}_j(A) = \bigcup \{U \in \tau_j : U \subseteq A\}, \text{ and}$$

$$\overline{T}_j(A) = \bigcap \{H : H^c \in \tau_j \text{ and } A \subseteq H\}.$$

It can be seen that $\underline{T}_j(A)$ and $\overline{T}_j(A)$ represent the interior and closure points of A , respectively. As a special case, if R is an equivalence relation, then $\underline{T}_j(A)$ and $\overline{T}_j(A)$ represent the lower and upper approximations in sense of Pawlak.

The regions and measures of subsets were formulated using $\underline{T}_j(A)$ and $\overline{T}_j(A)$ following a similar manner to their counterparts given in Definitions 2 and 3.

Remark 2 It has been benefited from some topological ideas to develop the approximations and measures of subsets. For example, it was applied near-open sets and ideals to decrease the size of the boundary region and increase the value of the accuracy measure of rough subsets, which overall leads to obtaining a more accurate decision. For more details, see Amer et al. (2017), Hosny (2020) and Nawar et al. (2022).

Recently, Al-Odhari (2015) have introduced a novel extension of topology called “infra-topology”; it was defined on a nonempty set W as a subfamily \mathcal{I} of the power set of W satisfying two conditions 1) $\emptyset, W \in \mathcal{I}$, and 2) \mathcal{I} is closed under finite intersection. Then, Witczak (2022) reconstructed the infra-topology concepts and deeply investigated main properties. He also showed the topological characterizations that are invalid through infra-topology structures such as the union of infra-open sets and intersection of infra-closed sets need not be infra-open and infra-closed, respectively.

3 Infra-topology generated by an arbitrary relation

In this section, we build some infra-topologies utilizing an arbitrary relation. We explore the main features of these infra-topologies and reveal the relationships between them. To support the given results, elucidative example is provided.

We begin with the following lemma which sets forth that the class of N_j -neighborhoods is not closed under the intersection operation.

Lemma 1 *If N_j -neighborhood systems are induced from an approximation space (W, R) , then $N_j(A \cap B) \subseteq N_j(A) \cap N_j(B)$ for each $A, B \subseteq W$.*

Proof Let $a \in N_j(A \cap B)$. Then there exists $b \in A \cap B$ such that $a \in N_j(b)$. Accordingly, $N_j(b) \subseteq N_j(A)$ and $N_j(b) \subseteq N_j(B)$. Hence, we obtain $a \in N_j(A) \cap N_j(B)$, as required. \square

In Example 1, consider $A = \{a\}$ and $B = \{b\}$ as subsets of W . For each j except for $\langle l \rangle$ and $\langle i \rangle$, we have $N_j(A \cap B) = N_j(\emptyset) = \emptyset$. But $N_j(A) \cap N_j(B) = \{a, b\}$, which means that the converse of the Lemma 1 is generally false.

The next result presents a method of generating infra-topology structures from N_j -neighborhood systems.

Table 1 N_j -neighborhoods

	a	b	c	d
N_r	$\{a, b, d\}$	$\{a, b, c\}$	$\{c\}$	$\{d\}$
N_l	$\{a, b\}$	$\{a, b\}$	$\{b, c\}$	$\{a, d\}$
N_i	$\{a, b\}$	$\{a, b\}$	$\{c\}$	$\{d\}$
N_u	$\{a, b, d\}$	$\{a, b, c\}$	$\{b, c\}$	$\{a, d\}$
$N_{(r)}$	$\{a, b\}$	$\{a, b\}$	$\{c\}$	$\{d\}$
$N_{(l)}$	$\{a\}$	$\{b\}$	$\{b, c\}$	$\{a, d\}$
$N_{(i)}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{d\}$
$N_{(u)}$	$\{a, b\}$	$\{a, b\}$	$\{b, c\}$	$\{a, d\}$

Proposition 3 Assume (W, R) is an approximation space and let $N_j(a)$ be the j -neighborhood of $a \in W$. Then, the collection \mathcal{I}_j generated by the intersections of the members of the family $\{\emptyset, W, N_j(a) : a \in W\}$ forms a j -infra-topology on W .

Proof Straightforward. \square

Definition 11 We call a triple system (W, R, \mathcal{I}_j) a j -infra-topological space (briefly, $jITS$), where \mathcal{I}_j is a j -infra-topology on W produced by a technique given in the above proposition.

We call a subset of W a j -infra-open set if it is a member of \mathcal{I}_j , and we call a subset of W a j -infra-closed set if its complement is a member of \mathcal{I}_j . The family of all j -infra-closed subsets of W will be denoted by \mathcal{I}_j^c .

The next example demonstrates how to produce j -infra-topologies from an approximation space.

Example 1 Consider (W, R) as an approximation space, where $W = \{a, b, c, d\}$ and $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (a, d)\}$. We first compute the different types of j -neighborhoods of each point in W as given in Table 1.

According to Definition 11, the j -infra-topologies \mathcal{I}_j generated from these neighborhoods are the following.

$$\left\{ \begin{array}{l} \mathcal{I}_r = \{\emptyset, W, \{c\}, \{d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}; \\ \mathcal{I}_l = \{\emptyset, W, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}\}; \\ \mathcal{I}_i = \{\emptyset, W, \{c\}, \{d\}, \{a, b\}\}; \\ \mathcal{I}_u = \{\emptyset, W, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \\ \quad \{a, b, d\}\}; \\ \mathcal{I}_{(r)} = \{\emptyset, W, \{c\}, \{d\}, \{a, b\}\}; \\ \mathcal{I}_{(l)} = \{\emptyset, W, \{a\}, \{b\}, \{a, d\}, \{b, c\}\}; \\ \mathcal{I}_{(i)} = \{\emptyset, W, \{a\}, \{b\}, \{c\}, \{d\}\}; \\ \mathcal{I}_{(u)} = \{\emptyset, W, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}\}. \end{array} \right. \quad (1)$$

Remark 3 Note that all the infra-topology structures given in (1), except for \mathcal{I}_u , are not topological spaces.

Remark 4 It follows from (1) that $\mathcal{I}_i \neq \mathcal{I}_r \cap \mathcal{I}_l$ and $\mathcal{I}_u \neq \mathcal{I}_r \cup \mathcal{I}_l$. Also, $\mathcal{I}_{(i)} \neq \mathcal{I}_{(r)} \cap \mathcal{I}_{(l)}$ and $\mathcal{I}_{(u)} \neq \mathcal{I}_{(r)} \cup \mathcal{I}_{(l)}$.

Proposition 4 Let (W, R, \mathcal{I}_j) be a $jITS$. Then $\mathcal{I}_{(r)} \subseteq \mathcal{I}_r$ and $\mathcal{I}_{(l)} \subseteq \mathcal{I}_l$.

Proof For the case of $\langle r \rangle$, let $F \in \mathcal{I}_{(r)}$. According to Proposition 3, F is given by the intersections of the family $\{\emptyset, W, N_{(r)}(a) : a \in W\}$. It follows from (i) of Definition 6 that each $N_{(r)}$ -neighborhood is empty or the intersection of some N_r -neighborhoods. Therefore, F , according to Proposition 3, belongs to \mathcal{I}_r . Thus, $\mathcal{I}_{(r)} \subseteq \mathcal{I}_r$. The case of $\langle l \rangle$ can be proved in the same way. \square

It follows from the infra-topologies given in (1) that the converse of Proposition 4 fails. Also, Proposition 4 does not hold for the cases of $\langle i \rangle$ and $\langle u \rangle$.

To topologically investigate these structures, we need to put forward the counterparts of interior and closure topological operators.

Definition 12 The infra- j -interior and infra- j -closure points of a subset A of a $jITS (W, R, \mathcal{I}_j)$ are defined, respectively, by

$$\begin{aligned} Int_j(A) &= \bigcup \{G \in \mathcal{I}_j : G \subseteq A\}, \text{ and} \\ Cl_j(A) &= \bigcap \{H \in \mathcal{I}_j^c : A \subseteq H\}. \end{aligned}$$

The next result provides a unique characteristic of infra- j -interior and infra- j -closure operators.

Proposition 5 Let A be a subset of a $jITS (W, R, \mathcal{I}_j)$. Then

- (i) $Int_r(A) = Int_{(r)}(A)$ and $Int_l(A) = Int_{(l)}(A)$.
- (ii) $Cl_r(A) = Cl_{(r)}(A)$ and $Cl_l(A) = Cl_{(l)}(A)$.

Proof (i): It follows from Proposition 4 that $Int_{(r)}(A) \subseteq Int_r(A)$. Conversely, let $a \in Int_r(A)$. Then there exists a member G of \mathcal{I}_r such that $a \in G \subseteq A$. This implies that there exist some N_r -neighborhoods contain a ; say, $N_r(b_1), N_r(b_2), \dots, N_r(b_n)$. Now, $a \in N_{(r)}(a) = \bigcap_{k=1}^n N_r(b_k)$. Obviously, $N_{(r)}(a) \subseteq G$. Therefore, $a \in Int_{(r)}(A)$. Thus, $Int_r(A) \subseteq Int_{(r)}(A)$. Hence, we obtain $Int_r(A) = Int_{(r)}(A)$, as required. The other cases can be proved in the same way. \square

The following two results show under what condition some infra- j -interior (infra- j -closure) operators are identical.

Proposition 6 Let (W, R, \mathcal{I}_j) be a $jITS$ such that R is symmetric. Then

- (i) $\mathcal{I}_r = \mathcal{I}_l = \mathcal{I}_i = \mathcal{I}_u$.
- (ii) $\mathcal{I}_{(r)} = \mathcal{I}_{(l)} = \mathcal{I}_{(i)} = \mathcal{I}_{(u)}$.

Proof Follows from the fact that $N_r(a) = N_l(a)$ and $N_{(r)}(a) = N_{(l)}(a)$ under a symmetric relation. \square

Corollary 1 Let A be a subset of a $jITS$ (W, R, \mathcal{I}_j) . If R is symmetric, then

- (i) $Int_u(A) = Int_r(A) = Int_l(A) = Int_i(A) = Int_{(u)}(A) = Int_{(r)}(A) = Int_{(l)}(A) = Int_{(i)}(A)$.
- (ii) $Cl_u(A) = Cl_r(A) = Cl_l(A) = Cl_i(A) = Cl_{(u)}(A) = Cl_{(r)}(A) = Cl_{(l)}(A) = Cl_{(i)}(A)$.

Proof (i): By Proposition 6, we find that $Int_u(A) = Int_r(A) = Int_l(A) = Int_i(A)$ and $Int_{(u)}(A) = Int_{(r)}(A) = Int_{(l)}(A) = Int_{(i)}(A)$. It follows from Proposition 5 that $Int_r(A) = Int_{(r)}(A)$. Hence, we obtain the desired result. Following similar argument, one can prove (ii). \square

Proposition 7 Let (W, R, \mathcal{I}_j) be a $jITS$ such that R is quasiorder (reflexive and transitive). Then

- (i) $\mathcal{I}_r = \mathcal{I}_{(r)}$.
- (ii) $\mathcal{I}_l = \mathcal{I}_{(l)}$.

Proof (i): We need to prove that $N_r(a) = N_{(r)}(a)$ for each $a \in W$. Obviously, $N_{(r)}(a) \subseteq N_r(a)$ because R is reflexive. Conversely, let $b \in N_r(a)$, i.e., $(a, b) \in R$. Assume that there exists $x \in W$ such that $a \in N_r(x)$, i.e., $(x, a) \in R$. By transitivity of R , we find $b \in N_r(x)$. This implies that $b \in N_{(r)}(a)$. Therefore, $N_r(a) \subseteq N_{(r)}(a)$. Hence, we obtain the desired result. Following similar arguments, (ii) is proved. \square

Corollary 2 Let (W, R, \mathcal{I}_j) be a $jITS$ such that R is quasiorder. Then

- (i) $\mathcal{I}_i = \mathcal{I}_{(i)}$.
- (ii) $\mathcal{I}_u = \mathcal{I}_{(u)}$.

Corollary 3 Let A be a subset of a $jITS$ (W, R, \mathcal{I}_j) such that R is quasiorder. Then

- (i) $Int_j(A) = Int_{(j)}(A)$.
- (iv) $Cl_j(A) = Cl_{(j)}(A)$.

Example 1 confirms that the condition of symmetry in Proposition 6 and Corollary 1 is necessary and the conditions of reflexivity and transitivity in Proposition 7, Corollary 2 and Corollary 3 are indispensable.

4 New rough models generated by infra-topology

In this section, we will establish novel rough models depending on j -infra-topologies induced from N_j -neighborhood

systems. We investigate their main properties and give an algorithm to illustrate how the infra-accuracy values are calculated. To show the importance of these models, we elucidate that they improve the approximations and produce accuracy better than some previous ones such that those introduced in Abd El-Monsef et al. (2014), Allam et al. (2005, 2006) and Yao (1996, 1998) if the relation is reflexive.

4.1 j -lower and j -upper approximations

Definition 13 We define infra- j -lower approximation $\underline{\mathcal{H}}_j$ and infra- j -upper approximation $\overline{\mathcal{H}}_j$ of a subset A of a $jITS$ (W, R, \mathcal{I}_j) as follows.

$$\underline{\mathcal{H}}_j(A) = \bigcup \{G \in \mathcal{I}_j : G \subseteq A\}, \text{ and}$$

$$\overline{\mathcal{H}}_j(A) = \bigcap \{H \in \mathcal{I}_j^c : A \subseteq H\}.$$

It is clear that $\underline{\mathcal{H}}_j(A) = Int_j(A)$ and $\overline{\mathcal{H}}_j(A) = Cl_j(A)$. Accordingly, we obtain $a \in \overline{\mathcal{H}}_j(A)$ iff $G \cap A \neq \emptyset$ for each $G \in \mathcal{I}_j$ containing a .

Foremost, we prove the first advantage of the current approximations which is to preserve all properties of Pawlak approximations.

Proposition 8 Let A and B be subsets of a $jITS$ (W, R, \mathcal{I}_j) . Then the next properties are satisfied.

- (i) $\underline{\mathcal{H}}_j(A) \subseteq A$.
- (ii) $\underline{\mathcal{H}}_j(\emptyset) = \emptyset$.
- (iii) $\underline{\mathcal{H}}_j(W) = W$.
- (iv) If $A \subseteq B$, then $\underline{\mathcal{H}}_j(A) \subseteq \underline{\mathcal{H}}_j(B)$.
- (v) $\underline{\mathcal{H}}_j(A \cap B) = \underline{\mathcal{H}}_j(A) \cap \underline{\mathcal{H}}_j(B)$.
- (vi) $\underline{\mathcal{H}}_j(A) \cup \underline{\mathcal{H}}_j(B) \subseteq \underline{\mathcal{H}}_j(A \cup B)$.
- (vii) $\underline{\mathcal{H}}_j(A^c) = (\overline{\mathcal{H}}_j(A))^c$.
- (viii) $\underline{\mathcal{H}}_j(\underline{\mathcal{H}}_j(A)) = \underline{\mathcal{H}}_j(A)$.

Proof The proofs of (i) and (ii) come from Definition 13.

The proof of (iii) comes from the fact that W is the largest infra-open subset of a $jITS$ (W, R, \mathcal{I}_j) .

(iv): Let $A \subseteq B$. Then $\bigcup \{G \in \mathcal{I}_j : G \subseteq A\} \subseteq \bigcup \{G \in \mathcal{I}_j : G \subseteq B\}$ and so $\underline{\mathcal{H}}_j(A) \subseteq \underline{\mathcal{H}}_j(B)$.

(v): It follows from (iv) that $\underline{\mathcal{H}}_j(A \cap B) \subseteq \underline{\mathcal{H}}_j(A) \cap \underline{\mathcal{H}}_j(B)$. Conversely, let $a \in \underline{\mathcal{H}}_j(A) \cap \underline{\mathcal{H}}_j(B)$. Then there exist infra-open sets F and G such that $a \in F \subseteq A$ and $a \in G \subseteq B$. Since \mathcal{I}_j is an infra-topology for each j , $F \cap G$ is an infra-open set in \mathcal{I}_j such that $a \in F \cap G \subseteq A \cap B$, which means that $a \in \underline{\mathcal{H}}_j(A \cap B)$. Hence, we obtain the required equality.

(vi): Directly follows from (iv).

(vii): Let $a \in \underline{\mathcal{H}}_j(A^c)$. Then there exists an infra-open set G such that $a \in G \subseteq A^c$. Therefore, $G \cap A = \emptyset$, which means that $a \notin \overline{\mathcal{H}}_j(A)$. Thus, $a \in (\overline{\mathcal{H}}_j(A))^c$. Conversely, let

$a \in (\overline{\mathcal{H}}_j(A))^c$. Then $a \notin \overline{\mathcal{H}}_j(A)$, which means there exists an infra-open set G such that $a \in G$ and $G \cap A = \emptyset$. So $a \in G \subseteq A^c$. Hence $a \in \underline{\mathcal{H}}_j(A^c)$.

(viii): From (i) we obtain $\underline{\mathcal{H}}_j(\underline{\mathcal{H}}_j(A)) \subseteq \underline{\mathcal{H}}_j(A)$. On the other hand, let $a \in \underline{\mathcal{H}}_j(A)$. Then there exists an infra-open set G such that $a \in G \subseteq A$. It follows from (iv) that $\underline{\mathcal{H}}_j(G) \subseteq \underline{\mathcal{H}}_j(A)$. According to Definition 13 we have $G = \underline{\mathcal{H}}_j(G)$, so $a \in \underline{\mathcal{H}}_j(G) \subseteq \underline{\mathcal{H}}_j(\underline{\mathcal{H}}_j(A))$. Thus, $\underline{\mathcal{H}}_j(A) \subseteq \underline{\mathcal{H}}_j(\underline{\mathcal{H}}_j(A))$. Hence, the proof is complete. \square

In an *lITS* (W, R, \mathcal{I}_l) given in equation (1) consider $A = \{a, c\}$ and $B = \{d\}$. Then $\underline{\mathcal{H}}_l(A) = \{a\} \subset A$. Also, $\underline{\mathcal{H}}_l(B) = \emptyset \subset \underline{\mathcal{H}}_l(A)$ whereas $A \not\subseteq B$. Moreover, $\underline{\mathcal{H}}_l(A) \cup \underline{\mathcal{H}}_l(B) = \{a\} \subset \underline{\mathcal{H}}_l(A \cup B) = \{a, d\}$. These computations confirm that the inclusion relations of (i),(iv) and (vi) of Proposition 8 are proper.

Proposition 9 *Let A and B be subsets of a $jITS$ (W, R, \mathcal{I}_j) . Then the next properties are satisfied.*

- (i) $A \subseteq \overline{\mathcal{H}}_j(A)$.
- (ii) $\overline{\mathcal{H}}_j(\emptyset) = \emptyset$.
- (iii) $\overline{\mathcal{H}}_j(W) = W$.
- (iv) If $A \subseteq B$, then $\overline{\mathcal{H}}_j(A) \subseteq \overline{\mathcal{H}}_j(B)$.
- (v) $\overline{\mathcal{H}}_j(A \cap B) \subseteq \overline{\mathcal{H}}_j(A) \cap \overline{\mathcal{H}}_j(B)$.
- (vi) $\overline{\mathcal{H}}_j(A) \cup \overline{\mathcal{H}}_j(B) = \overline{\mathcal{H}}_j(A \cup B)$.
- (vii) $\overline{\mathcal{H}}_j(A^c) = (\underline{\mathcal{H}}_j(A))^c$.
- (viii) $\overline{\mathcal{H}}_j(\overline{\mathcal{H}}_j(A)) = \overline{\mathcal{H}}_j(A)$.

Proof Similar to the proof of Proposition 8. \square

In an *rITS* (W, R, \mathcal{I}_r) given in equation (1), consider $A = \{b, c, d\}$ and $B = \{a, c\}$. Then $A \subset \overline{\mathcal{H}}_r(A) = W$. Also, $\overline{\mathcal{H}}_r(B) = \{a, b, c\} \subset \overline{\mathcal{H}}_r(A)$, whereas $B \not\subseteq A$. Moreover, $\overline{\mathcal{H}}_r(A \cap B) = \{c\} \subset \overline{\mathcal{H}}_r(A) \cap \overline{\mathcal{H}}_r(B) = \{a, b, c\}$. These computations confirm that the inclusion relations of (i) and (iv–v) of Proposition 9 are proper.

Definition 14 The infra- j -accuracy and infra- j -roughness measures of a set A in a $jITS$ (W, R, \mathcal{I}_j) are defined, respectively, by

$$\mathcal{O}_j(A) = \frac{|\underline{\mathcal{H}}_j(A)|}{|\overline{\mathcal{H}}_j(A)|}, \text{ where } A \neq \emptyset.$$

$$E_j(A) = 1 - \mathcal{O}_j(A).$$

Obviously, $\mathcal{O}_j(A), E_j(A) \in [0, 1]$ for every subset A of W .

Definition 15 We call the $jITS$ s $(W, R_1, \mathcal{I}_{1j})$ and $(W, R_2, \mathcal{I}_{2j})$ have the property of monotonicity accuracy (resp. monotonicity roughness) if $\mathcal{O}_{j1}(A) \geq \mathcal{O}_{j2}(A)$ (resp., $E_{j1}(A) \leq E_{j2}(A)$) whenever $\mathcal{I}_{2j} \subseteq \mathcal{I}_{1j}$.

In what follows, we show that the infra- j -accuracy and infra- j -roughness measures satisfying the monotonicity property.

Proposition 10 *Let $(W, R_1, \mathcal{I}_{1j})$ and $(W, R_2, \mathcal{I}_{2j})$ be two $jITS$ s such that $\mathcal{I}_{1j} \subseteq \mathcal{I}_{2j}$. Then $\mathcal{O}_{2j}(A) \geq \mathcal{O}_{1j}(A)$ for every subset A .*

Proof As we show that $\underline{\mathcal{H}}_j(A) = Int_j(A)$ and $\overline{\mathcal{H}}_j(A) = Cl_j(A)$, since $\mathcal{I}_{1j} \subseteq \mathcal{I}_{2j}$, it follows that $|\underline{\mathcal{H}}_{1j}(A)| \leq |\underline{\mathcal{H}}_{2j}(A)|$ and $\frac{1}{|\overline{\mathcal{H}}_{1j}(A)|} \leq \frac{1}{|\overline{\mathcal{H}}_{2j}(A)|}$. Thus, we obtain $\frac{|\underline{\mathcal{H}}_{1j}(A)|}{|\overline{\mathcal{H}}_{1j}(A)|} \leq \frac{|\underline{\mathcal{H}}_{2j}(A)|}{|\overline{\mathcal{H}}_{2j}(A)|}$. Hence, $\mathcal{O}_{1j}(A) \leq \mathcal{O}_{2j}(A)$, as required. \square

Corollary 4 *Let $(W, R_1, \mathcal{I}_{1j})$ and $(W, R_2, \mathcal{I}_{2j})$ be $jITS$ s such that $\mathcal{I}_{1j} \subseteq \mathcal{I}_{2j}$. Then $E_{1j}(A) \geq E_{2j}(A)$ for every subset A .*

Definition 16 The infra- j -positive, infra- j -boundary and infra- j -negative regions of a set A in a $jITS$ (W, R, \mathcal{I}_j) are defined, respectively, by

$$POS_j(A) = \underline{\mathcal{H}}_j(A),$$

$$B_j(A) = \overline{\mathcal{H}}_j(A) \setminus \underline{\mathcal{H}}_j(A), \text{ and}$$

$$NEG_j(A) = W \setminus \overline{\mathcal{H}}_j(A).$$

Proposition 11 *Let $(W, R_1, \mathcal{I}_{1j})$ and $(W, R_2, \mathcal{I}_{2j})$ be two ITS s such that $\mathcal{I}_{1j} \subseteq \mathcal{I}_{2j}$. Then for every subset A , the following results hold true.*

- (i) $B_{2j}(A) \subseteq B_{1j}(A)$.
- (ii) $NEG_{1j}(A) \subseteq NEG_{2j}(A)$.

Proof Follows from Proposition 10. \square

Definition 17 A subset A of a $jITS$ (W, R, \mathcal{I}_j) is called infra- j -exact if $\underline{\mathcal{H}}_j(A) = \overline{\mathcal{H}}_j(A) = A$. Otherwise, it is called an infra- j -rough set.

Proposition 12 *A subset A of a $jITS$ (W, R, \mathcal{I}_j) is infra- j -exact iff $B_j(A) = \emptyset$.*

Proof Assume that A is an infra- j -exact set. Then $B_j(A) = \overline{\mathcal{H}}_j(A) \setminus \underline{\mathcal{H}}_j(A) = \overline{\mathcal{H}}_j(A) \setminus \overline{\mathcal{H}}_j(A) = \emptyset$. On the other hand, let $B_j(A) = \emptyset$. Then $\overline{\mathcal{H}}_j(A) \setminus \underline{\mathcal{H}}_j(A) = \emptyset$, which means that $\overline{\mathcal{H}}_j(A) \subseteq \underline{\mathcal{H}}_j(A)$. But it is well known $\underline{\mathcal{H}}_j(A) \subseteq \overline{\mathcal{H}}_j(A)$; therefore, $\overline{\mathcal{H}}_j(A) = \underline{\mathcal{H}}_j(A)$. Thus, A is infra- j -exact. \square

In Algorithm 1, we elaborate how we can determine whether a subset of a j -infra-topology is infra- j -exact or infra- j -rough.

Proposition 13 *Let A be a subset of a $jITS$ (W, R, \mathcal{I}_j) . Then*

Input : An approximation space (W, R) .
Output: Determination whether a subset of a j -infra-topology is infra- j -exact or infra- j -rough.

```

1 Input the binary relation  $R$  that associated the elements of the
  universal set  $W$ ;
2 for each  $j$  do
3   Compute  $N_j$ -neighborhoods induced from  $R$  of each  $a \in W$ ;
4   Generate an infra-topology  $\mathcal{I}_j$  using Proposition 3
5 end
6 for each nonempty subset  $A$  of  $W$  do
7   Compute  $\underline{\mathcal{H}}_j(A)$  using formula given in Definition 13;
8   if  $\underline{\mathcal{H}}_j(A) = \emptyset$  then
9     return  $A$  is an infra- $j$ -rough set
10  else
11    Compute  $\overline{\mathcal{H}}_j(A)$  using formula given in Definition 13;
12    Calculate  $\mathcal{O}_j(A) = \frac{|\underline{\mathcal{H}}_j(A)|}{|\overline{\mathcal{H}}_j(A)|}$ ;
13    if  $\mathcal{O}_j(A) = 1$  then
14      return  $A$  is an infra- $j$ -exact set
15    else
16      return  $A$  is an infra- $j$ -rough set
17    end
18  end
19 end
  
```

Algorithm 1: Algorithm of determining infra- j -exact and infra- j -rough subsets of j -infra-topologies.

- (i) $\underline{\mathcal{H}}_r(A) = \underline{\mathcal{H}}_{\langle r \rangle}(A)$.
- (ii) $\underline{\mathcal{H}}_l(A) = \underline{\mathcal{H}}_{\langle l \rangle}(A)$.
- (iii) $\overline{\mathcal{H}}_r(A) = \overline{\mathcal{H}}_{\langle r \rangle}(A)$.
- (iv) $\overline{\mathcal{H}}_l(A) = \overline{\mathcal{H}}_{\langle l \rangle}(A)$.

Proof From the equalities $\underline{\mathcal{H}}_j(A) = Int_j(A)$ and $\overline{\mathcal{H}}_j(A) = Cl_j(A)$ as well as Proposition 5, the proof follows. \square

Corollary 5 Let A be a subset of a $jITS$ (W, R, \mathcal{I}_j) . Then

- (i) $\mathcal{O}_r(A) = \mathcal{O}_{\langle r \rangle}(A)$.
- (ii) $\mathcal{O}_l(A) = \mathcal{O}_{\langle l \rangle}(A)$.

Proposition 14 Let A be a subset of a $jITS$ (W, R, \mathcal{I}_j) such that R is symmetric. Then

- (i) $\underline{\mathcal{H}}_u(A) = \underline{\mathcal{H}}_r(A) = \underline{\mathcal{H}}_l(A) = \underline{\mathcal{H}}_i(A) = \underline{\mathcal{H}}_{\langle u \rangle}(A) = \underline{\mathcal{H}}_{\langle r \rangle}(A) = \underline{\mathcal{H}}_{\langle l \rangle}(A) = \underline{\mathcal{H}}_{\langle i \rangle}(A)$.
- (ii) $\overline{\mathcal{H}}_u(A) = \overline{\mathcal{H}}_r(A) = \overline{\mathcal{H}}_l(A) = \overline{\mathcal{H}}_i(A) = \overline{\mathcal{H}}_{\langle u \rangle}(A) = \overline{\mathcal{H}}_{\langle r \rangle}(A) = \overline{\mathcal{H}}_{\langle l \rangle}(A) = \overline{\mathcal{H}}_{\langle i \rangle}(A)$.

Proof From the equalities $\underline{\mathcal{H}}_j(A) = Int_j(A)$ and $\overline{\mathcal{H}}_j(A) = Cl_j(A)$ as well as Corollary 1, the proof follows. \square

Corollary 6 Let A be a subset of a $jITS$ (W, R, \mathcal{I}_j) such that R is symmetric. Then $\mathcal{O}_u(A) = \mathcal{O}_r(A) = \mathcal{O}_l(A) = \mathcal{O}_i(A) = \mathcal{O}_{\langle u \rangle}(A) = \mathcal{O}_{\langle r \rangle}(A) = \mathcal{O}_{\langle l \rangle}(A) = \mathcal{O}_{\langle i \rangle}(A)$.

Proposition 15 Let A be a subset of a $jITS$ (W, R, \mathcal{I}_j) such that R is quasiorder. Then

- (i) $\underline{\mathcal{H}}_j(A) = \underline{\mathcal{H}}_{\langle j \rangle}(A)$.
- (ii) $\overline{\mathcal{H}}_j(A) = \overline{\mathcal{H}}_{\langle j \rangle}(A)$.

Proof From the equalities $\underline{\mathcal{H}}_j(A) = Int_j(A)$ and $\overline{\mathcal{H}}_j(A) = Cl_j(A)$ as well as Corollary 3, the proof follows. \square

Corollary 7 Let A be a subset of a $jITS$ (W, R, \mathcal{I}_j) such that R is quasiorder. Then $\mathcal{O}_j(A) = \mathcal{O}_{\langle j \rangle}(A)$.

4.2 Comparison of our approach with the previous ones

In the following results, we prove that the current method produces higher accuracy and better approximations than those given (Abd El-Monsef et al. 2014; Allam et al. 2005, 2006; Yao 1996, 1998) under a reflexive relation.

Proposition 16 Let (W, R) be an approximation space and $A \subseteq W$. If R is reflexive, then

- (i) $\underline{R}_j(A) \subseteq \underline{\mathcal{H}}_j(A)$.
- (ii) $\overline{\mathcal{H}}_j(A) \subseteq \overline{R}_j(A)$.

Proof Let $a \in \underline{R}_j(A)$. According to Definition 5, $N_j(a) \subseteq A$. Since R is reflexive, $a \in N_j(a) \subseteq A$. According to Definition 11, $N_j(a)$ is an infra-open set in \mathcal{I}_j . This means that $a \in Int_j(A) = \underline{\mathcal{H}}_j(A)$. Hence, the proof is complete.

Following similar arguments, (ii) is proved. \square

Corollary 8 Let (W, R) be an approximation space such that R is reflexive. Then $M_j(A) \leq \mathcal{O}_j(A)$ for each $A \subseteq W$.

To validate that the current method produces higher accuracy and better approximations than those given Abd El-Monsef et al. (2014), Allam et al. (2005, 2006) and Yao (1996, 1998), we calculate, in Tables 2 and 3 the approximations and accuracy values of each subset of a $jITS$ (W, R, \mathcal{I}_j) given in Example 1. Note that the relation R in this example is reflexive. For the sake of brevity, we make these computations for the cases u and $\langle u \rangle$.

5 Rough analysis of some medical diseases using the infra-topology approach

In this section, we make two contributions, first, we examine the performance of our method to analyze the data of dengue fever disease and prove it is better than the previous ones given in Abd El-Monsef et al. (2014), Allam et al. (2005, 2006) and Yao (1996, 1998). Second, we design a strategy to classify the risk of infection with COVID-19. Additionally, we provide an algorithm to show how this strategy is applied.

Table 2 Approximations and accuracy values induced from N_u -neighborhood and u -infra-topology

A	$\underline{R}_u(A)$	$\overline{R}_u(A)$	$M_u(A)$	$\underline{\mathcal{H}}_u(A)$	$\overline{\mathcal{H}}_u(A)$	$\mathcal{O}_u(A)$
$\{a\}$	\emptyset	$\{a, b, d\}$	0	$\{a\}$	$\{a, d\}$	$\frac{1}{2}$
$\{b\}$	\emptyset	$\{a, b, c\}$	0	$\{b\}$	$\{b, c\}$	$\frac{1}{2}$
$\{c\}$	\emptyset	$\{b, c\}$	0	\emptyset	$\{c\}$	0
$\{d\}$	\emptyset	$\{a, d\}$	0	\emptyset	$\{d\}$	0
$\{a, b\}$	\emptyset	W	0	$\{a, b\}$	W	$\frac{1}{2}$
$\{a, c\}$	\emptyset	W	0	$\{a\}$	$\{a, c, d\}$	$\frac{1}{3}$
$\{a, d\}$	$\{d\}$	$\{a, b, d\}$	$\frac{1}{3}$	$\{a, d\}$	$\{a, d\}$	1
$\{b, c\}$	$\{c\}$	$\{a, b, c\}$	$\frac{1}{3}$	$\{b, c\}$	$\{b, c\}$	1
$\{b, d\}$	\emptyset	W	0	$\{b\}$	$\{b, c, d\}$	$\frac{1}{3}$
$\{c, d\}$	\emptyset	W	0	\emptyset	$\{c, d\}$	0
$\{a, b, c\}$	$\{b, c\}$	W	$\frac{1}{2}$	$\{a, b, c\}$	W	$\frac{3}{4}$
$\{a, b, d\}$	$\{a, d\}$	W	$\frac{1}{2}$	$\{a, b, d\}$	W	$\frac{3}{4}$
$\{a, c, d\}$	$\{d\}$	W	$\frac{1}{4}$	$\{a, d\}$	$\{a, c, d\}$	$\frac{2}{3}$
$\{b, c, d\}$	$\{c\}$	W	$\frac{1}{4}$	$\{b, c\}$	$\{b, c, d\}$	$\frac{2}{3}$
W	W	W	1	W	W	1

Table 3 Approximations and accuracy values induced from $N_{\langle u \rangle}$ -neighborhood and $\langle u \rangle$ -infra-topology

A	$\underline{R}_{\langle u \rangle}(A)$	$\overline{R}_{\langle u \rangle}(A)$	$M_{\langle u \rangle}(A)$	$\underline{\mathcal{H}}_{\langle u \rangle}(A)$	$\overline{\mathcal{H}}_{\langle u \rangle}(A)$	$\mathcal{O}_{\langle u \rangle}(A)$
$\{a\}$	\emptyset	$\{a, b, d\}$	0	$\{a\}$	$\{a, d\}$	$\frac{1}{2}$
$\{b\}$	\emptyset	$\{a, b, c\}$	0	$\{b\}$	$\{b, c\}$	$\frac{1}{2}$
$\{c\}$	\emptyset	$\{c\}$	0	\emptyset	$\{c\}$	0
$\{d\}$	\emptyset	$\{d\}$	0	\emptyset	$\{d\}$	0
$\{a, b\}$	$\{a, b\}$	W	0	$\{a, b\}$	W	$\frac{1}{2}$
$\{a, c\}$	\emptyset	W	0	$\{a\}$	$\{a, c, d\}$	$\frac{1}{3}$
$\{a, d\}$	$\{d\}$	$\{a, b, d\}$	$\frac{1}{3}$	$\{a, d\}$	$\{a, d\}$	1
$\{b, c\}$	$\{c\}$	$\{a, b, c\}$	$\frac{1}{3}$	$\{b, c\}$	$\{b, c\}$	1
$\{b, d\}$	\emptyset	W	0	$\{b\}$	$\{b, c, d\}$	$\frac{1}{3}$
$\{c, d\}$	\emptyset	$\{c, d\}$	0	\emptyset	$\{c, d\}$	0
$\{a, b, c\}$	$\{a, b, c\}$	W	$\frac{3}{4}$	$\{a, b, c\}$	W	$\frac{3}{4}$
$\{a, b, d\}$	$\{a, b, d\}$	W	$\frac{3}{4}$	$\{a, b, d\}$	W	$\frac{3}{4}$
$\{a, c, d\}$	$\{d\}$	W	$\frac{1}{4}$	$\{a, d\}$	$\{a, c, d\}$	$\frac{2}{3}$
$\{b, c, d\}$	$\{c\}$	W	$\frac{1}{4}$	$\{b, c\}$	$\{b, c, d\}$	$\frac{2}{3}$
W	W	W	1	W	W	1

5.1 Infra-topology model of dengue fever

In this subsection, we study the dengue fever disease which is a global problem. It is transmitting to humans by virus-carrying Dengue mosquitoes (Prabhat 2019). The symptoms of this disease mostly start from the third day of infection. The period of recovery takes a few days; usually, 2–7 days (Prabhat 2019). According to the statistics of the World Health Organization (WHO), it spreads in more than 120 nations and causes a huge number of deaths around the world; in particular, Asia and South America (World 2016). Accordingly, this disease occupies an important place worldwide,

which motivates us to analyze it by the approach introduced in this manuscript.

The data displayed in Table 4 decide this disease such that the columns give the symptoms of dengue fever as follows joint and muscle aches Q_1 , headache with puke Q_2 , skin rashes Q_3 , a temperature Q_4 with three levels (normal (n), high (h) and very high (vh)) and finally the decision D of infected or not. In contrast, the rows represents the patients under study $W = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$. The mark “ \checkmark ” (resp., “ \times ”) denotes the patient has a symptom (resp., the patient has no symptom).

Now, the descriptions of attributes $\{Q_i : i = 1, 2, 3, 4\}$ will be transmitted into quantity values showing the degree of

Table 4 Information system of dengue fever

W	Q_1	Q_2	Q_3	Q_4	Dengue fever
a_1	✓	✓	✓	h	✓
a_2	✓	✗	✗	h	✗
a_3	✓	✗	✗	h	✓
a_4	✗	✗	✗	vh	✗
a_5	✗	✓	✓	h	✗
a_6	✓	✓	✗	vh	✓
a_7	✓	✓	✗	n	✓
a_8	✓	✓	✗	vh	✓

Table 5 Similarity degrees between patients' symptoms

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
a_1	1	1/2	1/2	0	3/4	1/2	1/2	1/2
a_2	1/2	1	1	1/2	1/4	1/2	1/2	1/2
a_3	1/2	1	1	1/2	1/4	1/2	1/2	1/2
a_4	0	1/2	1/2	1	1/4	1/2	1/4	1/2
a_5	3/4	1/4	1/4	1/4	1	1/4	1/4	1/4
a_6	1/2	1/2	1/2	1/2	1/4	1	3/4	1
a_7	1/2	1/2	1/2	1/4	1/4	3/4	1	3/4
a_8	1/2	1/2	1/2	1/2	1/4	1	3/4	1

similarities among the patients' symptoms; see Table 5. We calculate similarity degree function between the patients a, b , denoted by $s(a, b)$, with respect to m conditions attributes by the next formula.

$$s(a, b) = \frac{\sum_{j=1}^m (A_j(a) = A_j(b))}{m} \tag{2}$$

The next procedure is proposing a relation; it is given according to the requirements of the standpoint of system's experts. In this example, we propose the following relation

$$(a, b) \in R \iff 1/2 \leq s(a, b) < 1.$$

Note that the given relations $\leq, <$ and numbers $1/2, 1$ can be replaced according to the conceptions of system's experts. It is clear that the suggested relation R is symmetric, so it produces two types of N_j -neighborhood systems. But R is neither reflexive nor transitive. Moreover, it is irreflexive because $s(a_i, a_i) = 1$ for each $a_i \in W$.

In Table 6, we compute the two types of N_j and $N_{\langle j \rangle}$ neighborhoods for each patient a_i .

To confirm the performance of our approach, we consider two subsets $F = \{a_1, a_4, a_6, a_7, a_8\}$ and $G = \{a_2, a_3, a_5\}$. Then, the approximations and accuracy measures of these two sets are computed with respect to N_j -neighborhoods and j -infra-topology.

Table 6 N_j and $N_{\langle j \rangle}$ of each $a_i \in W$

	N_j	$N_{\langle j \rangle}$
a_1	$\{a_2, a_3, a_5, a_6, a_7, a_8\}$	$\{a_1\}$
a_2	$\{a_1, a_4, a_6, a_7, a_8\}$	$\{a_2, a_3\}$
a_3	$\{a_1, a_4, a_6, a_7, a_8\}$	$\{a_2, a_3\}$
a_4	$\{a_2, a_3, a_6, a_8\}$	$\{a_1, a_4, a_6, a_7, a_8\}$
a_5	$\{a_1\}$	$\{a_2, a_3, a_5, a_6, a_7, a_8\}$
a_6	$\{a_1, a_2, a_3, a_7\}$	$\{a_6, a_8\}$
a_7	$\{a_1, a_2, a_3, a_6, a_8\}$	$\{a_7\}$
a_8	$\{a_1, a_2, a_3, a_7\}$	$\{a_6, a_8\}$

According to Table 7, the approximations and accuracy measures induced from j -infra-topology are better than those induced from N_j -neighborhoods.

Similarly, we compute the approximations and accuracy measures of the subsets with respect to $N_{\langle j \rangle}$ -neighborhoods and $\langle j \rangle$ -infra-topology.

5.2 Infra-topology model of COVID-19 pandemic

Nowadays, the COVID-19 pandemic is given a great interest in all countries of the world; it causes troubles in all human activities starting from the health and economic systems to food systems and the world of work. As it is well known today, individuals' contact or nearness physically is the major way of infection by COVID-19. Unfortunately, a successful remedy does not discover up to now. Only, there are some simple precautions and recommendations that can help to stay safe, according to WHO, such as physical distancing, avoiding crowds, wearing a mask and personal cleaning. With respect to the duration of quarantine of suspicious individuals, the WHO specified it by fourteen days renewable if the symptoms are still during the first duration. Otherwise, it is finished the duration of quarantine taking into account the procedures of healthy infection prevention.

Herein, we apply N_j -neighborhood systems and j -infra-topological structure to assort any community sample (like school staff, medical staff, etc.) in terms of suspicion of infected them by COVID-19. This proposed method can be applied to any infectious disease like influenza.

To illustrate the followed technique to quarantine the patients who suspicion of COVID-19, assume that W is a group of people who works together at a specified facility like bank, hospital, school, etc. To guarantee the safety of employees, the administration checks their medical cases in terms of being infected with COVID-19 by doing a periodic test for them in different durations depending on available capabilities.

Firstly, we quarantine the individuals having a positive test of COVID-19, say a group $A \subseteq W$. Then, we classify

Table 7 Accuracy measures induced from N_j -neighborhoods and j -infra-topology of the sets F and G

Set	N_j -neighborhoods		j -infra-topology			
	\underline{R}_j	\overline{R}_j	M_j	$\underline{\mathcal{H}}_j$	$\overline{\mathcal{H}}_j$	\mathcal{O}_j
F	$\{a_2, a_3, a_5\}$	W	$3/8$	F	$\{a_1, a_4, a_5, a_6, a_7, a_8\}$	$5/6$
G	\emptyset	$\{a_1, a_4, a_6, a_7, a_8\}$	0	$\{a_2, a_3\}$	$\{a_2, a_3, a_5\}$	$2/3$

the members in a suspicion set $A^c = W \setminus A$ according to the degree of infection suspicion. To do this, according to our model, we define a relation R among all the individuals of W who had contact together, i.e., $R = \{(x, y) : x \text{ contact with } y\}$; this relation inspired by the fact that the infection of COVID-19 mainly spreads among the individuals by contact. From this relation, we compute N_j -neighborhoods of each member in the suspicion set, and since we need to determine the size of contact with the infected set A by each member x of A^c , we define the associated K_j -neighborhoods to restrict the N_j -neighborhoods on A as follows $K_j(x) = N_j(x) \cap A$. We are now in a position to compute the contact degree for each $x \in A^c$ by the formula $c_j(x) = \frac{|K_j(x)|}{|A|}$ which will be the first evaluation criteria to specify the degree of suspicion according to the boundary numbers α which will be determined by experts' system. After that, we build a j -infra-topological structure from the family of associated K_j -neighborhoods; the importance of this step is to obtain an accuracy measure $\mathcal{O}(K_j(x))$ of each associated K_j -neighborhoods better than its counterparts induced directly from associated K_j -neighborhoods $M_j(K_j(x))$. This value will be the second evaluation criteria to specify the degree of suspicion according to the boundary numbers β given by experts' system as well. Finally, we classify the members in a suspicion set A^c in three groups according to the proposed boundary numbers α, β of evaluation as follows.

Group 1 (high suspicion): The individuals of this group have $c_j(x) \geq \alpha$ and $\mathcal{O}(K_j(x)) \geq \beta$, which means that they have a large number of contacts with the individuals infected by COVID-19 as well as a high accuracy measure for their associated K_j -neighborhoods. Accordingly, the probability of their infection is high. Therefore, the individuals of this group are classified under a high suspicion.

Group 2 (medium suspicion): The individuals of this group have $c_j(x) \geq \alpha$ and $\mathcal{O}(K_j(x)) < \beta$, or $c_j(x) < \alpha$ and $\mathcal{O}(K_j(x)) \geq \beta$. This implies that we have two cases

- An individual has a large number of contacts with the individuals infected by COVID-19, which increases his/her probability of infection. On the other hand, he/she has a low accuracy measure for his/her associated K_j -neighborhoods, which makes the completeness of knowledge obtained from his/her class is low.

- An individual has a small number of contacts with the individuals infected by COVID-19, which decreases his/her probability of infection. On the other hand, he/she has a high accuracy measure for his/her associated K_j -neighborhoods, which makes the completeness of knowledge obtained from his/her class is high.

Therefore, the individuals of this group are classified under a medium suspicion.

Group 3 (low suspicion): The individuals of this group have $c_j(x) < \alpha$ and $\mathcal{O}(K_j(x)) < \beta$, which means that they have a small number of contacts with the individuals infected by COVID-19 as well as a low accuracy measure for their associated K_j -neighborhoods. Accordingly, the probability of their infection is low. Therefore, the individuals of this group are classified under a low suspicion.

The role of our method stops here. The next (final) step "which one of the above groups will be quarantined?" is based on the capabilities available for the facility. That is, the facility suffices by quarantining the persons who are in the high suspicion group if the potential is not enough. Otherwise, it imposes the quarantine for the persons who are in the high and medium suspicion groups.

Remark 5 Note that the proposed relation R is symmetric because a is in contact with b if and only if b is in contact with a . According to Proposition 6, there are only two kinds of N_j -neighborhood systems. On the other hand, R is not a transitive relation because the contacts between a and b and between b and c do not guarantee the contact between a and c , so that R is not an equivalence relation, which implies that we cannot apply the Pawlak approximation model.

We close this section by presenting Algorithm 2, arising from the investigation above, in order to classify the members with respect to the degree/probability of their infection with COVID-19.

6 Discussions: strengths and limitations

This section demonstrates the main advantages of technique followed herein as well as shows its limitations.

Input : The individuals of a specific facility W , the group of individuals A who have a positive test of COVID-19 in a given time, the group of individuals $B = W \setminus A$ who have a negative test of COVID-19 in the same given time, the boundary numbers α, β of evaluation

Output: The classification of B in three suspicion groups: high, medium and low.

```

1 Classify the set  $W$  relying on COVID-19 tests into two groups:
  positive (symbolized by  $A$ ) and negative (symbolized by  $B$ );
2 if  $A = \emptyset$  then
3   return all members in  $W$  are classified as non-suspicious
4 else
5   Define the binary relation  $R$  on the set  $W$ :  $R = \{(x, y) : x$ 
   contact with  $y\}$ ;
6   Compute  $N_j$ -neighborhood of each  $x \in B = W \setminus A$ ;
7   Define associated  $K_j$ -neighborhood of each  $x \in B$ :
    $K_j(x) = N_j(x) \cap A$ ;
8   Calculate the  $j$ -contact degree  $c_j$  of each  $x \in B$ :
    $c_j(x) = \frac{|K_j(x)|}{|A|}$ ;
9   Build a  $j$ -infra-topology  $\mathcal{I}_j$  on  $A$  from the family of
    $\{\emptyset, K_j(x) : x \in B\}$ ;
10  for  $x \in B$  do
11    compute infra- $j$ -upper approximation  $\overline{\mathcal{H}}$  of  $K_j(x)$ ;
12    compute infra- $j$ -accuracy  $\mathcal{O}$  of  $K_j(x)$ :
    $\mathcal{O}(K_j(x)) = \frac{|\mathcal{H}(K_j(x))=K_j(x)|}{|\overline{\mathcal{H}}(K_j(x))|}$ ;
13  end
14  Classify the elements of  $B$  in terms of  $j$ -contact degree  $c_j(x)$ 
   and infra- $j$ -accuracy  $\mathcal{O}(K_j(x))$  into three groups:
15    High suspicion: The individuals who have  $c_j(x) \geq \alpha$  and
    $\mathcal{O}(K_j(x)) \geq \beta$ ;
16    Medium suspicion: The individuals who have  $c_j(x) \geq \alpha$ 
   and  $\mathcal{O}(K_j(x)) \leq \beta$  or  $c_j(x) \leq \alpha$  and  $\mathcal{O}(K_j(x)) \geq \beta$ ;
17    Low suspicion: The individuals who have  $c_j(x) \leq \alpha$  and
    $\mathcal{O}(K_j(x)) \leq \beta$ ;
18  return the obtained classification;
19 end

```

Algorithm 2: Algorithm for determining the risk of infection.

– Strengths

1. The current method keeps the monotonic property for both measures accuracy and roughness, whereas these measures lose this property in some foregoing topological approaches like those studied in Amer et al. (2017) and Mareay (2016).
2. The current method preserves all Pawlak properties of lower and upper approximations as proved in Proposition 8 and Proposition 9. On the other hand, some of these properties are losing in some previous approaches induced from topological structures such as Al-shami (2021b).
3. All the approximation operators and accuracy measures produced by the current approach are identical under a symmetric relation, whereas the identity obtained by the approaches existing in published lit-

erature under a symmetric relation is for the disjoint index sets $\{r, l, i, u\}$ and $\{\langle r \rangle, \langle l \rangle, \langle i \rangle, \langle u \rangle\}$.

4. The current method improves the approximations and accuracy measures compared to some foregoing methods introduced in Abd El-Monsef et al. (2014), Allam et al. (2005, 2006) and Yao (1996, 1998) under a reflexive relation.
5. The structure of infra-topology that we rely on to produce the novel rough set models in this manuscript is more relaxed than topological structure. That is, we can dispense with some topological conditions that are not convenient to describe the phenomena under study.

– limitations

It cannot be compared between the different types of approximation operators and accuracy measures generated from the different types of j -infra-topologies. In fact, this matter is satisfied for approximation operators and accuracy measures directly generated from N_j -neighborhoods.

7 Conclusion

The notions inspired by abstract structures like topology and ideal are considered a vital methods to reconstruct the rough set concepts as well as they overcome some failures aspects regarding approximation operators and accuracy values. This manuscript contributes to this path of research by providing novel rough set models motivated by the concept of “infra-topology.” These models are more relaxed than their counterparts induced from topology because we do without arbitrary union condition, so we can apply these models to simulate more real-life issues.

Through this article, we have proposed a method to build infra-topology structure from an approximation space under arbitrary relation. We have established their main properties that helped us to characterize the rough set models we have define in Sect. 4. With the help of an abstract example, we have shown that the current models lead to improve approximation operators and accuracy values more than some previously discussed models suggested in Abd El-Monsef et al. (2014), Allam et al. (2005, 2006) and Yao (1996, 1998). As an applications, we have studied the followed technique to describe two medical diseases are dengue fever and COVID-19 pandemic. In the end, we have explained the advantages and disadvantages of the our approach compared with the foregoing ones.

As upcoming works, we plan to investigate the next topics.

- (i) Investigate the manuscript concepts with respect to different types of neighborhoods such as containment and subset rough neighborhoods.
- (ii) As we know that adding ideal structures to topology guarantees good performance for approximation operators and accuracy values, so we will examine the structure consisting of ideal and infra-topology following similar approach displayed in Al-shami and Hosny (2022), Hosny (2020), Hosny et al. (2021), Hosny and Al-shami (2022), Kandil et al. (2020), Nawar et al. (2022) and Tantawy and Mustafa (2013).
- (iii) Study how the results and conceptions given herein can be improved using some infra-topology ideas such as generalizations of infra-open sets.
- (iv) Reformulate the manuscript concepts in some frames such as soft rough set and fuzzy rough set.

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