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FOUNDATION, ALGEBRAIC, AND ANALYTICAL METHODS IN SOFT COMPUTING

Bipolar fuzzy concepts reduction using granular-based weighted entropy

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Abstract
The bipolar fuzzy concept lattice has given a way to analyze the uncertainty in soft data set beyond the unipolar space. In this process, a problem is addressed while dealing with large number of bipolar fuzzy concepts and its importance for adequate decision-making process. It may create randomness in the decision due to bipolarity and its existence in customer feedback, or expert opinion. To overcome from this issue, the current paper tried to measure the randomness in bipolar fuzzy concepts using the properties of Shannon entropy. The importance of bipolar fuzzy concept is decided based on defined window of granulation (α_1, α_2) for its computed weight with an illustrative example. The obtained results are also compared with recently available approaches on data with bipolar fuzzy attributes for validation.

Keywords Bipolar fuzzy set · Bipolar fuzzy concept · Formal fuzzy concept · Fuzzy concept lattice · Granular computing · Soft data · Uncertainty measurement

1 Introduction

Knowledge discovery and representation from the given data sets are considered as one of the major tasks by research communities. The introduction of concept lattice theory by Wille (1982) has given a way to solve this issue via investigation of extent–intent pair in the given context, due to which the mathematical algebra of concept lattice (Ganter and Wille 1999) extended from unipolar (Vychodil 2005a; Ghosh et al. 2010) to bipolar fuzzy space (Singh and Cherukuri 2014a, b) for adequate analysis of soft data. The bipolar fuzzy graph representation of concept lattice given a way to deal with positive and negative side of data simultaneously (Ali et al. 2021, Pal and Mondal 2019; Marsala and Bouchon 2000; Singh 2019a, 2020). In this process, a problem is addressed about selection of some important bipolar fuzzy concepts as discussed by Singh (2019a) or finding bipolar fuzzy attribute implications as shown by Singh (2022b) for multi-decision process based on user requirements (Singh 2022a). The reason is validation of information content available in the given bipolar fuzzy concepts and its accuracy is one of the difficult tasks. To resolve this issue, current paper focuses on introducing the entropy theory for randomness measurement of bipolar fuzzy attributes, such that some important bipolar fuzzy concepts can be selected based on user-defined information granules. In this way, the proposed method will be helpful while analysis of uncertainty in bipolar attributes at micro–macro-level of granulation.

The bipolar information used to found frequently in any soft data sets like customer feedback, opinion of people which is based on human quantum Turiyam awareness (Singh 2022a). It contains both positive and negative simultaneously to represent any event as discussed by Dubois and Prade (2012). It is used to be visualized as symmetry of positive and negative side as shown in Fig. 1, as for example in Chinese Yin–Yang of Taoism discussed by Welch H (1957) or Yoni–Linga, Rudra–Akasha or Yama–Yami in Sanskrit.[1] To deal with these types of human thoughts based on its existence (i.e., 1) or nonexistence (i.e., 0) Boolean logic {0, 1} is introduced by Boole (1854) motivated from Pingala binary system[2]. In this case, a problem is addressed while dealing the

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[1] https://asiasociety.org/education/shakti-power-feminine

[2] https://en.wikipedia.org/wiki/Pingala

uncertainty and randomness in data sets which was precisely handled by Shannon (1948) using combination of probabilistic and logarithmic model [0, 1]. Zadeh (1965) introduced a set to compute with the uncertainty in human cognition by a defined fuzzy membership values in [0, 1] as non-dualism.[3] The L-fuzzy set represents the acceptation and rejection part via a single-valued membership which was unable to represent the bipolar information (Goguen 1967). Zhang (1994) introduced bipolar fuzzy set, i.e., $[-1, 0) \times (0, 1]$ for precise representation of bipolar information where $[-1, 0)$ represents negative side and $(0, 1]$ represents the positive side, independently. This set has given a chance to represent the character of love–hate, action–reaction, particle–antiparticle as independent in bipolar fuzzy space as shown in Fig. 1 rather than dependent true and false region of intuitionistic fuzzy set. Same time intuitionistic fuzzy set provides a way to compute hesitant part which is not available in bipolar fuzzy set. The another difference is complement operation which is available in intuitionistic fuzzy set but not in bipolar fuzzy set. In this way, the bipolar fuzzy set provides a way to represent true and false membership values independently. It can be observed in sports data where performance of player can be analyzed using intuitionistic fuzzy set (Singh 2022c) but its celebration of expression can be represented by bipolar fuzzy set independently for India and Pakistan supporters. The reason is celebration of expression for India–Pakistan match is different for same wicket out like Indians may feel happiness whereas Pakistani feels pain. In this case, celebration of Indians increases does not means that the pain of Pakistani supporter decreases as per intuitionistic fuzzy set and vice versa. This type of bipolar relations exists, independently without any hesitant part. Hence, the bipolarity was mostly found in these types of international relations[4] which cannot be represented via intuitionistic fuzzy set due to independent true and false membership values. These types of bipolar relations as for example USA–Russia relations, India–Pakistan, Philistine–Israel do not contain any hesitant part, i.e., $1 - t - f$. It is totally independent from true and false membership values. It can be represented only via two independent mapping constituting same information as : $\mu^P: Z \rightarrow [0, 1]$ and $\mu^N: Z \rightarrow [-1, 0]$ where $\mu^P(z)$ represents the somewhat satisfaction degree of attributes whereas $\mu^N(z)$ represents implicit counter-property. The membership value 0 represents that the given attribute is somewhat irrelevant to given context. These information can be precisely represented by a defined bipolar fuzzy set $Z= \{(z, \mu^P(z), \mu^N(z)) | z \in Z\}$ (Zhang and Zhang 2004). It can be visualized via a defined bipolar fuzzy graph $G = (V, E)$ (Akram 2011, 2013) for applications in several fields (Gulistan et al. 2021; Rashmanlou et al. 2014a, b). The extensive

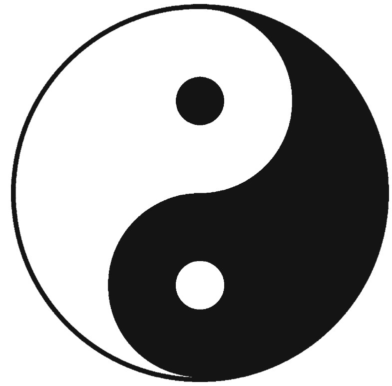

Fig. 1 A bipolar representation of given data set using Yin–Yang theory

properties of bipolar fuzzy graph (Rashmanlou et al. 2015) and its product have given a way to utilize its algebra for various applications (Rashmanlou et al. 2016; Talebi and Rashmanlou 2014; Tahmasbpour et al. 2016) Motivated from these studies, Singh and Cherukuri (2014a; 2014b) provided hierarchical ordering among the bipolar fuzzy concepts for knowledge processing task. In this process, a problem is addressed while dealing randomness in bipolar fuzzy concepts. The problem becomes more complex when an expert wants to refine the bipolar fuzzy concepts based on his/her requirement for decision-making process. There is less attention toward this direction till now as shown in Table 1. This paper tried to fill this gap via introduction of Shannon entropy for dealing with bipolar fuzzy attributes with a defined weighted granulation.

Table 1 shows that randomness measurement in bipolar fuzzy attributes is considered as one of the major issues while dealing the data with bipolar fuzzy attributes. This issue arises when the large number of bipolar fuzzy concepts generated from any given context as discussed recently by Singh(2019a). In this case, uncertainty or randomness may generate while analyzing the relevant information based on expert requirements. To deal with this issue, researchers tried to pay attention toward fuzzy concept lattice (Vychodil 2005a; Ghosh et al. 2010; Pocs 2012) and its reduction (Cherukuri and Srinivas 2010; Kang et al. 2012; Vychodil 2005a) for finding some relevant concept (Zacpal 2005; Macko 2001). It can be investigated using the object or attribute attributed multi-adjoint concept lattice (Medina 2012). In case the expert unable to draw the concept lattice weighted-based method can be useful as an alternative tool for the content measurement of the formal concept (Wu et al. 2009; Zhang et al. 2012). Another reason is that this method provides a flexibility to select some of the relevant concepts based on the user requirements as discussed by Singh et al. (2017). In this regard, entropy theory introduced by Shannon (1948) played an important role while

[3] https://en.wikipedia.org/wiki/Nondualism

[4] https://en.wikipedia.org/wiki/Polarity_(international_relations)

Table 1 The basic understanding of research trends on the current topic

Author year-wise work	Research findings	Pitfall
Boole (1854)	Law of thought {0, 1}	Uncertainty measurement
Shannon (1948)	Uncertainty in thought [0, 1]	Unipolar space
Welch (1957)	Taoism, i.e., Yin–Yang {−, +}	Randomness measurement
Zadeh (1965)	Fuzzy set (0, 1) for acceptance	Bipolarity measurement
Goguen (1967)	L-Fuzzy set	Bipolarity measurement
Zhang (1994)	Bipolar fuzzy set $[-1, 0) \times (0, 1]$	Graphs and lattice
Zacpal (2005)	Fuzzy concept lattice	Bipolarity measurement
Ghosh et al. (2010)	Fuzzy graph-based lattice	Bipolarity measurement
Cherukuri and Srinivas (2010)	Concept lattice reduction	Bipolarity measurement
Macko (2001)	Selection of important concepts	Uncertainty measurement
Bloch (2011)	Lattices of bipolar fuzzy	Uncertainty measurement
Akram (2011)	Lattices of bipolar fuzzy	Uncertainty measurement
Kang et al. (2012)	Fuzzy lattice at granulation	Bipolarity measurement
Li et al. (2012)	Knowledge reduction	Uncertainty measurement
Bloch (2012)	Morphology of bipolar set	Randomness measurement
Zhang et al. (2012)	Weighted concepts	Bipolarity measurement
Li et al. (2013)	Entropy-based concepts	Bipolarity measurement
Akram (2013)	Bipolar fuzzy graph	Lattice visualization
Singh (2014a; 2014b)	Bipolar fuzzy concept lattice	Randomness measurement
Talebi and Rashmanlou (2014)	Complement of bipolar graphs	Lattice visualization
Li et al. (2015)	Granular-based concepts	Unipolar space
Singh and Gani (2015)	Entropy-based fuzzy lattice	Bipolarity measurement
Tahmasbpour et al. (2016)	Bipolar f-morphism	Randomness measurement
Singh et al. (2017)	Selection of fuzzy concepts	Bipolarity measurement
Sumangali et al. (2017)	Entropy-based concepts	Bipolarity measurement
Sarwar and Akram (2018)	Bipolar fuzzy applications	Randomness measurement
Singh (2019a)	Bipolar fuzzy concept distance	Randomness measurement
Pal and Mondal (2019)	Bipolar fuzzy matrix	Randomness measurement
Singh (2019b)	Vague entropy	Bipolarity measurement
Ali et al. (2019)	Bipolar parameter reduction	Randomness measurement
Ali et al. (2020)	Bipolar attribute reduction	Randomness measurement
Marsala and Bouchon	Bipolar and intuitionistic entropy	Knowledge processing
Akram et al. (2020)	Bipolar fuzzy TOPSIS method	Randomness measurement
Akram et al. (2021b)	Bipolar fuzzy graphs	Lattice visualization
Riaz and Therim (2021)	Bipolar fuzzy connection number	Bipolarity measurement
Singh (2022a)	Bipolar multi-fuzzy concept	Randomness measurement
Singh (2022b)	Bipolar fuzzy implication	Randomness measurement
Singh (2022d)	Complex fuzzy lattice entropy	Bipolarity measurement

measuring the information loss. It provides the average information weight for the generated concepts either in data with binary attributes (Li et al. 2013; Singh et al. 2017; Sumangali et al. 2017), fuzzy attributes (Singh and Gani 2015; Singh 2018), vague attributes (Singh 2019b) or complex attributes (Singh 2022d). The lossless compression and encoding of concepts via entropy theory motivated the author to introduce it for handling data with bipolar fuzzy attributes as discussed by Singh and Gani (2015). The objective is select some of the relevant bipolar fuzzy concepts based on user-required (α_1, α_2)-granulation independently for true and false values (Singh 2019b, 2022b). One of the significant outputs of the proposed method is that provides flexibility of each user to choose relevant concepts based on his/her required information granules rather than drawing the concept lattice. In addition, the proposed method encodes the obtained bipolar fuzzy concepts which can be useful for data security purpose. In last, the analysis derived from the proposed method is compared with subset-based bipolar fuzzy concept lattice (Singh and Cherukuri 2014a, b) and its Euclidean distance method (Singh 2019a) to validate the results. In this way, the proposed method is distinct any of the available approaches in following aspects:

(i) It provides an information content measurement for data with bipolar fuzzy attributes using Shannon entropy, and

(ii) It provides a way to encode the bipolar fuzzy concepts for knowledge processing tasks which can be useful in data security.

The rest of the paper is organized as follows: Section 2 provides some basic mathematics to understand the data with bipolar fuzzy attributes and its representation. Section 3 contains the proposed method with its illustration in Sect. 4. Section 5 provides discussion followed by conclusions, acknowledgments and references.

2 Data with bipolar fuzzy attributes

This section provides preliminaries about bipolar fuzzy context, bipolar fuzzy concepts and its concept lattice for better understanding as given below:

Definition 1 (*Formal fuzzy context*) (Burusco and Fuentes-Gonzales 1994): The triplet $\mathbf{K} = (X, Y, R)$ is called as formal fuzzy context, in which X represents set of objects, Y represents set of fuzzy attributes and R is an L-relation among X and Y as: $X \times Y \rightarrow L$. In this way, the fuzzy relation $R(x, y) \in L$ represents the single-valued fuzzy membership value at which the object $x \in X$ has the fuzzy attribute $y \in Y$ in [0, 1]. The L-relation is a support set of a residuated lattice $\mathbf{L}$ which defines the generalization and specialization among them. To achieve this goal, several algorithms are proposed to generate the pattern from a given fuzzy context called as formal fuzzy concepts.

Definition 2 (*Formal fuzzy concepts*) (Vychodil 2005b): The formal fuzzy concepts can be generated for any $\mathbf{L}$-set $A \in L^X$ of objects, and $B \in L^Y$ of fuzzy attributes then a $\mathbf{L}$-set $A^{\uparrow} \in L^Y$ of attributes and $\mathbf{L}$-set $B^{\downarrow} \in L^X$ of objects can be investigated as follows:

(1) $A^{\uparrow}(y) = \wedge_{x \in X}(A(x) \rightarrow \tilde{R}(x, y))$,
(2) $B^{\downarrow}(x) = \wedge_{y \in Y}(B(y) \rightarrow \tilde{R}(x, y))$.

The $A^{\uparrow}(x)$ is interpreted as the $\mathbf{L}$-set of attribute $y \in Y$ shared by all objects from A. Similarly, $B^{\downarrow}(x)$ is interpreted as the $\mathbf{L}$-set of all objects $x \in X$ having the attributes from B in common. The formal fuzzy concepts is a pair of $(A, B) \in L^X \times L^Y$ satisfying $A^{\uparrow} = B$ and $B^{\downarrow} = A$. The fuzzy set of objects A called as an extent and the fuzzy set of attributes B is called as an intent. This pattern can be generated for the given bipolar fuzzy context based on positive and negative membership values as discussed by Singh and Cherukuri (2014b).

Definition 3 (*Bipolar fuzzy context*) Singh and Cherukuri (2014a; 2014b) : A bipolar fuzzy context is an extension of fuzzy context $\mathbf{F} = (X, Y, \tilde{R})$ where X is a set of objects, Y is a set of bipolar fuzzy attributes and $\tilde{R}$ represents bipolar fuzzy relation among X and Y as: $X \times Y \rightarrow [-1, 1]$. In this way, $\tilde{R}(x, y)$ represents the bipolar fuzzy membership value at which the object $x \in X$ has the fuzzy attribute $y \in Y$ in (0, 1] or not in [-1, 0). In case, the attribute the irrelevant then it can be represented using (0, 0). It means the entries of bipolar fuzzy matrix can be written as : $\tilde{R}=(\mu_{\tilde{R}}^P, \mu_{\tilde{R}}^N)$:$X \times Y \rightarrow$[-1, 0]$\times$[0, 1] where $\mu_{\tilde{R}}^P(x_1, y_1) \in [0, 1]$ and $\mu_{\tilde{R}}^N(x_1, y_1) \in [-1, 0]$. The problem arises while investigating some useful pattern from the given bipolar fuzzy context for knowledge processing tasks.

Definition 4 (*Bipolar fuzzy concepts*) Singh and Cherukuri (Singh and Cherukuri 2014a, b) : A bipolar fuzzy concepts is a pair (A, B) which contains set of bipolar fuzzy set of objects and bipolar fuzzy set of attributes which shares maximal positive and negative membership values in the given bipolar fuzzy context. The bipolar fuzzy set of objects and attributes set (A, B) is called as bipolar fuzzy concepts iff:

$(B, (\mu^P(B), \mu^N(B)))^{\downarrow} = (A, min(\mu^P(A), \mu^N(A)))$ and,
$(A, (\mu^P(A), \mu^N(A)))^{\uparrow} = (B, min(\mu^P(B), \mu^N(B)))$.

The ($\downarrow$) operator can be applied on the set of bipolar fuzzy attributes to find the maximal covering bipolar fuzzy set of objects while integrating the information from positive and negative membership value. Consequently, the operator ($\uparrow$) can be applied on the bipolar fuzzy set constituted by these covering objects set resulting from integrating the positive and negative membership values among objects and attributes set. It means ($\downarrow$, $\uparrow$) provides a pair of maximal bipolar fuzzy set of objects and attributes with respect to integrating the information from them in given bipolar fuzzy space $[-1, 0) \times (0, 1]$. After that, any bipolar fuzzy set of attributes (or objects) cannot be found which can make the membership value of the obtained pair bigger, if the pair of the set of objects and the set constituted by its covered attributes forms a bipolar formal fuzzy concept. It can be easily visualization via Hasse diagram as discussed by Zhang and Zhang (2004) for dealing the quantum data (Zhang 2021) and its changes as discussed by Singh (2020).

Definition 5 (*Complete lattice*) (Ganter and Wille 1999) : The complete lattice provides an infimum and a supremum for any given two formal concepts which are generated from the given context. It can be discovered as follows :

- $\wedge_{j \in J}(A_j, B_j) = (\bigcap_{j \in J} A_j, (\bigcup_{j \in J} B_j)^{\downarrow\uparrow})$,

Table 2 A bipolar fuzzy subset of V for Example 1

	v_1	v_2	v_3
μ_I^P	0.5	0.7	0.6
μ_I^N	− 0.3	− 0.4	− 0.5

Table 3 A bipolar fuzzy subset of E for Example 1

	$v_1 v_2$	$v_2 v_3$	$v_3 v_1$
μ_J^P	0.5	0.6	0.5
μ_J^N	− 0.3	− 0.4	− 0.3

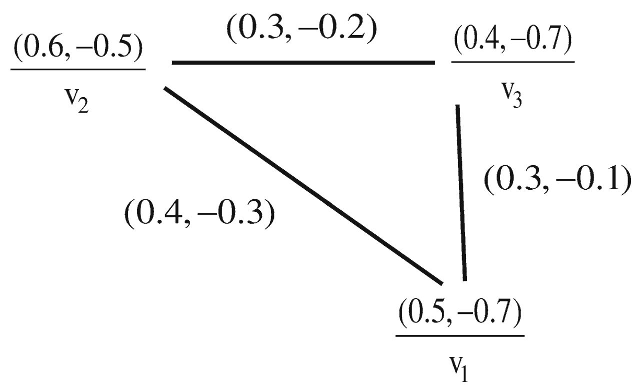

Fig. 2 A bipolar fuzzy complete graph representation for Tables 2 and 3

- $\vee_{j\in J}(A_j, B_j) = ((\bigcup_{j\in J} A_j)^{\uparrow\downarrow}, \bigcap_{j\in J} B_j)$.

The complete lattice provides specialization and generalization among the concepts which is useful in multi-decision process. In similar way, the complete lattice of bipolar fuzzy concepts can be visualized using the algebra of bipolar fuzzy complete graph as discussed by Singh and CHerukuri (2014a; 2014b).

Definition 6 (*Bipolar fuzzy complete graph*) (Akram 2011) : A bipolar fuzzy graph G=(I, J) is complete iff:

$\mu_J^P(\{v_1, v_2\}) = \min(\mu_I^P(v_1), \mu_I^P(v_2))$ and,

$\mu_J^N(\{v_1, v_2\}) = \max(\mu_I^N(v_1), \mu_I^N(v_2))$

for all $v_1, v_2 \in V$ and $(v_1, v_2) \in V \times V$.

Example 1 Let us suppose, a graph G having set of vertices V= (v_1, v_2, v_3), and set of edges E= (v_1v_2, v_2v_3, v_3v_1). The bipolar fuzzy set based on given vertices I and their edges E can be written through the properties of bipolar fuzzy set as shown in Tables 2 and 3. These sets can be visualized through a fuzzy complete graph shown in Fig. 2 (Akram 2013).

The bipolar fuzzy complete graph can be useful to visualize the lattices of bipolar fuzzy set as discussed by (Bloch 2011); 2013) and others (Singh and Cherukuri 2014a, b). The problem arises when the bipolar fuzzy concept lattice and its graphical visualization become huge even for small

Table 4 Some of the defined nomenclature used for the proposed method in this paper

Nomenclature	Meaning
K	Formal fuzzy context
L	Scale of truth degree
X	Set of objects
x	An object
Y	Set of attributes
y	An attribute
P	Probability
Z	Bipolar fuzzy set
p_i	i^{th}-object Probability
R	L-relation between X and R
$\tilde{R}$	Bipolar fuzzy relation between X and R
μ_P	Positive membership values
μ_N	Negative membership values
$(\uparrow, \downarrow)$	Galois connection
A	Extent
B	Intent
$\bigcup$	Union
$\bigcap$	Intersection
$\wedge$	Infimum
$\vee$	Supremum
α_1, α_2	Window of Granulation
E	Average information weight
$\sum$	Summation
m	Total number of attributes
w_j	Weight of attribute
W_j	Weight of jth formal concept
C1–C6	Bipolar fuzzy concept
$FC_{\mathbf{F}}$	Set of fuzzy formal concepts
x_i	i^{th}-object
y_j	j^{th}-attribute

bipolar context. In this case, selection of some important bipolar fuzzy concepts (Singh 2019a) or attributes reduction (Ali et al. 2020) become major issues. To resolve this issue, entropy (Shannon 1948) and granular computing (Yao 2004) is considered as one of the prominent tools for controlling the formal concepts (Li et al. 2013; Sumangali et al. 2017), fuzzy concepts (Singh et al. 2017) and its lattice structure (Singh and Gani 2015; Singh 2022d). These studies motivated the author to utilize the Shannon entropy for selection of bipolar fuzzy concepts in the next section. To understand the proposed method, some nomenclature and its meaning are comprised in Table 4.

3 Proposed method

It is well known that the concept lattice reduction is one of the major issues for knowledge processing tasks as discussed by (2010). This inherited to its extension into bipolar space also as discussed by Singh and Cherukuri (2014a; 2014b). In this section, a new method is proposed to measure the information content of the given bipolar fuzzy concepts using the calculus of Shannon entropy. To select some of the relevant bipolar fuzzy concepts based on user-required granulation for the computed weight, the mathematical paradigm of granular computing provides a way to zoom in and zoom out the given context based on user-required window of information as granules (Yao 2004). The information granules are nothing but collections of relevant information to refine the knowledge. It means the relevant of information can be based on computed weight (Singh 2018), similarity index (Macko 2001) or indistinguishable attributes Singh (2019a). In this way, the granulation provides several window, i.e., (α_1, α_2) to modularize the complex problem into a series of well-defined sub-problems at micro–macro-level of defined window ($0 \leq \alpha_1 \leq \alpha_2 \leq 1$) (Singh 2022c). In this way, the granular computing provides another alternative way to analyze the average information content and its quantification for multi-decision process (Singh 2022a), due to which the algebra of granular computing is used for controlling the binary concept lattice (Wu et al. 2009; Zhang et al. 2012), fuzzy concept lattice (Singh et al. 2017; Singh 2022d), bipolar fuzzy concept lattice (Singh and Cherukuri 2014a,b) as well as Plithogenic context (Singh 2022c). This paper focuses on dealing the data with bipolar fuzzy attributes using Shannon entropy as given below:

Definition 7 (*Entropy*) (Shannon 1948): It measures the randomness and uncertainty in the given attributes. The more the randomness higher the entropy and vice versa. It is considered as one of the important metrics to quantify the diversity of randomness with lossless compression. The advantages of entropy are that it compresses the data including same information with less redundancy, less time and space as: $E(p_i) = \text{-}p_i \log_2(p_i)$. It represents the objects possessing the corresponding j^{th}-attribute $\frac{(1,0)}{y_j} \in Y$ can be computed by $p_i = \mathrm{P}\left(\frac{\frac{(1,0)}{y_j}}{\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i}}\right)$. Recently, it is applied for dealing fuzzy concept lattice (Singh and Gani 2015) and its extension to complex space for extraction of important concepts (Singh 2022d). This paper focused on utilizing its properties for reducing the bipolar fuzzy concepts using the paradigm of granular computing.

The entropy and granular computing can be used as follows to select some of the relevant bipolar fuzzy concepts based on user requirements for knowledge processing tasks:

Let us consider a bipolar fuzzy concepts (A, B) having j^{th}-attribute $\frac{(1,0)}{y_j} \in Y$ which covers bipolar fuzzy set of object (A), i.e., $\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i} \in X$ in the given bipolar fuzzy context $\mathbf{F}$. The probability of bipolar fuzzy set of objects—$\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i}$ possessing the corresponding j^{th}-attribute $\frac{(1,0)}{y_j} \in Y$ can be computed by $p_i = \mathrm{P}\left(\frac{\frac{(1,0)}{y_j}}{\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i}}\right)$. It provides number of $\frac{(1,0)}{y_j} \in Y$ covers the bipolar fuzzy set of objects $\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i}$ divided by the total number of objects having the membership $\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i}$. The number of attributes having bipolar fuzzy relation as $(\mu^P(x_i, y_j) \geq \mu^P(x_i))$ and $(\mu^P(x_i, y_j) \leq \mu^N(x_i))$ at particular row–column. The information weight for this bipolar objects, i.e., $\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i} \in X$ to possess the attribute $\frac{(1,0)}{y_j}$ can be computed by Entropy $E(p_i) = \text{-}p_i \log_2(p_i)$. In case the concept (B) contains k-number of objects to possess the attributes $\frac{(1,0)}{y_j} \in Y$. The total information weight can be computed as, i.e., $w_j = \sum_{i=1}^{n} E(p_i)$. The average information weight can be computed as $Weight(B_k) = \frac{\sum(w_i)}{n}$ where n is number of bipolar fuzzy set of objects which possess the attributes. In this way, the randomness in bipolar fuzzy attributes can be considered as computed using Shannon entropy within interval [0, 1] for knowledge processing tasks as shown in Fig. 3.

The steps of the proposed method can be defined as follows based on the flowchart shown in Fig. 3:

Step 1 Let us consider, a bipolar fuzzy concepts (A, B) having j^{th}-attribute $\frac{(1,0)}{y_j} \in Y$.

Step 2 List all of its covering bipolar fuzzy set of objects $\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i} \in X$ possess the attribute $\frac{(1,0)}{y_j} \in Y$.

Step 3 Compute its probability to possess the j^{th}-attribute as: $p_i = \mathrm{P}\left(\frac{\frac{(1,0)}{y_j}}{\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i}}\right)$.

Step 4 The number of attributes having bipolar fuzzy relation as $(\mu^P(x_i, y_j) \geq \mu^P(x_i))$ and $(\mu^P(x_i, y_j) \leq \mu^N(x_i))$ at particular row–column.

Step 5 Compute the information weight for the object as: $E(p_i) = \text{-}\mathrm{P}\left(\frac{\frac{(1,0)}{y_j}}{\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i}}\right) \log_2(\mathrm{P}\left(\frac{\frac{(1,0)}{y_j}}{\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i}}\right))$. The n represents total number of bipolar fuzzy set of objects which possess the attributes, i.e., $\frac{(1,0)}{Y}$.

Step 6 Sum the information weight for each bipolar fuzzy set of objects (A) which possess the attribute $\frac{(1,0)}{y_j} \in Y$ as $w_j = \sum_{i=1}^{n} E(p_i)$

Step 7 The average information weight for the bipolar fuzzy concepts (A, B) using their intent (B) can be computed as $W_j = \frac{\sum(w_j)}{n}$. The n represents total number of bipolar fuzzy attributes which covers the attribute $\frac{(1,0)}{y_j} \in Y$.

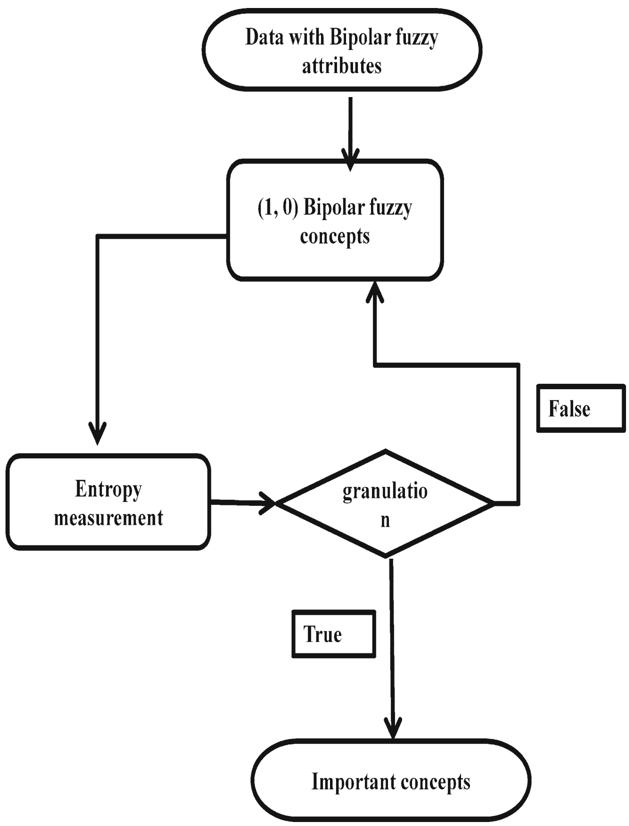

Fig. 3 The flowchart of the proposed method

Step 8 In similar way the information weight for each bipolar fuzzy concepts can be found.

Step 9 The proposed method can be applied using the extent also as vice versa via fixing (1, 0) for object set.

Step 10 The bipolar fuzzy concepts having maximum average information weight can be considered as an important for knowledge processing tasks and others can be removed.

Step 11 In case the expert wants to select some specific bipolar fuzzy concepts then windows for granulation can be defined (α_1, α_2) to refine the concepts based on their weight.

Step 12 The bipolar fuzzy concepts can be selected iff the computed weight in the defined window as: $(\alpha_1 \leq w_j \leq \alpha_2)$.

Step 13 The bipolar fuzzy concepts which weight are not in the defined window of granulation can be removed.

Step 14 Interpret the obtained bipolar fuzzy concepts for knowledge processing tasks.

Step 15 The pseudocode for the proposed algorithm is shown in Table 5.

Complexity: Let us suppose the bipolar fuzzy context **F** contains n—number of objects and m—number of bipolar fuzzy attributes. In this case, the proposed method may take O $(n \times log(n))$ time to compute the entropy weight for the maximal acceptance of attributes $\frac{(1,0)}{y_j} \in Y$. In addition, it computes the bipolar fuzzy relations and its computation for m—number of attributes. In this case, the maximum time complexity that can be taken by the proposed method is $O(m \times n \times log(n))$.

4 Illustration

Recent time dealing the data with bipolar fuzzy attributes is considered as one of the major issues (Singh 2019a, b; Riaz and Therim 2021) for multi-decision process (Ali et al. 2019; Akram et al. 2020) and other applications (Akram et al. 2021a, b; Sarwar and Akram 2018; Singh 2022a, c). In this process, a problem arises while dealing with large number of bipolar fuzzy concepts as discussed by Singh and Cherukuri (2014a; 2014b). The problem becomes more crucial in case the expert wants some of the relevant bipolar fuzzy concepts based on his/her requirement to refine the pattern as discussed by Singh 2019a. To achieve this goal, entropy theory introduced by Shannon (1948) has played a major role in computing the information weight of formal concepts (Singh and Gani 2015; Singh et al. 2017; Singh 2018; Sumangali et al. 2017; Singh 2022d). Motivated from these studies, the current paper focused on measuring the information content of bipolar fuzzy attributes using Shannon entropy and its selection by user-defined information granules. To achieve this goal, a method is proposed in Sect. 3 which is illustrated step by step in this section as given below:

Example 2 Let us suppose, a company manufactures set of cars (objects)= $\{x_1, x_2, x_3\}$ considering some parameters (y_1=Modern Technology, y_2=Cost, y_3=Fuel efficient) using the customer feedback as discussed in Singh and Cherukuri (2014(a,b). Now suppose, 30 percent customers given positive feedback about car x_1 considering its modern technology y_1, 20 percent satisfied with price details (y_2) and only 10 percent customer convinced due to its fuel efficiency (y_3). This type of positive feedback or rating can be collected for other cars (x_2, x_3) which can be written in a matrix format as shown in Table 6. In case, the 30 percent customer given positive feedback means 70 percent rejected or given negative feedback about the car. The negative side also need to be considered for precise analysis of decision making as shown in Table 7.

It is well known that both positive and negative side of feedback are integral part of cars (x_1, x_2, x_3) which can be represented using a bipolar fuzzy context as shown in Table 8. The following problem arises with Car company that need to analyze the customer feedback which car is more preferable or relevant for the given customer. It will be helpful to increase the production of car as well as profit. Same time company wants that which car production need to be stopped due to customer negative feedback. This issue can be resolved when some useful pattern from Table 8 as discussed by Singh and Cherukuri (2014a; 2014b). In this process, a problem

Table 5 A Proposed algorithm for computing the weight of bipolar fuzzy concepts

Input: A bipolar fuzzy concepts having attribute $\frac{(1,0)}{y_j} \in Y$ Outputs: Weight of bipolar concepts W_j	
Steps of the proposed method	Time complexity
(1) Let us consider the bipolar concept (W_j) having the attribute $\frac{(1,0)}{y_j} \in Y$.	O (1)
(2) Write the objects which possess the $\frac{(1,0)}{y_j}$ as: $\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i} \in X$.	O(n)
(3). **for** $i = 1, ..., n$ where n represents total number of bipolar objects.	O($n \times m$)
(4). Compute the probability to possess the attribute $\frac{(1,0)}{y_j}$: $p_i = \mathrm{P}\left(\frac{\frac{(1,0)}{y_j}}{\frac{(\mu^P(x_i),\mu^N(x_i))}{x_i}}\right)$.	
(5). Information weight, i.e., $E(p_i)$=-$p_i \log_2(p_i)$.	O($n \times log(n)$)
(6). Weight, i.e., $w_j = \sum_{i=1}^{n} E(p_i)$.	O($m \times n \times log(n)$)
(7). Average Weight, i.e., $W_j = \frac{\sum(w_j)}{n}$.	O($m \times n \times log(n)$)
(8). Return the weight W_j	
(9). end **for**	
(10). Set the granulation window $[\alpha_1, \alpha_2]$ in [0, 1].	O(n)
(11). **if** $(\alpha_1 \le W_j \le \alpha_2)$	O($m \times n$)
(12). Select the concepts.	
(13). **else**	
(14). Remove the concepts.	
(15). Interpret the concepts.	
Total time complexity of the proposed method	O($m \times n \times log(n)$)

Table 6 A positive feedback of customer toward the car

	y_1	y_2	y_3
x_1	0.3	0.2	0.1
x_2	0.5	1.0	0.6
x_3	0.4	1.0	0.3

Table 7 The negation of context shown in Table 6 as negative feedback

	y_1	y_2	y_3
x_1	–0.7	–0.8	–0.9
x_2	–0.5	–0.0	–0.4
x_3	–0.6	–0.0	–0.7

Table 8 The bipolar fuzzy context representation of Table 6 and 7

	y_1	y_2	y_3
x_1	(0.3, –0.7)	(0.2, –0.8)	(0.1, –0.9)
x_2	(0.5, –0.5)	(1.0, –0.0)	(0.6, –0.4)
x_3	(0.4, –0.6)	(1.0, –0.0)	(0.3, –0.7)

is addressed while dealing with large number of concepts. Same time the problem arises while investigation of some relevant bipolar fuzzy concepts based on user requirement as discussed by Singh (2019a). It means the relevancy may change based on each customer. The current paper addressed this problem and try to fix it using the entropy theory and its computed weight for distinct granulation. To achieve this goal, a method is proposed in Sect. 3 . The obtained results from proposed method are compared with its concept lattice structure for validating the information.

Let us consider the bipolar fuzzy context shown in Table 8. The subset-based method was introduced by Singh and Cherukuri (2014a; 2014b) to generate the distinct bipolar fuzzy concepts using maximal acceptance of attributes (1, 0) as shown in Fig. 4. It represents following information:

C1 $\{\frac{(1,0)}{x_1} + \frac{(1,0)}{x_2} + \frac{(1,0)}{x_3}, \emptyset\}$.

Information: It shows that none of car satisfied each parameter of the customer.

C2 $\{\frac{(0.2,-0.8)}{x_1} + \frac{(1.0,0.0)}{x_2} + \frac{(1.0,0.0)}{x_3}, \frac{(1.0,0.0)}{y_2}\}$.

Information: The customer prefers Car x_2 and x_3 due to its cost.

C3 $\{\frac{(0.3,-0.7)}{x_1} + \frac{(0.5,-0.5)}{x_2} + \frac{(0.4,-0.6)}{x_3}, \frac{(1.0,0.0)}{y_1}\}$.

Information: The Car x_2 contains half traditional and half modern for better understanding of old and young customers.

C4 $\{\frac{(0.2,-0.7)}{x_1} + \frac{(0.5,0.0)}{x_2} + \frac{(0.4,0.0)}{x_3}, \frac{(1.0,0.0)}{y_1} + \frac{(1.0,0.0)}{y_2}\}$.

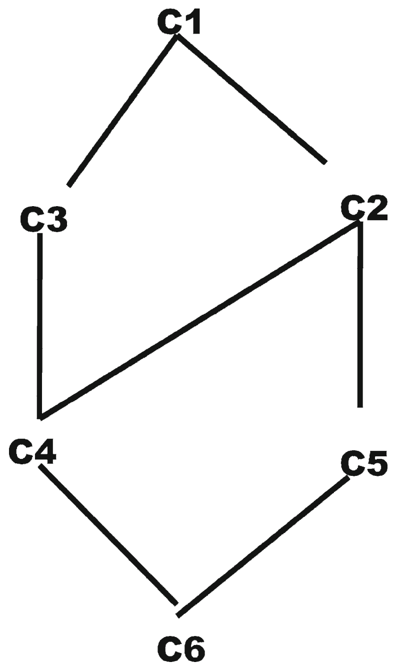

Fig. 4 A bipolar fuzzy concept lattice generated from Table 8

Information: The Car x_2 is preferred more by half young and half old people due to its cost and technology.

C5 $\{\frac{(0.1,-0.7)}{x_1}+\frac{(0.5,0)}{x_2}+\frac{(0.3,0.0)}{x_3}, \frac{(1.0,0.0)}{y_1}+\frac{(1.0,0.0)}{y_2}+\frac{(1.0,0.0)}{y_3}\}$

Information: The Car x_2 is first preference of the customers based on each parameters.

C6 $\{\frac{(0.1,-0.8)}{x_1}+\frac{(0.6,0.0)}{x_2}+\frac{(0.3,0.0)}{x_3}\}, \{\frac{(1.0,0.0)}{y_2}+\frac{(1.0,0.0)}{y_3}\}$

Information: The Car x_2 preferred 60 percent by customers due to its cost and fuel efficiency.

The above generated bipolar fuzzy concepts represent that Car (x_2) will be first preference whereas Car x_3 will be second preference of customer based on the given feedback. Very few customers want to purchase the Car (x_1) due to its traditional technology, cost and less fuel efficiency. This will help the production team while manufacturing the car as well as sells team.

The problem arises when the expert wants to analyze cars and its relevancy based on customer requirements and its given feedback. In this case, the expert requires single-valued weighted representation to deal the bipolar opinion given by customer. To deal with this problem, Shannon entropy is considered as one of the prominent information theoretical measurements as discussed by Li et al. (2013) as well as Singh and Gani (2015). The current paper utilized it for dealing the data with bipolar fuzzy attributes in this paper. To achieve this goal, a method is proposed in Sect. 3 which is illustrated in this section. The proposed method is illustrated using extent as formal concepts can be uniquely identified either by extent or intent vice versa.

Table 9 Extent(A) for fuzzy formal concepts shown in Fig. 4

Bipolar Concepts	Extent shown in Fig. 4
C1	$\{\frac{(1,0)}{x_1}+\frac{(1,0)}{x_2}+\frac{(1,0)}{x_3}\}$
C2	$\{\frac{(0.2,-0.8)}{x_1}+\frac{(1.0,0.0)}{x_2}+\frac{(1.0,0.0)}{x_3}\}$
C3	$\{\frac{(0.3,-0.7)}{x_1}+\frac{(0.5,-0.5)}{x_2}+\frac{(0.4,-0.6)}{x_3}\}$
C4	$\{\frac{(0.2,-0.7)}{x_1}+\frac{(0.5,0.0)}{x_2}+\frac{(0.4,0.0)}{x_3}\}$
C5	$\{\frac{(0.1,-0.7)}{x_1}+\frac{(0.5,0)}{x_2}+\frac{(0.3,0.0)}{x_3}\}$
C6	$\{\frac{(0.1,-0.8)}{x_1}+\frac{(0.6,0.0)}{x_2}+\frac{(0.3,0.0)}{x_3}\}$

Table 10 Computing probability and entropy of each bipolar objects of Table 9 using Table 8

Bipolar object	P(x_i)	$E(x_i)$
$\frac{(0.1,-0.7)}{x_1}$	1	0
$\frac{(0.1,-0.8)}{x_1}$	1	0
$\frac{(0.2,-0.7)}{x_1}$	0.66	0.40
$\frac{(0.2,-0.8)}{x_1}$	0.66	0.40
$\frac{(0.3,-0.7)}{x_1}$	0.33	0.53
$\frac{(1.0,0.0)}{x_1}$	0	0
$\frac{(0.5,-0.5)}{x_2}$	1	0
$\frac{(0.5,0.0)}{x_2}$	1	0
$\frac{(0.6,0.0)}{x_2}$	0.66	0.40
$\frac{(1.0,0.0)}{x_2}$	0.33	0.53
$\frac{(0.3,0.0)}{x_3}$	1	0
$\frac{(0.4,-0.6)}{x_3}$	0.66	0.40
$\frac{(0.4,0.0)}{x_3}$	0.66	0.40
$\frac{(1.0,0.0)}{x_3}$	0.33	0.53

To measure the information of bipolar fuzzy concepts shown in Fig. 4, its extent is shown in Table 9. Table 10 represents the computation of information entropy for each bipolar fuzzy set of objects shown in Table 8. Table 11 represents the average information weight computation of each bipolar fuzzy concepts shown in Table 9. Now the some of the relevant bipolar fuzzy concepts based on user-required information granules can be selected for knowledge processing tasks as shown in Table 12.

The information weight of Fig. 4 is 1.55 as per the proposed method. It is one of the advantages of the proposed method which transforms the bipolar fuzzy concepts into numerical format. It will be helpful in encoding the data derived from bipolar fuzzy context.

In case the expert wants to select of the important concepts from Fig. 4 then the concept having maximum weight in Table 11 can be considered an important. It can be observed that the concept **C2** $\{\frac{(0.2,-0.8)}{x_1}+\frac{(1.0,0.0)}{x_2}+\frac{(1.0,0.0)}{x_3}, \frac{(1.0,0.0)}{y_2}\}$ having maximum weight, i.e., 0.49 as per Table 11. It represents that each customer gave first importance to cost while purchasing the car. In similar way the second important parameter can be analyzed. These types of important bipolar fuzzy concepts cannot be selected using the subset-based method discussed in Singh and Cherukuri (2014a; 2014b). Same time the company wants to select some of the important concepts based on their closed window of granulation (α_1, α_2) in [0, 1]. It can be also achieved using the proposed method as shown in Table 12. To achieve this goal, the proposed method just take $O(m \times n \times log(n))$ rather than 2^m. It is one of the major significant advantages of the proposed method while dealing with bipolar fuzzy attributes.

Table 12 The reduction of bipolar concepts based on window $(\alpha_1, \alpha_2)$of granulation

Weight at granulation ($W_(j)$)	Selected bipolar fuzzy concepts
$0 \leq W_j \leq 1$	**C1, C2, C3, C4, C5, C6**
$0 \leq W_j \leq 0.48$	**C1, C3, C4, C5, C6**
$0 \leq W_j \leq 0.34$	**C3, C4, C5, C6**
$0 \leq W_j \leq 0.30$	**C4, C5, C6**
$0 \leq W_j \leq 0.26$	**C5, C6**
$0 \leq W_j \leq 0.12$	**C5**

5 Discussion

The precise analysis of human thought is one of the major concern for research communities. The first problem arises with its precise representation due to uncertainty and randomness as discussed by Shannon (1948). Boole (1854) tried to introduce a method for representation of human thought using the binary logic {0, 1}. In this case, the problem arises while dealing the soft words like tall and young. To compute these types of linguistics words, Zadeh (1965) introduced fuzzy logic. The problem arises with fuzzy logic is that it used to represent the acceptation and rejection part via a single-valued membership values. The Taoism theory stated that positive–negative sides exist simultaneously to represent the information (Welch 1957). This type of bipolarity is depicted in Sanskrit as an example called as character of half man–half woman https://en.wikipedia.org/wiki/Ardhanarishvara. Same time action–reaction, love–hate and many bipolar information exists in dark data sets as discussed by (Singh 2022a). It became indeed requirements to deal with bipolar information while measuring the soft data sets as discussed by Akram et al. (Akram et al. 2021a, b) recently. The problem arises when the experts want to analyze the uncertainty and randomness in customer feedback for multi-decision process. To deal with these type of information the algebra of bipolar fuzzy sets in [-1, 0) × [0, 1) (Zhang 1994) and its graphical representation (Akram 2011) is introduced for bipolar concept lattice visualization (Singh and Cherukuri 2014a, b). In this case, a problem is addressed while dealing the large number of bipolar fuzzy concepts generated from a given context. One of the solutions is to find some of the relevant bipolar fuzzy concepts based on its similarity or indistinguishably as discussed by Singh (2019a). The another solution is information measurement using the Shannon (1948) entropy as applied for dealing the data with fuzzy attributes (Singh and Gani 2015; Singh 2018); Singh et al. (2017); (Singh 2022d)) and vague attributes Singh (2022b). It is considered that the information measurement may save the information loss (Li et al. 2013; Sumangali et al. 2017) while considering the attribute reduction method (Ali et al. 2020). This motivated to introduce the entropy theory for dealing with data of bipolar fuzzy attributes to tackle the following issues:

- Is it possible to measure the information content in the bipolar fuzzy concept lattice ?
- Is it possible to select some of the important bipolar fuzzy concepts?
- Is it possible to reduce the bipolar fuzzy concepts based on user-required granulation?

To deal with above issues, the current paper proposed following methods in this paper:

Table 11 Computed average weight $W_(j)$ for bipolar extent shown in Table 10

Concept–Extent(A)	$w_j = \sum_{i=1}^{n} E(p_i)$	$W_j = \frac{\sum(w_j)}{n}$
C1 $\{\frac{(1,0)}{x_1} + \frac{(1,0)}{x_2} + \frac{(1,0)}{x_3}\}$	0+0.53+0.53=1.06	0.35
C2 $\{\frac{(0.2,-0.8)}{x_1} + \frac{(1.0,0.0)}{x_2} + \frac{(1.0,0.0)}{x_3}\}$	0.40+0.53+0.53=1.46	0.49
C3 $\{\frac{(0.3,-0.7)}{x_1} + \frac{(0.5,-0.5)}{x_2} + \frac{(0.4,-0.6)}{x_3}\}$	0.53+0+0.40=0.93	0.31
C4 $\{\frac{(0.2,-0.7)}{x_1} + \frac{(0.5,0.0)}{x_2} + \frac{(0.4,0.0)}{x_3}\}$	0.40+0+0.40=0.80	0.27
C5 $\{\frac{(0.1,-0.7)}{x_1} + \frac{(0.5,0)}{x_2} + \frac{(0.3,0.0)}{x_3}\}$	0+0+0=0	0
C6 $\{\frac{(0.1,-0.8)}{x_1} + \frac{(0.6,0.0)}{x_2} + \frac{(0.3,0.0)}{x_3}\}$	0+0.40+0=0.40	0.13
Total computed weight of Fig. 4		1.55

Table 13 Comparison of the proposed method with recent studies on bipolar fuzzy concepts

	Singh and Cherukuri (2014a, b)	Singh (2019a, b)	Proposed algorithm
1	Subset-based method	Euclidean distance	Bipolar entropy
2.	Bipolar concept lattice	Similar bipolar concepts	Similar bipolar concepts
3.	(α, β)-cut	(α, β)-cut	(α_1, α_2)-cut
4.	Specialization and Generalization	Ordering of Bipolar concepts	Important bipolar Fuzzy concepts
5.	Introduced bipolar Concept lattice	Introduced neighbor Bipolar concepts	Introduced average Information weight
6.	Small context	Small or Medium context	Small or Large context
7.	Resembled information From Table 7	Resembled information From Table 7	Resembled information From Table 7
8.	Do not Encode Fig. 4	Encode the Fig. 4	Encode the Fig. 4
9.	Takes more time	Takes less time	take less space and time
10.	Graph visualization	No graph	No graph
11.	Pattern	Similar concepts	Randomness measurement
12.	$(2^m \times n)$	$(m^2 \times n)$	$O(m \times n \times log(n))$

- The proposed method measures the average information weight of the bipolar fuzzy concepts using entropy as shown in Sect. 3 ,
- The proposed method provides a way to decide the importance of bipolar fuzzy concepts based on their computed weight as shown in Sect. 4,
- In last, the important bipolar fuzzy concepts and its selection at user-defined (α_1, α_2)-level of granulation as shown in Table 12.

The proposed method shown in Sect. 3 is illustrated for the bipolar fuzzy concepts generated from Table 8 as shown in Sect. 4. The obtained results from the proposed method is compared with its bipolar fuzzy concept lattice shown in Fig. 4. It can be observed that both of the information echo with each other whereas the proposed method provides a way to select some of the relevant concepts based on user-required granulation. To achieve this goal, the proposed method does not draw any graph or concept lattice. It just measures the information content of given bipolar fuzzy attributes within logarithmic time complexity. In this way, the proposed method is distinct from any of the available approaches on data with bipolar fuzzy attributes as shown in Table 13.

It can be observed that the proposed method in this paper is distinct from each of the available approaches for handling data with bipolar fuzzy attributes as shown in Table 13. The proposed method tried to resolve the issue of randomness arises in bipolar fuzzy concepts due to expert opinion. Same time it provides an alternative way to select some of the important bipolar fuzzy concepts based on its information content within $O(m \times n \times log(n))$ time complexity. It can be observed that the knowledge discovered by the proposed method resembled with subset-based method (Singh and Cherukuri 2014a,b) and bipolar Euclidean distance (Singh 2019a,b) with less time complexity. Subsequently, the proposed method provides a way to convert the information weight of bipolar fuzzy concept lattice shown in Fig. 4 into numerical format. It is one of the major advantages of the proposed method which will help in lossless bipolar fuzzy concepts compression and its encoding. However, the proposed method provides single-valued measurement due to algebra of Shannon entropy. In this case, the biasness measurement in expert decision and its interpretation may be complex. To resolve this issue, author will focus on exploring other metric (Akram et al. 2021a; Riaz and Therim 2021) for controlling the bipolar fuzzy concept and its implications (Singh 2022a). Same time the author will focus on dealing the superposition of human quantum cognition and its dynamic changes at given phase of time Singh (2020)

6 Conclusions and Future Research

This paper aimed at measuring the information weight in bipolar fuzzy concepts using Shannon entropy in [0, 1]-interval. The analysis derived from the proposed method is compared with recently available approaches on bipolar fuzzy graph representation of concept lattice as shown in Table 12. It can be observed that the proposed method resembled the obtained information within time complexity $O(m \times n \times log(n))$ rather than $m \times 2^m$. Same time the important bipolar fuzzy concepts are selected based on their computed weight at defined window of granulation (α_1, α_2).

In addition, the proposed method encodes the bipolar fuzzy concept lattice into numerical format which will be useful in encoding. It is one of the major advantages of the proposed method while dealing space-time trade-off. It distinguishes the proposed method from any of the available approaches as shown in Table 13.

In near future, the author work will focus on other applications of bipolar fuzzy concepts, bipolar fuzzy entropy and its dynamic changes at given phase of time for dealing the superposition of human cognition.

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Declarations

Conflict of Interest The author declares that there is no conflict of interest for this paper.

Ethical approval This article does not contain any studies with human or animal participants.

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