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METHODOLOGIES AND APPLICATION

A novel multicriteria decision making (MCDM) approach for precise decision making under a fuzzy environment

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Abstract
The existing crisp and fuzzy multicriteria decision making (MCDM) methods exhibit consistency, complexity, and reliability issues. To address these challenges, we propose a new MCDM method called fuzzy technique for best–worst analysis (FTBWA). In FTBWA, a decision-maker (DM) first identifies a set of criteria and then determines the best–worst criteria. Next, the DM performs the fuzzy reference comparisons between the best-to-other (BtO) and the others-to-worst (OtW) criteria using the linguistic expressions. The process results in fuzzy BtO and fuzzy OtW vectors, which are then defuzzified to obtain quantifiable values. Afterward, a maximin problem is built and solved to obtain the weights of criteria and alternatives. The best alternative can be selected based on the final score obtained by aggregating the weights of different sets of criteria and alternatives. Further, we propose a consistency ratio to check the reliability of the results of FTBWA. To verify the practicality and consistency of FTBWA, we perform two illustrative case studies. Moreover, we perform a comprehensive analysis considering a comparative analysis, rank reversal analysis, and support for group decision making. From the results, we observe that FTBWA outperforms existing fuzzy/crisp MCDM methods.

Keywords Fuzzy best–worst method · Fuzzy multicriteria decision making · Fuzzy reference comparisons · Decision-making methods · Soft computing · Consistency ratio

1 Introduction

According to Triantaphyllou (2000), “probably, the most perpetual intellectual challenge in science and engineering is how to make the optimal decision in a given situation, this is a problem as old as mankind.” We occasionally come across decision problems where we must make decisions that may significantly impact our future. Such decisions require careful analysis and consideration of multiple conflicting criteria, for example, selection of an appropriate cloud service provider (CSP) or Internet service, investment of savings, making appropriate career selection, buying a house or allocation of funds for a research project, etc.

Decision making is a process of choosing the best among multifarious substitutes keeping in sight the heterogenous decision criterion and priorities of the decision-makers (Rezaei 2016). The decision problems that require evaluation of multiple contradictory criteria fall in the realm of multicriteria decision making (MCDM). It is a discipline related to operations research that helps in decision making based on multiple decision criteria (Whaiduzzaman et al. 2014).

There are two important areas of decision making, namely multi-objective decision making (MODM) and multi-attribute decision making (MADM) also known as MCDM. MODM deals with the problems where decision space is continuous. The set of decision alternatives are designed using a mathematical framework. MODM problems may involve many alternatives. Each alternative is

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evaluated based on the degree to which it satisfies a constraint or multiple constraints. In contrast, MADM (also known as MCDM) focuses on the discrete aspect of the decision space. It is a popular approach for decision making where more than one decision criteria are involved. However, it considers only finite alternatives. A decision is made based on the attributes of decision alternatives (Rezaei 2015; Triantaphyllou 2000).

In the past, several MCDM approaches have been proposed to assist decision-makers in the process of decision making. These approaches include analytical hierarchy process (AHP) (Saaty 1977, 1994), analytical network process (ANP) (Saaty 2004; Saaty and Vargas 2013), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) (Hwang and Yoon 1981), Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) (Brans and Vincke 1985; Brans et al. 1986), Elimination and Choice Expressing Reality (ELECTRE) (Figueira et al. 2013; Govindan and Jepsen 2016; Roy 1978, 1991), VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) (Chang 2010; Jingzhu and Xiangyi 2008; Opricovic and Tzeng 2004) and so on. For further reading regarding MCDM methods, readers may refer to Hendriks et al. (1992), Kahraman (2008), Mardani et al. (2015), Triantaphyllou (2000), and Tzeng and Huang (2011).

The best–worst method (BWM) (Rezaei 2015) is a relatively recent addition to the fleet of MCDM methods. In contrast to a full matrix-based pairwise comparison in AHP, the BWM advocates vector-based comparison of best-to-others and others-to-worst criteria. Thus, it significantly reduces the number of comparisons. Based on the best–worst vector comparisons, a maximin mathematical problem is formulated and solved to compute optimal weights of criteria. Since the two vectors (i.e., best–worst) are more structured compared to a full matrix, the BWM produces more reliable and consistent results compared to AHP (Rezaei et al. 2016). BWM has two variants, a nonlinearly constrained model (Rezaei 2015) and a linearly constrained model (Rezaei 2016).

Despite several advantages, the implementation of crisp MCDM methods like BWM may confront significant limitations while handling real-world situations. This is because of the inability of these methods to handle imprecise, ambiguous, and vague information. Such situations in decision making can be addressed with the help of fuzzy set theory that provides an effective way to pamper imprecise and fuzzy data (Kahraman 2008). In the past, researchers have proposed several fuzzy MCDM methods including Fuzzy TOPSIS (Chen and Tsao 2008), Fuzzy AHP (Ouma et al. 2015; Torfi et al. 2010; Wang et al. 2008), Fuzzy ELECTRE (Chen et al. 2015; Hatami-Marbini and Tavana 2011), and so on.

Like fuzzy extensions of other MCDM methods, BWM has also been extended by researchers to cater to the fuzziness of human judgment. For instance, Aboutorab et al. (2018), Guo and Zhao (2017), and Mou et al. (2016) proposed the nonlinearly constrained fuzzy extensions of BWM, whereas Hafezalkotob and Hafezalkotob (2017) proposed a linearly constrained fuzzy extension of BWM. However, these fuzzy extensions of BWM possess several potential limitations as follows. (1) As the number of criteria increases, the implementation complexity of these methods increases manifolds. (2) Unsuitability for group decision making due to a further upsurge in implementation complexity as the number of decision-makers increases. (3) In case of inconsistent results, these methods require more time/effort of the decision-maker to revise the comparison. Given these limitations, further research is required to better utilize the capabilities of BWM under a fuzzy environment.

To address the limitations of present methods, in this paper, we propose a novel technique, namely the fuzzy technique for best–worst analysis (FTBWA). The FTBWA is based on the nonlinearly constrained model of BWM. In the proposed technique, first, the decision-makers identify decision criteria. Then, the decision-makers identify the best (most important) and the worst (least important) criteria. Next, the decision-makers perform fuzzy reference comparisons of best-to-others and others-to-worst criteria, which results in two vectors, namely fuzzy best-to-other (FBtO) and fuzzy others-to-worst (FOtW). To perform the fuzzy reference comparisons, the decision-makers use natural language expressions represented by triangular fuzzy numbers (TFNs). Next, the FBtO and FOtW vectors are converted to crisp vectors to obtain crisp best-to-other (CBtO) and crisp others-to-worst (COtW) vector. Based on the CBtO and COtW vectors, a maximin mathematical problem is formulated and solved for calculation of optimal weights. We propose a consistency ratio to check the reliability of the results of the proposed technique. The results of the comprehensive analysis show that compared to the existing methods, the proposed technique not only significantly reduces implementation and computational complexity but also produces more consistent and reliable results. The salient features of the proposed technique supported by the results of the comprehensive analysis are as follows. The proposed technique (1) offers more consistent and reliable results under a fuzzy environment; (2) significantly reduces implementation complexity; (3) appropriately handles the rank reversal problem; and (4) provides adequate support for group decision making.

The paper contributes to the literature in many ways. Some salient contributions of this paper are summarized below.

1. We propose a novel nonlinearly constrained fuzzy MCDM approach, namely FTBWA.
2. We propose a consistency ratio to check the reliability of the results of FTBWA. We also propose a ranking mechanism to rank the alternatives using the proposed technique.
3. We design three robust algorithms to support the real-world programming of FTBWA.
4. We present two case studies to appreciate the performance, practicability, expediency, and viability of FTBWA. The results show FTBWA produces highly consistent and reliable results.
5. We validate FTBWA using a comprehensive analysis considering the following. (1) A comparative analysis with existing methods (crisp/fuzzy). (2) Rank reversal analysis. (3) Adequacy of FTBWA to support group decision making. The results favor the proposed technique.

The rest of the paper is organized as follows. In Sect. 2, we briefly review the literature along with preliminary concepts of fuzzy set theory. In Sect. 3, we present the proposed technique along with the pseudocode to simplify its programming. In Sect. 4, we present two illustrative case studies to demonstrate the practical applications of FTBWA. Section 5 presents a comprehensive analysis. Finally, in Sect. 6, we conclude the paper and discuss future research directions.

2 Literature review

This section presents a brief survey of the BWM along with its fuzzy extensions. We also present a concise review of the related work. Moreover, we discuss preliminary concepts related to fuzzy set theory and TFNs.

2.1 Related work

BWM (Rezaei 2015) is relatively a new MCDM method. According to BWM, firstly, the decision-makers identify a set of decision criteria. Once the decision criteria are identified, decision-makers determine the best and the worst criteria. This is followed by the comparison of the best to all other and all other-to-worst criteria using a 1-9 integer scale. Finally, a maximin mathematical problem is formulated, which is solved to obtain optimal weights of decision criteria. The reliability of the outcome is assessed based on the consistency ratio of the comparison. Since its inception, the BWM method has attracted immense attention of the research community. Mi et al. (2019) conducted a state-of-the-art survey of the BWM. They performed a comprehensive analysis of the publications from 2015 to 2019 that utilized and extended this method. BWM has been utilized in several papers to support the decision-making process. Salimi and Rezaei (2018) used BWM to evaluate the research and development performance of fifty high-tech SMEs in the Netherlands utilizing data collected through surveys from R&D experts and SMEs. Gupta and Barua (2017) presented a three-phase methodology for supplier selection and applied BWM in the second phase of the framework to rank the selection criteria. Rezaei et al. (2015) used the BWM method and proposed a framework to evaluate and segment suppliers based on capabilities and willingness. They argued that their framework would assist organizations to efficiently allocate their managerial resources. Rezaei et al. (2016) applied BWM in the second stage of a three-phase methodology to choose optimal suppliers. Salimi and Rezaei (2016) used BWM to measure the efficiency of university-industry Ph.D. projects. BWM has also been used in combination with other methods by many authors to solve individual and group decision-making problems. You, Chen, and Yang (2016) combined BWM with ELECTRE and proposed a framework to solve multicriteria group decision-making problems. Serrai et al. (2017) explored a new mechanism for web-based service selection using skyline for scrutinization, BWM to assign weights to criteria, and VIKOR to rank the web services. Kheybari et al. (2019) evaluated several decision-making criteria using BWM to identify the best location for the production of bioethanol. Hussain et al. (2020) used BWM in combination with several other techniques and proposed an integrated approach to select cloud services with consensus while considering both QoS and quality of experience of the cloud customers.

In real-world problems, the crisp nature of BWM hinders its ability to handle incomplete and imprecise information arising from the qualitative judgment of decision-makers. For this reason, some researchers have proposed fuzzy extensions of BWM. For example, Mou et al. (2016) proposed intuitionistic fuzzy multiplicative BWM (IFMBWM) using intuitionistic multiplicative preference relations. They utilized the proposed approach to evaluate the weights of criteria regarding the severity of emphysema-infected patients. Guo and Zhao (2017) proposed FBWM based on the nonlinearly constrained model of BWM. FBWM performs fuzzy reference comparison of the best–worst vectors to compute fuzzy weights and then converts fuzzy weights to crisp weights using graded mean integration representation. Hafezalkotob and Hafezalkotob (2017) suggested an individual and group decision-making approach based on BWM. Aboutorab et al. (2018) proposed a fuzzy extension of BWM using Z-numbers and performed a case study to validate the proposed approach. Liao et al. (2019) proposed a BWM-based MCDM method utilizing hesitant fuzzy linguistic information. To illustrate the viability of their proposed method, they presented a case study relating to the evaluation of hospital performance.

As a result of the detailed analysis of BWM and its fuzzy extensions, we have identified several limitations that require attention and further exploration. Table 1 presents the identified limitations of BWM and its fuzzy extensions. In this research, to address the limitations of existing methods, we propose an innovative fuzzy MCDM technique, namely FTBWA.

2.2 Preliminary concepts

In this subsection, we discuss the basic concepts of fuzzy set theory (Zadeh 1996). The theory of fuzzy sets effectively takes care of the element of impreciseness and vagueness in problem solving.

2.2.1 Basic definitions

Definition 1 A fuzzy set is rooted in a nonfuzzy macrocosm of discourse. A fuzzy set $\tilde{H}$ in a macrocosm of discourse, W, is represented by a characteristic function $\mu_H = W \to [0,1]$. It relates with every element w in the universe W a number $\mu_{\tilde{H}}(w)$ in the interval [0, 1], where $\mu_{\tilde{H}}(w)$ represents the membership grade of w in $\tilde{H}$. Closer the value of $\mu_H(w)$ to unity makes membership grade of w in $\tilde{H}$ higher (Zadeh 1996).

Definition 2 A number is said to be fuzzy if a fuzzy set $\tilde{H}$ on W satisfies the following conditions. (1) Must be a normal fuzzy set. (2) Should be closed interval for every α ∈ (0,1]. (3) The support of $\tilde{H}$ should be bounded (George and Yuan 1995)

Definition 3 A triangular fuzzy number (TFN) is illustrated in the form of trinity, i.e., $\tilde{H}$ = (x, y, z). The characteristic function $\mu_{\tilde{H}}(w) : W \to [0,1]$ of a TFN $\tilde{H}$ can be framed mathematically as shown. Figure 1 represents the TFN characteristic function.

$$\mu_{\tilde{H}}(w) = \begin{cases} \dfrac{w-x}{y-x} & \text{if } x \le w \le y \\ \dfrac{z-w}{z-y} & \text{if } y \le w \le z \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

For further details regarding TFNs and other technical information, readers can refer to (Kauffman and Gupta 1985; Shyamal and Pal 2007; Yong 2009)

Definition 4 The defuzzification of the triangular fuzzy set $\tilde{H} = (x, y, z)$ can be achieved using the following equation (Kumar et al. 2011).

$$D(\tilde{H}) = \frac{1}{4}(x + 2y + z) \tag{2}$$

Definition 5 Linguistic variables are everyday speech lexes utilized to delineate a certain fuzzy set in a given problem, i.e., "very hot," "hot," "warm," or "normal," etc. (Zadeh 1984).

2.2.2 TFN arithmetic operations

The following arithmetic operations can be performed between two TFNs defined on a universal set of real number R (Kaufmann and Gupta 1991). Let suppose $\tilde{Y} = (h, i, j)$, and $\tilde{Z} = (k, l, m)$ are two TFNs then

Table 1 BWM and fuzzy extensions along with some limitations

References	Proposed	Validation	Limitations
Rezaei (2015, 2016)	BWM	Numerical examples	Inability to handle imprecise/inexact information
Mou et al. (2016)	Intuitionistic fuzzy BWM	Case study	(1) The asymmetrical scale leads to confusion. (2) High computational cost due to the complex comparison system. (3) Difficult to revise comparison in case of low consistency. (4) Increased time, effort, and analysis complexity
Guo and Zhao (2017)	Fuzzy BWM	Performed three case studies	(1) Increased implementation complexity. (2) High computational complexity. (3) Inadequate for group decision making
Hafezalkotob and Hafezalkotob (2017)	GI-FBWM	Numerical examples	(1) The complexity of the already complex model increases with the increase in the size of the expert panel. (2) Complexity further increases with an increase in the number of criteria
Aboutorab et al. (2018)	Z-BWM	Performed a case study	(1) Subjectivity issues in the fuzzy part of Z-numbers. (2) High computational complexity. (3) Complex constraint and problem formulation. (4) Inadequate for group decision making

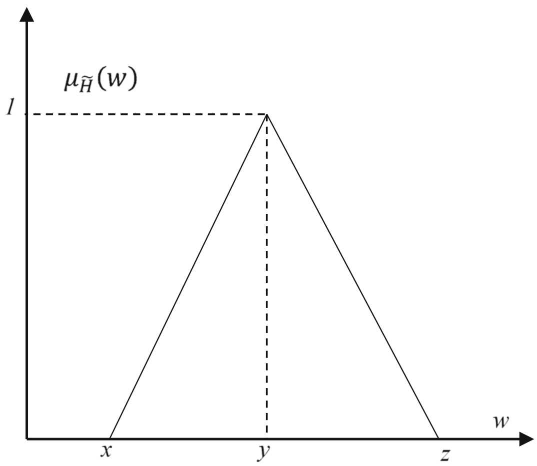


Fig. 1 TFN characteristic function

1. $\tilde{Y}+\tilde{Z}=(h,i,j)+(k,l,m)=(h+k,i+l,j+m)$,
2. $\tilde{Y}-\tilde{Z}=(h,i,j)-(k,l,m)=(h-m,i-l,j-k)$,
3. $-\tilde{Y}=-(h,i,j)=(-h,-i,-j)$,
4. Let $\tilde{Y}$ = $(h,\ i,\ j)$ be any TFN, and $\tilde{Z}=(k,l,m)$ be a non-negative TFN then,

$$\tilde{Y}\times\tilde{Z}\cong\begin{cases}(hk,il,jm),a\geq 0.\\(hm,il,jm),a<0,c\geq 0,\\(az,by,cx),c>0\end{cases}$$

3 Proposed fuzzy technique: FTBWA

In this section, we present the proposed FTBWA. It is a new MCDM method that is based on the nonlinearly constrained model of the BWM.

3.1 Transformation rules for linguistic expressions

If we assume n decision criteria (dc) for a decision-making problem, where we need to perform pairwise comparisons, then according to definition (5), we can perform such comparisons based on decision-makers' preferences in the form of ordinary language. We can then transform the linguistic expressions into TFNs. Table 2 shows the transformation rules. We adapt the transformation scale for linguistic variables from Guillaume and Charnomordic (2004).

3.2 Fuzzy reference comparison

The decision-makers perform the fuzzy reference comparison of best-to-other and other-to-worst criteria using linguistic expressions (Table 2). As a result of the comparison, the following fuzzified comparison matrix can be developed.

$$\tilde{A}=\begin{array}{c}\\ da_1\\ da_2\\ \vdots\\ da_m\end{array}\begin{array}{c}\begin{array}{cccc}dc_1 & dc_2 & \cdots & dc_n\end{array}\\ \begin{bmatrix}\tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n}\\ \tilde{a}_{21} & \tilde{a}_{22} & \ddots & \tilde{a}_{2n}\\ \vdots & \vdots & & \vdots\\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn}\end{bmatrix}\end{array}\quad(3)$$

where $\{da_1, da_2 \ldots da_m\}$ is set of alternatives and $\{dc_1, dc_2, \ldots, dc_n\}$ is a set of decision criteria, $\tilde{a}_{ij}$ depicts fuzzy relative priority of criteria i over criteria j, as these criteria are TFNs, $\tilde{a}_{ij}=(1,1,1)$ *when* $i = j$.

In our proposed technique, for $\tilde{A}$, there are 2n-3 fuzzy reference comparisons. These include n-2 best criteria to other criteria fuzzy reference comparisons; plus, n-2 other criteria to worst criteria fuzzy reference comparisons; plus, one best criterion to worst criteria fuzzy reference comparison.

Unlike BWM, in our proposed technique, the preferences of FBtO ($\tilde{a}_{bj}$) (i.e., stage 3) and FOtW ($\tilde{a}_{jw}$) vectors (i.e., stage 4) are TFNs based on linguistic expressions of decision-makers. The proposed technique uses a TFN transformation scale (Table 2) to perform comparisons based on the fuzzified priorities of the decision-makers (i.e., $\tilde{a}_{bj}$ and $\tilde{a}_{jw}$ vectors). Next, the TFN-based vectors (i.e., $\tilde{a}_{bj}$ and $\tilde{a}_{jw}$) are converted to crisp vectors, i.e., (CBtO and COtW). Finally, a maximin problem is formulated. The optimization problem is then solved to get optimal weights of decision criteria.

Definition 6 A pairwise fuzzy comparison $\tilde{a}_{ij}$ may be expressed as a fuzzified reference comparison, where i represents the best and j represents the worst element.

Table 2 Linguistic expressions transformation scale (Guillaume and Charnomordic 2004)

Linguistic/verbal expression	TFN representation
Equally vital (EV)	(1/2, 1, 3/2)
Weakly more vital (WMV)	(1, 3/2, 2)
Intensely more vital (IMV)	(3/2, 2, 5/2)
Very intensely more vital (VIMV)	(2, 5/2, 3)
Absolutely more vital (AMV)	(5/2, 3, 7/2)

3.3 Stages of the proposed FTBWA

The proposed technique comprises six stages as follows.

Stage 1. Frame the set of the decision-making criterion. This stage deals with the determination and consideration of the decision criteria (dc). Let us assume that there is 'n' decision criterion, i.e., {dc_1, dc_2, dc_3, …. dc_n}.

Stage 2. Identify the best (most preferred) and worst (least preferred) decision-criteria. This stage requires no comparison. A decision-maker determines/identifies the best and worst criteria in general.

Stage 3. Decide the fuzzified preference of best criteria over all other criteria using linguistic expressions. This stage requires careful execution of the fuzzified reference comparison. The best criteria determined by the decision-maker are compared to all other criteria using the linguistic expressions of the decision-maker (i.e., EV, WMV, IMV, VIMV, and AMV) (Guillaume and Charnomordic 2004), where the decision-maker assigns the preference of the criteria. The linguistic terms chosen in this process are then transformed to TFNs using the transformation scale presented in Table 2. The resultant FBtO vector is given below.

$$\tilde{A_B} = (\tilde{a}_{B1}, \tilde{a}_{B2}, \tilde{a}_{B3}, \ldots, \tilde{a}_{Bn}) \tag{4}$$

In Eq. (4), $\tilde{A_B}$ represents the FBtO vector $\tilde{a}_{Bj}$, where $\tilde{a}_{Bj}$ points to the fuzzy priority of best decision criterion B over all other criteria criterion j. It is clear that $\tilde{a}_{BB}$ = (1, 1, 1).

Stage 4. Decide the fuzzified preference of all-other criterion over the worst criterion using linguistic expressions. This stage requires the determination of fuzzified priorities of all other criteria over the worst criterion using the linguistic terms mentioned (Table 2). These fuzzified preferences are then converted to TFNs using the transformation scale presented in Table 2. In this way, the following FOtW vector is obtained.

$$\tilde{A_w} = (\tilde{a}_{1w}, \tilde{a}_{2w}, \tilde{a}_{3w}, \ldots, \tilde{a}_{nw}) \tag{5}$$

In above Eq. (5), $\tilde{A_w}$ shows the FOtW vector $\tilde{a}_{jw}$, which points to the fuzzy priority of other criteria j over the worst criteria W. It is clear here that $\tilde{a}_{ww}$ = (1, 1, 1).

Stage 5. Defuzzification of the FBtO ($\tilde{A_B}$) and FOtW ($\tilde{A_w}$). In this stage, we defuzzify the FBtO and FOtW vectors to obtain crisp data. In the FBtO vector, as shown in Eq. (4), the $\tilde{a}_{Bj}$ refers to the fuzzy preferences of best-to-others criteria in the form of TFNs, i.e., $\tilde{a}_{Bj} = \left(x_{bj}, y_{bj}, z_{bj}\right)$. Similarly, in the FOtW vector, as shown in Eq. (5), the $\tilde{a}_{jw}$ refers to the fuzzy preferences of others-to-worst criteria in the form of TFNs, i.e., $\tilde{a}_{jw} = \left(x_{jw}, y_{jw}, z_{jw}\right)$. To proceed further, we need crisp values of FBtO and FOtW vectors. Therefore, by Definition 4, we can use Eq. (2) to defuzzify the FBtO and FOtW vectors. In this way, we obtain the crisp best-to-other vector (CBtO) and crisp others-to-worst (COtW) vector, as follows.

$$\text{CBtO} = C_B = (c_{B1}, c_{B2}, c_{B3}, \ldots, c_{Bn}) \tag{6}$$

$$\text{COtW} = C_w = (c_{1w}, c_{2w}, c_{3w}, \ldots, c_{nw}) \tag{7}$$

Stage 6. Compute the optimal weights (w_1^*, w_2^*, …, w_n^*). The stage concerns the calculation of the optimal weights of criteria. The weight of decision criteria is termed as optimal where for each pair w_B/w_j and w_j/w_W the following two conditions are true (1) $w_B/w_j = c_{Bj}$ and (2) $w_j/w_W = c_{jw}$. To satisfy these conditions for all j, we need to find a solution where the maximum absolute difference $\left|\frac{w_B}{w_j} - c_{Bj}\right|$ and $\left|\frac{w_j}{w_W} - c_{jw}\right|$ for all j is minimized. It is worth mentioning here that the proposed technique is different from the crisp BWM in the sense that c_{Bj} and c_{jw} refer to CBtO and COtW vectors that we obtained by the defuzzification of the TFN-based FBtO and FOtW. The nonlinearly constrained optimization problem can be formulated as follows.

$$\begin{gathered} \min \max_j \left\{ \left|\frac{w_B}{w_j} - c_{Bj}\right|, \left|\frac{w_j}{w_W} - c_{jw}\right| \right\} \\ \text{s.t.} \\ \sum_j w_j = 1, \\ w_j \geq 0, \ \text{for all}\, j. \end{gathered} \tag{8}$$

Equation (8) can be transformed to the following non-linear-programming problem.

$$\min \xi$$
$$\text{s.t.} \quad \begin{cases} \left|\dfrac{w_B}{w_j} - c_{Bj}\right| \leq \xi, \ \text{for all}\, j \\ \left|\dfrac{w_j}{w_w} - c_{jw}\right| \leq \xi, \ \text{for all}\, j \\ \sum_j w_j = 1, \\ w_j \geq 0, \ \text{for all}\, j. \end{cases} \tag{9}$$

By solving Eq. (9) optimal weights $\left(w_1^*, w_2^*, \ldots, w_n^*\right)$ can be obtained. In Sect. 3.5, we propose a consistency ratio to check the reliability level of the comparison performed using the proposed technique.

3.4 Ranking of alternatives using FTBWA

In this subsection, we present a mechanism for the ranking of the alternatives using FTBWA. Consider a decision-making problem k where we have an alternative value i pertaining to criteria j (k_{ij}), for example, in the cloud service selection problem where CPU performance is a criterion. We can obtain quantitative information about the CPU performance of all the alternatives. However, in some decision-making problems, the values k_{ij} are not available. For instance, consider the same service selection problem where service reputation is a criterion. There is no quantitative measure of reputation. In such problems where the values of k_{ij} are not available, the steps of FTBWA described above are also applied for alternatives (comparison of alternatives against each criterion) to find k_{ij} (the weight of alternative i against criterion j). Once the weights (w_j) of criteria and scores of all the alternatives (k_{ij}) are available, we then calculate the overall score of alternative i using the following equation.

$$S_i = \sum_{j=1}^{n} w_j k_{ij} \tag{10}$$

Sorting the value of S_i for all I, we can identify the most suitable alternative.

3.5 Consistency ratio for FTBWA

In this subsection, we propose a way to determine the consistency ratio for the proposed FTBWA.

Definition 7 When $\tilde{a}_{Bj} \times \tilde{a}_{jW} = \tilde{a}_{BW}$, the comparison is fully consistent, where $\tilde{a}_{Bj}$ represents the best-to-other criteria fuzzy preference, $\tilde{a}_{jW}$ represents others-to-worst criteria fuzzy preference, and $\tilde{a}_{BW}$ represents the fuzzy preference of best-to-worst criteria.

As shown in Table 2, the maximum possible fuzzy value of $\tilde{a}_{BW}$ is (5/2, 3, 7/2), which relates to the linguistic expression AMV (Table 2). The consistency of the comparison reduces when $\tilde{a}_{Bj} \times \tilde{a}_{jW} \neq \tilde{a}_{BW}$, i.e., $\tilde{a}_{Bj} \times \tilde{a}_{jW}$ is lower or higher than $\tilde{a}_{BW}$. The highest inconsistency occurs when the maximum value of $\tilde{a}_{Bj}$ and $\tilde{a}_{jW}$ becomes equal to $\tilde{a}_{BW}$. This results in ξ. As we know, $(w_B/w_j) \times (w_j/w_W) = w_B/w_W$. Given the greatest inequality because of assigning the maximum value by $\tilde{a}_{Bj}$ and $\tilde{a}_{jW}$, ξ is the value that needs to be subtracted from $\tilde{a}_{Bj}$ and $\tilde{a}_{jW}$ and added to $\tilde{a}_{BW}$ or evenly:

$$\left(\tilde{a}_{Bj} - \xi\right) \times \left(\tilde{a}_{jW} - \xi\right) = \left(\tilde{a}_{BW} + \xi\right) \tag{11}$$

For the minimum fuzzy consistency $\tilde{a}_{Bj} = \tilde{a}_{jw} = \tilde{a}_{BW}$, Eq. (11) can be framed as follows.

$$\left(\tilde{a}_{BW} - \xi\right) \times \left(\tilde{a}_{BW} - \xi\right) = \left(\tilde{a}_{BW} + \xi\right) \tag{12}$$

From Eq. (12), we can derive the following relation.

$$\xi^2 - \left(1 + 2\tilde{a}_{BW}\right)\xi + \left(\tilde{a}_{BW}^2 - \tilde{a}_{BW}\right) = 0 \tag{13}$$

For $\tilde{a}_{BW} = (x_{BW}, y_{BW}, z_{BW})$, the maximum possible fuzzy value is (5/2, 3, 7/2), which shows $x_{BW} = 5/2$, $y_{BW} = 3$, and $z_{BW} = 7/2$. This also indicates that the maximum value of x_{BW}, y_{BW}, and z_{BW} cannot be greater than 7/2. In this case, if we utilize the upper limit z_{BW} to compute the consistency index. The same is also applicable to other cases like $\tilde{a}_{BW} =$ (1/2, 1, 3/2), $\tilde{a}_{BW} =$ (1, 3/2, 2), $\tilde{a}_{BW} =$ (3/2, 2, 5/2), and $\tilde{a}_{BW} =$ 2, 5/2, 3). Therefore, Eq. (13) can be written as follows.

$$\xi^2 - \left(1 + 2z_{BW}\right)\xi + \left(z_{BW}^2 - z_{BW}\right) = 0 \tag{14}$$

where z_{BW} = 3/2, 2, 5/2, 3, and 7/2.

Solving Eq. (14) for different values of z_{BW}, we can obtain the maximum possible ξ. Next, we can use the maximum values as the consistency index (CI) as given in Table 3. We can obtain the consistency ratio (CR) based on the CI and ξ^* as follows.

$$\mathrm{CR} = \frac{\xi^*}{\mathrm{CI}} \tag{15}$$

CR belongs to ∈ 0, 1. The lower the value of the CR, the comparisons are considered more consistent, which leads to more reliable results.

3.6 Pseudocode for FTBWA

In this section, we present three algorithms for implementing the proposed technique. Algorithm 1 (Cal_weight) is the main algorithm that returns optimal weights obtained using FTBWA. It calls Algorithm 2 and Algorithm 3 to perform fuzzy reference comparisons. Algorithm 2 performs FBtO comparison, and Algorithm 3 performs FOtW comparisons. Next, we describe each algorithm in detail.

Algorithm 1 (Cal_weight) concerns the calculation of the weights using FTBWA. At line 1, it assigns a set of decision criteria (dc). At lines 2 and 3, it determines the best and the worst criteria. At lines 4 and 5, it calls Algorithm 2 and Algorithm 3, respectively, to perform fuzzy reference comparisons. At lines 6 and 7, this algorithm performs defuzzification to obtain CBtO and COtW vectors. At line 10, it makes use of OptW_FTBWA functions to calculate weights of criteria using the proposed technique. Finally, in line 12, it returns the optimal weights of criteria.

Algorithm 1. Cal_weight.

Input: Linguistic Expression: *LE*,

Output: Optimal Weights //optimal weights of criteria

1. *dc* ← assign a set of dc (step 1);
2. *BDC* ← Determine the Best Decision Criteria (*BDC*) (step 2)
3. *WDC* ← Determine the Worst Decision Criteria (*WDC*) (step 2)
4. $\tilde{\boldsymbol{A}}_{\boldsymbol{B}}$ ← FBtO_Comparison (*dc, BDC, WDC, LE*) (step 3)
5. $\tilde{\boldsymbol{A}}_{\boldsymbol{W}}$← FOtW_Comparison (*dc, BDC, WDC, LE*) (step 4)
6. $\boldsymbol{C}_{\boldsymbol{B}}$← Convert_FBtO_to_CBtO ($\tilde{\boldsymbol{A}}_{\boldsymbol{B}}$) (step 5)
7. $\boldsymbol{C}_{\boldsymbol{W}}$← Convert_FOtW_to_COtW ($\tilde{\boldsymbol{A}}_{\boldsymbol{W}}$) (step 5)
8. *m* ← |*dc*|
9. *for each i* in *m do*
10. *OptW_FTBWA*← Opt_weight ($\boldsymbol{C}_{\boldsymbol{B}}$, $\boldsymbol{C}_{\boldsymbol{W}}$) // (step 6)
11. *end for*
12. *return OptW_FTBWA*

Algorithm 2 concerns the FBtO reference comparison. From lines 3 to 6 it performs fuzzy reference comparisons of best-to-other decision criteria. At line 8, it returns the FBtO vector, i.e., $\tilde{A}_B$. Likewise, Algorithm 3 concerns the FOtW reference comparison. From lines 3 to 6, it performs a fuzzy reference comparison of others-to-worst decision criteria. At line 8, it returns the FOtW vector, i.e., $\tilde{A}_W$.

Algorithm 2. FBtO_Comparison (*dc, BDC, WDC, LE*)

Input: Decision-maker's Preference (DMP)

Output: FBtO vector

1. *m* ← |*dc*|
2. *for each i* in *m do*
3. *println*← ('Please assign a preference of' BDC 'over' dc_i '=')
4. *println*← ('Please use transformation scale to assign preferences' LE)
5. DMP*i*← Decision-maker's input
6. $\tilde{\boldsymbol{A}}_{\boldsymbol{B}}\boldsymbol{i}$← TFN_Transform (*DMPi*)
7. *end*
8. *return* $\tilde{\boldsymbol{A}}_{\boldsymbol{B}}$

Algorithm 3. FOtW_Comparison (dc, BDC, WDC, LE)

Input: Decision-maker's Preference (DMP)

Output: OtW vector

1. $m \leftarrow |\mathrm{DC}|$

2. *for each i* in *m do*

3. *println*← ('Please assign a preference of' $\mathrm{dc_i}$ 'over' WDC '=')

4. *println*← ('Please use transformation scale to assign preferences' LE)

5. *DMPi*← Decision-maker's input

6. $\boldsymbol{\widetilde{A}_W i} \leftarrow$TFN_Transform (*DMPi*)

7. *end*

8. return $\boldsymbol{\widetilde{A}_W}$

4 Case studies

This section presents two case studies using the proposed technique. The case studies describe the practical applications of FTBWA. The first case study involves the selection of the best CSP for a company that intends to shift from in-house to cloud-based computing. The second case study deals with the supplier segmentation problem. We adopt this case from Rezaei et al. (2015).

4.1 Case study 1: CSP selection

This case study involves the selection of a CSP for an organization. CSP selection is an important decision for any organization that intends to adopt internet-based computing. The decision-makers need to consider multiple quality of service (QoS) criteria according to the requirements of their organization. The decision-makers' judgment regarding the preference of such criteria may contain ambiguous and imprecise information. The proposed fuzzy technique can appropriately handle vagueness and impreciseness of decision-makers' judgment when making such critical decisions. The stage-wise implementation of the case study using the FTBWA is as follows.

Stage 1 Identify decision criteria. Table 4 presents the criteria identified for CSP selection.

Stage 2 determines the best and the worst criteria for CSP selection. Cost (dc3) is identified as the best and "security" (dc1) as the worst criterion.

Stage 3 Using the linguistic expressions (Table 2), the fuzzy reference comparisons of best-to-other criteria are performed. Table 5 presents the linguistic expressions for FBtO criteria comparison for this case study.

Based on the FBtO criteria comparison given in Table 5, we can use the corresponding TFN representation given in Table 2 to obtain the FBtO vector as follows.

$$\tilde{A_B} = \left[\left(\frac{5}{2},3,\frac{7}{2}\right),\left(1,\frac{3}{2},2\right),\left(\frac{1}{2},1,\frac{3}{2}\right),\left(2,\frac{5}{2},3\right),\left(1,\frac{3}{2},2\right),\left(\frac{3}{2},2,\frac{5}{2}\right)\right]$$

Stage 4 Again, using the linguistic terms (Table 2), the fuzzy reference comparison of others-to-worst criteria is performed. Table 6 shows the linguistic terms for the FOtW criteria comparison. Based on the fuzzy reference comparison (Table 6), we can use the corresponding TFN representation given in Table 2 to obtain the FOtW vector as follows.

$$\tilde{A_w} = \left[\left(\frac{1}{2},1,\frac{3}{2}\right),\left(\frac{3}{2},2,\frac{5}{2}\right),\left(\frac{5}{2},3,\frac{7}{2}\right),\left(\frac{3}{2},2,\frac{5}{2}\right),\left(1,\frac{3}{2},2\right),\left(1,\frac{3}{2},2\right)\right]$$

Stage 5 Obtain the CBtO and COtW vectors. In this stage, we defuzzify the FBtO vector (obtained in stage 3) and FOtW vector (obtained in stage 4) to obtain CBtO and COtW vectors, respectively. For this purpose, as mentioned in Sect. 3 (stage 5), we use Eq. (2) to convert TFN-based fuzzy preferences of decision-makers to crisp data. The resultant CBtO and COtW vectors are given below.

Table 3 CI for FTBWA

Linguistic expressions	EV	WMV	IMV	VIMV	AMV
$\tilde{a}_{BW}$	(1/2,1,3/2)	(1,3/2,2)	(3/2,2,5/2)	(2,5/2,3)	(5/2, 3, 7/2)
CI	3.80	4.56	5.29	6.00	6.69

Table 4 CSP selection criteria

Criteria	Description
dc1	Security (SC)
dc2	Data control (DC)
dc3	Cost (CO)
dc4	Reliability (RL)
dc5	Performance (PR)
dc6	Reputation (RP)

$$\mathrm{CBtO} = C_w = \left[\frac{1}{4} * (2.5 + 2 * 3 + 3.5), \frac{1}{4} * (1. + 2 * 1.5 + 2), \frac{1}{4} * (0.5 + 2 * 1 + 1.5), \frac{1}{4} * (2 + 2 * 2.5 + 3), \frac{1}{4} * (1 + 2 * 1.5 + 2), \frac{1}{4} * (1.5 + 2 * 2 + 2.5)\right]$$

$$\mathrm{CBtO} = C_w = (3, 1.5, 1, 2.5, 1.5, 2)$$

$$\mathrm{COtW} = C_w = \left[\frac{1}{4} * (0.5 + 2 * 1 + 1.5), \frac{1}{4} * (1.5 + 2 * 2 + 2.5), \frac{1}{4} * (2.5 + 2 * 3 + 3.5), \frac{1}{4} * (1.5 + 2 * 2 + 2.5), \frac{1}{4} * (1 + 2 * 1.5 + 2), \frac{1}{4} * (1 + 2 * 1.5 + 2)\right]$$

$$\mathrm{COtW} = C_w = (1, 2, 3, 2, 1.5, 1.5)$$

Stage 6 Compute the optimal weights of criteria. In the stage, we apply the proposed technique FTBWA to compute optimal weights. We formulate nonlinearly constrained problem (16) according to the procedure described in Sect. 3.3 (stage 6) of this paper.

Problem (16) can be solved to obtain optimal weights of decision criteria. Based on the results, the optimal weights of six decision criteria are dc1 = 0.0837, dc2 = 0.2002, dc3 = 0.2839, dc4 = 0.1346, dc5 = 0.1501, and dc6 = 0.1475, whereas the value of $k*$ is 0.3915. Because $a_{BW} = a_{31}$ = (5/2, 3, 7/2), the consistency index for this case is 6.69 (Table 4). Therefore, based on Eq. (15), the consistency ratio (ξ^*) for this case is 0.3915/6.69 = 0.0585. Closer the values of the ξ^* to zero means a higher level of consistency.

Table 5 Linguistic expressions for FBtO criteria comparison

Criteria	dc1	dc2	dc3	dc4	dc5	dc6
Best criteria (dc3)	AMV	WMV	EV	VIMV	WMV	IMV

$$\min k^*$$
$$\text{s.t.} \begin{cases} -k \leq \dfrac{w_{dc3}}{w_{dc1}} - 3 \leq k \\ -k \leq \dfrac{w_{dc3}}{w_{dc2}} - 1.5 \leq k \\ -k \leq \dfrac{w_{dc3}}{w_{dc4}} - 2.5 \leq k \\ -k \leq \dfrac{w_{dc3}}{w_{dc5}} - 1.5 \leq k \\ -k \leq \dfrac{w_{dc3}}{w_{dc6}} - 2 \leq k \\ -k \leq \dfrac{w_{dc2}}{w_{dc1}} - 2 \leq k \\ -k \leq \dfrac{w_{dc4}}{w_{dc1}} - 2 \leq k \\ -k \leq \dfrac{w_{dc5}}{w_{dc1}} - 1.5 \leq k \\ -k \leq \dfrac{w_{dc6}}{w_{dc1}} - 1.5 \leq k \\ w_{dc1} + w_{dc2} + w_{dc3} + w_{dc4} + w_{dc5} + w_{dc6} = 1 \\ w_{dc1}, w_{dc2}, w_{dc3}, w_{dc4}, w_{dc5}, w_{dc6} > 0 \\ k > = 0; \end{cases} \tag{16}$$

4.2 Case Study 2: Supplier segmentation problem (SSP)

To perform this case study, we adapt the supplier segmentation case discussed by Rezaei et al. (2015). The case

Table 6 Linguistic expressions for FOtW criteria comparison

Decision criteria	Worst criteria (dc1)
dc1	EV
dc2	IMV
dc3	AMV
dc4	IMV
dc5	WMV
dc6	WMV

Table 7 Supplier segmentation/performance evaluation criteria (Rezaei et al. 2015)

Identified criteria	Description
wc1	"Willingness to improve performance"
wc2	"Willingness to share information"
wc3	"Willingness to rely on each other"
wc4	"Willingness to become involved in a long-term relationship"

Table 8 Best to all other willingness criteria fuzzified preference, case study 2

Criteria	wc1	wc2	wc3	wc4
Best (wc1)	EV	VIMV	WMV	WMV

Table 9 All other-to-worst willingness criteria fuzzified preference, case study 2

Criteria	Worst criteria (wc2)
wc1	VIMV
wc2	EV
wc3	IMV
wc4	WMV

argues that capabilities and supplier's willingness to collaborate are of paramount importance for evaluation and subsequent segmentation of suppliers. They identified four willingness criteria for the evaluation of supplier performance. Table 7 shows the willingness criteria. Now, we utilize the usual six-step proposed technique FTBWA to compute optimal weights.

Stage 1 Table 7 shows the identified criteria for supplier evaluation.

Stage 2 "Willingness to improve performance" is identified as the best criterion. "Willingness to share information" is identified as the worst criteria for this case study.

Stage 3 Using Table 2 the fuzzy reference comparison is performed to obtain the fuzzy preference of the best criteria over all other criteria. Table 8 presents the linguistic expressions for fuzzy priorities of the best-to-other criterion.

The FBtO vector based on the information given in Table 8 and using the corresponding TFN representation (Table 2) can be obtained as follows.

$$\tilde{A_B} = \left[\left(\frac{1}{2}, 1, \frac{3}{2}\right), \left(2, \frac{5}{2}, 3\right), \left(1, \frac{3}{2}, 2\right), \left(1, \frac{3}{2}, 2\right)\right]$$

Stage 4 The fuzzy preference of others-to-worst criteria is obtained by performing the fuzzy reference comparison using Table 2. Table 9 shows the linguistic expressions for the fuzzified preference of all-the-other criteria compared to the worst criteria.

Based on the information given in Table 9 and using the matching TFN representation (Table 2), we can obtain the FOtW vector as follows.

$$\tilde{A_W} = \left[\left(2, \frac{5}{2}, 3\right), \left(\frac{1}{2}, 1, \frac{3}{2}\right), \left(\frac{3}{2}, 2, \frac{5}{2}\right), \left(1, \frac{3}{2}, 2\right)\right]$$

Stage 5 In this stage, we defuzzify the FBtO and FOtW vectors obtained in stage 3 and 4 to obtain the CBtO and COtW vectors. Based on the procedure described in stage 5 of the proposed technique, the CBtO and COtW vectors for SSP are given below.

$$\mathrm{CBtO} = C_w = \left[\frac{1}{4} * (0.5 + 2 * 1 + 1.5), \frac{1}{4} * (2 + 2 * 2.5 + 3), \frac{1}{4} * (1 + 2 * 1.5 + 2), \frac{1}{4} * (1 + 2 * 1.5 + 2)\right]$$

$$\mathrm{CBtO} = C_w = (1, 2.5, 1.5, 1.5)$$

$$\mathrm{COtW} = C_w = \left[\frac{1}{4} * (2 + 2 * 2.5 + 3), \frac{1}{4} * (0.5 + 2 * 1 + 1.5), \frac{1}{4} * (1.5 + 2 * 2 + 2.5), \frac{1}{4} * (1 + 2 * 1.5 + 2)\right]$$

$$\mathrm{COtW} = C_w = (2.5, 1, 2, 1.5)$$

Stage 6 Compute the weights of criteria. For this case study, using CBtO (C_B) and COtW (C_W) vectors obtained in stage 5, we formulate the following problem based on FTBWA to compute the optimal weights of criteria.

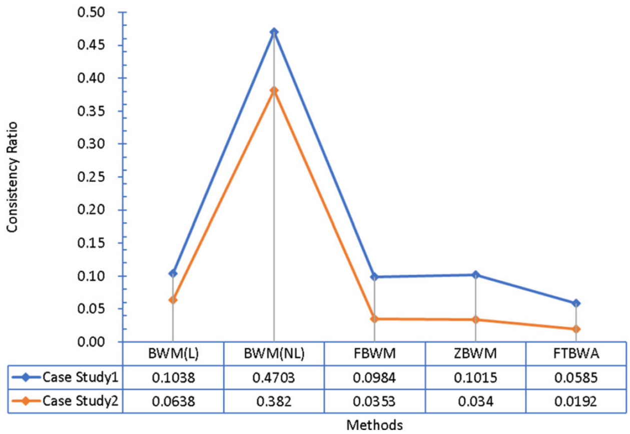


Fig. 2 Comparison of consistency ratio

$$\min k^*$$
$$\text{s.t.} \begin{cases} -k \leq \dfrac{w_{wc1}}{w_{wc2}} - 2.5 \leq k \\ -k \leq \dfrac{w_{wc1}}{w_{wc3}} - 1.5 \leq k \\ -k \leq \dfrac{w_{wc1}}{w_{wc4}} - 1.5 \leq k \\ -k \leq \dfrac{w_{wc3}}{w_{wc2}} - 2 \leq k \\ -k \leq \dfrac{w_{wc4}}{w_{wc2}} - 1.5 \leq k \\ w_{wc1} + w_{wc2} + w_{wc3} + w_{wc4} = 1 \\ w_{dc1}, w_{dc2}, w_{dc3}, w_{dc4} > 0 \\ k > 0 \end{cases} \tag{17}$$

We can solve Problem (17) to calculate the optimal weights of decision criteria. The calculated optimal weights are as follows, $w^*_{wc1} = 0.3670, w^*_{wc2} = 0.1407, w^*_{wc3} = 0.2651, w^*_{wc4} =$ 0.2272, and k^* = 0.1154. Because $a_{BW} = a_{12} = (2, 5/2, 3)$, the consistency index for this case is 6.00 (Table 4). Therefore, based on Eq. (15), the consistency ratio (ξ^*) for this case is 0.1154/6.00 = 0.0192. Closer the values of the ξ^* to zero means a higher level of consistency.

Based on the results of both the case studies, we can conclude that the proposed technique produces highly consistent and reliable results. To validate the performance of FTBWA, in the next section, we perform a comprehensive analysis.

5 Comprehensive analysis

In this section, we perform a comprehensive analysis of FTBWA. We analyze FTBWA considering (1) comparative analysis, (2) rank reversal analysis, and (3) adequacy to support group decision making.

5.1 Comparative analysis

To perform the comparative analysis, we consider (1) consistency ratio, (2) complexity of constraint formulation, (3) number of iterations to reach an optimal solution, and (4) elapsed runtime and memory usage.

5.1.1 Consistency ratio

To appreciate the performance of FTBWA in terms of the consistency ratio of the comparisons, we compare FTBWA with BWM (linear and nonlinear models) (Rezaei 2015, 2016). We also compare our proposed technique with two important fuzzy extensions of BWM, namely FBWM (Guo and Zhao 2017) and ZBWM (Aboutorab et al. 2018).

To perform this comparison, we use our case studies presented in Sect. 4 of this paper. We perform this comparison using similar preferences of decision criteria based on respective scales of each of the considered methods. To conduct a fair comparison, we keep the best and the worst criteria alike for all methods. Figure 2 presents the results of this comparison. In Fig. 2, BWM(L) represents the BWM linear model, and BWM(NL) represents the BWM nonlinear model.

We can observe from Fig. 2 that for case study 1, the consistency ratio obtained using our proposed technique is very close to zero (i.e., 0.0585). This is significantly better compared to BWM linear model (0.1038), BWM nonlinear model (0.4703), FBWM (0.0984), and ZBWM (0.1015). Likewise, for case study 2, the proposed technique (i.e., FTBWA) again produces highly consistent results with a consistency ratio of 0.0192. The results again favor our proposed technique compared to the linear BWM (0.068), nonlinear BWM (0.382), FBWM (0.0353), and ZBWM (0.034). Therefore, based on the results of this analysis, we can conclude that the proposed technique produces significantly more consistent and reliable results compared to the BWM (linear and nonlinear model), FBWM, and ZBWM.

5.1.2 The complexity of constraints formulation

In this section, we compare FTBWA with two well-known fuzzy extensions of BWM in terms of the complexity of constraint formulation. For this comparison, we design ten experiments where the value of n (criteria) ranges from three to thirty. Figure 3 presents the results of the comparisons. We can observe from Fig. 3 that the proposed technique requires a significantly fewer number of constraints compared to the existing methods. From Fig. 3, we observe that our proposed technique needs 70–73% fewer constraints, yet produces significantly more consistent and

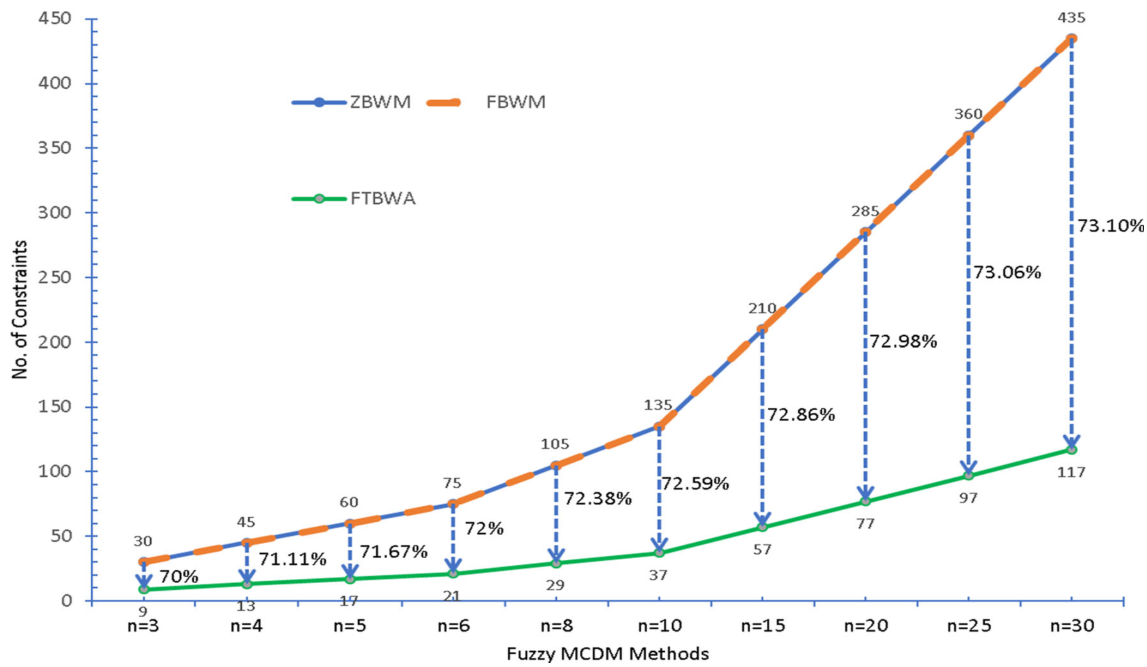


Fig. 3 Comparison of the number of constraints (fewer the better, the value of n is a random number)

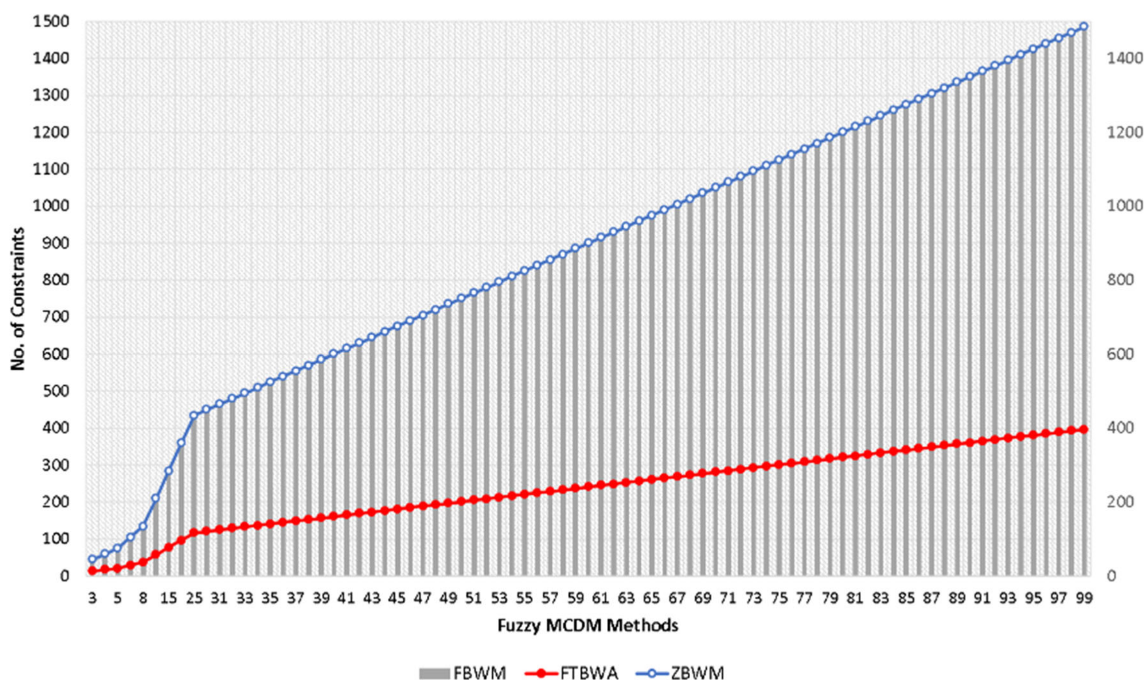


Fig. 4 Comparison of the number of constraints (fewer the better, the value of n ranges from 3 to 100)

reliable results as shown in Sect. 5.1.1. Moreover, we observe that as the problem size increases, the complexity of our proposed technique decreases compared to other fuzzy methods. For example, we can see from Fig. 3 that for $n = 3$ our proposed technique required 70% fewer constraints compared to FBWM and ZBWM, which further reduced to 72.59% for n = 10 and 73.10% for n = 30. To further strengthen the argument, we also performed experiments with the value of n ranging from 3 to 100. The results of all the experiments, as shown in Fig. 4, favor our proposed technique.

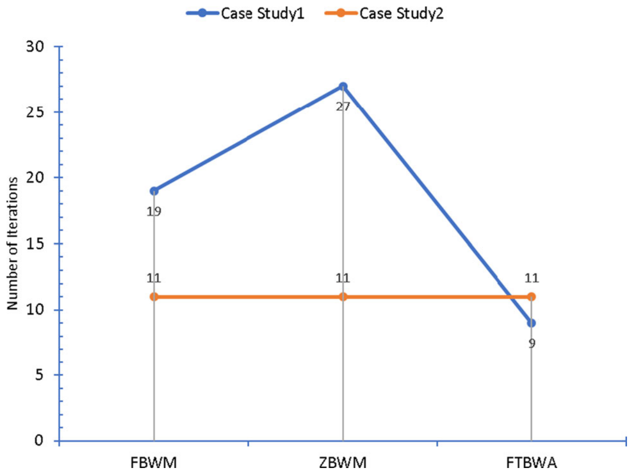


Fig. 5 Number of iterations to reach an optimal solution (lower the better)

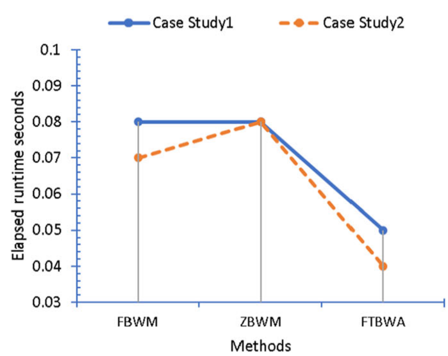
5.1.3 The number of iterations to reach an optimal solution

In this analysis, we compare our proposed technique with FBWM and ZBWM in terms of the number of iterations to reach an optimal solution. For each of our case studies (see Sect. 4), we note the number of iterations for each of the above-mentioned methods to reach the optimal solution. Figure 5 presents the results of this analysis. From the results, we can observe that for case study 1, compared to FBWM and ZBWM, the proposed technique requires a significantly lesser number of iterations to reach an optimal solution. For case study 2, FTBWA, FBWM, and ZBWM each require 11 iterations. It is quite interesting to note that as the problem size increases (as in the case of case study 1) the performance of the proposed technique compared to other methods improves significantly. For example, in case study 1 where the number of considered criteria (n) is 6 (i.e., $n = 6$), the proposed technique reaches an optimal solution in nine (9) iterations, whereas FBWM and ZBWM require 19 and 27 iterations, respectively. This shows that the proposed technique outperforms existing methods in terms of the number of iterations required to achieve an optimal solution.

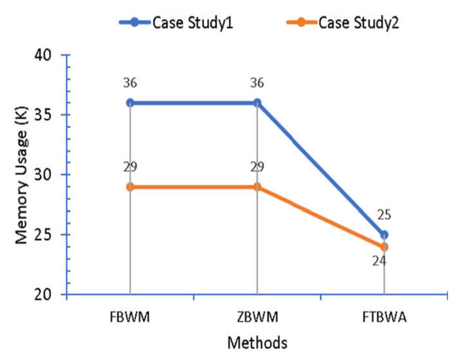
5.1.4 Elapsed runtime and memory usage

In this subsection, we compare FTBWA with existing methods in terms of elapsed runtime and memory usage. For this comparison, we use our case studies (see Sect. 4). Figure 6 presents the results of these comparisons. From Fig. 6a we observe that in both case studies, FTBWA performs better in terms of elapsed runtime compared to

Fig. 6 Comparison between proposed technique and other methods (lower the better)



(a) Comparison of elapsed runtime



(b) Comparison of memory usage

Table 10 Rank reversal analysis, CSP ranking under different scenarios

Scenario	Alternatives	Results (ranks obtained)
The initial test, three alternatives	CSP1, CSP2, CSP3	CSP3 > CSP2 > CSP1
Introduced near-identical copy (CSP4) of nonoptimal alternative (CSP1)	CSP1, CSP2, CSP3, CSP4	CSP3 > CSP2 > CSP1 > CSP4
Replaced nonoptimal alternative (CSP1) with the worst alternative (CSPx)	CSPx, CSP2, CSPx	CSP3 > CSP2 > CSPx
Removed alternative CSP2	CSP1, CSP3	CSP3 > CSP1
Removed alternative CSP1	CSP2, CSP3	CSP3 > CSP2
Removed alternative CSP3	CSP1, CSP2	CSP2 > CSP1

FBWM and ZBWM. Likewise, from Fig. 6b we observe the FTBWA requires lesser resources compared to FBWM and ZBWM.

From the results of this comparison, we observe, as the problem size increases, the FBWM and ZBWM require more time to reach an optimal solution compared to our proposed technique (FTBWA). Likewise, for larger problems, FTBWA requires fewer resources compared to FBWM and ZBWM.

5.2 Rank reversal analysis

Rank reversal analysis was proposed by Belton and Gear (1983). They noted that while making a decision using an MCDM method addition or deletion of an alternative to the existing set of alternatives may reverse the ranks. In this section, we test the adequacy of the proposed technique to handle rank reversal by adding and removing alternatives. For this testing, we use the same criteria as given in our first case study (Sect. 4.1). In addition, we initially consider three alternatives, i.e., CSP1, CSP2, CSP3. We perform comparisons of alternatives based on the weights computed in case study 1 and the performance scores of alternatives against each criterion.

To perform this analysis, we design different scenarios and perform comparisons using the proposed technique. Table 10 presents the results for each scenario. We can observe that under different scenarios including the addition and removal of alternatives, the proposed technique maintains the original ranks of alternatives. Thus, we can conclude that the method did not exhibit evidence regarding the reversal of ranks. However, according to Saaty and Vargas (1984) since the rank order of alternatives is dependent upon the relationship between new and existing alternatives under each criterion, the need for rank preservation is not a dogma (Saaty and Vargas 1984). The authors further argue that the “Rank reversal can be a good thing. That is how a new and important attribute can alter previous preferences. There is no law of nature that prohibits such a way of thinking.”

5.3 Support for group decision making

The existing fuzzy extensions of BWM like FBWM (Guo and Zhao 2017), ZBWM (Aboutorab et al. 2018), IFMBWM (Mou et al. 2016), and the model proposed by (Hafezalkotob and Hafezalkotob 2017) have many shortcomings that make them less suitable for group decision making. The problem formulation process of the above-mentioned methods is extremely complex, laborious, and burdensome. These methods require more time and effort of a decision-maker to process and analyze data. This increases the implementation complexity of these methods. The process becomes even more complex as the number of criteria and the number of decision-makers increase. In group decision making, data collection and analysis complexity of these methods increase manifolds because of (1) the involvement of multiple decision-makers, (2) increased number of criteria, and (3) increased complexity of the mathematical model. In contrast, our proposed technique is more suitable for group decision making under a fuzzy environment for the following reasons. (1) Simple constraint formulation. (2) Reduced implementation complexity. (3) Less computational complexity and better performance. (4) Better consistency ratio of the comparison.

6 Conclusion and future research

In this paper, we propose an innovative MCDM technique, namely FTBWA for precise decision making under a fuzzy environment. The FTBWA is based on the nonlinear model of the BWM. Unlike BWM, the proposed technique can effectively address the element of impreciseness and vagueness in the decision-making process. In our proposed technique, the decision-makers use linguistic expressions to obtain FBtO and FOtW vectors. The linguistic expressions are then converted to TFNs using a transformation scale. The fuzzified preferences of the decision-makers are converted to crisp values using defuzzification, and a maximin problem is formulated. The optimization problem is then solved to get optimal weights of the decision criteria. We propose a constancy ratio to check the reliability level of results. Further, we design three robust algorithms to facilitate the programming of the proposed technique.

To appreciate the efficacy, practicability, and consistency of the proposed technique, we present two illustrative case studies including a CSP selection problem and supplier segmentation problem. The results show, the proposed technique is practical, produces highly consistent results, and is viable for precise decision making under a fuzzy environment. We also perform a comprehensive analysis of the proposed technique considering (1) comparative analysis, (2) rank reversal analysis, and (3) suitability for group decision making. From the results of comparative analysis, we observe that compared to existing methods (i.e., FBWM, ZBWM, etc.), FTBWA has the following salient features. (1) FTBWA performs significantly better in terms of the consistency ratio of the comparison, which makes the results of FTBWA more steadfast and trustworthy. (2) The proposed method requires significantly fewer constraints. This substantially reduces the time and effort of the DMs in the decision-making process. In other words, FTBWA considerably reduces implementation complexity. (3) The proposed technique performs considerably better in terms of execution time, the number of iterations to achieve an optimal solution, and memory consumption. This makes the decision-making process swift and less burdensome in terms of computing resources. (4) FTBWA is more suitable for group decision making due to lower complexity, reduced computing overhead, and better consistency of the comparison.

In future work, uncertainty and subjectivity management techniques may be utilized to further enhance FTBWA. We plan to apply the proposed fuzzy technique to solve various real-world decision problems like service selection, resource allocation, supplier selection, etc. Moreover, we will combine the proposed technique with other MCDM approaches to further verify its usefulness in individual and group decision-making.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki Declaration and its later amendments or comparable ethical standards.

Informed consent Informed consent was obtained from all individual participants involved in this study.

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