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FOUNDATIONS

European option pricing under multifactor uncertain volatility model

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Abstract
This paper presents an uncertain stock model under the multifactor uncertain volatility framework. Based on the uncertainty theory, some closed-form and analytical formulas presented to value a European call and put option under the multifactor uncertain volatility model. Numerical tests are reported to highlight how the proposed model provides interesting results on pricing a European option. Finally, we summarize the theoretical results and some numerical experiments.

Keywords Uncertainty theory · Liu process · Multifactor model · European option

1 Introduction

Option is an important instrument in financial markets which derivates from underlying securities such as stocks and gives its holder the opportunity to purchase or sell—depending on the type of contract they hold—particular underlying stock at a specified strike price on or before the option expiration date. Option pricing models are mathematical models to estimate the fair value of an option. The valuation of the fair price of an option can help finance professionals to adjust their trading strategies. Therefore, option pricing models are powerful tools for finance professionals involved in options trading. In stochastic approach, the Black–Scholes model was one of the first models for investors to derive a fair price for options. The economists Fischer Black and Scholes (1973) provided this model which assumed that volatility is constant, but in reality, it is never constant in the long term. In fact, the volatility of asset prices changes throughout the trading day. Let us focus on Heston model and its extension. The Heston model as a stochastic volatility model was developed by Steven Heston (1993). The point that volatility is arbitrary rather than constant is the key factor that makes stochastic volatility models applicable. In order to match precisely the market implied volatility surface, it turns out that Heston model does not have enough parameters. Hence, Christoffersen et al. (2009) introduced the model providing more flexible modeling of the volatility term structure which called double Heston model. A number of articles in the literature are concerned with the development of the double Heston model. See, for example, Saber and Mehrdoust (2015) and Fallah and Mehrdoust (2019a, b).

In probability theory, we need a large amount of historical data to estimate a probability distribution and the mentioned models are based on probability theory. Besides, when no samples are available, some domain experts estimate the belief degree for each event. Degrees of belief represent the strength with which we believe the truth of various propositions (Huber and Schmidt-Petri 2009). Uncertainty theory is interpreted as personal belief degree, and it was founded by Liu (2007). This theory is a branch of mathematics and based on normality, duality, subadditivity and product axioms. The study of uncertain process was started by Liu (2008). Then, he presented a canonical Liu process as an uncertain counterpart of Wiener process (2009). In 2014, Liu introduced the concept of uncertainty distribution and inverse uncertainty distribution to describe uncertain variable. Uncertain calculus was developed by Liu (2009) to deal with differential equations and integration of function of uncertain process. Chen and Liu (2010), Yao (2013), Liu (2012), Wang (2012) and Yao and Chen (2013) presented some analytical and numerical methods to solve the uncertain differential

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equations. The proof of existence and uniqueness theorem of solution of uncertain differential equation was developed by Chen and Liu (2010). Liu (2009) proved stability of uncertain differential equation. Yao (2014) extended uncertain calculus to multi-dimensional with Liu process. Uncertain differential equation was proposed by Liu (2008). Then, Li et al. (2015) proposed a type of multi-factor uncertain differential equation throughout uncertain theory.

In finance, Liu (2009) introduced first uncertain stock model and provided some formulas for European option value. Peng and Yao (2011), Yu (2012), Chen et al. (2013), Yao (2015), Wang and Ning (2017), Sun et al. (2018), Jiao and Yao (2015), Yang et al. (2019), Tian et al. (2019a, b), Ji and Zhou (2015), Gao, Yang and Fu (2018) and Tian et al. (2019a, b) investigated widely valuing derivatives in uncertain financial markets. Also, Wang (2019) combined stochastic calculus and uncertainty theory and proposed a currency model, in which the exchange rate follows an uncertain differential equation and the interest rates obey stochastic differential equations. Hassanzadeh and Mehrdoust (2018) proposed a new stock model which is an uncertain counterpart of Heston model. The idea of modeling the variance by a variable of higher dimension motivates us to present double Heston model in uncertain framework. Indeed, in order to acquire a more realistic description of volatility dynamics in the Heston model, the number of factors that drives the volatility levels must be increased, and therefore, we extend the uncertain counterpart of the Heston model to multifactor extensions of the Heston model by incorporating two factors of volatility to the Heston model.

The main goal of this work is to provide a new uncertain stock model by using multifactor uncertain volatility model. In fact, this stock model is an uncertain counterpart of the two-factor Heston model. Uncertain volatility models feature an instantaneous variance of the asset price, the volatility. Multifactor uncertain volatility model is defined throughout uncertain multi-dimensional dynamics. We also study the behavior of European option price under the proposed model. Some theorems are proved, and a numerical method for price of a European option is derived.

The rest of this paper is organized as follows. In Sect. 2, we provide some basic definitions and theorems of uncertainty theory. Uncertain differential equation is presented in Sect. 3. Besides in Sect. 4, we defined two-factor structure for the uncertain volatility and the new stock model is presented. The value of a European call and put options is discussed in Sect. 5. Finally, some algorithms are provided in Sect. 6.

2 Preliminary

Liu (2007) established uncertainty theory as a branch of mathematics based on normality, duality, subadditivity and product axioms. In this section, we introduce some fundamental concepts and properties in uncertain theory.

Definition 1 (Liu 2007) Let $\mathcal{L}$ be a σ-algebra on a nonempty set Γ. Each element $\Lambda \in \mathcal{L}$ is called an event. A set function $\mathcal{M} : \mathcal{L} \to [0,1]$ is called an uncertain measure if it satisfies the following axioms:

Axiom 1 (*Normality axiom*) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.

Axiom 2 (*Duality axiom*) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.

Axiom 3 (*Subadditivity axiom*) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

The triple $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space. Besides, the product uncertain measure on the product σ-algebra was defined by Liu (2009) as follows:

Axiom 4 (*Product axiom*) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \ldots$. Then, the product uncertain measure $\mathcal{M}$ is an uncertain measure on product σ-algebra $\mathcal{L}_1 \times \mathcal{L}_2 \times \ldots$ satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\}$$

where Λ_k are the arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \ldots$, respectively.

Definition 2 (Liu 2007) An uncertain variable is a function X from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that $\{X \in B\}$ is an event for any Borel set B of real numbers. The uncertainty distribution $\Phi : \mathbb{R} \to [0,1]$ of an uncertain variable X is defined as

$$\Phi(x) = \mathcal{M}\{X \le x\}$$

for any real number x.

Definition 3 (Liu 2010) An uncertainty distribution $\Phi(x)$ is said to be regular if it is a continuous and strictly increasing function with respect to x at which $0 < \Phi(x) < 1$, and

$$\lim_{x \to -\infty} \Phi(x) = 0, \quad \lim_{x \to +\infty} \Phi(x) = 1.$$

Definition 4 (Liu 2015) An uncertain variable X is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(\mu - x)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad x \in \mathbb{R}$$

denoted by $\mathcal{N}(\mu, \sigma)$ where μ and σ are the real numbers with $\sigma > 0$.

Definition 5 (Liu 2007) Let X be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then, the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of X.

Definition 6 (Liu 2007) Let X be an uncertain variable. Then, the expected value of X is defined by

$$E[X] = \int_0^{+\infty} \mathcal{M}\{X \geq x\}\mathrm{d}x - \int_{-\infty}^{0} \mathcal{M}\{X \leq x\}\mathrm{d}x$$

provided that at least one of the two integrals is finite.

Theorem 1 (Liu 2010) *Let X be an uncertain variable with regular uncertainty distribution Φ. Then, we have*

$$E[X] = \int_0^1 \Phi^{-1}(\alpha)\mathrm{d}\alpha.$$

An uncertain process is essentially a sequence of uncertain variables indexed by time. The concept of uncertain process was introduced by Liu (2008).

Definition 7 (Liu 2008) Let $(\Gamma, \mathcal{L}, \mathcal{M})$ be an uncertainty space, and let T be a totally ordered set (e.g., time). An uncertain process is a function $X_t(\gamma)$ from $T \times (\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that $\{X_t \in B\}$ is an event for any Borel set B of real numbers at each $t \in T$. An uncertain process X_t is said to have independent increments if

$$X_{t_1}, X_{t_2} - X_{t_1}, \ldots, X_{t_k} - X_{t_{k-1}}$$

are independent uncertain variables where $t_1, t_2, \ldots, t_k$ are any times with $t_1 < t_2 < \cdots < t_k$.

Definition 8 (Liu 2014) An uncertain process X_t is said to have an uncertainty distribution $\Phi_t(x)$ if at each time t, the uncertain variable X_t has the uncertainty distribution $\Phi_t(x)$.

Definition 9 (Liu 2009) Let X_t be an uncertain process. For any partition of closed interval $[a, b]$ with $a = t_1 < t_2 < \cdots < t_{k+1} = b$, the mesh is written as

$$\Delta = \max_{1 \leq i \leq k} |t_{i+1} - t_i|.$$

Then, the time integral of X_t with respect to t is

$$\int_a^b X_t \mathrm{d}t = \lim_{\Delta \to 0} \sum_{i=1}^{k} X_{t_i}.(t_{i+1} - t_i)$$

provided that the limit exists almost surely and is finite. X_t is said to be time integrable.

Definition 10 (Liu 2009) An uncertain process C_t is said to be a canonical Liu process if.

(1) $C_0 = 0$ and almost all sample paths are Lipschitz continuous,
(2) C_t has stationary and independent increments,
(3) The increment of $C_{s+t} - C_s$ is a normal uncertain variable with expected value 0 and variance t^2.

The uncertainty distribution of C_t is

$$\Phi_t(x) = \left(1 + \exp\left(-\frac{\pi x}{\sqrt{3}t}\right)\right)^{-1}, \quad x \in \mathbb{R}$$

and its inverse uncertainty distribution is as follows

$$\Phi_t^{-1}(\alpha) = \frac{t\sqrt{3}}{\pi} \ln \frac{\alpha}{1 - \alpha}.$$

Definition 11 (Liu 2009) Let X_t be an uncertain process, and let C_t be a canonical Liu process. For any partition of closed interval $[a, b]$ with $a = t_1 < t_2 < \cdots < t_{k+1} = b$, the mesh is written as

$$\Delta = \max_{1 \leq i \leq k} |t_{i+1} - t_i|.$$

Then, Liu integral of X_t with respect to C_t is defined as

$$\int_a^b X_t \mathrm{d}C_t = \lim_{\Delta \to 0} \sum_{i=1}^{k} X_{t_i} \cdot (C_{t_{i+1}} - C_{t_i})$$

provided that the limit exists almost surely and is finite. By this definition, the uncertain process X_t is said to be integrable.

Definition 12 (Chen and Ralescu 2013) Let C_t be a canonical Liu process, and let V_t be an uncertain process. If there exist uncertain processes μ_t and σ_t such that

$$V_t = V_0 + \int_0^t \mu_s \mathrm{d}s + \int_0^t \sigma_s \mathrm{d}C_s$$

for any $t \geq 0$, then V_t is called a Liu process with drift μ_t and diffusion σ_t. Furthermore, V_t has an uncertain differential as follows

$$\mathrm{d}V_t = \mu_t \mathrm{d}t + \sigma_t \mathrm{d}C_t.$$

2.1 Uncertain differential equation

Uncertain differential equation is a type of differential equation involving uncertain processes. In this section, we discuss about existence and uniqueness of solutions of uncertain differential equations, Yao–Chen formula and multifactor uncertain differential equation.

Definition 13 (Liu 2008) Suppose C_t is a canonical Liu process, and f and g are two functions. Then,

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + g(t, X_t)\mathrm{d}C_t \tag{1}$$

is called an uncertain differential equation. A solution is a Liu process X_t that satisfies (1) identically in t.

Definition 14 (Yao and Chen 2013) Let α be a number with $0<\alpha<1$. An uncertain differential equation

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + g(t, X_t)\mathrm{d}C_t$$

is said to have an α-path X_t^α, if it solves the corresponding ordinary differential equation

$$\mathrm{d}X_t^\alpha = f\left(t, X_t^\alpha\right)\mathrm{d}t + \left|g\left(t, X_t^\alpha\right)\right|\Phi^{-1}(\alpha)\mathrm{d}t$$

where $\Phi^{-1}(\alpha)$ is the inverse standard normal uncertainty distribution, i.e.,

$$\Phi^{-1}(\alpha) = \frac{\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}.$$

In this case, X_t is called a contour process.

Theorem 2 (Yao and Chen 2013) *Let X_t and X_t^α be the solution and α-path of the uncertain differential equation*

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + g(t, X_t)\mathrm{d}C_t,$$

respectively. Then,

$$\mathcal{M}\left\{X_t \le X_t^\alpha, \forall t\right\} = \alpha,$$
$$\mathcal{M}\left\{X_t > X_t^\alpha, \forall t\right\} = 1-\alpha.$$

Theorem 3 (Yao and Chen 2013) *Let X_t and X_t^α be the solution and α-path of the uncertain differential equation*

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + g(t, X_t)\mathrm{d}C_t,$$

respectively. Then, the solution X_t has an inverse uncertainty distribution $\Psi_t^{-1}(\alpha) = X_t^\alpha$.

Theorem 4 (Yao and Chen 2013) *Let X_t and X_t^α be the solution and α-path of the uncertain differential equation*

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + g(t, X_t)\mathrm{d}C_t,$$

respectively. Then, for any monotone function J, we have

$$E[J(X_t)] = \int_0^1 J\left(X_t^\alpha\right)\mathrm{d}\alpha.$$

Definition 15 (Liu 2014) The uncertain processes $X_{1t}, X_{2t}, \ldots, X_{nt}$ are said to be independent if for any positive integer k and any times $t_1, t_2, \ldots, t_k$, the uncertain vector

$$\xi_i = (X_{it_1}, X_{it_2}, \ldots, X_{it_k}), \quad i = 1, 2, \ldots, n$$

is independent, i.e., for any Borel sets $B_1, B_2, \ldots, B_n$ of k-dimensional real vectors, we have

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}(\xi_i \in B_i)\right\} = \min_{1\le i\le n}\mathcal{M}\{\xi_i \in B_i\}.$$

Theorem 5 (Liu 2014) *Let $X_{1t}, X_{2t}, \ldots, X_{nt}$ be independent uncertain processes with regular uncertainty distributions $\Phi_{1t}, \Phi_{2t}, \ldots, \Phi_{nt}$, respectively. If $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then*

$$X_t = f(X_{1t}, X_{2t}, \ldots, X_{nt})$$

has an inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f\Big(\Phi_{1t}^{-1}(\alpha), \ldots, \Phi_{mt}^{-1}(\alpha), \Phi_{(m+1)t}^{-1}(1-\alpha), \ldots, \Phi_{nt}^{-1}(1-\alpha)\big).$$

Definition 16 (Yao 2014) Let C_t be an n-dimensional canonical Liu process. Suppose $f(t,x)$ is a vector-valued function from $T \times \mathbb{R}^m$ to $\mathbb{R}^m$, and $g(t,x)$ is a matrix-valued function from $T \times \mathbb{R}^n$ to $\mathbb{R}^n$ the set of $m \times n$ matrices. Then,

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + g(t, X_t)\mathrm{d}C_t \tag{2}$$

is called a multi-dimensional uncertain differential equation driven by a multi-dimensional canonical Liu process. A solution is an m-dimensional uncertain process that satisfies (2) identically at each $t \in \mathbb{R}$.

Theorem 6 (Li et al. 2015) *Assume that f and g_i, $i = 1, 2, .., n$, are continuous functions of two variables and $C_{1t}, C_{2t}, \ldots, C_{nt}$ are independent canonical processes. Let X_t and X_t^α be the solution and α-path of the uncertain differential equation*

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + \sum_{i=1}^{n} g_i(t, X_t)\mathrm{d}C_{it},$$

respectively. Then,

$$\mathcal{M}\left\{X_t \le X_t^\alpha, \forall t\right\} = \alpha,$$
$$\mathcal{M}\left\{X_t > X_t^\alpha, \forall t\right\} = 1-\alpha.$$

Theorem 7 (Hassanzadeh and Mehrdoust 2018) *Suppose that Y_t and Y_t^α be the solution and α-path of an uncertain differential equation*

$$\mathrm{d}Y_t = f_1(t, Y_t)\mathrm{d}t + g_1(t, Y_t)\mathrm{d}C_{1t},$$

respectively. Let $|h(t,y)|$ be a continuous increasing function. Then, the solution X_t of an uncertain differential equation

$$\mathrm{d}X_t = f_2(t, X_t)\mathrm{d}t + h(t, Y_t)g_2(t, X_t)\mathrm{d}C_{2t}$$

is a contour process with an α-path X_t^α that solves the corresponding ordinary differential equation

$$\mathrm{d}X_t^\alpha = f_2\left(t, X_t^\alpha\right)\mathrm{d}t + \left|h\left(t, Y_t^\alpha\right)g_2\left(t, X_t^\alpha\right)\right|\Phi^{-1}(\alpha)\mathrm{d}t$$

where

$$\Phi^{-1}(\alpha) = \frac{\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}, \alpha \in (0,1)$$

and C_{1t} and C_{2t} are the independent canonical Liu processes. In other words,

$$\mathcal{M}\left\{X_t \le X_t^\alpha, \forall t\right\} = \alpha,$$
$$\mathcal{M}\left\{X_t > X_t^\alpha, \forall t\right\} = 1-\alpha.$$

3 The stock model with a multifactor uncertain volatility

Liu (2009) first supposed that the stock price follows an uncertain differential equation and presented an *uncertain stock model* as follows

$$\begin{cases} \mathrm{d}X_t = rX_t\mathrm{d}t \\ \mathrm{d}Y_t = \mu Y_t\mathrm{d}t + \sigma Y_t\mathrm{d}C_t \end{cases}$$

where X_t is the bond price, Y_t denotes the stock price, r is the riskless interest rate, μ is the log-drift, σ is the log-diffusion, and C_t is a canonical Liu process. Volatility as a measure for movement of underlying asset in short is assumed to be constant, but it is never constant in the long term and changes with time. Therefore, Hassanzadeh and Mehrdoust (2018) proposed a stock model as an uncertain counterpart of Heston model which uncertain volatility has been described in an uncertain dynamic as follows

$$\begin{cases} \mathrm{d}B_t = rB_t\mathrm{d}t \\ \mathrm{d}S_t = S_t\left(\mu\mathrm{d}t + \sqrt{\sigma_t}\mathrm{d}C_{1t}\right) \\ \mathrm{d}\sigma_t = \kappa(\theta - \sigma_t)\mathrm{d}t + \sigma\sqrt{\sigma_t}\mathrm{d}C_{2t} \end{cases}$$

where C_{1t} and C_{2t} are the two independent canonical Liu processes, S_t is the stock price at time t, B_t is the bond price at time t, σ_t is volatility of the stock price, r is risk-free interest rate, μ denotes the log-drift of the stock price, κ is rate of reversion to the long-term price variance, θ is long-term price variance, and σ is volatility of the volatility.

In this paper, we consider that a stock price described as an uncertain model which its behavior satisfies uncertain differential equation and define two-factor structure for the volatility as follows

$$\begin{cases} \mathrm{d}B_t = rB_t\mathrm{d}t \\ \mathrm{d}S_t = \mu S_t\mathrm{d}t + \sqrt{\sigma_{1t}}S_t\mathrm{d}C_{1t} + \sqrt{\sigma_{2t}}S_t\mathrm{d}C_{2t} \\ \mathrm{d}\sigma_{1t} = \kappa_1(\theta_1 - \sigma_{1t})\mathrm{d}t + \sigma_1\sqrt{\sigma_{1t}}\mathrm{d}C_{3t} \\ \mathrm{d}\sigma_{2t} = \kappa_2(\theta_2 - \sigma_{2t})\mathrm{d}t + \sigma_2\sqrt{\sigma_{2t}}\mathrm{d}C_{4t} \end{cases} \tag{3}$$

where C_{1t}, C_{2t}, C_{3t} and C_{4t} are the independent canonical Liu processes. S_t and $\sigma_{it}, i = 1,2$, denote the price and volatilities of asset price at time t, B_t is the bond price at time t, r is the risk-free interest rate, μ denotes the log-drift of the stock price, $\kappa_i, i = 1,2$, are the rate of reversion to the long-term price variance, $\theta_i, i = 1,2$, are the long-term price variance, $\sigma_i, i = 1,2$, are the volatility of the volatility.

We will study the numerical solution for this model by the α-path method and generalize Yao–Chen formula in the next theorem.

Theorem 8 *Assume that for $1 \le i \le 2, f_{i1}, g_{i1}, g_{i2}$ and f are continuous functions and C_{i1t} and C_{i2t} are independent canonical Liu processes. Suppose that Y_{it} and Y_{it}^α be the solution and α-path of an uncertain differential equation*

$$\mathrm{d}Y_{it} = f_{i1}(t, Y_{it})\mathrm{d}t + g_{i1}(t, Y_{it})\mathrm{d}C_{i1t}, i = 1,2$$

respectively. Let $|h_i(t,y)|$ be a continuous increasing function. Then, the solution X_t of an uncertain differential equation

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + \sum_{i=1}^{2} h_i(t, Y_{it})g_{i2}(t, X_t)\mathrm{d}C_{i2t}$$

is a contour process with an α-path X_t^α that solves the corresponding ordinary differential equation

$$\mathrm{d}X_t^\alpha = f\left(t, X_t^\alpha\right)\mathrm{d}t + \sum_{i=1}^{2}\left|h_i\left(t, Y_{it}^\alpha\right)g_{i2}\left(t, X_t^\alpha\right)\right|\Phi^{-1}(\alpha)\mathrm{d}t$$

where

$$\Phi^{-1}(\alpha) = \frac{\sqrt{3}}{\pi}ln\frac{\alpha}{1-\alpha}, \alpha \in (0,1).$$

In other words,

$$\mathcal{M}\left\{X_t \le X_t^\alpha, \forall t\right\} = \alpha$$
$$\mathcal{M}\left\{X_t > X_t^\alpha, \forall t\right\} = 1-\alpha.$$

Proof Given $\alpha \in (0,1)$. We define the following sets

$$T_i^+ = \{t \in [0,T] | h_i(t, Y_{it}^\alpha) g_{i2}(t, X_t^\alpha) \geq 0\},$$
$$T_i^- = \{t \in [0,T] | h_i(t, Y_{it}^\alpha) g_{i2}(t, X_t^\alpha) < 0\},$$

$i = 1,2$. Then, $T_i^+ \cup T_i^- = [0,T]$ and $T_i^+ \cap T_i^- = \emptyset$ for $i = 1,2$. Also, for each $1 \leq i \leq 2$ and for $s, c \in [0,T]$, write

$$\Lambda_{i1}^+ = \left\{\lambda | \frac{\mathrm{d}C_{i1t}(\lambda)}{\mathrm{d}t} \leq \Phi^{-1}(\alpha), \forall t \in (0,s]\right\},$$
$$\Lambda_{i1}^- = \left\{\lambda | \frac{\mathrm{d}C_{i1t}(\lambda)}{\mathrm{d}t} \geq \Phi^{-1}(1-\alpha), \forall t \in (0,s]\right\},$$
$$\Lambda_{i2}^+ = \left\{\lambda | \frac{\mathrm{d}C_{i2t}(\lambda)}{\mathrm{d}t} \leq \Phi^{-1}(\alpha), \forall t \in (0,c]\right\}$$

and

$$\Lambda_{i2}^- = \left\{\lambda | \frac{\mathrm{d}C_{i2t}(\lambda)}{\mathrm{d}t} \geq \Phi^{-1}(1-\alpha), \forall t \in (0,c]\right\},$$

where Φ^{-1} is the inverse uncertainty distribution of $\mathcal{N}(0,1)$. For $1 \leq i \leq 2$, T_i^+ and T_i^- are disjoint sets and C_{i1t} and C_{i2t} are independent increment processes, and by assumption, we have

$$\mathcal{M}\{\Lambda_{i1}^+\} = \alpha, \quad \mathcal{M}\{\Lambda_{i1}^-\} = \alpha, \quad \mathcal{M}\{\Lambda_{i1}^+ \cap \Lambda_{i1}^-\} = \alpha,$$
$$\mathcal{M}\{\Lambda_{i2}^+\} = \alpha, \quad \mathcal{M}\{\Lambda_{i2}^-\} = \alpha, \quad \mathcal{M}\{\Lambda_{i2}^+ \cap \Lambda_{i2}^-\} = \alpha.$$

Let $\Lambda_{i1} = \Lambda_{i1}^+ \cap \Lambda_{i1}^-$ and $\Lambda_{i2} = \Lambda_{i2}^+ \cap \Lambda_{i2}^-$. Since for $i = 1,2$, C_{i1t} and C_{i2t} are independent, we can write

$$\mathcal{M}\{\Lambda_{i1} \cap \Lambda_{i2}\} = \min\{\mathcal{M}\{\Lambda_{i1}^+ \cap \Lambda_{i1}^-\}, \mathcal{M}\{\Lambda_{i2}^+ \cap \Lambda_{i2}^-\}\} = \alpha$$

For any $\lambda \in \Lambda_{i1} \cap \Lambda_{i2}, i = 1,2$, we have

$$h_i(t, Y_{it}) g_{i2}(t, X_t) \frac{\mathrm{d}C_{i2t}(\lambda)}{\mathrm{d}t} \leq |h_i(t, Y_{it}^\alpha) g_{i2}(t, X_t^\alpha)| \Phi^{-1}(\alpha), \quad \forall t \in [0,T], \quad i = 1,2$$

Also,

$$\mathcal{M}\left\{\bigcap_{i=1}^{2} (\Lambda_{i1} \cap \Lambda_{i2})\right\} = \min_{1 \leq i \leq 2}\{\mathcal{M}\{\Lambda_{i1}^+ \cap \Lambda_{i1}^-\}, \mathcal{M}\{\Lambda_{i2}^+ \cap \Lambda_{i2}^-\}\} = \alpha$$

For any $\beta \in \bigcap_{i=1}^{2} (\Lambda_{i1} \cap \Lambda_{i2})$, we have

$$\sum_{i=1}^{2} h_i(t, Y_{it}) g_{i2}(t, X_t) \frac{\mathrm{d}C_{i2t}(\beta)}{\mathrm{d}t} \leq \sum_{i=1}^{2} |h_i(t, Y_{it}^\alpha) g_{i2}(t, X_t^\alpha)| \Phi^{-1}(\alpha), \quad \forall t \in [0,T]$$

Then,

$$X_t \leq X_t^\alpha, \quad \forall t \in [0,T].$$

Hence, we can write

$$\mathcal{M}\{X_t \leq X_t^\alpha, \forall t \in [0,T]\} \geq \mathcal{M}\left\{\bigcap_{i=1}^{2} (\Lambda_{i1} \cap \Lambda_{i2})\right\} = \alpha. \tag{4}$$

Besides, for each $1 \leq i \leq 2$ and for $s, c \in [0,T]$, define

$$\Sigma_{i1}^+ = \left\{\gamma | \frac{\mathrm{d}C_{i1t}(\gamma)}{\mathrm{d}t} > \Phi^{-1}(\alpha), \forall t \in (0,s]\right\},$$
$$\Sigma_{i1}^- = \left\{\gamma | \frac{\mathrm{d}C_{i1t}(\gamma)}{\mathrm{d}t} < \Phi^{-1}(1-\alpha), \forall t \in (0,s]\right\},$$
$$\Sigma_{i2}^+ = \left\{\gamma | \frac{\mathrm{d}C_{i2t}(\gamma)}{\mathrm{d}t} > \Phi^{-1}(\alpha), \forall t \in (0,c]\right\}$$

and

$$\Sigma_{i2}^- = \{\gamma | \frac{\mathrm{d}C_{i2t}(\gamma)}{\mathrm{d}t} < \Phi^{-1}(1-\alpha), \forall t \in (0,c]\},$$

where Φ^{-1} is the inverse uncertainty distribution of $\mathcal{N}(0,1)$. For $1 \leq i \leq 2$, C_{i1t} and C_{i2t} are independent increment processes, and by assumption, we have

$$\mathcal{M}\{\Sigma_{i1}^+\} = 1-\alpha, \quad \mathcal{M}\{\Sigma_{i1}^-\} = 1-\alpha, \quad \mathcal{M}\{\Sigma_{i1}^+ \cap \Sigma_{i1}^-\} = 1-\alpha,$$
$$\mathcal{M}\{\Lambda_{i2}^+\} = \alpha, \quad \mathcal{M}\{\Lambda_{i2}^-\} = \alpha, \quad \mathcal{M}\{\Lambda_{i2}^+ \cap \Lambda_{i2}^-\} = \alpha$$

Let $\Sigma_{i1} = \Sigma_{i1}^+ \cap \Sigma_{i1}^-$ and $\Sigma_{i2} = \Sigma_{i2}^+ \cap \Sigma_{i2}^-$, $i = 1,2$. Since for $i = 1,2$, C_{i1t} and C_{i2t} are independent, we can write

$$\mathcal{M}\{\Sigma_{i1} \cap \Sigma_{i2}\} = \min\{\mathcal{M}\{\Sigma_{i1}^+ \cap \Sigma_{i1}^-\}, \mathcal{M}\{\Sigma_{i2}^+ \cap \Sigma_{i2}^-\}\} = 1-\alpha$$

For any $\gamma \in \Sigma_{i1} \cap \Sigma_{i2}, i = 1,2$, we have

$$h_i(t, Y_{it}) g_{i2}(t, X_t) \frac{\mathrm{d}C_{i2t}(\gamma)}{\mathrm{d}t} > |h_i(t, Y_{it}^\alpha) g_{i2}(t, X_t^\alpha)| \Phi^{-1}(\alpha), \quad \forall t \in [0,T], \quad i = 1,2.$$

Also,

$$\mathcal{M}\left\{\bigcap_{i=1}^{2} (\Sigma_{i1} \cap \Sigma_{i2})\right\} = \min_{1 \leq i \leq 2}\{\mathcal{M}\{\Sigma_{i1}^+ \cap \Sigma_{i1}^-\}, \mathcal{M}\{\Sigma_{i2}^+ \cap \Sigma_{i2}^-\}\} = 1-\alpha$$

For any $\zeta \in \bigcap_{i=1}^{2} (\Sigma_{i1} \cap \Sigma_{i2})$, we have

$$\sum_{i=1}^{2} h_i(t, Y_{it}) g_{i2}(t, X_t) \frac{\mathrm{d}C_{i2t}(\zeta)}{\mathrm{d}t} > \sum_{i=1}^{2} |h_i(t, Y_{it}^\alpha) g_{i2}(t, X_t^\alpha)| \Phi^{-1}(\alpha), \forall t \in [0,T]$$

Then,

$$X_t > X_t^\alpha, \forall t \in [0,T].$$

Hence, we can write

$$\mathcal{M}\{X_t > X_t^\alpha, \forall t \in [0,T]\} \geq \mathcal{M}\left\{\bigcap_{i=1}^{2}(\Sigma_{i1} \cap \Sigma_{i2})\right\} = 1-\alpha. \tag{5}$$

By duality axiom,

$$\mathcal{M}\{X_t \leq X_t^\alpha, \forall t \in [0,T]\} + \mathcal{M}\left\{X_t \nleq X_t^\alpha, \forall t \in [0,T]\right\} = 1.$$

Besides,

$$\{X_t > X_t^\alpha, \forall t \in [0,T]\} \subset \left\{X_t \nleq X_t^\alpha, \forall t \in [0,T]\right\}.$$

So, we have

$$\mathcal{M}\{X_t \leq X_t^\alpha, \forall t \in [0,T]\} + \mathcal{M}\{X_t > X_t^\alpha, \forall t \in [0,T]\} \leq 1, \tag{6}$$

From inequalities (4), (5) and (6), we have

$$\mathcal{M}\{X_t \leq X_t^\alpha, \forall t \in [0,T]\} = 1-\alpha,$$
$$\mathcal{M}\{X_t > X_t^\alpha, \forall t \in [0,T]\} = 1-\alpha.$$

□

4 European option pricing

In this section, we propose a numerical method to value a European option based on proposed stock model (3). A European call option is a contract between a buyer and a seller, which gives its buyer the right but not the obligation to buy a prescribed stock in a certain price at a determined time in future.

Assume the European call option with a strike price K and maturity date T. Then, the price of the call option is as follows

$$C = \mathrm{e}^{-rT}E\left[(S_T-K)^+\right],$$

where S_T is the stock price at time T.

Theorem 9 *The price of a European call option for the stock model (3) with expiration date T and strike price K is as follows*

$$C = \mathrm{e}^{-rT}\int_0^1 \left(S_T^\alpha - K\right)^+ \mathrm{d}\alpha$$

where

$$S_T^\alpha = S_0\exp\left(\mu T + \int_0^T\left(\frac{\sqrt{\sigma_{1t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha} + \frac{\sqrt{\sigma_{2t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\right)\mathrm{d}t\right)$$

and σ_{1t}^α and σ_{1t}^α are the solution of the following ordinary differential equations

$$\mathrm{d}\sigma_{1t}^\alpha = \kappa_1\left(\theta_1 - \sigma_{1t}^\alpha\right)\mathrm{d}t + \frac{\sigma_1\sqrt{\sigma_{1t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\mathrm{d}t,$$

$$\mathrm{d}\sigma_{2t}^\alpha = \kappa_2\left(\theta_2 - \sigma_{2t}^\alpha\right)\mathrm{d}t + \frac{\sigma_2\sqrt{\sigma_{2t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\mathrm{d}t,$$

where κ_i, θ_i and σ_i are the some constants with $i = 1, 2$.

Proof According to Theorem 6, S_t is a contour process and its α-path is the solution of the following ordinary differential equation

$$\mathrm{d}S_t^\alpha = S_t^\alpha\left(\mu + \frac{\sqrt{\sigma_{1t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha} + \frac{\sqrt{\sigma_{2t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\right)\mathrm{d}t$$

where σ_{1t}^α is the solution of the following ordinary differential equation

$$\mathrm{d}\sigma_{1t}^\alpha = \kappa_1\left(\theta_1 - \sigma_{1t}^\alpha\right)\mathrm{d}t + \frac{\sigma_1\sqrt{\sigma_{1t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\mathrm{d}t,$$

and σ_{2t}^α is the solution of the following ordinary differential equation

$$\mathrm{d}\sigma_{2t}^\alpha = \kappa_2\left(\theta_2 - \sigma_{2t}^\alpha\right)\mathrm{d}t + \frac{\sigma_2\sqrt{\sigma_{2t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\mathrm{d}t.$$

Besides, the price of a European call option is

$$C = \mathrm{e}^{-rT}E\left[(S_T-K)^+\right].$$

Then, by Theorems 1 and 5, the theorem is proved. □

A European put option is a contract that gives its holder the right but not the obligation to sell a prescribed underlying asset in a certain price at a determined time in future.

Assume a European put option with a strike price K and maturity date T. Then, its price is

$$P = \mathrm{e}^{-rT}E[(K-S_T)^+]$$

where S_T is the stock price at time T.

Theorem 10 *The price of a European put option for the stock model (3) with expiration date T and strike price K is as follows*

$$P = \mathrm{e}^{-rT}\int_0^1 \left(K - S_T^\alpha\right)^+ \mathrm{d}\alpha$$

where

$$S_T^\alpha = S_0 \exp\left(\mu T + \int_0^T \left(\frac{\sqrt{\sigma_{1t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha} + \frac{\sqrt{\sigma_{2t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\right)\mathrm{d}t\right)$$

and σ_{1t}^α and σ_{2t}^α are the solution of the following ordinary differential equations

$$\mathrm{d}\sigma_{1t}^\alpha = \kappa_1\left(\theta_1 - \sigma_{1t}^\alpha\right)\mathrm{d}t + \frac{\sigma_1\sqrt{\sigma_{1t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\mathrm{d}t,$$

$$\mathrm{d}\sigma_{2t}^\alpha = \kappa_2\left(\theta_2 - \sigma_{2t}^\alpha\right)\mathrm{d}t + \frac{\sigma_2\sqrt{\sigma_{2t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\mathrm{d}t,$$

where κ_i, θ_i and σ_i are the some constants with $i = 1,2$.

Proof Based on Theorem 6, S_t is a contour process and its α-path is the solution of the following ordinary differential equation

$$\mathrm{d}S_t^\alpha = S_t^\alpha\left(\mu + \frac{\sqrt{\sigma_{1t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha} + \frac{\sqrt{\sigma_{2t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\right)\mathrm{d}t$$

where σ_{1t}^α is the solution of the following ordinary differential equation

$$\mathrm{d}\sigma_{1t}^\alpha = \kappa_1\left(\theta_1 - \sigma_{1t}^\alpha\right)\mathrm{d}t + \frac{\sigma_1\sqrt{\sigma_{1t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\mathrm{d}t,$$

and σ_{2t}^α is the solution of the following ordinary differential equation

$$\mathrm{d}\sigma_{2t}^\alpha = \kappa_2\left(\theta_2 - \sigma_{2t}^\alpha\right)\mathrm{d}t + \frac{\sigma_2\sqrt{\sigma_{2t}^\alpha}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\mathrm{d}t.$$

Besides, the price of a European put option is

$$P = \mathrm{e}^{-rT}E\left[(K - S_T)^+\right].$$

Then, by Theorems 1 and 5, the theorem is proved.

5 Numerical results

In what follows, we designed a numerical method for calculating European call and put options based on Theorems 9 and 10. All the numerical results have been performed with the parameters in Table 1 which are selected from Ahlip et al. (2018).

Table 1 The parameters

Risk-free interest rate	6.50×10^{-4}
Log-drift of the stock price	6.50×10^{-4}
Initial volatility 1	0.0179
Initial volatility 2	0.0221
Volatility of the volatility 1	0.9565
Volatility of the volatility 2	1.8167
Long-term price variance 1	0.0256
Long-term price variance 2	4.0097×10^{-4}
Mean revision 1	2.7994
Mean revision 2	18.4552

The following algorithm calculates the European call option under the stock price model (3) (Fig. 1).

Step 0 Fix the exercise date at time T, fix the volatility at time zero $\sigma_{10}^\alpha = \sigma_{10}, \sigma_{20}^\alpha = \sigma_{20}$, and choose $N = 100$ and set $i = 1, 2, \ldots, N-1$.

Step 1 Set $\alpha \leftarrow \frac{i}{N}$.

Step 2 Set $i \leftarrow i + 1$.

Step 3 Solve the corresponding ordinary differential equation via Runge–Kutta scheme (Shen and Yang 2015),

$$\mathrm{d}\sigma_{1t}^{\alpha_i} = \kappa_1\left(\theta_1 - \sigma_{1t}^{\alpha_i}\right)\mathrm{d}t + \frac{\sigma_1\sqrt{\sigma_{1t}^{\alpha_i}}\sqrt{3}}{\pi}\ln\frac{\alpha_i}{1-\alpha_i}\mathrm{d}t,$$

$$\mathrm{d}\sigma_{2t}^{\alpha_i} = \kappa_2\left(\theta_2 - \sigma_{2t}^{\alpha_i}\right)\mathrm{d}t + \frac{\sigma_2\sqrt{\sigma_{2t}^{\alpha_i}}\sqrt{3}}{\pi}\ln\frac{\alpha_i}{1-\alpha_i}\mathrm{d}t$$

and

$$\mathrm{d}S_t^{\alpha_i} = S_t^\alpha\left(\mu + \frac{\sqrt{\sigma_{1t}^{\alpha_i}}\sqrt{3}}{\pi}\ln\frac{\alpha_i}{1-\alpha_i} + \frac{\sqrt{\sigma_{2t}^{\alpha_i}}\sqrt{3}}{\pi}\ln\frac{\alpha_i}{1-\alpha_i}\right)\mathrm{d}t,$$

respectively. Then, obtain $\sigma_{1T}^{\alpha_i}$, $\sigma_{2T}^{\alpha_i}$ and $S_T^{\alpha_i}$ for $i = 1, 2, \ldots, 99$.

Step 4 Calculate the positive deviation between the stock price and strike price at time T

$$(S_T^{\alpha_i} - K)^+ = \max(0, S_T^{\alpha_i} - K).$$

Step 5 Calculate

$$\exp(-rT)(S_T^{\alpha_i} - K)^+.$$

If $i < N-1$, return to step 2.

Step 6: Calculate the value of the European call option

$$C \leftarrow \frac{1}{N-1}\exp(-rT)\sum_{i=1}^{N-1}(S_T^{\alpha_i} - K)^+,$$

Let us consider a European put option under the stock model (3). According to Theorem 10, the following algorithm is designed to price a mentioned option (Fig. 2).

Step 0 Fix the exercise date at time T, fix the volatility at time zero $\sigma_{10}^\alpha = \sigma_{10}, \sigma_{20}^\alpha = \sigma_{20}$, and choose $N = 100$ and set $i = 1, 2, \ldots, N-1$.

Step 1 Set $\alpha \leftarrow \frac{i}{N}$.

Step 2 Set $i \leftarrow i + 1$.

Step 3 Solve the corresponding ordinary differential equation via Runge–Kutta scheme (Shen and Yang 2015),

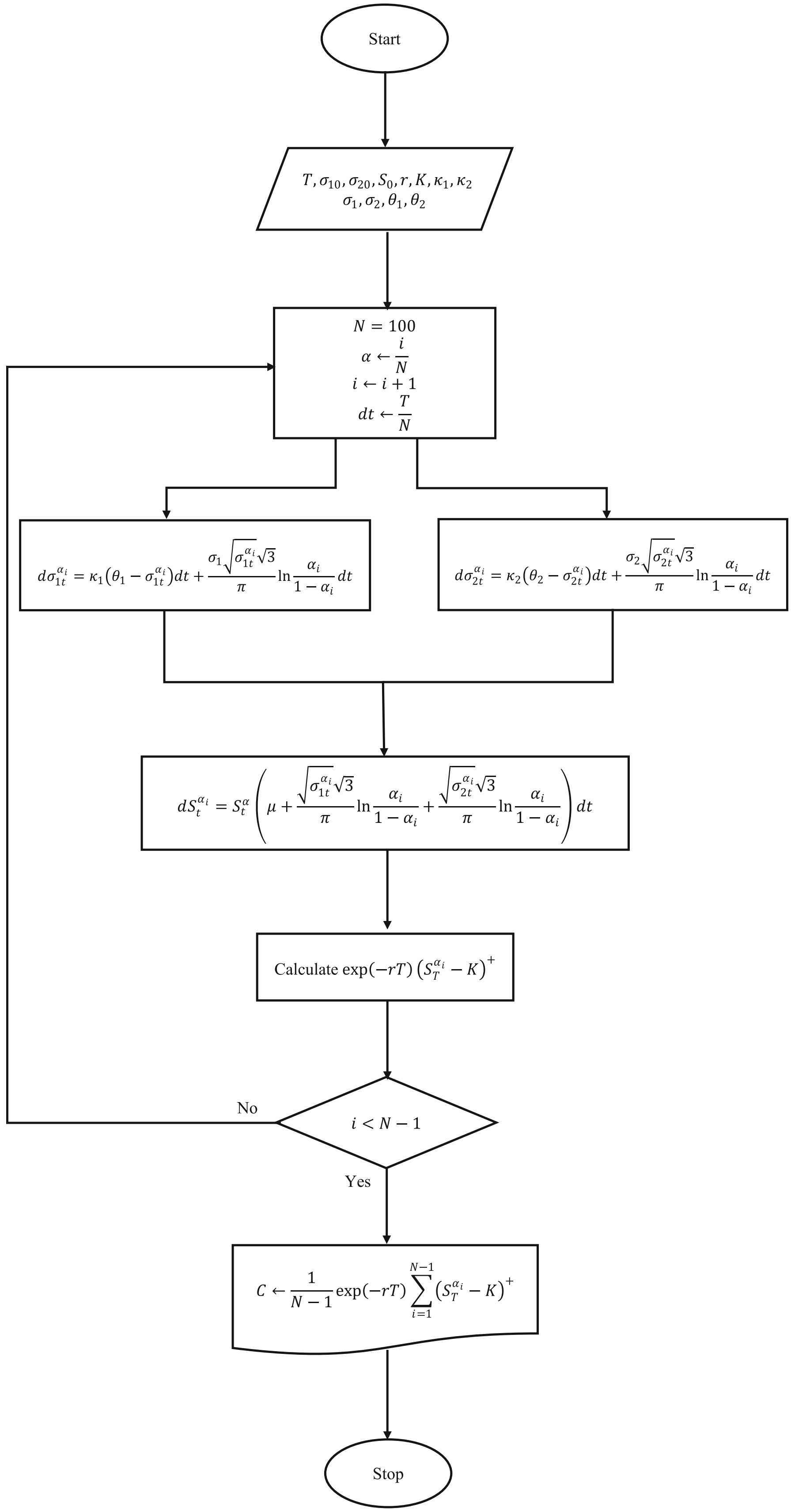
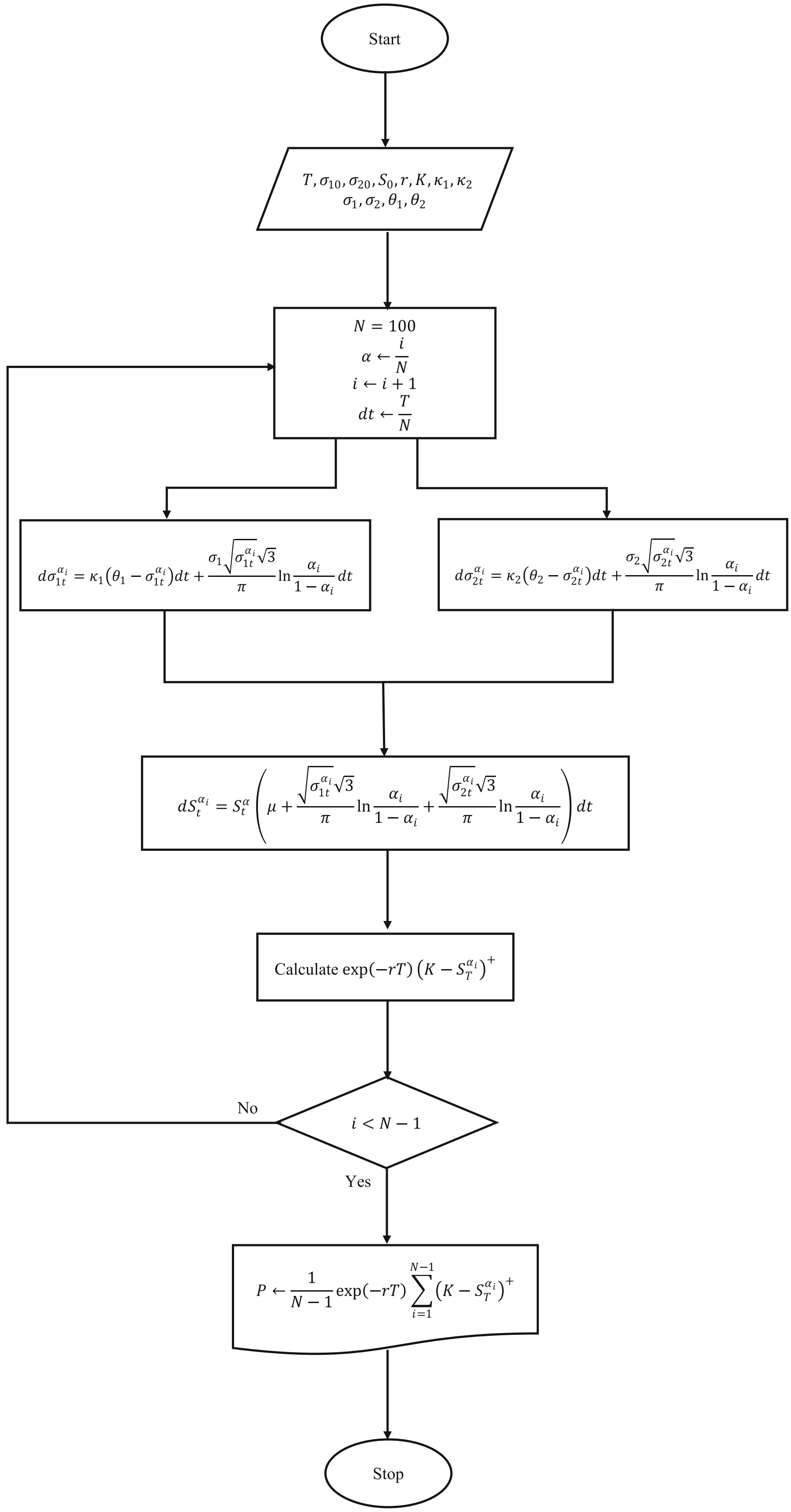


Fig. 1 The algorithm of a European call option pricing

Fig. 2 The algorithm of a European put option pricing



$$\mathrm{d}\sigma_{1t}^{\alpha_i} = \kappa_1\left(\theta_1 - \sigma_{1t}^{\alpha_i}\right)\mathrm{d}t + \frac{\sigma_1\sqrt{\sigma_{1t}^{\alpha_i}}\sqrt{3}}{\pi}\ln\frac{\alpha_i}{1-\alpha_i}\mathrm{d}t,$$

$$\mathrm{d}\sigma_{2t}^{\alpha_i} = \kappa_2\left(\theta_2 - \sigma_{2t}^{\alpha_i}\right)\mathrm{d}t + \frac{\sigma_2\sqrt{\sigma_{2t}^{\alpha_i}}\sqrt{3}}{\pi}\ln\frac{\alpha_i}{1-\alpha_i}\mathrm{d}t$$

and

$$\mathrm{d}S_t^{\alpha_i} = S_t^{\alpha}\left(\mu + \frac{\sqrt{\sigma_{1t}^{\alpha_i}}\sqrt{3}}{\pi}\ln\frac{\alpha_i}{1-\alpha_i} + \frac{\sqrt{\sigma_{2t}^{\alpha_i}}\sqrt{3}}{\pi}\ln\frac{\alpha_i}{1-\alpha_i}\right)\mathrm{d}t,$$

respectively. Then, obtain $\sigma_{1T}^{\alpha_i}$, $\sigma_{2T}^{\alpha_i}$ and $S_T^{\alpha_i}$ for $i = 1, 2, \ldots, 99$.

Step 4 Calculate the positive deviation between the stock price and strike price K at time T

$$(K - S_T^{\alpha_i})^+ = \max(0, K - S_T^{\alpha_i}).$$

Step 5 Calculate

$$\exp(-rT)(K - S_T^{\alpha_i})^+.$$

If $i < N-1$, return to step 2.

Step 6 Calculate the value of the European put option

$$P \leftarrow \frac{1}{N-1}\exp(-rT)\sum_{i=1}^{N-1}(K - S_T^{\alpha_i})^+.$$

Example 1 Assume that spot price is 120, maturity date is 135 days, the risk-free interest rate and the log-drift are 6.50 × 10^{-4} and strike price is 124. Let $\kappa_1 = 2.7994$, $\theta_1 = 0.0256$, $\sigma_{10} = 0.0179$, $\sigma_1 = 0.9565$, $\kappa_2 = 18.4552$, $\theta_2 = 4.0097 \times 10^{-4}$, $\sigma_{20} = 0.0221$, $\sigma_2 = 1.8167$. Then, the price of a European call option is $C = 7.3660$.

Example 2 Assume that spot price is 120, maturity date is 135 days, the risk-free interest rate and the log-drift are 6.50 × 10^{-4} and strike price is 136. Let $\kappa_1 = 2.7994$, $\theta_1 = 0.0256$, $\sigma_{10} = 0.0179$, $\sigma_1 = 0.9565$, $\kappa_2 = 18.4552$, $\theta_2 = 4.0097 \times 10^{-4}$, $\sigma_{20} = 0.0221$, $\sigma_2 = 1.8167$. Then, the price of a European put option is $P = 3.5013$.

The market price of a call option with a lower strike price will be higher than the market price for a call option on the same security with the same expiration date but with a higher strike price. Put options work in reverse to call options. As we can see in Figs. 3 and 4, the y-axis is the call option premium for each strike, and the x-axis is the strike price. The numerical results show clearly the relation between the strike price and the option price.

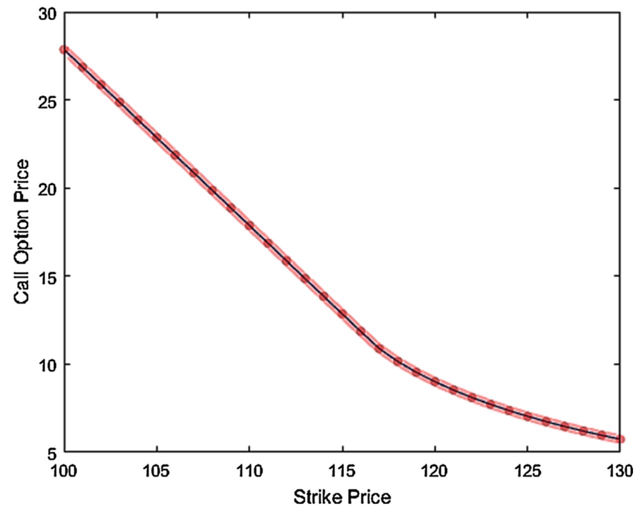


Fig. 3 The price of European call option for 130-day period

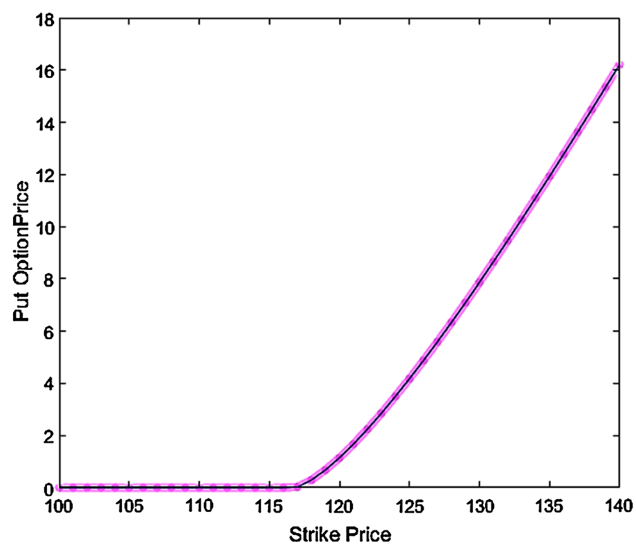


Fig. 4 The price of European put option for 140-day period

6 Remarks and conclusions

In this paper, we have introduced a new stock model as an uncertain counterpart of double Heston model. Besides, we extended Yao–Chen formula and also presented a numerical method to find a fair price of European call and put options when the underlying asset price is driven by uncertain two-factor Heston model. To support the model, we have provided some numerical examples. Numerical results show that option pricing under two-factor uncertain volatility model can be reasonable. The results that we found in this research make us optimistic about the knowledge that could obtain from further exploration of this new model in uncertainty theory.

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Compliance with ethical standards

Conflict of interest We declare that we have no conflict of interest.

Ethical approval This article does not contain any studies with human participants performed by any of the authors.

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