Soft Computing (2020) 24:13955–13977
https://doi.org/10.1007/s00500-020-04770-6

METHODOLOGIES AND APPLICATION

Goal programming technique for solving fully interval-valued intuitionistic fuzzy multiple objective transportation problems

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Published online: 28 February 2020

Abstract

In transportation problems, the cost depends on various irresistible factors like climatic conditions, fuel expenses, etc. Consequently, the transportation problems with crisp parameters fail to handle such situations. However, the construction of the problems under an imprecise environment can significantly tackle these circumstances. The intuitionistic fuzzy number associated with a point is framed by two parameters, namely membership and non-membership degrees. The membership degree determines its acceptance level, while the non-membership measures its non-belongingness (rejection level). However, a person, because of some hesitation, instead of giving a fixed real number to the acceptance and rejection levels, may assign them intervals. This new construction not only generalizes the concept of intuitionistic fuzzy theory but also gives wider scope with more flexibility. In the present article, a balanced transportation problem having all the parameters and variables as interval-valued intuitionistic fuzzy numbers is formulated. Then, a solution methodology based on goal programming approach is proposed. This algorithm not only cares to maximize the acceptance level of the objective functions but simultaneously minimizes the deviational variables attached with each goal. To tackle the interval-valued intuitionistic fuzzy constraints corresponding to each objective function, three membership and non-membership functions, linear, exponential and hyperbolic, are used. Further, a numerical example is solved to demonstrate the computational steps of the algorithm, and a comparison is drawn amidst linear, exponential and hyperbolic membership functions.

Keywords Multi-objective programming · Interval-valued triangular intuitionistic fuzzy numbers · Fuzzy goal programming · Expected value · Membership functions

Abbreviations

TP	Transportation problem
LPP	Linear programming problem
DM	Decision maker
MOTP	Multi-objective transportation problem
GP	Goal programming
MOLPP	Multi-objective linear programming problem
IF	Intuitionistic fuzzy
IFTP	Intuitionistic fuzzy transportation problem
IVIF	Interval-valued intuitionistic fuzzy
IVTIFN	Interval-valued triangular intuitionistic fuzzy number
IVTIF	Interval-valued triangular intuitionistic fuzzy
IVIFN	Interval-valued intuitionistic fuzzy number
IVIFTP	Interval-valued intuitionistic fuzzy transportation problem

Communicated by V. Loia.

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1 Introduction

A TP is one of the most significant applications of LPPs. Hitchcock (1941) had first developed the basic TP as a standard LPP. The objective of TPs is to transport units of commodities from various origins to different destinations so as to minimize the total transportation cost. Simplex algorithm is not much suitable to solve a TP due to the structure of its model; so, Charnes and Cooper (1954) had developed the stepping stone method to solve a TP. Later, Diaz (1979) proposed an algorithm to tackle MOTP.

GP technique is a very powerful and effective tool for the modelling and solution of multi-objective optimization problems. It was developed by Charnes and Cooper in 1961. It requires to set the aspiration goals for the various objectives involved in the problem. Ringuest and Rinks (1987) proposed an interactive algorithm for solving the linear MOTP. Lee and Li (1993) considered a MOLPP and used the ideal and anti-ideal solutions to define the membership functions. Zangiabadi and Maleki (2013) applied the fuzzy GP approach by adopting nonlinear membership functions to solve the MOTP. Later, Singh and Yadav (2018) developed a method to handle the IF MOLPP using various membership functions.

The customary TPs use the values of all the parameters as crisp values. But in real-life situations, the various parameters of a TP are not exactly known as crisp values due to various uncontrollable factors such as climatic conditions, uncertainty in judgement, road conditions and economic conditions. So, Zadeh (1965) introduced the concept of fuzzy sets by associating the membership degree with each element of the set. Many researchers had proposed various approaches to deal with the TPs in fuzzy environment. Chanas and Kuchta (1996) introduced a method for finding the optimal solution of the TP with coefficients as the fuzzy numbers. It was observed that the limitation of the fuzzy sets is that they use a unique or exact real number to represent the membership grade. However in practical situations, the membership value may not be known to us as a crisp number. Then, Atanassov (1986) originated the notion of IF sets using the membership as well as the non-membership functions. The IF sets include the hesitation or uncertainty involved with an element of the set. Angelov (1997) had broadened the fuzzy optimization into IF optimization. Later on, Hussain and Kumar (2012) solved a TP where supply and demand values were given by IF numbers. Gani and Abbas (2013), Singh and Yadav (2015) developed various approaches for finding the optimal solution of the IFTP. Later, Ebrahimnejad and Verdegay (2018) proposed an efficient computation approach for solving the balanced IFTP where cost, supply and demand were expressed by trapezoidal IF numbers. Recently, Mahajan and Gupta (2019) introduced an efficient algorithm to tackle the fully IF MOTP using various membership and non-membership functions.

It was observed in several cases that membership and non-membership degrees associated with an element may not be available as exact values. Therefore, Atanassov and Gargov (1989) established the idea of IVIF sets, so that membership and non-membership degrees are represented by intervals instead of crisp numbers. Many researchers had applied the concept of IVIF sets in decision-making problems. Nayagam et al. (2008) proposed the ranking of IVIF sets by making use of an accuracy function. Later, Sahin (2016) proposed a new accuracy function for IVIF sets and applied it on multi-criteria decision-making problems in fuzzy environment. Bharati and Singh (2018) proposed the solution algorithm for the TP where transportation costs, supply and demands are represented by IVTIFNs. Recently, Bharati et al. (2017) proposed an algorithm for solving fully fuzzy MOLPP based on deviation degree between two trapezoidal fuzzy numbers. Ishibuchi and Tanaka (1990) had first examined the multi-objective programming problem where objective function coefficients are given by intervals rather than crisp numbers. Later on, Jiuping (2011) considered a MOLPP based on interval-valued fuzzy sets. Li (2010) applied the IVIF sets theory in multi-attribute decision-making problems. Narayanamoorthy and Anukokila (2014) used the fuzzy GP technique for finding the solution of the MOTP with interval cost. Recently, Bharati and Singh (2019) introduced an approach for solving a MOLPP using IVIF environment.

In the literature, there exist several approaches to solve MOTP which are described in Table 1.

In the present article, we have developed a powerful method to solve MOTP by combining three approaches, viz interactive, GP and IVIF so as to treat the interval fuzziness in the input data, achieving the aspiration levels given by the DM and to quickly reach a preferred solution. This study is organized as follows: Sect. 2 introduces some basic terminologies related to fuzzy, IF and IVIF set theory. In Sect. 3, the fully fuzzy-balanced MOTP under IVIF situations has been formulated and some related theorems are established in support of our proposed methodology. Sect. 4 includes the major shortcomings of the existing studies. Sect. 5 deals with the GP approach for obtaining the optimal solution of the multi-objective fully IVIFTP (described in Sect. 3). Sect. 6 characterizes linear, exponential and hyperbolic membership functions to tackle the IVIF constraints connected with each objective. In Sect. 7, main advantages of our proposed algorithm are discussed. Section 8 illustrates the applicability of the proposed solution methodology by a practical example. Finally, Sect. 9 includes the main conclusions of the study along with the future research scope.

2 Preliminaries

In this section, we present basic definitions and notations used in the paper.

Definition 1 (Zadeh 1965) A fuzzy set $\tilde{A}$ in a non-empty universe X is a set of the form $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}}(x) \in [0, 1]$ represents the degree of membership of the element $x \in X$ being in $\tilde{A}$, and $\mu_{\tilde{A}} : X \to [0, 1]$ is called the membership function.

Definition 2 (Mahajan and Gupta 2019) An IF set $\tilde{A}^I$ in X is a set of ordered triples $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)) : x \in X\}$, where $\mu_{\tilde{A}^I}(x) : X \to [0, 1]$ and $\nu_{\tilde{A}^I}(x) : X \to [0, 1]$

Table 1 Different approaches to solve MOTP

Approach	References	Features	Limitations
Interactive	Ringuest and Rinks (1987)	Control of search direction is in the hands of DM	Convergence of solution depends on DM It is difficult to evaluate the set of efficient solutions in large-scale problems
Non-interactive	Diaz (1979), Isermann (1979) and Kasana and Kumar (2000)	Find the set of efficient solutions	It may take long time to find the efficient solutions DM's difficulty in selecting the preferred solution due to inexperience or incomplete information
GP	Lee and Moore (1973), Hemaida and Kwak (1994)	Gives satisfactory solution and simultaneously inspects the multiple goals	Unsophisticated assigning of weights lead to erroneous results Problem may arise in determining the aspiration levels also
Fuzzy GP	Zangiabadi and Maleki (2013), Narayanamoorthy and Anukokila (2014), Singh and Yadav (2018)	Effective tool to handle the incomplete information given by the DM	It changes the well-known structure of a TP model It does not guarantee an efficient solution
IF GP	Mahajan and Gupta (2019)	Takes care of both acceptance and rejection levels	Membership and non-membership degrees are taken to be crisp real numbers

represent the degree of membership and degree of non-membership of the element $x \in X$ being in $\tilde{A}^I$, respectively, such that $\forall\, x \in X, 0 \le \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1$.

An IF set $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)) : x \in X\}$ in X

- the value of $h_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x) - \nu_{\tilde{A}^I}(x)$ is called the degree of non-determinacy (hesitancy) of the element $x \in X$ to $\tilde{A}^I$.
- is normal if there exists $x_0,\ x_1 \in X$ such that $\mu_{\tilde{A}^I}(x_0) = 1$ and $\nu_{\tilde{A}^I}(x_1) = 1$.
- is convex if $\forall\, x_1, x_2 \in X,\ 0 \le \lambda \le 1$,

$$\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \ge \min\{\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2)\} \text{ and}$$
$$\nu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \le \max\{\nu_{\tilde{A}^I}(x_1), \nu_{\tilde{A}^I}(x_2)\}.$$

Definition 3 (Ebrahimnejad and Verdegay 2018) An IF set $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)) : x \in \mathbb{R}\}$ of the real number $\mathbb{R}$ is called an IF number if

(i) $\tilde{A}^I$ is normal and convex IF set,
(ii) $\mu_{\tilde{A}^I}$ is upper semicontinuous and $\nu_{\tilde{A}^I}$ is lower semicontinuous and
(iii) $\text{Supp}\tilde{A}^I = \{x \in \mathbb{R} : \nu_{\tilde{A}^I}(x) < 1\}$ is bounded.

Definition 4 (Atanassov and Gargov 1989) Let Int[0, 1] denote the set of all subintervals of the interval [0, 1]. An IVIF set is defined as a set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) : x \in X\}$, where $\mu_{\tilde{A}} : X \to \text{Int}[0, 1]$ and $\nu_{\tilde{A}} : X \to \text{Int}[0, 1]$ represent the interval-valued membership and non-membership functions, respectively, provided $\text{Sup}(\mu_{\tilde{A}}(x)) + \text{Sup}(\nu_{\tilde{A}}(x)) \le 1$ $\forall\, x \in X$.

Definition 5 (Bharati and Singh 2018) An IVTIFN is denoted by $\tilde{A} = \{(a_1^U, a_1^L, a_2, a_3^L, a_3^U), (b_1^L, b_1^U, a_2, b_3^U, b_3^L)\}$, and its membership and non-membership degrees are defined as follows:

- Lower and upper membership functions, respectively, are defined as:

$$\mu^L_{\tilde{A}}(x) = \begin{cases} 1, & \text{if } x = a_2, \\ \dfrac{x - a_1^L}{a_2 - a_1^L}, & \text{if } a_1^L < x < a_2, \\ \dfrac{a_3^L - x}{a_3^L - a_2}, & \text{if } a_2 < x < a_3^L, \\ 0, & \text{otherwise}, \end{cases} \quad \text{and}$$

$$\mu^U_{\tilde{A}}(x) = \begin{cases} 1, & \text{if } x = a_2, \\ \dfrac{x - a_1^U}{a_2 - a_1^U}, & \text{if } a_1^U < x < a_2, \\ \dfrac{a_3^U - x}{a_3^U - a_2}, & \text{if } a_2 < x < a_3^U, \\ 0, & \text{otherwise}, \end{cases}$$

- Lower and upper non-membership functions, respectively, are given by:

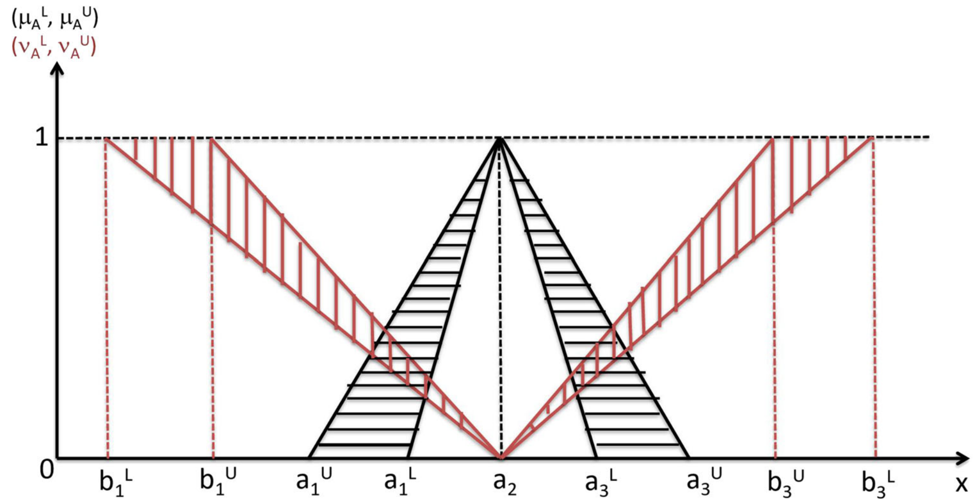


Fig. 1 Interval-valued triangular intuitionistic fuzzy number

$$\nu_{\tilde{A}}^L(x) = \begin{cases} 0, & \text{if } x = a_2, \\ \dfrac{a_2 - x}{a_2 - b_1^L}, & \text{if } b_1^L < x < a_2, \\ \dfrac{a_2 - x}{a_2 - b_3^L}, & \text{if } a_2 < x < b_3^L, \\ 1, & \text{otherwise,} \end{cases} \quad \text{and}$$

$$\nu_{\tilde{A}}^U(x) = \begin{cases} 0, & \text{if } x = a_2, \\ \dfrac{x - a_2}{b_1^U - a_2}, & \text{if } b_1^U < x < a_2, \\ \dfrac{x - a_2}{b_3^U - a_2}, & \text{if } a_2 < x < b_3^U, \\ 1, & \text{otherwise,} \end{cases}$$

where $b_1^L \leq b_1^U \leq a_1^U \leq a_1^L \leq a_2 \leq a_3^L \leq a_3^U \leq b_3^U \leq b_3^L$. The diagrammatic representation of IVTIFN is given in Fig. 1.

Remark 1 If $b_1^L = b_1^U$, $a_1^U = a_1^L$, $a_3^L = a_3^U$, $b_3^U = b_3^L$, then the IVTIFN $\tilde{A}$ reduces to a triangular IF number $\{(a_1^L, a_2, a_3^L), (b_1^L, a_2, b_3^L)\}$.

Definition 6 Let $\tilde{A} = \{(a_1^U, a_1^L, a_2, a_3^L, a_3^U), (b_1^L, b_1^U, a_2, b_3^U, b_3^L)\}$ and $\tilde{B} = \{(c_1^U, c_1^L, c_2, c_3^L, c_3^U), (d_1^L, d_1^U, c_2, d_3^U, d_3^L)\}$ be two IVTIFNs. Then

(a) $\tilde{A} \oplus \tilde{B} = \{(a_1^U + c_1^U, a_1^L + c_1^L, a_2 + c_2, a_3^L + c_3^L, a_3^U + c_3^U), (b_1^L + d_1^L, b_1^U + d_1^U, a_2 + c_2, b_3^U + d_3^U, b_3^L + d_3^L)\}$.

(b) $$k\tilde{A} = \begin{cases} \{(ka_1^U, ka_1^L, ka_2, ka_3^L, ka_3^U), \\ (kb_1^L, kb_1^U, ka_2, kb_3^U, kb_3^L)\}, & \text{if } k \geq 0, \\ \{(ka_3^U, ka_3^L, ka_2, ka_1^L, ka_1^U), \\ (kb_3^L, kb_3^U, ka_2, kb_1^U, kb_1^L)\}, & \text{if } k < 0. \end{cases}$$

(c) $\tilde{A} \ominus \tilde{B} = \{(a_1^U - c_3^U, a_1^L - c_3^L, a_2 - c_2, a_3^L - c_1^L, a_3^U - c_1^U), (b_1^L - d_3^L, b_1^U - d_3^U, a_2 - c_2, b_3^U - d_1^U, b_3^L - d_1^L)\}$.

(d) $\tilde{A} \otimes \tilde{B} = \{(e_1^U, e_1^L, e_2, e_3^L, e_3^U), (f_1^L, f_1^U, e_2, f_3^U, f_3^L)\}$

where

$$\begin{aligned} e_1^U &= \min\{a_1^U c_1^U, a_1^U c_3^U, a_3^U c_1^U, a_3^U c_3^U\}, \\ e_3^U &= \max\{a_1^U c_1^U, a_1^U c_3^U, a_3^U c_1^U, a_3^U c_3^U\}, \\ e_1^L &= \min\{a_1^L c_1^L, a_1^L c_3^L, a_3^L c_1^L, a_3^L c_3^L\}, \\ e_3^L &= \max\{a_1^L c_1^L, a_1^L c_3^L, a_3^L c_1^L, a_3^L c_3^L\}, \\ f_1^L &= \min\{b_1^L d_1^L, b_1^L d_3^L, b_3^L d_1^L, b_3^L d_3^L\}, \\ f_3^L &= \max\{b_1^L d_1^L, b_1^L d_3^L, b_3^L d_1^L, b_3^L d_3^L\}, \\ f_1^U &= \min\{b_1^U d_1^U, b_1^U d_3^U, b_3^U d_1^U, b_3^U d_3^U\}, \\ f_3^U &= \max\{b_1^U d_1^U, b_1^U d_3^U, b_3^U d_1^U, b_3^U d_3^U\}, \\ e_2 &= a_2 c_2. \end{aligned}$$

Definition 7 An IVTIFN $\tilde{A} = \{(a_1^U, a_1^L, a_2, a_3^L, a_3^U), (b_1^L, b_1^U, a_2, b_3^U, b_3^L)\}$ is said to be a nonnegative IVTIFN *iff* $b_1^L \geq 0$.

Remark 2 If $\tilde{A}$ and $\tilde{B}$ be two nonnegative IVTIFNs, then

$$\tilde{A} \otimes \tilde{B} = \{(a_1^U c_1^U, a_1^L c_1^L, a_2 c_2, a_3^L c_3^L, a_3^U c_3^U), (b_1^L d_1^L, b_1^U d_1^U, a_2 c_2, b_3^U d_3^U, b_3^L d_3^L)\}.$$

Definition 8 Two IVTIFNs $\tilde{A} = \{(a_1^U, a_1^L, a_2, a_3^L, a_3^U), (b_1^L, b_1^U, a_2, b_3^U, b_3^L)\}$ and $\tilde{B} = \{(c_1^U, c_1^L, c_2, c_3^L, c_3^U), (d_1^L, d_1^U, c_2, d_3^U, d_3^L)\}$ are said to be equal, i.e. $\tilde{A} \simeq \tilde{B}$ *iff* $a_1^U = c_1^U, a_1^L = c_1^L, a_2 = c_2, a_3^L = c_3^L, a_3^U = c_3^U$, $b_1^L = d_1^L, b_1^U = d_1^U, b_3^U = d_3^U$ and $b_3^L = d_3^L$.

Definition 9 (Bharati and Singh 2018) If $\tilde{A}=\{(a_1^U, a_1^L, a_2, a_3^L, a_3^U), (b_1^L, b_1^U, a_2, b_3^U, b_3^L)\}$ be a IVTIFN, then its expected value is given by:

$$EV(\tilde{A})=\frac{a_1^U+a_1^L+b_1^L+b_1^U+8a_2+a_3^L+a_3^U+b_3^U+b_3^L}{16}.$$

Theorem 1 (Bharati and Singh 2018) *The expected value function $EV: IVIF(\mathbb{R}) \to \mathbb{R}$ is a linear function, where IVIF($\mathbb{R}$) denotes the set of all IVIFNs over $\mathbb{R}$.*

Definition 10 *(Ordering of IVTIFNs)* Let $\tilde{A}$ and $\tilde{B}$ be two IVTIFNs. Then,

(i) $\tilde{A} \preceq \tilde{B} \iff EV(\tilde{A}) \leq EV(\tilde{B})$.
(ii) $\tilde{A} \succeq \tilde{B} \iff EV(\tilde{A}) \geq EV(\tilde{B})$.

3 Fully IVIF multi-objective transportation problem

The mathematical formulation of a TP having K-objectives (MOTP) in crisp environment does not depict the real-life situations appropriately, when there is vagueness/ambiguity in the objectives and/or constraints of the model. In such situations, the formulation of MOTP where all the involved parameters and variables are expressed by IVIFNs seems to be viable and gives more flexibility to the DM. The MOTP under IVIF situation can be formulated as follows:

$$\textbf{(IVIFTP)}\ \text{Minimize}\ \tilde{Z}(x)=\{\tilde{Z}_1(x), \tilde{Z}_2(x), \ldots, \tilde{Z}_K(x)\}$$
$$\text{where}\quad \tilde{Z}_k(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^k \otimes \tilde{x}_{ij};\quad k=1,2,\ldots,K$$
$$\text{subject to}\ \sum_{j=1}^{n}\tilde{x}_{ij}=\tilde{a}_i,\quad i=1,2,\ldots,m,$$
$$\sum_{i=1}^{m}\tilde{x}_{ij}=\tilde{b}_j,\quad j=1,2,\ldots,n,$$
$$\tilde{x}_{ij}\succeq\tilde{0},\qquad i=1,2,\ldots,m,\ j=1,2,\ldots,n$$

where $\tilde{a}_i=\{(a_{i1}^U, a_{i1}^L, a_{i2}, a_{i3}^L, a_{i3}^U), (a_{i1}'^L, a_{i1}'^U, a_{i2}, a_{i3}'^U, a_{i3}'^L)\}$ is the IVTIF supply of the commodity at the ith origin, $\tilde{b}_j=\{(b_{j1}^U, b_{j1}^L, b_{j2}, b_{j3}^L, b_{j3}^U), (b_{j1}'^L, b_{j1}'^U, b_{j2}, b_{j3}'^U, b_{j3}'^L)\}$ is the IVTIF demand of the commodity at the jth destination, $\tilde{c}_{ij}^k=\{(c_{ij1}^{kU}, c_{ij1}^{kL}, c_{ij2}^k, c_{ij3}^{kL}, c_{ij3}^{kU}), (c_{ij1}'^{kL}, c_{ij1}'^{kU}, c_{ij2}^k, c_{ij3}'^{kU}, c_{ij3}'^{kL})\}$ is the IVTIF penalty associated with transporting a unit of the commodity from ith origin to the jth destination according to the kth penalty criterion, and $\tilde{x}_{ij}=\{(x_{ij1}^U, x_{ij1}^L, x_{ij2}, x_{ij3}^L, x_{ij3}^U), (x_{ij1}'^L, x_{ij1}'^U, x_{ij2}, x_{ij3}'^U, x_{ij3}'^L)\}$ is the IVTIF quantity of the commodity that should be transported from the ith origin to the jth destination.

It is assumed that all the parameters involved in the problem (IVIFTP) are nonnegative and $\sum_{i=1}^m \tilde{a}_i=\sum_{j=1}^n \tilde{b}_j$. Let S_F denote the feasible region for (IVIFTP).
Now, using Remark 2 and putting $\tilde{x}_{ij}=\{(x_{ij1}^U, x_{ij1}^L, x_{ij2}, x_{ij3}^L, x_{ij3}^U), (x_{ij1}'^L, x_{ij1}'^U, x_{ij2}, x_{ij3}'^U, x_{ij3}'^L)\}$, the problem (IVIFTP) can be rewritten as:

$$\begin{aligned}
&\text{Minimize}\ \tilde{Z}(x)\\
&=\Bigg\{\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\big(c_{ij1}^{1U}x_{ij1}^U, c_{ij1}^{1L}x_{ij1}^L, c_{ij2}^1x_{ij2}, c_{ij3}^{1L}x_{ij3}^L, c_{ij3}^{1U}x_{ij3}^U\big),\\
&\big(c_{ij1}'^{1L}x_{ij1}'^L, c_{ij1}'^{1U}x_{ij1}'^U, c_{ij2}^1x_{ij2}, c_{ij3}'^{1U}x_{ij3}'^U, c_{ij3}'^{1L}x_{ij3}'^L\big)\Big),\\
&\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\big(c_{ij1}^{2U}x_{ij1}^U, c_{ij1}^{2L}x_{ij1}^L, c_{ij2}^2x_{ij2}, c_{ij3}^{2L}x_{ij3}^L,\\
&c_{ij3}^{2U}x_{ij3}^U\big), \big(c_{ij1}'^{2L}x_{ij1}'^L, c_{ij1}'^{2U}x_{ij1}'^U, c_{ij2}^2x_{ij2}, c_{ij3}'^{2U}x_{ij3}'^U, c_{ij3}'^{2L}x_{ij3}'^L\big)\Big),\\
&\vdots\\
&\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\big(c_{ij1}^{KU}x_{ij1}^U, c_{ij1}^{KL}x_{ij1}^L, c_{ij2}^Kx_{ij2}, c_{ij3}^{KL}x_{ij3}^L, c_{ij3}^{KU}x_{ij3}^U\big),\\
&\big(c_{ij1}'^{KL}x_{ij1}'^L, c_{ij1}'^{KU}x_{ij1}'^U, c_{ij2}^Kx_{ij2}, c_{ij3}'^{KU}x_{ij3}'^U, c_{ij3}'^{KL}x_{ij3}'^L\big)\Big)\Bigg\}\\
&\text{subject to}\ \sum_{j=1}^{n}\tilde{x}_{ij}=\tilde{a}_i,\ \ i=1,2,\ldots,m,\\
&\sum_{i=1}^{m}\tilde{x}_{ij}=\tilde{b}_j,\ \ j=1,2,\ldots,n,\\
&\tilde{x}_{ij}\succeq\tilde{0},\ \ \text{for all } i \text{ and } j.
\end{aligned}\tag{1}$$

Applying expected value function on all the components of objective function and using definitions 7 and 8, we further get

$$\begin{aligned}
&\textbf{(IVIFTP1)}\ \text{Min}\ EV(\tilde{Z}(x))\\
&=\Bigg\{EV\Bigg(\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\big(c_{ij1}^{1U}x_{ij1}^U, c_{ij1}^{1L}x_{ij1}^L, c_{ij2}^1x_{ij2}, c_{ij3}^{1L}x_{ij3}^L, c_{ij3}^{1U}x_{ij3}^U\big),\\
&\big(c_{ij1}'^{1L}x_{ij1}'^L, c_{ij1}'^{1U}x_{ij1}'^U, c_{ij2}^1x_{ij2}, c_{ij3}'^{1U}x_{ij3}'^U, c_{ij3}'^{1L}x_{ij3}'^L\big)\Big)\Bigg),\\
&EV\Bigg(\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\big(c_{ij1}^{2U}x_{ij1}^U, c_{ij1}^{2L}x_{ij1}^L, c_{ij2}^2x_{ij2}, c_{ij3}^{2L}x_{ij3}^L, c_{ij3}^{2U}x_{ij3}^U\big),\\
&\big(c_{ij1}'^{2L}x_{ij1}'^L, c_{ij1}'^{2U}x_{ij1}'^U, c_{ij2}^2x_{ij2}, c_{ij3}'^{2U}x_{ij3}'^U, c_{ij3}'^{2L}x_{ij3}'^L\big)\Big)\Bigg),\\
&\vdots\\
&EV\Bigg(\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(\big(c_{ij1}^{KU}x_{ij1}^U, c_{ij1}^{KL}x_{ij1}^L, c_{ij2}^Kx_{ij2}, c_{ij3}^{KL}x_{ij3}^L, c_{ij3}^{KU}x_{ij3}^U\big),\\
&\big(c_{ij1}'^{KL}x_{ij1}'^L, c_{ij1}'^{KU}x_{ij1}'^U, c_{ij2}^Kx_{ij2}, c_{ij3}'^{KU}x_{ij3}'^U, c_{ij3}'^{KL}x_{ij3}'^L\big)\Big)\Bigg)\Bigg\}
\end{aligned}$$

subject to

$$\left.\begin{array}{l}
\sum_{j=1}^{n} x_{ij1}^{U}=a_{i1}^{U},\quad \sum_{j=1}^{n} x_{ij1}^{L}=a_{i1}^{L},\quad \sum_{j=1}^{n} x_{ij2}=a_{i2},\quad \sum_{j=1}^{n} x_{ij3}^{L}=a_{i3}^{L},\\
\sum_{j=1}^{n} x_{ij3}^{U}=a_{i3}^{U},\quad \sum_{j=1}^{n} x_{ij1}^{\prime L}=a_{i1}^{\prime L},\quad \sum_{j=1}^{n} x_{ij1}^{\prime U}=a_{i1}^{\prime U},\quad \sum_{j=1}^{n} x_{ij3}^{\prime U}=a_{i3}^{\prime U},\\
\sum_{j=1}^{n} x_{ij3}^{\prime L}=a_{i3}^{\prime L},\quad \sum_{i=1}^{m} x_{ij1}^{U}=b_{j1}^{U},\quad \sum_{i=1}^{m} x_{ij1}^{L}=b_{j1}^{L},\quad \sum_{i=1}^{m} x_{ij2}=b_{j2},\\
\sum_{i=1}^{m} x_{ij3}^{L}=b_{j3}^{L},\quad \sum_{i=1}^{m} x_{ij3}^{U}=b_{j3}^{U},\quad \sum_{i=1}^{m} x_{ij1}^{\prime L}=b_{j1}^{\prime L},\quad \sum_{i=1}^{m} x_{ij1}^{\prime U}=b_{j1}^{\prime U},\\
\sum_{i=1}^{m} x_{ij3}^{\prime U}=b_{j3}^{\prime U},\quad \sum_{i=1}^{m} x_{ij3}^{\prime L}=b_{j3}^{\prime L},\\
x_{ij1}^{\prime L}\geq 0,\ \ x_{ij1}^{\prime U}-x_{ij1}^{\prime L}\geq 0,\ \ x_{ij1}^{U}-x_{ij1}^{\prime U}\geq 0,\ \ x_{ij1}^{L}-x_{ij1}^{U}\geq 0,\\
x_{ij2}-x_{ij1}^{L}\geq 0,\ \ x_{ij3}^{L}-x_{ij2}\geq 0,\ \ x_{ij3}^{U}-x_{ij3}^{L}\geq 0,\ \ x_{ij3}^{\prime U}-x_{ij3}^{U}\geq 0,\\
x_{ij3}^{\prime L}-x_{ij3}^{\prime U}\geq 0,\qquad i=1,2,\ldots,m,\ \ j=1,2,\ldots,n.
\end{array}\right\} \tag{2}$$

In the rest of the paper, we denote $EV(\tilde{Z}_k(x)) = Z'_k(x),\ k = 1, 2, \ldots, K$.

Theorem 2 *If* $\tilde{\tilde{x}}_{ij} = \{(\bar{x}_{ij1}^{U}, \bar{x}_{ij1}^{L}, \bar{x}_{ij2}, \bar{x}_{ij3}^{L}, \bar{x}_{ij3}^{U}), (\bar{x}_{ij1}^{\prime L}, \bar{x}_{ij1}^{\prime U}, \bar{x}_{ij2}, \bar{x}_{ij3}^{\prime U}, \bar{x}_{ij3}^{\prime L})\}$ *is the optimal solution of the problem* (2), *then it is a nonnegative IVIFN.*

Proof Let $\tilde{\tilde{x}}_{ij} = \{(\bar{x}_{ij1}^{U}, \bar{x}_{ij1}^{L}, \bar{x}_{ij2}, \bar{x}_{ij3}^{L}, \bar{x}_{ij3}^{U}), (\bar{x}_{ij1}^{\prime L}, \bar{x}_{ij1}^{\prime U}, \bar{x}_{ij2}, \bar{x}_{ij3}^{\prime U}, \bar{x}_{ij3}^{\prime L})\}$ be the optimal solution of (IVIFTP1). So, $\tilde{\tilde{x}}_{ij}$ satisfy the nonnegativity constraints of model (IVIFTP1). Hence,

$$\begin{aligned}
&\bar{x}_{ij1}^{\prime L}\geq 0,\ \ \bar{x}_{ij1}^{\prime U}-\bar{x}_{ij1}^{\prime L}\geq 0,\ \ \bar{x}_{ij1}^{U}-\bar{x}_{ij1}^{\prime U}\geq 0,\\
&\quad \bar{x}_{ij1}^{L}-\bar{x}_{ij1}^{U}\geq 0,\ \ \bar{x}_{ij2}-\bar{x}_{ij1}^{L}\geq 0,\ \ \bar{x}_{ij3}^{L}-\bar{x}_{ij2}\geq 0,\\
&\bar{x}_{ij3}^{U}-\bar{x}_{ij3}^{L}\geq 0,\quad \bar{x}_{ij3}^{\prime U}-\bar{x}_{ij3}^{U}\geq 0,\quad \bar{x}_{ij3}^{\prime L}-\bar{x}_{ij3}^{\prime U}\geq 0,\\
&\quad i=1,2,\ldots,m;\quad j=1,2,\ldots,n
\end{aligned}$$

which implies that

$$\begin{aligned}
&\bar{x}_{ij1}^{\prime L}\geq 0,\ \ \bar{x}_{ij1}^{\prime L}\leq \bar{x}_{ij1}^{\prime U}\leq \bar{x}_{ij1}^{U}\leq \bar{x}_{ij1}^{L}\leq \bar{x}_{ij2}\leq \bar{x}_{ij3}^{L}\\
&\quad \leq \bar{x}_{ij3}^{U}\leq \bar{x}_{ij3}^{\prime U}\leq \bar{x}_{ij3}^{\prime L},\\
&\quad\ i=1,2,\ldots,m;\ j=1,2,\ldots,n
\end{aligned}$$

Hence, the result. □

Theorem 3 *The optimal value of objectives* $\tilde{\tilde{Z}}_k = \{(\bar{Z}_1^{kU}, \bar{Z}_1^{kL}, \bar{Z}_2^{k}, \bar{Z}_3^{kL}, \bar{Z}_3^{kU}), (\bar{Z}_1^{\prime kL}, \bar{Z}_1^{\prime kU}, \bar{Z}_2^{k}, \bar{Z}_3^{\prime kU}, \bar{Z}_3^{\prime kL})\}, k = 1, 2, \ldots K$, *is a nonnegative IVIFN.*

Proof Let $\tilde{\tilde{x}}_{ij} = \{(\bar{x}_{ij1}^{U}, \bar{x}_{ij1}^{L}, \bar{x}_{ij2}, \bar{x}_{ij3}^{L}, \bar{x}_{ij3}^{U}), (\bar{x}_{ij1}^{\prime L}, \bar{x}_{ij1}^{\prime U}, \bar{x}_{ij2}, \bar{x}_{ij3}^{\prime U}, \bar{x}_{ij3}^{\prime L})\}$ be the optimal solution of (IVIFTP1). By Theorem 2, we have

$$\begin{aligned}
&\bar{x}_{ij1}^{\prime L}\geq 0,\ \ \bar{x}_{ij1}^{\prime L}\leq \bar{x}_{ij1}^{\prime U}\leq \bar{x}_{ij1}^{U}\leq \bar{x}_{ij1}^{L}\leq \bar{x}_{ij2}\leq \bar{x}_{ij3}^{L}\leq \bar{x}_{ij3}^{U}\\
&\quad \leq \bar{x}_{ij3}^{\prime U}\leq \bar{x}_{ij3}^{\prime L},\\
&\quad\ i=1,2,\ldots,m;\ j=1,2,\ldots,n.
\end{aligned}$$

Further, as $\tilde{c}_{ij}^{k}$ are also nonnegative IVTIFNs, therefore

$$\begin{aligned}
&c_{ij1}^{\prime kL}\geq 0,\ \ c_{ij1}^{\prime kL}\leq c_{ij1}^{\prime kU}\leq c_{ij1}^{kU}\leq c_{ij1}^{kL}\leq c_{ij2}^{k}\leq c_{ij3}^{kL}\\
&\quad \leq c_{ij3}^{kU}\leq c_{ij3}^{\prime kU}\leq c_{ij3}^{\prime kL};\\
&\quad\ i=1,2,\ldots,m;\ j=1,2,\ldots,n \text{ and } \forall\, k=1,2,\ldots,K.
\end{aligned}$$

Hence,

$$\begin{aligned}
&c_{ij1}^{\prime kL}\bar{x}_{ij1}^{\prime L}\geq 0,\ \ c_{ij1}^{\prime kL}\bar{x}_{ij1}^{\prime L}\leq c_{ij1}^{\prime kU}\bar{x}_{ij1}^{\prime U}\leq c_{ij1}^{kU}\bar{x}_{ij1}^{U}\leq c_{ij1}^{kL}\bar{x}_{ij1}^{L}\\
&\quad \leq c_{ij2}^{k}\bar{x}_{ij2}\leq c_{ij3}^{kL}\bar{x}_{ij3}^{L}\leq c_{ij3}^{kU}\bar{x}_{ij3}^{U}\leq c_{ij3}^{\prime kU}\bar{x}_{ij3}^{\prime U}\leq c_{ij3}^{\prime kL}\bar{x}_{ij3}^{\prime L},\\
&\quad\ i=1,2,\ldots,m;\ j=1,2,\ldots,n.
\end{aligned}$$

The components of $\tilde{\tilde{Z}}_k$ are given by

$$\begin{aligned}
&\bar{Z}_1^{\prime kL}=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij1}^{\prime kL}\bar{x}_{ij1}^{\prime L},\ \ \bar{Z}_1^{\prime kU}=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij1}^{\prime kU}\bar{x}_{ij1}^{\prime U},\\
&\quad \bar{Z}_1^{kU}=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij1}^{kU}\bar{x}_{ij1}^{U},\\
&\bar{Z}_1^{kL}=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij1}^{kL}\bar{x}_{ij1}^{L},\ \ \bar{Z}_2^{k}=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij2}^{k}\bar{x}_{ij2},\\
&\quad \bar{Z}_3^{kL}=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij3}^{kL}\bar{x}_{ij3}^{L},\\
&\bar{Z}_3^{kU}=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij3}^{kU}\bar{x}_{ij3}^{U},\ \ \bar{Z}_3^{\prime kU}=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij3}^{\prime kU}\bar{x}_{ij3}^{\prime U},\\
&\quad \bar{Z}_3^{\prime kL}=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij3}^{\prime kL}\bar{x}_{ij3}^{\prime L};\quad k=1,2,\ldots,K.
\end{aligned}$$

It follows that

$$\tilde{\tilde{Z}}_k \succeq 0\ \ \forall\, k=1,2,\ldots,K.$$

This proves the theorem. □

Definition 11 A point $\tilde{\tilde{x}}$ is said to be an efficient or Pareto optimal solution of IVIFTP if there does not exist any $\tilde{x} \in S_F$ such that $\tilde{Z}_k(\tilde{\tilde{x}}) \succeq \tilde{Z}_k(\tilde{x})\ \forall\, k$ and $\tilde{Z}_k(\tilde{\tilde{x}}) \succ \tilde{Z}_k(\tilde{x})$ for at least one k.

Theorem 4 *The efficient solution of problem* (2) *is an efficient solution of model* (1).

Proof Let $\tilde{\tilde{x}} = \Big\{\{(\hat{x}_{ij1}^{U}, \hat{x}_{ij1}^{L}, \hat{x}_{ij2}, \hat{x}_{ij3}^{L}, \hat{x}_{ij3}^{U}), (\hat{x}_{ij1}^{\prime L}, \hat{x}_{ij1}^{\prime U}, \hat{x}_{ij2}, \hat{x}_{ij3}^{\prime U}, \hat{x}_{ij3}^{\prime L})\},\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n\Big\}$ be an efficient solution of (2). Hence, by feasibility conditions, we have

$$\sum_{j=1}^{n}\hat{x}_{ij1}^{U}=a_{i1}^{U},\quad \sum_{j=1}^{n}\hat{x}_{ij1}^{L}=a_{i1}^{L},\quad \sum_{j=1}^{n}\hat{x}_{ij2}=a_{i2},$$

$$\sum_{j=1}^{n} \hat{x}_{ij3}^{L} = a_{i3}^{L}, \quad \sum_{j=1}^{n} \hat{x}_{ij3}^{U} = a_{i3}^{U},$$
$$\sum_{j=1}^{n} \hat{x}_{ij1}^{\prime L} = a_{i1}^{\prime L}, \quad \sum_{j=1}^{n} \hat{x}_{ij1}^{\prime U} = a_{i1}^{\prime U}, \quad \sum_{j=1}^{n} \hat{x}_{ij3}^{\prime U} = a_{i3}^{\prime U},$$
$$\sum_{j=1}^{n} \hat{x}_{ij3}^{\prime L} = a_{i3}^{\prime L}, \quad i = 1, 2, \ldots, m,$$
$$\sum_{i=1}^{m} \hat{x}_{ij1}^{U} = b_{j1}^{U}, \quad \sum_{i=1}^{m} \hat{x}_{ij1}^{L} = b_{j1}^{L}, \quad \sum_{i=1}^{m} \hat{x}_{ij2} = b_{j2},$$
$$\sum_{i=1}^{m} \hat{x}_{ij3}^{L} = b_{j3}^{L}, \quad \sum_{i=1}^{m} \hat{x}_{ij3}^{U} = b_{j3}^{U},$$
$$\sum_{i=1}^{m} \hat{x}_{ij1}^{\prime L} = b_{j1}^{\prime L}, \quad \sum_{i=1}^{m} \hat{x}_{ij1}^{\prime U} = b_{j1}^{\prime U}, \quad \sum_{i=1}^{m} \hat{x}_{ij3}^{\prime U} = b_{j3}^{\prime U},$$
$$\sum_{i=1}^{m} \hat{x}_{ij3}^{\prime L} = b_{j3}^{\prime L}, \quad j = 1, 2, \ldots, n,$$

$$\hat{x}_{ij1}^{\prime L} \geq 0, \quad \hat{x}_{ij1}^{\prime U} - \hat{x}_{ij1}^{\prime L} \geq 0, \quad \hat{x}_{ij1}^{U} - \hat{x}_{ij1}^{\prime U} \geq 0, \quad \hat{x}_{ij1}^{L} - \hat{x}_{ij1}^{U} \geq 0,$$
$$\hat{x}_{ij2} - \hat{x}_{ij1}^{L} \geq 0, \quad \hat{x}_{ij3}^{L} - \hat{x}_{ij2} \geq 0,$$
$$\hat{x}_{ij3}^{U} - \hat{x}_{ij3}^{L} \geq 0, \quad \hat{x}_{ij3}^{\prime U} - \hat{x}_{ij3}^{U} \geq 0,$$
$$\hat{x}_{ij3}^{\prime L} - \hat{x}_{ij3}^{\prime U} \geq 0, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n.$$

The above constraints give

(i) $\sum_{j=1}^{n} \tilde{\hat{x}}_{ij} = \tilde{a}_i, \quad i = 1, 2, \ldots, m,$
(ii) $\sum_{i=1}^{m} \tilde{\hat{x}}_{ij} = \tilde{b}_j, \quad j = 1, 2, \ldots, n,$
(iii) $\tilde{\hat{x}}_{ij} \succeq \tilde{0}, \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n.$

Thus, (i), (ii) *and* (iii) give $\tilde{\hat{x}}$ is a feasible solution of (1). Now, as $\tilde{\hat{x}}$ is an efficient solution of (2), there does not exist any IVTIFN $\tilde{\bar{x}}$ such that $EV(\tilde{Z}_k(\tilde{\bar{x}})) \leq EV(\tilde{Z}_k(\tilde{\hat{x}}))$, for all $k = 1, 2, \ldots, K$ and $EV(\tilde{Z}_k(\tilde{\bar{x}})) < EV(\tilde{Z}_k(\tilde{\hat{x}}))$, for at least one k, which using definition (10) yields $\tilde{\hat{x}}$ is an efficient solution of (1) also.
Hence proved. □

4 Shortcomings of the existing studies

Various researchers have studied the transportation problems under fuzzy, IF and IVIF situations. Some of the major shortcomings of their studies are listed below:

1. Hussain and Kumar (2012) and Singh and Yadav (2016) proposed the methodologies for finding the solution of fully IFTPs. In their studies, all the parameters, i.e. transportation costs, supply and demand, were taken to be positive IF numbers. But their approaches yields negative quantities for some of the IF variables. Moreover, negative terms are also appeared in the objective function values which have no physical meaning. But, our study overcomes this limitation.
2. The existing studies of dealing with multi-objective programming models with IF (Singh and Yadav 2018) and IVIF (Bharati and Singh 2019) environments encounters non-membership degree in parameters of TP only; however, in real-life situations, the vagueness may also creep in the variables. In the present formulation, the variables are also considered as IVIFNs.
3. The method in Bharati and Singh (2018) was developed to deal with the single-objective balanced TP of the type:

$$\min \ \tilde{Z}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes x_{ij}$$
$$\text{s.t.} \sum_{j=1}^{n} x_{ij} = \tilde{a}_i, \quad i = 1, 2, \ldots, m,$$
$$\sum_{i=1}^{m} x_{ij} = \tilde{b}_j, \quad j = 1, 2, \ldots, n,$$
$$x_{ij} \geq 0, \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n$$

where all the parameters of a TP were represented by IVTIFNs and the variables were assumed to be crisp. However in the present study, we introduce a methodology to deal with multi-objective fully IVIFTPs which are more general and have wide number of applications in comparison with single-objective problems.
4. Gupta and Kumar (2012) developed a solution methodology for solving linear MOTP with parameters represented by (λ, ρ) interval-valued fuzzy numbers. A balanced TP is always feasible, but the solution obtained by their approach is not even feasible.
5. The approach proposed by Mahajan and Gupta (2019) deals with MOTPs in IF environment. But in practical situations, membership and non-membership degrees may not be crisp. Our study takes care of this limitation by considering the MOTP in IVIF scenario which is more realistic and general.
6. The method dealing with MOTPs given in Abd El-Wahed and Lee (2006), all the constraints connected with the objective goals become redundant, whereas the current study takes care of it by minimizing the sum of all deviational variables.
7. The existing approaches (Narayanamoorthy and Anukokila 2014; Singh and Yadav 2018; Mahajan and Gupta 2019) to solve multiple objective optimization problems, either maximize the difference of acceptance and rejective level or minimize the sum of deviations but our approach considers both the situations simultaneously.

Table 2 Payoff table

X	Z'_1	Z'_2	Z'_3	…	Z'_K
X_1	$Z'_1(X_1)$	$Z'_2(X_1)$	$Z'_3(X_1)$	…	$Z'_K(X_1)$
X_2	$Z'_1(X_2)$	$Z'_2(X_2)$	$Z'_3(X_2)$	…	$Z'_K(X_2)$
X_3	$Z'_1(X_3)$	$Z'_2(X_3)$	$Z'_3(X_3)$	…	$Z'_K(X_3)$
⋮	⋮	⋮		⋮	⋮
X_K	$Z'_1(X_K)$	$Z'_2(X_K)$	$Z'_3(X_K)$	…	$Z'_K(X_K)$

5 Proposed approach

In this section, we present the weighted GP approach in IVIF environment for finding the optimal solution of the problem (IVIFTP). All the optimization models in the paper are solved directly by using a software "LINGO−17.0" on a MacBook Air system with 1.8 GHz Dual-Core Intel Core i5 processor and 8 GB RAM.
The steps involved in the algorithm to solve (IVIFTP) are as follows:

Step 1 Develop the model as described in (IVIFTP).

Step 2 Split the problem (IVIFTP1) in K-subproblems. The kth subproblem is given by:

$$\textbf{(IVIFTP-}k\textbf{)} \quad \text{Minimize } Z'_k(x) = EV(\tilde{Z}_k(x))$$
$$= EV\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\left(c_{ij1}^{kU}x_{ij1}^{U}, c_{ij1}^{kL}x_{ij1}^{L}, c_{ij2}^{k}x_{ij2}, c_{ij3}^{kL}x_{ij3}^{L}, c_{ij3}^{kU}x_{ij3}^{U}\right),\right.\right.$$
$$\left.\left.\left(c_{ij1}^{\prime kL}x_{ij1}^{\prime L}, c_{ij1}^{\prime kU}x_{ij1}^{\prime U}, c_{ij2}^{k}x_{ij2}, c_{ij3}^{\prime kU}x_{ij3}^{\prime U}, c_{ij3}^{\prime kL}x_{ij3}^{\prime L}\right)\right)\right)$$

subject to all the constraints of (2).

Let the optimal solution of (IVIFTP-k) be X_k. Find $Z'_1(X_k)$, $Z'_2(X_k), \ldots, Z'_K(X_k)$.

Step 3 Continue Step 2 for $k = 1, 2, \ldots, K$, and assume that the corresponding optimal solutions be $X_1,\ X_2,\ \ldots,\ X_K$. If all the solutions $X_k,\ k = 1, 2, \ldots, K$ are same, then it is the optimal compromise solution, otherwise go to Step 4.

Step 4 Construct a payoff matrix using all the solutions obtained in Steps 2 and 3 as shown in Table 2.

Remark 3 For $k = 1$, X_1 will be the optimal solution and Z_1 will be the required optimal transportation cost of the corresponding single-objective TP. For $k > 1$, go to step 5.

Step 5 From the above payoff matrix of order $K \times K$, whose (i, j)th element is equal to $Z'_j(X_i) = EV(\tilde{Z}_j(X_i))$, evaluate $U_k = \max_{1\le p\le K} Z'_k(X_p)$ and $L_k = Z'_k(X_k),\ k = 1, 2, \ldots, K$. Here, we call L_k to be the most acceptable level and U_k to be the worst acceptable level of achievement for the kth objective function.

Step 6 Formulate a model to find $x_{ij1}^U, x_{ij1}^L, x_{ij2}, x_{ij3}^L, x_{ij3}^U, x_{ij1}^{\prime L}, x_{ij1}^{\prime U}, x_{ij3}^{\prime U}, x_{ij3}^{\prime L},\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n$ such that

$$\textbf{(COP)} \quad Z'_k(x) \sim L_k,\ k = 1, 2, \ldots, K$$

along with the constraints of (2)

where $Z'_k(x) \sim L_k,\ k = 1, 2, \ldots, K$ is an IVIF equality which can be handled using a membership function. Choice of this function depends on the DM. It can be taken as linear, exponential or hyperbolic.

Step 7 Using a membership function and the concept of IVIF decision, to achieve the aspiration level G_k for the kth objective, we minimize the difference between Z'_k and G_k $(k = 1, 2, \ldots, K)$ by including the deviational variables d_k^+ and d_k^-, which are defined as follows:

$$d_k^+ = \max\{0, Z'_k - G_k\} = \frac{1}{2}\left[Z'_k - G_k + |Z'_k - G_k|\right], \text{ and}$$
$$d_k^- = \max\{0, G_k - Z'_k\} = \frac{1}{2}\left[G_k - Z'_k + |G_k - Z'_k|\right].$$

Following Abd El-Wahed and Lee (2006), Bharati and Singh (2019), the model (COP) reduces to:

$$\begin{aligned}
\textbf{(COM)}\quad & \text{Max } \left(\theta\alpha + (1-\theta)\beta - \theta\gamma - (1-\theta)\delta - (w_1(d_1^+ + d_1^-)\right.\\
& \left.+ w_2(d_2^+ + d_2^-) + \cdots + w_K(d_K^+ + d_K^-))\right)\\
\text{s.t.}\quad & \mu_k^U\left(Z'_k(x)\right) \ge \theta\alpha + (1-\theta)\beta,\\
& \mu_k^L\left(Z'_k(x)\right) \ge \alpha,\\
& \nu_k^U\left(Z'_k(x)\right) \le \theta\gamma + (1-\theta)\delta,\\
& \nu_k^L\left(Z'_k(x)\right) \le \gamma,\\
& Z'_k(x) - d_k^+ + d_k^- = G_k,\\
& \theta\alpha + (1-\theta)\beta + \theta\gamma + (1-\theta)\delta \le 1,\\
& \beta + \delta \le 1,\quad \beta \ge \alpha,\quad \delta \ge \gamma,\\
& \gamma \ge 0,\quad \alpha \ge 0,\quad 0 \le \theta \le 1,\\
& d_k^-,\ d_k^+ \ge 0,\quad k = 1, 2, \ldots, K\\
& \text{and all the constraints of (2)}
\end{aligned}$$

where α is the minimum degree of acceptance, γ is the maximum degree of rejection and $w_1,\ w_2, \ldots, w_K$ are the weights based on the priority levels of objectives.

Step 8 Finally, $\tilde{x} = \left\{\{(x_{ij1}^U, x_{ij1}^L, x_{ij2}, x_{ij3}^L, x_{ij3}^U), (x_{ij1}^{\prime L}, x_{ij1}^{\prime U}, x_{ij2}, x_{ij3}^{\prime U}, x_{ij3}^{\prime L})\}\ \ i = 1, 2, \ldots, m;\ \ j = 1, 2, \ldots, n\right\}$ gives IVIF optimal solution. Further, on substituting $\tilde{x}$ in $\tilde{Z}_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^k \otimes \tilde{x}_{ij};\ k = 1, 2, \ldots, K$, we get the optimal objective function value of the multi-objective IVIFTP.

The flow chart in Fig. 2 depicts the proposed methodology.

6 Membership functions

(Bharati and Singh 2019; Mahajan and Gupta 2019)

Fig. 2 Flow chart depicting the steps of proposed algorithm

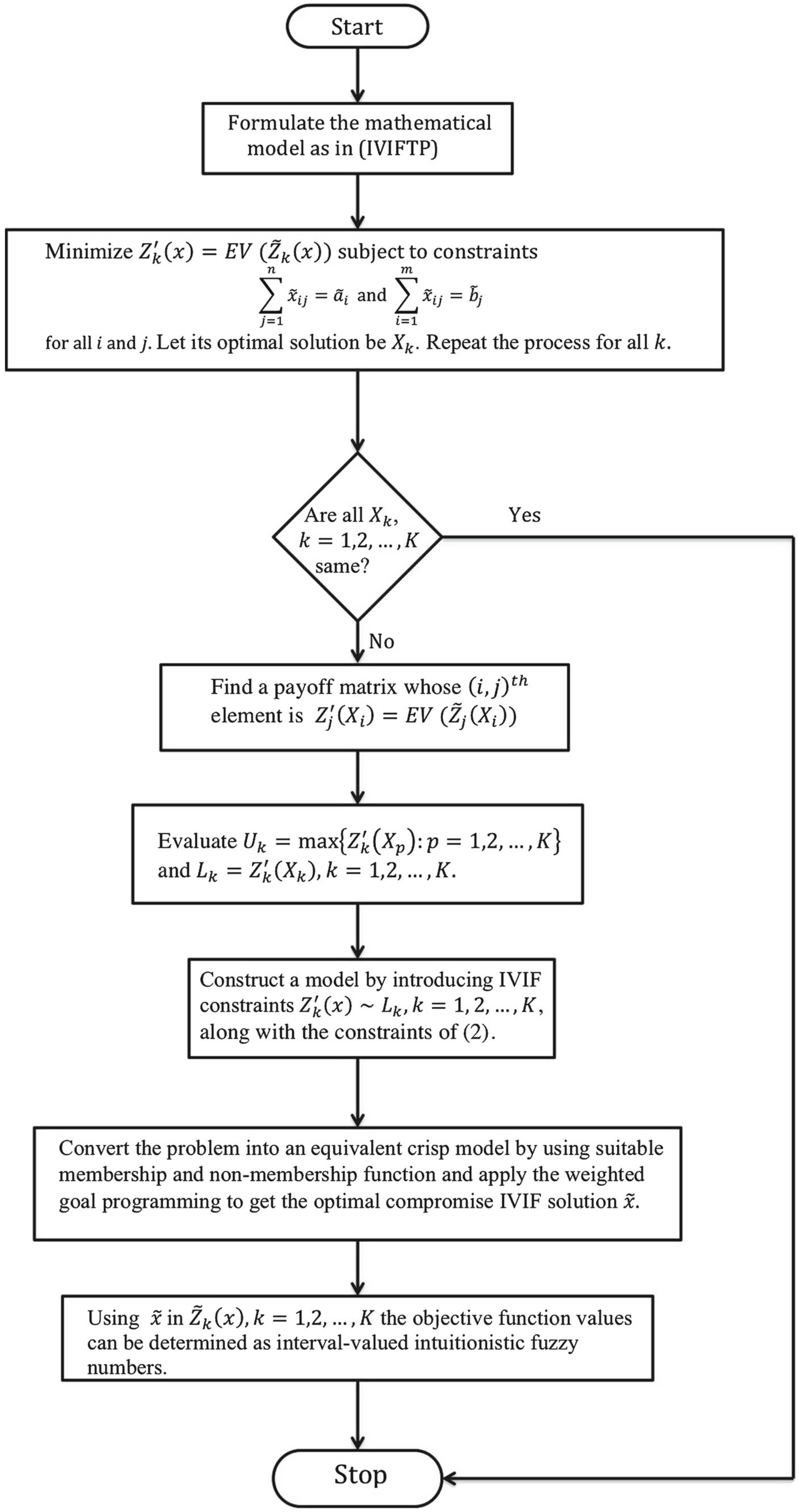
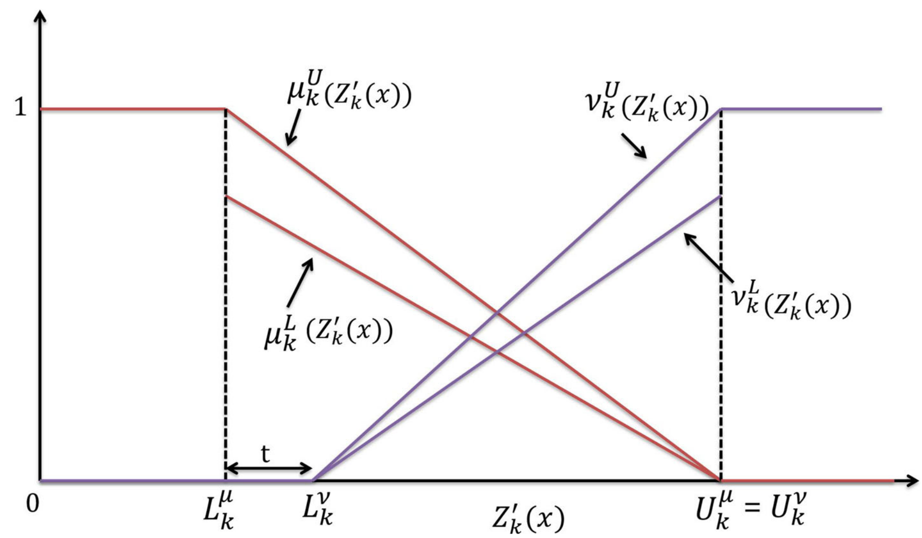


Fig. 3 Lower and upper linear membership and non-membership functions



To handle the IVIF constraints,

$$Z_k^{'}(x) \sim L_k,\ k = 1, 2, \ldots, K,$$

one can take linear, exponential or hyperbolic membership and non-membership functions. A linear membership function is most commonly used because of its simplicity. However, to deal with the real-life situations, the linear membership function may not be applicable. As a result, one can opt for nonlinear membership functions which can depict the practical situation in a better way.

6.1 Linear membership function

Upper and lower linear membership functions $\mu_k^U(Z_k'(x))$ and $\mu_k^L(Z_k'(x))$ (Fig. 3) for the kth objective are defined as:

$$\mu_k^U\left(Z_k'(x)\right) = \begin{cases} 1 & \text{if } Z_k'(x) \le L_k^\mu, \\ \dfrac{U_k^\mu - Z_k'(x)}{U_k^\mu - L_k^\mu} & \text{if } L_k^\mu < Z_k'(x) < U_k^\mu, \\ 0 & \text{if } Z_k'(x) \ge U_k^\mu, \end{cases}$$

$$\mu_k^L\left(Z_k'(x)\right) = \begin{cases} 1 & \text{if } Z_k'(x) \le L_k^\mu, \\ \eta\left(\dfrac{U_k^\mu - Z_k'(x)}{U_k^\mu - L_k^\mu}\right) & \text{if } L_k^\mu < Z_k'(x) < U_k^\mu, \\ 0 & \text{if } Z_k'(x) \ge U_k^\mu. \end{cases}$$

The upper and lower linear non-membership functions $\nu_k^U(Z_k'(x))$ and $\nu_k^L(Z_k'(x))$ (Fig. 3) for the kth objective are given by:

$$\nu_k^U\left(Z_k'(x)\right) = \begin{cases} 0 & \text{if } Z_k'(x) \le L_k^\nu, \\ \dfrac{Z_k'(x) - L_k^\nu}{U_k^\nu - L_k^\nu} & \text{if } L_k^\nu < Z_k'(x) < U_k^\nu, \\ 1 & \text{if } Z_k'(x) \ge U_k^\nu, \end{cases}$$

$$\nu_k^L\left(Z_k'(x)\right) = \begin{cases} 0 & \text{if } Z_k'(x) \le L_k^\nu, \\ \eta\left(\dfrac{Z_k'(x) - L_k^\nu}{U_k^\nu - L_k^\nu}\right) & \text{if } L_k^\nu < Z_k'(x) < U_k^\nu, \\ 1 & \text{if } Z_k'(x) \ge U_k^\nu, \end{cases}$$

where $0 \le \eta \le 1$, $k = 1, 2, \ldots, K$ and $L_k^\mu = L_k,\ L_k^\nu = L_k^\mu + t(U_k^\mu - L_k^\mu);\ \ 0 < t < 1$.

Here, t is called the tolerance. The approach used is pessimistic in which the decision maker is likely to be ready for extra acceptance. In other words, if the degree of rejection is zero, the DM is not accessible to accept it fully.

In the remaining part of the paper, we define $U_k^\mu = U_k^\nu = U_k$.

The intervals $[\mu^L(x), \mu^U(x)]$ and $[\nu^L(x), \nu^U(x)]$ denote the acceptance and rejection degrees, respectively.

Now, on applying the above upper and lower membership and non-membership functions in all the objectives and using the concept of weighted GP, the problem (COM) is converted to the following model:

$$
\begin{aligned}
\textbf{(LMF)}\quad \text{Max}\ &\big(\theta\alpha+(1-\theta)\beta-\theta\gamma-(1-\theta)\delta-(w_1(d_1^+ + d_1^-)\\
&+w_2(d_2^+ + d_2^-)+\cdots+w_K(d_K^+ + d_K^-))\big)\\
\text{s.t.}\ &U_k^\mu - Z_k'(x) \ge (U_k^\mu - L_k^\mu)(\theta\alpha+(1-\theta)\beta),\\
&\eta(U_k^\mu - Z_k'(x)) \ge (U_k^\mu - L_k^\mu)\alpha,\\
&Z_k'(x) - L_k^\nu \le (U_k^\nu - L_k^\nu)(\theta\gamma+(1-\theta)\delta),\\
&\eta(Z_k'(x) - L_k^\nu) \le (U_k^\nu - L_k^\nu)\gamma,\\
&Z_k'(x) - d_k^+ + d_k^- = G_k,\\
&\theta\alpha+(1-\theta)\beta+\theta\gamma+(1-\theta)\delta \le 1,\\
&\beta+\delta\le 1,\quad \beta\ge\alpha,\quad \delta\ge\gamma,\\
&\gamma\ge 0,\quad \alpha\ge 0,\quad 0\le\theta\le 1,\\
&d_k^-,\ d_k^+\ge 0,\quad k=1,2,\ldots,K\\
&\text{and all the constraints of (2).}
\end{aligned}
$$

6.2 Exponential membership function

The functions representing the two ends of exponential membership function $\mu_k^U(Z_k'(x))$ and $\mu_k^L(Z_k'(x))$ (Fig. 4) for the kth objective are given by:

$$
\mu_k^U\left(Z_k'(x)\right) = \begin{cases} 1 & \text{if } Z_k'(x) \le L_k^\mu,\\ \dfrac{e^{-S_k\left((Z_k'(x)-L_k^\mu)/(U_k^\mu-L_k^\mu)\right)} - e^{-S_k}}{1-e^{-S_k}} & \text{if } L_k^\mu < Z_k'(x) < U_k^\mu,\\ 0 & \text{if } Z_k'(x) \ge U_k^\mu, \end{cases}
$$

$$
\mu_k^L\left(Z_k'(x)\right) = \begin{cases} 1 & \text{if } Z_k'(x) \le L_k^\mu,\\ \eta\left(\dfrac{e^{-S_k\left((Z_k'(x)-L_k^\mu)/(U_k^\mu-L_k^\mu)\right)} - e^{-S_k}}{1-e^{-S_k}}\right) & \text{if } L_k^\mu < Z_k'(x) < U_k^\mu,\\ 0 & \text{if } Z_k'(x) \ge U_k^\mu. \end{cases}
$$

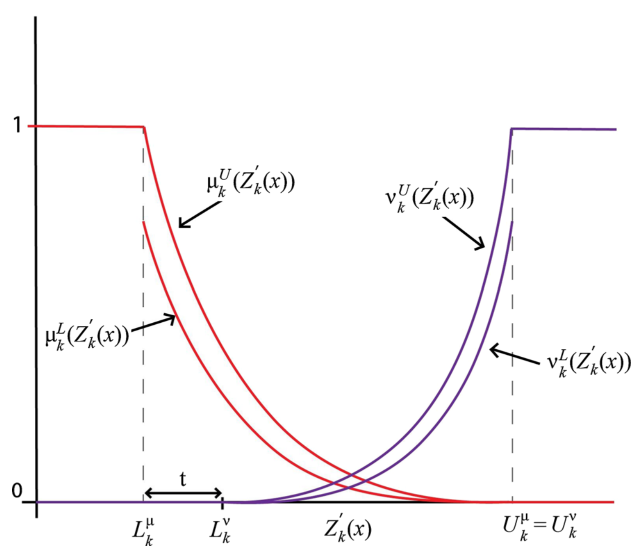


Fig. 4 Lower and upper exponential membership and non-membership functions

The functions representing upper and lower exponential non-membership functions $\nu_k^U(Z_k'(x))$ and $\nu_k^L(Z_k'(x))$ (Fig. 4) for the kth objective are defined by:

$$
\nu_k^U\left(Z_k'(x)\right) = \begin{cases} 0 & \text{if } Z_k'(x) \le L_k^\nu,\\ \dfrac{e^{-S_k\left((U_k^\nu-Z_k'(x))/(U_k^\nu-L_k^\nu)\right)} - e^{-S_k}}{1-e^{-S_k}} & \text{if } L_k^\nu < Z_k'(x) < U_k^\nu,\\ 1 & \text{if } Z_k'(x) \ge U_k^\nu, \end{cases}
$$

$$
\nu_k^L\left(Z_k'(x)\right) = \begin{cases} 0 & \text{if } Z_k'(x) \le L_k^\nu,\\ \eta\left(\dfrac{e^{-S_k\left((U_k^\nu-Z_k'(x))/(U_k^\nu-L_k^\nu)\right)} - e^{-S_k}}{1-e^{-S_k}}\right) & \text{if } L_k^\nu < Z_k'(x) < U_k^\nu,\\ 1 & \text{if } Z_k'(x) \ge U_k^\nu, \end{cases}
$$

where $0 \le \eta \le 1$ and S_k, $k = 1, 2, \ldots, K$ are the shape parameters decided by the DM.

Finally, applying the exponential membership function, the model (COM) becomes:

$$
\begin{aligned}
\textbf{(EMF)}\quad \text{Max}\ &\big(\theta\alpha+(1-\theta)\beta-\theta\gamma-(1-\theta)\delta-(w_1(d_1^+ + d_1^-)\\
&+w_2(d_2^+ + d_2^-)+\cdots+w_K(d_K^+ + d_K^-))\big)\\
\text{s.t.}\ &e^{-S_k\left((Z_k'(x)-L_k^\mu)/(U_k^\mu-L_k^\mu)\right)} - (1-e^{-S_k})(\theta\alpha+(1-\theta)\beta)\\
&\ge e^{-S_k},\\
&\eta\left(e^{-S_k\left((Z_k'(x)-L_k^\mu)/(U_k^\mu-L_k^\mu)\right)}\right) - (1-e^{-S_k})\alpha \ge \eta e^{-S_k},\\
&e^{-S_k\left((U_k^\nu-Z_k'(x))/(U_k^\nu-L_k^\nu)\right)} - (1-e^{-S_k})(\theta\gamma+(1-\theta)\delta)\\
&\le e^{-S_k},\\
&\eta\left(e^{-S_k\left((U_k^\nu-Z_k'(x))/(U_k^\nu-L_k^\nu)\right)}\right) - (1-e^{-S_k})\gamma \le \eta e^{-S_k},\\
&Z_k'(x) - d_k^+ + d_k^- = G_k,\\
&\theta\alpha+(1-\theta)\beta+\theta\gamma+(1-\theta)\delta \le 1,\\
&\beta+\delta\le 1,\quad \beta\ge\alpha,\quad \delta\ge\gamma,\\
&\gamma\ge 0,\quad \alpha\ge 0,\quad 0\le\theta\le 1,\\
&d_k^-,\ d_k^+\ge 0,\quad k=1,2,\ldots,K\\
&\text{and all the constraints of (2).}
\end{aligned}
$$

6.3 Hyperbolic Membership function

The upper and lower hyperbolic membership $\mu_k^U(Z_k'(x))$ and $\mu_k^L(Z_k'(x))$ (Fig. 5) for the kth objective are defined by the following functions:

$$
\mu_k^U\left(Z_k'(x)\right) = \begin{cases} 1 & \text{if } Z_k'(x) \le L_k^\mu,\\ \dfrac{1}{2}\tanh\left(\alpha_k^1\left(\dfrac{U_k^\mu+L_k^\mu}{2} - Z_k'(x)\right)\right) + \dfrac{1}{2} & \text{if } L_k^\mu < Z_k'(x) < U_k^\mu,\\ 0 & \text{if } Z_k'(x) \ge U_k^\mu, \end{cases}
$$

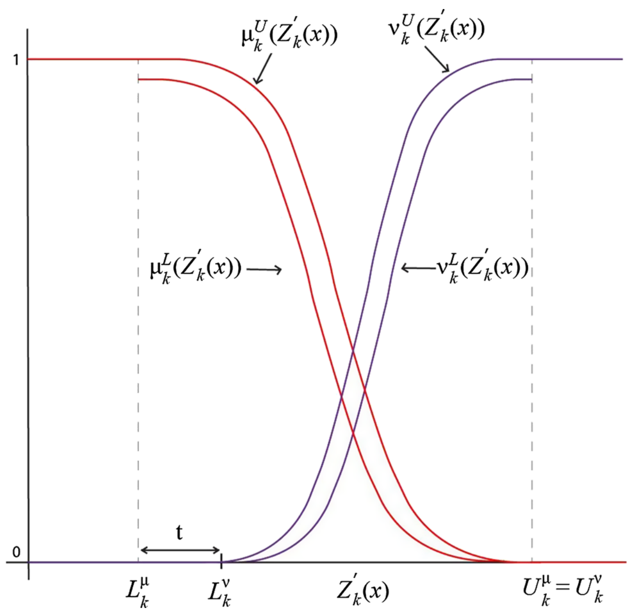


Fig. 5 Lower and upper hyperbolic membership and non-membership functions

$$\mu_k^L\left(Z_k'(x)\right) = \begin{cases} 1 & \text{if } Z_k'(x) \le L_k^\mu, \\ \eta\left(\frac{1}{2}\tanh\left(\alpha_k^1\left(\frac{U_k^\mu + L_k^\mu}{2} - Z_k'(x)\right)\right) + \frac{1}{2}\right) & \text{if } L_k^\mu < Z_k'(x) < U_k^\mu, \\ 0 & \text{if } Z_k'(x) \ge U_k^\mu, \end{cases}$$

where $\alpha_k^1 = \dfrac{6}{U_k^\mu - L_k^\mu}$, $k = 1, 2, \ldots, K$.

The upper and lower hyperbolic ends of the non-membership $\nu_k^U(Z_k'(x))$ and $\nu_k^L(Z_k'(x))$ (Fig. 5) for the kth objective are given by the following functions:

$$\nu_k^U\left(Z_k'(x)\right) = \begin{cases} 0 & \text{if } Z_k'(x) \le L_k^\nu, \\ \frac{1}{2}\tanh\left(\alpha_k^2\left(Z_k'(x) - \frac{U_k^\nu + L_k^\nu}{2}\right)\right) + \frac{1}{2} & \text{if } L_k^\nu < Z_k'(x) < U_k^\nu, \\ 1 & \text{if } Z_k'(x) \ge U_k^\nu, \end{cases}$$

$$\nu_k^L\left(Z_k'(x)\right) = \begin{cases} 0 & \text{if } Z_k'(x) \le L_k^\nu, \\ \eta\left(\frac{1}{2}\tanh\left(\alpha_k^2\left(Z_k'(x) - \frac{U_k^\nu + L_k^\nu}{2}\right)\right) + \frac{1}{2}\right) & \text{if } L_k^\nu < Z_k'(x) < U_k^\nu, \\ 1 & \text{if } Z_k'(x) \ge U_k^\nu, \end{cases}$$

where $0 \le \eta \le 1$ and $\alpha_k^2 = \dfrac{6}{U_k^\nu - L_k^\nu}$, $k = 1, 2, \ldots, K$.

The model (COM) becomes:

$$\begin{aligned} \textbf{(HMF)}\ \ \text{Max}\ & \big(\theta\alpha + (1-\theta)\beta - \theta\gamma - (1-\theta)\delta - (w_1(d_1^+ + d_1^-) \\ & + w_2(d_2^+ + d_2^-) + \cdots + w_K(d_K^+ + d_K^-))\big) \\ \text{s.t. } & \alpha_k^1 Z_k'(x) + \tanh^{-1}\left(2(\theta\alpha + (1-\theta)\beta) - 1\right) \\ & \le \frac{\alpha_k^1}{2}(U_k^\mu + L_k^\mu), \\ & \alpha_k^1 Z_k'(x) + \tanh^{-1}\left(\frac{2\alpha}{\eta} - 1\right) \le \frac{\alpha_k^1}{2}(U_k^\mu + L_k^\mu), \\ & \alpha_k^2 Z_k'(x) - \tanh^{-1}\left(2(\theta\gamma + (1-\theta)\delta) - 1\right) \\ & \le \frac{\alpha_k^2}{2}(U_k^\nu + L_k^\nu), \\ & \alpha_k^2 Z_k'(x) - \tanh^{-1}\left(\frac{2\gamma}{\eta} - 1\right) \\ & \le \frac{\alpha_k^2}{2}(U_k^\nu + L_k^\nu), \\ & Z_k'(x) - d_k^+ + d_k^- = G_k, \\ & \theta\alpha + (1-\theta)\beta + \theta\gamma + (1-\theta)\delta \le 1, \\ & \beta + \delta \le 1,\ \ \beta \ge \alpha,\ \ \delta \ge \gamma, \\ & \gamma \ge 0,\ \ \alpha \ge 0,\ \ 0 \le \theta \le 1, \\ & d_k^-,\ d_k^+ \ge 0,\quad k = 1, 2, \ldots, K \\ & \text{and all the constraints of (2),} \end{aligned}$$

where $\alpha_k^1 = \dfrac{6}{U_k^\mu - L_k^\mu}$, $\alpha_k^2 = \dfrac{6}{U_k^\nu - L_k^\nu}$, $k = 1, 2, \ldots, K$.

Remark 4 The above models are solved in IVIF environment but they can be reduced into IF and fuzzy environment as discussed below:

(i) If $\theta = 1$ and $\eta = 1$, the above models (LMF), (EMF) and (HMF) become equivalent models in IF sense.
(ii) If $\alpha = \beta$, $\gamma = \delta = 0$ and $\eta = 1$, then the models reduce to problems in fuzzy environment.

7 Major advantages of the proposed algorithm

The main advantages of the proposed study over the existing studies are as follows:

1. Because IVIF set theory is the more general concept, consequently, the proposed method can be reduced for solving both fuzzy transportation problems as well as IFTPs.
2. The ranking method used is universal; hence, it can be utilized for ordering of all IVIFNs. Therefore, the proposed algorithm can be applied to solve a general IVIFTP.
3. In comparison with the existing studies, the proposed approach can be successfully used for solving single-objective as well as the multi-objective fully IVIFTP, where all the parameters of the model are expressed by IVTIFNs.
4. The proposed approach gives optimal solutions as non-negative IVTIFNs, i.e. no negative part appears in the

IVIF quantities of the commodity or in the values of various objectives involved in the problem under consideration.

5. The solution obtained are in the form of IVTIFNs and transportation quantities are in IVTIFNs form instead of crisp values. This provides the DM more freedom which is very impressive in view of practical applications.
6. In the GP approach to solve MOTP, we not only maximize the acceptance level of the objectives but also minimize deviational variables associated with each goal, which provide better solution to the problem in terms of satisfaction level of DM.

8 Numerical illustration

The applicability and flexibility of the proposed approach is authenticated by completing the optimal shipping plan from two origins to three destinations along with the objectives of minimizing the total transportation cost, labour working hours and wastage of raw material based on uncertain data represented by IVTIFNs.

Let us consider the problem of a company manager who would like to transport the cartons of shoes from two different factories situated at S_1 and S_2 to three different major retail stores located at D_1, D_2 and D_3 (Fig. 6). The aim of manager is to

(a) minimize the total transportation cost,
(b) minimize the total labour working hours,
(c) minimize the wastage of raw material used,
(d) find the number of cartons to be transported from different origins to destinations.

The production of shoes depends on various factors such as condition of machines, labourers skill and power cuts, and the demand also varies due to various factors like seasons, locality and lifestyle of locals. The cartons are to be transported by trucks. There are variations in the transportation cost due to various irresistible factors such as climatic conditions, diesel expenses and traffic jams. The data dealing with the required labour working hours and the estimated data of wastage of raw material are uncertain.

Table 3 summarizes all the data showing transportation costs due to fuel expenses $\tilde{c}^1_{ij}$, labour working hours $\tilde{c}^2_{ij}$ and wastage of raw material $\tilde{c}^3_{ij}$.

The values of $\tilde{c}^1_{ij}, \tilde{c}^2_{ij}$ and $\tilde{c}^3_{ij}$ for $i = 1, 2;\ j = 1, 2, 3$ are:

$$\tilde{c}^1_{11} = \{(15, 20, 25, 30, 35), (5, 10, 25, 40, 45)\},$$
$$\tilde{c}^1_{12} = \{(40, 45, 50, 55, 60), (30, 35, 50, 65, 70)\},$$
$$\tilde{c}^1_{13} = \{(75, 80, 85, 90, 95), (65, 70, 85, 100, 105)\},$$
$$\tilde{c}^1_{21} = \{(35, 40, 45, 50, 55), (25, 30, 45, 60, 65)\},$$
$$\tilde{c}^1_{22} = \{(50, 55, 60, 65, 70), (40, 45, 60, 75, 80)\},$$
$$\tilde{c}^1_{23} = \{(25, 30, 35, 40, 45), (15, 20, 35, 50, 55)\},$$
$$\tilde{c}^2_{11} = \{(6, 8, 10, 12, 14), (2, 4, 10, 16, 18)\},$$
$$\tilde{c}^2_{12} = \{(20, 25, 30, 35, 40), (10, 15, 30, 45, 50)\},$$
$$\tilde{c}^2_{13} = \{(22, 23, 24, 25, 26), (20, 21, 24, 27, 28)\},$$
$$\tilde{c}^2_{21} = \{(30, 35, 40, 45, 50), (20, 25, 40, 55, 60)\},$$
$$\tilde{c}^2_{22} = \{(12, 16, 20, 24, 28), (4, 8, 20, 32, 36)\},$$
$$\tilde{c}^2_{23} = \{(25, 35, 45, 55, 65), (5, 15, 45, 75, 85)\},$$
$$\tilde{c}^3_{11} = \{(3, 4, 5, 6, 7), (1, 2, 5, 8, 9)\},$$
$$\tilde{c}^3_{12} = \{(12, 14, 16, 18, 20), (8, 10, 16, 22, 24)\},$$
$$\tilde{c}^3_{13} = \{(15, 20, 25, 30, 35), (5, 10, 25, 40, 45)\},$$
$$\tilde{c}^3_{21} = \{(18, 19, 20, 21, 22), (16, 17, 20, 23, 24)\},$$
$$\tilde{c}^3_{22} = \{(9, 12, 15, 18, 21), (3, 6, 15, 24, 27)\},$$
$$\tilde{c}^3_{23} = \{(26, 28, 30, 32, 34), (22, 24, 30, 36, 38)\}.$$

The availabilities $\tilde{a}_i$ for $i = 1, 2$ and demands $\tilde{b}_j$ for $j = 1, 2, 3$ are given as:

$$\tilde{a}_1 = \{(110, 120, 130, 140, 150), (90, 100, 130, 160, 170)\},$$
$$\tilde{a}_2 = \{(50, 60, 70, 80, 90), (30, 40, 70, 100, 110)\},$$
$$\tilde{b}_1 = \{(40, 50, 60, 65, 70), (30, 35, 60, 75, 80)\},$$
$$\tilde{b}_2 = \{(40, 45, 50, 55, 60), (20, 30, 50, 65, 75)\},$$
$$\tilde{b}_3 = \{(80, 85, 90, 100, 110), (70, 75, 90, 120, 125)\}.$$

It is to be noted that $\tilde{a}_1 + \tilde{a}_2 = \tilde{b}_1 + \tilde{b}_2 + \tilde{b}_3$.

Solution:

Step 1 Mathematically, the multi-objective IVIFTP is formulated as follows:

$$\textbf{(P1)}\quad \text{Minimize } \tilde{Z}(x) = \{\tilde{Z}_1(x),\ \tilde{Z}_2(x),\ \tilde{Z}_3(x)\}$$
$$\text{where } \tilde{Z}_p(x) = \sum_{i=1}^{2}\sum_{j=1}^{3} \tilde{c}^p_{ij} \otimes \tilde{x}_{ij};\quad p = 1, 2, 3$$
$$\text{subject to } \sum_{j=1}^{3} \tilde{x}_{ij} = \tilde{a}_i,\quad i = 1, 2,$$
$$\sum_{i=1}^{2} \tilde{x}_{ij} = \tilde{b}_j,\quad j = 1, 2, 3,$$
$$\tilde{x}_{ij} \succsim \tilde{0},\quad \forall\, i, j.$$

Step 2 Divide the problem (P1) into three subproblems, (S1), (S2), (S3), and substitute $\tilde{x}_{ij} = \{(x^U_{ij1}, x^L_{ij1}, x_{ij2}, x^L_{ij3}, x^U_{ij3}), (x'^L_{ij1}, x'^U_{ij1}, x_{ij2}, x'^U_{ij3}, x'^L_{ij3})\}$, with the common set of constraints (3) given below:

Table 3 Data related to shoe factory TP

Sources	Destinations			Availabilities
	D_1	D_2	D_3	$\tilde{a}_i$
S_1	$\{\tilde{c}_{11}^1, \tilde{c}_{11}^2, \tilde{c}_{11}^3\}$	$\{\tilde{c}_{12}^1, \tilde{c}_{12}^2, \tilde{c}_{12}^3\}$	$\{\tilde{c}_{13}^1, \tilde{c}_{13}^2, \tilde{c}_{13}^3\}$	$\tilde{a}_1$
S_2	$\{\tilde{c}_{21}^1, \tilde{c}_{21}^2, \tilde{c}_{21}^3\}$	$\{\tilde{c}_{22}^1, \tilde{c}_{22}^2, \tilde{c}_{22}^3\}$	$\{\tilde{c}_{23}^1, \tilde{c}_{23}^2, \tilde{c}_{23}^3\}$	$\tilde{a}_2$
Demands ($\tilde{b}_j$)	$\tilde{b}_1$	$\tilde{b}_2$	$\tilde{b}_3$	

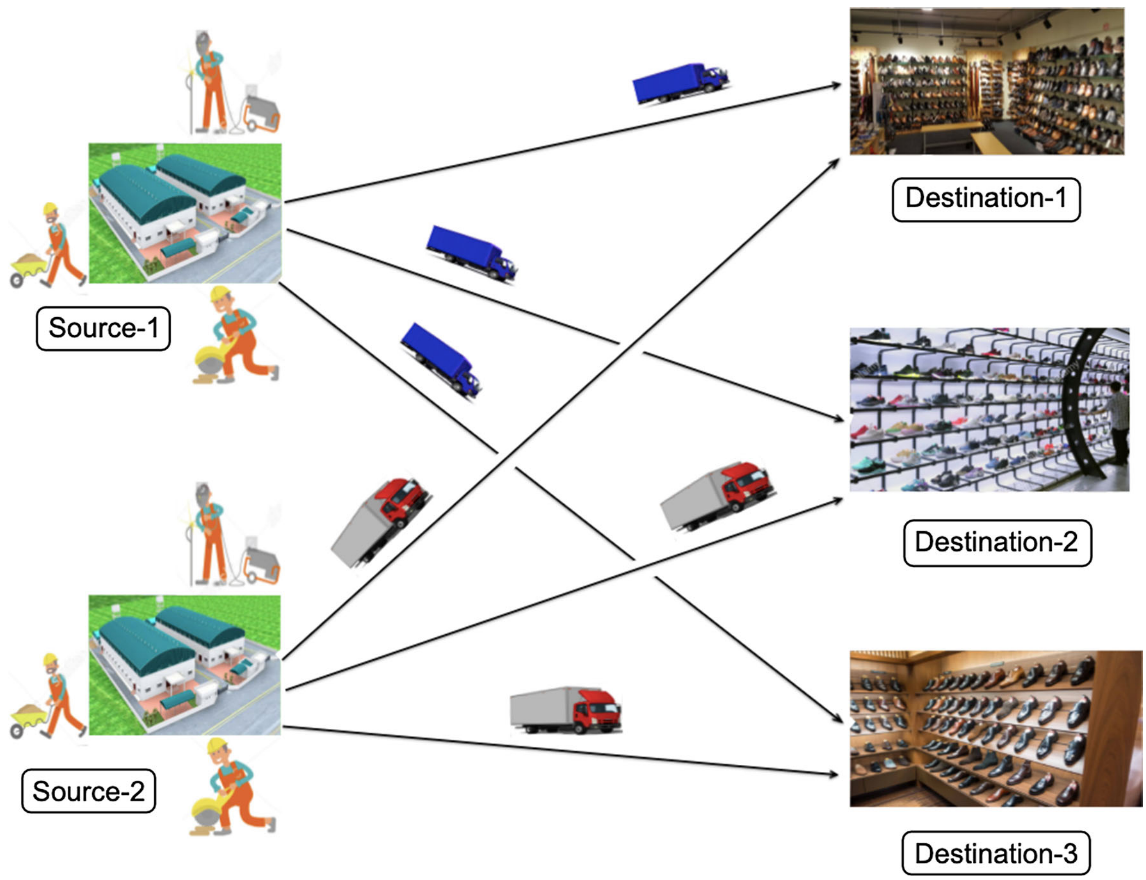


Fig. 6 Pictorial representation of the shoe company problem

$$
\left.\begin{array}{l}
x_{111}^{U}+x_{121}^{U}+x_{131}^{U}=110,\;\; x_{111}^{L}+x_{121}^{L}+x_{131}^{L}=120,\\
x_{112}+x_{122}+x_{132}=130,\\
x_{113}^{L}+x_{123}^{L}+x_{133}^{L}=140,\;\; x_{113}^{U}+x_{123}^{U}+x_{133}^{U}=150,\\
x_{111}^{\prime L}+x_{121}^{\prime L}+x_{131}^{\prime L}=90,\\
x_{111}^{\prime U}+x_{121}^{\prime U}+x_{131}^{\prime U}=100,\;\; x_{113}^{\prime U}+x_{123}^{\prime U}+x_{133}^{\prime U}=160,\\
x_{113}^{\prime L}+x_{123}^{\prime L}+x_{133}^{\prime L}=170,\\
x_{211}^{U}+x_{221}^{U}+x_{231}^{U}=50,\;\; x_{211}^{L}+x_{221}^{L}+x_{231}^{L}=60,\\
x_{212}+x_{222}+x_{232}=70,\\
x_{213}^{L}+x_{223}^{L}+x_{233}^{L}=80,\;\; x_{213}^{U}+x_{223}^{U}+x_{233}^{U}=90,\\
x_{211}^{\prime L}+x_{221}^{\prime L}+x_{231}^{\prime L}=30,\\
x_{211}^{\prime U}+x_{221}^{\prime U}+x_{231}^{\prime U}=40,\;\; x_{213}^{\prime U}+x_{223}^{\prime U}+x_{233}^{\prime U}=100,\\
x_{213}^{\prime L}+x_{223}^{\prime L}+x_{233}^{\prime L}=110,\\
x_{111}^{U}+x_{211}^{U}=40,\;\; x_{111}^{L}+x_{211}^{L}=50,\;\; x_{112}+x_{212}=60,\\
x_{113}^{L}+x_{213}^{L}=65,\\
x_{113}^{U}+x_{213}^{U}=70,\;\; x_{111}^{\prime L}+x_{211}^{\prime L}=30,\;\; x_{111}^{\prime U}+x_{211}^{\prime U}=35,\\
x_{113}^{\prime U}+x_{213}^{\prime U}=75,\\
x_{113}^{\prime L}+x_{213}^{\prime L}=80,\;\; x_{121}^{U}+x_{221}^{U}=40,\;\; x_{121}^{L}+x_{221}^{L}=45,\\
x_{122}+x_{222}=50,\\
x_{123}^{L}+x_{223}^{L}=55,\;\; x_{123}^{U}+x_{223}^{U}=60,\;\; x_{121}^{\prime L}+x_{221}^{\prime L}=20,\\
x_{121}^{\prime U}+x_{221}^{\prime U}=30,\\
x_{123}^{\prime U}+x_{223}^{\prime U}=65,\;\; x_{123}^{\prime L}+x_{223}^{\prime L}=75,\;\; x_{131}^{U}+x_{231}^{U}=80,\\
x_{131}^{L}+x_{231}^{L}=85,\\
x_{132}+x_{232}=90,\;\; x_{133}^{L}+x_{233}^{L}=100,\;\; x_{133}^{U}+x_{233}^{U}=110,\\
x_{131}^{\prime L}+x_{231}^{\prime L}=70,\\
x_{131}^{\prime U}+x_{231}^{\prime U}=75,\;\; x_{133}^{\prime U}+x_{233}^{\prime U}=120,\\
x_{133}^{\prime L}+x_{233}^{\prime L}=125,\\
x_{1j1}^{\prime L}\ge 0,\;\; x_{1j1}^{\prime U}-x_{1j1}^{\prime L}\ge 0,\;\; x_{1j1}^{U}-x_{1j1}^{\prime U}\ge 0,\\
x_{1j1}^{L}-x_{1j1}^{U}\ge 0,\;\; x_{1j2}-x_{1j1}^{L}\ge 0,\\
x_{1j3}^{L}-x_{1j2}\ge 0,\;\; x_{1j3}^{U}-x_{1j3}^{L}\ge 0,\;\; x_{1j3}^{\prime U}-x_{1j3}^{U}\ge 0,\\
x_{1j3}^{\prime L}-x_{1j3}^{\prime U}\ge 0;\;\; j=1,2,3,\\
x_{2j1}^{\prime L}\ge 0,\;\; x_{2j1}^{\prime U}-x_{2j1}^{\prime L}\ge 0,\;\; x_{2j1}^{U}-x_{2j1}^{\prime U}\ge 0,\\
x_{2j1}^{L}-x_{2j1}^{U}\ge 0,\;\; x_{2j2}-x_{2j1}^{L}\ge 0,\\
x_{2j3}^{L}-x_{2j2}\ge 0,\;\; x_{2j3}^{U}-x_{2j3}^{L}\ge 0,\;\; x_{2j3}^{\prime U}-x_{2j3}^{U}\ge 0,\\
x_{2j3}^{\prime L}-x_{2j3}^{\prime U}\ge 0;\;\; j=1,2,3.
\end{array}\right\} \tag{3}
$$

The problems (S1), (S2) and (S3) are:

(S1) $\min\ Z_1'(x)=\frac{1}{16}\Big[15x_{111}^{U}+20x_{111}^{L}+30x_{113}^{L}$
$+35x_{113}^{U}+200x_{112}+5x_{111}^{\prime L}+10x_{111}^{\prime U}$
$+40x_{113}^{\prime U}+45x_{113}^{\prime L}+40x_{121}^{U}+45x_{121}^{L}+55x_{123}^{L}$
$+60x_{123}^{U}+400x_{122}+30x_{121}^{\prime L}+35x_{121}^{\prime U}$
$+65x_{123}^{\prime U}+70x_{123}^{\prime L}+75x_{131}^{U}+80x_{131}^{L}+90x_{133}^{L}$
$+95x_{133}^{U}+680x_{132}+65x_{131}^{\prime L}$
$+70x_{131}^{\prime U}+100x_{133}^{\prime U}+105x_{133}^{\prime L}+35x_{211}^{U}+40x_{211}^{L}$
$+50x_{213}^{L}+55x_{213}^{U}+360x_{212}$
$+25x_{211}^{\prime L}+30x_{211}^{\prime U}+60x_{213}^{\prime U}+65x_{213}^{\prime L}+50x_{221}^{U}$
$+55x_{221}^{L}+65x_{223}^{L}+70x_{223}^{U}+480x_{222}$
$+40x_{221}^{\prime L}+45x_{221}^{\prime U}+75x_{223}^{\prime U}+80x_{223}^{\prime L}+25x_{231}^{U}$
$+30x_{231}^{L}+40x_{233}^{L}+45x_{233}^{U}+280x_{232}$
$+15x_{231}^{\prime L}+20x_{231}^{\prime U}+50x_{233}^{\prime U}+55x_{233}^{\prime L}\Big]$

subject to constraints (3).

(S2) $\min\ Z_2'(x)=\frac{1}{16}\Big[6x_{111}^{U}+8x_{111}^{L}+12x_{113}^{L}$
$+14x_{113}^{U}+80x_{112}+2x_{111}^{\prime L}+4x_{111}^{\prime U}+16x_{113}^{\prime U}$
$+18x_{113}^{\prime L}+20x_{121}^{U}+25x_{121}^{L}+35x_{123}^{L}+40x_{123}^{U}$
$+240x_{122}+10x_{121}^{\prime L}+15x_{121}^{\prime U}$
$+45x_{123}^{\prime U}+50x_{123}^{\prime L}+22x_{131}^{U}+23x_{131}^{L}+25x_{133}^{L}$
$+26x_{133}^{U}+192x_{132}+20x_{131}^{\prime L}+21x_{131}^{\prime U}$
$+27x_{133}^{\prime U}+28x_{133}^{\prime L}+30x_{211}^{U}+35x_{211}^{L}+45x_{213}^{L}$
$+50x_{213}^{U}+320x_{212}+20x_{211}^{\prime L}$
$+25x_{211}^{\prime U}+55x_{213}^{\prime U}+60x_{213}^{\prime L}+12x_{221}^{U}+16x_{221}^{L}$
$+24x_{223}^{L}+28x_{223}^{U}+160x_{222}+4x_{221}^{\prime L}$
$+8x_{221}^{\prime U}+32x_{223}^{\prime U}+36x_{223}^{\prime L}+25x_{231}^{U}+35x_{231}^{L}$
$+55x_{233}^{L}+65x_{233}^{U}+360x_{232}+5x_{231}^{\prime L}$
$+15x_{231}^{\prime U}+75x_{233}^{\prime U}+85x_{233}^{\prime L}\Big]$

subject to constraints (3).

(S3) $\min\ Z_3'(x)=\frac{1}{16}\Big[3x_{111}^{U}+4x_{111}^{L}+6x_{113}^{L}+7x_{113}^{U}$
$+40x_{112}+x_{111}^{\prime L}+2x_{111}^{\prime U}+8x_{113}^{\prime U}+9x_{113}^{\prime L}$
$+12x_{121}^{U}+14x_{121}^{L}+18x_{123}^{L}+20x_{123}^{U}+128x_{122}$
$+8x_{121}^{\prime L}+10x_{121}^{\prime U}+22x_{123}^{\prime U}+24x_{123}^{\prime L}$
$+15x_{131}^{U}+20x_{131}^{L}+30x_{133}^{L}+35x_{133}^{U}+200x_{132}$
$+5x_{131}^{\prime L}+10x_{131}^{\prime U}+40x_{133}^{\prime U}+45x_{133}^{\prime L}$
$+18x_{211}^{U}+19x_{211}^{L}+21x_{213}^{L}+22x_{213}^{U}+160x_{212}$
$+16x_{211}^{\prime L}+17x_{211}^{\prime U}+23x_{213}^{\prime U}+24x_{213}^{\prime L}$
$+9x_{221}^{U}+12x_{221}^{L}+18x_{223}^{L}+21x_{223}^{U}+120x_{222}$
$+3x_{221}^{\prime L}+6x_{221}^{\prime U}+24x_{223}^{\prime U}+27x_{223}^{\prime L}$
$+26x_{231}^{U}+28x_{231}^{L}+32x_{233}^{L}+34x_{233}^{U}+240x_{232}$
$+22x_{231}^{\prime L}+24x_{231}^{\prime U}+36x_{233}^{\prime U}+38x_{233}^{\prime L}\Big]$

subject to constraints (3).

Step 3 The optimal solutions of the models (S1), (S2) and (S3), respectively, are:

$$X_1 = \begin{Bmatrix} \tilde{x}_{11} = \{(40, 50, 60, 65, 70), (30, 35, 60, 75, 80)\}, \\ \tilde{x}_{12} = \{(30, 30, 30, 35, 40), (20, 25, 30, 45, 50)\}, \\ \tilde{x}_{13} = \{(40, 40, 40, 40, 40), (40, 40, 40, 40, 40)\}, \\ \tilde{x}_{21} = \{(0, 0, 0, 0, 0), (0, 0, 0, 0, 0)\}, \\ \tilde{x}_{22} = \{(10, 15, 20, 20, 20), (0, 5, 20, 20, 25)\}, \\ \tilde{x}_{23} = \{(40, 45, 50, 60, 70), (30, 35, 50, 80, 85)\}. \end{Bmatrix}$$

$$X_2 = \begin{Bmatrix} \tilde{x}_{11} = \{(40, 50, 60, 60, 60), (30, 35, 60, 60, 65)\}, \\ \tilde{x}_{12} = \{(0, 0, 0, 0, 0), (0, 0, 0, 0, 0)\}, \\ \tilde{x}_{13} = \{(70, 70, 70, 80, 90), (60, 65, 70, 100, 105)\}, \\ \tilde{x}_{21} = \{(0, 0, 0, 5, 10), (0, 0, 0, 15, 15)\}, \\ \tilde{x}_{22} = \{(40, 45, 50, 55, 60), (20, 30, 50, 65, 75)\}, \\ \tilde{x}_{23} = \{(10, 15, 20, 20, 20), (10, 10, 20, 20, 20)\}. \end{Bmatrix}$$

$$X_3 = \begin{Bmatrix} \tilde{x}_{11} = \{(40, 50, 60, 65, 70), (30, 35, 60, 75, 80)\}, \\ \tilde{x}_{12} = \{(0, 0, 0, 5, 10), (0, 0, 0, 15, 20)\}, \\ \tilde{x}_{13} = \{(70, 70, 70, 70, 70), (60, 65, 70, 70, 70)\}, \\ \tilde{x}_{21} = \{(0, 0, 0, 0, 0), (0, 0, 0, 0, 0)\}, \\ \tilde{x}_{22} = \{(40, 45, 50, 50, 50), (20, 30, 50, 50, 55)\}, \\ \tilde{x}_{23} = \{(10, 15, 20, 30, 40), (10, 10, 20, 50, 55)\}. \end{Bmatrix}$$

Step 4 The payoff matrix

$$Z = \begin{pmatrix} 9709.375 & 5648.75 & 3782.1875 \\ 11837.5 & 4499.375 & 3692.8125 \\ 11453.12 & 4713.4375 & 3621.56 \end{pmatrix}$$

where $Z'_j(X_i) = EV(\tilde{Z}_j(X_i))\ \forall\, i$ and j.

Step 5 From the payoff matrix Z, we get $U_1 = 11837.5$, $L_1 = 9709.375$, $U_2 = 5648.75$, $L_2 = 4499.375$, $U_3 = 3782.1875$, $L_3 = 3621.56$. Set $G_1 = 11453.12$, $G_2 = 4713.4375$, $G_3 = 3692.8125$.

Step 6 We find $x^U_{ij1}, x^L_{ij1}, x_{ij2}, x^L_{ij3}, x^U_{ij3}, x'^L_{ij1}, x'^U_{ij1}, x'^U_{ij3}, x'^L_{ij3}$, $i = 1, 2,\ \ j = 1, 2, 3$ such that

$$Z'_1(x) \sim 9709.375, \quad \text{(A)}$$
$$Z'_2(x) \sim 4499.375, \quad \text{(B)}$$
$$Z'_3(x) \sim 3621.56, \quad \text{(C)}$$

subject to constraints (3)

where $\sim$ is an IVIF equality, which can be handled using different membership functions.

Step 7 The model (COM) can be formulated as crisp model given by:

(i) Using linear membership function:

$$\begin{aligned}
\textbf{(LMF)}\ \text{Max}\ & \big(\theta\alpha + (1-\theta)\beta - \theta\gamma - (1-\theta)\delta - 0.6d_1^+ - 0.6d_1^- \\
& -0.3d_2^+ - 0.3d_2^- - 0.1d_3^+ - 0.1d_3^-\big) \\
\text{s.t.}\ & 11837.5 - Z'_1(x) \ge 2128.125(\theta\alpha + (1-\theta)\beta), \\
& 5648.75 - Z'_2(x) \ge 1149.375(\theta\alpha + (1-\theta)\beta), \\
& 3782.1875 - Z'_3(x) \ge 160.6275(\theta\alpha + (1-\theta)\beta), \\
& \eta(11837.5 - Z'_1(x)) \ge 2128.125\alpha, \\
& \eta(5648.75 - Z'_2(x)) \ge 1149.375\alpha, \\
& \eta(3782.1875 - Z'_3(x)) \ge 160.6275\alpha, \\
& Z'_1(x) - 9975.39 \le 1862.11(\theta\gamma + (1-\theta)\delta), \\
& Z'_2(x) - 4643.05 \le 1005.7(\theta\gamma + (1-\theta)\delta), \\
& Z'_3(x) - 3641.64 \le 140.5475(\theta\gamma + (1-\theta)\delta), \\
& \eta(Z'_1(x) - 9975.39) \ge 1862.11\gamma, \\
& \eta(Z'_2(x) - 4643.05) \ge 1005.7\gamma, \\
& \eta(Z'_3(x) - 3641.64) \ge 140.5475\gamma, \\
& Z'_1(x) - d_1^+ + d_1^- = 11453.12, \\
& Z'_2(x) - d_2^+ + d_2^- = 4713.4375, \\
& Z'_3(x) - d_3^+ + d_3^- = 3692.8125, \\
& \theta\alpha + (1-\theta)\beta + \theta\gamma + (1-\theta)\delta \le 1, \\
& \beta + \delta \le 1,\ \ \beta \ge \alpha,\ \ \delta \ge \gamma, \\
& \gamma \ge 0,\ \ \alpha \ge 0,\ \ 0 \le \theta,\ \eta \le 1, \\
& d_1^-,\ d_1^+,\ d_2^-,\ d_2^+,\ d_3^-,\ d_3^+ \ge 0, \\
& \text{along with set of constraints (3), taking t=0.125.}
\end{aligned}$$

(ii) Using exponential membership function:

$$\begin{aligned}
\textbf{(EMF)}\ \text{Max}\ & \big(\theta\alpha + (1-\theta)\beta \\
& -\theta\gamma - (1-\theta)\delta - 0.6d_1^+ - 0.6d_1^- \\
& -0.3d_2^+ - 0.3d_2^- - 0.1d_3^+ - 0.1d_3^-\big) \\
\text{s.t.}\ & e^{-\left((Z'_1(x) - 9709.375)/2128.125\right)} \\
& -(1-e^{-1})(\theta\alpha + (1-\theta)\beta) \ge e^{-1}, \\
& e^{-\left((Z'_2(x) - 4499.375)/1149.375\right)} \\
& -(1-e^{-1})(\theta\alpha + (1-\theta)\beta) \ge e^{-1}, \\
& e^{-\left((Z'_3(x) - 3621.56)/160.6275\right)} \\
& -(1-e^{-1})(\theta\alpha + (1-\theta)\beta) \ge e^{-1}, \\
& \eta\Big(e^{-\left((Z'_1(x) - 9709.375)/2128.125\right)}\Big) \\
& -(1-e^{-1})\alpha \ge \eta e^{-1}, \\
& \eta\Big(e^{-\left((Z'_2(x) - 4499.375)/1149.375\right)}\Big) \\
& -(1-e^{-1})\alpha \ge \eta e^{-1}, \\
& \eta\Big(e^{-\left((Z'_3(x) - 3621.56)/160.6275\right)}\Big) \\
& -(1-e^{-1})\alpha \ge \eta e^{-1}, \\
& e^{-\left((11837.5 - Z'_1(x))/1862.11\right)} \\
& -(1-e^{-1})(\theta\gamma + (1-\theta)\delta) \le e^{-1}, \\
& e^{-\left((5648.75 - Z'_2(x))/1005.7\right)} \\
& -(1-e^{-1})(\theta\gamma + (1-\theta)\delta) \le e^{-1}, \\
& e^{-\left((3782.1875 - Z'_3(x))/140.5475\right)} \\
& -(1-e^{-1})(\theta\gamma + (1-\theta)\delta) \le e^{-1}, \\
& \eta\Big(e^{-\left((11837.5 - Z'_1(x))/1862.11\right)}\Big) \\
& -(1-e^{-1})\gamma \le \eta e^{-1}, \\
& \eta\Big(e^{-\left((5648.75 - Z'_2(x))/1005.7\right)}\Big) \\
& -(1-e^{-1})\gamma \le \eta e^{-1},
\end{aligned}$$

Table 4 Value of objective function for (LMF) model

θ	η									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.1	−0.6129	−0.6129	−0.6129	−0.6129	−0.6129	−0.6129	−0.6129	−0.6129	−0.6129	−0.6129
0.2	−0.6248	−0.6212	−0.6176	−0.6140	−0.6129	−0.6129	−0.6129	−0.6129	−0.6129	−0.6129
0.3	−0.6437	−0.6382	−0.6328	−0.6274	−0.6220	−0.6166	−0.6129	−0.6129	−0.6129	−0.6129
0.4	−0.6625	−0.6553	−0.6480	−0.6408	−0.6336	−0.6264	−0.6191	−0.6129	−0.6129	−0.6129
0.5	−0.6813	−0.6723	−0.6633	−0.6542	−0.6452	−0.6362	−0.6272	−0.6181	−0.6129	−0.6129
0.6	−0.7002	−0.6893	−0.6785	−0.6677	−0.6568	−0.6460	−0.6351	−0.6243	−0.6135	−0.6129
0.7	−0.7190	−0.7064	−0.6937	−0.6811	−0.6684	−0.6558	−0.6431	−0.6305	−0.6179	−0.6129
0.8	−0.7378	−0.7234	−0.7089	−0.6945	−0.6801	−0.6656	−0.6511	−0.6367	−0.6150	−0.6222
0.9	−0.7567	−0.7404	−0.7242	−0.7079	−0.6916	−0.6754	−0.6591	−0.6429	−0.6266	−0.6185
0.95	−0.7661	−0.7489	−0.7318	−0.7146	−0.6975	−0.6803	−0.6631	−0.6460	−0.6288	−0.6202

$$\begin{aligned}
&\eta\left(\mathrm{e}^{-\left((3782.1875-Z_3'(x))/140.5475\right)}\right)\\
&-(1-\mathrm{e}^{-1})\gamma \le \eta\mathrm{e}^{-1},\\
&Z_1'(x)-d_1^+ + d_1^- = 11453.12,\\
&Z_2'(x)-d_2^+ + d_2^- = 4713.4375,\\
&Z_3'(x)-d_3^+ + d_3^- = 3692.8125,\\
&\theta\alpha+(1-\theta)\beta+\theta\gamma+(1-\theta)\delta \le 1,\\
&\beta+\delta\le 1,\quad \beta\ge\alpha,\quad \delta\ge\gamma,\\
&\gamma\ge 0,\quad \alpha\ge 0,\quad 0\le\theta,\ \eta\le 1,\\
&d_1^-,\ d_1^+,\ d_2^-,\ d_2^+,\ d_3^-,\ d_3^+\ge 0,
\end{aligned}$$

along with set of constraints (3), taking $t = 0.125,\ S_k = 1\ \forall\ k$.

(iii) Using hyperbolic membership function:

$$\begin{aligned}
\textbf{(HMF)}\ \ \text{Max}\ &\left(\theta\alpha+(1-\theta)\beta-\theta\gamma-(1-\theta)\delta-0.6d_1^+\right.\\
&\left.-0.6d_1^- -0.3d_2^+ -0.3d_2^- -0.1d_3^+ -0.1d_3^-\right)\\
\text{s.t. }&0.0028Z_1'(x)+\tanh^{-1}\left(2(\theta\alpha+(1-\theta)\beta)-1\right) \le 30.1656,\\
&0.0052Z_2'(x)+\tanh^{-1}\left(2(\theta\alpha+(1-\theta)\beta)-1\right) \le 26.3851,\\
&0.0373Z_3'(x)+\tanh^{-1}\left(2(\theta\alpha+(1-\theta)\beta)-1\right) \le 138.08,\\
&0.0028Z_1'(x)+\tanh^{-1}\left(\frac{2\alpha}{\eta}-1\right)\le 30.1656,\\
&0.0052Z_2'(x)+\tanh^{-1}\left(\frac{2\alpha}{\eta}-1\right)\le 26.3851,\\
&0.0373Z_3'(x)+\tanh^{-1}\left(\frac{2\alpha}{\eta}-1\right)\le 138.08,\\
&0.0032Z_1'(x)-\tanh^{-1}\left(2(\theta\gamma+(1-\theta)\delta)-1\right) \le 34.9006,\\
&0.006Z_2'(x)-\tanh^{-1}\left(2(\theta\gamma+(1-\theta)\delta)-1\right) \le 30.8754,\\
&0.0427Z_3'(x)-\tanh^{-1}\left(2(\theta\gamma+(1-\theta)\delta)-1\right) \le 158.4987,\\
&0.0032Z_1'(x)-\tanh^{-1}\left(\frac{2\gamma}{\eta}-1\right)\le 34.9006,\\
&0.006Z_2'(x)-\tanh^{-1}\left(\frac{2\gamma}{\eta}-1\right)\le 30.8754,\\
&0.0427Z_3'(x)-\tanh^{-1}\left(\frac{2\gamma}{\eta}-1\right)\le 158.4987,
\end{aligned}$$

$$\begin{aligned}
&Z_1'(x)-d_1^+ + d_1^- = 11453.12,\\
&Z_2'(x)-d_2^+ + d_2^- = 4713.4375,\\
&Z_3'(x)-d_3^+ + d_3^- = 3692.8125,\\
&\theta\alpha+(1-\theta)\beta+\theta\gamma+(1-\theta)\delta \le 1,\\
&\beta+\delta\le 1,\quad \beta\ge\alpha,\quad \delta\ge\gamma,\\
&\gamma\ge 0,\quad \alpha\ge 0,\quad 0\le\theta\le 1,\quad 0<\eta\le 1,\\
&d_1^-,\ d_1^+,\ d_2^-,\ d_2^+,\ d_3^-,\ d_3^+\ge 0,
\end{aligned}$$

along with set of constraints (3), takingt=0.125.

Step 8 Solving the problems (LMF), (EMF) and (HMF) for various values of parameters θ and η, the values of objective function are tabulated in Tables 4, 5 and 6, respectively. Performing the experimental simulation on the models (LMF), (EMF) and (HMF), we have concluded that the maximum value of the objective function is obtained at

(i) $\theta = 0.3$ and $\eta = 0.7$ using linear membership function.
(ii) $\theta = 0.6$ and $\eta = 0.3$ with exponential membership function.
(iii) $\theta = 0$ and $\eta = 0.6076$ using hyperbolic membership function.

The corresponding surface plots of objective function values for (LMF), (EMF) and (HMF) models showing the variation of objective function values with respect to θ and η are shown in Figs. 7, 8 and 9, respectively.
The IVTIF optimal point and the corresponding objective function values for problem (P1) at $\theta = 0.4$ and $\eta = 0.8$, using different membership functions for (LMF), (EMF) and (HMF) models are given in Table 7.

Under IF environment:

In view of Remark 4 (i), the models (LMF), (EMF) and (HMF) can be modelled in IF environment as follows:

(i) Using linear membership approach:

Table 5 Value of objective function for (EMF) model

θ	η									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.1	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897
0.2	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897
0.3	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897
0.4	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897
0.5	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897
0.6	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897
0.7	– 0.6084	– 0.6003	– 0.5922	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897	– 0.5897
0.8	– 0.6367	– 0.6275	– 0.6183	– 0.6091	– 0.5998	– 0.5906	– 0.5897	– 0.5897	– 0.5897	– 0.5897
0.9	– 0.6651	– 0.6547	– 0.6443	– 0.6340	– 0.6236	– 0.6132	– 0.6028	– 0.5925	– 0.5897	– 0.5897
0.95	– 0.6792	– 0.6683	– 0.6573	– 0.6464	– 0.6355	– 0.6245	– 0.6136	– 0.6026	– 0.5938	– 0.5897

Table 6 Value of objective function for (HMF) model

θ	η									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.1	Infeasible	infeasible	Infeasible	Infeasible	– 2180.7	– 581.5	– 506.3	– 462.5	– 431.7	– 419.1
0.2	Infeasible	Infeasible	Infeasible	Infeasible	– 2180.7	– 581.5	– 506.3	– 462.5	– 431.7	– 419.1
0.3	Infeasible	Infeasible	Infeasible	Infeasible	– 2180.7	– 581.5	– 506.3	– 462.5	– 431.7	– 419.1
0.4	Infeasible	Infeasible	Infeasible	Infeasible	– 2260.6	– 581.5	– 506.3	– 462.5	– 431.7	– 419.1
0.5	Infeasible	infeasible	Infeasible	Infeasible	– 2180.7	– 581.5	– 506.3	– 462.5	– 431.7	– 419.1
0.6	Infeasible	Infeasible	Infeasible	Infeasible	– 2221.9	– 581.5	– 506.3	– 462.5	– 431.7	– 419.1
0.7	Infeasible	Infeasible	Infeasible	Infeasible	– 2340.3	– 581.5	– 506.3	– 462.5	– 431.7	– 419.1
0.8	Infeasible	Infeasible	Infeasible	Infeasible	– 2448.4	– 581.5	– 506.3	– 462.5	– 431.7	– 419.1
0.9	Infeasible	infeasible	Infeasible	Infeasible	– 2524.6	– 581.5	– 506.3	– 462.5	– 431.7	– 419.1
0.95	Infeasible	Infeasible	Infeasible	Infeasible	– 2693.5	– 581.5	– 506.3	– 462.5	– 431.7	– 419.1

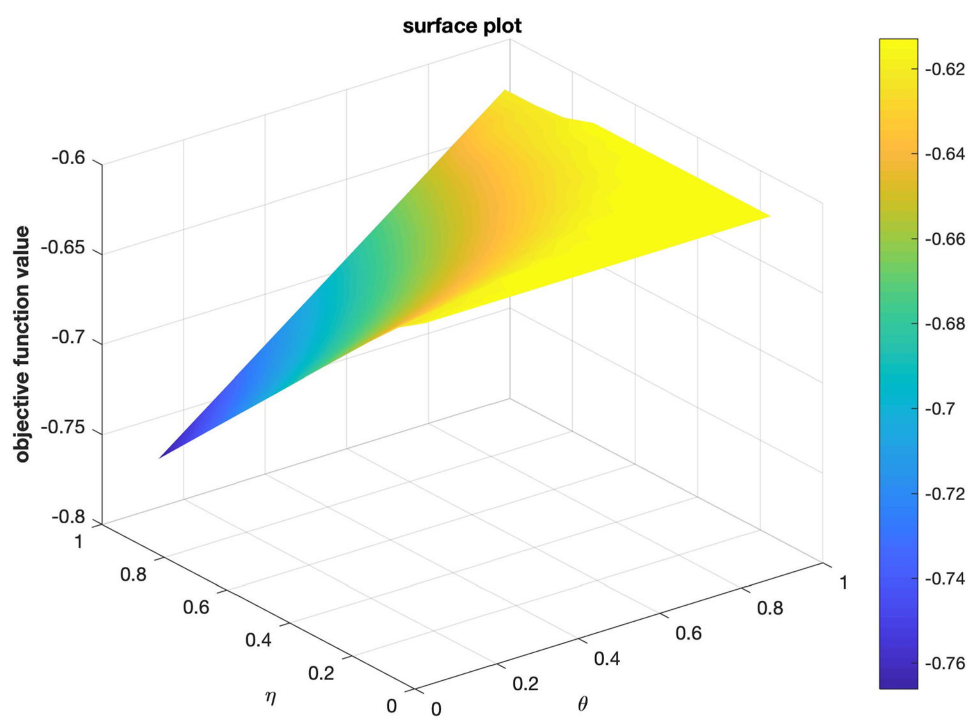


Fig. 7 Surface plot of objective function for (LMF) model

Fig. 8 Surface plot of objective function for (EMF) model

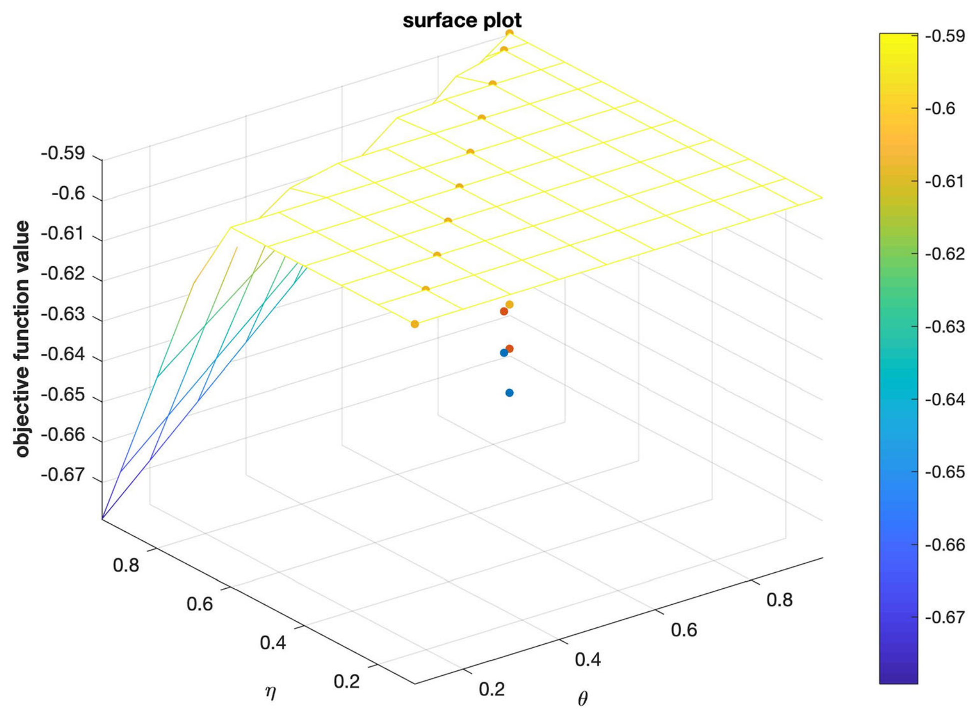


Fig. 9 Surface plot of objective function for (HMF) model

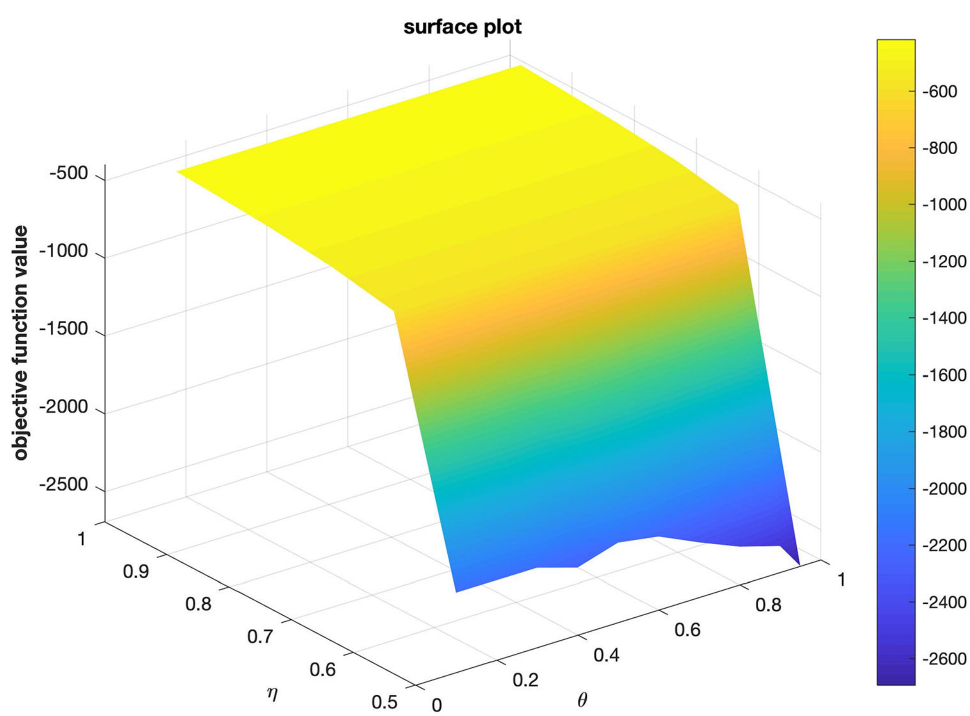


Table 7 Optimal solution of (LMF), (EMF) and (HMF) models for $\theta = 0.4$ and $\eta = 0.8$

	(LMF)	(EMF)	(HMF)
$\tilde{x}_{11}$	{(35,40,50,50,50), (30,35,50,55,55)}	{(35,42,52,57,62), (30,30,52,67,70)}	{(30,35,40,40,41), (30,30,40,42,42)}
$\tilde{x}_{12}$	{(15,15,15,20,20), (0,5,15,25,30)}	{(35,38,38,43,47), (20,30,38,52,58)}	{(35,40,45,50,55), (20,30,45,60,70)}
$\tilde{x}_{13}$	{(60,65,65,70,80), (60,60,65,80,85)}	{(40,40,40,40,41), (40,40,40,41,42)}	{(45,45,45,50,54), (40,40,45,58,58)}
$\tilde{x}_{21}$	{(5,10,10,15,20), (0,0,10,20,25)}	{(5,8,8,8,8), (0,5,8,8,10)}	{(10,15,20,25,29), (0,5,20,33,38)}
$\tilde{x}_{22}$	{(25,30,35,35,40), (20,25,35,40,45)}	{(5,7,12,12,13), (0,0,12,13,17)}	{(5,5,5,5,5), (0,0,5,5,5)}
$\tilde{x}_{23}$	{(20,20,25,30,30), (10,15,25,40,40)}	{(40,45,50,60,69), (30,35,50,79,83)}	{(35,40,45,50,56), (30,35,45,62,67)}
$\tilde{Z}_1(x)$	{(7550,9325,10950,13125,15800), (5000,6150,10950,18025,20925)}	{(6350,7805,9430,11255,13340), (3800,5000,9430,15565,18195)}	{(6700,8175,9850,12025,14330), (3800,5000,9850,16835,19435)}
$\tilde{Z}_2(x)$	{(2780,3720,4735,6215,7650), (1390,1900,4735,9545,11390)}	{(3000,4173,5430,7137,9063), (1210,2060,5430,11300,13603)}	{(3105,4320,5755,7475,9408), (1210,2060,5755,11563,14035)}
$\tilde{Z}_3(x)$	{(2020,2780,3590,4665,5850), (610,1230,3590,7050,8375)}	{(2300,2996,3708,4620,5604), (1050,1685,3708,6660,7765)}	{(2320,3065,3870,4855,5924), (1050,1685,3870,7087,8261)}

(LMF1)
$$\begin{aligned}
\text{Max } F = {} & \alpha - \gamma - 0.6d_1^+ - 0.6d_1^- - 0.3d_2^+ - 0.3d_2^- - 0.1d_3^+ - 0.1d_3^- \\
\text{s.t. } & 2128.125\alpha + Z_1'(x) \le 11837.5, \\
& 1149.375\alpha + Z_2'(x) \le 5648.75, \\
& 160.6275\alpha + Z_3'(x) \le 3782.1875, \\
& Z_1'(x) - 1862.11\gamma \le 9975.39, \\
& Z_2'(x) - 1005.7\gamma \le 4643.05, \\
& Z_3'(x) - 140.5475\gamma \le 3641.64, \\
& Z_1'(x) - d_1^+ + d_1^- = 11453.12, \\
& Z_2'(x) - d_2^+ + d_2^- = 4713.4375, \\
& Z_3'(x) - d_3^+ + d_3^- = 3692.8125, \\
& \alpha + \gamma \le 1, \quad \alpha \ge \gamma, \quad \gamma \ge 0, \\
& d_1^-,\ d_1^+,\ d_2^-,\ d_2^+,\ d_3^-,\ d_3^+ \ge 0,
\end{aligned}$$
along with set of constraints (3), taking $t = 0.125$.

(ii) Using exponential membership function:

(EMF1)
$$\begin{aligned}
\text{Max } F = {} & \alpha - \gamma - 0.6d_1^+ - 0.6d_1^- - 0.3d_2^+ - 0.3d_2^- - 0.1d_3^+ - 0.1d_3^- \\
\text{s.t. } & e^{-\left((Z_1'(x)-9709.375)/2128.125\right)} - (1-e^{-1})\alpha \ge e^{-1}, \\
& e^{-\left((Z_2'(x)-4499.375)/1149.375\right)} - (1-e^{-1})\alpha \ge e^{-1}, \\
& e^{-\left((Z_3'(x)-3621.56)/160.6275\right)} - (1-e^{-1})\alpha \ge e^{-1}, \\
& e^{-\left((11837.5-Z_1'(x))/1862.11\right)} - (1-e^{-1})\gamma \le e^{-1}, \\
& e^{-\left((5648.75-Z_2'(x))/1005.7\right)} - (1-e^{-1})\gamma \le e^{-1}, \\
& e^{-\left((3782.1875-Z_3'(x))/140.5475\right)} - (1-e^{-1})\gamma \le e^{-1}, \\
& Z_1'(x) - d_1^+ + d_1^- = 11453.12, \\
& Z_2'(x) - d_2^+ + d_2^- = 4713.4375, \\
& Z_3'(x) - d_3^+ + d_3^- = 3692.8125, \\
& \alpha + \gamma \le 1, \quad \alpha \ge \gamma, \quad \gamma \ge 0, \\
& d_1^-,\ d_1^+,\ d_2^-,\ d_2^+,\ d_3^-,\ d_3^+ \ge 0,
\end{aligned}$$
along with set of constraints (3), taking $t = 0.125, \ S_k = 1, \ \forall\, k$

(iii) Using hyperbolic membership function:

(HMF1)
$$\begin{aligned}
\text{Max } F = {} & \alpha - \gamma - 0.6d_1^+ - 0.6d_1^- - 0.3d_2^+ - 0.3d_2^- - 0.1d_3^+ - 0.1d_3^- \\
\text{s.t. } & 0.0028Z_1'(x) + \tanh^{-1}(2\alpha - 1) \le 30.1656, \\
& 0.0052Z_2'(x) + \tanh^{-1}(2\alpha - 1) \le 26.3851, \\
& 0.0373Z_3'(x) + \tanh^{-1}(2\alpha - 1) \le 138.08, \\
& 0.0032Z_1'(x) - \tanh^{-1}(2\gamma - 1) \le 34.9006, \\
& 0.006Z_2'(x) - \tanh^{-1}(2\gamma - 1) \le 30.8754, \\
& 0.0427Z_3'(x) - \tanh^{-1}(2\gamma - 1) \le 158.4987, \\
& Z_1'(x) - d_1^+ + d_1^- = 11453.12, \\
& Z_2'(x) - d_2^+ + d_2^- = 4713.4375, \\
& Z_3'(x) - d_3^+ + d_3^- = 3692.8125, \\
& \alpha + \gamma \le 1, \quad \alpha \ge \gamma, \quad \gamma \ge 0, \\
& d_1^-,\ d_1^+,\ d_2^-,\ d_2^+,\ d_3^-,\ d_3^+ \ge 0,
\end{aligned}$$
along with set of constraints (3), taking $t = 0.125$.

Solving (LMF1), (EMF1) and (HMF1) models, we get the solutions shown in Table 8.

From Tables 7 and 8, the expected value of the objective functions $\tilde{Z}_1(x)$, $\tilde{Z}_2(x)$ and $\tilde{Z}_3(x)$ under IVIF and IF environments using linear, exponential and hyperbolic membership functions is given in Table 9.

It is clear from the Table 9 that in most of the cases, the expected value of the objective functions $\tilde{Z}_1(x)$, $\tilde{Z}_2(x)$ and $\tilde{Z}_3(x)$ under IVIF is either lesser or equal to the corresponding values with IF environment. Hence, solving the problem under IVIF situations provides better solution and more flexibility.

It is to be noted that all the optimal solutions are presented up to four decimal places. Using LINGO−17.0, the average

Table 8 Optimal solution for (LMF1), (EMF1) and (HMF1) models

	(LMF1)	(EMF1)	(HMF1)
F	− 365.2471	− 365.2471	− 407.8149
α	0.4667	0.3461	0.5
γ	0.4667	0.3461	0.5
d_1^-	608.7452	608.7452	679.6914
d_1^+	0	0	0
d_2^-	0	0	0
d_2^+	0	0	0
d_3^-	0	0	0
d_3^+	0	0	0
$\tilde{x}_{11}$	{(35,40,45,45,45), (30,35,45,50,50)}	{(35,42,52,57,62), (30,30,52,67,70)}	{(20,27,37,40,40), (20,20,37,40,40)}
$\tilde{x}_{12}$	{(35,35,40,45,50), (20,25,40,55,60)}	{(35,38,38,43,47), (20,30,38,52,58)}	{(39,41,42,44,48), (19,29,42,54,64)}
$\tilde{x}_{13}$	{(40,45,45,50,55), (40,40,45,55,60)}	{(40,40,40,40,41), (40,40,40,41,42)}	{(51,51,51,56,61), (51,51,51,66,66)}
$\tilde{x}_{21}$	{(5,10,15,20,25), (0,0,15,25,30)}	{(5,8,8,8,8), (0,5,8,8,10)}	{(20,22,23,25,30), (10,15,23,35,40)}
$\tilde{x}_{22}$	{(5,10,10,10,10), (0,5,10,10,15)}	{(5,7,12,12,13), (0,0,12,13,17)}	{(1,4,8,11,11), (1,1,8,11,11)}
$\tilde{x}_{23}$	{(40,40,45,50,55), (30,35,45,65,65)}	{(40,45,50,60,69), (30,35,50,79,83)}	{(29,34,39,44,49), (19,24,39,54,59)}
$\tilde{Z}_1(x)$	{(6350,8125,9800,11975,14350), (3800,4950,9800,16575,19475)}	{(6350,7805,9430,11255,13340), (3800,5000,9430,15565,18195)}	{(7160,8585,10240,12385,14700), (4560,5760,10240,17335,19935)}
$\tilde{Z}_2(x)$	{(3000,4140,5555,7255,9165), (1210,1920,5555,11330,13445)}	{(3000,4173,5430,7137,9063), (1210,2060,5430,11300,13603)}	{(3359,4438,5689,7229,9059), (1549,2329,5689,11179,13579)}
$\tilde{Z}_3(x)$	{(2300,2980,3790,4780,5870), (1050,1590,3790,6965,8185)}	{(2300,2996,3708,4620,5604), (1050,1685,3708,6660,7765)}	{(2416,3120,3882,4843,5932), (1008,1677,3882,7161,8365)}

Table 9 Expected value of $\tilde{Z}_1(x)$, $\tilde{Z}_2(x)$ and $\tilde{Z}_3(x)$ under IVIF and IF situations

	(LMF)	(EMF)	(HMF)
Z'_1	11468.75	9796.875	10318.75
Z'_2	5154.375	5936.625	6201
Z'_3	3831.25	3896.5	4075.4375
	(LMF1)	(EMF1)	(HMF1)
Z'_1	10250	9796.875	10771.25
Z'_2	5994.0625	5936.625	6139.5625
Z'_3	4002.5	3896.5	4098.625

Table 10 Average CPU time, memory used and number of solver iterations for various models

Model	Average CPU time (in s)	Memory used (in KB)	Number of solver iterations
(LMF)	0.2535	55	12–17
(EMF)	0.2612	57	20–28
(HMF)	0.842	56	12–44
(LMF1)	0.19	50	13
(EMF1)	0.23	55	28
(HMF1)	0.19	50	10

Table 11 Comparative study under IVIF environment

Membership function	Range of objective function value
Linear	− 0.7661 to − 0.6129
Exponential	− 0.6792 to − 0.5897
Hyperbolic	− 2693.4920 to − 407.8149

Table 12 Comparative study under IF environment

Membership function	α	γ	The value of objective function
Linear	0.4667	0.4667	− 365.2471
Exponential	0.3461	0.3461	− 365.2471
Hyperbolic	0.5	0.5	− 407.8149

elapsed runtime (CPU time), generator memory and number of solver iterations used for solving the various problems are listed in Table 10.

9 Conclusions and future direction

To the best of our knowledge, construction and development of solution methodology for a balanced MOTP, in which all the parameters and variables are IVTIFNs, have not been discussed so far in the literature. Formulation of such type of problems not only gives more flexibility to the DM but also generalizes the earlier results and models. While assigning two parameters, membership and non-membership associated with cost, demand and availability in TP, a person due to some hesitation may allot incorrect values, which may not yield satisfactory results for a realistic problem. However, this deficiency has been removed by giving the degrees of the membership and non-membership in terms of intervals. In the present article, a goal programming approach to tackle the fully IVIF balanced TP with multiple objectives is proposed. The methodology not only handles the maximization of the acceptance level but simultaneously minimizes the deviational variables along with priorities attached with the aspiration levels corresponding to each objective. The method is well-explained step-wise by illustrating an example of shoe industry. To deal with the IVIF constraints related with the objective functions, linear, exponential and hyperbolic membership/non-membership functions have been considered. The Tables 11 and 12 represent the comparative study using these three membership and non-membership functions in IVIF and IF environments for the numerical example considered in Sect. 8:

From Table 11 in the light of Tables 4, 5 and 6, we can predict that

Exponential approach > Linear approach > Hyperbolic approach.

However, Table 12 yields

Exponential approach = Linear approach > Hyperbolic approach.

Therefore, one can observe that the exponential membership function provides the more satisfaction to the DM in contrast to linear or hyperbolic cases. However, on comparing the corresponding results for all approaches in IVIF and IF environments, IVIF equality constraints give much better results than IF equality constraints. Moreover, the results using IVIF constraints provides more flexibility to the DM by considering membership and non-membership degrees as intervals instead of crisp real numbers. The work presented in the paper can be further extended to deal with unbalanced IVIF multi-objective transportation problems.

Acknowledgements We would like to express our sincere thanks to the anonymous referees for their valuable comments and suggestions which helped us to improve the quality and clarity of the paper. The first author is also grateful to the Ministry of Human Resource Development, India, for financial support, to carry out this work.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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