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METHODOLOGIES AND APPLICATION

Fuzzy applications of Best–Worst method in manufacturing environment

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Abstract

High-strength steel alloys, titanium, ceramics, composites are in the group of materials that are hard to machine. Conventional manufacturing techniques are not sufficient to machine these materials. For this reason, these materials are generally machined with non-conventional manufacturing methods. In this study, a fuzzy application of Best–Worst method and a novel hybrid decision-making model (Best–Worst decision-making approach with fuzzy TOPSIS) are proposed to solve different non-traditional machining method selection problems which were taken from the literature. Using these models, the Best–Worst method shortens the steps of solutions in the fuzzy environment compared to the AHP/ANP-based fuzzy solutions in the literature. The proposed models produce successful results.

Keywords Best–Worst method · Fuzzy TOPSIS · TOPSIS · Non-traditional machining · Fuzzy numbers

1 Introduction

Important changes have occured in the production sector because of the rapid development of technology. The use of laser, water jet, electric discharge and ultrasonic processing techniques increases compared to turning, milling, etc. In particular, complex- shaped parts and hard-to-machine materials are processed with non-conventional manufacturing techniques (Rajurkar and Ross 1992; Yao et al. 2005; Mardani et al. 2015).

Recently, a large number of studies have been performed in multi-criteria decision-making models (MCDM) in the literature. There are several articles in the area of material science (Jahan et al. 2011; Chatterjee et al. 2009, 2011), production technologies (Streimikiene et al. 2012), mass production (Chang et al. 2013), manufacturing sector (Bagočius et al. 2013), manufacturing systems (Jana et al. 2013), global production (Tzeng and Huang 2012) and production strategies (Yurdakul 2004). Buyurgan and Saygin (2008) have worked on part routing and real-time programming via multi-criteria decision-making (MCDM) methods. İç et al. (2012) used the analytic hierarchy process (AHP) method and Yurdakul and İç (2009) developed a TOPSIS model in machine selection problems. Numerous works have been conducted using the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Elimination and Choice Translating Reality (ELECTRE), Preference Ranking Organization Method (PROMETHEE), Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) and so on (Chatterjee and Chakraborty 2012; Jahan and Edwards 2013; Khorshidi and Hassani 2013). Yurdakul (2004) and Çalişkan et al. (2013) analyzed the selection of cutting tool problem with AHP, analytic network process (ANP), TOPSIS, VIKOR and The Extended PROMETHEE (EXPROM-2). Several new studies investigating fuzzy MCDM methods have been carried out on different engineering problems in civil engineering (Bagočius et al. 2014), industrial engineering (Avikal et al. 2014), computer science (Kaya and Kahraman 2010), electrical engineering (Kurt 2014) and mechanical engineering (Azadnia et al. 2014).

The fuzzy set theory with TOPSIS method has been developed by Zadeh because decision-making methods require linguistic terms (Zadeh 1965). Many studies were reported where fuzzy TOPSIS was deployed (Boran et al. 2009; Dağdeviren et al. 2009). Fuzzy TOPSIS procedure has been used in various topics (road safety assessment,

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green supply chain, supplier selection, price strategy selection, renewable energy supply system selection, landfill site selection, etc.) (Beskese et al. 2015; Arabzad et al. 2015; Sengul et al. 2015).

The problem of non-conventional machining method selection is often different and contradictory as an MCDM problem (Chakraborty and Dey 1977). Thus, the need for systems to help for the non-conventional machining method selection increases and this requires the use of MCDM systematic approach. One of these methods is the goal programming to minimize deviation from the objective (Dağdeviren et al. 2009). Chakroborty and Dey (2007) developed an expert system using quality function deployment (QFD) for non-conventional machining method selection. Chakladar and Chakraborty (2008) proposed an AHP-TOPSIS method with the creation of normalization of decision matrix and decision matrix, determination of the criteria with AHP and ranking of the alternatives. Krohling and Campanharo (2011) proposed a method that uses classic TOPSIS and Fuzzy TOPSIS together in group decision making. Chakladar et al. (2009) suggested an expert system for selecting a non-conventional machining method selection using digraph theory. Das and Chakraborty (2011) suggested ANP process for selecting a non-conventional machining method. Traditional methods have some limitations due to inaccurate/incomplete information of criteria (Kahraman 2008). According to Bellman and Zadeh (1970), most real-world decision-making problems take place in an environment where the limitations of the goals and the results of possible actions are not known at all. Zimmermann and Zysno (1985) applied the theory of fuzzy sets to MCDM problems to add uncertainty. With the introduction of fuzzy clusters into the MCDM area, the classical MCDM methods such as AHP, TOPSIS and PROMETHEE were revised by the fuzzy set theory. One of the studies on the contributions of fuzzy methods to the results of sorting is the selection of the computer numerical control (CNC) turning center with fuzzy ANP by Duran and Aguilo (2007). The weighting process of six different criteria, called flexibility, ease of operation, reliability, quality, ease of installation and maintenance, was performed with fuzzy triangular numbers. The authors stated that the criteria they use in the calculation of the relative weight of the criteria were modeled with fuzzy triangular numbers and the use of fuzzy numbers provides significant benefits.

Best–Worst method (BWM) is a new method that has been presented in the literature recently (Rezaei 2015). It is a new method, and it has some advantages compared to AHP. This new method makes calculations easy during criteria weighting. Also, the technique combines subjective and objective calculations. The benefits of BWM are given as follows:

1. BWM is a vector-based approach that provides lower comparisons compared to AHP.
2. BWM offers more consistent solutions compared to AHP.
3. BWM can also be hybridized with other decision-making techniques.

When these advantages are considered, Best–Worst method in the fuzzy MCDM approach can be used. Also, the model has not been performed in the selection of non-traditional machining method selection before. When the advantages of this new method are taken into consideration, the developed models contribute to the fuzzy MCDM theory. BWM is a semi-objective weighting method, and it is easy to calculate for a decision maker. In the literature, when subjective and objective weighting methods are compared, these methods produce different results for the same problem. Therefore, semi-objective methods are superior compared to these methods. The obtained results are more consistent for BWM compared to the other ways (AHP, ANP, etc.). Proposed models are used to take advantage of the BWM in the fuzzy environment.

In this study, two different MCDM models are proposed: (1) a new fuzzy decision-making method (fuzzy Best–Worst Method) with TOPSIS and (2) Best–Worst method with the fuzzy TOPSIS method. In the first model, Best–Worst method with TOPSIS model is developed by the fuzzy numbers. In Best–Worst-fuzzy TOPSIS model, criteria weights are obtained using Best–Worst method and these weights are used in the fuzzy TOPSIS approach. Various non-conventional manufacturing method selection problems are taken from the literature and discussed. Developed models are tested by using these problems. In the second section, the decision-making methods are given. In the next part, non-traditional manufacturing method selection problems are explained. Then, results and discussion are given.

2 Methods used in the study

2.1 Best–Worst method (BWM)

Best–Worst technique is one of the novel techniques to calculate the weights of criteria (Rezaei 2015). The calculation steps are given as follows:

1. Define decision-making criteria ($c_1, c_2 \ldots c_n$).
2. Define the best and the worst criterion.
3. Score the best criterion versus the other criteria.

   $a_{Bj} = (a_{B1}, a_{B2} \ldots a_{Bn})$.

   a_{Bj} comparison scores of the best criterion B with jth criteria

4. Score the other criteria versus the worst criterion

   $a_{jw} = (a_{1w}, a_{2w} \ldots a_{nw})^T$.

   a_{jw} comparison scores of the worst criterion w with jth criteria.
5. Calculate optimum weights (w_1^*, w_2^*, $w_3^* \ldots\ldots w_n^*$) and index for consistency ratio (ε^*)

The developed model is shown as follows (Eqs. 1–4):
Min ε
subject to

$$\left|\frac{w_B}{w_j} - a_{Bj}\right| \le \epsilon \tag{1}$$

$$\left|\frac{w_j}{w_w} - a_{jw}\right| \le \epsilon \tag{2}$$

$$\sum_j w_j = 1 \tag{3}$$

$$w_j \ge 0 \tag{4}$$

Table 1 gives the consistency index values, and Eq. 5 gives the formula of the consistency ratio:

$$\text{Consistency ratio} = \frac{\epsilon^*}{\text{Consistency index}} \tag{5}$$

2.2 TOPSIS algorithm

This algorithm is based on the principle of ideal solution proximity of the decision-making points (ELECTRE approach). The steps are explained as follows (Hwang and Yoon 1981).

2.2.1 Specify the decision matrix

Equation 6 presents the decision matrix of a problem. m shows the number of decision-making points and n indicates the number of evaluation factors.a_{ij} shows the values of decision-making points according to the evaluation criteria. (j = 1, 2, 3… n and i = 1, 2, 3.…m).

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix} \tag{6}$$

Table 1 Table of consistency index

a_{BW}	1	2	3	4	5	6	7	8	9
Consistency index	0	0.44	1	1.63	2.3	3	3.73	4.47	5.23

2.2.2 Calculate the standard decision matrix

Equation 7 is the formula of the standard decision-making matrix elements:

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{k=1}^{m} a_{kj}^2}} \tag{7}$$

Equation 8 shows the standard decision-making matrix (R_{ij}):

$$R_{ij} = \begin{bmatrix} r_{11} & r_{12} & \ldots & r_{1n} \\ r_{21} & r_{22} & \ldots & r_{2n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ r_{m1} & r_{m2} & \ldots & r_{mn} \end{bmatrix} \tag{8}$$

2.2.3 Calculate the weighted decision matrix

Equation 9 gives the weighted decision-making matrix (V_{ij}), which is created by the multiplication of weights (w_j):

$$V_{ij} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \ldots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \ldots & w_n r_{2n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ w_1 r_{m1} & w_2 r_{m2} & \ldots & w_n r_{mn} \end{bmatrix} \tag{9}$$

2.2.4 Calculate the ideal and negative ideal solutions

Equation 10 gives the ideal solution set (A^*):

$$A^* = \left\{ (\max_i v_{ij} \big| j \in J), (\min_i v_{ij} \big| j \in J') \right\} \tag{10}$$

The ideal solution: $A^* = \{v_1^*, v_2^*, \ldots, v_n^*\}$.

Equation 11 shows the negative ideal solution set (A^-):

$$A^- = \left\{ (\min_i v_{ij} \big| j \in J), (\max_i v_{ij} \big| j \in J') \right\} \tag{11}$$

The negative ideal solution: $A^- = \{v_1^-, v_2^-, \ldots, v_n^-\}$.

J shows cluster (maximization) and J' shows cluster (minimization).

2.2.5 Calculate the distinction measure

The ideal distinction (S_i^*) is given in Eq. 12, and the negative ideal distinction (S_i^-) is computed in Eq. 13:

$$S_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2} \quad (12)$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2} \quad (13)$$

2.2.6 Calculate the proximity values relative to the ideal solution

Closeness values (C_i^*) are calculated in Eq. 14:

$$C_i^* = \frac{S_i^-}{S_i^- + S_i^*} \quad (14)$$

2.3 Fuzzy TOPSIS model

Chen developed a fuzzy TOPSIS algorithm in 2000. The steps are explained as follows (Chen 2000).

2.3.1 Specify the decision matrix

Equations 15 and 16 show the decision-making matrix (D) of a problem:

$$D = [\widetilde{x_{ij}}] \quad (15)$$

$$\widetilde{x_{ij}} = (a_{ij}, b_{ij}, c_{ij}) \quad (16)$$

x_{ij} elements show ith decision-making points according to the jth evaluation criteria. Triangular fuzzy numbers describe these linguistic variables, n shows the number of criteria and m represents the number of alternatives (j = 1, 2, 3…n and i = 1, 2, 3…m).

2.3.2 Calculate the standard decision matrix

The normalized fuzzy decision matrix is denoted by $R = [\widetilde{r_{ij}}]_{m \times n}$. Standard decision-making matrix is computed (Eqs. 17, 18). Benefit and cost criteria are B and C.

$$\widetilde{r_{ij}} = \frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}.c_j^* = \max c_{ij} \quad \text{if } j \in B \quad (17)$$

$$\widetilde{r_{ij}} = \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}} \cdot a_j^- = \min a_{ij} \quad \text{if } j \in C \quad (18)$$

Equation 19 gives the standard decision-making matrix (R_{ij}):

$$R_{ij} = [\widetilde{r_{ij}}]_{m \times n} \quad (19)$$

2.3.3 Calculate the weighted decision matrix

Using Eqs. 20, 21, the standard decision-making matrix is multiplied by the weights (w_j) to obtain the weighted decision-making matrix (V_{ij}):

$$V_{ij} = [\widetilde{v_{ij}}]_{m \times n} \quad (20)$$

$$\widetilde{v_{ij}} = \widetilde{r_{ij}}(\cdot)\widetilde{w_j} \quad (21)$$

2.3.4 Calculate the ideal and negative ideal solutions

Equations 22 and 23 give the calculation of ideal solution set (A^*):

$$A^* = (\widetilde{v_1}^*, \widetilde{v_2}^* \ldots \widetilde{v_n}^*) \quad (22)$$

$$\widetilde{v_j}^* = (1, 1, 1) \quad (23)$$

Equations 24 and 25 show the calculation of the negative ideal solution set (A^-):

$$A^- = (\widetilde{v_1}^-, \widetilde{v_2}^- \ldots \widetilde{v_n}^-) \quad (24)$$

$$\widetilde{v_j}^- = (0, 0, 0) \quad (25)$$

2.3.5 Calculate the distinction measure

Equations 26 and 27 give the calculation of ideal distinction (S_i^*) measure and negative ideal distinction measure (S_i^-). d_v () shows the distance measurement between fuzzy numbers.

$$S_i^* = \sum_{j=1}^{n} d_v(\widetilde{v_{ij}}, \widetilde{v_j}^*) \quad (26)$$

$$S_i^- = \sum_{j=1}^{n} d_v(\widetilde{v_{ij}}, \widetilde{v_j}^-) \quad (27)$$

2.3.6 Calculate the proximity values relative to the ideal solution

In order to find closeness values (C_i^*), ideal and negative ideal distinction measures are used (Eq. 28):

$$C_i^* = \frac{S_i^-}{S_i^- + S_i^*} \quad (28)$$

2.4 Proposed models

The flow diagrams of the proposed algorithms are given in Fig. 1. Two different models are proposed. For the first

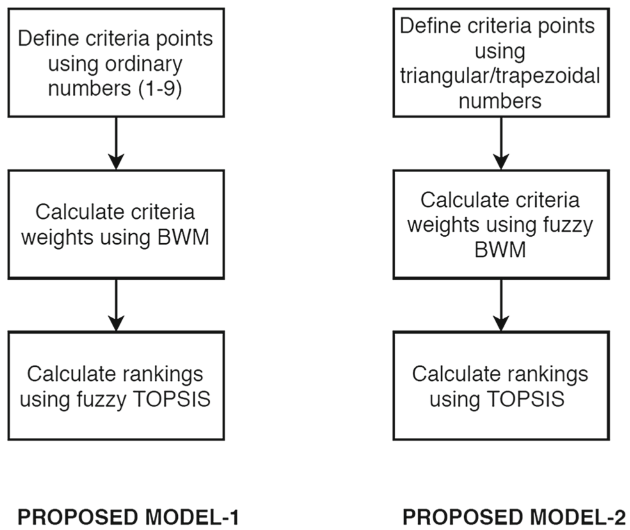

Fig. 1 Flow diagrams of the proposed methods

Table 2 Abbreviation of the non-conventional machining methods

AJM	Abrasive jet machining
USM	Ultrasonic machining
ECM	Electrochemical machining
EDM	Electrical discharge machining
EBM	Electron beam machining
LBM	Laser beam machining
CHM	Chemical machining
AWJM	Abrasive water-jet machining
RUSM	Rotary ultrasonic machining

model, three case studies are considered, whereas two case studies are used for the second model.

3 Case studies

Several case studies are taken from the literature studies (Kul et al. 2014; Yurdakul and Çoğun 2003). A detailed explanation of the problems is given in these studies. Abbreviations are provided in Table 2 for non-traditional machining processes.

4 Results and discussion

4.1 Best–Worst-fuzzy TOPSIS

4.1.1 Case study-1

Drilling operation is carried out for of the turbine engine combustion chamber. Generally, EBM method is used. Process-related requirements are given as follows:

Workpiece material	Superalloy
Process	Hole drilling process

Comparison of the best and worst criteria according to the other criteria is given in Table 3 for case study-1. According to case study-1 (Table 23), the cost is chosen as the best criterion, whereas surface damage is selected as the worst criterion. Based on Kul et al.'s (2014) study, criteria are scored.

The criteria weights of case study-1 are given in Table 4. The cost has the highest criterion weight. Material removal rate, workpiece material, taper, surface finish and surface damage are sorted in descending order, respectively. In Table 4, the objective function value and consistency ratio are given. The consistency ratio is lower than 0.1. Therefore, the analysis is consistent.

In this stage, fuzzy TOPSIS is performed by using the criteria weights obtained from the Best–Worst method (Table 4). The scores and rankings for case study-1 are given in Table 5. Electrochemical machining is the best alternative, whereas ultrasonic machining is the worst alternative.

Spearman rank correlation test results of case study-1 are given in Table 6. According to the correlation test results, all rankings are nearly the same except AHP + TOPSIS ranking. Also, there is no difference between rankings at the 5% significance level.

4.1.2 Case study-2

Electrochemical machining is used to machine multiple pockets simultaneously. The sample parts contain 16 pocket pieces. All pockets are machined with ECM for less than 6 min. The material is 4140 steel. Process requirements are as follows:

Workpiece material	Steel (4140)
Process	Pocket machining

Table 3 Pairwise comparison of case study-1

	Surface finish	Surface damage	Taper	Material removal rate (MRR)	Workpiece material (WM)	Cost
Worst criterion: surface damage	0.5	1	1	0.33	0.25	0.14
Best criterion: cost	5	7	3	2	3	1

Table 4 Weights of criteria and the other indices of case study-1

Criteria	Weights	Parameters	
Surface damage	0.055	Objective function	0.058
Surface finish	0.0805	Consistency ratio	0.013
Taper	0.0997		
Workpiece material	0.1745		
Material removal rate	0.2094		
Cost	0.3808		

The pairwise comparison of case study-2 is presented in Table 7. Based on case study-2 (Table 24), the worst criterion is surface damage, whereas surface finish is chosen as the best criterion. Based on the Kul et al.'s (2014) study, criteria are scored.

The criteria weights of the case study-2 are given in Table 8. Surface finish, cost and material removal rate have the highest criteria weights, whereas surface damage, corner radius, taper and workpiece material have the lowest criteria weights. Table 8 shows the objective function value and consistency ratio. Consistency ratio is lower than 0.1. Therefore, the analysis is consistent.

In this stage, fuzzy TOPSIS is performed by using the criteria weights obtained from the Best–Worst method (Table 8). The scores of the case study-2 study and rankings are given in Table 9 with a comparison of the other studies. Electrochemical machining is the best alternative, whereas electrical discharge machining is the worst alternative.

Spearman rank correlation test results of case study-2 are given in Table 10. Correlation test results show that according to the generalized average method, the rankings are the same. However, based on best non-fuzzy method, the calculated ranking is different, and it is not significant at 5% level.

4.1.3 Case study-3

Custom USM is used to machine multiple holes simultaneously. An insulator ceramic material (aluminum oxide) with 0.64 mm thickness is used in the electronics industry. USM is used for machining of 0.64 mm diameter of 930 holes and 1.53 mm diameter of 30 holes simultaneously.

Table 5 Comparison of rankings for case study-1

Final scores	Final ranking	Chen method (Kul et al. 2014)	Generalized mean (Kul et al. 2014)	Best non-fuzzy performance method (Kul et al. 2014)	Fuzzy AHP + TOPSIS (Kul et al. 2014)	AHP + TOPSIS (Kul et al. 2014)
0.413598	4	2	2	3	4	2
0.407786	9	9	8	9	6	5
0.422462	1	1	1	1	1	1
0.411912	6	7	7	8	9	9
0.415539	2	4	4	4	3	6
0.412163	5	6	6	6	8	8
0.411106	7	5	5	5	7	7
0.414554	3	3	3	2	2	3
0.408056	8	8	9	7	5	4

Table 6 Spearman correlation test of different rankings for case study-1

Spearman's rho		Chen method (Kul et al. 2014)	Generalized mean (Kul et al. 2014)	Best non-fuzzy performance method (Kul et al. 2014)	Fuzzy AHP + TOPSIS (Kul et al. 2014)	AHP + TOPSIS (Kul et al. 2014)
Final ranking	Correlation Coefficient	.883	.867	.867	.683	.417
	Sig. (2-tailed)	.002	.002	.002	.042	.265
	N	9	9	9	9	9

Table 7 Pairwise comparison of case study-2

	Surface finish	Surface damage	Corner radius	Taper	Material removal rate	Workpiece material	Cost
Best criterion: surface finish	1	6	6	6	1	6	1
Worst criterion: surface damage	0.17	1	1	1	0.17	1	0.17

Table 8 Weights of criteria and the other indices of case study-2

Criteria	Weights	Parameters	
Surface damage	0.04545	Objective function	5.1989e−7
Corner radius	0.04545	Consistency ratio	1.73e−7
Taper	0.04545		
Material removal rate	0.2727		
Workpiece material	0.04545		
Cost	0.2727		
Surface finish	0.2727		

Table 9 Comparison of ranking results for case study-2

Final scores	Final ranking	Best non-fuzzy performance method (Kul et al. 2014)	Generalized average method (Kul et al. 2014)
0.678419	2	3	2
0.683758	1	1	1
0.654304	4	4	4
0.677881	3	2	3

Table 10 Spearman correlation test of different rankings for case study-2

Spearman's rho	Best non-fuzzy performance method (Kul et al. 2014)	Generalized average method (Kul et al. 2014)
Final ranking		
Correlation coefficient	.800	1.000
Sig. (2-tailed)	.200	0.000
N	4	4

The operation takes nearly 8.5 min with 320-grit boron carbide abrasive and stainless steel tool.

Workpiece material	Ceramic
Operation	Hole drilling

The pairwise comparison of case study-3 is given in Table 11. Based on case study-3 (Table 25), the worst criterion is surface damage, whereas tolerance is chosen as the best criterion. According to Kul et al.'s (2014) study, criteria are scored. In Table 12, tolerance has the highest criteria weight, which is 31%, whereas surface damage and taper have the lowest criteria weight, which is 4%. Table 12 shows the objective function value and consistency ratio. The consistency ratio is nearly 0.1, so the analysis is consistent.

In this step, fuzzy TOPSIS is performed by using the criteria weights obtained from the Best–Worst method (Table 12). Table 13 gives the scores of the case study-3, and rankings are presented with a comparison of other studies. Rotary ultrasonic machining is recommended.

Spearman rank correlation test results of case study-3 are given in Table 14. According to the results, there is no difference between rankings at the 5% significance level.

4.2 Fuzzy Best–Worst-TOPSIS

Table 15 shows the ordinary numbers, triangular and trapezoidal numbers used in the Best–Worst method. The ordinary numbers are taken from a previous study (Yurdakul and Çoğun 2013).

In this study, fuzzy numbers are calculated separately, and final weights are obtained. Final weights for triangular and trapezoidal numbers are given in Eqs. 29 and 30, respectively:

Table 11 Pairwise comparison of case study-3

	Tolerance	Surface finish	Surface damage	Taper	Material removal rate	Workpiece material	Cost
Best criterion: tolerance	1	2	7	7	2	3	3
Worst criterion: surface damage	0.14	0.2	1	1	0.2	0.33	0.33

Table 12 Weights of criteria and the other indices of case study-3

Criteria	Weights	Parameters	
Surface damage	0.042	Objective function	0.3944
Surface finish	0.1934	Consistency ratio	0.1045
Taper	0.042		
MRR	0.1934		
W. material	0.1094		
Cost	0.1094		
Tolerance	0.3105		

Table 13 Comparison of ranking results for case study-3

Final scores	Final ranking	Chen method (Kul et al. 2014)	Best non-fuzzy performance method (Kul et al. 2014)	Generalized mean (Kul et al. 2014)
0.687591	5	4	4	4
0.7157	2	2	2	2
0.675189	7	7	7	7
0.685036	6	6	6	6
0.69761	4	5	5	5
0.71156	3	3	3	3
0.719245	1	1	1	1

Table 14 Spearman correlation test of different rankings for case study-3

Spearman's rho	Chen method (Kul et al. 2014)	Best non-fuzzy performance method (Kul et al. 2014)	Generalized mean (Kul et al. 2014)
Final ranking			
Correlation coefficient	.964	.964	.964
Sig. (2-tailed)	.000	.000	.000
N	7	7	7

$$\text{final weight}_{\text{triangular}} = \frac{w_l + 2x w_m + w_u}{4} \tag{29}$$

$$\text{final weight}_{\text{trapezoidal}} = \frac{w_l + w_m + w_n + w_u}{4} \tag{30}$$

Lower limit, average and upper limit values are expressed as follows:

Table 15 Ordinary, triangular and trapezoidal fuzzy numbers used in the Best–Worst method

Ordinary numbers	Triangular numbers				Trapezoidal numbers			
	l	*m*	*m*	*u*	*l*	*m*	*n*	*u*
1	1	1	1	1	1	1	1	1
2	1	2	2	3	1	2	3	4
3	2	3	3	4	2	3	4	5
4	3	4	4	5	3	4	5	6
5	4	5	5	6	4	5	6	7
6	5	6	6	7	5	6	7	8
7	6	7	7	8	6	7	8	9
8	7	8	8	9	7	8	9	10
9	8	9	9	10	8	9	10	10
10	9	10	10	10	9	10	10	10

Table 16 Results of the model with triangular numbers

Triangular numbers	*l*	*u*	*m*	
Objective function value	1.71e−8	1.45e−7	5.2e−7	
Consistency index	2.3	3.73	3	
Consistency ratio	7.43e−9	3.89e−8	1.73e−7	
Criteria	Weights			Final weights
Surface damage	0.05	0.04	0.05	0.0475
Corner radius	0.05	0.04	0.05	0.0475
Taper	0.05	0.04	0.05	0.0475
Material removal rate	0.26	0.28	0.27	0.27
Workpiece material	0.05	0.04	0.05	0.0475
Cost	0.26	0.28	0.27	0.27
Surface finish	0.26	0.28	0.27	0.27

Lower limit (L)	It means the process values obtained in cases where the process is applied in unfavorable conditions
Average (M)	Process values given by the process in general. It can also be expressed as application values
Upper limit (U)	Process values obtained by experienced users in very favorable conditions

4.2.1 Case study-1

In case study-1 (pockets machining into a hardened bearing surface) (Table 26), the consistency ratio is considered.

Table 17 Results of the model with trapezoidal numbers

Trapezoidal numbers	*l*	*n*	*m*	*u*
Objective function value	1.71e−8	1.45e−7	5.2e−7	3.01e−12
Consistency index	2.3	3.73	3	4.47
Consistency ratio	7.43e−9	3.89e−8	1.73e−7	6.73e−13
Criteria	Weights				Final weights
Surface damage	0.05	0.04	0.05	0.04	0.04
Corner radius	0.05	0.04	0.05	0.04	0.04
Taper	0.05	0.04	0.05	0.04	0.04
Material removal rate	0.26	0.28	0.27	0.29	0.28
Workpiece material	0.05	0.04	0.05	0.04	0.04
Cost	0.26	0.28	0.27	0.29	0.28
Surface finish	0.26	0.28	0.27	0.29	0.28

Table 18 Rankings and TOPSIS scores

Triangular	Ranking	Trapezoidal	Ranking	Yurdakul and Çoğun (2003)
0.436	2	0.435	2	2
0.859	1	0.859	1	1
0.291	3	0.288	3	3

Table 19 Spearman correlation test

r/*p* value	Triangular	Trapezoidal
Yurdakul and Çoğun (2003)	1/0.000	1/0.000

Table 20 Results of the model with triangular numbers

Triangular numbers	*l*	*u*	*m*	
Objective function value	0.83	1.39	0.06	
Consistency index	3	4.47	3.73	
Consistency ratio	0.28	0.31	0.015	
Criteria	Weights			Final weights
Surface damage	0.07	0.05	0.05	0.06
Surface finish	0.07	0.08	0.08	0.08
Taper	0.13	0.12	0.10	0.11
Material removal rate	0.19	0.13	0.21	0.19
Workpiece material	0.18	0.17	0.17	0.17
Cost	0.36	0.45	0.38	0.39

Table 21 Rankings and TOPSIS scores

Triangular	Ranking	Yurdakul and Çoğun (2003)
0.463	5	5
0.419	6	6
0.686	1	1
0.480	4	4
0.369	7	7
0.570	3	3
0.574	2	2

Table 22 Spearman correlation test

r/p value	Triangular
Yurdakul and Çoğun (2003)	1/0.000

Objective function values and criteria weights of the fuzzy Best–Worst method with triangular numbers are given in Table 16. Cost, surface finish and material removal rate have the highest criteria weight which is nearly 27%, while surface damage, taper, workpiece material and corner radius have the lowest criteria weight which is nearly 5%. Consistency ratio is lower than 0.1, so the analysis is consistent.

Objective function values and criteria weights of the fuzzy Best–Worst method with trapezoidal numbers are given in Table 17. Cost, surface finish and material removal rate are effective criteria, whereas surface damage, taper, corner radius and workpiece material are less effective. The consistency ratio is lower than 0.1, so the analysis is consistent.

In Table 18, the rankings and scores of the TOPSIS model are given. Electrochemical machining is chosen as the best alternative. Compared to the study in the literature, there is no difference between rankings at the 5% significance level (Table 19).

4.2.2 Case study-2

In case study-2 (drilling of turbine engine combustor domes) (Table 27), the consistency ratio is not taken into consideration. Objective function values and criteria weights of the fuzzy Best–Worst method with triangular numbers are given in Table 20. The cost has the highest criteria weight, which is nearly 39%, while surface damage has the lowest criteria weight, which is nearly 6%.

In Table 21, the rankings and scores of the TOPSIS model are given. Compared to the study in the literature, there is no difference between rankings at 5% significance level (Table 22).

5 Conclusions

In this research, new hybrid fuzzy decision-making models are suggested (fuzzy Best–Worst method with TOPSIS and Best–Worst method and fuzzy TOPSIS). Triangular and trapezoidal fuzzy numbers are used in the fuzzy Best–Worst method. The obtained results show that calculated rankings are nearly the same. Best–Worst method is used as an alternative to the AHP method, and it is more practical compared to AHP. The calculation steps are simple. Also, it includes both subjective and objective calculation to weight criteria. Therefore, it contributes to the fuzzy TOPSIS model to make calculation easy compared to the other MCDM approaches. The developed models can be used as a practical method in material/manufacturing method selection.

Compliance with ethical standards

Conflict of interest The author declares there is no conflict of interest.

Human and animal rights This article does not contain any studies with human participants or animals performed by any of the authors.

Appendix

See Tables 23, 24, 25, 26 and 27.

Table 23 Criteria-alternative matrix for case study-1

	Surface finish			Surface damage			Taper			MRR			WM			Cost		
	l	*m*	*u*	*l*	*m*	*u*	*l*	*m*	*u*	*l*	*m*	*u*	*l*	*m*	*u*	*l*	*m*	*u*
1.AJM	1.25	0.6	0.25	0.03	0.025	0.02	0.006	0.005	0.004	20	50	200	8	9	10	12	17	22
2.USM	0.75	0.5	0.25	0.03	0.025	0.02	0.005	0.004	0.003	300	600	2100	2	3	4	20	25	30
3.ECM	1.5	1	0.2	0.006	0.005	0.004	0.003	0.002	0.001	500	2000	14000	8	9	10	31	36	41
4.CHM	2.5	2	0.5	0.006	0.005	0.004	0.4	0.3	0.2	15	40	140	5	6	7	16	21	26
5.EDM	3	2	0.3	0.03	0.02	0.01	0.003	0.002	0.001	100	800	1300	8	9	10	27	32	37
6.EBM	4	3	1	0.03	0.025	0.025	0.25	0.2	0.15	0.3	2	6	5	6	7	19	24	29
7.LBM	1.5	1	0.4	0.15	0.1	0.05	0.06	0.05	0.04	0.1	2	5	5	6	7	17	22	27
8.AWJM	0.4	0.3	0.2	0.03	0.025	0.02	0.004	0.003	0.003	300	600	2000	8	9	10	13	19	24
9.RUSM	0.75	0.5	0.25	0.03	0.025	0.025	0.005	0.004	0.003	400	800	2400	2	3	4	22	27	32

Table 24 Criteria-alternative matrix for case study-2

	Surface finish			Surface damage			Corner radius			Taper			MRR			WM			Cost		
	l	*m*	*u*	*l*	*m*	*u*	*l*	*m*	*u*	*l*	*m*	*u*	*l*	*m*	*u*	*l*	*m*	*u*	*l*	*m*	*u*
1.AJM	1.25	0.6	0.25	0.03	0.025	0.02	0.25	0.2	0.15	0.006	0.005	0.004	20	50	200	5	6	7	12	17	22
2.ECM	1.5	1	0.2	0.006	0.005	0.004	0.25	0.2	0.15	0.003	0.002	0.001	500	2000	14000	8	9	10	31	36	41
3.EDM	3	2	0.3	0.03	0.02	0.01	0.5	0.4	0.3	0.003	0.002	0.001	100	800	1300	8	9	10	27	32	37
4.RUSM	0.75	0.5	0.25	0.03	0.025	0.025	0.09	0.08	0.07	0.005	0.004	0.003	400	800	2400	5	6	7	22	27	32

Table 25 Criteria-alternative matrix for case study-3

	Tolerance			Surface finish			Surface damage			Taper			MRR			WM			Cost		
	l	m	u	l	m	u	l	m	u	l	m	u	l	m	u	l	m	u	l	m	u
1.AJM	0.12	0.05	0.03	1.25	0.6	0.25	0.03	0.025	0.02	0.006	0.005	0.004	20	50	200	8	9	10	12	17	22
2.USM	0.025	0.013	0.01	0.75	0.5	0.25	0.03	0.025	0.02	0.005	0.004	0.003	300	600	2100	7	8	9	20	25	30
3. CHM	0.08	0.03	0.01	2.5	2	0.5	0.006	0.005	0.004	0.4	0.3	0.2	15	40	140	2	3	4	16	21	26
4.EBM	0.03	0.02	0.01	4	3	1	0.03	0.025	0.025	0.25	0.2	0.15	0.3	2	6	8	9	10	19	24	29
5.LBM	0.03	0.02	0.01	1.5	1	0.4	0.15	0.1	0.05	0.06	0.05	0.04	0.1	2	5	8	9	10	17	22	27
6.AWJM	0.06	0.04	0.026	0.4	0.3	0.2	0.03	0.025	0.02	0.004	0.003	0.003	300	600	2000	8	9	10	13	19	24
7.RUSM	0.022	0.012	0.008	0.75	0.5	0.25	0.03	0.025	0.025	0.005	0.004	0.003	400	800	2400	8	9	10	22	27	32

Table 26 Criteria-alternative matrix for case study-1

	Surface finish	Surface damage	Corner radii	Taper	MRR	WM	Cost
1.AJM	0.6	2.5	0.3	0.005	50	2	4
2.ECM	1	0	0.2	0.001	2000	3	4
3.EDM	2	20	0.4	0.001	800	3	7

Table 27 Criteria-alternative matrix for case study-2

	Surface finish	Surface damage	Taper	MRR	WM	Cost
1.AJM	0.6	2.5	0.005	50	3	4
2.USM	0.5	25	0.005	500	1	5
3.ECM	1	0	0.001	2000	3	4
4.CHM	2	5	0.3	40	2	2
5.EDM	2	20	0.001	800	3	7
6.EBM	3	100	0.02	2	2	1
7.LBM	1	100	0.05	2	2	1

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