Some new information measures for hesitant fuzzy PROMETHEE method and application to green supplier selection

Shu-ping Wan[1] · Wen-chang Zou[1] · Long-gen Zhong[1] · Jiu-ying Dong[2]

Abstract

Due to the increasing complexity and uncertainty in green supplier selection, there would be some hesitations for decision makers (DMs) to provide evaluation information of suppliers. Hesitant fuzzy set is a suitable tool to model such hesitations. This paper develops a hesitant fuzzy Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) for multi-criteria group decision-making and applies to green supplier selection. First, a new hesitancy index of hesitant fuzzy element (HFE) is defined. Then, a generalized hesitant fuzzy Hausdorff distance is proposed considering the individual deviation of membership values and the hesitancy index simultaneously. A combined hesitant fuzzy entropy is presented integrating the defined fuzziness entropy and hesitancy entropy of HFEs. Subsequently, a linear programming model is established to derive DMs' weights objectively. To determine the criteria weights for each DM, a nonlinear programming model is built through minimizing the relative entropy. The PROMETHEE is employed to obtain individual ranking of alternatives for each DM. To obtain the collective ranking of alternatives, a multi-objective assignment model is constructed and transformed into a single-objective assignment model for resolution. Thereby, a hesitant fuzzy PROMETHEE method is presented. A green supplier selection example is demonstrated to validate the proposed method.

Keywords Multi-criteria group decision-making · Hesitant fuzzy elements · PROMETHEE · Information measures · Green supplier selection

1 Introduction

Over the past few decades, the environment pollution has become more and more serious. Environment sustainable development has attracted much attention. Pressure from customers and societies is dramatic (Govindan et al. 2014). Many companies have focused on the environmental impact of their production operations. To equilibrate the economic profits and the environment sustainable development, companies conducted the green supply chain management (GSCM) on their productions (Sarkis 1999). The green supplier selection is very important in the

✉ Shu-ping Wan
shupingwan@163.com

1 School of Information Technology, Jiangxi University of Finance and Economics, Nanchang 330013, China

2 School of Statistics, Jiangxi University of Finance and Economics, Nanchang 330013, China

GSCM (Blome et al. 2014). Since the green supplier selection involves various criteria and lots of decision makers (DMs), it is always treated as a type of multi-criteria group decision-making (MCGDM) problems (Qin et al. 2017). In practical green supplier selection, the supplier and manufacturer cannot share enough information of suppliers (Zarandi and Moghadam 2017). Thus, the practical green supplier selection process is filled with uncertainty and DMs usually hesitate to give evaluation among several values. The traditional fuzzy sets are insufficient to model the green supplier selection problem because the hesitancy and uncertainty always occur in green supplier selection practice. Hesitant fuzzy set (HFS) (Torra 2010) is viewed as a strong tool to characterize the DM's hesitancy and subjective uncertainty.

In 1982, Brans proposed the PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) (Brans 1982), which is a useful method to deal with the decision-making problems. Compared with other MCGDM methods, there exist three merits: (1) The PROMETHEE does not require to aggregate different kinds of criteria

values of an alternative into a comprehensive value; (2) Different from ELECTRE (ELimination Et Choix Traduisant la REalite) (Chen and Xu 2015), it is unnecessary for DMs to provide the relative parameter a priori, which can reduce DMs' subjective randomness; (3) Relative to QUALIFLEX (qualitative flexible multiple criteria method) (Zhang and Xu 2015), the PROMETHEE is easy to compute and is more time-saving. Recently, many studies have extended PROMETHEE to the fuzzy environments (Li and Li 2010; Liao and Xu 2014; Yilmaz and Dağdeviren 2011). However, only few investigations were reported on extending the PROMETHEE to the hesitant fuzzy environment (Mahmoudi et al. 2016; Feng et al. 2015).

Due to the uncertainty and hesitancy inherent in green supplier selection problems, the HFS is a suitable tool to solve such problems. Meanwhile, the PROMETHEE can handle the conflicting criteria and has some merits stated as above. Hence, it is necessary and worthwhile to extend PROMETHEE to accommodate the hesitant fuzzy environment and applied to green supplier selection. Noticeably, to extend PROMETHEE for MCGDM with HFSs, some difficulties and challenges need to be solved: (1) To measure the deviation of two hesitant fuzzy elements (HFEs), the distance between two HFEs needs to be defined considering hesitancy index of HFE and the deviation of values in intersectional position between two HFEs; (2) a new hesitant fuzzy entropy measure must be defined to measure the fuzziness and uncertainty of HFEs; (3) some new methods should be investigated to determine the criteria weights and the DMs' weights objectively. Therefore, this paper develops some new information measures of HFEs to extend PROMETHEE for MCGDM with HFSs and applies to green supplier selection.

The structure of this paper is organized as follows. Section 2 reviews green supplier evaluation and selection methods as well as some related literature about HFSs. Section 3 recalls the concept of HFEs and the classical PROMETHEE. Section 4 proposes a generalized hesitant fuzzy Hausdorff distance and a combined hesitant fuzzy entropy of HFEs. In Sect. 5, DMs' weights and the criteria weights are determined. Then, an extended PROMETHEE method is presented for MCGDM with HFSs. In Sect. 6, an example of green supplier selection is demonstrated to verify the validity and feasibility of the proposed method. Section 7 presents the conclusions.

2 Literature review

This section reviews the evaluation criteria and selection methods for green supplier selection as well as the information measures of HFSs. Then, the main work and features of this paper are outlined.

2.1 Evaluation criteria for green supplier selection

A variety of quantitative and qualitative criteria have appeared in the supplier selection problems. Lots of researchers studied the evaluating criteria including economic criteria and green criteria (Govindan et al. 2015; Govindan and Jepsen 2016). For the *economic criteria*, Dickson (1966) deemed that price, quality and delivery were the most popular criteria by an investigation. After surveying supplier selection at 1966–1990, Webber (1991) summarized that quality, delivery, price, production facilities and capacity were the most relevant criteria. Other literature (Yang and Tzeng 2011; Thiruchelvam and Tookey 2011; Banaeian et al. 2018) revealed that the price, service performance and quality were the most frequently used criteria. For the *green criteria*, the recent review in (Govindan et al. 2015; Nielsen et al. 2014) identified the environment management system as the most comprehensive and flexible environmental criteria. Recently, references (Banaeian et al. 2018; Ghorabaee et al. 2016; Kannan et al. 2014) performed the supplier selection of different industries, in which the environment management system is an important evaluating factor. In addition, environmental competences have been widely used in many green supplier selection problems (Grisi et al. 2010; Awasthi et al. 2010; Darabi and Heydari 2016). For the detailed description on environment management system and competences, please refer to Table 1. Table 1 presents a combination of economic criteria and green criteria to evaluate the green supplier selection.

2.2 Green supplier selection methods

Up to now, a large number of methods have been proposed to deal with green supplier selection problems. They are simply categorized as three groups.

The first group is multi-criteria decision-making (MCDM) techniques. Based on fuzzy AHP (Analytical hierarchy process), Grisi et al. (2010) applied a seven-step approach to dealing with green supplier selection. Hashemi et al. (2015) utilized ANP (analytic network process) to drive the criteria weights and improved GRA (grey relational analyse) to rank the suppliers. Roshandel et al. (2013) proposed hierarchical fuzzy TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) for supplier selection in detergent production industry. Based on GSCM practices, Kannan and Jabbour (2014) applied fuzzy TOPSIS to select the green supplier for a Brazilian electronics company. Hsu et al. (2013) utilized the DEMATEL (Decision-making Trial and Evaluation Laboratory) to propose a carbon management method of supplier selection. Based

Table 1 Evaluating criteria of green supplier selection

Literature	Evaluating criteria	Description
Economic criteria	Price	It covers all kinds of prices needed in the supply chain and determines the total cost
	Service performance	It includes the service quality in the supply process. For instance, supply capacity, after-sales service and on time delivery
	Quality	It indicates the quality of the product provided to the customer
Green criteria	Environmental management system	It is referred to the effectiveness of the policy to protect environment, organizational structure for environmental management
	Environmental competences	It shows the capability to reduce the impact to environment in the process of production. It can be expressed by use of ecological materials, availability of "clean" technologies, ability to respond adequately, etc.

on PROMETHEE, Govindan et al. (2017) constructed a group compromise ranking method for green supplier selection in food supply chain. Gupta and Barua (2017) used BWM (Best and Worst Method) and fuzzy TOPSIS to select supplier among small and medium enterprises on the basis of their green innovation ability. Awasthi and Kannan (2016) investigated green supplier development program selection using NGT (Nominal Group Technique) and VIKOR (Vlsekriterijumska Optimizacija I Kompromisno Resenje) under fuzzy environment. Chou and Chang (2008) studied a decision support system for supplier selection based on a strategy-aligned fuzzy SMART (strategy-aligned fuzzy simple multi-attribute rating technique) approach. Li and Li (2010) extended PROMETHEE II to generalized fuzzy numbers. Liao and Xu (2014) extended PROMETHEE to intuitionistic fuzzy sets for multi-criteria decision-making. Yilmaz and Dağdeviren (2011) proposed a combined approach for equipment selection: F-PROMETHEE method and zero–one goal programming. Although references (Li and Li 2010; Liao and Xu 2014; Yilmaz and Dağdeviren 2011) have extended PROMETHEE to the fuzzy environments, these extensions are invalid for hesitant fuzzy environment. Up to now, only two references (Mahmoudi et al. 2016; Feng et al. 2015) extended the PROMETHEE to the hesitant fuzzy environment. Mahmoudi et al. (2016) extended PROMETHEE in the context of the typical hesitant fuzzy sets to solve multi-attribute decision-making problem. Feng et al. (2015) developed a PROMETHEE method for hesitant fuzzy multi-criteria decision-making based on possibility degree. However, methods in Mahmoudi et al. (2016) and Feng et al. (2015) are only suitable for single-person decision-making. They cannot be used to solve MCGDM. In addition, Govindan et al. (2017) failed to consider the uncertainty and hesitancy. They just directly employed PROMETHEE to rank the suppliers.

The second group is the mathematical programming (MP) techniques. Integrating the CMS (Carbon Market Sensitive) and DEA (Data Envelopment Analysis), Jain et al. (2016) developed a CMS and a green decision

making approach based on DEA called CMS–GDEA. To determine the environmental factors, Dobos and Vörösmarty (2014) combined DEA and CWA (common weights analysis) to investigate a weight system. Kannan et al. (2013) constructed some FMOP (fuzzy multi-objective programming) models and applied them to green supplier selection problems in automobile manufacturing company. Integrating ABC (activity-based costing) and performance evaluation, Tsai and Hung (2009) developed a FGP (fuzzy goal programming) approach to selecting optimal green supplier in a value-chain structure. Jauhar and Pant (2017) integrated DEA with DE and MODE for sustainable supplier selection. Hamdan and Cheaitou (2017) put forward a MCDM and multi-objective optimization approach for supplier selection and order allocation with green criteria. Du et al. (2015) designed a Pareto supplier selection algorithm for minimum the life cycle cost of complex product system.

The third group is artificial intelligence (AI) techniques. To solve supplier selection problems, lots of AI methods have been developed. Kuo et al. (2010) proposed an ANN (artificial neural network) method. Yan (2009) integrated GA (genetic algorithm) and AHP to select green supplier. Yeh and Chuang (2011) used multi-objective genetic algorithm for partner selection in green supply chain problems. In order to assess environmental management performance of suppliers, Humphreys et al. (2003) developed CBS (case-based reasoning) method. Hosseini and Barker (2016) built a BN (Bayesian network) model to select resilience-based suppliers.

A summary of existing methods for green supplier selection is listed in Table 2.

The MCDM techniques are the most frequently used methods. Especially, AHP, ANP and TOPSIS are very popular among theses green supplier selection methods. Meanwhile, it can be seen that many fuzzy methods have been employed to solve green supplier selection. However, little attention has been paid to deal with green supplier selection problems in hesitant fuzzy environment. In addition, the extensive literature review shows that

Table 2 Summary of existing green supplier selection methods

Category	Methods	Literature
MCDM techniques	AHP	Grisi et al. (2010), Kannan et al. (2013)
	ANP	Hashemi et al. (2015)
	TOPSIS	Roshandel et al. (2013), Kannan and Jabbour (2014), Gupta and Barua (2017)
	VIKOR	Awasthi and Kannan (2016)
	PROMETHEE	Govindan et al. (2017)
	DEMATEL	Hsu et al. (2013)
	SMART	Chou and Chang (2008)
	TODIM	Qin et al. (2017)
MP techniques	DEA	Jain et al. (2016), Jauhar and Pant (2017)
	FGP	Tsai and Hung (2009).
	FMOP Pareto algorithm	Kannan et al. (2013), Hamdan and Cheaitou (2017), Du et al. (2015)
	ANN	Kuo et al. (2010).
AI techniques	GA	Chou and Chang (2008), Yeh and Chuang (2011), Du et al. (2015)
	CBS	Humphreys et al. (2003)
	BN	Hosseini and Barker (2016)

comparatively little research has focused on employing the outranking method to green supplier selection although they have been proven to be very effective in other applications (Govindan and Jepsen 2016; Behzadian et al. 2010).

2.3 Review of information measures of HFSs

DMs usually express hesitation when evaluating alternatives, criteria, variables, etc. Based on such thinking, Torra (2010) introduced the hesitant fuzzy set (HFS). There are many fruits on HFSs (Zhu and Xu2013; Xu and Xia 2011a, b; Peng et al. 2016; Li et al. 2015; Farhadinia 2014; Hu et al. 2016; Beg and Rashid 2017; Zhang et al. 2018; Xu and Xia 2012; Farhadinia 2013; Jin et al. 2016; Quirós et al. 2015; Wei et al. 2016; Torra et al. 2014; Xia and Xu 2011; Zhang and Xu 2014; Bedregal et al. 2014b, c; Xu and Zhang 2013; He et al. 2016; Zhang 2013), and some new concepts combining HFS with linguistic terms were proposed by Dong et al. (2016), Wu et al. (2018), Dong et al. (2015, 2018), Liu et al. (2018, 2019). Especially, the distance and entropy measures are two important focuses for HFSs (Xu and Xia 2011a, b; Peng et al. 2016; Li et al. 2015; Farhadinia 2014; Hu et al. 2016; Beg and Rashid 2017; Zhang et al. 2018; Xu and Xia 2012; Farhadinia 2013; Jin et al. 2016; Quirós et al. 2015; Wei et al. 2016; Torra et al. 2014).

The distance measure is mainly used to discriminate the diverse information. Xu and Xia (2011a, b) provided the axiom definitions of distance and similarity measures of HFSs. Peng et al. (2016) developed the generalized hesitant fuzzy synergetic weighted distance measure. Considering

the DM's hesitancy, Li et al. (2015) investigated some new distance formulas for HFSs containing hesitance degrees of HFEs. They presented a hesitancy index of HFEs and proposed the distance and similarity measures for HFSs. It is noticed that these distance measures of HFSs have some limitations: (1) The length of different HFEs must be the same by adding some accurate values. However, it is not easy to avoid subjective randomness for DM when determining which values to be added; (2) sorting the elements in each HFE is an extra burden work. To overcome these deficiencies, Farhadinia (2014) applied Hausdorff metric to the distance measures of HFSs and proposed various distance measures for higher order HFSs. Hu et al. (2016) proposed a variety of distance formulas of HFEs and HFSs without need to rearrange the elements in HFEs and add values. Based on the Hausdorff metric, Peng et al. (2016) defined the Hamming–Hausdorff and Euclidean–Hausdorff distance measures. Beg and Rashid (2017) proposed the Hausdorff-based distance. These studies (Peng et al. 2016; Li et al. 2015; Hu et al. 2016; Beg and Rashid 2017) effectively circumstanced the aforesaid two shortcomings. However, their distance measures overlook the DM's hesitancy in decision-making, which is an important factor in distance measures. Recently, Zhang et al. (2018) proposed an interesting distance measure for HFSs by considering the cardinal number of HFEs. It provides a new perspective to uniform the length of different HFSs. However, such a method makes the length of HFSs too large, which is more complicated. Meanwhile, they also ignored the hesitancy occurred in decision-making.

Entropy is the measure of uncertainty related to a specific fuzzy set. Recently, the entropy measure of HFSs

has attracted much attention. Xu and Xia (2012) first presented the concept of entropy and cross entropy of HFEs and meanwhile constructed some formulas for them. However, Farhadinia (2013) deemed that the definition of hesitant fuzzy entropy proposed in Xu and Xia (2012) is unable to distinguish a part of HFEs. Then, he discussed the relationship among distance, similarity and entropy measures and thus proposed a distance-based entropy measure of HFSs. Hu et al. (2016) proposed various entropy measures of HFSs, such as similarity measures-based entropy, distance measures-based entropy and hesitant-operation-based entropy. Using the continuous ordered weighted averaging operators, Jin et al. (2016) constructed some information measure formulas in the context of interval-valued HFSs (IVHFSs). It is noticed that most of the existing definitions of hesitant fuzzy entropy only reflect the information from one aspect, namely "fuzziness." Quirós et al. (2015) defined an entropy measure of IVHFSs based on three measures, including "fuzziness," "hesitancy" and "lack of knowledge." Wei et al. (2016) proposed an entropy formula considering both the fuzziness and hesitancy of HFEs. Literature review reveals that little attention has been focused on the hesitancy of the HFEs although it is an important aspect of the entropy measures of the HFSs (Bedregal et al. 2014).

2.4 Main work and features of this paper

To avoid the shortcomings of existing methods on green supplier selection and information measures of HFEs, this paper first develops some new information measures of HFEs, including a generalized hesitant fuzzy Hausdorff distance considering hesitancy index of HFEs, a combined hesitant fuzzy entropy and relative closeness degree. Then, by minimizing group inconsistency, a new linear programming model is established to derive DMs' weights explicitly. Through minimizing the relative entropy of the criteria weights, a nonlinear optimization model is established to obtain the criteria weights for each DM. Subsequently, the individual ranking of alternatives for each DM is derived under the framework of PROMETHEE. To obtain the collective ranking of alternatives, we establish a multi-objective assignment model and transform it into a single-objective assignment model for resolution. Thus, a hesitant fuzzy PROMETHEE method is put forward for MCGDM. The major contributions and features of this work are simplified as four aspects:

1. A new hesitancy index for HFEs is defined. It can effectively describe the hesitancy of HFEs and possess some desirable properties. Then, a generalized hesitant fuzzy Hausdorff distance is defined considering the hesitancy degree of HFEs and the deviation of values

in intersectional position between two HFEs simultaneously. It does not need to extend HFEs into the same length and can overcome the limitations of existing distance definitions. Hence, the generalized hesitant fuzzy Hausdorff distance is more reasonable and comprehensive. This contribution can solve the first challenge aforesaid in Introduction (i.e., (1) *To measure the deviation of two hesitant fuzzy elements (HFEs), the distance between two HFEs needs to be defined considering hesitancy index of HFE and the deviation of values in intersectional position between two HFEs*).

2. The fuzziness entropy and hesitancy entropy of HFEs are developed, respectively. Then, taking these two kinds of measures into account, a combined hesitant fuzzy entropy measure is further presented. Compared with existing entropy of Farhadinia (2013), the proposed combined hesitant fuzzy entropy integrates the fuzziness and hesitancy of HFS. It incorporates the BUM function and DM' risk attitude simultaneously. Therefore, it can provide more flexibility for DM in practical applications. This contribution can solve the second challenge aforesaid in Introduction (i.e., *(2) A new hesitant fuzzy entropy measure must be defined to measure the fuzziness and uncertainty of HFEs*).

3. By minimizing the relative entropy of the criteria weights, a nonlinear optimization model is constructed to determine the criteria weights for each DM. Moreover, through minimizing the group inconsistency, a linear program model is constructed to derive DMs' weights objectively. This contribution can solve the third challenge aforesaid in Introduction (i.e., *(3) Some new methods should be investigated to determine the criteria weights and the DMs' weights objectively.*).

4. The individual ranking order of alternatives for each DM is generated by extending PROMETHEE. Then, to obtain the group ranking order of alternatives, a multi-objective assignment model is established. Although Mahmoudi et al. (2016) and Feng et al. (2015) also extended the PROMETHEE to the hesitant fuzzy environment, there exist remarkable differences between their methods and the proposed method of this paper: (i) Their methods only consider the single decision-making problems, whereas this paper addresses MCGDM problems. Moreover, the proposed method can also be applied to deal with the single decision-making problems. (ii) In their methods, the criteria weights are given a priori. However, we set up a nonlinear optimization model to objectively derive the criteria weights for each DM.

3 Preliminary

This section briefly recalls some concepts of HFEs and introduces the classical PROMETHEE.

3.1 Concept and operations of HFEs

Definition 3.1 Torra (2010): A hesitant fuzzy set A on a fixed set $X = \{x_1, x_2, \ldots, x_n\}$ is defined by a function $h_A(x)$ when applied to X returns a subset of $[0, 1]$.

Xia and Xu (2011) mathematically represented a HFS A as follows:

$$A = \{ <x, h_A(x)> | x \in X\}, \tag{1}$$

where $h_A(x)$ is a set, possessing several possible values in $[0, 1]$ and representing the degree of an element x to a given set A. Xia and Xu 2011called $h_A(x)$ a hesitant fuzzy element (HFE) denoted by $h = \{\gamma^s | s = 1, 2, \ldots, l(h)\}$, where $l(h)$ is the number of membership values in a HFE. Zhang and Xu (2014) called HFE $h_A(x) = 0$ a hesitant empty fuzzy element and $h_A(x) = 1$ a hesitant full fuzzy element. The complement of HFE h is $h^c = \cup_{\gamma \in h}\{1 - \gamma\}$.

Recently, Bedregal et al. (2014b, c) defined typical hesitant fuzzy set, which merely focuses on those finite and nonempty subsets of interval [0,1]. The existing studies on hesitant fuzzy sets almost consider the typical HFSs and HFEs. Hence, HFSs and HFEs used in this paper are all referred to as typical HFSs and HFEs, respectively. It is noticed that the ideal of HFS is similar to the fuzzy multiset (Bedregal et al. 2014) and the concept of n-dimensional fuzzy set put forward by Shang et al. (2010).

Xu and Zhang (2013) provided the principle of extending the length for shorter HFEs until all HFEs have the same length. Assume h^+ and h^- are the maximum and minimum values in h, respectively. Then, the adding value is $h' = \eta h^+ + (1 - \eta)h^-$, where parameter η reflects DM's risk preference. If $\eta = 1$, i.e., the DM is risk-seeking and optimistic, then the maximum value h^+ is added to h. If $\eta = 0.5$, i.e., the DM is risk-neutral, then the middle value of h is added to h. If $\eta = 0$, i.e., the DM is risk-averse and pessimistic, then the minimum value h^- is added to h.

Definition 3.2 Xu and Xia 2011: For any two HFEs $h_i = \{\gamma_i^s | s = 1, 2, \ldots, l(h_i)\}$ $(i = 1, 2)$, the generalized hesitant fuzzy normalized distance, hesitant fuzzy normalized Hamming distance, hesitant fuzzy normalized Euclidean distance, hesitant fuzzy Hamming–Hausdorff distance and hesitant fuzzy Euclidean–Hausdorff distance are defined, respectively, as follows:

$$d_1(h_1, h_2) = \frac{1}{l} \sum_{j=1}^{l} |\gamma_1^{\sigma(j)} - \gamma_2^{\sigma(j)}|, \tag{2}$$

$$d_2(h_1, h_2) = \sqrt{\frac{1}{l} \sum_{j=1}^{l} |\gamma_1^{\sigma(j)} - \gamma_2^{\sigma(j)}|^2}, \tag{3}$$

$$d_3(h_1, h_2) = \max_j |\gamma_1^{\sigma(j)} - \gamma_2^{\sigma(j)}|, \tag{4}$$

$$d_4(h_1, h_2) = \max_j |\gamma_1^{\sigma(j)} - \gamma_2^{\sigma(j)}|^2, \tag{5}$$

where $\gamma_1^{\sigma(j)}$ and $\gamma_2^{\sigma(j)}$ are the jth smallest values in h_1 and h_2, respectively, and l is the maximum number between $l(h_1)$ and $l(h_2)$.

Although Definition 3.2 is wonderful and pioneering, there exist some limitations. In Eqs. (2)–(5), HFEs h_1 and h_2 should be extended into the same length based on DM's risk attitude. During the decision-making process, it is not easy for a DM to evaluate his/her risk preference by a precise numerical value. Thus, influenced by DM's risk preference, the distances of Definition 3.2 would be not accurate or unreasonable to some degree. On the other hand, when extending the length of shorter HFE, the number of the values in the HFE is altered. The information about the length of a HFE would be ignored and distorted in the distance measures of Definition 3.2. What's more important, these distance measures only consider the deviations between elements from the same position in two HFEs and ignore those from intersectional position in two HFEs.

3.2 Classical PROMETHEE

The PROMETHEE, initiated by Brans (1982), is one of the outranking methods. It often takes place when two alternatives are not comparable on particular criterion. PROMETHEE II consists of two phases: Calculate the net flow and obtain a complete ranking of alternatives. Denote the set of alternatives by $A = \{A_1, A_2, \ldots, A_m\}$ and the set of the criteria by $C = \{c_1, c_2, \ldots, c_n\}$. The evaluation of A_i on c_j is represented as $c_j(A_i)$ which is a positive real number. The main steps of PROMETHEE are involved as:

Step 1 Compute the deviation by comparing the pair-wise alternatives as

$$d_j(A_i, A_l) = c_j(A_i) - c_j(A_l), \tag{6}$$

where $d_j(A_i, A_l)$ reflects the deviation between the evaluations of alternatives A_i and A_l on criterion c_j.

Step 2 Calculate the preference degree of A_i over A_l on criterion c_j by the function as:

$$P_j(A_i, A_l) = f(d_j(A_i, A_l)), \tag{7}$$

where f is a preference function which transforms deviation $d_j(A_i, A_l)$ into preference degree $P_j(A_i, A_l)$ satisfying $0 \le P_j(A_i, A_l) \le 1$. Generally, if $d_j(A_i, A_l) = 0$, then $P_j(A_i, A_l) = 0$; if $d_j(A_i, A_l)$ is big enough, then $P_j(A_i, A_l)$ tends to be 1.

Step 3 Calculate the total preference degree of A_i over A_l

$$\pi(A_i, A_l) = \sum_{j=1}^{n} \omega_j P_j(A_i, A_l), \tag{8}$$

where ω_j is the weight of criterion c_j, satisfying the conditions $0 \le \omega_j \le 1$ and $\sum_{j=1}^{n} \omega_j = 1$.

Step 4 Calculate the outgoing flow $\Phi^+(A_i)$ and the incoming flow $\Phi^-(A_i)$, respectively, as

$$\Phi^+(A_i) = \frac{1}{m-1} \sum_{l \in M, l \ne i} \pi(A_i, A_l) \ (i \in M), \tag{9}$$

$$\Phi^-(A_i) = \frac{1}{m-1} \sum_{l \in M, l \ne i} \pi(A_l, A_i) \ (i \in M). \tag{10}$$

Step 5 The net flow $\Phi(A_i)$ can be calculated as follows:

$$\Phi(A_i) = \Phi^+(A_i) - \Phi^-(A_i). \tag{11}$$

Then, a complete ranking of alternatives is derived by the net flows of all alternatives.

4 Some new information measures of HFEs

In this section, a new hesitant fuzzy distance measure, a relative closeness degree and a new hesitant fuzzy entropy of HFEs are proposed. Some desirable properties are also discussed in detail.

4.1 A new distance measure of HFEs

To conquer the aforementioned shortcomings of the distances of Definition 3.2, Beg and Rashid (2017) proposed a hesitant fuzzy Hausdorff distance below.

Definition 4.1 For two HFEs $h_1 = \{\gamma_1^s | s = 1, 2, \ldots, l(h_1)\}$ and $h_2 = \{\gamma_2^t | t = 1, 2, \ldots, l(h_2)\}$, a hesitant fuzzy Hausdorff distance is defined as

$$d_{nhd}(h_1, h_2) = \max\{d^*(h_1, h_2), d^*(h_2, h_1)\}, \tag{12}$$

where $d^*(h_1, h_2) = \max_{\gamma_1^s \in h_1} \min_{\gamma_2^t \in h_2} |\gamma_1^s - \gamma_2^t|$ and $d^*(h_2, h_1) = \max_{\gamma_2^t \in h_2} \min_{\gamma_1^s \in h_1} |\gamma_2^t - \gamma_1^s|$.

Theorem 4.1 *For three HFEs* $h_i = \{\gamma_i^s | s = 1, 2, \ldots, l(h_i)\}$ $(i = 1, 2, 3)$, *the distance of* Definition 4.1 *possesses some properties as follows*:

1. $0 \le d_{nhd}(h_1, h_2) \le 1$;
2. $d_{nhd}(h_1, h_2) = 0$ iff $h_1 = h_2$;

3. $d_{nhd}(h_1, h_2) = d_{nhd}(h_2, h_1)$.

Proof 1. Since $0 \le |\gamma_1^s - \gamma_2^t| \le 1$, one has $0 \le d^*(h_1, h_2) \le 1$ and $0 \le d^*(h_2, h_1) \le 1$. Because $d_{nhd}(h_1, h_2) = \max\{d^*(h_1, h_2), d^*(h_2, h_1)\}$, it is concluded that $0 \le d_{nhd}(h_1, h_2) \le 1$.

2. If $d_{nhd}(h_1, h_2) = 0$, we have $d^*(h_1, h_2) = \max_{\gamma_1^s \in h_1} \min_{\gamma_2^t \in h_2} |\gamma_1^s - \gamma_2^t| = 0$ and $d^*(h_2, h_1) = \max_{\gamma_2^t \in h_2} \min_{\gamma_1^s \in h_1} |\gamma_2^t - \gamma_1^s| = 0$. Then, for any value γ_1^s in h_1, there exists a value γ_2^t in h_2 such that $\gamma_1^s = \gamma_2^t$, and vice versa. Thus, $h_1 = h_2$. If $h_1 = h_2$, it can be easily calculated that $d_{nhd}(h_1, h_2) = 0$. Hence, it holds that $d_{nhd}(h_1, h_2) = 0$ if and only if $h_1 = h_2$.

3. By Eq. (12), one has

$$\begin{aligned} d_{nhd}(h_1, h_2) &= \max\{d^*(h_1, h_2), d^*(h_2, h_1)\} \\ &= \max\{d^*(h_2, h_1), d^*(h_1, h_2)\} = d_{nhd}(h_2, h_1) \end{aligned}$$

It holds that $d_{nhd}(h_1, h_2) = d_{nhd}(h_2, h_1)$.
This completes the proof. $\square$

Compared with Definitions 3.2, 4.1 does not need to extend the length, which can not only preserve the original information and but also reduce the uncertainty in decision-making. On the other hand, Definition 4.1 focuses on the intersectional information in two HFEs and considers the deviation between the values in different positions.

Definition 4.2 For a HFE $h = \{\gamma^s | s = 1, 2, \ldots, l(h)\}$, a new hesitancy index of h is defined as:

$$u(h) = \begin{cases} \frac{1}{l(h)} \sum_{s=1}^{l(h)} |\gamma^s - \bar{h}|, & l(h) > 1 \\ 0, & l(h) = 1 \end{cases} \tag{13}$$

where $\bar{h} = \frac{1}{l(h)} \sum_{s=1}^{l(h)} \gamma^s$ represents the mean value of h.

The mean value $\bar{h}$ of a HFE represents the collective rating because it is the maximum compromise and signifies the highest decision level for a decision organization. While for individual, the mean value is a balance for his or her own preference, which is most possibly accepted by DM. The deviations between the several possible values in HFE and the mean value of the HFE can well depict the hesitancy of the HFE. Therefore, the new hesitancy index $u(h)$ has reasonability from the point of view of a decision organization or individual.

Property 4.1 For a HFE $h = \{\gamma^s | s = 1, 2, \ldots, l(h)\}$, let h^c be the complement of h. Then, the hesitancy indexes of h and h^c satisfy the following properties:

(1) $0 \le u(h) \le \frac{1}{2}$; (2) $u(h) = u(h^c)$.

Proof (1) It is straightforward.

(2) If $l(h) = 1$, one has $u(h) = 0 = u(h^c)$. If $l(h) > 1$, we have

$$u(h) = \frac{1}{l(h)} \sum_{s=1}^{l(h)} \left| \gamma^s - \frac{1}{l(h)} \sum_{j=1}^{l(h)} \gamma^j \right|$$

$$= \frac{1}{l(h)} \sum_{s=1}^{l(h)} \left| \frac{1}{l(h)} (\gamma^s l(h) - \sum_{j=1}^{l(h)} \gamma^j) \right|.$$

Moreover, for any value γ^s in h, it yields that

$$\left| \frac{1}{l(h)} (\gamma^s l(h) - \sum_{j=1}^{l(h)} \gamma^j) \right| = \left| \frac{1}{l(h)} [\gamma^s (l(h) - 1) - \sum_{j=1, j \neq s}^{l(h)} \gamma^j] \right|,$$

$$u(h^c) = \frac{1}{l(h)} \sum_{s=1}^{l(h)} |(1 - \gamma^s) - \frac{1}{l(h)} \sum_{j=1}^{l(h)} (1 - \gamma^j)|$$

$$= \frac{1}{l(h)} \sum_{s=1}^{l(h)} \left| \frac{1}{l(h)} [(1 - \gamma^s) l(h) - \sum_{j=1}^{l(h)} (1 - \gamma^j)] \right|$$

Additionally, for any value γ^s in h, one has

$$\left| \frac{1}{l(h)} [(1 - \gamma^s) l(h) - \sum_{j=1}^{l(h)} (1 - \gamma^j)] \right|$$

$$= \left| \frac{1}{l(h)} [\gamma^s (1 - l(h)) + \sum_{j=1, j \neq s}^{l(h)} \gamma^j] \right|.$$

It is obvious that $\left| \frac{1}{l(h)} [\gamma^s (l(h) - 1) - \sum_{j=1, j \neq s}^{l(h)} \gamma^j] \right|$ $= \left| \frac{1}{l(h)} [\gamma^s (1 - l(h)) + \sum_{j=1, j \neq s}^{l(h)} \gamma^j] \right|$, and thus, $u(h) = u(h^c)$. □

Example 4.1 In practical decision-making, assume that an organization which is comprised of several DMs assesses an alternative. The DMs hesitate among 0.3, 0.7 and 0.8. Such a rating can be represented by a HFE $\{0.3, 0.7, 0.8\}$. Then, by Eq. (13), the hesitancy index of the organization is calculated as:

$$u(h) = \frac{1}{3} (|0.3 - 0.6| + |0.7 - 0.6| + |0.8 - 0.6|) = 0.2$$

Thus, the hesitancy degree of the organization is 0.2, which is described by the deviation between the possible value and the mean value. Totally, the larger the departure from all the possible values to the mean value of a HFE, the bigger the hesitancy degree of the HFE.

The hesitancy degree of the HFE is a critical index in hesitant fuzzy decision-making. It should be considered in the distance of HFEs; otherwise, an unreasonable result will be caused.

Example 4.2 Assume there are two patterns h_1 and h_2 represented by two HFEs $h_1 = \{0.8, 0.75, 0.7\}$ and $h_2 = \{0.4, 0.5\}$, respectively. Consider which pattern an object $h_3 = \{0.65, 0.63, 0.55\}$ should belong to. By using Eq. (12), one gets $d_{nhd}(h_3, h_1) = 0.15$ and $d_{nhd}(h_3, h_2) = 0.15$. Obviously, it is difficult to judge the pattern of h_3 by using the hesitant fuzzy Hausdorff distance

of Definition 4.1, i.e., the value-based Hausdorff distance cannot recognize the patterns of HFEs whose deviation of value is the same.

Motivated by the idea of (Li et al. 2015), a new generalized hesitant fuzzy Hausdorff distance is defined by taking the hesitancy degree of HFE into account below.

Definition 4.3 For two HFEs $h_1 = \{\gamma_1^s | s = 1, 2, \ldots, l(h_1)\}$ and $h_2 = \{\gamma_2^t | t = 1, 2, \ldots, l(h_2)\}$, a generalized hesitant fuzzy Hausdorff distance considering hesitancy index is defined as:

$$d_{ghhd}(h_1, h_2)$$
$$= \left[\frac{1}{2} (\max\{ \max_{\gamma_1^s \in h_1} \min_{\gamma_2^t \in h_2} |\gamma_1^s - \gamma_2^t|^\lambda, \max_{\gamma_2^t \in h_2} \min_{\gamma_1^s \in h_1} |\gamma_2^t - \gamma_1^s|^\lambda \} \right.$$
$$\left. + |u(h_1) - u(h_2)|^\lambda) \right]^{\frac{1}{\lambda}},$$

(14)

where λ is a parameter satisfying $\lambda > 0$.

If $\lambda = 1$, Eq. (14) reduces to a hesitant fuzzy Hamming–Hausdorff distance:

$$d_{hhhd}(h_1, h_2) = \frac{1}{2}$$
$$\left[\max\{ \max_{\gamma_1^s \in h_1} \min_{\gamma_2^t \in h_2} |\gamma_1^s - \gamma_2^t|, \max_{\gamma_2^t \in h_2} \min_{\gamma_1^s \in h_1} |\gamma_2^t - \gamma_1^s| \} + |u(h_1) - u(h_2)| \right].$$

(15)

If $\lambda = 2$, Eq. (14) degenerates to a hesitant fuzzy Euclidean–Hausdorff distance:

$$d_{ehhd}(h_1, h_2)$$
$$= \left[\frac{1}{2} (\max\{ \max_{\gamma_1^s \in h_1} \min_{\gamma_2^t \in h_2} (\gamma_1^s - \gamma_2^t)^2, \max_{\gamma_2^t \in h_2} \min_{\gamma_1^s \in h_1} (\gamma_2^t - \gamma_1^s)^2 \} + (u(h_1) - u(h_2))^2) \right]^{\frac{1}{2}}.$$

(16)

Theorem 4.2 *The distance $d_{ghhd}(h_1, h_2)$ satisfies the desirable properties:*

1. $0 \leq d_{ghhd}(h_1, h_2) \leq 1$;
2. $d_{ghhd}(h_1, h_2) = 0$ *if and only if $h_1 = h_2$*;
3. $d_{ghhd}(h_1, h_2) = d_{ghhd}(h_2, h_1)$.

Proof (i) Since $0 \leq |\gamma_1^s - \gamma_2^t| \leq 1$, one has $0 \leq d^*(h_1, h_2) \leq 1$ and $0 \leq d^*(h_2, h_1) \leq 1$. According to Property 4.1, one has $0 \leq |u(h_1) - u(h_2)| \leq \frac{1}{2}$. Thus, it is concluded that $0 \leq d_{ghhd}(h_1, h_2) \leq 1$.

(ii) If $d_{ghhd}(h_1, h_2) = 0$, we have
$$d^*(h_1, h_2) = \max_{\gamma_1^s \in h_1} \min_{\gamma_2^t \in h_2} |\gamma_1^s - \gamma_2^t| = 0,$$

$$d^*(h_2, h_1) = \max_{\gamma_2^t \in h_2} \min_{\gamma_1^s \in h_1} |\gamma_2^t - \gamma_1^s| = 0, \text{ and } |u(h_1) - u(h_2)| = 0. \quad \square$$

Then, for any value γ_1^s in h_1, there exists a value γ_2^t in h_2 such that $\gamma_1^s = \gamma_2^t$, and vice versa. Thus, $h_1 = h_2$.

If $h_1 = h_2$, it can be easily calculated that $d_{ghhd}(h_1, h_2) = 0$. Hence, it can be concluded that $d_{ghhd}(h_1, h_2) = 0$ if and only if $h_1 = h_2$.

(iii) By Eq. (14), one has

$$d_{ghhd}(h_1, h_2)$$
$$= \left[\frac{1}{2} (\max\{\max_{\gamma_1^s \in h_1} \min_{\gamma_2^t \in h_2} |\gamma_1^s - \gamma_2^t|^\lambda, \max_{\gamma_2^t \in h_2} \min_{\gamma_1^s \in h_1} |\gamma_2^t - \gamma_1^s|^\lambda \} + |u(h_1) - u(h_2)|^\lambda) \right]^{\frac{1}{\lambda}}$$
$$= \left[\frac{1}{2} (\max_{\gamma_2^t \in h_2} \min_{\gamma_1^s \in h_1} |\gamma_2^t - \gamma_1^s|^\lambda, \max\{\max_{\gamma_1^s \in h_1} \min_{\gamma_2^t \in h_2} |\gamma_1^s - \gamma_2^t|^\lambda \} + |u(h_2) - u(h_1)|^\lambda) \right]^{\frac{1}{\lambda}}$$
$$= d_{ghhd}(h_2, h_1).$$

It holds that $d_{nhd}(h_1, h_2) = d_{nhd}(h_2, h_1)$.

This completes the proof of Theorem 4.2.

Example 4.3 Consider Example 4.2. By Eq. (15), one has $d_{hhhd}(h_3, h_1) = 0.153$ and $d_{hhhd}(h_3, h_2) = 0.155$. Since $d_{hhhd}(h_3, h_1) < d_{hhhd}(h_3, h_2)$, the object h_3 should belong to the pattern h_1.

Remark 4.1 If the hesitancy degree is not considered, then the generalized hesitant fuzzy Hausdorff distance of Definition 4.3 reduces to $d_{ghhd}(h_1, h_2) = [\frac{1}{2}(\max\{\max_{\gamma_1^s \in h_1} \min_{\gamma_2^t \in h_2} | \gamma_1^s - \gamma_2^t|^\lambda, \max_{\gamma_2^t \in h_2} \min_{\gamma_1^s \in h_1} |\gamma_2^t - \gamma_1^s|^\lambda\})]^{\frac{1}{\lambda}}$, which generalizes the hesitant fuzzy Hausdorff distance of Definition 4.1. Thus, the generalized hesitant fuzzy Hausdorff distance inherits the advantage of the hesitant fuzzy Hausdorff distance. In addition, compared with Definition 3.2, Definition 4.3 does not need to extend HFEs into the same length. Moreover, Definition 4.3 simultaneously considers the individual deviation of membership values and the hesitancy degree of HFEs, which overcomes the limitations of Definition 3.2. Therefore, the generalized hesitant fuzzy Hausdorff distance is more reasonable and comprehensive.

4.2 Relative closeness degree of HFEs

Let the full HFE $h^* = \{1\}$ be a positive ideal HFE, and the null HFE $h^- = \{0\}$ be a negative ideal HFE. Using Eq. (16), the hesitant fuzzy Euclidean–Hausdorff distance between HFE $h = \{\gamma^s | s = 1, 2, ..., l(h)\}$ and positive ideal HFE $h^* = \{1\}$ is calculated as

$$d_{ehhd}(h, h^*) = \left[\frac{1}{2} (\max_{\gamma^s \in h} (\gamma^s - 1)^2 + u(h)^2) \right]^{\frac{1}{2}}, \quad (17)$$

the hesitant fuzzy Euclidean–Hausdorff distance between HFE $h = \{\gamma^s | s = 1, 2, ..., l(h)\}$ and negative ideal HFE $h^- = \{0\}$ is calculated as

$$d_{ehhd}(h, h^-) = \left[\frac{1}{2} (\max_{\gamma^s \in h} (\gamma^s)^2 + u(h)^2) \right]^{\frac{1}{2}}. \quad (18)$$

Motivated by the idea of TOPSIS, the closer a HFE h to positive ideal HFE h^* and the farther from negative ideal HFE h^-, the better the HFE h. Thus, a relative closeness degree of h is given below.

Definition 4.4 Let $h = \{\gamma^s | s = 1, 2, ..., l(h)\}$ be a HFE. A relative closeness degree of HFE h is defined as

$$r(h) = \frac{d_{ehhd}(h, h^-)}{d_{ehhd}(h, h^-) + d_{ehhd}(h, h^*)}. \quad (19)$$

The larger the relative closeness degree $r(h)$, the better the HFE h. The relative closeness degree will be employed to the hesitant fuzzy PROMETHEE in subsection 5.4.

4.3 A new entropy measure of HFEs

In this section, a new entropy measure for HFEs is put forward from the perspective of the fuzziness and hesitancy of HFEs.

(1) Fuzziness entropy measure of HFEs

The fuzziness entropy measure of HFEs should manifest the deviations between HFEs and numerical values. The distance between HFEs and numerical values can be applied to calculate the fuzziness entropy measure. By extending the axioms of interval-valued intuitionistic fuzzy entropy in (Zhang et al. 2014), the axioms of fuzziness entropy of HFEs are defined in what follows.

Definition 4.5 Let h be a HFE. A function E_F is called a fuzziness entropy of a HFE, if it meets the following properties:

1. $0 \le E_F(h) \le 1$;
2. $E_F(h) = 0$, if h is a crisp set, i.e., $h = \{0\}$ or $h = \{1\}$;
3. $E_F(h) = 1 \Leftrightarrow h = \{\frac{1}{2}\}$;
4. $E_F(h) = E_F(h^c)$;
5. If $d_{nhd}(h_1, \{\frac{1}{2}\}) \ge d_{nhd}(h_2, \{\frac{1}{2}\})$, then $E_F(h_1) \le E_F(h_2)$.

where d_{nhd} is the hesitant fuzzy Hausdorff distance of Definition 4.1.

The first property indicates that the fuzziness entropy is nonnegative. The second property indicates that HFE h has no fuzziness if h is a real number 0 or 1. The third property shows that the maximum fuzziness appears when HFE h is $\frac{1}{2}$. The fourth property implies that a HFE h and its complement have the same fuzziness. The last property means that the farther from $\{\frac{1}{2}\}$ a HFE h, the smaller the fuzziness entropy E_F of h.

Remark 4.2 It is noticed that the fuzziness entropy of HFE only focuses on the fuzziness of HFEs (i.e., the deviation between HFEs and real numbers) instead of the hesitancy of HFEs. Thus, the distance used in Definition 4.5 only considers the value-based hesitant fuzzy Hausdorff distance of Definition 4.1 instead of the generalized Hausdorff distance of HFEs of Definition 4.3.

Theorem 4.3 *Let $E_F(h)$ be a mapping: $HFE(h) \rightarrow [0,1]$. Fuzziness entropy $E_F(h)$ related to the hesitant fuzzy Hausdorff distance can be symbolized by the following formula:*

$$E_F(h) = 1 - 2d_{nhd}(h, \{\tfrac{1}{2}\}), \tag{20}$$

where d_{nhd} is the hesitant fuzzy Hausdorff distance by Eq. (12).

Proof It is sufficient to prove that $E_F(h)$ of Eq. (20) satisfies the properties in Definition 4.5.

1. It is straightforward that $0 \leq E_F(h) = 1 - 2d_{nhd}(h, \{\tfrac{1}{2}\}) \leq 1$.
2. If h is a crisp set, i.e., $h = \{0\}$ or $h = \{1\}$, then using Eq. (12), we conclude that $E_F(h) = 1 - 2d_{nhd}(\{0\}, \{\tfrac{1}{2}\}) = 1 - 2d_{nhd}(\{1\}, \{\tfrac{1}{2}\}) = 0$.
3. Given that $E_F(h) = 1$, then $1 - 2d_{nhd}(h, \{\tfrac{1}{2}\}) = 1 \Leftrightarrow d_{nhd}(h, \{\tfrac{1}{2}\}) = 0$. It is no doubt that $h = \{\tfrac{1}{2}\}$. If $h = \{\tfrac{1}{2}\}$, then $1 - 2d_{nhd}(h, \{\tfrac{1}{2}\}) = 1$, i.e., $E_F(h) = 1$. Thus, $E_F(h) = 1 \Leftrightarrow h = \{\tfrac{1}{2}\}$.
4. According to Eq. (12), it has $d_{nhd}(h, \{\tfrac{1}{2}\}) = d_{nhd}(h^c, \{\tfrac{1}{2}\})$. Thus, $E_F(h) = 1 - 2d_{nhd}(h, \{\tfrac{1}{2}\}) = 1 - 2d_{nhd}(h^c, \{\tfrac{1}{2}\}) = E_F(h^c)$ is satisfied.
5. Since $d_{nhd}(h_1, \{\tfrac{1}{2}\}) \geq d_{nhd}(h_2, \{\tfrac{1}{2}\})$, it yields that $1 - 2d_{nhd}(h_1, \{\tfrac{1}{2}\}) \leq 1 - 2d_{nhd}(h_2, \{\tfrac{1}{2}\})$. Thus, one has $E(h_1) \leq E(h_2)$. $\quad\square$

This completes the proof of Theorem 4.3.

(2) Hesitancy entropy measure of HFEs

Hesitancy is an important index to express the uncertainty, which is ignored in existing entropy measures of HFEs. In what follows, a hesitancy entropy is defined by the deviation between several possible values in a HFE and the mean value in the HFE, i.e., the hesitancy degree defined in Definition 4.2.

Definition 4.6 Let h be a HFE and E_H be a mapping: $HFE(h) \rightarrow [0,1]$. Then, E_H is called a hesitancy entropy of HFE h if E_H satisfies:

1. $E_H(h) = 0 \Leftrightarrow u(h) = 0$;
2. $E_H(h) = 1 \Leftrightarrow u(h) = \tfrac{1}{2}$;
3. $E_H(h) = E_H(h^c)$;
4. $E_H(h_1) \leq E_H(h_2)$, if and only if $u(h_1) \leq u(h_2)$.

The first property states that the hesitancy entropy appears to be zero when the hesitancy degree is zero (i.e., the number of values in h is only one). The second property implies when the hesitancy degree of h is equal to 0.5, $E_H(h)$ is 1. The third property indicates that HFE h and its complement h^c have the same hesitancy entropy. The last property shows that the larger the deviation between the

several possible values in a HFE and the mean value in the HFE, the larger the hesitancy entropy of h.

Theorem 4.5 *For a HFE h, if there exists a mapping $y: \mathbb{N} \rightarrow [0,1]$ such that*

$$E_H(h) = y(2u(h)), \tag{21}$$

then $E_H(h)$ is a hesitancy entropy, where y is a BUM function satisfying three conditions: (1) $y(a) = 0 \Leftrightarrow a = 0$;, (2) $y(a) = 1 \Leftrightarrow a = 1$; and 3) y is monotone increasing. $\mathbb{N}$ denotes the set of nonnegative real numbers.

Proof It is sufficient to prove that $E_H(h)$ of Eq. (21) meets the properties in Definition 4.6.

1. If $E_F(h) = 0$, i.e., $y(2u(h)) = 0$, one gets $2u(h) = 0$. In other words, $E_H(h) = 0 \Rightarrow u(h) = 0$, and vice versa. Thus, $E_H(h) = 0 \Leftrightarrow u(h) = 0$ is satisfied.
2. If $E_F(h) = 1$, i.e., $y(2u(h)) = 1$, then one gets $u(h) = \tfrac{1}{2}$. Thus, $E_F(h) = 1 \Rightarrow u(h) = \tfrac{1}{2}$. If $u(h) = \tfrac{1}{2}$, then $E_F(h) = y(2u(h)) = 1$. Hence, $E_H(h) = 1 \Leftrightarrow u(h) = \tfrac{1}{2}$.
3. For $u(h) = u(h^c)$, it has $E_H(h) = E_H(h^c)$.
4. Given $u(h_1) \leq u(h_2)$ and y is monotone increasing, one has $y(2u(h_1)) \leq y(2u(h_2))$, i.e., $E_H(h_1) \leq E_H(h_2)$. $\quad\square$

This completes the proof of Theorem 4.5.

(3) Combined hesitant fuzzy entropy of HFEs

Considering the fuzziness entropy and the hesitancy entropy of HFEs simultaneously, a combined hesitant fuzzy entropy measure is defined as follows:

Definition 4.7 Combined the fuzziness entropy with the hesitancy entropy, a combined hesitant fuzzy entropy of HFE h is defined as:

$$E(h) = \alpha E_F(h) + (1 - \alpha)E_H(h), \tag{22}$$

where α reflects DM's risk attitude satisfying $0 \leq \alpha \leq 1$.

DM's risk attitude parameter α can be determined according to the need and characteristics of real-life decision-making problem. If $\alpha = 0$, it means that the combined hesitant fuzzy entropy is reduced to the hesitancy entropy in Definition 4.6. If $\alpha = 1$, then the combined hesitant fuzzy entropy is degenerated to the fuzzy entropy in Definition 3.5. In addition, the combined hesitant fuzzy entropy possesses properties: (1) $0 \leq E(h) \leq 1$; (2) $E(h) = E(h^c)$.

For a HFS $A = \{ <x_i, h_A(x_i)> | x_i \in X \}$, the fuzziness entropy, hesitancy entropy and combined hesitant fuzzy entropy are, respectively, defined as follows:

$$E_F(A) = \frac{1}{n} \sum_{i=1}^{n} E_F(h_A(x_i)),$$

$$E_H(A) = \frac{1}{n} \sum_{i=1}^{n} E_H(h_A(x_i)), \; E(A) = \frac{1}{n} \sum_{i=1}^{n} E(h_A(x_i))$$

Remark 4.2 Farhadinia (2013) transformed distance or similarity measures into the entropy of HFS. For a HFS $A = \{<x_i, h_A(x_i)> | x_i \in X\}$ with $h_A(x_i) = \{\gamma_{x_i}^j | j = 1, 2, \ldots, l_{x_i}\}$, the entropy transformed from distance is as follows (Farhadinia 2013):

$$E_{dhnh}(A) = 1 - \frac{2}{n} \sum_{i=1}^{n} \left[\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left| \gamma_{x_i}^j - \frac{1}{2} \right| \right]. \tag{23}$$

Apparently, the entropy $E_{dhnh}(A)$ defined by Farhadinia (2013) is just similar to the fuzziness entropy $E_F(A)$ of this paper. In other words, the entropy defined by Farhadinia (2013) just considers the fuzziness while neglects the hesitancy. The combined hesitant fuzzy entropy of Definition 4.7 considers the entropy measure from the perspective of fuzziness and hesitancy simultaneously. Moreover, the combined hesitant fuzzy entropy takes DMs' risk attitude into account sufficiently.

Remark 4.3 The links of the above-defined information measures of HFSs are as follows: The new hesitancy index $u(h)$ is used to define the generated hesitant fuzzy Hausdorff distance $d_{ghhd}(h_1, h_2)$, while the latter is utilized to compute the relative closeness degree $r(h)$. Meanwhile, $u(h)$ is also employed to construct the hesitancy entropy $E_H(h)$, and the hesitant fuzzy Hausdorff distance $d_{nhd}(h_1, h_2)$ is employed to obtain the fuzziness entropy $E_F(h)$. The combined hesitant fuzzy entropy $E(h)$ is the convex combination of $E_F(h)$ and $E_H(h)$.

Example 4.4 For a HFS $A = \{<x_1, \{0.3, 0.7, 0.8\}>, <x_2, \{0.1, 0.2, 0.3\}>\}$, the entropy measure of HFSs (Farhadinia 2013) is calculated by Eq. (23) as $E_{dhnh}(A) = 0.4667$.

In the sequel, different BUM functions y are selected, and the corresponding fuzziness entropy $E_F(A)$, hesitant entropy $E_H(A)$ and combined hesitant fuzzy entropy $E(A)$ are computed and presented in Tables 3, 4 and 5.

From Tables 3, 4 and 5, it is easily seen that the hesitant entropy can be changed with the parameter t of BUM functions, and the combined hesitant fuzzy entropy can be changed with the parameter t and the risk attitude α. DM can choose different BUM functions to compute the combined hesitant fuzzy entropy of HFS according to his (her) risk attitude. This shows that the combined hesitant fuzzy entropy proposed in this paper can provide more flexibility for DM in actual applications.

5 A hesitant fuzzy PROMETHEE method for MCGDM

In this section, a hesitant fuzzy PROMETHEE is developed for MCGDM.

5.1 Problem description for MCGDM with HFSs

Let $E = \{e_1, e_2, \ldots, e_t\}$ be a set of DMs, $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives, and $C = \{c_1, c_2, \ldots, c_n\}$ be a set of all criteria. Denote the rating of alternative A_i on criterion c_j given by DM e_k by a HFE h_{ij}^k. Hence, a hesitant fuzzy decision matrix $\boldsymbol{H}^k = (h_{ij}^k)_{m \times n}$ is elicited. Therefore, MCGDM problems can be concisely described by matrices $\boldsymbol{H}^k = (h_{ij}^k)_{m \times n}$ $(k = 1, 2, \ldots, t)$.

Since the criteria consist of benefit criteria and cost criteria, the individual matrix $\boldsymbol{H}^k = (h_{ij}^k)_{m \times n}$ is transformed into the normalized matrix $\boldsymbol{F}^k = (f_{ij}^k)_{m \times n}(k = 1, 2, \ldots, t)$ where

$$f_{ij}^k = \begin{cases} h_{ij}^k, & \text{if } c_j \text{ is a benefit criterion} \\ (h_{ij}^k)^c, & \text{if } c_j \text{ is a cost criterion} \end{cases} \tag{24}$$

Let be $\boldsymbol{w} = (w_1, w_2, \ldots, w_t)^{\mathrm{T}}$ be DMs' weight vector, where $0 \le w_k \le 1$ $(k = 1, 2, \ldots, t)$ and $\sum_{k=1}^{t} w_k = 1$. Owing to the increasing complexity and uncertainty of the real-life decision-making problems, it is always hard to measure the importance of DMs accurately. In general, the information of DMs' weights is usually incompletely known. It has five basic forms:

Form 1 A weak ranking: $\{w_i \ge w_j\}$;
Form 2 A strict ranking: $\{w_i - w_j \ge \alpha_i\}$ $(\alpha_i > 0)$;
Form 3 A ranking of differences: $\{w_i - w_j \ge w_k - w_l\}$, for $j \ne k \ne l$;
Form 4 A ranking with multiples: $\{w_i \ge \alpha_i w_j\}$ $\{0 \le \alpha_i \le 1\}$;
Form 5 An interval form: $\{\alpha_i \le w_i \le \alpha_i + \varepsilon_i\}$ $(0 \le \alpha_i \le \alpha_i + \varepsilon_i \le 1)$.

Let Ω be a set of the incomplete information of DMs' weights which may consist of several basic forms.

DMs differ in their experience and knowledge. They are often good at some subjects, but not at the others. In their views, different criteria have diverse importance degrees. Thus, it is reasonable to assign different criteria weights for different DMs. Denote the criteria weight vector for DM e_k by $\boldsymbol{\omega}^k = (\omega_1^k, \omega_2^k, \ldots, \omega_n^k)^{\mathrm{T}}$, where $\boldsymbol{\omega}^k$ satisfies $0 \le \omega_j^k \le 1$ $(j = 1, 2, \ldots, n)$ and $\sum_{j=1}^{n} \omega_j^k = 1$.

Table 3 Computation results for BUM function $y(s) = s^t$ with different values of parameter t

t	$E_F(A)$	$E_H(A)$	$E(A)$ with $\alpha = 0$	$E(A)$ with $\alpha = 0.5$	$E(A)$ with $\alpha = 1$
0	0.3	1	1	0.65	0.3
0.5	0.3	0.4988	0.4988	0.3994	0.3
1	0.3	0.2667	0.2667	0.2833	0.3
2	0.3	0.0889	0.0889	0.1944	0.3
$+\infty$	0.3	0	0	0.15	0.3

Table 4 Computation results for BUM function $Q(s) = \left(\frac{1-e^{-s}}{1-e^{-1}}\right)^t$ with different values of parameter $t > 0$

t	$E_F(A)$	$E_H(A)$	$E(A)$ with $\alpha = 0$	$E(A)$ with $\alpha = 0.5$	$E(A)$ with $\alpha = 1$
0	0.3	1	1	0.65	0.3
0.5	0.3	0.5833	0.5833	0.4416	0.3
1	0.3	0.3595	0.3595	0.3298	0.3
2	0.3	0.1555	0.1555	0.2278	0.3
$+\infty$	0.3	0	0	0.15	0.3

Table 5 Computation results for BUM function $Q(s) = (\sin(\frac{1}{2}\pi s))^t$ with different values of parameter t

t	$E_F(A)$	$E_H(A)$	$E(A)$ with $\alpha = 0$	$E(A)$ with $\alpha = 0.5$	$E(A)$ with $\alpha = 1$
0	0.3	1	1	0.65	0.3
1	0.3	0.3978	0.3978	0.3489	0.3
2	0.3	0.1944	0.1944	0.2472	0.3
$+\infty$	0.3	0	0	0.15	0.3

5.2 Determination of DMs' weights

Inspired by method (He et al. 2016), the DM whose rating is close to the mean rating of decision group would be assigned larger weight, while the DM whose rating is far from the mean rating would be given lower weight. Group inconsistency measures the deviation between the individual rating and the mean rating on each criterion. It can be used to determine DMs' weights. The mean rating also represents the collective rating because it is the maximum compromise for all DMs. Moreover, it represents the highest decision level of the group which is most possibly received by DMs (He et al. 2016). With this understanding in mind, a new method is put forward to derive DMs' weights below.

According to the normalized matrices $\boldsymbol{F}^k = (f_{ij}^k)_{m \times n}$ $(k = 1, 2, \ldots, t)$, a matrix $\boldsymbol{F}'^j$ with respect to criterion c_j is defined as:

$$
\boldsymbol{F}'^j = \begin{array}{c} \\ e_1 \\ e_2 \\ \ldots \\ e_t \end{array} \begin{array}{cccc} A_1 & A_2 & \ldots & A_m \\ \left(\begin{array}{cccc} f_{1j}^1 & f_{2j}^1 & \ldots & f_{mj}^1 \\ f_{1j}^2 & f_{2j}^2 & \ldots & f_{mj}^2 \\ \ldots & \ldots & \ldots & \ldots \\ f_{1j}^t & f_{2j}^t & \ldots & f_{mj}^t \end{array} \right), \end{array} \tag{25}
$$

where f_{ij}^k signifies the degree to which DM e_k is satisfied with alternative A_i on criterion c_j.

The mean rating $\bar{f}_{ij}$ of alternative A_i on criterion c_j is defined as follows:

$$
\bar{f}_{ij} = \frac{1}{t} \sum\nolimits_{k=1}^t f_{ij}^k \tag{26}
$$

The group inconsistency ϑ_{ij} represents the deviation between the individual rating and the mean rating of alternative A_i on criterion c_j. It is defined as

$$
\vartheta_{ij} = \sum_{k=1}^t w_k d_{ehhd}(f_{ij}^k, \bar{f}_{ij}) \ (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n) \tag{27}
$$

where d_{ehhd} is the hesitant fuzzy Euclidean–Hausdorff distance calculated by Eq. (16).

Hence, the group inconsistency ϑ^j on criterion c_j is defined as

$$
\vartheta^j = \sum_{i=1}^m \sum_{k=1}^t w_k d_{ehhd}(f_{ij}^k, \bar{f}_{ij}) \ (j = 1, 2, \ldots, n), \tag{28}
$$

which denotes the deviation between the individual rating and the mean rating on criteria c_j.

Therefore, the total group inconsistency ϑ can be defined by

$$
\vartheta = \sum\nolimits_{i=1}^m \sum\nolimits_{j=1}^n \sum\nolimits_{k=1}^t w_k d_{ehhd}(h_{kj}, \bar{h}_{ij}), \tag{29}
$$

which represents the total deviation between the individual rating and the mean rating.

Generally, the lower the group inconsistency ϑ, the more reliable the decision result. Thus, to determine the DMs' weights, a linear programming model of minimizing the group inconsistency ϑ is established:

$$\min \theta = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{t} w_k d_{ehhd}(f_{ij}^k, \bar{f}_{ij})$$

$$s.t. \begin{cases} \sum_{k=1}^{t} w_k = 1, w_k \geq 0 \quad (k = 1, 2, \ldots, t) \\ \boldsymbol{w} \in \Omega \end{cases} \quad (30)$$

Solving Eq. (30) by the simplex method, the DMs' weight vector $\boldsymbol{w} = (w_1, w_2, \ldots, w_t)^{\mathrm{T}}$ can be obtained.

5.3 Determination of criteria weights for each DM

In most of the literature (Xu and Zhang 2013; Wei et al. 2011), the criteria weights through different DMs are viewed as the same. However, DMs differ in their own capabilities of knowledge and experience and may be professional in some criteria, but not in other criteria. This paper determines the criteria weights for each DM from the perspective of the relative entropy.

Definition 5.1 Dong and Wan 2016 Let $\boldsymbol{X} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ and $\boldsymbol{Y} = (y_1, y_2, \ldots, y_n)^{\mathrm{T}}$ be two vectors which satisfy $x_i \geq 0$, $y_i \geq 0$, and $\sum_{i=1}^{n} x_i \geq \sum_{i=1}^{n} y_i$. Then, the relative entropy from $\boldsymbol{X}$ to $\boldsymbol{Y}$ is defined as

$$R(\boldsymbol{X}, \boldsymbol{Y}) = \sum_{i=1}^{n} x_i \ln(x_i/y_i). \quad (31)$$

It holds that:

(1) $R(\boldsymbol{X}, \boldsymbol{Y}) \geq 0$; (2) $R(\boldsymbol{X}, \boldsymbol{Y}) = \sum_{i=1}^{n} x_i \ln(x_i/y_i) = 0$, if and only if $x_i = y_i$ for all i.

It can be easily seen that $R(\boldsymbol{X}, \boldsymbol{Y})$ is an appropriate consistent measure that depicts the consistency between the vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$. The larger the relative entropy $R(\boldsymbol{X}, \boldsymbol{Y})$, the smaller the consistency between $\boldsymbol{X}$ and $\boldsymbol{Y}$. Especially, the vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$ are completely consistent if $\boldsymbol{X} = \boldsymbol{Y}$, i.e., $R(\boldsymbol{X}, \boldsymbol{Y}) = 0$ iff $\boldsymbol{X} = \boldsymbol{Y}$.

According to Eq. (22), the individual normalized matrix $\boldsymbol{F}^k = (f_{ij}^k)_{m \times n}$ for DM e_k can be converted to a combined hesitant fuzzy entropy matrix:

$$\boldsymbol{E}^k = (E_{ij}^k)_{m \times n} = \begin{pmatrix} E_{11}^k & E_{12}^k & \cdots & E_{1n}^k \\ E_{21}^k & E_{22}^k & \cdots & E_{2n}^k \\ \cdots & \cdots & \cdots & \cdots \\ E_{m1}^k & E_{m2}^k & \cdots & E_{mn}^k \end{pmatrix} \quad (32)$$

where E_{ij}^k indicates the combined hesitant fuzzy entropy of alternative A_i on criterion x_j for DM e_k. Normalize the vector $\boldsymbol{E}_i^k = (E_{i1}^k, E_{i2}^k, \ldots, E_{in}^k)^{\mathrm{T}}$ into $\boldsymbol{E}_i^{k\prime} = (E_{i1}^{k\prime}, E_{i2}^{k\prime}, \ldots, E_{in}^{k\prime})^{\mathrm{T}}$ by $E_{ij}^{k\prime} = E_{ij}^k / \sum_{i=1}^{m} E_{ij}^k$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$. The criteria weight vector $\boldsymbol{\omega}^k$ can be objectively obtained by the amount of information provided by DM e_k which is implied in the combined hesitant fuzzy entropy matrix $\boldsymbol{E}^k$. Thus, it is reasonable that the criteria weight vector $\boldsymbol{\omega}^k = (\omega_1^k, \omega_2^k, \ldots, \omega_n^k)^{\mathrm{T}}$ should be consistent with the combined hesitant fuzzy entropy vectors $\boldsymbol{E}_i^{k\prime} = (E_{i1}^{k\prime}, E_{i2}^{k\prime}, \ldots, E_{in}^{k\prime})^{\mathrm{T}}$ $(i = 1, 2, \ldots, m)$. Therefore, the more consistent between $\boldsymbol{\omega}^k$ and $\boldsymbol{E}_i^{k\prime}$ $(i = 1, 2, \ldots, m)$, the smaller the relative entropy $R(\boldsymbol{\omega}^k, \boldsymbol{E}_i^{k\prime})$ $(i = 1, 2, \ldots, m)$.

According to Eq. (31), the relative entropy from $\boldsymbol{\omega}^k$ to $\boldsymbol{E}_i^{k\prime}$ is derived as

$$R(\boldsymbol{\omega}^k, \boldsymbol{E}_i^{k\prime}) = \sum_{j=1}^{n} \omega_j^k \ln(\omega_j^k / E_{ij}^{k\prime}) \ (i = 1, 2, \ldots, m;$$
$$k = 1, 2, \ldots, t)$$

If $R(\boldsymbol{\omega}^k, \boldsymbol{E}_i^{k\prime}) = 0$, then $\boldsymbol{\omega}^k$ is completely consistent with $\boldsymbol{E}_i^{k\prime}$. Thus, it is reasonable to derive the criteria weights through making the relative entropy between $\boldsymbol{\omega}^k$ and $\boldsymbol{E}_i^{k\prime}$ as small as possible. With this idea in mind, a nonlinear optimization model is established for DM e_k as:

$$\min \ \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \omega_j^k \ln\left(\omega_j^k \Big/ E_{ij}^{k\prime}\right) \right\}$$

$$s.t. \begin{cases} \sum_{j=1}^{n} \omega_j^k = 1 \\ \omega_j^k > 0 \ (j = 1, 2, \ldots, n) \end{cases} \quad (33)$$

To solve Eq. (33), a Lagrange function is established as

$$L(\boldsymbol{\omega}^k, \lambda) = \sum_{i=1}^{m} \sum_{j=1}^{n} \omega_j^k \ln(\omega_j^k / e_{ij}^{k\prime}) + \lambda(\sum_{j=1}^{n} \omega_j^k - 1),$$
$$(34)$$

where λ is the Lagrange multiplier. The global optimal solution can be derived by taking partial derivatives of ω_j^k and λ such that

$$\partial L(\boldsymbol{\omega}^k, \lambda) \Big/ \partial \omega_j^k = \sum_{i=1}^{m} [\ln(\omega_j^k / e_{ij}^{k\prime}) + 1] + \lambda = 0, \quad (35)$$

$$\partial L(\boldsymbol{\omega}^k, \lambda) / \partial \lambda = \sum_{j=1}^{n} \omega_j^k - 1 = 0. \quad (36)$$

Solving the above equalities, the solution can be obtained:

$$\omega_j^k = \exp\{\frac{1}{m} \sum_{i=1}^{m} \ln E_{ij}^{k\prime} - (1 + \lambda)\}, \quad (37)$$

$$\lambda = \ln\{\sum_{i=1}^{n} \exp(\frac{1}{m} \sum_{i=1}^{m} \ln E_{ij}^{k\prime} - 1)\}. \quad (38)$$

Then, one has

$$\omega_j^k = \frac{\exp(\frac{1}{m} \sum_{i=1}^{m} \ln E_{ij}^{k\prime} - 1)}{\sum_{j=1}^{n} \exp(\frac{1}{m} \sum_{i=1}^{m} \ln E_{ij}^{k\prime} - 1)} \ (j = 1, 2, \ldots, n) \quad (39)$$

Thus, the criteria weight vectors $\boldsymbol{\omega}^k = (\omega_1^k, \omega_2^k, \ldots, \omega_n^k)^{\mathrm{T}}$ $(k = 1, 2, \ldots, t)$ are obtained for all DMs.

5.4 A hesitant fuzzy PROMETHEE for MCGDM

This section describes the steps of the proposed hesitant fuzzy PROMETHEE for MCGDM.

Step 1 Form the individual hesitant fuzzy decision matrix $\boldsymbol{H}^k = (h_{ij}^k)_{m \times n}$ and transform it into the normalized matrix $\boldsymbol{F}^k = (f_{ij}^k)_{m \times n}$ $(k = 1, 2, \ldots, t)$ by Eq. (24).

Step 2 Derive DMs' weights $\boldsymbol{w} = (w_1, w_2, \ldots, w_t)^{\mathrm{T}}$ using Eq. (30).

Step 3 Utilize Eq. (39) to derive the criteria weights $\boldsymbol{\omega}^k = \{\omega_1^k, \omega_2^k, \ldots, \omega_n^k\}^{\mathrm{T}}$ for DM e_k $(k = 1, 2, \ldots, t)$.

Step 4 A weighted normalized individual matrix $\boldsymbol{G}^k = (g_{ij}^k)_{m \times n}$ is derived from the normalized matrix $\boldsymbol{F}^k = (f_{ij}^k)_{m \times n}$, where $g_{ij}^k = w_k f_{ij}^k = \cup_{\gamma \in f_{ij}^k} \{w_k \gamma\}$.

Step 5 A deviation $d_j^k(A_i, A_l)$ between the evaluations of alternative A_i and A_l on criterion c_j for DM e_k is calculated as

$$d_j^k(A_i, A_l) = r_j^k(A_i) - r_j^k(A_l), \tag{40}$$

where $r_j^k(A_i)$ indicates the relative closeness degree calculated by Eq. (19).

Step 6 A preference function is defined as:

$$P_j^k(A_i, A_l) = \begin{cases} d_j^k(A_i, A_l), & \text{if } r_j^k(A_i) > r_j^k(A_l) \\ 0, & \text{if } r_j^k(A_i) < r_j^k(A_l) \end{cases} \tag{41}$$

If $r_j^k(A_i) > r_j^k(A_l)$, alternative A_i is superior to alternative A_l on criterion c_j and the value of preference degree $P_j^k(A_i, A_l)$ is equal to $d_j^k(A_i, A_l)$. Otherwise, the preference degree is zero.

Step 7 Calculate the total preference degree of A_i over A_l for DM e_k:

$$\pi^k(A_i, A_l) = \sum_{j=1}^n \omega_j^k P_j(A_i, A_l). \tag{42}$$

Step 8 Calculate the outgoing flow $\Phi_k^+(A_i)$ and the incoming flow $\Phi_k^-(A_i)$ for DM e_k, respectively:

$$\Phi_k^+(A_i) = \frac{1}{m-1} \sum_{l \in M, l \neq i} \pi^k(A_i, A_l) \ (i \in M), \tag{43}$$

$$\Phi_k^-(A_i) = \frac{1}{m-1} \sum_{l \in M, l \neq i} \pi^k(A_l, A_i) \ (i \in M). \tag{44}$$

Step 9 The net flow $\Phi^k(A_i)$ is calculated as follows:

$$\Phi^k(A_i) = \Phi_k^+(A_i) - \Phi_k^-(A_i). \tag{45}$$

By descending the net flow $\Phi^k(A_i)$, the individual ranking of alternatives for DM e_k is obtained.

Step 10 Using matrix $\boldsymbol{X}^k = (x_{ij}^k)_{m \times m}$ to describe the ranking permutation for DM e_k, where

$$x_{ij}^k = \begin{cases} 1, & \text{if DM } e_k \text{ ranks alternative } A_i \text{ in the } jth \text{ position} \\ 0, & \text{otherwise} \end{cases}. \tag{46}$$

For instance, $x_{41}^k = 1$ means that DM e_k ranks alternative A_4 the first.

Step 11 Denote the collective ranking matrix by $\boldsymbol{X} = (x_{ij})_{m \times m}$, which needs to be derived, where

$$x_{ij} = \begin{cases} 1, & \text{if the decision group ranks alternative } A_i \text{ in the } jth \text{ position} \\ 0, & \text{otherwise} \end{cases} \tag{47}$$

By minimizing the deviation between the individual order and the collective one, a multi-objective assignment model is set up:

$$\min Z^k = \sum_{i=1}^m \sum_{j=1}^m |x_{ij}^k - x_{ij}| \ (k = 1, 2, \ldots, t)$$
$$s.t. \begin{cases} \sum_{i=1}^m x_{ij} = 1 \ (j = 1, 2, \ldots, m) \\ \sum_{j=1}^m x_{ij} = 1 \ (i = 1, 2, \ldots, m) \\ x_{ij} = 0 \text{ or } 1 \ (i, j = 1, 2, \ldots, m) \end{cases} \tag{48}$$

where the constraints $\sum_{i=1}^m x_{ij} = 1 \ (j = 1, 2, \ldots, m)$ assure that each alternative is sorted in only one position and the constraints $\sum_{j=1}^m x_{ij} = 1 \ (i = 1, 2, \ldots, m)$ guarantee that each position is placed by one alternative.

Equation (48) is converted into a single-objective assignment mode:

$$\min Z = \sum_{k=1}^t \left(w_k \sum_{i=1}^m \sum_{j=1}^m |x_{ij}^k - x_{ij}| \right)$$
$$s.t. \begin{cases} \sum_{i=1}^m x_{ij} = 1 \ (j = 1, 2, \ldots, m) \\ \sum_{j=1}^m x_{ij} = 1 \ (i = 1, 2, \ldots, m) \\ x_{ij} = 0 \text{ or } 1 \ (i, j = 1, 2, \ldots, m) \end{cases} \tag{49}$$

After solving Eq. (49) by Hungarian method, we can derive the collective ranking matrix which is used to rank the alternatives and choose the best one.

Remark 5.1 In the classical PROMETHEE (Brans and Vincle 1985), the evaluation values are real numbers, the criteria weights are known a priori, and the criteria weights are the same for different DMs. The deviation between pair-wise alternatives on criterion is the subtraction of the evaluation values (see Eq. (6)). Thus, the classical PROMETHEE is only suitable for the single decision-making problems with crisp evaluation values. However, the evaluation values are HFEs in this paper. The criteria weights for each DM and DMs' weights are determined objectively. Then, the PROMETHEE is generalized to hesitant fuzzy environment. Furthermore, the deviation between pair-wise alternatives on criterion is calculated by the subtraction of the relative closeness degree of HFEs
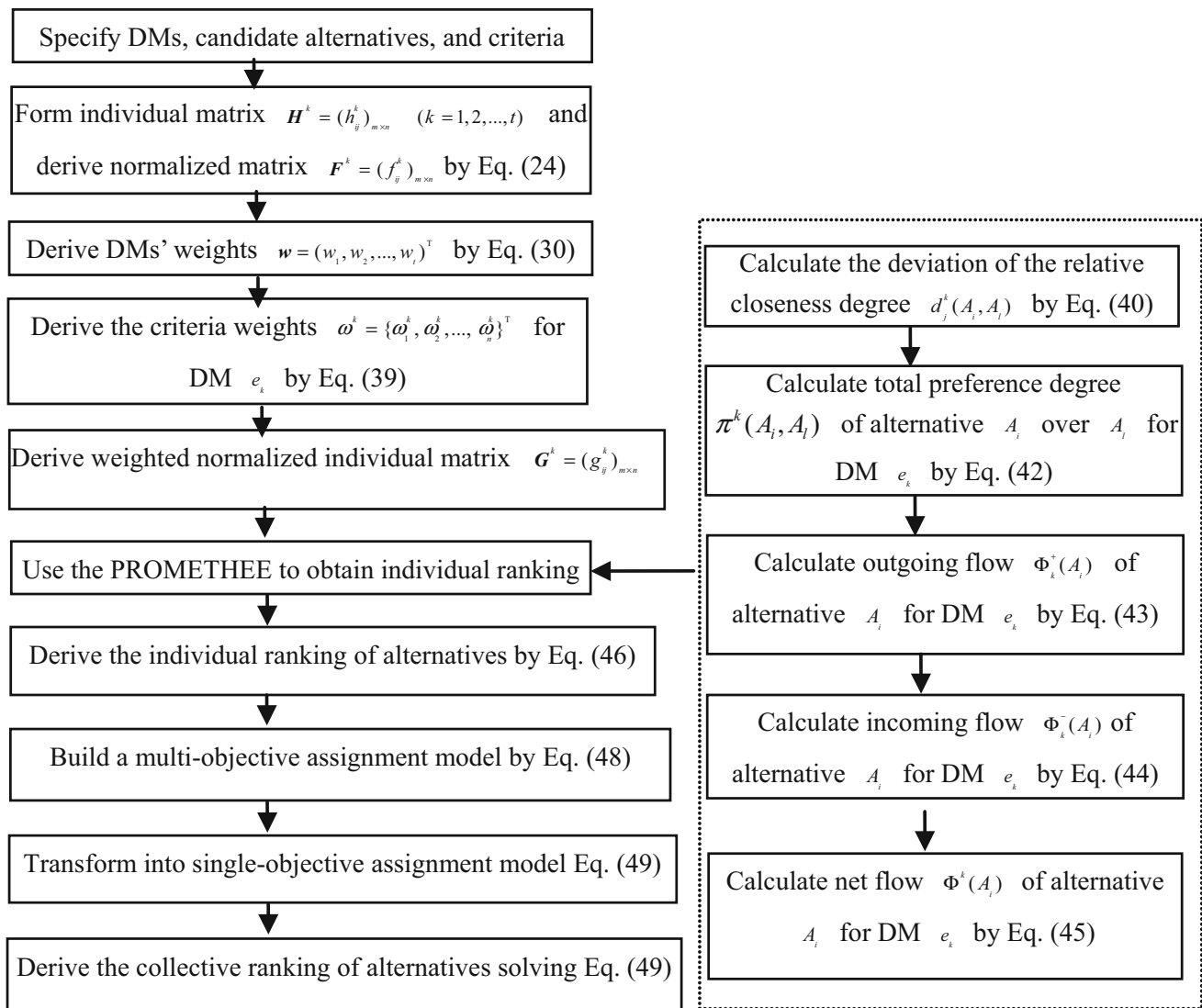
Fig. 1 Hesitant fuzzy PROMETHEE for MCGDM

(see Eq. (40)). The individual ranking of alternatives for each DM is derived by the net flow in Eq. (45) and the corresponding individual ranking matrix is generated. Then, a multi-objective assignment model is set up to obtain the group ranking of alternatives. Therefore, the classical PROMETHEE is extended to solve the HF-MCGDM problems. If the evaluation values degenerate to numerical numbers, the proposed method is reduced to the classical PROMETHEE.

The proposed hesitant fuzzy PROMETHEE for MCGDM is depicted in Fig. 1.

6 Green supplier selection example and comparative analyses

In this section, an example of green supplier selection is implemented using the proposed method. The comparative analyses are also conducted.

6.1 Green supplier selection example

Shuanghui Group Co., Ltd is a large-scale food company mainly engaged in meat processing. It has built many processing bases throughout the nationwide 18 provinces of China. The subsidiaries of the company include meat processing, bioengineering, chemical packaging, Shuanghui logistics, etc. Shuanghui Group Co., Ltd possesses Chinese largest meat processing, the total assets of which

Table 6 Decision matrices for DMs e_1, e_2, and e_3

DM	supplier	c_1	c_2	c_3	c_4	c_5
e_1	A_1	{0.1,0.3}	{0.3,0.5,0.7}	{0.1,0.5,0.6}	{0.3,0.4}	{0.5,0.7}
	A_2	{0.3,0.4,0.5}	{0.1,0.2,0.4}	{0.5,0.7,0.8}	{0.3,0.5}	{0.5,0.7}
	A_3	{0.5,0.6}	{0.2,0.3}	{0.4,0.7}	{0.5,0.6,0.7}	{0.4,0.5,0.7}
	A_4	{0.3,0.4}	{0.5,0.6,0.7}	{0.2,0.5}	{0.4,0.6,0.7}	{0.7,0.8}
	A_5	{0.2,0.5,0.6}	{0.5,0.8,0.9}	{0.6,0.7,0.8}	{0.8,0.9}	{0.5,0.6,0.8}
e_2	A_1	{0.1,0.2}	{0.2,0.3,0.6}	{0.1,0.4,0.5}	{0.1,0.3,0.4}	{0.4,0.5}
	A_2	{0.5,0.6}	{0.7,0.8}	{0.6,0.7,0.8}	{0.75,0.8,0.9}	{0.75,0.8,0.9}
	A_3	{0.2,0.6,0.7}	{0.5,0.6,0.8}	{0.5,0.7,0.8}	{0.4,0.5,0.7}	{0.5,0.7,0.8}
	A_4	{0.4,0.6,0.8}	{0.2,0.6,0.7}	{0.2,0.5,0.8}	{0.6,0.7,0.9}	{0.1,0.2}
	A_5	{0.1,0.5,0.6}	{0.1,0.4,0.5}	{0.3,0.4,0.5}	{0.1,0.2,0.5}	{0.3,0.4}
e_3	A_1	{0.35, 0.6,0.75}	{0.5,0.6}	{0.5,0.6}	{0.7,0.85}	{0.4,0.5,0.6}
	A_2	{0.4,0.5}	{0.1,0.3,0.4}	{0.3,0.4}	{0.1, 0.3,0.5}	{0.2,0.5,0.7}
	A_3	{0.5,0.6}	{0.2,0.4,0.6}	{0.5,0.6}	{0.2,0.4}	{0.4,0.5}
	A_4	{0.1,0.4,0.5}	{0.3,0.4,0.5}	{0.3,0.4,0.5}	{0.4,0.5,0.6}	{0.1,0.2}
	A_5	{0.5,0.9}	{0.5,0.6,0.8}	{0.5,0.6,0.8}	{0.6,0.7}	{0.4,0.5}

are approximately 200 billion Yuan and 65,000 employees. In 2010, Shuanghui Group Co., Ltd ranked 160 in Chinese top 500 enterprises and its brand value reached 19.652 billion Yuan.

With the continuous development of environment protection and commercial competitions, Shuanghui Group Co., Ltd has to improve the quality of GSCM and reduce the cost of the product. To achieve such requirements, the managers of company plan to change its green equipment suppliers for new product processing. After preliminary screening, five potential green suppliers $\{A_1, A_2, A_3, A_4, A_5\}$ remain to be further evaluated. These green suppliers are Zhucheng Zhongyun Industry and Trade Co., Ltd. (A_1, ZZIT for short), Sichuan Provincial Agricultural and Sideline Products Processing Technology Development Company in Chengdu (A_2, SPAS for short), Hangzhou Hangsheng Machinery Equipment Co., Ltd. (A_3, HHME for short), Shandong Kaile Feng Food Machinery Co., Ltd. (A_4, SKFFM for short) and Zhaoqing Yonghui Machinery Co., Ltd. (A_5, ZYMC for short). The ZZIT (A_1) is an old-fashioned food machinery manufacturer with a history of more than 20 years. It is a high-tech enterprise integrating R&D (Research & Development), design, production, sales and after-sales services. Especially, its smoke furnace is famous for its good quality to control the smoke volume and temperature in the furnace, which sufficiently considers the environment protection. The SPAS (A_2) is a nationalized business engaged in the agricultural research and sideline product processing technology. Over the years, the company has been devoted to the research of deep meat processing equipment considering the ecological restoration and energy saving. It is currently the most complete enterprise that provides agricultural and sideline products processing projects in China. The HHME (A_3) is a

comprehensive large-scale mechanical equipment group company. The main business of this company is to operate a variety of meat equipment, integrating environmental monitoring, data services and comprehensive management. The SKFFM (A_4) possesses strong technical force, excellent machine processing equipment, advanced production process, complete inspection facilities and after-sales service teams. Also, its organizational structure for environmental management is outstanding. The ZYMC (A_5) is a technology-based enterprise integrating R&D, manufacturing and sales. The company focuses on all kinds of meat processing and has a good capability to clean production waste. Due to the excellent performances, good reputations and specialties in environmental protection of these five companies, we choose these companies as potential alternatives.

Five criteria have been identified through literature review (see Table 1), which are c_1: price, c_2: service performance, c_3: quality, c_4: environmental management system and c_5: environmental competences. Note that these five criteria are all benefit criteria. A committee for supplier selection consists of three DMs $E = \{e_1, e_2, e_3\}$ coming from the purchase department, production department and food safety department, respectively. The ratings on each supplier are represented by HFEs, which are listed in Table 6. The incomplete information of DMs' weights Ω is furnished as: $\Omega = \{ \boldsymbol{w} = (w_1, w_2, \ldots, w_t)^T | w_1 \geq w_3, \ 0.2 \leq w_2 \leq 0.4, \ w_3 \leq 0.3\}$.

Step 1 Form the individual hesitant fuzzy decision matrix given by each DM as shown in Table 6. Since all criteria are benefit criteria, the normalized matrix $\boldsymbol{F}^k = (f_{ij}^k)_{m \times n}$ is still the individual hesitant fuzzy decision matrix in Table 6.

Step 2 Applying Eqs. (29) and (30), the following model is obtained:

Table 7 Combined hesitant fuzzy entropy matrices for DMs e_1, e_2 and e_3

DM	supplier	c_1	c_2	c_3	c_4	c_5
e_1	A_1	0.2	0.53	0.24	0.50	0.52
	A_2	0.51	0.20	0.36	0.52	0.52
	A_3	0.66	0.34	0.54	0.51	0.52
	A_4	0.50	0.51	0.38	0.52	0.34
	A_5	0.38	0.22	0.35	0.18	0.36
e_2	A_1	0.18	0.38	0.22	0.20	0.66
	A_2	0.66	0.34	0.35	0.18	0.18
	A_3	0.40	0.36	0.36	0.52	0.36
	A_4	0.37	0.40	0.40	0.20	0.18
	A_5	0.24	0.22	0.51	0.22	0.50
e_3	A_1	0.46	0.66	0.66	0.27	0.67
	A_2	0.66	0.20	0.50	0.21	0.39
	A_3	0.66	0.37	0.66	0.36	0.66
	A_4	0.22	0.51	0.51	0.67	0.18
	A_5	0.24	0.36	0.37	0.50	0.66

$$\min \quad \vartheta = 6.27w_1 + 6.67w_2 + 6.12w_3$$

$$s.t. \begin{cases} w_1 + w_2 + w_3 = 1, \ w_k \geq 0 \ (k = 1, 2, 3) \\ w_1 \geq w_3, \ 0.2 \leq w_2 \leq 0.4, \ w_3 \leq 0.3 \end{cases}$$

The DMs' weight vector is derived as $w = (0.5, 0.2, 0.3)^{\mathrm{T}}$ by solving the above model.

Step 3 For simplicity, set $y(a) = a$ and $\alpha = 0.8$. By Eq. (23), three combined hesitant fuzzy entropy matrices for all DMs are obtained in Table 7.

Then, the normalized entropy matrices for all DMs are derived in Table 8.

By Eq. (39), the criteria weight vectors for different DMs are calculated as follows:

$$\omega^1 = (0.196, 0.195, 0.204, 0.196, 0.207)^{\mathrm{T}},$$
$$\omega^2 = (0.195, 0.211, 0.208, 0.197, 0.189)^{\mathrm{T}}$$
$$\omega^3 = (0.194, 0.201, 0.212, 0.199, 0.194)^{\mathrm{T}}.$$

Step 4 The weighted normalized matrices for all DMs are obtained in Table 9.

Step 5 The relative closeness matrices are obtained by Eq. (19) as shown in Table 10.

The deviations between the evaluations for each DM are calculated and shown in Tables 11, 12 and 13.

Step 6 As per Eq. (41), the preference degrees $P_j^k(A_i, A_l)$ for each DM are obtained as shown in Tables 14, 15 and 16.

Step 7 By Eq. (42), the total preference degrees $\pi^k(A_i, A_l)$ for different DMs are calculated and listed in Table 17.

Step 8 The outgoing flows of alternatives for different DMs are computed by Eq. (43) and listed in Table 18.

By Eq. (44), obtain the incoming flows of alternatives for different DMs shown in Table 19.

Step 9 Derive the net flows of alternatives for different DMs by Eq. (45) shown in Table 20.

Thus, the individual ranking orders of five suppliers are: $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$ for DM e_1, $A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$ for DM e_2, $A_1 \succ A_5 \succ A_3 \succ A_4 \succ A_2$ for DM e_3.

Step 10 Then, the individual ranking matrices for different DMs are obtained as follows:

$$X^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \ X^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \ X^3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Table 8 Normalized entropy matrices for DMs e_1, e_2 and e_3

DM	supplier	c_1	c_2	c_3	c_4	c_5
e_1	A_1	0.09	0.30	0.13	0.22	0.23
	A_2	0.23	0.11	0.19	0.23	0.23
	A_3	0.29	0.19	0.29	0.23	0.23
	A_4	0.22	0.28	0.20	0.24	0.15
	A_5	0.17	0.12	0.19	0.08	0.16
e_2	A_1	0.10	0.22	0.12	0.15	0.35
	A_2	0.36	0.20	0.19	0.14	0.10
	A_3	0.22	0.21	0.20	0.39	0.19
	A_4	0.20	0.23	0.22	0.15	0.10
	A_5	0.13	0.13	0.28	0.17	0.27
e_3	A_1	0.20	0.31	0.24	0.13	0.26
	A_2	0.29	0.10	0.19	0.11	0.15
	A_3	0.29	0.18	0.24	0.18	0.26
	A_4	0.10	0.24	0.19	0.33	0.07
	A_5	0.11	0.17	0.14	0.25	0.26

Table 9 Weighted normalized matrices for DMs e_1, e_2 and e_3

DM	supplier	c_1	c_2	c_3	c_4	c_5
e_1	A_1	{0.05,0.15}	{0.15,0.25,0.35}	{0.05,0.25,0.3}	{0.15,0.2}	{0.25,0.35}
	A_2	{0.15,0.2,0.25}	{0.05,0.1,0.2}	{0.25,0.35,0.4}	{0.15,0.25}	{0.25,0.35}
	A_3	{0.25,0.3}	{0.1,0.15}	{0.2,0.35}	{0.25,0.3,0.35}	{0.2,0.25,0.35}
	A_4	{0.15,0.2}	{0.25,0.3,0.35}	{0.1,0.25}	{0.2,0.3,0.35}	{0.35,0.4}
	A_5	{0.1,0.25,0.3}	{0.25,0.4,0.45}	{0.3,0.35,0.4}	{0.4,0.45}	{0.25,0.3,0.4}
e_2	A_1	{0.02,0.04}	{0.04,0.06,0.12}	{0.02,0.08,0.1}	{0.02,0.06,0.08}	{0.08,0.1}
	A_2	{0.1,0.12}	{0.14,0.16}	{0.12,0.14,0.16}	{0.15,0.16,0.18}	{0.15,0.16,0.18}
	A_3	{0.04,0.12,0.14}	{0.1,0.12,0.16}	{0.1,0.14,0.16}	{0.08,0.1,0.14}	{0.1,0.14,0.16}
	A_4	{0.08,0.12,0.16}	{0.04,0.12,0.14}	{0.04,0.1,0.16}	{0.12,0.14,0.18}	{0.02,0.04}
	A_5	{0.02,0.1,0.12}	{0.02,0.08,0.1}	{0.06,0.08,0.1}	{0.02,0.04,0.1}	{0.06,0.08}
e_3	A_1	{0.11,0.18,0.23}	{0.15,0.18}	{0.15,0.18}	{0.21,0.255}	{0.12,0.15,0.18}
	A_2	{0.12,0.15}	{0.03,0.09,0.12}	{0.09,0.12}	{0.03,0.09,0.15}	{0.06,0.15,0.21}
	A_3	{0.15,0.18}	{0.06,0.12,0.18}	{0.15,0.18}	{0.06,0.12}	{0.12,0.15}
	A_4	{0.03,0.12,0.15}	{0.09,0.12,0.15}	{0.09,0.12,0.15}	{0.12,0.15,0.18}	{0.03,0.06}
	A_5	{0.15,0.27}	{0.15,0.18,0.24}	{0.12,0.18,0.24}	{0.18,0.21}	{0.12,0.15}

Step 11 By Eq. (49), the assignment model is constructed as follows:

$$
\begin{aligned}
\min = {} & 0.5(x_{11} + x_{12} + x_{13} + x_{14} + 1 - x_{15} + x_{21} \\
& + x_{22} + x_{23} + 1 - x_{24} + x_{25} + x_{31} + x_{32} \\
& 1 - x_{33} + x_{34} + x_{35} + x_{41} + 1 - x_{42} + x_{43} \\
& + x_{44} + x_{45} + 1 - x_{51} + x_{52} + x_{53} + x_{54} + x_{55}) \\
& + 0.2(x_{11} + x_{12} + x_{13} + x_{14} + 1 - x_{15} + 1 - x_{21} \\
& + x_{22} + x_{23} + x_{24} + x_{25} + x_{31} + 1 - x_{32} \\
& + x_{33} + x_{34} + x_{35} + x_{41} + x_{42} + 1 - x_{43} + x_{44} \\
& + x_{45} + x_{51} + x_{52} + x_{53} + 1 - x_{54} + x_{55}) \\
& + 0.3(1 - x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{21} \\
& + x_{22} + x_{23} + x_{24} + 1 - x_{25} + x_{31} + x_{32} \\
& + 1 - x_{33} + x_{34} + x_{35} + x_{41} + x_{42} + x_{43} \\
& + 1 - x_{44} + x_{45} + x_{51} + 1 - x_{52} + x_{53} + x_{54} + x_{55})
\end{aligned}
$$

(50)

Table 10 Relative closeness degree matrices for DMs e_1, e_2 and e_3

DM	supplier	c_1	c_2	c_3	c_4	c_5
e_1	A_1	0.266	0.500	0.407	0.365	0.581
	A_2	0.420	0.314	0.612	0.419	0.581
	A_3	0.548	0.275	0.537	0.582	0.537
	A_4	0.419	0.582	0.391	0.537	0.725
	A_5	0.424	0.636	0.664	0.814	0.612
e_2	A_1	0.186	0.432	0.364	0.314	0.455
	A_2	0.545	0.725	0.664	0.779	0.779
	A_3	0.469	0.612	0.612	0.537	0.612
	A_4	0.569	0.469	0.500	0.686	0.186
	A_5	0.407	0.364	0.418	0.364	0.365
e_3	A_1	0.534	0.545	0.545	0.734	0.500
	A_2	0.455	0.314	0.365	0.363	0.468
	A_3	0.545	0.431	0.545	0.338	0.455
	A_4	0.364	0.418	0.418	0.500	0.186
	A_5	0.631	0.612	0.569	0.635	0.455

Table 11 Deviations between the evaluations for DM e_1

DM e_1	c_1	c_2	c_3	c_4	c_5	DM e_1	c_1	c_2	c_3	c_4	c_5
$d(A_1,A_2)$	-0.154	0.186	-0.205	-0.054	0.000	$d(A_3,A_4)$	0.128	-0.307	0.146	0.045	-0.188
$d(A_1,A_3)$	-0.282	0.225	-0.130	-0.217	0.044	$d(A_3,A_5)$	0.124	-0.361	-0.128	-0.232	-0.075
$d(A_1,A_4)$	-0.153	-0.082	0.016	-0.173	-0.144	$d(A_4,A_1)$	0.153	0.082	-0.016	0.173	0.144
$d(A_1,A_5)$	-0.158	-0.136	-0.258	-0.449	-0.031	$d(A_4,A_2)$	-0.001	0.268	-0.221	0.118	0.144
$d(A_2,A_1)$	0.154	-0.186	0.205	0.054	0.000	$d(A_4,A_3)$	-0.128	0.307	-0.146	-0.045	0.188
$d(A_2,A_3)$	-0.127	0.039	0.075	-0.163	0.044	$d(A_4,A_5)$	-0.005	-0.053	-0.274	-0.276	0.113
$d(A_2,A_4)$	0.001	-0.268	0.221	-0.118	-0.144	$d(A_5,A_1)$	0.158	0.136	0.258	0.449	0.031
$d(A_2,A_5)$	-0.004	-0.322	-0.052	-0.395	-0.031	$d(A_5,A_2)$	0.004	0.322	0.052	0.395	0.031
$d(A_3,A_1)$	0.282	-0.225	0.130	0.217	-0.044	$d(A_5,A_3)$	-0.124	0.361	0.128	0.232	0.075
$d(A_3,A_2)$	0.127	-0.039	-0.075	0.163	-0.044	$d(A_5,A_4)$	0.005	0.053	0.274	0.276	-0.113

Table 12 Deviations between the evaluations for DM e_2

DM e_2	c_1	c_2	c_3	c_4	c_5	DM e_2	c_1	c_2	c_3	c_4	c_5
$d(A_1,A_2)$	-0.359	-0.293	-0.300	-0.465	-0.324	$d(A_3,A_4)$	-0.100	0.143	0.112	-0.149	0.426
$d(A_1,A_3)$	-0.283	-0.180	-0.248	-0.223	-0.157	$d(A_3,A_5)$	0.062	0.248	0.194	0.173	0.247
$d(A_1,A_4)$	-0.383	-0.037	-0.136	-0.372	0.269	$d(A_4,A_1)$	0.383	0.037	0.136	0.372	-0.269
$d(A_1,A_5)$	-0.221	0.068	-0.053	-0.050	0.090	$d(A_4,A_2)$	0.148	-0.052	0.015	0.004	-0.492
$d(A_2,A_1)$	0.359	0.293	0.300	0.465	0.324	$d(A_4,A_3)$	0.100	-0.143	-0.112	0.149	-0.426
$d(A_2,A_3)$	-0.089	-0.004	-0.003	0.136	0.066	$d(A_4,A_5)$	0.162	0.104	0.082	0.322	-0.179
$d(A_2,A_4)$	-0.148	0.052	-0.015	-0.004	0.492	$d(A_5,A_1)$	0.221	-0.068	0.053	0.050	-0.090
$d(A_2,A_5)$	-0.021	0.197	0.211	0.267	0.353	$d(A_5,A_2)$	0.021	-0.197	-0.211	-0.267	-0.353
$d(A_3,A_1)$	0.283	0.180	0.248	0.223	0.157	$d(A_5,A_3)$	-0.062	-0.248	-0.194	-0.173	-0.247
$d(A_3,A_2)$	0.089	0.004	0.003	-0.136	-0.066	$d(A_5,A_4)$	-0.162	-0.104	-0.082	-0.322	0.179

Table 13 Deviations between the evaluations for DM e_3

DM e_3	c_1	c_2	c_3	c_4	c_5	DM e_3	c_1	c_2	c_3	c_4	c_5
$d(A_1,A_2)$	0.079	0.231	0.180	0.371	0.032	$d(A_3,A_4)$	0.181	0.013	0.127	− 0.162	0.269
$d(A_1,A_3)$	− 0.011	0.114	0.000	0.396	0.045	$d(A_3,A_5)$	− 0.086	− 0.181	− 0.024	− 0.297	0.000
$d(A_1,A_4)$	0.170	0.127	0.127	0.234	0.314	$d(A_4,A_1)$	− 0.170	− 0.127	− 0.127	− 0.234	− 0.314
$d(A_1,A_5)$	− 0.097	− 0.067	− 0.024	0.099	0.045	$d(A_4,A_2)$	− 0.091	0.104	0.053	0.137	− 0.282
$d(A_2,A_1)$	− 0.079	− 0.231	− 0.180	− 0.371	− 0.032	$d(A_4,A_3)$	− 0.181	− 0.013	− 0.127	0.162	− 0.269
$d(A_2,A_3)$	− 0.090	− 0.117	− 0.180	0.024	0.014	$d(A_4,A_5)$	− 0.267	− 0.194	− 0.151	− 0.135	− 0.269
$d(A_2,A_4)$	0.091	− 0.104	− 0.053	− 0.137	0.282	$d(A_5,A_1)$	0.097	0.067	0.024	− 0.099	− 0.045
$d(A_2,A_5)$	− 0.176	− 0.298	− 0.204	− 0.273	0.014	$d(A_5,A_2)$	0.176	0.298	0.204	0.273	− 0.014
$d(A_3,A_1)$	0.011	− 0.114	0.000	− 0.396	− 0.045	$d(A_5,A_3)$	0.086	0.181	0.024	0.297	0.000
$d(A_3,A_2)$	0.090	0.117	0.180	− 0.024	− 0.014	$d(A_5,A_4)$	0.267	0.194	0.151	0.135	0.269

Table 14 Preference degrees for DM e_1

DM e_1	c_1	c_2	c_3	c_4	c_5	DM e_1	c_1	c_2	c_3	c_4	c_5
$P(A_1,A_2)$	0.000	0.186	0.000	0.000	0.000	$P(A_3,A_4)$	0.128	0.000	0.146	0.045	0.000
$P(A_1,A_3)$	0.000	0.225	0.000	0.000	0.044	$P(A_3,A_5)$	0.124	0.000	0.000	0.000	0.000
$P(A_1,A_4)$	0.000	0.000	0.016	0.000	0.000	$P(A_4,A_1)$	0.153	0.082	0.000	0.173	0.144
$P(A_1,A_5)$	0.000	0.000	0.000	0.000	0.000	$P(A_4,A_2)$	0.000	0.268	0.000	0.118	0.144
$P(A_2,A_1)$	0.154	0.000	0.205	0.054	0.000	$P(A_4,A_3)$	0.000	0.307	0.000	0.000	0.188
$P(A_2,A_3)$	0.000	0.039	0.075	0.000	0.044	$P(A_4,A_5)$	0.000	0.000	0.000	0.000	0.113
$P(A_2,A_4)$	0.001	0.000	0.221	0.000	0.000	$P(A_5,A_1)$	0.158	0.136	0.258	0.449	0.031
$P(A_2,A_5)$	0.000	0.000	0.000	0.000	0.000	$P(A_5,A_2)$	0.004	0.322	0.052	0.395	0.031
$P(A_3,A_1)$	0.282	0.000	0.130	0.217	0.000	$P(A_5,A_3)$	0.000	0.361	0.128	0.232	0.075
$P(A_3,A_2)$	0.127	0.000	0.000	0.163	0.000	$P(A_5,A_4)$	0.005	0.053	0.274	0.276	0.000

Table 15 Preference degrees for DM e_2

DM e_2	c_1	c_2	c_3	c_4	c_5	DM e_2	c_1	c_2	c_3	c_4	c_5
$P(A_1,A_2)$	0.000	0.000	0.000	0.000	0.000	$P(A_3,A_4)$	0.000	0.143	0.112	0.000	0.426
$P(A_1,A_3)$	0.000	0.000	0.000	0.000	0.000	$P(A_3,A_5)$	0.062	0.248	0.194	0.173	0.247
$P(A_1,A_4)$	0.000	0.000	0.000	0.000	0.269	$P(A_4,A_1)$	0.383	0.037	0.136	0.372	0.000
$P(A_1,A_5)$	0.000	0.068	0.000	0.000	0.090	$P(A_4,A_2)$	0.148	0.000	0.015	0.004	0.000
$P(A_2,A_1)$	0.359	0.293	0.300	0.465	0.324	$P(A_4,A_3)$	0.100	0.000	0.000	0.149	0.000
$P(A_2,A_3)$	0.000	0.000	0.000	0.136	0.066	$P(A_4,A_5)$	0.162	0.104	0.082	0.322	0.000
$P(A_2,A_4)$	0.000	0.052	0.000	0.000	0.492	$P(A_5,A_1)$	0.221	0.000	0.053	0.050	0.000
$P(A_2,A_5)$	0.000	0.197	0.211	0.267	0.353	$P(A_5,A_2)$	0.021	0.000	0.000	0.000	0.000
$P(A_3,A_1)$	0.283	0.180	0.248	0.223	0.157	$P(A_5,A_3)$	0.000	0.000	0.000	0.000	0.000
$P(A_3,A_2)$	0.089	0.004	0.003	0.000	0.000	$P(A_5,A_4)$	0.000	0.000	0.000	0.000	0.179

After solving Eq. (50) by Hungarian method, the optimal solution is generated as $x_{15} = 1, x_{24} = 1, x_{33} = 1, x_{42} = 1, x_{51} = 1$. Hence, the collective ranking is obtained as $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$, and the best supplier is A_5. Since the derived DMs' weight vector in Step 2 is $w = (0.5, 0.2, 0.3)^T$, the DM e_1 has the highest importance. Thus, the collective ranking $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$ should be in accordance with the individual ranking $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$ for DM e_1. It can be seen from Table 9 that most of the criteria values of A_5 are

the biggest relative to other alternatives. Meanwhile, most of the criteria values of A_4 are larger than those of alternatives A_3, A_2 and A_1. Therefore, it is reasonable to obtain that the best supplier is A_5, and the sub-optimal supplier is A_4.

6.2 Sensitivity analysis

This subsection conducts the sensitive analysis through altering the values of α (the DMs' risk attitude parameter in

Table 16 Preference degrees for DM e_3

DM e_3	c_1	c_2	c_3	c_4	c_5	DM e_3	c_1	c_2	c_3	c_4	c_5
$P(A_1,A_2)$	0.079	0.231	0.180	0.371	0.032	$P(A_3,A_4)$	0.181	0.013	0.127	0.000	0.269
$P(A_1,A_3)$	0.000	0.114	0.000	0.396	0.045	$P(A_3,A_5)$	0.000	0.000	0.000	0.000	0.000
$P(A_1,A_4)$	0.170	0.127	0.127	0.234	0.314	$P(A_4,A_1)$	0.000	0.000	0.000	0.000	0.000
$P(A_1,A_5)$	0.000	0.000	0.000	0.099	0.045	$P(A_4,A_2)$	0.000	0.104	0.053	0.137	0.000
$P(A_2,A_1)$	0.000	0.000	0.000	0.000	0.000	$P(A_4,A_3)$	0.000	0.000	0.000	0.162	0.000
$P(A_2,A_3)$	0.000	0.000	0.000	0.024	0.014	$P(A_4,A_5)$	0.000	0.000	0.000	0.000	0.000
$P(A_2,A_4)$	0.091	0.000	0.000	0.000	0.282	$P(A_5,A_1)$	0.097	0.067	0.024	0.000	0.000
$P(A_2,A_5)$	0.000	0.000	0.000	0.000	0.014	$P(A_5,A_2)$	0.176	0.298	0.204	0.273	0.000
$P(A_3,A_1)$	0.011	0.000	0.000	0.000	0.000	$P(A_5,A_3)$	0.086	0.181	0.024	0.297	0.000
$P(A_3,A_2)$	0.090	0.117	0.180	0.000	0.000	$P(A_5,A_4)$	0.267	0.194	0.151	0.135	0.269

Table 17 Total preference degrees for different DMs

	DM e_1	DM e_2	DM e_3		DM e_1	DM e_2	DM e_3
$\pi(A_1,A_2)$	0.036	0.000	0.180	$\pi(A_3,A_4)$	0.064	0.134	0.097
$\pi(A_1,A_3)$	0.053	0.000	0.126	$\pi(A_3,A_5)$	0.024	0.185	0.015
$\pi(A_1,A_4)$	0.003	0.051	0.175	$\pi(A_4,A_1)$	0.110	0.184	0.015
$\pi(A_1,A_5)$	0.000	0.031	0.044	$\pi(A_4,A_2)$	0.106	0.033	0.075
$\pi(A_2,A_1)$	0.083	0.347	0.015	$\pi(A_4,A_3)$	0.099	0.049	0.048
$\pi(A_2,A_3)$	0.032	0.039	0.023	$\pi(A_4,A_5)$	0.023	0.134	0.015
$\pi(A_2,A_4)$	0.045	0.104	0.070	$\pi(A_5,A_1)$	0.205	0.064	0.034
$\pi(A_2,A_5)$	0.000	0.205	0.018	$\pi(A_5,A_2)$	0.158	0.004	0.173
$\pi(A_3,A_1)$	0.125	0.218	0.015	$\pi(A_5,A_3)$	0.158	0.000	0.116
$\pi(A_3,A_2)$	0.057	0.019	0.077	$\pi(A_5,A_4)$	0.122	0.034	0.166

Table 18 Outgoing flows of alternatives for different DMs

DM	$\Phi^+(A_1)$	$\Phi^+(A_2)$	$\Phi^+(A_3)$	$\Phi^+(A_4)$	$\Phi^+(A_5)$
e_1	0.093	0.160	0.270	0.338	0.642
e_2	0.082	0.695	0.557	0.400	0.102
e_3	0.525	0.127	0.205	0.153	0.488

Table 19 Incoming flows of alternatives for different DMs

DM	$\Phi^-(A_1)$	$\Phi^-(A_2)$	$\Phi^-(A_3)$	$\Phi^-(A_4)$	$\Phi^-(A_5)$
e_1	0.522	0.357	0.342	0.234	0.048
e_2	0.813	0.056	0.088	0.322	0.556
e_3	0.080	0.505	0.312	0.508	0.093

Table 20 Net flows of alternatives for different DMs

DM	$\Phi(A_1)$	$\Phi(A_2)$	$\Phi(A_3)$	$\Phi(A_4)$	$\Phi(A_5)$
e_1	− 0.430	− 0.197	− 0.072	0.104	0.595
e_2	− 0.731	0.639	0.468	0.078	− 0.454
e_3	0.445	− 0.378	− 0.107	− 0.355	0.395

Eq. (22)). Table 21 lists the corresponding computation results for different values of parameter α.

It is easily seen that the ranking order of manufacturing firms is not sensitive to the value of α as far as this example is concerned. In other words, despite the alteration in the DMs' risk attitude α, the ranking order of suppliers remains unchanged.

Table 21 Ranking orders of manufacturing firms for different values of parameter α

α	Ranking order of alternatives	α	Ranking order of alternatives
0	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$	0.6	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$
0.1	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$	0.7	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$
0.2	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$	0.8	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$
0.3	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$	0.9	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$
0.4	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$	1	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$
0.5	$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$		

6.3 Comparative analyses

To highlight the superiority of the proposed method of this paper, we make comparisons with the hesitant fuzzy power operator aggregated method (Zhang 2013), hesitant fuzzy TOPSIS method (Xu and Zhang 2013) and hesitant fuzzy QUALIFLEX method (Zhang and Xu 2015).

1. Assume that DMs' weight vector is $\omega = (0.4, 0.3, 0.3)^{\mathrm{T}}$, the criteria weight vector is adjusted as $w = (0.3, 0.2, 0.2, 0.2, 0.1)^{\mathrm{T}}$ and DMs are pessimistic. Using method (Zhang 2013) to solve the above green supplier selection example, the ranking order of suppliers is generated as $A_5 \succ A_2 \succ A_3 \succ A_4 \succ A_1$.
2. Method (Xu and Zhang 2013) presented a hesitant fuzzy TOPSIS method and method (Zhang and Xu 2015) developed a hesitant fuzzy QUALIFLEX method. Both methods only considered only one DM. Without loss of generality, only the evaluation information of DM e_3 is used to perform the comparative analyses. Employing methods (Zhang and Xu 2015; Xu and Zhang 2013), the ranking order of suppliers is generated as $A_5 \succ A_1 \succ A_3 \succ A_2 \succ A_4$ and $A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$, respectively.

The ranking orders obtained by the above three methods are significantly different from that obtained by the proposed method of this paper. Compared with these methods (Zhang and Xu 2015; Xu and Zhang 2013; Zhang 2013), the proposed method has some desirable merits as follows:

1. Method (Zhang 2013) gave DMs' weights a priori and failed to consider the determination of DMs' weights, while this paper determines the DMs' weights objectively by a linear program. On the other hand, methods (Zhang and Xu 2015; Xu and Zhang 2013; Zhang 2013) assumed that the criteria weights are the same for diverse DMs. Methods (Zhang and Xu 2015; Zhang 2013) gave the criteria weights a priori. However, this paper derives the criteria weights for each DM objectively by a nonlinear program and the criteria weights have different values for different DMs, which is more consistent with real-life decision situation.

2. Method (Zhang and Xu 2015) added the minimum value to extend the length of HFE. The distance measure defined in this paper does not need to extend the length, which can well preserve the original decision information and effectively reduce the uncertainty in decision-making. In addition, the proposed distance focuses on the intersectional information in two HFEs and considers the hesitancy degrees of HFEs.

3. The proposed method is powerful in ranking alternatives with those incomparable criteria, such as the criteria "Quality" and "Price" in green supplier selection example. Zhang (Zhang 2013) aggregated different criteria values into overall values, and Xu and Zhang (2013) directly aggregated the distances to the ideal solution with weighted summarization, which is a little bit unreasonable. Relative to the hesitant fuzzy QUALIFLEX method (Zhang and Xu 2015), the proposed PROMETHEE method is easy to compute and is more time-saving. Method (Zhang and Xu 2015) must consider all possible permutations of alternatives, and the computation complexity will increase dramatically with the increase of number alternatives.

7 Conclusions

To equilibrate the economic profits and the environment sustainable development, more and more companies performed the GSCM on their organizational and technological projects. In this paper, the green supplier selection is formulated as a kind of MCGDM problems with HFSs. Therefore, this paper developed a hesitant fuzzy PROMETHEE method for MCGDM and applied to green supplier selection.

1. Some new information measures of HFEs are proposed. First, a hesitancy index is proposed to characterize the hesitant degree of HFE. Considering such a hesitancy index, a generalized hesitant fuzzy Hausdorff distance is defined. Then, a new combined hesitant fuzzy entropy is presented to depict the hesitancy and fuzziness of HFE. Relative closeness degree of HFE is also introduced.

2. Two optimization models are built to obtain the criteria weights and the DMs' weights objectively. The DMs' weights are obtained by a linear programming model of minimizing the group inconsistency. The criteria weights for each DM are derived by a nonlinear optimization model of minimizing the relative entropy.

3. The individual ranking of alternatives for each DM is derived by the net flow, and the corresponding individual ranking matrix is then generated. A multi-objective assignment model is set up to obtain the collective ranking order of alternatives. Thereby, an extended PROMETHEE method is developed for MCGDM with HFSs and applied to green supplier selection.

This paper provides a novel perspective to handle green supplier selection problems. Motivated by hesitant linguistic information initiated by Dong et al. (2015, 2016), Wu et al. (2018), we will propose some new information measures for hesitant linguistic information and apply them to the practical MCGDM problems. Furthermore, references (Dong et al. 2018), Liu et al. (2018, 2019) pointed out that DMs are often dishonest in MADM and MCGDM. Hence, how to involve the strategic weight manipulation in hesitant fuzzy decision-making is also an interesting issue, which deserves to be further studied. In addition, this paper does not consider the heterogeneous information (Li et al. 2016), classification and clustering algorithms (Kou et al. 2012, 2014), preference relations (Zhang et al. 2019; Kou et al. 2014, 2016; Kou and Lin 2014) in group decision-making. They are very critical issues that we will investigate them in future.

Compliance with ethical standards

Conflict of interest Authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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