Soft Computing (2020) 24:17–40
https://doi.org/10.1007/s00500-019-04360-1

FOCUS

Dynamic adaptation of the PID's gains via Interval type-1 non-singleton type-2 fuzzy logic systems whose parameters are adapted using the backpropagation learning algorithm

Gerardo M. Méndez[1] · P. Noradino Montes Dorantes[2] · M. Aracelia Alcorta[3]

Published online: 3 October 2019

Abstract

This work presents a new design to dynamically adapt the proportional, the integral and the derivative (PID) controller's gains using three interval type-1 non-singleton type-2 fuzzy logic systems (IT2 NSFLS-1), one fuzzy system for each gain of the PID, being the first main contribution of this proposal. This assembly is named as hybrid IT2 NSFLS-1 PID. Each IT2 NSFLS-1 system requires two non-singleton input values each period of discrete time $(\boldsymbol{k})$, (1) the error $\boldsymbol{e}(\boldsymbol{k})$ and its standard deviation $\boldsymbol{\sigma e}(\boldsymbol{k})$, and (2) the change of error $\Delta\boldsymbol{e}(\boldsymbol{k})$ and its standard deviation $\boldsymbol{\sigma}\Delta\boldsymbol{e}(\boldsymbol{k})$, to calculate the corresponding adjustment $\Delta\boldsymbol{KP}(\boldsymbol{k})$, $\Delta\boldsymbol{KI}(\boldsymbol{k})$, and $\Delta\boldsymbol{KD}(\boldsymbol{k})$ for the PID controller's gains $\boldsymbol{K_p}(\boldsymbol{k})$, $\boldsymbol{K_i}(\boldsymbol{k})$, and $\boldsymbol{K_d}(\boldsymbol{k})$. The second main contribution of this proposal is that the parameters of each IT2 NSFLS-1 system are tuned each period of discrete time $(\boldsymbol{k})$ by the non-singleton backpropagation (BP) algorithm using the plant output error and its standard deviation, which are processed as non-singleton values together with its non-singleton partial derivatives with respect to each IT2 fuzzy system parameter. Then these updated gains are used by the PID controller to calculate the best control signal for the plant under control. The uncertainty and the mean value of the measurement are used to calculate the non-singleton error which is processed as (a) input and (b) as gradient vector by each of the three IT2 NSFLS-1 systems. Simulation results show that the proposed hybrid assembly presents the better performance than the next five benchmarking control systems (a) the classic Zeigler–Nichols PID controller, and (b) four hybrid assemblies using PID controller and fuzzy systems with fixed fuzzy rule bases (T1 SFLS, T1 NSFLS, IT2 SFLS, IT2 NSFLS-1). The proposed assembly produces the better performance in a shortest period of time and it maintains a stable behavior on the output of the second-order plant model subject to variations and noise.

Keywords IT2 fuzzy logic systems · PID control algorithm · Singleton numbers · Type-1 non-singleton numbers · PID IT2 fuzzy self-tuning · IT2 NSFLS-1

Communicated by O. Castillo, D. K. Jana.

✉ P. Noradino Montes Dorantes
pascualresearch@gmail.com; cualmontes@hotmail.com

Gerardo M. Méndez
gmm_paper@yahoo.com

M. Aracelia Alcorta
maria.alcortagr@uanl.edu.mx

[1] Instituto Tecnológico de Nuevo León, Av. Eloy Cavazos 2001, Guadalupe, Nuevo León, Mexico

[2] División de Estudios de Posgrado e Investigación, Universidad Autónoma del Noreste (UANE), Blvd. José Musa de León y General Medardo de la Peña S/N Col. Los Pinos, CP 25100 Saltillo, Coahuila, Mexico

[3] Physical and Mathematics School, Universidad Autónoma de Nuevo León (UANL), Ave. Universidad s/n, CP 66455 San Nicolás de los Garza, Nuevo León, Mexico

1 Introduction

Nowadays, the large amount of uncertainty that is inherent in most of industrial processes is a major concern in the process control systems. For this reason, the importance of the fuzzy logic field has been increased with many works involving hybrid systems between fuzzy logic, PID and complex algorithms, such as bee colony (Kumar and Kumar 2017b), big-bang big-crunch optimization (Yesil 2014), gradient descend methods (Sakalli et al. 2016), cuckoo search (Fatihu Hamza et al. 2017) and granular computing (Castillo et al. 2016b). There is a big amount of work being done to implement fuzzy logic controllers (FLC) that are replacing the PID controllers in both types. On type-1 singleton (Jie et al. 2010; Kumbasar and Hagras 2015; Álvarez et al. 2018; Arghavani et al. 2017; Karasakal et al. 2013; Sakalli et al. 2014a, b; Yesil and Guzay 2014; Savran et al. 2015; Var et al. 2015; Souran et al. 2014; Kumar and Kumar 2017a; Kosari et al. 2017; Reyes et al. 2016; Kumbasar and Hagras 2017; Nayak et al. 2018; Kudinov et al. 2017; Mohanty et al. 2016; Kumar et al. 2018). On type-1 non-singleton (Ramos et al. 2016; Reyes et al. 2018) and type-2 singleton (Kumbasar 2014a, b, 2016; Kumbasar et al. 2013; Sakalli et al. 2014a, b; Kumbasar and Hagras 2015; Olivas et al. 2019; Aliasghary et al. 2012; Mehndiratta et al. 2016; Castro et al. 2008; Sahin and Kumbasar 2018; Kumar and Kumar 2017a, b, c, d; Khosla et al. 2014; El-Bardini and El-Nagar 2014a, b; Beirami and Zerafat 2015; Yesil 2014; Fatihu Hamza et al. 2017; Sanchez et al. 2015a, b; Ontiveros-Robles et al. 2018; Cervantes and Castillo, 2015; Fatihu Hamza et al. 2017; Yesil et al. 2014). However, the usage of the hybrid PID assembled with fuzzy logic systems in order to calculate and update the PID gains is gaining places on industrial control applications, where the PID performance is substantially enhanced with the uncertainty processing of the fuzzy logic systems. These hybrid controllers can be considered as an emerging class of intelligent hybrid controllers, see Table 1. The uncertainties of the measurement processes in the industrial facilities are the motivation to use the type-2 fuzzy models. Their nature is capable to manage and process the uncertainties present in the measurements and provide accurate results. These facts motivate the use of the type-2 fuzzy models to tuning and actualize the parameters of the PID controller.

The literature analysis shows that the hybrid model based on non-singleton type-1 fuzzy logic systems (T1 NSFLS) using non-singleton numbers practically was not used to model PID controllers. Only two papers have shown their use and represented 3.6% of the publications found in the literature. In Ramos et al. (2016), type-1 non-singleton fuzzy logic system (T1 NSFLS) is applied on an Atmega 2560 for controlling a stepper motor. The uncertainties presented in the processes are not treated because of the usage of the T1 SFLS models in the 40% of the proposals studied; only the 3.6% use the T1 NSFLS. The uncertainties are "treated" in some manner by the usage of the interval singleton type-2 fuzzy models (IT2 SFLS). In Reyes et al. (2018), we find the hybrid use of T1 NSFLS to update the PID gains in a second-order plant in a faster way when it is compared to a type-1 singleton models.

In the control field, a lot of work is found in the literature. Ontiveros-Robles et al. (2018) study and compare the results of the robustness of the type-2 models against type-1. Castillo et al. (2016a) present a comparison between type-1, type-2 and General type-2 fuzzy models (GT2) to show the efficiency and the performance of different models in a controller. Castillo et al. (2016a, b) add the granularity to the GT2 model to divide the controller in several phases using alpha planes to the implementation. Also, Sanchez et al. (2015a, b) use the advantage of the type-2 fuzzy sets to add the uncertainties and process them in granules that later are optimized by a metaheuristic as Cuckoo search. Zarandi et al. (2019) use the GT2 in design and diagnosis using z-slices and other methods to optimize and model the system in several phases. Sanchez et al. (2015a, b) use GT2 to demonstrate that this model is capable to outperform the type-1 and type-2 fuzzy models over external perturbations with the handle of uncertainties presented. Castro et al. (2008) present a tool to help the modeling of IT2 FLS that simplifies the generation of the rules and the inference process. Cervantes and Castillo (2015) present a multiple IT2 fuzzy controller to adjust the outputs and the global results of a controller.

The usage of fuzzy hybrid PID controllers is extensive, e.g., a control model of an inverted pendulum applies an interval singleton type-2 fuzzy logic system (IT2 SFLS) controller that stabilizes the pendulum position (Khosla et al. 2014). A hybrid PID controller that uses an IT2 SFLS is analyzed on El-Bardini and El-Nagar (2014a) where the fuzzy system directly calculates the gains of the PID. It uses triangular fuzzy sets in the input-output values, and a method is proposed to reduce the computational complexity of the outputs. In El-Bardini and El-Nagar (2014b), it is proposed an interval type-2 fuzzy proportional integral derivative controller for controlling an inverted pendulum with an uncertain model called simplified type-reduction method, where the controller handles uncertainties due to the structure of the IT2 SFLS system. A fuzzy self-tuning PID is proposed in Jie et al. (2010), where the error and change of error are the inputs to a fuzzy logic system to obtain the correction to the gains of PID controller. The BP algorithm is used for dynamic calculating and updating the gains of a PID controller (Reyes et al. 2016), and the

Table 1 Hybrid PID controllers using T1 and IT2 fuzzy systems

Topic	System					
	T1 SFLS	T1 NSFLS	IT2 SFLS	GT2 FLS	IT2 NSFLS-1	IT2 NSFLS-1
	Singleton	Non-singleton	Singleton		Type-1 Non-singleton	Type-1 Non-singleton BP tuning
Tuning and updating	Jie et al. (2010), Kumbasar and Hagras (2015), Álvarez et al. (2018), Meza et al. (2012), Trautzsch and Dawson (2002), Arghavani et al. (2017), Tang et al. (2001), Karasakal et al. (2013), Sakalli et al. (2014a, b), Yesil and Guzay (2014), Savran et al. (2015)	Ramos et al. (2016)	Kumbasar (2014b), Kumbasar (2016), Kumbasar et al. (2013), Sakali et al. (2014a, b), Kumbasar and Hagras (2015), Olivas et al. (2019)			
Design	Var et al. (2015)		Kumbasar (2016), Aliasghary et al. (2012), Mehndiratta et al. (2016), Sakalli et al. (2014a, b), Castro et al. (2008)	Castillo et al. (2016a, b), Zarandi et al. (2019)		
Parameter estimation	Var et al. (2015), Souran et al. (2014), Kumar and Kumar (2017a)		Kumbasar (2014a), Sahin and Kumbasar (2018), Kumar and Kumar (2017b), Castro et al. (2008)	Zarandi et al. (2019)		
Calculate	Souran et al. (2014)					
Maneuver	Kosari et al. (2017)					
Control and system stabilization	Reyes et al. (2016), Kumbasar and Hagras (2017), Trautzsch and Dawson (2002), Deepak and Cheolkeun (2013), Nayak et al. (2018)	Ramos et al. (2016); Reyes et al. (2018)	Khosla et al. (2014), El-Bardini and El-Nagar (2014a), Kumbasar (2014a), Beirami and Zerafat (2015)			
System Optimization	Kudinov et al. (2017), Mohanty et al. (2016).		Kumar and Kumar (2017a), Yesil (2014), Fatihu Hamza et al. (2017), Kumar and Kumar (2017c), Sanchez et al. (2015a, b)	Zarandi et al. (2019)		
Compare	Meza et al. (2012), Souran et al. (2014) and Kumar et al. (2018)		Ontiveros-Robles et al. (2018)	Sanchez et al. (2015a, b)		
Analyze			El-Bardini and El-Nagar (2014b), Ontiveros-Robles et al. (2018)			
Adjustment	Alberto (2000)		Khosla et al. (2014), Cervantes and Castillo (2015)			
Robot manipulation	Meza et al. (2012)		Fatihu Hamza et al. 2017, Alberto (2000) and Kumar and Kumar (2017d)			
Improve			Yesil et al. (2014)			

performance is compared with that of an IT2 SFLS and with professional PID controllers obtaining the better response of these. In Reyes et al. (2018), the PID gains are updated by a T1 NSFLS and it is compared with a T1 SFLS controller obtaining better responses and lesser computational time. Also, the T1 SFLS and the IT-2 SFLS are used in partial form for tuning and updating the gains in a top control and system stabilization in Kumbasar and Hagras

(2017), for tuning and updating the PI gains. In Kumbasar (2014a), the interval type-2 is used to control and stabilize the PD gains.

The literature shows work related to PID controllers and IT2 FLS systems using only singleton numbers. Kumbasar (2014b) uses the IT2 SFLS to design and tuning the parameters of a PID controller. In Kumbasar (2016), it uses the IT2 to online tuning the PID controller using T1 singleton systems and later scale to IT2 SFLS model. Kumbasar et al. (2013) use the IT2 SFLS systems to adjust the fuzzy rule base with the elimination or acceptation criteria of activate or inactive rule. Sakalli et al. (2016) use the IT2 SFLS combined with a gradient descend method and Kalman filters to improve the performance of the fuzzy PID controller. Kumbasar and Hagras (2015) use the IT2 SFLS to generate the universe of disclosure and reduce them into z-slices to obtain a self-tuning technique via IT2 model. Kumbasar (2016) presents a self-tuning mechanism based on type-1 models that extend onto type-2 model to study their structure based on a 3×3 rule base in a singleton type-1 and type 2 models. Aliasghary et al. (2012) proposes a methodology to assemble interval singleton type-2 fuzzy model in a partial PID controller using only proportional and integral part of the PID controller. Mehndiratta et al. (2016) validate the construction and use of the footprint of uncertainty on a helicopter to tuning PID controller using type-2 PID fuzzy model. Sakalli et al. (2014a, b) present the development of IT2 PID controller with a four rule base that could be visualized by metaheuristics and later is analyzed around the steady state and demonstrate the analogy between conventional and fuzzy PID controllers. Kumbasar (2014a) studied the robustness and stability of the PD singleton IT2 controller but missing the integral part. Sahin and Kumbasar (2018) use a generic type-2 fuzzy logic controller to avoid the uncertainties present in a computer games. These uncertainties are defined as obstacles in the game board. Kumar and Kumar (2017b) present a singleton IT2 TSK fractional-order PID controller to study and measure the time response and the load of the disturbance via bee colony and genetic algorithms to optimize the parameters. Khosla et al. (2014) use the IT2 to develop and adjust the parameters to maintain the stability position of the inverted pendulum. El-Bardini and El-Nagar (2014a) develop a controller model based on IT2 to show the robustness of the proposal against the classic PID controllers. Beirami and Zerafat (2015) use the IT2 to model the time delay and uncertainties present in a reactor tank via an adaptive adjustment of the parameters of a PID controller. Kumar and Kumar (2017a) implement the PID controller with IT2 fuzzy to avoid the nonlinearities, the uncertainties and external disturbances of the process of a manipulator robot optimized by metaheuristics as bee colony and genetic algorithms. Yesil (2014) uses the IT2 to optimize the load frequency problem (LFP) and to avoid the complexities of the LFP using big-bang big-crunch optimization to scaling the factors and to minimize the deviations via the IT2 model. Fatihu Hamza et al. (2017) use the cuckoo search algorithm to optimize the gains of the PID model due to the uncertainties. Kumar and Kumar (2017c) argument the use of IT2 due the uncertainties, nonlinearity's, perturbations and random noise of the signals of robotic systems and to deal with these characteristics use the genetic algorithms to optimize and scaling the IT2 PD + I controller; El-Bardini and El-Nagar (2014b) applied the IT2 model in a PID control to an inverted pendulum positioned in a cart; Alberto (2000) uses the IT2 to adapt the control of a robot manipulator; Kumar and Kumar (2017d) use the artificial bee colony algorithm to optimize the antecedents of the fuzzy rules of an IT2 model of a PID controller. Yesil et al. (2014) uses the IT2 model to improve the system performance and to provide a self-tuning mechanism to scale the PID controller in the transient phase. Cervantes and Castillo (2015) use the IT2 model to provide a global result in a multiphase model and a hierarchical architecture to adjust the result of the model in an airplane flight control.

Also was found a little work on general type-2 fuzzy systems: Kumbasar and Hagras (2015) use the IT2 in the training phase to adjust, tuning and updating the gains but only in proportional and integral gains in a type-2 fuzzy singleton model. Castillo et al. (2016a) use the type-2 to compare their results against the GT2 model and show their advantages and enhancements in their performance. Castillo et al. (2016a, b) design a tool to help the researchers in the modeling phase in particular to generate the fuzzy rules. Cervantes and Castillo (2015) use the IT2 to adjust the outputs of a controller via multiple IT2 systems to provide a global result of the mentioned controller. Zarandi et al. (2019) approach the type-2 model to provide an interval capable to diagnosis that is adjusted for optimization and it is done in posterior phase. Sanchez et al. (2015a, b) approach the capabilities of the type-2 models to add the uncertainties present in the data to the model and process them. Ontiveros-Robles et al. (2018) use the IT2 fuzzy to compare the robustness of the model against type-1 fuzzy. Sanchez et al. (2015a, b) approach the interval type-2 to generate granules of value to that later are optimize by metaheuristic models Olivas et al. (2019) propose a type-2 fuzzy model with dynamic adaptation in metaheuristic models, among others).

There are no publications in the literature on using a non-singleton BP method as a learning mechanism for antecedent's and consequent's parameter tuning of each one of the three IT2 NSFLS-1 systems, which are used to calculate as interval sets the corresponding adjustments for the three gains of the PID controller, the $\boldsymbol{K_p}(\boldsymbol{k})$, $\boldsymbol{K_i}(\boldsymbol{k})$, and

$\boldsymbol{K_d(k)}$, being the first main contribution of this paper. The usage of type-1 non-singleton numbers represents the chance to manage, as the mean of an interval set, the numerical values of each gain of the PID controller and represents an additional contribution.

The non-singleton BP training algorithm uses the plant output error measurement, which is processed as non-singleton number, using its mean and its standard deviation values. Each IT2 NSFLS-1 uses the non-singleton error and its non-singleton partial to update its Gaussians non-singleton parameter's values. This approach is not found in the literature and constitutes the second main contribution of this proposal.

This proposal also presents the math formulation of the non-singleton BP method that uses the non-singleton partial derivatives of the non-singleton error of the output of the second-order plant with respect to each parameter of the IT2 NSFLS-1 fuzzy system.

There is one IT2 NSFLS-1 system to calculate as the mean of an interval value the adjustment $\boldsymbol{\Delta KP(k)}$ for the $\boldsymbol{K_p(k)}$ gain; one IT2 NSFLS-1 system to calculate as the mean of an interval value the adjustment $\boldsymbol{\Delta KI(k)}$ for the $\boldsymbol{K_i(k)}$ gain; and one IT2 NSFLS-1 system to calculate the mean of an interval value the adjustment $\boldsymbol{\Delta KD(k)}$ for the $\boldsymbol{K_d(k)}$ gain.

Each IT2 NSFLS-1 uses two input variables, granulated as non-singleton values: (1) the error $e(k)$ and its standard deviation $\sigma e(k)$, and (2) the change of error $\Delta e(k)$ and its standard deviation $\sigma\Delta e(k)$. The error is calculated as the difference between two non-singleton Gaussians numbers, the set point value, and the value measured by the sensor located at the output section of the plant.

During the experimental modeling process, the antecedent's and consequent's parameters of the rule base remain fixed on each of the next four different benchmarking assemblies composed by a PID controller and by a specific fuzzy system: (1) a PID controller, which gains adjustments are calculated by a T1 SFLS, which inputs are singleton or crisp values; (2) a PID controller, which gains adjustments are calculated by a T1 NSFLS, which inputs are non-singleton values; (3) a PID controller, which gains adjustments are calculated by an IT2 SFLS, which inputs are singleton or crisp values; and (4) a PID controller, which gains adjustments are calculated by an IT2 NSFLS-1, which inputs are non-singleton values.

The classic PID Ziegler–Nichols' (PID ZN), which gains remain fixed and its inputs are singletons or crisp values is also used as a benchmarking control system.

Table 1 contains a resume of the references found in the literature and is arranged according to the application of the proposal, and by the fuzzy model used to calculate the adjustment for each PID gains.

This paper is organized as follows. Section 2 describes the problem statement of the systems modeled as second-order process, the PID controller formulations, along with a basic explanation of the T1 and IT2 FLS systems. The proposed method is described in Sect. 3. Section 3.4 shows the simulation, and the results are presented in Sect. 5. Finally the conclusions are presented in Sect. 4.

2 Problem statement

2.1 The second-order model of the plant

A second-order transfer function model was used to represent the plant under control. This model is considered without time delay. Figure 1 shows that

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{(T_1 s + 1)(T_2 s + 1)} \tag{1}$$

where $\boldsymbol{G(s)}$ represents the transfer function of the plant, $\boldsymbol{K}$ represents the gain of the process, $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ are the time constants of the process, and $\boldsymbol{s}$ is the complex variable. Its equivalent difference equation in discrete period of time (Gaidhane et al. 2019) can be expressed as:

$$y(k) = K_2 u(k) - a_1 y(k-1) - a_2 y(k-2) \tag{2}$$

where

$$\alpha_1 = e^{-T_s/T_1} \tag{3}$$

$$\alpha_2 = e^{-T_s/T_2} \tag{4}$$

$$a_1 = -(\alpha_1 + \alpha_2) \tag{5}$$

$$a_2 = \alpha_1 * \alpha_2 \tag{6}$$

$$K_2 = K * T_s \frac{\alpha_1 - \alpha_2}{T_1 - T_2} \tag{7}$$

where $\boldsymbol{T_s} = \frac{T}{20}$ is the sampling time or the measurement period, and $\boldsymbol{T}$ is the minimum value of the time constants of the plant, min $[\boldsymbol{T}_1, \boldsymbol{T}_2]$, $\boldsymbol{y(k)}$ is the output value of the plant at the discrete time $\boldsymbol{k}$, and $\boldsymbol{u(k)}$ is the PID control value, which is the input of the plant.

2.2 PID controller

The PID controller has several versions, which are widely used in the industry (Gaidhane et al. 2019). Due to its simplicity and robustness to execute rational decisions, which take into account the present, the past and the desired states of the plant under control, the PID is the industry's preferred controller (proportional, integral and derivative controllers). The positional version of the PID's discrete algorithm is used in this proposal:

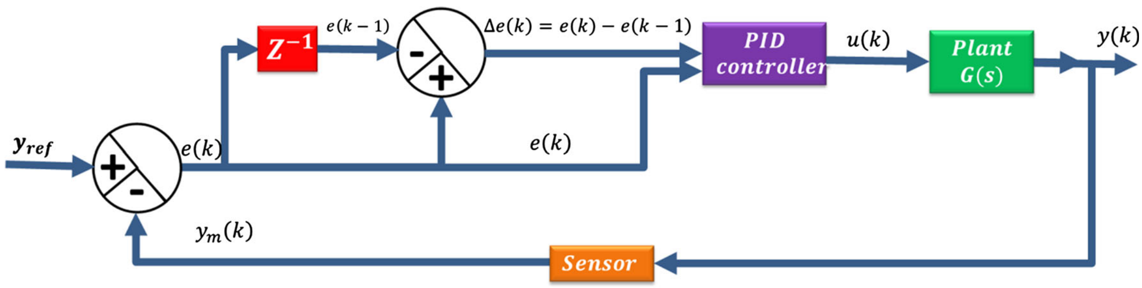

Fig. 1 Block diagram of the classic PID controller

$$u(k) = K_p\left[e(k) + \frac{T_c}{T_i}\sum_{i=0}^{k} e(i) + \frac{T_d}{T_c}[e(k) - e(k-1)]\right] \quad (8)$$

$$u(k) = K_p e(k) + K_i \sum_{i=0}^{k} e(i) + K_d[e(k) - e(k-1)] \quad (9)$$

where $\boldsymbol{u}(\boldsymbol{k})$ is the control variable, $\boldsymbol{e}(\boldsymbol{k})$ is the error of the output in the discrete time k, $\boldsymbol{e}(\boldsymbol{k}-1)$ represents the error in the discrete time $\boldsymbol{k}-1$, $\boldsymbol{K_p}$ is the proportional gain, $\boldsymbol{Ti}$ represents the integral time, $\boldsymbol{T_d}$ represents the derivative time, and $\boldsymbol{T_c}$ is the control interval time. According to (Gaidhane et al. 2019) in order to avoid oscillations in the behavior of the PID controllers, the control period should be selected as:

$$\boldsymbol{T_c} \geq 5.0 * \boldsymbol{T_m} \quad (10)$$

where $\boldsymbol{T_m} = \boldsymbol{T_s}$ is the measurement interval time.

Using

$$\boldsymbol{e}(\boldsymbol{k}) = \boldsymbol{y_{ref}} - \boldsymbol{y_m}(\boldsymbol{k}) \quad (11)$$

where $\boldsymbol{y_m}$ is the feedback, the measurement of the output of the plant, and $\boldsymbol{y_{ref}}$ is the set point value, the PID controller (2) can be expressed as:

$$u(k) = \left(K_p + K_i * k\right)y_{ref} - \left(K_p + K_i + K_d\right)y(k-1) - (K_i + K_d)y(k-2) - (K_i)\sum_{i=0}^{k-3} y(i), \quad (12)$$

where

$$\boldsymbol{K_i} = \boldsymbol{K_p}\frac{\boldsymbol{T_c}}{\boldsymbol{T_i}} \quad (13)$$

$$\boldsymbol{K_d} = \boldsymbol{K_p}\frac{\boldsymbol{T_d}}{\boldsymbol{T_c}} \quad (14)$$

$\boldsymbol{y}(\boldsymbol{k})$, $\boldsymbol{y}(\boldsymbol{k}-1)$, and $\boldsymbol{y}(\boldsymbol{k}-2)$ are the output of the plant in the discrete time $\boldsymbol{k}$, $\boldsymbol{k}-1$, and $\boldsymbol{k}-2$.

2.3 T1 FLS systems (Mendel 2001)

A type-1 (T1) fuzzy set, A, is a generalization of a crisp set $\boldsymbol{x} \in \boldsymbol{X}$. It is defined on a universe of discourse $\boldsymbol{X}$, and it means the space or interval where $\boldsymbol{X}$ could get a value in a specification and it is characterized by the membership function $\boldsymbol{\mu_A}(\boldsymbol{x})$ that takes on values in the interval $[0, 1]$. A membership function provides a measure of the degree of membership of an element in X to the fuzzy set, which is really the membership of x in A. Such a set may be represented as:

$$A = \{(x, \mu_A(x))/\forall x \in X\} \quad (15)$$

T1 membership function $\mu_A(x)$ is constrained to be between 0 and 1 for all $x \in X$, and is a 2D Gaussian function. The T1 fuzzy rule base consists of a set of IF–THEN rules that represents the model of the system. The type-1 FLS has p inputs, $x_1 \in X$, $x_2 \in X$,…, $x_p \in X$, and one output $y \in Y$, which have a rule base size of M rules of the form:

$$R^i : IF x_1 \text{ is } F_1^i \text{ and } x_2 \text{ is } F_2^i \ldots \text{and } x_p \text{ is } F_p^i \; THEN \; y^i \text{ is } G^i \quad (16)$$

where i = 1,2,…M, F_1^i is the antecedent fuzzy set of the x_1, F_2^i is the antecedent fuzzy set of x_2, …, F_p^i is the antecedent fuzzy set of x_p, G^i is the consequent fuzzy set of the output y^i, and $F_p^i = \mu_{A_p}^i(x)$. These rules represent a fuzzy relation between the input space $X_1 \times X_2 \times \ldots \times X_p$ and the output space Y, and it is complete, consistent and continuous (Wang 1999). See Fig. 2 for singleton model and Fig. 3 for non-singleton model.

The membership function for each antecedent is a Gaussian function as:

$$\mu_{Ak}^i(x_q) = F_q^i = \exp\left[-\frac{1}{2}\left[\frac{x_q - m_q^i}{\sigma_q^i}\right]\right] \quad (17)$$

where m_q^i is mean, σ_q^i is the standard deviation, q = 1, 2, …, p, p is the number of inputs, and i = 1,2,…M. The means of the antecedent fuzzy sets were uniformly distributed over the entire input space.

The membership function for each consequent is a Gaussian function with the form:

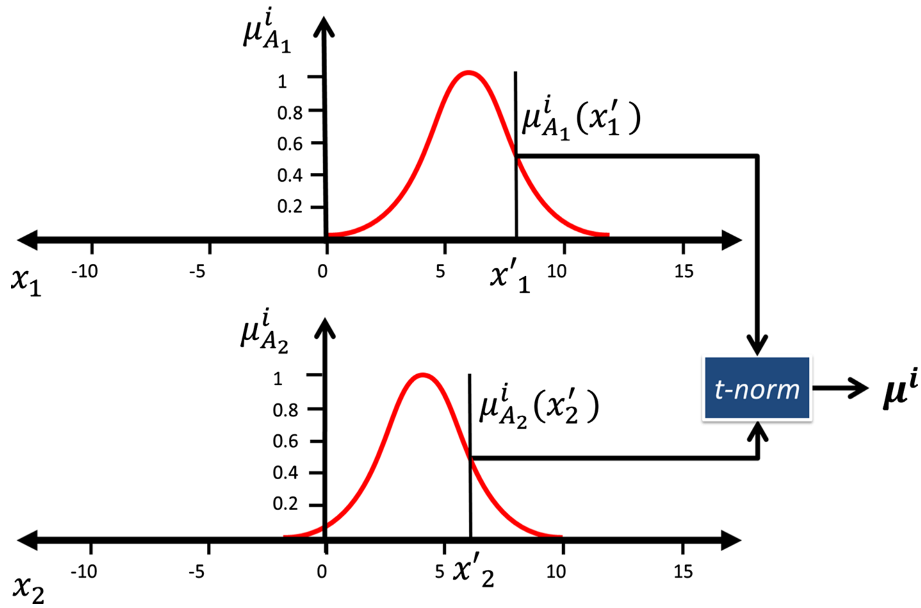

Fig. 2 Pictorial description of singleton input and antecedent operations for T1 SFLS. Adapted from Mendel (2001)

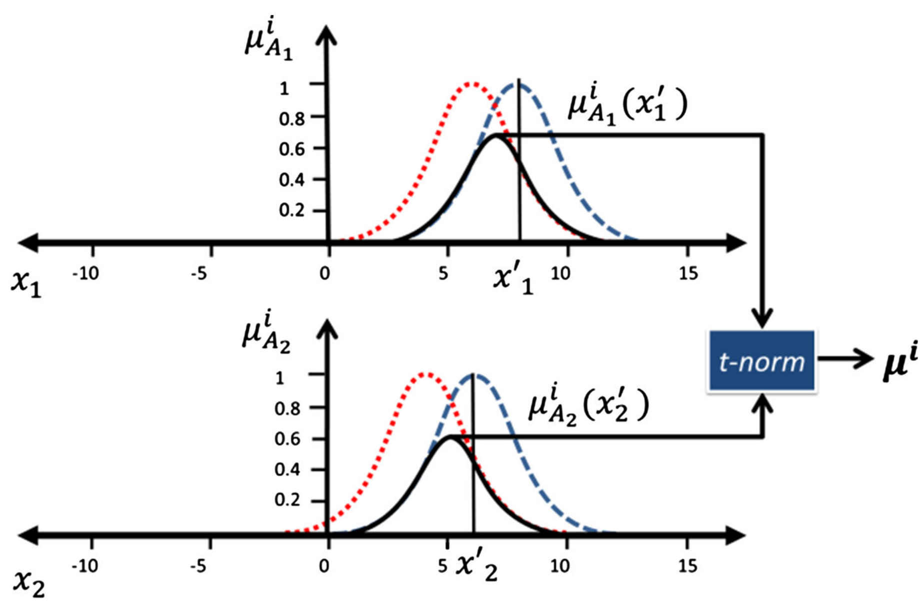

Fig. 3 Pictorial description of non-singleton input and antecedent operations for a T1 NSFLS. Adapted from Mendel (2001)

$$\mu^i_G(y^i) = G^i = \exp\left[-\frac{1}{2}\left[\frac{y^i - m^i_y}{\sigma^i_y}\right]\right] \tag{18}$$

where m^i_y is the mean, σ^i_y is the standard deviation of the output y^i, and $i = 1,2,\ldots M$.

If $\boldsymbol{\mu}^i$ is the firing level of the consequent of the ith rule, then the centroid defuzzifier is given as:

$$y_c(x) = \frac{\sum_{i=1}^{M} y_i \mu^i}{\sum_{i=1}^{M} \mu^i} \tag{19}$$

where y_i is the mean value of the Gaussian function of the i^{th} consequent, and y_c is the calculated value of the fuzzy system.

2.3.1 T1 SFLS

The singleton T1 FLS are those systems which inputs are modeled as crisp numbers:

$$x_q = x'_q \tag{20}$$

where $q = 1,2\ldots p$, with p the number of inputs, and the antecedents and consequents are fuzzy sets, are Gaussians of type-1, as (17) and (18).

The singleton fuzzification operation or value transformation mechanism in a type-1 fuzzy set is shown in Fig. 2. The vertical axis represents the membership grade, $\boldsymbol{\mu}_{A_q}(\boldsymbol{x'}_q) \in [0, 1]$ of the crisp input value $\boldsymbol{x'}_q$. The horizontal axis represents the real values of the universe of disclosure of each crisp input.

2.3.2 T1 NSFLS

The T1 NSFLS systems are those systems, which inputs are modeled as Gaussians functions of type-1, non-singleton values with mean and standard deviation, and these functions can be used to handle the uncertainty of the inputs:

$$\mu_{X_q}(x_q) = \exp\left[-\frac{1}{2}\left[\frac{x_q - m_{X_q}}{\sigma_{X_q}}\right]\right] \quad (21)$$

where $\boldsymbol{m}_{X_q}$ is the mean of each input centered at $\boldsymbol{x'}_q$, $\boldsymbol{m}_{X_q} = \boldsymbol{x'}_q$, $\boldsymbol{\sigma}_{X_q}$ is the standard deviation of each input, q = 1,2…p. These systems have antecedents and consequents modeled as fuzzy sets which use Gaussians of type-1, as (17) and (18).

The non-singleton fuzzification operation in a type-1 fuzzy set is shown in Fig. 3. In this type of inputs, their uncertainty values σ_{X_q} are taking into account together with their mean values $x_q = x_q^{'}$.

The T1 NSFLS systems have their inputs modeled as type-1 fuzzy numbers that can handle measurements uncertainties when applied to antecedents and consequents parameters modeled as Gaussians of type-1 fuzzy values. The measured data that come from the sensors of the process are uncertain, but there is no way to account for any uncertainty in the antecedent and consequent membership functions of T1 NSFLS systems. These systems only take into account for the noise through the filtering action of the non-singleton fuzzification operations.

2.4 IT2 FLS systems (Mendel 2001)

Consider the transition from an ordinary set to fuzzy sets. When it is not easy to determine the membership of an element in a set as 0 or 1, fuzzy sets of type-1 are used. Similarly, when the circumstances are so fuzzy that there is trouble to determine the membership grade even as a crisp number in [0, 1], fuzzy sets of type-2 are used.

A general type-2 fuzzy set, denoted by $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x, u)$, where$x \in X$ and $u \in J_x \subseteq [0, 1]$:

$$\tilde{A} = \left\{\left((x, u), \mu_{\tilde{A}}(x, u)\right) \mid \forall x \in X,\ \forall u \in J_x \subseteq [0, 1]\right\} \quad (22)$$

and $0 \leq \mu_{\tilde{A}} \leq 1$

The amplitude of a secondary membership function is called a secondary grade. In (2), $\mu_{\tilde{A}}(x, u)$ for $x \in X$, $u \in J_x$ is the secondary grade.

When $\mu_{\tilde{A}}(x', u') = 1, \forall u \in J_x \subseteq [0, 1]$ in (11), then the secondary membership functions are interval sets, and if this is true for $\forall x \in X$, then is the case of an interval type-2 membership function. They reflect a uniform uncertainty at the primary memberships of x.

A Mamdani interval type-2 fuzzy logic system having p inputs $x_1 \in X_1, \ldots, x_p \in X_p$ and the output space $y \in Y$ is represented by fuzzy IF–THEN rules that represent input–output relations of a system and can be expressed as:

$$R^i : IF\ x_1 \text{ is } \tilde{F}_1^i \text{ and } x_2 \text{ is } \tilde{F}_2^i \ldots \text{ and } x_p \text{ is } \tilde{F}_p^i\ THEN\ y^i \text{ is} \tilde{G}^I \quad (23)$$

where y^i is the consequent output of the ith rule with an interval type-2 fuzzy set value $\tilde{G}^i$ (i = 1,…,M rules), and $\tilde{F}_q^i$ (q = 0, 1, …, p, with p inputs) are interval type-2 antecedents fuzzy sets. This rule represents a type-2 relation between the input space $X_1 \times X_2 \times \ldots \times X_p$, and the output space Y of the type-2 FLS.

The membership function for each antecedent is a Gaussian function as:

$$\mu_{\tilde{A}q}^i(x_q) = \tilde{F}_q^i = \exp\left[-\frac{1}{2}\left[\frac{x_q - m_q^i}{\sigma_q^i}\right]\right] \quad (24)$$

where $m_q^i \in \left[m_{q1}^i, m_{q2}^i\right]$ is the uncertain mean, q = 1,2,…,p (p the number of inputs) and i = 1,2,…,M (the number of M rules), and σ_q^i is the standard deviation. The means of the antecedent fuzzy sets are uniformly distributed over the entire input space. It contains the uncertainty contained in the antecedent fuzzy sets.

The membership function for each consequent is a Gaussian function with the form:

$$\mu_{\tilde{G}}^i(y^i) = \tilde{G}^i = \exp\left[-\frac{1}{2}\left[\frac{y^i - m_y^i}{\sigma_y^i}\right]\right] \quad (25)$$

where $\boldsymbol{m}_y^i \in \left[\boldsymbol{m}_{y_1}^i, \boldsymbol{m}_{y_2}^i\right]$ is the uncertain mean, i = 1,2,…M (The number of M rules), $\boldsymbol{\sigma}_k^i$ is the standard deviation. It contains the uncertainty of the consequent fuzzy sets. $\tilde{\boldsymbol{G}}^i$ of the ith can be represented by an interval set $[\boldsymbol{y}_l^i, \boldsymbol{y}_r^i]$. If $\bar{\boldsymbol{f}}^i$ and $\underline{\boldsymbol{f}}^i$ are the firing interval of the consequent of the ith rule, see Figs. 4 and 5, then the type reduction and defuzzification can be given as:

$$y_l = \frac{\sum_{i=1}^{L} \bar{f}^i y_l^i + \sum_{i=L+1}^{M} \underline{f}^i y_l^i}{\sum_{i=1}^{L} \bar{f}^i + \sum_{i=L+1}^{M} \underline{f}^i} \quad (26)$$

$$y_r = \frac{\sum_{i=1}^{R} \underline{f}^i y_r^i + \sum_{j=R+1}^{M} \bar{f}^i y_l^j}{\sum_{i=1}^{R} \underline{f}^i + \sum_{i=R+1}^{M} \bar{f}^i} \quad (27)$$

$$y_c = f_{ns2-1} = \frac{y_l + y_r}{2} \quad (28)$$

where L and R are calculated values by optimization means, $[y_l, y_r]$ is the interval set calculated value of the

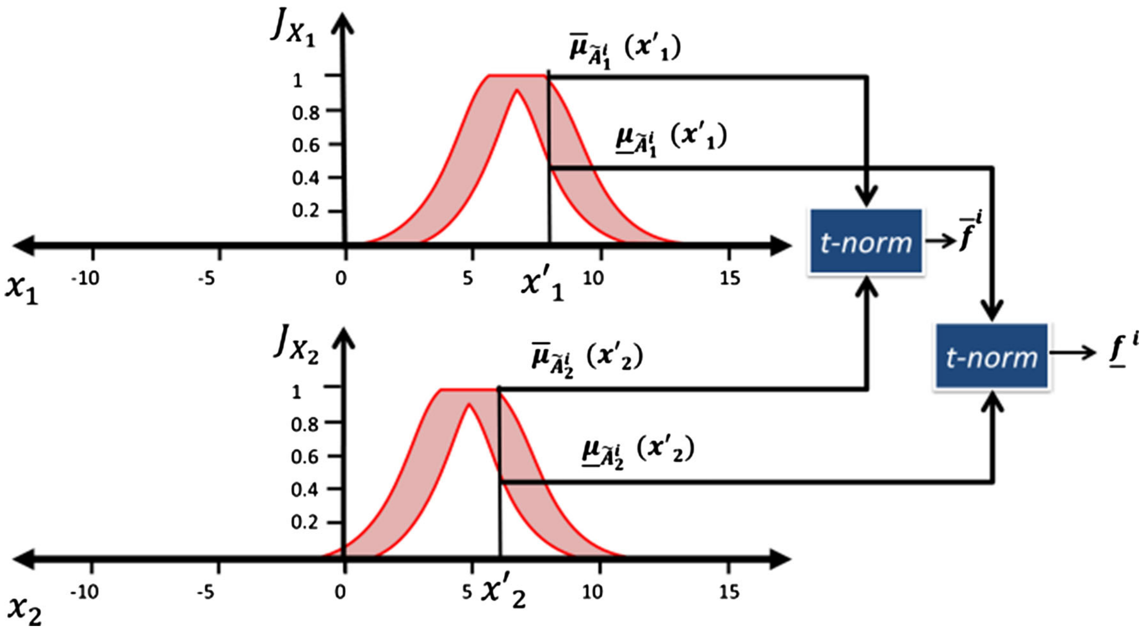

Fig. 4 Pictorial description of singleton input and antecedent operations for an IT2 SFLS. Adapted from Mendel (2001)

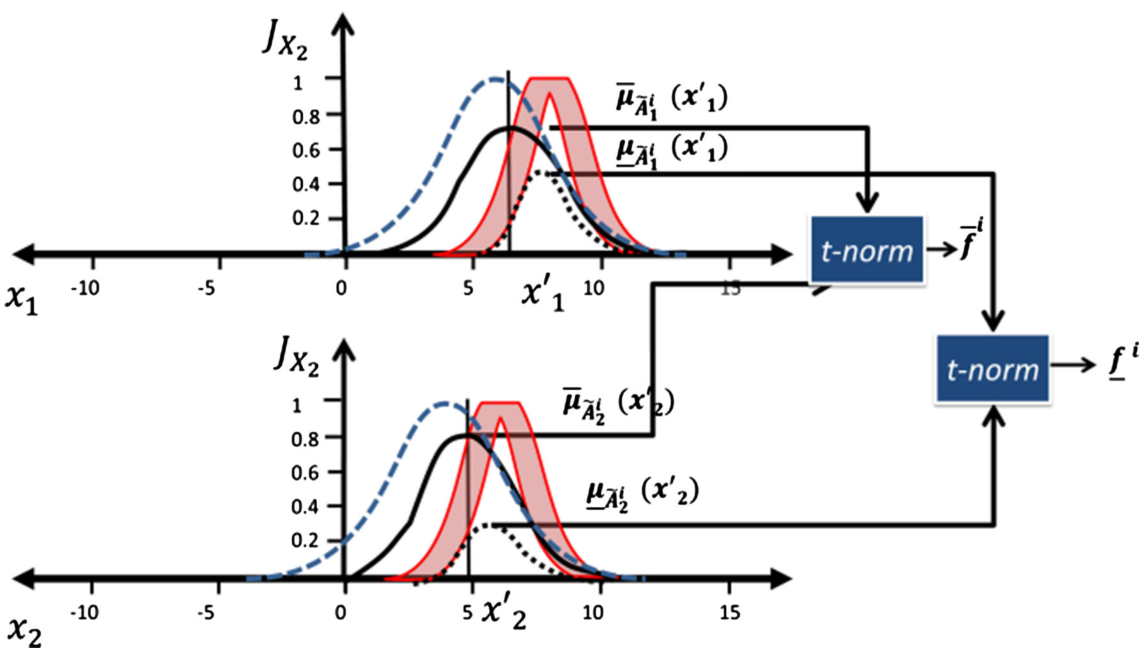

Fig. 5 Pictorial description of non-singleton input and antecedent operations for an IT2 NSFLS-1. Adapted from Mendel (2001)

type-2 fuzzy system, and y_c is the mean value of the interval set.

IT2 FLS takes into account the rule (antecedent and consequent) uncertainties due to construction and initial training with noisy data.

2.4.1 IT2 SFLS

The singleton IT2 SFLS are those systems which inputs are modeled as crisp numbers:

$$x_q = x_q^{'} \tag{29}$$

where $q = 1,2\ldots p$, with p as the number of inputs. The antecedents and consequents are modeled as fuzzy sets, which use Gaussians of type-2, as (24) and (25), respectively.

The singleton fuzzification operation in an interval type-2 fuzzy set is shown in Fig. 4. The vertical axis represents the primary membership of $\boldsymbol{u} = \boldsymbol{J}_{x_q}$ of $\boldsymbol{x}'_q, \boldsymbol{J}_{x_q} \in [0, 1]$, of the crisp input value x'_q.The horizontal axis represents the real values of the universe of disclosure of crisp each input.

2.4.2 IT2 NSFLS-1

The interval type-1 non-singleton type-2 FLS are those systems, which inputs are modeled as Gaussians functions of type-1, as non-singleton values with mean and standard deviation, and these functions can be used to handle the uncertainty of the inputs:

$$\mu_{X_q}(x_q) = \exp\left[-\frac{1}{2}\left[\frac{x_q - m_{X_q}}{\sigma_{X_q}}\right]\right] \quad (30)$$

where $\boldsymbol{m}_{X_q}$ is the mean of each input centered at $\boldsymbol{m}_{X_q} = \boldsymbol{x}'_q, \boldsymbol{\sigma}_{X_q}$ is the standard deviation of each input, q = 1,2…p. The antecedents and consequents are fuzzy sets, which use Gaussians of type-2, as (24) and (25).

The non-singleton fuzzification operation in an interval type-2 fuzzy set is shown in Fig. 5. In this type of inputs, the uncertainty values σ_{X_q} are taken into account together with their mean values $x_q = x'_q$.

The IT2 NSFLS-1 systems take into account all the uncertainties that can be presented in a fuzzy logic system: (a) the antecedent and consequent uncertainties due to the heuristic construction of the fuzzy rule base and the estimation of the first values of their parameters using noisy data, and (b) the uncertainties due to the measurements that are used during the control and feedback processes.

3 Proposed method

The parameters of the PID controller are initially calculated using the Ziegler–Nichols tuning rule, based on the parameters of the second-order plant, and on its experimental step-function response. The initial values of the PID gains contain uncertainties due to the uncertainty generated during the identification process of the plant and due to the use of Ziegler–Nichols rule. The fuzzy rule base of the proposed IT2 NSFLS-1 systems is constructed using a heuristic method that incorporates uncertainty in the antecedents and consequents fuzzy sets. The proposed assembly uses three IT2 NSFLS-1 systems to calculate the adjustment for each the three gains of the PID controller, (2), in order to reduce the variations and the transient time on the desired output of the plant until the stopping criteria established by the user is accomplished.

3.1 Proposed dynamic updating of the PID gains using an IT2 NSFLS-1 system

Three IT2 NSFLS-1 systems update the gains of the PID controller (2), through the processing of two type-1 fuzzy inputs: (a) the error $\boldsymbol{e}(\boldsymbol{k})$ that is calculated using the feedback measurement of the output of the plant $\boldsymbol{y_m}(\boldsymbol{k})$, and the reference value $\boldsymbol{y_{ref}}$ or set point, (31), and its standard deviation $\boldsymbol{\sigma e}(\boldsymbol{k})$; and (b) the change of error $\Delta\boldsymbol{e}(\boldsymbol{k})$ that is calculated using the errors $\boldsymbol{e}(\boldsymbol{k}), \boldsymbol{e}(\boldsymbol{k}-1)$ in the discrete times k and $k-1$, $\boldsymbol{k} = 1, 2, 3, \ldots$, and its standard deviation, $\boldsymbol{\sigma}\Delta\boldsymbol{e}(\boldsymbol{k})$, (32).

$$\begin{aligned}\boldsymbol{e}(\boldsymbol{k}) &= \boldsymbol{y_{ref}} - \boldsymbol{y_m}(\boldsymbol{k}) = \boldsymbol{N}(\boldsymbol{y_{ref}}, 0) - \boldsymbol{N}\left[\boldsymbol{y_m}(\boldsymbol{k}), \boldsymbol{\sigma}^2_{y_m}(\boldsymbol{k})\right] \\ &= \boldsymbol{N}\left[\boldsymbol{y_{ref}} - \boldsymbol{y_m}(\boldsymbol{k}), \boldsymbol{\sigma}^2_{y_m}(\boldsymbol{k})\right]\end{aligned} \quad (31)$$

$$\begin{aligned}\Delta\boldsymbol{e}(\boldsymbol{k}) &= \boldsymbol{e}(\boldsymbol{k}) - \boldsymbol{e}(\boldsymbol{k}-1) = -\boldsymbol{N}\left[\boldsymbol{y_m}(\boldsymbol{k}), \boldsymbol{\sigma}^2_{y_m}(\boldsymbol{k})\right] \\ &\quad + \boldsymbol{N}\left[\boldsymbol{y_m}(\boldsymbol{k}-1), \boldsymbol{\sigma}^2_{y_m}(\boldsymbol{k}-1)\right] \\ &= \boldsymbol{N}\left[-\boldsymbol{y_m}(\boldsymbol{k}) + \boldsymbol{y_m}(\boldsymbol{k}-1), \boldsymbol{\sigma}^2_{y_m}(\boldsymbol{k}) + \boldsymbol{\sigma}^2_{y_m}(\boldsymbol{k}-1)\right]\end{aligned} \quad (32)$$

The next three rules of thumb are used in Beirami and Zerafat (2015), to construct the fuzzy rule base of each IT2 NSFLS-1 system.

- If $|\boldsymbol{e}(\boldsymbol{k})|$ is big, then $\Delta\boldsymbol{KP}(\boldsymbol{k})$ is big, then $\Delta\boldsymbol{KD}(\boldsymbol{k})$ is small, and $\Delta\boldsymbol{KI}(\boldsymbol{k})$ is small too.
- If $|\boldsymbol{e}(\boldsymbol{k})|$ is medium, then $\Delta\boldsymbol{KP}(\boldsymbol{k})$ is small, then $\Delta\boldsymbol{KD}(\boldsymbol{k})$ is medium, and $\Delta\boldsymbol{KI}(\boldsymbol{k})$ is medium.
- If $|\boldsymbol{e}(\boldsymbol{k})|$ is small, then $\Delta\boldsymbol{KP}(\boldsymbol{k})$ and $\Delta\boldsymbol{KI}(\boldsymbol{k})$ are big.
- If $|\Delta\boldsymbol{e}(\boldsymbol{k})|$ is small, then $\Delta\boldsymbol{KD}(\boldsymbol{k})$ is big and reverse.

The fuzzy sets of each antecedent of the IT2 NSFLS-1 rule base has two non-singleton values: (a) the uncertain error (interval mean error $\boldsymbol{e}(\boldsymbol{k}) = [\boldsymbol{e_l}(\boldsymbol{k}), \boldsymbol{e_r}(\boldsymbol{k})]$ and its standard deviation $\boldsymbol{\sigma_e}$; and (b) the uncertain change of error (interval mean change of error $\Delta\boldsymbol{e}(\boldsymbol{k}) = [\Delta\boldsymbol{e_l}(\boldsymbol{k}), \Delta\boldsymbol{e_r}(\boldsymbol{k})]$ and its standard deviation $\boldsymbol{\sigma}_{\Delta e}$.

Negative big left and right ([Nbl, Nbr], [nbl, nbr], NBl, NBr]), negative medium left and right ([Nml, Nmr], [nml, nmr], [NMl, NMr]), negative small left and right ([Nsl, Nsr], [nsl, nsr], [NSl, NSr]), zero left and right ([Zcl, Zcr], [zcl, zcr], [Z0 l, Z0r]), positive small left and right ([Psl, Psr], [psl, psr], [PSl, PSr]), positive medium left and right ([Pml, Pmr], [pml, pmr], [PMl, PMr]), and positive big left and right ([Pbl, Pbr], [pbl, pbr], [PBl, PBr]) are the uncertain linguistic variables used for the IT2 NSFLS-1 fuzzy sets constructions (Tables 2, 3, 4).

In the proposed assembly, three IT2 NSFLS-1 systems are used to calculate the gains, $\Delta\boldsymbol{KP}(\boldsymbol{k})$, $\Delta\boldsymbol{KI}(\boldsymbol{k})$ and $\Delta\boldsymbol{KD}(\boldsymbol{k})$.

Table 2 IT2 NSFLS-1 fuzzy rule base for (a) KPl(k) gain adjustment, (b) KPr(k) gain adjustment

$\Delta KPl(k)$		$\Delta el(k)$						
		nbl	nml	nsl	zcl	psl	pml	pbl
	Nbl	PBl	PBl	PMl	PMl	PSl	Z0 1	Z0 1
	Nml	PBl	PBl	PMl	PSl	PSl	Z0 1	NSl
$el(k)$	Nsl	PMl	PMl	PMl	PSl	Z0 1	NSl	NSl
	Zcl	PMl	PMl	PSl	Z0 1	NSl	NMl	NMl
	Psl	PSl	PSl	Z0 1	NSl	NSl	NMl	NMl
	Pml	PSl	Z0 1	NSl	NMl	NMl	NMl	NBl
	Pbl	Z0 1	Z0 1	NMl	NMl	NMl	NBl	NBl
$\Delta KPr(k)$		$\Delta er(k)$						
		nbr	nmr	nsr	zcr	psr	pmr	pbr
	Nbr	PBr	PBr	PMr	PMr	PSr	Z0r	Z0r
	Nmr	PBr	PBr	PMr	PSr	PSr	Z0r	NSr
$er(k)$	Nsr	PMr	PMr	PMr	PSr	Z0r	NSr	NSr
	Zcr	PMr	PMr	PSr	Z0r	NSr	NMr	NMr
	Psr	PSr	PSr	Z0r	NSr	NSr	NMr	NMr
	Pmr	PSr	Z0r	NSr	NMr	NMr	NMr	NBr
	Pbr	Z0r	Z0r	NMr	NMr	NMr	NBr	NBr

Table 3 IT2 NSFLS-1 fuzzy rule base for (a) KIl(k) gain adjustment, (b) KIr(k) gain adjustment

$\Delta KIl(k)$		$\Delta el(k)$						
		nbl	nml	nsl	zcl	psl	pml	pbl
	Nbl	NBl	NBl	NMl	NMl	NSl	Z0 1	Z0 1
	Nml	NBl	NBl	NMl	NSl	NSl	Z0 1	Z0 1
$el(k)$	Nsl	NBl	NMl	NSl	NSl	Z0 1	PSl	PSl
	Zcl	NMl	NMl	NSl	Z0 1	PSl	PMl	PMl
	Psl	NMl	NSl	Z0 1	PSl	PSl	PMl	PBl
	Pml	Z0 1	Z0 1	PSl	PSl	PMl	PBl	PBl
	Pbl	Z0 1	Z0 1	PSl	PMl	PMl	PBl	PBl
$\Delta KIr(k)$		$\Delta er(k)$						
		nbr	nmr	nsr	zcr	psr	pmr	pbr
	Nbr	NBr	NBr	NMr	NMr	NSr	Z0r	Z0r
	Nmr	NBr	NBr	NMr	NSr	NSr	Z0r	Z0r
$er(k)$	Nsr	NBr	NMr	NSr	NSr	Z0r	PSr	PSr
	Zcr	NMr	NMr	NSr	Z0r	PSr	PMr	PMr
	Psr	NMr	NSr	Z0r	PSr	PSr	PMr	PBr
	Pmr	Z0r	Z0r	PSr	PSr	PMr	PBr	PBr
	Pbr	Z0r	Z0r	PSr	PMr	PMr	PBr	PBr

Table 4 IT2 NSFLS-1 fuzzy rule base for (a) KDl(k) gain adjustment, (b) KDr(k) gain adjustment

$\Delta KDl(k)$		$\Delta el(k)$						
		nbl	nml	nsl	zcl	psl	pml	pbl
	Nbl	PSl	NSl	NBl	NBl	NBl	NMl	PSl
	Nml	PSl	NSl	NBl	NMl	NMl	NSl	Z0 1
$el(k)$	Nsl	Z0 1	NSl	NMl	NMl	NSl	NSl	Z0 1
	Zcl	Z0 1	NSl	NSl	NSl	NSl	NSl	Z0 1
	Psl	Z0 1	Z0 1	Z0 1	Z0 1	Z0 1	Z0 1	Z0 1
	Pml	PBl	NSl	PSl	PSl	PSl	PSl	PBl
	Pbl	PBl	PMl	PMl	PMl	PSl	PSl	PBl
$\Delta KDr(k)$		$\Delta er(k)$						
		nbr	nmr	nsr	zcr	psr	pmr	pbr
	Nbr	PSr	NSr	NBr	NBr	NBr	NMr	PSr
	Nmr	PSr	NSr	NBr	NMr	NMr	NSr	Z0r
$er(k)$	Nsr	Z0r	NSr	NMr	NMr	NSr	NSr	Z0r
	Zcr	Z0r	NSr	NSr	NSr	NSr	NSr	Z0r
	Psr	Z0r	Z0r	Z0r	Z0r	Z0r	Z0r	Z0r
	Pmr	PBr	NSr	PSr	PSr	PSr	PSr	PBr
	Pbr	PBr	PMr	PMr	PMr	PSr	PSr	PBr

The IT2 NSFLS-1 fuzzy rule for $\Delta \boldsymbol{KP}(\boldsymbol{k})$ calculation has the following form:

$$\text{IF } |\boldsymbol{e}(\boldsymbol{k})| \text{ is } \widetilde{\boldsymbol{Pm}} \text{ and } \Delta \boldsymbol{e}(\boldsymbol{k}) \text{ is } \widetilde{\boldsymbol{ps}} \text{ THEN } \Delta \boldsymbol{KP}(\boldsymbol{k}) \text{ is } \widetilde{\boldsymbol{NM}} \tag{33}$$

The IT2 NSFLS-1 fuzzy rule for $\Delta \boldsymbol{KI}(\boldsymbol{k})$ calculation has the form:

$$\text{IF } |\boldsymbol{e}(\boldsymbol{k})| \text{ is } \widetilde{\boldsymbol{Pm}} \text{ and } \Delta \boldsymbol{e}(\boldsymbol{k}) \text{ is } \widetilde{\boldsymbol{ps}} \text{ THEN } \Delta \boldsymbol{KI}(\boldsymbol{k}) \text{ is } \widetilde{\boldsymbol{PM}} \tag{34}$$

The IT2 NSFLS-1 fuzzy rule for $\Delta \boldsymbol{KD}(\boldsymbol{k})$ calculation has the form:

$$\text{IF } |\boldsymbol{e}(\boldsymbol{k})| \text{ is } \widetilde{\boldsymbol{Pm}} \text{ and } \Delta \boldsymbol{e}(\boldsymbol{k}) \text{ is } \widetilde{\boldsymbol{ps}} \text{ THEN } \Delta \boldsymbol{KD}(\boldsymbol{k}) \text{ is } \widetilde{\boldsymbol{PS}} \tag{35}$$

Each input variable contains seven fuzzy sets, resulting in 49 IT2 fuzzy rules, per each fuzzy rule base.

3.2 The PID controller

The first estimation of the starting values of the PID gains is using the ZN closed-loop method that gives an amplitude ratio between subsequent oscillations after a step change of the set point equal to ¼, and this is often a guarantee as a medium range stability (Alberto, 2000). After that the gains are adjusted by trial and error observing the best behavior of the response of the plant. This method surely induces uncertainty to the starting point of the PID gains values and in the control signal estimation.

The PID gains are updated after each control cycle by the next calculations:

$$\boldsymbol{K_p(k) = K_p(k-1) + \Delta KP(k)} \tag{36}$$

$$\boldsymbol{K_i(k) = K_i(k-1) + \Delta KI(k)} \tag{37}$$

$$\boldsymbol{K_d(k) = K_d(k-1) + \Delta KD(k)} \tag{38}$$

where $\boldsymbol{K_p(k)}$ is the proportional gain, $\boldsymbol{K_i(k)}$ is the integral gain, and $\boldsymbol{K_d(k)}$ is the derivative gain of the PID controller at the discrete time $\boldsymbol{k}$. $\boldsymbol{\Delta KP(k)}$, $\boldsymbol{\Delta KI(k)}$ and $\boldsymbol{\Delta KD(k)}$ adjustments are calculated by each IT2 NSFLS-2 system as interval sets values.

Equations (36), (37) and (38) are computed each control cycle in the discrete time k and are used to update each gain of the PID (2) controller. Figure 6 shows the interaction for every component of the proposed assembly that uses the IT2 NSFLS-1 for dynamic PID gains updating.

3.3 The backpropagation algorithm for IT2 NSFLS-1 parameters adjustment

As explained in Mendel (2001), an objective function $e(\theta)$ may have a nonlinear form with respect to an adjustable parameter θ. In iterative descent methods, the next point θ_{next} is determined by step down from the current point θ_{now} in the negative direction of the gradient of the function $e(\theta)$:

$$\theta_{next} = \theta_{now} - \alpha g \tag{39}$$

where α is the learning rate, and g is the vector of the first partial derivatives of $E(\theta)$

$$g(\theta) = \left[\frac{\partial e(\theta)}{\partial \theta_1}, \frac{\partial(\theta)}{\partial \theta_2}, \ldots, \frac{\partial(\theta)}{\partial \theta_n}\right]^T \tag{40}$$

Considering that given the control signal model, the second-order plant model, and the IT2 NSFLS-1 fuzzy systems, $j = k$, is the discrete time, then:

$$u(k) = \left(K_p + K_i * k\right)y_{ref} - \left(K_p + K_i + K_d\right)y(k-1) - \left(K_i + K_d\right)y(k-2) - \left(K_i\right)\sum_{i=0}^{j-3} y(i) \tag{41}$$

$$y(k) = K_2 u(k) - a_1 y(k-1) - a_2(k-2) \tag{42}$$

$$y_m(k) + \sigma y_m(k) = y(k) + \sigma y(k) \tag{43}$$

and defining the error function as in Eq. (11) the objective function, which is required to minimize its value in order to update the design parameters of the $f_{ns2-1}(x)$ fuzzy systems, Eq. (8), each one composed by M fuzzy rules, with the next form: $m_q^i \in \left[m_{q1}^i, m_{q2}^i\right]$ is the uncertain mean of the antecedents, σ_q^i is the standard deviation of antecedents, $\sigma_{X_q}^i$ is the standard deviation of type-1 non-singleton input data. Let be p = the number of input variables of the IT2 NSFLS-1 system, with $q = 1, 2, \ldots, p$, and $i = 1, 2, \ldots, M$, being the number of fuzzy rules, j and k being the discrete period of time.

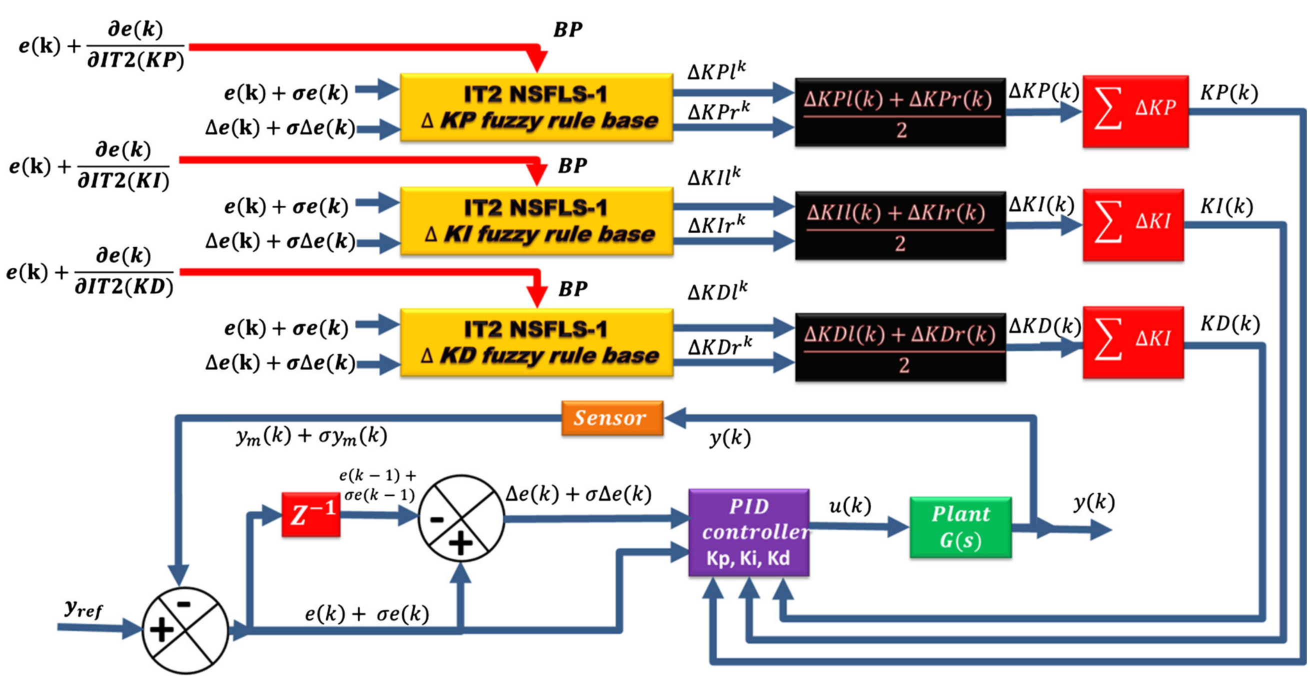

Fig. 6 Block diagram of proposal using the non-singleton values, and three IT2 NSFLS-1 fuzzy system to adjust the PID's gains

Equation (39) is used to establish the basic equations to update the antecedent parameters using the BP method:

$$m_{q1}^{i}(k+1) = m_{q1}^{i}(k) - \alpha \left.\frac{\partial e}{\partial m_{q1}^{i}}\right|_{j} \tag{44}$$

$$m_{q2}^{i}(k+1) = m_{q2}^{i}(k) - \alpha \left.\frac{\partial e}{\partial m_{q2}^{i}}\right|_{j} \tag{45}$$

$$\sigma_{q}^{i}(k+1) = \sigma_{q}^{i}(k) - \alpha \left.\frac{\partial e}{\partial \sigma_{q}^{i}}\right|_{j} \tag{46}$$

and is also used for update the consequent parameters:

$$y_{l}^{i}(k+1) = y_{l}^{i}(k) - \alpha \left.\frac{\partial e}{\partial y_{l}^{i}}\right|_{j} \tag{47}$$

$$y_{r}^{i}(k+1) = y_{r}^{i}(k) - \alpha \left.\frac{\partial e}{\partial y_{r}^{i}}\right|_{j} \tag{48}$$

Using the chain rule for partial derivatives

$$\left.\frac{\partial e}{\partial m_{q1}^{i}}\right|_{j} = \left[\left(\frac{\partial e}{\partial y_m}\right)\left(\frac{\partial y_m}{\partial u}\right)\left(\frac{\partial u}{\partial f_{ns2-1}}\right)\left(\frac{\partial f_{ns2-1}}{\partial m_{q1}^{i}}\right)\right]_{j} \tag{49}$$

$$\left.\frac{\partial e}{\partial m_{q2}^{i}}\right|_{j} = \left[\left(\frac{\partial e}{\partial y_m}\right)\left(\frac{\partial y_m}{\partial u}\right)\left(\frac{\partial u}{\partial f_{ns2-1}}\right)\left(\frac{\partial f_{ns2-1}}{\partial m_{q2}^{i}}\right)\right]_{j} \tag{50}$$

$$\left.\frac{\partial e}{\partial \sigma_{q}^{i}}\right|_{j} = \left[\left(\frac{\partial e}{\partial y_m}\right)\left(\frac{\partial y_m}{\partial u}\right)\left(\frac{\partial u}{\partial f_{ns2-1}}\right)\left(\frac{\partial f_{ns2-1}}{\partial \sigma_{q}^{i}}\right)\right]_{j} \tag{51}$$

$$\left.\frac{\partial e}{\partial y_{l}^{i}}\right|_{j} = \left[\left(\frac{\partial e}{\partial y_m}\right)\left(\frac{\partial y_m}{\partial u}\right)\left(\frac{\partial u}{\partial f_{ns2-1}}\right)\left(\frac{\partial f_{ns2-1}}{\partial y_{l}^{i}}\right)\right]_{j} \tag{52}$$

$$\left.\frac{\partial e}{\partial y_{r}^{i}}\right|_{j} = \left[\left(\frac{\partial e}{\partial y_m}\right)\left(\frac{\partial y_m}{\partial u}\right)\left(\frac{\partial u}{\partial f_{ns2-1}}\right)\left(\frac{\partial f_{ns2-1}}{\partial y_{r}^{i}}\right)\right]_{j} \tag{53}$$

the resulting derivative components are:

$$\left.\frac{\partial f_{ns2-1}}{\partial m_{q1}^{i}}\right|_{j} = \left[\left(\frac{\partial f_{ns2-1}}{\partial y_l}\right)\left(\frac{\partial y_l}{\partial m_{q1}^{i}}\right) + \left(\frac{\partial f_{ns2-1}}{\partial y_r}\right)\left(\frac{\partial y_r}{\partial m_{q1}^{i}}\right)\right]_{j} \tag{54}$$

$$\left.\frac{\partial y_l}{\partial \overline{f}^{i}}\right|_{j} = \begin{cases} \dfrac{c_l^i - y_l}{\sum_{i=1}^{L}\overline{f}^{i} + \sum_{i=L+1}^{M}\underline{f}^{i}} & \text{if} \quad i \le L \\ 0 & \text{if} \quad i > L \end{cases} \tag{55}$$

$$\left.\frac{\partial y_l}{\partial \underline{f}^{i}}\right|_{j} = \begin{cases} \dfrac{c_l^i - y_l}{\sum_{i=1}^{L}\overline{f}^{i} + \sum_{i=L+1}^{M}\underline{f}^{i}} & \text{if} \quad i > L \\ 0 & \text{if} \quad i \le L \end{cases} \tag{56}$$

$$\left.\frac{\partial y_r}{\partial \overline{f}^{i}}\right|_{j} = \begin{cases} \dfrac{c_r^i - y_r}{\sum_{i=1}^{R}\underline{f}^{i} + \sum_{i=R+1}^{M}\overline{f}^{i}} & \text{if} \quad i > R \\ 0 & \text{if} \quad i \le R \end{cases} \tag{57}$$

$$\left.\frac{\partial y_r}{\partial \underline{f}^{i}}\right|_{j} = \begin{cases} \dfrac{c_r^i - y_r}{\sum_{i=1}^{L}\underline{f}^{i} + \sum_{i=L+1}^{M}\overline{f}^{i}} & \text{if} \quad i \le R \\ 0 & \text{if} \quad i > R \end{cases} \tag{58}$$

$$\left.\frac{\partial y_l}{\partial m_{q1}^{i}}\right|_{j} = \left[\left(\frac{\partial y_l}{\partial \overline{f}^{i}}\right)\left(\frac{\partial \overline{f}^{i}}{\partial m_{q1}^{i}}\right) + \left(\frac{\partial y_l}{\partial \underline{f}^{i}}\right)\left(\frac{\partial \underline{f}^{i}}{\partial m_{q1}^{i}}\right)\right]_{j} \tag{59}$$

$$\left.\frac{\partial y_r}{\partial m_{q1}^{i}}\right|_{j} = \left[\left(\frac{\partial y_r}{\partial \overline{f}^{i}}\right)\left(\frac{\partial \overline{f}^{i}}{\partial m_{q1}^{i}}\right) + \left(\frac{\partial y_r}{\partial \underline{f}^{i}}\right)\left(\frac{\partial \underline{f}^{i}}{\partial m_{q1}^{i}}\right)\right]_{j} \tag{60}$$

$$\left.\frac{\partial \overline{f}^{i}}{\partial m_{q1}^{i}}\right|_{j} = \begin{cases} \dfrac{\left(x_q - m_{q1}^{i}\right)\overline{f}^{i}\left(x_{\mathrm{up}_q}\right)}{\left(\sigma_{X_q}\right)^2 + \left(\sigma_q^i\right)^2} & \text{if} \quad x_q < m_{q1}^{i} \\ 0 & \text{if} \quad x_q \ge m_{q2}^{i} \end{cases} \tag{61}$$

$$\left.\frac{\partial \underline{f}^{i}}{\partial m_{q1}^{i}}\right|_{j} = \begin{cases} 0 & \text{if} \quad x_q \le \dfrac{m_{q1}^{i} + m_{q2}^{i}}{2} - \dfrac{\sigma_{X_q}^2\left(m_{q2}^{i} - m_{q1}^{i}\right)}{2\left(\sigma_q^i\right)^2} \\ \dfrac{\left(x_q - m_{q1}^{i}\right)\underline{f}^{i}\left(x_{\mathrm{low}_q}\right)}{\left(\sigma_{X_q}\right)^2 + \left(\sigma_q^i\right)^2} & \text{if} \quad x_q > \dfrac{m_{q1}^{i} + m_{q2}^{i}}{2} + \dfrac{\sigma_{X_q}^2\left(m_{q2}^{i} - m_{q1}^{i}\right)}{2\left(\sigma_q^i\right)^2} \end{cases} \tag{62}$$

$$\left.\frac{\partial f_{ns2-1}}{\partial m_{q2}^{i}}\right|_{j} = \left[\left(\frac{\partial f_{ns2-1}}{\partial y_l}\right)\left(\frac{\partial y_l}{\partial m_{q2}^{i}}\right) + \left(\frac{\partial f_{ns2-1}}{\partial y_r}\right)\left(\frac{\partial y_r}{\partial m_{q2}^{i}}\right)\right]_{j} \tag{63}$$

$$\left.\frac{\partial y_l}{\partial m_{q2}^{i}}\right|_{j} = \left[\left(\frac{\partial y_l}{\partial \overline{f}^{i}}\right)\left(\frac{\partial \overline{f}^{i}}{\partial m_{q2}^{i}}\right) + \left(\frac{\partial y_l}{\partial \underline{f}^{i}}\right)\left(\frac{\partial \underline{f}^{i}}{\partial m_{q2}^{i}}\right)\right]_{j} \tag{64}$$

$$\left.\frac{\partial y_r}{\partial m_{q2}^{i}}\right|_{j} = \left[\left(\frac{\partial y_r}{\partial \overline{f}^{i}}\right)\left(\frac{\partial \overline{f}^{i}}{\partial m_{q2}^{i}}\right) + \left(\frac{\partial y_r}{\partial \underline{f}^{i}}\right)\left(\frac{\partial \underline{f}^{i}}{\partial m_{q2}^{i}}\right)\right]_{j} \tag{65}$$

$$\left.\frac{\partial \overline{f}^{i}}{\partial m_{q2}^{i}}\right|_{j} = \begin{cases} 0 & \text{if} \quad x_q \le m_{q1}^{i} \\ \dfrac{\left(x_q - m_{q2}^{i}\right)\overline{f}^{i}\left(x_{\mathrm{up}_q}\right)}{\left(\sigma_{X_q}\right)^2 + \left(\sigma_q^i\right)^2} & \text{if} \quad x_q > m_{q2}^{i} \end{cases} \tag{66}$$

$$\left.\frac{\partial \underline{f}^{i}}{\partial m_{q2}^{i}}\right|_{j} = \begin{cases} \dfrac{\left(x_q - m_{q2}^{i}\right)\underline{f}^{i}\left(x_{\mathrm{low}_q}\right)}{\left(\sigma_{X_q}\right)^2 + \left(\sigma_q^i\right)^2} & \text{if} \quad x_q \le \dfrac{m_{q1}^{i} + m_{q2}^{i}}{2} - \dfrac{\sigma_{X_q}^2\left(m_{q2}^{i} - m_{q1}^{i}\right)}{2\left(\sigma_q^i\right)^2} \\ 0 & \text{if} \quad x_q \le \dfrac{m_{q1}^{i} + m_{q2}^{i}}{2} - \dfrac{\sigma_{X_q}^2\left(m_{q2}^{i} - m_{q1}^{i}\right)}{2\left(\sigma_q^i\right)^2} \end{cases} \tag{67}$$

$$\left.\frac{\partial f_{ns2-1}}{\partial \sigma_{q}^{i}}\right|_{j} = \left[\left(\frac{\partial f_{ns2-1}}{\partial y_l}\right)\left(\frac{\partial y_l}{\partial \sigma_{q}^{i}}\right) + \left(\frac{\partial f_{ns2-1}}{\partial y_r}\right)\left(\frac{\partial y_r}{\partial \sigma_{q}^{i}}\right)\right]_{j} \tag{68}$$

$$\left.\frac{\partial y_l}{\partial \sigma_q^i}\right|_j = \left[\left(\frac{\partial y_l}{\partial \bar{f}^i}\right)\left(\frac{\partial \bar{f}^i}{\partial \sigma_q^i}\right) + \left(\frac{\partial y_l}{\partial \underline{f}^i}\right)\left(\frac{\partial \underline{f}^i}{\partial \sigma_q^i}\right)\right]_j. \tag{69}$$

$$\left.\frac{\partial y_r}{\partial \sigma_q^i}\right|_j = \left[\left(\frac{\partial y_r}{\partial \bar{f}^i}\right)\left(\frac{\partial \bar{f}^i}{\partial \sigma_q^i}\right) + \left(\frac{\partial y_r}{\partial \underline{f}^i}\right)\left(\frac{\partial \underline{f}^i}{\partial \sigma_q^i}\right)\right]_j \tag{70}$$

$$\left.\frac{\partial \bar{f}^i}{\partial \sigma_q^i}\right|_j = \begin{cases} \dfrac{\left(x_q - m_{q1}^i\right)^2 \underline{f}^i\left(x_{\mathrm{up}_q}\right)\sigma_q^i}{\left(\sigma_{X_q} + \sigma_q^i\right)^2} & \text{if} \quad x_q < m_{q1}^i \\ \dfrac{\left(x_q - m_{q2}^i\right)\underline{f}^i\left(x_{\mathrm{up}_q}\right)\sigma_q^i}{\left(\sigma_{X_q} + \sigma_q^i\right)^2} & \text{if} \quad x_q > m_{q2}^i \end{cases} \tag{71}$$

$$\left.\frac{\partial \underline{f}^i}{\partial \sigma_q^i}\right|_j = \begin{cases} \dfrac{\left(x_k - m_{q2}^i\right)\underline{f}^i\left(x_{\mathrm{low}_q}\right)\sigma_q^i}{\left(\sigma_{X_q} + \sigma_q^i\right)^2} & \text{if} \quad x_q < \dfrac{m_{q1}^i + m_{q2}^i}{2} \\ \dfrac{\left(x_q - m_{q2}^i\right)\underline{f}^i\left(x_{\mathrm{low}_q}\right)\sigma_q^i}{\left(\sigma_{X_q} + \sigma_q^i\right)^2} & \text{if} \quad x_q > \dfrac{m_{q1}^i + m_{q2}^i}{2} \end{cases} \tag{72}$$

$$\left.\frac{\partial f_{ns2-1}}{\partial y_l^i}\right|_j = \left[\left(\frac{\partial f_{ns2-1}}{\partial y_l}\right)\left(\frac{\partial y_l}{\partial y_l^i}\right) + \left(\frac{\partial f_{ns2-1}}{\partial y_r}\right)\left(\frac{\partial y_r}{\partial y_l^i}\right)\right]_j \tag{73}$$

$$\left.\frac{\partial f_{ns2-1}}{\partial y_r^i}\right|_j = \left[\left(\frac{\partial f_{ns2-1}}{\partial y_l}\right)\left(\frac{\partial y_l}{\partial y_r^i}\right) + \left(\frac{\partial f_{ns2-1}}{\partial y_r}\right)\left(\frac{\partial y_r}{\partial y_r^i}\right)\right]_j \tag{74}$$

$$\left.\frac{\partial y_l}{\partial y_l^i}\right|_j = \begin{cases} \dfrac{\bar{f}^i}{\sum_{i=1}^{L}\bar{f}^i + \sum_{i=L+1}^{M}\underline{f}^i} & \text{if} \quad i \le L \\ \dfrac{\bar{f}^i}{\sum_{i=1}^{L}\bar{f}^i + \sum_{i=L+1}^{M}\underline{f}^i} & \text{if} \quad i > L \end{cases} \tag{75}$$

$$\left.\frac{\partial y_r}{\partial y_r^i}\right|_j = \begin{cases} \dfrac{\underline{f}^i}{\sum_{i=1}^{R}\underline{f}^i + \sum_{i=R+1}^{M}\bar{f}^i} & \text{if} \quad i \le R \\ \dfrac{\bar{f}^i}{\sum_{i=1}^{R}\underline{f}^i + \sum_{i=R+1}^{M}\bar{f}^i} & \text{if} \quad i > R \end{cases} \tag{76}$$

$$\left.\frac{\partial e}{\partial y_m}\right|_j = -1 \tag{77}$$

$$\left.\frac{\partial y_m}{\partial u}\right|_j = K_2 \tag{78}$$

$$\left.\frac{\partial f_{ns2-1}}{\partial y_l}\right|_j = \frac{1}{2} \tag{79}$$

$$\left.\frac{\partial f_{ns2-1}}{\partial y_r}\right|_j = \frac{1}{2} \tag{80}$$

$$\left.\frac{\partial u}{\partial K_p}\right|_j = e(k-1) \text{ with } f_{ns2-1} = K_p \tag{81}$$

$$\left.\frac{\partial u}{\partial K_i}\right|_j = e(k-1) \text{ with } f_{ns2-1} = K_i \tag{82}$$

$$\left.\frac{\partial u}{\partial K_d}\right|_j = e(k-1) - e(k-2) \text{ with } f_{ns2-1} = K_d \tag{83}$$

As an example, for the case of the IT2 NSFLS-1 system that calculates the K_p gain, it is possible to obtain the recursive equation for m_{q1}^i parameter tuning using the BP method in the particular values of $i > L, x_q < m_{q1}^i$ if (44) is solved using the corresponding partial derivatives:

$$\left.\frac{\partial e}{\partial m_{q1}^i}\right|_j = \left[\left(\frac{\partial e}{\partial y_m}\right)\left(\frac{\partial y_m}{\partial u}\right)\left(\frac{\partial u}{\partial K_p}\right)\left(\frac{\partial K_p}{\partial m_{q1}^i}\right)\right]_j \tag{84}$$

$$\begin{aligned} m_{q1}^i(k+1) = m_{q1}^i(k) + \alpha * \frac{1}{2} * K_2 \\ * [e(k-1)]\left[\frac{\left[y_r^i - y_r\right]}{\sum_{i=1}^{R}\underline{f}^i + \sum_{i=R+1}^{M}\bar{f}^i}\right] \\ * \left[\frac{\left(x_k - m_{q1}^i\right)\bar{f}^i\left(x_{\mathrm{up}_q}\right)}{\left(\sigma_{X_q}\right)^2 + \left(\sigma_q^i\right)^2}\right] \end{aligned} \tag{85}$$

Also, for the case of the IT2 NSFLS-1 system that calculates the K_i gain of the PID controller, the recursive equation is derived. For the case of the IT2 NSFLS-1 system that calculates the K_d gain, the following partial derivative equation is obtained:

$$\begin{aligned} m_{q1}^i(k+1) = m_{q1}^i(k) + \alpha * \frac{1}{2} * K_2 \\ * [e(k-1) - e(k-2)]\left[\frac{\left[y_r^i - y_r\right]}{\sum_{i=1}^{R}\underline{f}^i + \sum_{i=R+1}^{M}\bar{f}^i}\right] \\ * \left[\frac{\left(x_q - m_{q1}^i\right)\bar{f}^i\left(x_{\mathrm{up}_q}\right)}{\left(\sigma_{X_q}\right)^2 + \left(\sigma_q^i\right)^2}\right] \end{aligned} \tag{86}$$

A similar procedure can be followed to compute (39), (40), (41) and (42).

The calculation of x_{up_q} and x_{low_q} depends on the location of x_q with respect to the values of $m_{q1}^i, m_{q2}^i, \sigma_q^i$, y_l^i and y_r^i as clearly explained in Ramos et al. (2016), resulting in five situations or states as shown in Table 5.

The equations for antecedent and consequent parameters tuning in the period of time j, $m_{q1}^i, m_{q2}^i, \sigma_q^i$, y_l^i and y_r^i depend on the relative position of x_q with respect to the values of $m_{q1}^i, m_{q2}^i, \sigma_q^i$, y_l^i and y_r^i as shown in Tables 5, 6, 7, 8 and 9 for the K_p and K_i gains adjustments. The corresponding derivative equations for the adjustment of the K_d gain are straightforward and can be calculated using (83) in the same way as the previous gains.

Table 5 Situations or states of $\mathbf{x_q}$ for calculation of $\mathbf{x_{up_q}}$ and $\mathbf{x_{low_q}}$

Case	State of x_q for x_{low_q} calculation	State of x_q for x_{up_q} calculation	$x^i_{low_q}$	$x^i_{up_q}$ calculation
1	$x_q < \frac{m^i_{q_1}+m^i_{q_2}}{2} - \frac{\sigma^2_{X_q}\left(m^i_{q_2}-m^i_{q_1}\right)}{2(\sigma^i_q)^2}$	$x_q < m^i_{q_1}$	$x_{low_q} = \frac{(\sigma_{X_q})^2 m^i_{q_2}+(\sigma^i_q)^2 x_q}{(\sigma_{X_q})^2+(\sigma^i_q)^2}$	$x_{up} = \frac{(\sigma_{X_q})^2 m^i_{q_1}+(\sigma^i_q)^2 x_q}{(\sigma_{X_q})^2+(\sigma^i_q)^2}$
2	$x_q < \frac{m^i_{q_1}+m^i_{q_2}}{2} - \frac{\sigma^2_{X_q}\left(m^i_{q_2}-m^i_{q_1}\right)}{2(\sigma^i_q)^2}$	$x_q \in \left[m^i_{q_1}, m^i_{q_2}\right]$	$x_{low_q} = \frac{(\sigma_{X_q})^2 m^i_{q_2}+(\sigma^i_q)^2 x_q}{(\sigma_{X_q})^2+(\sigma^i_q)^2}$	$x_{up_q} = x_q$
3	$x_q \in \left[\frac{m^i_{q_1}+m^i_{q_2}}{2} - \frac{\sigma^2_{X_q}\left(m^i_{q_2}-m^i_{q_1}\right)}{2(\sigma^i_q)^2}, \frac{m^i_{q_1}+m^i_{q_2}}{2} + \frac{\sigma^2_{X_q}\left(m^i_{q_2}-m^i_{q_1}\right)}{2(\sigma^i_q)^2}\right]$	$x_q \in \left[m^i_{q_1}, m^i_{q_2}\right]$	$x_{low_q} = \frac{m^i_{q_1}+m^i_{q_2}}{2}$	$x_{up_q} = x_q$
4	$x_q > \frac{m^i_{q_1}+m^i_{q_2}}{2} + \frac{\sigma^2_{X_q}\left(m^i_{q_2}-m^i_{q_1}\right)}{2(\sigma^i_q)^2}$	$x_q \in \left[m^i_{q_1}, m^i_{q_2}\right]$	$x_{low_q} = \frac{(\sigma_{X_q})^2 m^i_{q_1}+(\sigma^i_q)^2 x_q}{(\sigma_{X_q})^2+(\sigma^i_q)^2}$	$x_{up_k} = x_q$
5	$x_q > \frac{m^i_{q_1}+m^i_{q_2}}{2} + \frac{\sigma^2_{X_q}\left(m^i_{q_2}-m^i_{q_1}\right)}{2(\sigma^i_q)^2}$	$x_q > m^i_{q_2}$	$x_{low_q} = \frac{(\sigma_{X_q})^2 m^i_{q_1}+(\sigma^i_q)^2 x_q}{(\sigma_{X_q})^2+(\sigma^i_q)^2}$	$x_{up_q} = \frac{(\sigma_{X_q})^2 m^i_{q_2}+(\sigma^i_q)^2 x_q}{(\sigma_{X_q})^2+(\sigma^i_q)^2}$

3.4 Simulation and analysis

The MATLAB software was used to model and simulate the PID controllers using ZN, the T1 SFLS and NSFLS, the IT2 SFLS, the IT2 NSFLS-1, and the proposed assembly using IT2 NSFLS-1 with BP learning mechanism (PID IT2 NSFLS-1 BP) to calculate the PID gains dynamic adjustments. In this experiment was used the second-order model of a hydraulic actuator that operation range is 4–20 mA. The control signal opens and closes the valve. The reference value was selected as $y_{ref} = 20.0$ mA that means the valve is 100% opened. The set point can be selected as any value of the operational range 4–20 mA.

The simulation process followed the next steps:

(a) The parameters of the plant were identified through the experimental test, obtaining $K = 2.0$, $T_1 = 5.0$, $T_2 = 10.0$, $T_m = \frac{T1}{20.0} = \frac{5.0}{20.0} = 0.25$, $T_c = 1.0 \& T_c \leq 5.0 * T_m = 1.25$, $K_2 = 0.0024$, $a_1 = -1.9265$, $a_2 = 0.9277$.

The second-order model for the valve using (1) was the next:

$$G(s) = \frac{2}{(5s+1)(10s+1)} \tag{87}$$

$$y(k) = 0.0024 * u(k) - (-1.9265) * y(k-1) - (0.9277)y(k-2) \tag{88}$$

(b) The initial values of the gains of the PID controller were obtained through the ZN method using the previous parameters of the second-order plant; $T_c = 1.0, K_p = 6.0, T_i = 70.0, T_d = 6.7.$ In this experiment, the initial values of the PID gains were: $K_p(0) = 1.3$, $K_i(0) = 0.0153$, $K_d(0) = 15.6$

$$u(k) = \left(K_p + K_i * k\right)y_{ref} - \left(K_p + K_i + K_d\right)y(k-1) - (K_i - K_d)y(k-2) - (0.0153)\sum_{i=0}^{k-3} y(i) \tag{89}$$

$$u(k) = 1.3153 * y_{ref} - (16.9153)y(k-1) - (-15.5847)y(k-2) - (K_i)\sum_{i=0}^{k-3} y(i) \tag{90}$$

(c) c Each consequent was designed to have two fuzzy sets: (1) one fuzzy set for the input x_1, the error $e(k)$, was modeled using the mean value $\frac{\mu_{el}+\mu_{er}}{2}$ and the standard deviation value σ_e; and (2) a second fuzzy set for the input x_2, the change of error $\Delta e(k)$ modeled using the mean values $\frac{\mu_{\Delta el}+\mu_{\Delta er}}{2}$ and the standard deviation $\sigma_{\Delta e}$ of it. Both of them were modeled as Gaussians of type-1 fuzzy values.

(d) The numerical values of each uncertain fuzzy set of the PID controller using the IT2 NSFLS-1 with BP learning mechanism (PID IT2 NSFLS-1 BP) were initialized as shown in Table 10. Each consequent was designed to have two fuzzy sets: (1) one fuzzy set for the input x_1, the error $e(k)$ that was modeled using two mean values $[\mu_{el}, \mu_{er}]$, and the standard deviation value σ_e; and (2) a second fuzzy set for the input x_2, the change of error $\Delta e(k)$ modeled using two mean values $[\mu_{\Delta el}, \mu_{\Delta er}]$, and the standard deviation $\sigma_{\Delta e}$ of it. Both of them were modeled as Gaussians type-2 fuzzy values.

The values of the non-singleton inputs: (1) the uncertain error $e(k) + \sigma e(k)$, and (2) the uncertain change of error, $\Delta e(k) + \sigma \Delta e(k)$, were modeled as non-singleton Gaussians fuzzy values. The initial parameters values of the three IT2 NSFLS-1 systems are shown in Table 10.

Table 6 Selection of antecedent parameters of $\mathbf{y_r}$ using BP method for tuning $\mathbf{K_p}$ and $\mathbf{K_i}$ IT2 NSFLS-1 parameters

	Position of x_q		Parameter of antecedent membership function that contributes to the right-most
1	$x_q \le m_{q_1}^i$	$\bar{f}_r^i \in \left(\bar{f}^{R+1},\ldots,\bar{f}^M\right)$	$m_{q_1}^i(k+1) = m_{q_1}^i(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{[c_r^i - y_r]}{\sum_{i=1}^{R}\underline{f}^i+\sum_{i=R+1}^{M}\bar{f}^i}\right]\left[\frac{x_q - m_{q_1}^i}{(\sigma_{X_q})^2+(\sigma_q^i)^2}\right]\left[\bar{f}^i\left(x_{up_q}\right)\right]$
			$\sigma_q^i(k+1) = \sigma_q^i(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{[c_r^i - y_r]}{\sum_{i=1}^{R}\underline{f}^i+\sum_{i=R+1}^{M}\bar{f}^i}\right]\left[\sigma_q^i\right]\left[\frac{x_q - m_{q_1}^i}{(\sigma_{X_q})^2+(\sigma_q^i)^2}\right]^2\left[\bar{f}^i\left(x_{up_q}\right)\right]$
2	$x_q \ge m_{q_2}^i$	$\bar{f}_r^i \in \left(\bar{f}^{R+1},\ldots,\bar{f}^M\right)$	$m_{q_2}^i(k+1) = m_{q_2}^i(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{[c_r^i - y_r]}{\sum_{i=1}^{R}\underline{f}^i+\sum_{i=R+1}^{M}\bar{f}^i}\right]\left[\frac{x_q - m_{q_2}^i}{(\sigma_{X_q})^2+(\sigma_q^i)^2}\right]\left[\bar{f}^i\left(x_{up_q}\right)\right]$
			$\sigma_q^i(k+1) = \sigma_q^i(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{[c_r^i - y_r]}{\sum_{i=1}^{R}\underline{f}^i+\sum_{i=R+1}^{M}\bar{f}^i}\right]\left[\sigma_q^i\right]\left[\frac{x_q - m_{q_2}^i}{(\sigma_{X_q})^2+(\sigma_q^i)^2}\right]^2\left[\bar{f}^i\left(x_{up_q}\right)\right]$
3	$x_q \le \frac{m_{q_1}^i+m_{q_2}^i}{2} - \frac{\sigma_{X_q}^2\left(m_{q_2}^i - m_{q_1}^i\right)}{2(\sigma_q^i)^2}$	$\underline{f}_r^i \in \left(\underline{f}^1,\ldots,\underline{f}^R\right)$	$m_{q_2}^i(k+1) = m_{q_2}^i(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{[c_r^i - y_r]}{\sum_{i=1}^{R}\underline{f}^i+\sum_{i=R+1}^{M}\bar{f}^i}\right]\left[\frac{x_q - m_{q_2}^i}{(\sigma_{X_q})^2+(\sigma_q^i)^2}\right]\left[\underline{f}^i\left(x_{up_q}\right)\right]$
			$\sigma_q^i(k+1) = \sigma_q^i(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{[c_r^i - y_r]}{\sum_{i=1}^{R}\underline{f}^i+\sum_{i=R+1}^{M}\bar{f}^i}\right]\left[\sigma_q^i\right]\left[\frac{x_q - m_{q_2}^i}{(\sigma_{X_q})^2+(\sigma_q^i)^2}\right]^2\left[\underline{f}^i\left(x_{up_q}\right)\right]$
4	$x_q \ge \frac{m_{q_1}^i+m_{q_2}^i}{2} + \frac{\sigma_{X_q}^2\left(m_{q_2}^i - m_{q_1}^i\right)}{2(\sigma_q^i)^2}$	$\underline{f}_r^i \in \left(\underline{f}^1,\ldots,\underline{f}^R\right)$	$m_{q_1}^i(k+1) = m_{q_1}^i(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{[c_r^i - y_r]}{\sum_{i=1}^{R}\underline{f}^i+\sum_{i=R+1}^{M}\bar{f}^i}\right]\left[\frac{x_q - m_{q_1}^i}{(\sigma_{X_q})^2+(\sigma_q^i)^2}\right]\left[\underline{f}^i\left(x_{up_q}\right)\right]$
			$\sigma_q^i(k+1) = \sigma_q^i(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{[c_r^i - y_r]}{\sum_{i=1}^{R}\underline{f}^i+\sum_{i=R+1}^{M}\bar{f}^i}\right]\left[\sigma_q^i\right]\left[\frac{x_k - m_{q_1}^i}{(\sigma_{X_q})^2+(\sigma_q^i)^2}\right]^2\left[\underline{f}^i\left(x_{up_q}\right)\right]$

Table 7 Selection of antecedent parameters of y_l using BP method for tuning K_p and K_i IT2 NSFLS-1 parameters

	Position of x_q		Parameter of antecedent membership function that contributes to the left-most
1	$x_q \le m^i_{q_1}$	$\bar{f}^i_l \in \left(\bar{f}^1,\ldots,\bar{f}^L\right)$	$m^i_{q_1}(k+1) = m^i_{q_1}(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{\left[c^i_l - y_l\right]}{\sum_{i=1}^{L}\bar{f}^i + \sum_{i=L+1}^{M}\underline{f}^i}\right]\left[\frac{x_q - m^i_{q_1}}{(\sigma_{X_q})^2 + (\sigma^i_q)^2}\right]\left[\bar{f}^i\left(x_{\mathrm{low}_q}\right)\right]$
			$\sigma^i_q(k+1) = \sigma^i_q(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{\left[c^i_l - y_l\right]}{\sum_{i=1}^{L}\bar{f}^i + \sum_{i=L+1}^{M}\underline{f}^i}\right]\left[\sigma^i_q\right]\left[\frac{x_q - m^i_{q_1}}{(\sigma_{X_q})^2 + (\sigma^i_q)^2}\right]^2\left[\bar{f}^i\left(x_{\mathrm{low}_q}\right)\right]$
2	$x_q \ge m^i_{q_2}$	$\bar{f}^i_l \in \left(\bar{f}^1,\ldots,\bar{f}^L\right)$	$m^i_{q_2}(k+1) = m^i_{q_2}(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{\left[c^i_l - y_l\right]}{\sum_{i=1}^{L}\bar{f}^i + \sum_{i=L+1}^{M}\underline{f}^i}\right]\left[\frac{x_k - m^i_{q_2}}{(\sigma_{X_q})^2 + (\sigma^i_q)^2}\right]\left[\bar{f}^i\left(x_{\mathrm{low}_q}\right)\right]$
			$\sigma^i_q(k+1) = \sigma^i_q(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{\left[c^i_l - y_l\right]}{\sum_{i=1}^{L}\bar{f}^i + \sum_{i=L+1}^{M}\underline{f}^i}\right]\left[\sigma^i_q\right]\left[\frac{x_q - m^i_{q_2}}{(\sigma_{X_q})^2 + (\sigma^i_q)^2}\right]^2\left[\bar{f}^i\left(x_{low_q}\right)\right]$
3	$x_q < \frac{m^i_{q_1} + m^i_{q_2}}{2} - \frac{\sigma^2_{X_q}\left(m^i_{q_2} - m^i_{q_1}\right)}{2(\sigma^i_q)^2}$	$\underline{f}^i_l \in \left(\underline{f}^{L+1},\ldots,\underline{f}^M\right)$	$m^i_{q_2}(k+1) = m^i_{q_2}(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{\left[c^i_l - y_l\right]}{\sum_{i=1}^{L}\bar{f}^i + \sum_{i=L+1}^{M}\underline{f}^i}\right]\left[\frac{x_q - m^i_{q_2}}{(\sigma_{X_q})^2 + (\sigma^i_q)^2}\right]\left[\underline{f}^i\left(x_{low_q}\right)\right]$
			$\sigma^i_q(k+1) = \sigma^i_q(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{\left[c^i_l - y_l\right]}{\sum_{i=1}^{L}\bar{f}^i + \sum_{i=L+1}^{M}\underline{f}^i}\right]\left[\sigma^i_q\right]\left[\frac{x_q - m^i_{q_2}}{(\sigma_{X_q})^2 + (\sigma^i_q)^2}\right]^2\left[\underline{f}^i\left(x_{low_q}\right)\right]$
4	$x_q \ge \frac{m^i_{q_1} + m^i_{q_2}}{2} + \frac{\sigma^2_{X_q}\left(m^i_{q_2} - m^i_{q_1}\right)}{2(\sigma^i_q)^2}$	$\underline{f}^i_l \in \left(\underline{f}^{L+1},\ldots,\underline{f}^M\right)$	$m^i_{q_1}(k+1) = m^i_{q_1}(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{\left[c^i_l - y_l\right]}{\sum_{i=1}^{L}\bar{f}^i + \sum_{i=L+1}^{M}\underline{f}^i}\right]\left[\frac{x_q - m^i_{q_1}}{(\sigma_{X_q})^2 + (\sigma^i_q)^2}\right]\left[\underline{f}^i\left(x_{\mathrm{low}_q}\right)\right]$
			$\sigma^i_q(k+1) = \sigma^i_q(k) + \alpha\frac{1}{2}K_2 e(k-1)\left[\frac{\left[c^i_l - y_l\right]}{\sum_{i=1}^{L}\bar{f}^i + \sum_{i=L+1}^{M}\underline{f}^i}\right]\left[\sigma^i_q\right]\left[\frac{x_q - m^i_{q_1}}{(\sigma_{X_q})^2 + (\sigma^i_q)^2}\right]^2\left[\underline{f}^i\left(x_{\mathrm{low}_q}\right)\right]$

Table 8 Selection of consequent parameter of y_r using BP method for tuning the K_p and K_i IT2 NSFLS-1 parameters

	Parameter of consequent membership function that contributes to the right-most
$\underline{f}_r^i \in \left(\underline{f}^1, \ldots, \underline{f}^R\right)$	$y_r^i(k+1) = y_r^i(k) + \alpha \frac{1}{2} K_2 e(k-1) \left[\frac{\left[\bar{f}\left(x_{\mathrm{low}_q}\right)\right]}{\sum_{i=1}^{R} \underline{f}^i + \sum_{i=R+1}^{M} \bar{f}^i} \right]$
$\bar{f}_r^i \in \left(\bar{f}^{R+1}, \ldots, \bar{f}^{MR}\right)$	$y_r^i(k+1) = y_r^i(k) + \alpha \frac{1}{2} K_2 e(k-1) \left[\frac{\left[\bar{f}\left(x_{\mathrm{up}_q}\right)\right]}{\sum_{i=1}^{R} \underline{f}^i + \sum_{i=R+1}^{M} \bar{f}^i} \right]$

Table 9 Selection of consequent parameter of y_l using BP Method for tuning the K_p and K_i IT2 NSFLS-1 parameters

	Parameter of consequent membership function that contributes to the right-most
$\bar{f}_l^i \in \left(\bar{f}^1, \ldots, \bar{f}^L\right)$	$y_l^i(k+1) = y_l^i(k) + \alpha \frac{1}{2} K_2 e(k-1) \left[\frac{\bar{f}\left(x_{\mathrm{up}_q}\right)}{\sum_{i=1}^{L} \bar{f}^i + \sum_{i=L+1}^{M} \underline{f}^i} \right]$
$\underline{f}_l^i \in \left(\underline{f}^{L+1}, \ldots, \underline{f}^M\right)$	$y_l^i(k+1) = y_l^i(k) + \alpha \frac{1}{2} K_2 e(k-1) \left[\frac{\underline{f}\left(x_{\mathrm{low}_q}\right)}{\sum_{i=1}^{L} \bar{f}^i + \sum_{i=L+1}^{M} \underline{f}^i} \right]$

Table 10 Universe of disclosure and initial values of fuzzy sets for ΔKP(k), ΔKI(k), and ΔKD(k) estimation. There are three IT2 NSFLS-1 learning mechanism with BP learning mechanism

Input 1 $(\boldsymbol{e})$ $\boldsymbol{\mu_{el}}$	Nbl = − 22	Nml = − 16	Nsl = − 7	Zcl = − 2	Psl = 4	Pml = 12	Pbl = 18
Input 1 $(\boldsymbol{e})$ $\boldsymbol{\mu_{er}}$	Nbr = − 18	Nmr = − 14	Nsr = − 6	Zcr = 2	Psr = 8	Pmr = 16	Pbr = 22
Input 1 $(\boldsymbol{e})$ $\boldsymbol{\sigma_e}$	6	6	6	6	6	6	6
Input 2 $(\Delta\boldsymbol{e})$ $\boldsymbol{\mu_{\Delta el}}$	nbl = − 12	nml = − 7	nsl = − 4	zcl = − 2	psl = 0	pml = 3	pbl = 8
Input 2 $(\Delta\boldsymbol{e})$ $\boldsymbol{\mu_{\Delta er}}$	nbr = − 8	nm = − 3	nsr = − 1	zcr = 2	psr = 4	pmr = 7	pbr = 12
Input 2 $(\Delta\boldsymbol{e})$ $\boldsymbol{\sigma_{\Delta e}}$	2	2	2	2	2	2	2
Output $\boldsymbol{y_l^i}$	NBl = − 11	NMl = − 9	NSl = − 7	Z0 l = − 2	PSl = 7	PMl = 10	PBl = 16
Output $\boldsymbol{y_r^i}$	NBr = − 7	NMr = − 5	NSr = − 3	Z0r = 2	PSr = 11	PMr = 14	PBr = 20

The parameters are updated each control period of time using the BP's derivative equations as shown in Tables 5, 6, 7, 8 and 9.

3.4.1 Experimental modeling

The results of the experimental modeling process are shown in Fig. 7. The stabilization times at 20 mA, arbitrarily selected as the operating set point, are shown in Table 11. The proposed assembly PID IT2 NSFLS-1 BP controller stabilizes 1.5 times fast that the one controller that uses the PID ZN and these using the T1 SFLS, the T1 NSFLS, and the IT2 NSFLS1 systems. There is not overshooting in any PID controller as shown in caption a) of Fig. 7, which shows the transient period of the six PID controllers: classic ZN, the updated by T1 SFLS, T1 NSFLS, IT2 SFLS by IT2 NSFLS-1 and by the proposed IT2 NSFLS-1 with BP learning mechanism.

3.4.2 Experimental modeling with noise

The next experiment was performed with the purpose to know the behavior of the plant under noise disturbance with a signal-to-noise ratio (SNR) of $\cong$ 15 dB. When the valve was at the stable state zone of the set point, a noise was introduced. In the simulation process of the five PID controllers, the noise was added directly to the control signal as shown in (91), where $u_n(k)$ is the control signal of normal operation, and $r(k)$ is the imposed noise with a SNR of approximately 10% of the normal control signal value. The corresponding output can be calculated using next equations,

$$u(k) = u_n(k) + r(k) \tag{91}$$

$$y_m(k) = y_u(k) + y_{rn}(k) \tag{92}$$

where $y_u(k)$, and $y_{rn}(k)$ are the model outputs to $u_n(k)$, and $r(k)$ inputs.

In order to obtain a 10% of variation in the steady state, this simulation was performed with a control value $u(k)$ with a negative noise value of -2.0, $u(k) = u_n(k) - 2.0$, and was established during three cycles of time. After that, the control signal was set to its calculated value, $u(k) = u_n(k)$. The behavior of the PID controller is shown in caption (b) of Fig. 7. The caption shows the behavior of the controller systems with negative noise

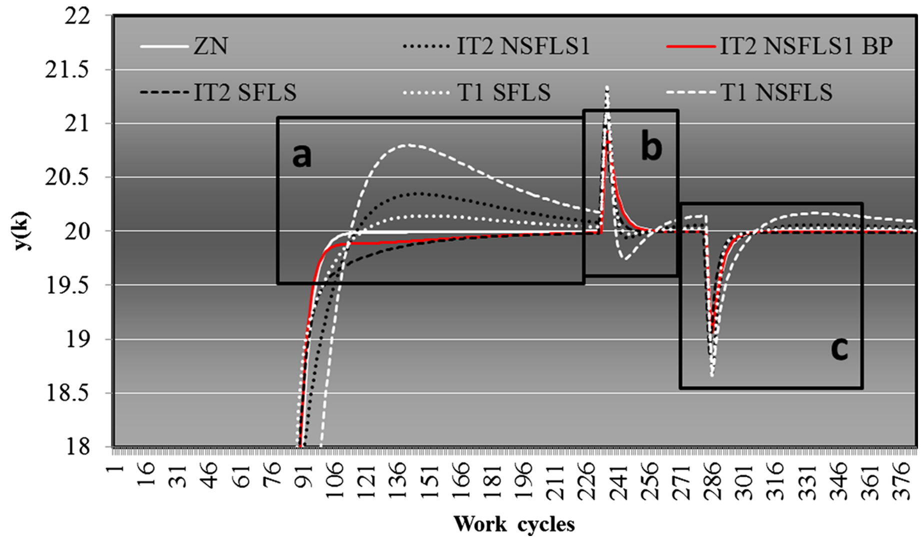

Fig. 7 Comparison of the performance among the PID controllers: the classic PID ZN controller, the one that uses gains update by T1 and IT2 fuzzy system, and the proposed PID using IT2 NSFLS-1 system with BP algorithm

Table 11 Comparison of transient periods

PID system	Cycles for stabilization
ZN	211 (stability remains by 138 cycles)
T1 SFLS	194 (stability remains by 8 cycles)
T1 NSFLS	190 (stability remains by 33 cycles)
IT2 SFLS	249 (stability remains by 54 cycles)
IT2 NSFLS1	245 (stability remains by 42 cycles)
IT2 NSFLS-1 with BP learning mechanism	211 (stability remains by 189 cycles)

Table 12 Stabilization for negative disturbance

PID system	Cycles for stabilization
ZN	61 (stability remains by 17 cycles)
T1 SFLS	339 (stability remains by 69 cycles)
T1 NSFLS	207 (stability remains by 13 cycles)
IT2 SFLS	15 (stability remains by 76 cycles)
IT2 NSFLS1	68 (stability remains by 135 cycles)
IT2 NSFLS-1 with BP learning mechanism	72 (stablity remains by 130 cycles)

perturbation. The values obtained from this experiment are shown in Table 12.

The response of the PID IT2 NSFLS-1 with BP learning mechanism is smooth and stable in comparison with the other benchmark PID controllers.

In the same manner, a positive noise of $u(k)$ with 2 mA of magnitude and 3 cycles of time was applied in all the benchmarking systems during the steady state as can be seen in caption (c) of Fig. 7, which shows the behavior of the PID systems with a positive noise perturbation.

Figure 8a shows the behavior of the proposed hybrid assembly and the benchmark models in the transient phase. There, it can be see that the stabilization takes around of 60 cycles with a smooth curve in contrast to the benchmark fuzzy models that presents overshoot and damping. Also when the model is under positive or negative disturbance the behavior of the model is equal as shown in the insets b, d, f of Fig. 8. The proposal assembly doesn't show overshoot and damping and it presents the best stability before and after the perturbation.

The behavior of the PID ZN and the PID T1 NSFLS systems under the positive disturbance reflects a slow recovery behavior in comparison with the proposed PID IT2 NSFLS-1 with BP learning mechanism performance. The stabilization times are shown in Table 13.

There is a difference between the behaviors of these five systems depending on the negative or a positive value of the noise. The analysis shows that the response for recovery of the proposed PID IT2 NSFLS-1 BP controller is

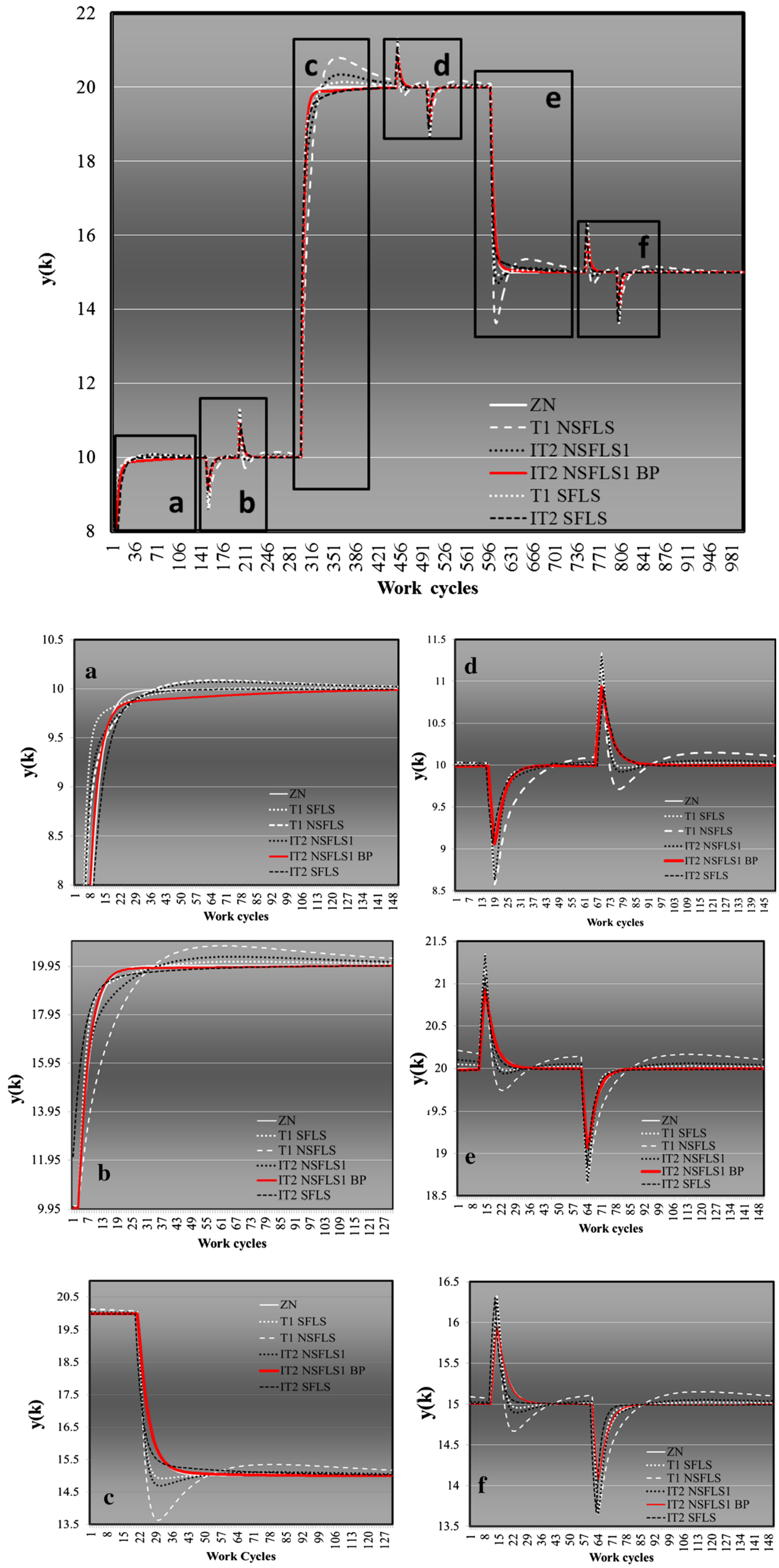

Fig. 8 Comparison of the performance among the NZ, T1 SFLS, T1 NSFLS, IT2 SFLS, IT2 NSFLS-1 hybrid PID controllers at different operating set points: 10, 15, 20 mA

Table 13 Stabilization for positive disturbance

PID system	Cycles to stabilization
ZN	66 (stability remains by 144 cycles)
T1 SFLS	108 (stability remains by 100 cycles)
T1 NSFLS	43 (stability remains by 24 cycles)
IT2 SFLS	15 (stability remains by 137 cycles)
IT2 NSFLS1	206 (stability remains by 25 cycles)
IT2 NSFLS-1 with BP learning mechanism	60 (stability remains by 144 cycles)

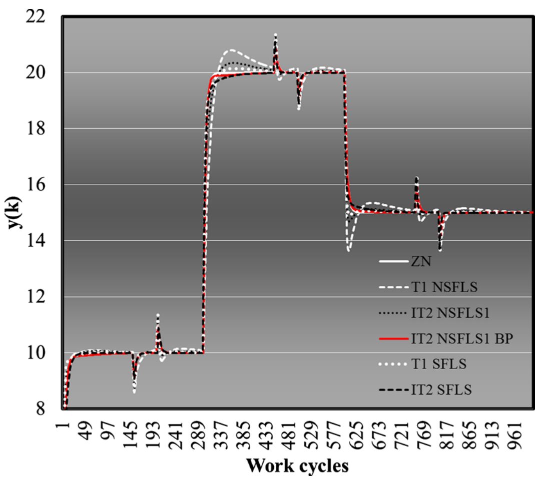

Fig. 9 Comparison of the performance among the PID controllers: the classic PID ZN controller, the one that uses gains update by T1 and IT2 FLS system, and the proposed PID using IT2 NSFLS-1 system for gains update

faster than ZN, T1 SFLS and IT2 NSFLS1 benchmarking controllers.

3.4.3 Experimental modeling with different set points

The fuzzy PID controllers were tested with three different operating set points with arbitrarily selected values as 10, 20 and 15 mA. Different set points were tested to observe the behavior of the systems under a perturbation and to observe the transient periods. In the stable period (Fig. 9), there were added a negative (− 2.0 mA) and positive (+ 2.0 mA) disturbances with 3 cycles of application. The results of the simulation are shown in Figs. 8 and 9. The captions (a), (b) and (c) in Fig. 8 show the stabilization times after the sequential change of the values of the operating set point. The captions from (d) to (f) of Fig. 8 show the behavior of the PID controllers under the influence of negative and positive disturbances.

4 Conclusion

At the end of the experimental phase of this work, the results show fast response and an improvement in the stabilization behavior of the output of the plant under the control of the proposed assembly. The performance of the proposal overcomes the performance of the five benchmarking assemblies: the PID ZN, the PID and the T1 SFLS, the PID and the T1 NSFLS, the PID and the IT2 SFLS, the PID and the IT2 NSFLS-1. Table 14 shows a complete comparison among these benchmarking controllers.

The proposal is a breakthrough in PID modeling. According to the literature, this is the first time that the PID can process the uncertainties and noise of the measurements thanks to the IT2 NSFLS1 systems that has its parameters and its output value in the form of non-singleton numbers, Gaussian numbers with its mean value and its standard deviation value.

Table 14 Comparison of the stabilization on negative–positive disturbances, and overshooting between the PID ZN, the PID T1 NSFLS & the IT2 NSFLS-1

	PID ZN	PID T1 NSFLS	PID IT2 NSFLS-1	Improvements of the PID IT2 NSFLS-1 over the PID T1 NSFLS
Stabilization time (cycles)	211	190	211	11% lesser
Overshooting	11.15%	0.03%	0	Without overshooting
Stabilization cycles in negative disturbance	61	207	72	287% lesser
Stabilization cycles in positive disturbance	66	43	60	39% superior

The best performance is obtained from the proposed assembly that uses the PID controller and the IT2 NSFLS-1 systems that calculate the specific adjustments for the PID's gains required to carry out the output of the plant to the desired set point in the shortest period of time and with the required condition of stability for the plant's response.

The most important characteristic of the proposed IT2 NSFLS1BP system is the capability to processes the uncertainty coming from the input data, the sensor, the modeling of the IT2 system and the initial values of their parameters. Uncertainties filtering provide a characteristic to the system to adapt their performance to the desired set point with small variations that provide stability to the control process, however, in the presence of perturbations, and arise as the better assembly to avoid the loss of stability.

Other PID approximations only work with the mean of the measures, without the standard deviation. This proposal works with central and dispersion statistic indicators of the measurements covering all uncertainties and noise in the measurement.

The results of the experiment show that the stability of the proposed assembly is clearly the best because the output remains almost two periods of time in contrast with other proposals or tests, and also the proposal of IT2 NSFLS1 BP maintains the stability in presence of a perturbations and this stability do not presents oscillations as overshoot and damping. In the assembly constructed using the PID controller and the T1 SFLS the stability remains only 9 cycles in contrast with the period of time of the proposal, which stability time remains by 189 cycles in the worst case. In the better case the stability of the proposed model remains 1.5 times in contrast to the PID and 19 times against to the T1 NSFLS. Also in the transient period it is observed a similar situation.

The proposed assembly of the PID controller using IT2 NSFLS-1 systems with BP shows the best stability and the best performance compared to the five assemblies used as benchmarking systems.

In the simulations results it can be seen patterns in the behavior of the response of the plant that provide the chance to predict the behavior of the model over a disturbance. Also, the variation of the proposal is completely stable with a variation of 10% over the rest of the models that have variations more than 15% over a controlled disturbance.

The stabilization of the proposed assembly is faster that their counterpart's of the time cycles to recover the stability.

As future work the authors:

- will apply the model to real industrial problems where the uncertainties are non-stationary in a hot strip mill facility located in Monterrey, N.L. MX., firstly in off line mode and secondly in real-time mode,
- will work in a proposal of the PID control model using artificial neural networks for the PID gains calculation,
- will apply the BP model onto a GT2 model to evaluate the performance in a PID controller,
- will propose other type of models for tuning with metaheuristics assemblies,
- will apply the proposed assembly in a plant modeled with time delay in its output.

Compliance in Ethical Standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study.

References

Alberto AB (2000) Temas de identificación y control adaptable. Instituto de Cibernética Matemática y Física (ICIMAF), La Habana, Cuba

Aliasghary M, Eksin I, Güzelkaya M, Kumbasar T (2012) Design of an interval type-2 fuzzy logic controller based on conventional PI controller. Presented at the 20th Mediterranean conference on control and automation (MED), July, pp 627–632

Álvarez A, Reyes D, Rincón EJ, Valderrama J, Noradino P, Méndez GM (2018) PID implemented by a type-1 fuzzy logic system with back-propagation algorithm for online tuning of its gains. In: Melin P, Castillo O, Kacprzyk J, Reformat M, Melek W (eds) Fuzzy logic in intelligent system design: theory and applications. Proceedings of NAFIPS 2017. Advances in intelligent systems and computing, vol 648. Springer, Cham, pp 256–263

Arghavani N, Almobaied M, Guzelkaya M, Eksin I (2017) On-line rule weighting for PID-type fuzzy logic controllers using extended Kalman filter. IFAC-PapersOnLine 50(1):7140–7145

Beirami H, Zerafat MM (2015) Self-tuning of an interval type-2 fuzzy PID controller for a heat exchanger system. Iran J Sci Technol Trans Mech Eng 39:113–129

Castillo O, Amador-Angulo L, Castro JR, Garcia-Valdez M (2016a) A comparative study of type-1 fuzzy logic systems, interval type-2 fuzzy logic systems and generalized type-2 fuzzy logic systems in control problems. Inf Sci 354:257–274

Castillo O, Cervantes L, Soria J, Sanchez M, Castro JR (2016b) A generalized type-2 fuzzy granular approach with applications to aerospace. Inf Sci 354:165–177

Castro JR, Castillo O, Melin P, Rodríguez-Díaz A (2008) Building fuzzy inference systems with a new interval type-2 fuzzy logic toolbox. Transactions on computational science I. Springer, Berlin, pp 104–114

Cervantes L, Castillo O (2015) Type-2 fuzzy logic aggregation of multiple fuzzy controllers for airplane flight control. Inf Sci 324:247–256

Deepak G, Cheolkeun H (2013) Control of a quadrotor using a smart self-tuning fuzzy PID controller. Int J Adv Robot Syst 1:1–9

El-Bardini M, El-Nagar A (2014a) Interval type-2 fuzzy PID controller: analytical structures and stability analysis. Arab J Sci Eng 39(10):7443–7458

El-Bardini M, El-Nagar AM (2014b) Interval type-2 fuzzy PID controller for uncertain nonlinear inverted pendulum system. ISA Trans 53(3):732–743

Fatihu Hamza M, Yapa H, Ahmed Choudhury I (2017) Cuckoo search algorithm based design of interval Type-2 Fuzzy PID. Eng Appl Artif Intell 62:134–151

Gaidhane PJ, Nigam MJ, Kumar A, Pradhan PM (2019) Design of interval type-2 fuzzy precompensated PID controller applied to two-DOF robotic manipulator with variable payload. ISA Trans 89:169–185

Jie SUN, Zhang DH, Xu L, Zhang J, Du DS (2010) Smith prediction monitor AGC system based on fuzzy self-tuning PID control. J Iron Steel Res Int 17(2):22–26

Karasakal O, Guzelkaya M, Eksin I, Yesil E, Kumbasar T (2013) Online tuning of fuzzy PID controllers via rule weighing based on normalized acceleration. Eng Appl Artif Intell 26(1):184–197

Khosla A, Leena G, Soni MK (2014) Interval type-2 fuzzy logic controller to control the velocity and angle of inverted pendulum. Int J Intell Syst Appl 6(7):44–51

Kosari A, Jahanshahi H, Razavi SA (2017) An optimal fuzzy PID control approach for docking maneuver of two spacecraft: orientational motion. Eng Sci Technol Int J 20(1):293–309

Kudinov YI, Kolesnikov VA, Paschenko FF, Paschenko AF, Papoc L, (2017) Optimization of fuzzy PID controller's parameters. In: 12th international symposium "intelligent systems", INTELS'16, 5 Oct 2016, Elsevier, Procedia Computer Sciences, vol 103, pp 618–622

Kumar A, Kumar V (2017a) Hybridized ABC-GA optimized fractional order fuzzy pre-compensated FOPID Control design for 2-DOF robot manipulator. AEU-Int J Electron Commun 79(2017):219–233

Kumar A, Kumar V (2017b) A novel interval type-2 fractional order fuzzy PID controller: design, performance evaluation, and its optimal time domain tuning. ISA Trans 68(2017):251–275

Kumar A, Kumar V (2017c) Evolving an interval type-2 fuzzy PID controller for the redundant robotic manipulator. Expert Syst Appl 73(2017):161–177

Kumar A, Kumar V (2017d) Artificial bee colony based design of the interval type-2 fuzzy PID controller for robot manipulator. Presented at TENCON 2017–2017 IEEE Region 10 conference, November, 2017, pp 602–607

Kumar A, Kumar V, Gaidhane PJ (2018) Optimal design of fuzzy fractional order PIλDμ controller for redundant robot. Procedia Comput Sci 125:442–448

Kumbasar T (2014a) Robust stability analysis of PD type single input interval type-2 fuzzy control systems. Presented at 2014 IEEE international conference on fuzzy systems (FUZZ-IEEE), pp 634–639

Kumbasar T (2014b) A simple design method for interval type-2 fuzzy PID controllers. Soft Comput 18(7):1293–1304

Kumbasar T (2016) Interval type-2 fuzzy PID controllers and an online self-tuning mechanism. Pamukkale Univ J Eng Sci 22(8):643–649

Kumbasar T, Hagras H (2015) A self-tuning zSlices-based general type-2 fuzzy PI controller. IEEE Trans Fuzzy Syst 23(4):991–1013

Kumbasar T, Hagras H (2017) A gradient descent based online tuning mechanism for pi type single input interval type-2 fuzzy logic controllers. Presented at IEEE international conference on fuzzy systems (FUZZ-IEEE), pp 1–6

Kumbasar T, Yesil E, Karasakal O (2013) Self-tuning interval type-2 fuzzy PID controllers based on online rule weighting. Presented at the 2013 IEEE international conference on fuzzy systems (FUZZ-IEEE), July, pp 1–6

Mehndiratta M, Kayacan E, Kumbasar T (2016) Design and experimental validation of single input type-2 fuzzy PID controllers as applied to 3 DOF helicopter testbed. Presented at the 2016 IEEE international conference on fuzzy systems (FUZZ-IEEE), July, 2014, pp 1584–1591

Mendel JM (2001) Uncertain rule-based fuzzy logic systems: introduction and new directions. Prentice Hall PTR, Upper Saddle River

Meza J, Santibáñez V, Soto R, Llama MA (2012) Fuzzy self-tuning PID semi global regulator for robot manipulators. IEEE Trans Ind Electron 59(6):2709–2717

Mohanty PK, Sahu BK, Pati TK, Panda S, Kar SK (2016) Design and analysis of fuzzy PID controller with derivative filter for AGC in multi-area interconnected power system. IET Gener Transm Distrib 10(15):3764–3776

Nayak JR, Shaw B, Sahu BK (2018) Application of adaptive-sos (asos) algorithm based interval-type-2 fuzzy-PID controller with derivative filter for automatic generation control of an interconnected power system. Eng Sci Technol Int J 21(3):465–485

Olivas F, Valdez F, Melin P, Sombra A, Castillo O (2019) Interval type-2 fuzzy logic for dynamic parameter adaptation in a modified gravitational search algorithm. Inf Sci 476:159–175

Ontiveros-Robles E, Melin P, Castillo O (2018) Comparative analysis of noise robustness of type 2 fuzzy logic controllers. Kybernetika 54(1):175–201

Ramos JM, Reyes E, Sanchez JL, Hernandez JI, Méndez GM (2016) A professional PID implemented using a non-singleton type-1 fuzzy logic system to control a stepper motor. Int J Eng Res Sci 2(2):94–101

Reyes E, Ramos J, Hernandez J, Sanchez L, Méndez GM (2016) Online tuning of the PID controller using the back-propagation algorithm. Int J Eng Res Sci 2(2):86–93

Reyes D, Álvarez A, Rincón EJ, Valderrama J, Noradino P, Méndez GM (2018) A PID using a non-singleton fuzzy logic system type 1 to control a second-order system. In: Melin P, Castillo O, Kacprzyk J, Reformat M, Melek W (eds) Fuzzy logic in intelligent system design: theory and applications. Proceedings of NAFIPS 2017. Advances in intelligent systems and computing, vol 648. Springer, Cham, pp 264–269

Sahin A, Kumbasar T (2018) Type-2 fuzzy logic control in computer games. In: John R, Hagras H, Castillo O (eds) Type-2 fuzzy logic and systems. Springer, Cham, pp 105–127

Sakalli A, Kumbasar T, Dodurka MF, Yesil E (2014a) The simplest interval type-2 fuzzy PID controller: structural analysis. Presented at the 2014 IEEE international conference on fuzzy systems (FUZZ-IEEE), July 2014, pp 626–633

Sakalli A, Kumbasar T, Yesil E, Hagras H (2014b) Analysis of the performances of type-1, self-tuning type-1 and interval type-2 fuzzy PID controllers on the magnetic levitation system. Presented at the 2014 IEEE international conference on fuzzy systems (FUZZ-IEEE), 2014, pp 1859–1866

Sakalli A, Beke A, Kumbasar T (2016) Gradient descent and extended Kalman filter based self-tuning interval type-2 fuzzy PID controllers. Presented at the 2016 IEEE international conference on fuzzy systems (FUZZ-IEEE), July, pp 1592–1598

Sanchez MA, Castillo O, Castro JR (2015a) Information granule formation via the concept of uncertainty-based information with interval type-2 fuzzy sets representation and Takagi–Sugeno–Kang consequents optimized with Cuckoo search. Appl Soft Comput 27:602–609

Sanchez MA, Castillo O, Castro JR (2015b) Generalized type-2 fuzzy systems for controlling a mobile robot and a performance

comparison with interval type-2 and type-1 fuzzy systems. Expert Syst Appl 42(14):5904–5914
Savran AI, Beke A, Kumbasar T, Yesil (2015). An IMC based fuzzy self-tuning mechanism for fuzzy PID controllers. Presented at the 2015 international symposium on innovations in intelligent systems and applications (INISTA), September, pp 1–7
Souran DM, Mir M, Mebrabian A, Razeghi B, Hatamian M, Sebtahmadi SS (2014) A performance comparison of classical PID, Type-1 and Type-2 fuzzy controller in a three tank level control system. In: IEEE international symposium on robotics and manufacturing automation (ROMA). IEEE, pp 86–91
Tang KS, Man KF, Guanrong C, Kwong S (2001) An optimal fuzzy PID controller. IEEE Trans Ind Electron 48(42):757–765
Trautzsch T, Dawson JG (2002) A stable self-tuning fuzzy logic control system for industrial temperature regulation. IEEE Trans Ind Appl 38(2):414–424
Var A, Kumbasar T, Yesil E (2015) An internal model control based design method for single input fuzzy PID controllers. Presented at IEEE international conference on fuzzy systems (FUZZ-IEEE), August, pp 1–7
Wang LX (1999) A course in fuzzy systems. Prentice-Hall Press, USA, pp 258–265
Yesil E (2014) Interval type-2 fuzzy PID load frequency controller using Big Bang-Big Crunch optimization. Appl Soft Comput 15:100–112
Yesil E, Guzay C (2014) A self-tuning fuzzy PID controller design using gamma aggregation operator. Presented at the IEEE international conference on fuzzy systems (FUZZ-IEEE), pp 2032–2038
Yesil E, Kumbasar T, Dodurka MF, Sakalli A (2014) Peak observer based self-tuning of type-2 fuzzy PID controllers. Presented at IFIP international conference on artificial intelligence applications and innovations, September, 2014, pp 487–497
Zarandi MF, Soltanzadeh S, Mohammadi A, Castillo O (2019) Designing a general type-2 fuzzy expert system for diagnosis of depression. Appl Soft Comput 80:329–341