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METHODOLOGIES AND APPLICATION

A new formulation for prediction of the shear capacity of FRP in strengthened reinforced concrete beams

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Abstract

The use of fiber-reinforced polymer (FRP) to strength the concrete beams is an efficient method in retrofitting of preexisting structures. The application of FRP sheets makes to have higher shear strength, but the common equations in determining the shear strength are no longer effective. In this paper, a new formulation is presented to predict the shear contribution of FRP in strengthened reinforced concrete beams. The formula is produced using the multigene genetic programming (MGP) machine. For this purpose, a set of experimental data is collected from the literature. The shear capacity of FRP in reinforced concrete (RC) beams is considered as the output data, while other variables are considered as the input data. MGP is trained with the experimental data and a formula is produced. The results of the proposed formula are compared with the experimental data to show the ability of the proposed formula. Also, these results are compared with those obtained from the available formulas, approximation models and published researches. Results show that the proposed formula is able to predict the shear capacity of FRP in strengthened RC beams with a higher precision than the other evaluated methods such as CIDAR, Fib.TG9.3, ACI and CSA. The mean absolute percentage error for the MGP formula was reduced about 74% in comparison with the CIDAR equations. Also, the root-mean-squared-error of the MGP formula was decreased near 71% in comparison with the Fib.TG9.3 equations.

Keywords FRP · Multigene genetic programming · Shear capacity · Soft computing method

1 Introduction

Design of non-strengthened reinforced concrete (RC) members is an interesting problem for researchers since there are different approaches to their analysis and design. To design non-strengthened RC members, designers should consider two important parameters: shear and flexural failures. When the internal shear forces in RC beams become large, it could lead to shear destruction, and therefore an abrupt brittle fracture might occur. This type of failure is not intended by designers. In fact, when the internal shear force is larger than the flexural ones, the shear strengthening of RC structures has an essential role to stop the shear destruction. Therefore, the measurement of the shear capacity of RC is an important factor to select a suitable method for the strengthening of RC structures (Naderpour and Alavi 2017).

Fiber-reinforced polymer (FRP) is a combination of high-strength fibers and matrix. The matrix binds these fibers to fabricate structural shapes. There are four common fiber types: aramid (AFRP), carbon (CFRP), glass (GFRP) and high-strength steel. Also, two common matrices exist named epoxy and ester (Karbhari 2015; Wu and Eamon 2017). The use of FRP in repairing and rehabilitation of the existing structures or in the construction of new structures has introduced a new area in civil engineering (Cai and Aref 2015; Xu et al. 2019). In addition to increasing the speed of the construction and creation of continuous confinement for the concrete member, FRP increases the

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member's resistance to corrosion without changing the shape or size of the cross section (De Lorenzis and Tepfers 2003). Therefore, FRP sheets are widely used in the world for the aim of repairing or retrofitting the structural members (Colomb et al. 2008; Niroomandi et al. 2010; Ozcan et al. 2008; Promis et al. 2009).

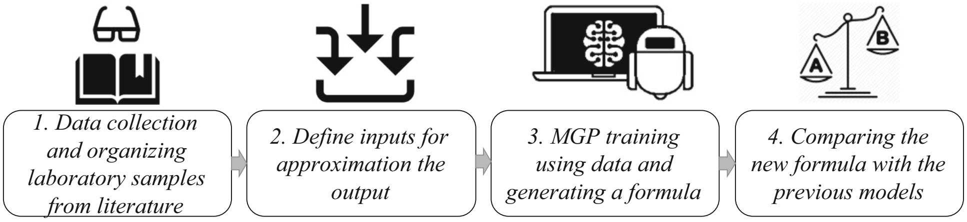
There are enormous researches on the behavior of the RC members retrofitted by FRP sheets. Berset (1992) has carried out preliminary research on the shear capacity of beams strengthened with FRP sheets. The study presented an analytical model based on some experimental data to predict the shear capacity of FRP in the strengthened reinforced concrete beams. Also, Uji (1992) has proposed a model to estimate the shear contribution of CFRP in the strengthened reinforced concrete members. In a similar way, Dolan et al. (1993) have been studied different types of composites (i.e., AFRP, CFRP, and GFRP) in the improvement in the shear capacity of concrete members. The effect of different types of FRP on the improvement in the shear capacity of concrete members has been widely investigated by various researchers. Three different bounded and unbounded FRP wraps have been also studied by Teng et al. (2009) in order to improve the shear capacity of RC beams. In addition, several CFRP sheets have been used by Gamino et al. (2010) to improve the load capacity and ductility of concrete members. Moreover, the effect of strip-width-to-strip-spacing ratio on the behavior of the RC beams retrofitted by FRP composites has been investigated (Mofidi and Chaallal 2011). Furthermore, Li and Leung (2016) have evaluated the effect of shear span–effective depth ratio on the shear capacity of the retrofitted RC beams by full-wrapping FRP strip.

Nowadays, intelligence approximate systems such as the artificial neural network (ANN), adaptive neuro-fuzzy inference system (ANFIS), genetic programming (GP) and so on have provided a special tool to solve the problems in engineering (e.g., FRP-strengthened concrete structure, water resources management, bridge engineering, structural optimization, vibration control, etc.).Various researchers work on these fields (Abualigah and Khader 2017; Abualigah et al. 2018a, b, c, 2019; Abualigah 2019; Abualigah and Hanandeh 2015; Al-Janabi 2018; Al-Janabi et al. 2019a, b; Al-Janabi and Alkaim 2019; Al-Janabi and Alwan 2017; Al-Janabi and Mahdi 2019; Ali 2012a, b, 2013; Alkaim and Al-Janabi 2019; Ebrahimpour Komleh and Maghsoudi 2015; Hüsken et al. 2005; Kaghed et al. 2006; Kamgar et al. 2018; Mahdi and Al-Janabi 2019; Salman and Ali 2007). These algorithms are applicable to the complex problems with time-consuming mathematical solutions and the experimental problems with no clear logical relation between their parameters. A review of the literature shows the ability of ANN, ANFIS and GP to predict the behavior of the studied systems because of their positive features. These features are: (I) controlling the existing uncertainties between a large number of complex data, (II) finding the sensitivity between the inputs data and finally (III) devoting a relation between input and output data to predict the behavior of the studied system. Proposing an optimized formula is the main advantage of GP in comparison with ANN and ANFIS. In ANN and ANFIS, no formula or algorithm is presented. They are a trained black box to predict the targets. The proposed formula by GP could be used with a calculator or Excel file. But, it is worth to note that a computer with professional software such as MATLAB should be used for applying the ANFIS and ANN.

The determination of the shear capacity of RC beams is still a big challenge. This complexity is even enhanced when FRP sheets are applied to augment the shear capacity of RC beams (Sas et al. 2009). The total shear capacity of RC beam can be computed by adding the contributions of the concrete (V_c), the steel stirrups (V_s) and the FRP sheets (V_f). So far, numerous numerical studies have been provided to produce a mathematical equation for the estimation of the shear contribution of the FRP sheets. These equations are used in the guidelines such as CIDAR, Fib.TG9.3, ACI and CSA. A comprehensive comparison was presented by Folani (Sas et al. 2009) to evaluate the reliability of the proposed analytical equations. They illustrated that the proposed equations have considerable errors when they are compared with the experimental results. This proves that the use of intelligent approximation systems in this field could be very effective. Also, Tanarslan (2011) proposed a model to predict the shear contribution of FRP using ANN based on the experimental data. Also, Naderpour and Alavi (2017) predicted the shear capacity of the FRP sheets using ANFIS.

In this paper, multigene genetic programming (MGP) is used to propose a formula for the prediction of the shear capacity of FRP in strengthened RC beams. For this purpose, four steps including data gathering, inputs definition, MGP training and comparing are done. These steps are briefly shown in Fig. 1. Also, the sections of the paper are organized as follows. In the second section, the conducted studies about the application of ANN, ANFIS and GP to predict the capacity of elements combined with the composite material such as FRP are presented. In the third section, the MGP theory, construction, limitation, hypothesis and definitions are presented. Then, MGP is trained using the experimental data, and a formula is produced. In the fourth section, the error of the proposed formula is investigated in comparison with the actual values. Also, the MGP model is compared with the ANFIS model proposed by Naderpour and Alavi (2017). In the fifth section, the equations proposed by various researchers and guidelines, and their parameters are defined. In the sixth section, the

Fig. 1 Main conducted steps



results of MGP and the previous equations are compared with each other, and their errors are calculated. The results prove the excellent performance of the MGP model.

2 Related work

As mentioned, predicting the capacity of composite elements is complex. The conducted studies have indicated that the ability of intelligence approximation systems to predict the strength capacity of composite elements is more than the ability of formulations obtained by experiences. A summary of these studies is presented in Table 1. Mousavi et al. (2010) have been presented a formulation to predict the compressive strength of carbon fiber-reinforced polymer-confined concrete cylinders using GP, simulated annealing (SA) and multi-expression programming (MEP) methods. The linear genetic programming (LGP) has been used by Gandomi et al. (2011) to evaluate the shear resistance of steel fiber-RC beams (SFRCB). The model has been trained using the experimental results including normal and high-strength concrete beams. Kara (2011) presented a simple improved model to predict the shear strength of concrete slender beams without stirrups and reinforced with FRP bars using GP. Lee and Lee (2014) used ANN to predict the shear strength of RC flexural members reinforced with FRP bars. A new formulation has been also presented to predict the curvature ductility factor of FRP-strengthened reinforced high-strength concrete beams by Ebrahimpour Komleh and Maghsoudi (2015) using ANFIS and multiple regression methods. Also, GP has been utilized by Kalfat et al. (2016) to present a formulation for the anchorage strength model. This model can be used to predict the bond performance of FRP which leads to a better performance in retrofitting the structures. In the same way, Al-Mosawe et al. (2017) have used GP to present a formulation to predict the bond strength of CFRP-steel double-strap joints subjected to the impact loads.

The prediction of the shear contribution of FRP in retrofitted RC beams was studied by Tanarslan (2011) and Naderpour and Alavi (2017). Tanarslan (2011) predicted the shear contribution of FRP by using ANN. The model was developed using a database containing 103 test results of FRP-RC beams including rectangular and T-shaped beams. The model was trained using 83 data sets, and then it was tested with 20 sets of test data. Guideline equations, Fib14, ACI 440.2R, CIDAR, CNR-DT 200 and CHBDC were selected for evaluation the ANN model. Naderpour and Alavi (2017) approximated the shear contribution of FRP by using ANFIS. They used a data set that includes 89 samples of rectangular RC beams strengthened with FRP sheets for training and testing the ANFIS model. The model was trained using 76 data sets (85%) and then tested using 13 test data sets (15%). The proposed model was compared with the existing guideline equations such as Fib-TG9.3, CIDAR, ACI 440.2R-08 and CSA-S806(12).

As shown in Table 1, two recent works are completely related to this study. In these studies, it proved that the use of the artificial intelligence methods (i.e., ANN and ANFIS) could be effective in the prediction of shear

Table 1 Overview of the related work

No.	Author	Element	Added composite	Used model	Predicted parameter
1	Mousavi et al. (2010)	Concrete cylinder	CFRP sheet	GP	Compressive strength
2	Gandomi et al. (2011)	Beam	Steel fibers (SF)	GP	Shear strength of SF
3	Kara (2011)	Beam	FRP bars	GP	Total shear strength
4	Lee and Lee (2014)	Beam	FRP bars	ANN	Total shear strength
5	Ebrahimpour Komleh and Maghsoudi (2015)	Beam	FRP sheet	ANFIS	Curvature ductility factor
6	Kalfat et al. (2016)	Beam	FRP sheet + Anchor	GP	Maximum laminate strain
7	Al-Mosawe et al. (2017)	Steel plate	CFRP laminate	GP	Bond strength
8	Tanarslan (2011)	Beam	FRP sheet	ANN	Shear strength of FRP
9	Naderpour and Alavi (2017)	Beam	FRP sheet	ANFIS	Shear strength of FRP

capacity of FRP sheets. The accuracy of the artificial intelligence in comparison with the formulas provided by guidelines is very suitable. In the study conducted by Tanarslan et al. (2012), some of the samples are belonged to the T-shape beams and contain the mechanical anchors, and they cause errors in their approximation and prediction. The ANFIS model presented by Naderpour and Alavi (2017) was trained based on the rectangular beams with no additional mechanical equipment such as anchors. It caused the errors of ANFIS reduced in comparison with the ANN model. However, in both models, no formula was presented and the trained systems (ANN and ANFIS) were a black box while the result of a trained GP model is a formula for predicting the output based on the inputs.

3 Multigene genetic programming

3.1 Theory and construction

In 1954, the genetic program (GP) was introduced by Nils Aall Barricelli (Koza 1992). GP is an evolutionary algorithm and a learning machine to generate a function between the inputs and output. This generation is done using the Darwinian principles in survival and reproductive theory and with genetic operators for the mating formulas (Koza 1994). If it is possible to have different formulas and relationships to solve a problem, GP is a system for creating a function to solve the problem. The diagram of GP for generating a formula (a function of a computer program) is indicated in Fig. 2.

GP creates a function based on tree form. Each tree (gene) has a value of depth that defines the length of the function. In multigene genetic programming, each individual includes a number of weighted genes ($d_1, d_2, \ldots, d_n$) and a bias term (d_0) (Brameier and Banzhaf 2007). Each tree (gene) involves the input variables (x, y, z, etc.) and predefined mathematical operators (see Fig. 3).

In the first step of MGP, the initial population is created based on the predefined values for depth, the number of genes and mathematical operators. This generation in the first iteration is random. The number of individuals in each generation is based on the population size. There are three generation methods named full method, growth method and half-and-half method (see Fig. 4). The half-and-half method applies the full method for 50% of the population, and for the other 50% the growth method is employed (see Fig. 4) (Nedjah et al. 2009).

The root-mean-squared-error (RMSE) between the output of MPG and real values are calculated as a fitness function. Based on the fitness function, the individuals are collected for the next generation. The tournament selection method is used to choose individuals for the next generation. This method randomly selects individuals from the population. Individuals participate in a tournament based on the best fitness function. The tournament size defines the number of individuals that have participated in

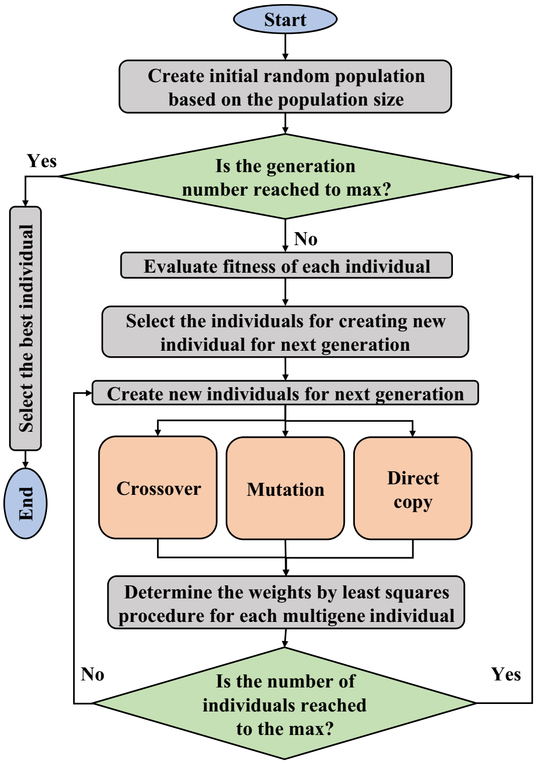
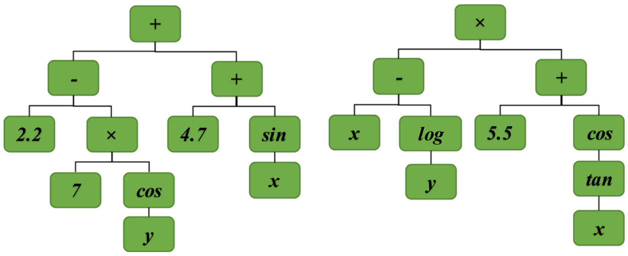


Fig. 2 Diagram of GP



$$Y = d_0 + d_1 \times Tree_1 + d_2 \times Tree_2 + \cdots + d_n \times Tree_n$$
$$Y = d_0 + d_1[(2.2 - 7 \times \cos(y)) + (4.7 + \sin(x)] + d_2[(x - \log(y)) \times (5.5 + \cos(\tan(x)))]$$

Fig. 3 A sample of multigene genetic programming

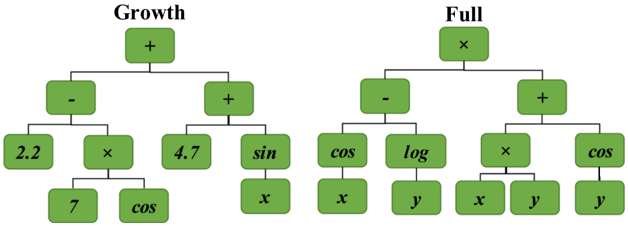


Fig. 4 Full and growth method (max depth = 3)

the tournament (Brameier and Banzhaf 2007; Searson et al. 2010).

Three genetic operators, including crossover, mutation and direct copy are applied to create the next generation. The crossover operator consists of low- and high-level forms. In the low-level form, two individuals are selected as parents (Searson et al. 2010). A tree from each parent is selected and a branch of them is swapped, and then the offsprings are created (see Fig. 5). In high-level form, one or more genes are swapped between the parents for the creation of the offsprings (see Fig. 6). In mutation operator, some parts of genes in each individual are randomly changed (see Fig. 7). These changes include: (a) subtree replacing, (b) switch terminals and (c) Gaussian perturbation constant. In the direct copy operator, the individual is transferred to the next generation without any change. These operators have a probability of occurrence for the individuals. The crossover is the main and stable operator in GP as in a reproductive of animals and humans. The increase in the probability of the mutation and direct copy leads to errors enhancement in approximation. So, the probability of the crossover should be considerably more than the mutation and direct copy operators (Nedjah et al. 2009; Searson et al. 2010).

Using the selected individuals and the genetic operators, the next generation is created. The number of individuals in each generation should be equal to the population size. This process is iteratively performed based on the number of generations. Simultaneously, the weights of trees (genes) in each individual are modified by the least squares procedure (Searson et al. 2010). The algorithm of GP is presented in Fig. 8.

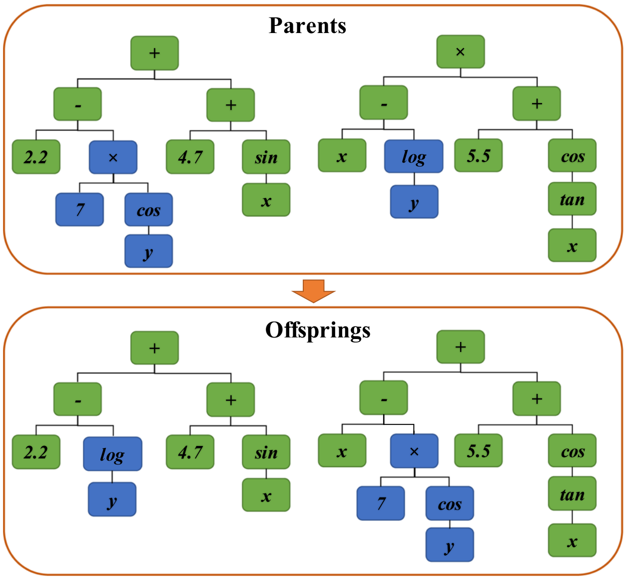


Fig. 5 Low-level crossover operator

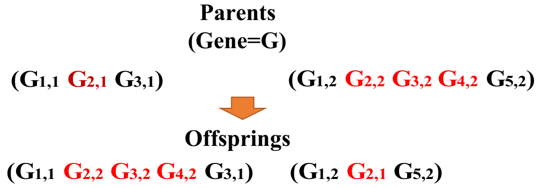


Fig. 6 High-level crossover operator

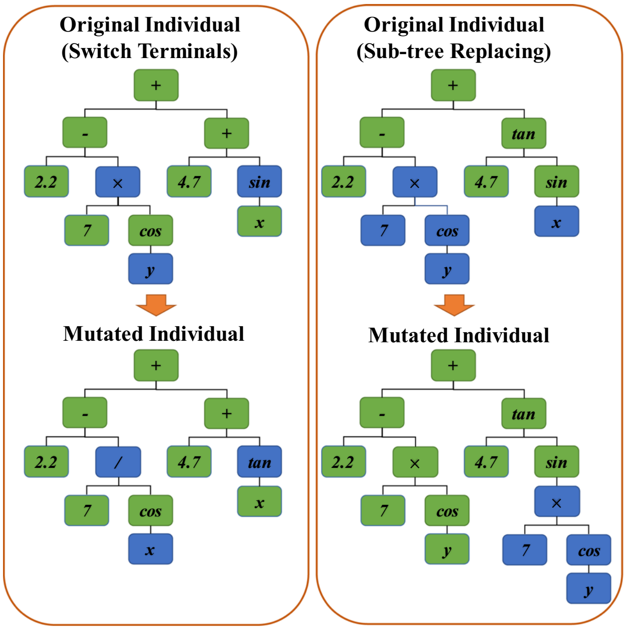


Fig. 7 Mutation operator

3.2 Limitations and hypothesis

The aim of the proposed MGP is to create a formula that is easy to use and has acceptable accuracy. Also, the formula should be compared with the ANFIS model presented by Naderpour and Alavi (2017). In addition, the training of MGP is based on the experimental results presented in past studies for rectangular beams. Based on these reasons, the following hypothesis is considered here:

1. Since there are extensive experimental data for FRP sheets in 45° and 90° (two common used degree in practice), the obtained formula is valid for these degrees.
2. The proposed formulation can be used to predict the shear capacity of FRP in strengthened reinforced concrete beams for all full and U-wrap models with a single equation.
3. In deriving the proposed formulation, only some of the mathematical operators (not all of them) have been used in MGP models which results in a formula with high accuracy and ability to calculate manually. So, the

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MGP algorithm for predicting the shear strength of FRP sheet
Input: Properties of concrete beam and FRP sheet (Table 2)
Output: The shear strength capacity of FRP sheet (Table 2)
• Step 1: definitions
    o Define training instance (Table A-1)
    o Define testing instance (Table A-1)
    o Define population size (Table 3)
    o Define number of max generation (Table 3)
    o Define tournament size (Table 3)
    o Define number of max genes in each individual (Table 3)
    o Define depth of tree (Table 3)
    o Define probability of genetic operators (crossover, mutation and direct copy) (Table 3)
    o Define mathematical operations (Table 4)
• Step 2: Create the initial papulation
    o Initial population is created randomly (1/2 using growth method and 1/2 using full method (Fig. 2))
• Step 3: Evaluate the fitness of each individual based on RMSE
    o Define RSME between the predicted shear capacity and real shear capacity for each created formula
      (individual)
• Step 4: Create next generation
    o Select the best individuals based on their fitness and using tournament method
    o Select a genetic operator based on the defined probability of genetic operators
        ▪ (85%) select two individuals as parents and create two children using crossover
            - (20%) High-level crossover (Fig. 5)
            - (80%) Low-level crossover (Fig. 6)
        ▪ (10%) Select one individual and create a new individual using mutation
            - (95%) Subtree method (Fig. 7)
            - (5% Switch input terminal (Fig. 7)
            - (5%) Gaussian perturbation
        ▪ (5%) Select on individual and copy to next generation directly
    o Determine weights of genes in individual ($d_0$, $d_1$, $d_2$, …, $d_n$) using least squares procedure
    o Step 4 should be repeated  until the population size satisfy
• Step 5: Is the number of generation reached to the max?
    o Yes, go to Step 6
    o No, go to Step 3
• Step 6: Select the best individual based on RMSE
• Step 7: Call testing instance and predict the shear capacity of FRP
• Step 8: End MGP algorithm
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Fig. 8 Multi-genetic programming algorithm

use of bi-sectional commands or If-then (conditional) commands is prevented.

4. Given that the proposed model should be compared with the ANFIS model, the samples selected for training and testing are in accordance with the study conducted by Naderpour and Alavi (2017).

3.3 Variables definition

Herein, the parameters of MGP are set. The defined values are based on the trial and errors to obtain the best results. The input variables are presented in Table 2. The training and testing data are defined according to Table 9 (see "Appendix"). For comparing the MGP model (in this study) and ANFIS model (Naderpour and Alavi 2017), the input variables and test data are selected according to the model proposed by Naderpour and Alavi (2017). The defined values of the MGP parameters and the used mathematical operators are presented in Tables 3 and 4, respectively.

3.4 Results

The values of RMSE for the best formula in each generation are shown in Fig. 9. The results illustrate that RMSE in the first iteration is equal to 40.43, while it reduces to 3.66 in the 300th iteration. The genes of the best formula at the end of the training procedure are presented in Table 5.

Table 2 Introducing data and input variables

Type	Parameters	Expression	Variable
Date	a	Beam shear span length	–
	d	Effective depth of the concrete beam	–
	W_f	Width of FRP	–
	S_f	Spacing steps of FRP strips	–
	α	Angle of inclination of FRP fibers	–
	E_f (GPa)	Elastic modulus of FRP	–
	t_f (mm)	Thickness of FRP layers	–
Input	ε_{fu}	The ultimate strain of FRP	x_1
	d_f (mm)	Effective depth of FRP	x_2
	f_c' (MPa)	Concrete compressive strength	x_3
	$C = a/d$	Combinated parameter	x_4
	$B = W_f/S_f\ [\sin(\alpha) + \cos(\alpha)]$	Combinated parameter	x_5
	R (GPa mm) $= E_f \times t_f$	Combinated parameter	x_6
Output	V_f (kN)	Shear capacity of FRP	y

Table 3 Defined parameters in MGP

Run parameter	Value
Number of input variables	6
Training instances	76
Testing instances	13
Population size	400
Max. generations	300
Tournament size	2
Max. genes	100
Max. tree depth	10
Crossover probability	85% (High-level: 20%, low-level: 80%)
Mutation probability	10% (Subtree: 95%, Switch input terminal: 5%, Gaussian perturbation: 5%)
Direct copy probability	5%

Table 4 Defined mathematical operators in MGP

Mathematical operator	Command
$X_i \times X_j$	TIMES (X_i, X_j)
$X_i - X_j$	MINUS (X_i, X_j)
$X_i + X_j$	PLUS (X_i, X_j)
X_i/X_j	RDIVIDE (X_i, X_j)
$X_i \times X_j \times X_k$	MULT3 (X_i, X_j, X_k)
$X_i + X_j + X_k$	ADD3 (X_i, X_j, X_k)
$-X$	NEG (X)
$\lvert X\rvert$	ABS (X)
X^2	SQUARE (X)
X^3	CUBE (X)
$\lvert X\rvert^{1/2}$	SQRT (X)
e^X	EXP (X)
$\sin(x)$, $\cos(x)$, $\tanh(x)$, $\log(x)$	$\sin(x)$, $\cos(x)$, $\tanh(x)$, $\log(x)$

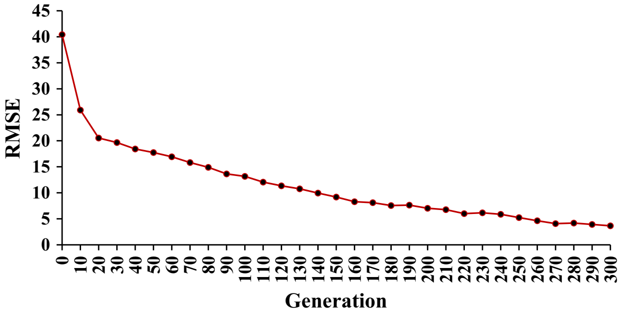


Fig. 9 Values of RMSE during MGP process

As shown, the number of genes has reached 67. It should be noted that the presented genes in Table 5 are the weighted genes ($d_n \times$ tree$_n$). The tree form of Gene17 is indicated in Fig. 10. It can be seen that the depth of the tree has reached 10. The weight of Gene17 is equal to − 30.37. The formula obtained by MGP has been presented in Fig. 11.

4 Comparing the MGP and ANFIS models

In this section, the proposed formula using MGP is compared with the ANFIS model proposed by Naderpour and Alavi (2017). For this purpose, the values predicted by MGP and ANFIS are plotted in Figs. 12 and 13 for the aims of training and testing data, and they have been compared with the actual values. The results illustrate that the MGP model has a good agreement with the actual values. In addition, the errors of the MGP and ANFIS models are calculated and compared in terms of mean absolute percentage error (MAPE), absolute fraction of

Table 5 Genes of the proposed formula by MGP

Term	Value	Term	Value
Bias	-0.0544	Gene 34	$72.8\log(x_5)$
Gene 1	$18.0\,x_5$	Gene 35	$98.6\,\mathrm{abs}(x_4)^{1/2} + 98.6\,x_1^{1/2}$
Gene 2	$-16.9\sin(\exp(x_2))$	Gene 36	$-0.0543\,\mathrm{abs}(x_6)^2$
Gene 3	$-5.27\cos((x_6^{1/2}\exp(x_6))^{1/2})$	Gene 37	$-402.0\log(x_3)$
Gene 4	$18.0\,x_5$	Gene 38	$266.0\,x_3^{1/2}$
Gene 5	$-3.09\cos(x_2)$	Gene 39	$-102.0\sin(\log(x_5))$
Gene 6	$-7.6\cos((x_5^{1/2}\exp(x_6))^{1/2} - 1.0\,x_5^2\,(x_1 - 1.0\,x_3))$	Gene 40	$2.5\,x_1 - 2.5\,x_3$
Gene 7	$0.00222\,x_5\,x_6(x_1 + x_2)$	Gene 41	$2.78\sin(\cos(\log(x_6)))$
Gene 8	$10.0\,x_5^{1/2}$	Gene 42	$2.47\,(x_1 - 1.0\,x_5 + \sin(x_5^2\,(x_1 - 1.0\,x_3)))^2$
Gene 9	$-286.0\,x_1^{1/2}$	Gene 43	$-8.32\sin(x_3\,x_5\,(x_1 + x_3)^2)$
Gene 10	$30.5\,x_2^{1/2}$	Gene 44	-291.0
Gene 11	$-21.8\,\mathrm{abs}(\sin(x_6))^{1/2}$	Gene 45	$8.89\,x_5^{1/4}$
Gene 12	$-0.972\,x_5^3\,(x_1 - 1.0\,x_3)$	Gene 46	$1.83\,x_2\,x_5^{1/2}$
Gene 13	$10.7\cos(1.0\,x_2\,x_5\,(x_1 + 73.4))$	Gene 47	$30.5\,x_2^{1/2}$
Gene 14	$-82.6\,(x_2\,x_5^{1/2})^{1/2}$	Gene 48	$0.558\,\mathrm{abs}(\sin(x_3^2\,x_5))$
Gene 15	$0.118\,(x_6 - 1.0\,x_1 + \sin(x_4))^2$	Gene 49	$-360.0\,x_6$
Gene 16	$2.09\sin(\cos(\log(-1.0\,x_5^2\,(x_1 - 1.0\,x_3))))$	Gene 50	$9.27\,\mathrm{abs}(x_6)^{1/2}$
Gene 17	$-30.3\cos((x_5^{1/2}\exp(\cos(\sin(x_2\,x_5^{1/2})$ $-1.0\,x_5^2\,(x_1 - 1.0\,x_3))))^{1/2})$	Gene 51	$-5.41\sin(\sin(x_1))$
Gene 18	$-23.2\,x_5\sin(x_2)$	Gene 52	$-0.454\cos((x_1\exp(x_6))^{1/2})$
Gene 19	$-(8.92\,x_2\,x_4)/(x_2\,x_4)^{1/2}$	Gene 53	$-34.2\tanh(\mathrm{abs}(x_5 - 1.0\,x_1 + \sin(x_4))^2)$
Gene 20	$-32.5\,\mathrm{abs}(\sin(\exp(x_2)))^{1/2}$	Gene 54	$-(0.0012\,x_2)/x_1$
Gene 21	$14.1\cos(x_3^3)$	Gene 55	$-5.56\,x_2$
Gene 22	$2.07\sin(x_2\,x_5)$	Gene 56	$5.39\cos((x_6^{3/2})^{1/2})$
Gene 23	$-5.89\cos((\exp(x_3)\,\mathrm{abs}(x_6))^{1/2})$	Gene 57	$(0.732\,x_4^2)/x_5$
Gene 24	$18.0\,x_5$	Gene 58	$-313.0\sin(\sin(x_4))$
Gene 25	$0.0199\cos(\mathrm{abs}(\sin(x_3^2\,x_5)))$	Gene 59	$-7.9\ \mathrm{x}_3$
Gene 26	$9.27\,\mathrm{abs}(x_6)^{1/2}$	Gene 60	$6.21\cos((\exp(x_6)\,\mathrm{abs}(x_6))^{1/2})$
Gene 27	$-2.51\cos(\exp(x_6))$	Gene 61	$-75.7\,\mathrm{abs}(\log(x_5))$
Gene 28	$-4.77\cos((\mathrm{abs}(x_6)\,(x_6 - 1.0\,x_1 + x_2\,x_5)2)^{1/2})$	Gene 62	$-8.04\cos(\sin(x_2\,x_5^{1/2})$ $-1.0\,x_5\,x_6^{1/2}\exp(x_6)\,(x_1 - 1.0\,x_3))$
Gene 29	$170.0\,\mathrm{abs}(x_4)^{1/2}$	Gene 63	$9.02\cos(\mathrm{abs}(x_4)^{1/2} - 1.0\,x_5^2\,(x_1 - 1.0\,x_3))$
Gene 30	$-0.0543\,\mathrm{abs}(x_6)^2$	Gene 64	$356.0\,((x_6 - 1.0\,x_1 + \sin(x_4))^2)^{1/2}$
Gene 31	$-6.03\cos(x_2\,x_5\,(2.0\,x_1 + x_3 + x_4))$	Gene 65	$4.46\,x_2 + 4.46\,\mathrm{abs}(x_5)^{1/2}$
Gene 32	$30.5\,x_2^{1/2}$	Gene 66	$8.89\,x_5^{1/4}$
Gene 33	$-5.3\cos(\sin(\mathrm{abs}(\sin(\exp(x_2)))))$ $-1.0\,x_5\,x_6\,(x_1 - 1.0\,x_3))$	Gene 67	$12.6\,x_1 + 12.6\,x_5$

variance (R^2) and root-mean-squared-error (RMSE) according to Eqs. (1)–(3).

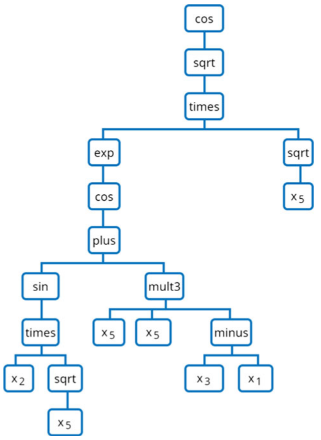
$$\mathrm{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{V_{\mathrm{f(predict)}} - V_{\mathrm{f(experiment)}}}{V_{\mathrm{f(experiment)}}}\right) \times 100 \qquad (1)$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(V_{\mathrm{f(predict)}} - V_{\mathrm{f(experiment)}}\right)^2}{\sum_{i=1}^{n}\left(V_{\mathrm{f(predict)}}\right)^2} \qquad (2)$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(V_{\mathrm{f(predict)}} - V_{\mathrm{f(experiment)}}\right)^2} \qquad (3)$$

As shown in Table 6, the values of MAPE and RMSE for the training data of the MGP model are reduced approximately by 60% in comparison with those of the ANFIS model. Also, these reductions are about 48% (MAPE) and 28% (RMSE) for the whole data.

Therefore, it can be concluded that MGP can predict the actual shear capacity of FRP by the proposed formula with reasonable accuracy. Proposing a formula is the main characteristic of MGP in comparison with ANFIS and ANN. It helps MGP to become very usable while ANFIS is a black box that cannot present any equation or algorithm.



$$\underbrace{-30.3}_{d_{17}}\ \underbrace{\cos((x_5^{1/2}\exp(\cos(\sin(x_2\, x_5^{1/2}) - 1.0\, x_5^2\,(x_1 - 1.0\, x_3))))^{1/2})}_{\text{Tree }17}$$

Fig. 10 Tree form of Gene 17

In fact, ANFIS should be used in computing for new samples. Also, ANFIS should be applied by software such as MATLAB or other programs. But, the proposed formula by MGP could be utilized easily using a calculator and simple software like Microsoft Excel.

5 Overview of some existing models

There are some formulas which can be used to approximate the shear capacity of FRP in strengthened reinforced concrete beams. In these formulas, some parameters are used as the input parameters [i.e., elasticity modulus of FRP (E_f), effective strain of FRP (ε_{fe}), ultimate strain of FRP (ε_{fu}), FRP reinforcement ratio (ρ_f), concrete compressive strength (f_c'), FRP thickness (t_f), effective depth of RC beam (d), minimum width of RC beam over the effective

$$\begin{aligned}
&y = 15.1\, x_1 - 1.1\, x_2 - 10.4\, x_3 + 66.6\, x_5 - 360.0\, x_6 - 4.77 \cos((\mathrm{abs}(x_6)\,(x_6 - x_1 + x_2 x_5)^2)^{1/2}) - 6.03 \cos(x_2 x_5\,(2.0\, x_1 + x_3 + x_4)) + \\
&14.1 \cos(x_3^3) - 2.51 \cos(\exp(x_6)) - 75.7\, \mathrm{abs}(\log(x_5)) - 16.9 \sin(\exp(x_2)) - 102 \sin(\log(x_5)) - 5.41 \sin(\sin(x_1)) - 313 \sin(\sin(x_4)) \\
&+ 9.02 \cos(\mathrm{abs}(x_4)^{1/2} - x_5^2\,(x_1 - x_3)) - 5.3 \cos(\sin(\mathrm{abs}(\sin(\exp(x_2))))) - x_5\, x_6\,(x_1 - x_3)) - 8.32 \sin(x_3 x_5\,(x_1 + x_3)^2) - \\
&0.454 \cos((x_1 \exp(x_6))^{1/2}) + 0.558\, \mathrm{abs}(\sin(x_3^2 x_5)) - 34.2 \tanh(\mathrm{abs}(x_5 - x_1 + \sin(x_4))^2) + 5.39 \cos((x_6^{3/2})^{1/2}) - \\
&7.6 \cos((x_5^{1/2} \exp(x_6))^{1/2} - x_5^2\,(x_1 - x_3)) - 8.04 \cos(\sin(x_2 x_5^{1/2}) - x_5 x_6^{1/2} \exp(x_6)\,(x_1 - x_3)) + 2.09 \sin(\cos(\log(-x_5^2\,(x_1 - x_3)))) - \\
&30.3 \cos((x_5^{1/2} \exp(\cos(\sin(x_2 x_5^{1/2}) - x_5^2\,(x_1 - x_3))))^{1/2}) + 2.78 \sin(\cos(\log(x_6))) + 2.07 \sin(x_2 x_5) + 10.7 \cos(x_2 x_5\,(x_1 + 73.4)) - \\
&5.27 \cos((x_6^{1/2} \exp(x_6))^{1/2}) - 5.89 \cos((\exp(x_3)\, \mathrm{abs}(x_6))^{1/2}) + 6.21 \cos((\exp(x_6)\, \mathrm{abs}(x_6))^{1/2}) - 3.09 \cos(x_2) - 402.0 \log(x_3) + \\
&72.8 \log(x_5) + 0.0199 \cos(\mathrm{abs}(\sin(x_3^2 x_5))) + (0.732\, x_4^2)/x_5 - 82.6\,(x_2 x_5^{1/2})^{1/2} - 32.5\, \mathrm{abs}(\sin(\exp(x_2)))^{1/2} + \\
&356.0\,((x_6 - x_1 + \sin(x_4))^2)^{1/2} + 269.0\, \mathrm{abs}(x_4)^{1/2} + 4.46\, \mathrm{abs}(x_5)^{1/2} - 0.109\, \mathrm{abs}(x_6)^2 + 18.5\, \mathrm{abs}(x_6)^{1/2} - 0.972\, x_5^3\,(x_1 - x_3) - \\
&21.8\, \mathrm{abs}(\sin(x_6))^{1/2} - (0.0012\, x_2)/x_1 + 1.83\, x_2 x_5^{1/2} + 0.118\,(x_6 - x_1 + \sin(x_4))^2 - 23.2\, x_5 \sin(x_2) - 188.0\, x_1^{1/2} + 91.4\, x_2^{1/2} + \\
&266.0\, x_3^{1/2} + 10.0\, x_5^{1/2} + 17.8\, x_5^{1/4} + 2.47\,(x_1 - x_5 + \sin(x_5^2\,(x_1 - x_3)))^2 + 0.00222\, x_5 x_6\,(x_1 + x_2) - (8.92\, x_2 x_4)/(x_2 x_4)^{1/2} - 291.0
\end{aligned}$$

Fig. 11 The proposed formula by MGP

Fig. 12 Comparison between the MGP, ANFIS and actual outputs for training data

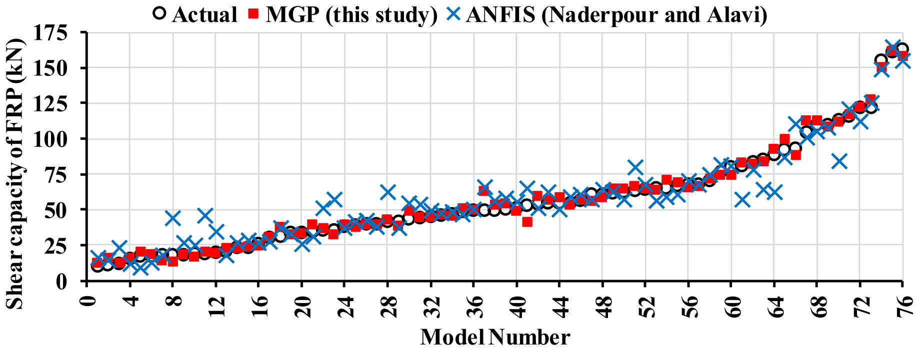


Fig. 13 Comparison between the MGP, ANFIS and actual outputs for testing data

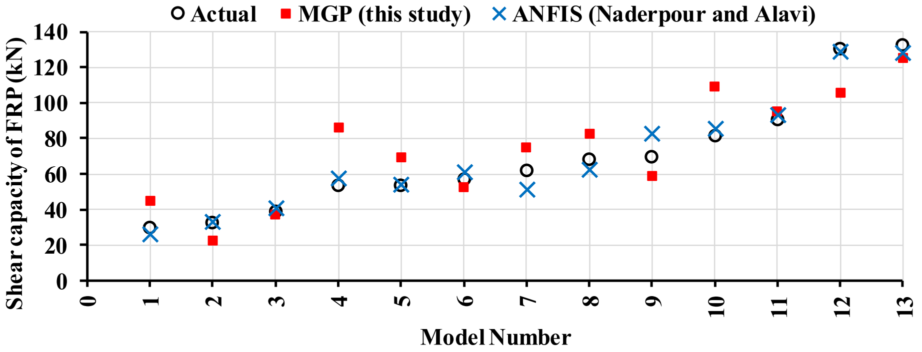


Table 6 Values of RMSE, R^2 and MAPE errors for MGP and ANFIS

Models	Training data			Testing data			Whole data		
	RMSE	R^2	MAPE	RMSE	R^2	MAPE	RMSE	R^2	MAPE
MGP	3.662	0.997	6.416	16.51	0.956	23.573	7.160	0.989	8.922
ANFIS	9.617	0.979	17.16	12.416	0.968	17.352	10.074	0.977	17.188

Table 7 Some of the existing and developed formulas to predict the shear capacity of FRP in strengthened RC beams

Guideline	Formula
CSA-S806-12 (2012)	$V_f = \frac{A_{fv} f_{fe} d_f (\sin\alpha + \cos\alpha)}{S_f}; \; A_{fv} = 2nt_f w_f; \; f_{fe} = \varepsilon_{fe}.E_f; \; \varepsilon_{fe}\begin{cases} \varepsilon_{fe} = 0.006 & \text{for full wrap} \\ \varepsilon_{fe} = K_v \varepsilon_{fu} \le 0.004 & \text{for U−wrap and side bonded} \end{cases}$ $K_v = \frac{k_1 k_2 L_e}{11900\varepsilon_{fu}} \le 0.75; \; L_e = \frac{23300}{(nt_f E_f)^{0.58}}; \; k_1 = \left[\frac{f'_c}{27}\right]^{\frac{2}{3}}; \; k_2 = \frac{d_f - L_e}{d_f}$
ACI Committee 440 (2008)	$V_f = \frac{A_{fv} f_{fe} d_f (\sin\alpha + \cos\alpha)}{S_f}; \; A_{fv} = 2nt_f w_f; \; f_{fe} = \varepsilon_{fe}.E_f; \; \varepsilon_{fe}\begin{cases} \varepsilon_{fe} = 0.004 \le 0.75\varepsilon_{fu} & \text{for full wrap} \\ \varepsilon_{fe} = K_v \varepsilon_{fu} \le 0.004 & \text{for U−wrap and side bonded} \end{cases}$ $K_v = \frac{k_1 k_2 L_e}{11900\varepsilon_{fu}} \le 0.75; \; L_e = \frac{23300}{(nt_f E_f)^{0.58}}; \; k_1 = \left[\frac{f'_c}{27}\right]^{\frac{2}{3}}; \; k_2 = \begin{cases} \frac{d_f - L_e}{d_f}; & \text{for U−wrap} \\ \frac{d_f - 2L_e}{d_f}; & \text{for side bonded} \end{cases}$
CIDAR (2006)	$V_f = 2 f_{fed} t_f \frac{w_f}{S_f} h_{fe} (\cot\theta + \cot\alpha) \sin\alpha; \; h_{fe} = z_b - z_t; \; z_b = 0.9d - d_{fb}; \; z_t = d_{ft}; \; f_{fed} = D_f f_{fd,max}$ $\text{For full wrap:} \; D_f = 0.5\left(1 + \frac{z_t}{z_b}\right); \; f_{fd,max} = \left\{\begin{matrix} \frac{1}{r_f}\phi_R f_{fu}; & \varepsilon_f \le 1.5\% \\ \frac{1}{r_f}\phi_R E_f \varepsilon_f; & \varepsilon_f > 1.5\% \end{matrix}\right\}; \; \phi_R = 0.8; \; \gamma_f = 1.25$ $\text{For U−wrap and side−bonded:} \; D_f = \begin{cases} \frac{2}{\lambda\pi} \frac{1 - \cos\left(\frac{\pi}{2}\lambda\right)}{\sin\left(\frac{\pi}{2}\lambda\right)}; & \lambda \le 1 \\ 1 - \frac{\pi - 2}{\pi\lambda}; & \lambda > 1 \end{cases} \; \lambda = \frac{L_{max}}{L_e}; \; L_{max} = \begin{cases} \frac{h_{fe}}{\sin\alpha}; & \text{for U−wrap} \\ \frac{h_{fe}}{2\sin\alpha}; & \text{for side−bonded} \end{cases};$ $L_e = \sqrt{\frac{E_f t_f}{\sqrt{f'_c}}}; \; f_{fd,max} = \min\begin{cases} \frac{1}{r_f}\phi_R f_{fu} \\ \frac{1}{r_f} 0.35 \beta_L \beta_W \sqrt{\frac{E_f \sqrt{f'_c}}{t_f}} \end{cases}; \; \beta_L = \begin{cases} \lambda; & \lambda \le 1 \\ 1; & \lambda > 1 \end{cases}; \; \beta_W = \sqrt{\frac{2 - \frac{W_f}{S_f \sin\alpha}}{1 + \frac{W_f}{S_f \sin\alpha}}}$
Fib-TG9.3 (2001)	$V_f = 0.9\varepsilon_{fe} E_f \rho_f b_w d (\cot\theta + \cot\alpha) \sin\alpha; \; \rho_f = \frac{2t_f w_f}{b_w S_f} \; \text{(Strips)}; \; \rho_f = \frac{2t_f \sin\alpha}{b_w} \; \text{(Cont.)}$ $\text{For full wrap (CFRP):} \; \varepsilon_{fe} = 0.17\left(\frac{f_{cm}^{\frac{2}{3}}}{E_f \rho_f}\right)^{0.3} \varepsilon_{fu}; \; \text{For full wrap (AFRP):} \; \varepsilon_{fe} = 0.048\left(\frac{f_{cm}^{\frac{2}{3}}}{E_f \rho_f}\right)^{0.47} \varepsilon_{fu}$ $\text{For U−wrap and side−bonded:} \; \varepsilon_{fe} = \min\left[0.65\left(\frac{f_{cm}^{\frac{2}{3}}}{E_f \rho_f}\right)^{0.56} \times 10^{-3}, \; 0.17\left(\frac{f_{cm}^{\frac{2}{3}}}{E_f \rho_f}\right)^{0.3} \varepsilon_{fu}\right]$
Proposed formulae	$y = 15.1 x_1 - 1.1 x_2 - 10.4 x_3 + 66.6 x_5 - 360.0 x_6 - 4.77 \cos((\mathrm{abs}(x_6) (x_6 - x_1 + x_2 x_5)^2)^{1/2}) - 6.03 \cos(x_2 x_5 (2.0 x_1 + x_3 + x_4)) + 14.1 \cos(x_3^3) - 2.51 \cos(\exp(x_6)) - 75.7 \mathrm{abs}(\log(x_5)) - 16.9 \sin(\exp(x_2)) - 102 \sin(\log(x_5)) - 5.41 \sin(\sin(x_1)) - 313 \sin(\sin(x_4)) + 9.02 \cos(\mathrm{abs}(x_4)^{1/2} - x_5^2(x_1 - x_3)) - 5.3 \cos(\sin(\mathrm{abs}(\sin(\exp(x_2))))) - x_5 x_6(x_1 - x_3)) - 8.32 \sin(x_3 x_5 (x_1 + x_3)^2) - 0.454 \cos((x_1 \exp(x_6))^{1/2}) + 0.558 \mathrm{abs}(\sin(x_3^2 x_5)) - 34.2 \tanh(\mathrm{abs}(x_5 - x_1 + \sin(x_4))^2) + 5.39 \cos((x_6^{3/2})^{1/2}) - 7.6 \cos((x_5^{1/2} \exp(x_6))^{1/2} - x_5^2 (x_1 - x_3)) - 8.04 \cos(\sin(x_2 x_5^{1/2}) - x_5 x_6^{1/2} \exp(x_6) (x_1 - x_3)) + 2.09 \sin(\cos(\log(- x_5^2 (x_1 - x_3)))) - 30.3 \cos((x_5^{1/2} \exp(\cos(\sin(x_2 x_5^{1/2}) - x_5^2 (x_1 - x_3))))^{1/2}) + 2.78 \sin(\cos(\log(x_6))) + 2.07 \sin(x_2 x_5) + 10.7 \cos(x_2 x_5 (x_1 + 73.4)) - 5.27 \cos((x_6^{1/2}\exp(x_6))^{1/2}) - 5.89 \cos((\exp(x_3) \mathrm{abs}(x_6))^{1/2}) + 6.21 \cos((\exp(x_6) \mathrm{abs}(x_6))^{1/2}) - 3.09 \cos(x_2) - 402.0 \log(x_3) + 72.8 \log(x_5) + 0.0199 \cos(\mathrm{abs}(\sin(x_3^2 x_5))) + (0.732 x_4^2)/x_5 - 82.6 (x_2 x_5^{1/2})^{1/2} - 32.5 \mathrm{abs}(\sin(\exp(x_2)))^{1/2} + 356.0 ((x_6 - x_1 + \sin(x_4))^2)^{1/2} + 269.0 \mathrm{abs}(x_4)^{1/2} + 4.46 \mathrm{abs}(x_5)^{1/2} - 0.109 \mathrm{abs}(x_6)^2 + 18.5 \mathrm{abs}(x_6)^{1/2} - 0.972 x_5^3 (x_1 - x_3) - 21.8 \mathrm{abs}(\sin(x_6))^{1/2} - (0.0012 x_2)/x_1 + 1.83 x_2 x_5^{1/2} + 0.118 (x_6 - x_1 + \sin(x_4))^2 - 23.2 x_5 \sin(x_2) - 188.0 x_1^{1/2} + 91.4 x_2^{1/2} + 266.0 x_3^{1/2} + 10.0 x_5^{1/2} + 17.8 x_5^{1/4} + 2.47 (x_1 - x_5 + \sin(x_5^2 (x_1 - x_3)))^2 + 0.00222 x_5 x_6 (x_1 + x_2) - (8.92 x_2 x_4)/(x_2 x_4)^{1/2} - 291.0$ where $x_1 = \varepsilon_{fu}$, $x_2 = d_f$ (mm), $x_3 = f_c'$ (MPa), $x_4 = C = a/d$, $x_5 = B = W_f/S_f [\sin(\alpha) + \cos(\alpha)]$, $x_6 = R$ (GPa mm) $= E_f \times t_f$, $y = V_f$ (kN)

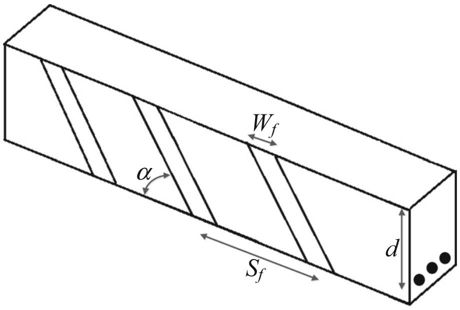


Fig. 14 A strengthened beam using FRP sheets

depth (b_w), FRP strips width (w_f), FRP strips spacing (s_f), effective stress of FRP (f_{fe}), ultimate stress of FRP (f_{fd}), angle of diagonal crack with respect to the member axis (θ) and the angle between fiber orientation and longitudinal axis member (α)]. The existing formula to compute the shear capacity of FRP in strengthened reinforced concrete beams is listed in Table 7. Also, Fig. 14 shows a beam strengthened by FRP sheets and the parameters that have been used to compute the shear capacity of FRP.

6 Comparison of the proposed formula with several existing formulas

To show the ability of the proposed formula, four common guidelines (ACI Committee 440 2008; CIDAR 2006; CSA-S806-12 2012; Fib-TG9.3 2001) are considered, and the results are compared. It should be noted that the existing guidelines (ACI Committee 440 2008; CIDAR 2006; CSA-S806-12 2012; Fib-TG9.3 2001) have two different formulas based on FRP-configuration in RC members (full-wrap and U-wrap), while the proposed formula by MPG covers these two configurations by a single formula. It can be considered as a positive point of the proposed formula. The values of shear capacity of FRP obtained by the proposed and the existing formulas are shown in Fig. 15. Also, Table 8 provides the values of MAPE, RMSE and R^2 for the studied formulas.

Based on Table 8, the values MAPE, RMSE and R^2 for the proposed formula are 8.92%, 7.16% and 0.989%, respectively. These values for the CSA S806, ACI 440.2R, CIDAR and Fib-TG9.3 methods are more than 35.91%, 25.06% and 0.302%, respectively. This indicates that for the proposed formula, the simulated results are exactly being consistent with the experimental results more than 91%. This shows the accuracy of the proposed formula while the accuracy of other existing formulas is lower than 64%. It needs to be noted that the proposed formula is able to predict the shear capacity of FRP with a different configuration for FRP (full-wrap and U-wrap). In fact, it can be concluded that the proposed formula has a higher precision compared with the existing formulas in the prediction of FRP shear capacity.

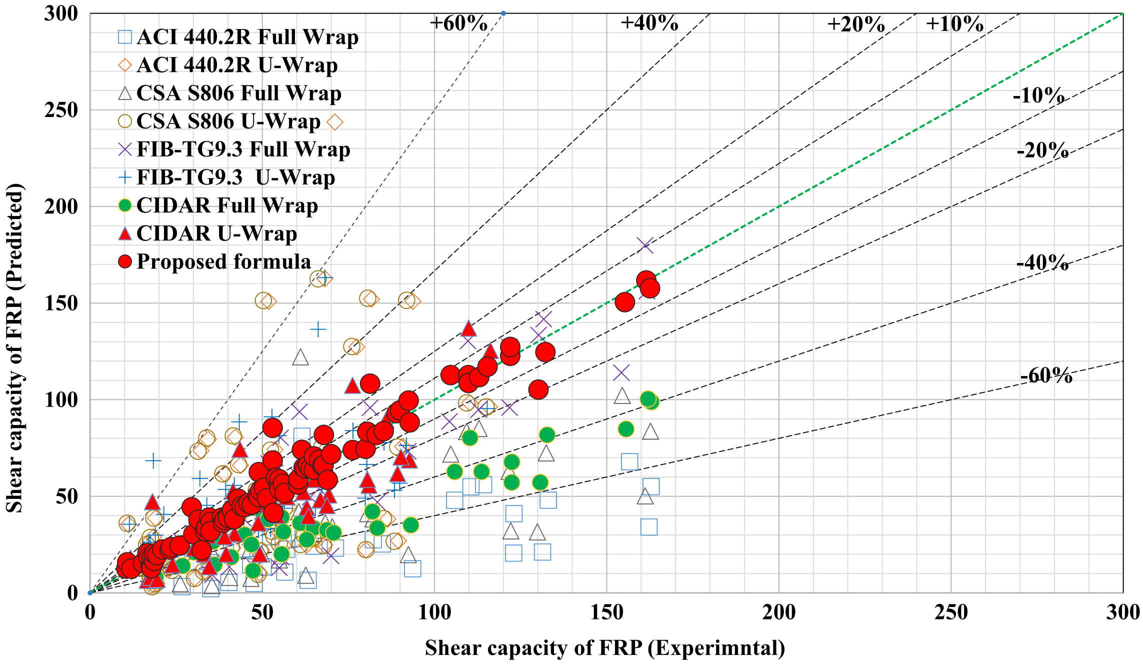


Fig. 15 Comparison between MGP and experimental results for the shear capacity of FRP

Table 8 Compression between the values of MAPE, RMSE and R^2 for the different studied models

Error	Method				
	CSA-S806-12 (2012) (%)	ACI Committee 440 (2008) (%)	CIDAR (2006) (%)	Fib-TG9.3 (2001) (%)	Proposed formula (%)
MAPE	57.60	65.20	35.91	39.07	**8.92**
RMSE	41.11	51.15	28.46	25.06	**7.16**
R^2	0.521	0.302	0.708	0.873	**0.989**

Bold values indicate better results than other existing methods

7 Conclusion

In this paper, the MGP method has been used to predict the shear capacity of FRP in strengthened RC beams. For this purpose, a set of previously published and available experimental data (89 instances) have been collected. Then, 85% of the data (76 instances) has been used for training the MGP and 15% (13 instances) for testing. Finally, a new formulation has been proposed utilizing MGP. The correlation coefficients of the proposed formula are equal to 0.997, 0.956 and 0.989 for training, testing and whole data, respectively. The results make it obvious that the MGP model has a good agreement with the actual values. To study the accuracy of the proposed formula, the results obtained by MGP have been compared with those of the ANFIS model. The errors of the MGP and ANFIS models have been calculated and compared in terms of MAPE, R^2 and RMSE. The results indicated that the errors were decreased in the MGP model for training and whole data in comparison with the ANFIS model. This decrease in training step was about 62% for MAPE error. For the whole data, MAPE and RMSE errors were reduced by about 48% and 29%, respectively. But the errors for testing data in the ANFIS model were less than the MGP model. The MAPE and RMSE errors in the ANFIS model were, respectively, reduced by about 36% and 33% in comparison with the MGP model. Furthermore, the proposed formula has been compared to the formulas from the available guidelines including CSA-S806, ACI 440.2R, CIDAR and Fib-TG9.3. The results reveal that the formula proposed by MGP is able to predict the shear capacity of FRP with higher precision compared to the existing formulas. The equations CIDAR and Fib.TG9.3 had more accuracy in the prediction of the shear capacity of FRP in comparison with the ACI and CSA equations. The MAPE error for the MGP formula was reduced by about 74% in comparison with the CIDAR equations. Also, the RMSE error of MGP formula was decreased near 71% in comparison with the Fib.TG9.3 equations.

Besides, it is shown that the MGP is a formula which could be used by a calculator, while the ANFIS model should be applied by a computer and professional programs. Also, the MGP formulation could be used for both full and U-wrap FPR models, while CSA S806, ACI 440.2R, CIDAR and Fib-TG9.3 have two different formulas for full and U-wrap models. There is a low experimental test on the retrofitting of the beam sections with FRP sheets by angle 30°, 60° and 75° that could be tested experimentally to propose a formula that covers all over the region of FRP angles. It can be worked in the future.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with animal/human participants performed by any of the authors.

Appendix

The collected data for training and testing are indicated in Table 9. The testing data are marked with "*" in Table 9.

Table 9 Collected data for training and testing

No.	Refs.	ε_f	d_f (mm)	W_f/S_f	α	f_c' (MPa)	a/d	E_f (GPa)	t_f (mm)	V_f (kN)
1	Gamino et al. (2010)	0.0150	185.00	0.286	90	57.27	2.64	235.00	0.110	49.00
2	Ono et al. (1997)	0.0138	260.00	1.000	90	24.00	1.54	248.00	0.110	113.0
3	Chajes et al. (1995)	0.0134	88.900	1.000	90	41.81	2.67	14.270	0.460	17.93
4	(Chajes et al. 1995)	0.0095	88.900	1.000	90	43.91	2.67	20.960	0.580	16.68
5	Chajes et al. (1995)	0.0095	88.900	1.000	90	47.15	2.67	20.960	0.580	17.61
6	Khalifa (1999)	0.0154	253.00	1.000	90	19.30	3.00	228.00	0.330	50.50
7	Khalifa (1999)	0.0154	255.00	1.000	90	19.30	4.00	228.00	0.330	80.50
8	Khalifa (1999)	0.0154	253.00	0.400	90	27.50	3.00	228.00	0.165	54.00
9*	Khalifa (1999)	0.0154	253.00	0.600	90	27.50	3.00	228.00	0.165	56.50
10	Khalifa (1999)	0.0154	253.00	1.000	90	27.50	3.00	228.00	0.165	67.50
11	Khalifa (1999)	0.0154	253.00	1.000	90	27.50	3.00	228.00	0.330	92.50
12	Khalifa (1999)	0.0154	255.00	0.400	90	27.50	4.00	228.00	0.165	62.50
13*	Khalifa (1999)	0.0154	255.00	1.000	90	27.50	4.00	228.00	0.165	90.00
14	Jayaprakash et al. (2008)	0.0170	310.00	0.533	90	27.38	2.50	230.00	0.090	38.25
15	Jayaprakash et al. (2008)	0.0170	310.00	0.533	45	16.73	4.00	230.00	0.090	44.90
16*	Monti (2007)	0.0077	252.00	1.000	45	9.630	3.48	390.00	0.220	69.00
17	Monti (2007)	0.0077	252.00	0.670	45	9.630	3.48	390.00	0.220	66.50
18	Monti (2007)	0.0077	252.00	0.333	45	9.630	3.48	390.00	0.220	60.50
19	Sato et al. (1996)	0.0151	260.00	0.500	90	41.30	2.69	230.00	0.110	55.00
20	Araki et al. (1997)	0.0151	336.00	0.230	90	24.50	1.56	230.00	0.110	34.60
21	Araki et al. (1997)	0.0151	336.00	0.410	90	24.60	1.56	230.00	0.111	60.40
22	Araki et al. (1997)	0.0151	336.00	0.700	90	24.90	1.56	230.00	0.111	104.7
23	Araki et al. (1997)	0.0151	336.00	1.000	90	25.10	1.56	230.00	0.111	155.3
24	Taerwe et al. (1997)	0.0125	395.00	0.250	90	28.36	3.16	280.00	0.110	41.20
25	Taerwe et al. (1997)	0.0125	395.00	1.000	90	28.36	3.16	280.00	0.110	115.4
26	Taerwe et al. (1997)	0.0125	395.00	0.125	90	28.36	3.16	280.00	0.110	33.40
27	Taerwe et al. (1997)	0.0125	395.00	0.0833	90	28.36	3.16	280.00	0.110	30.00
28	Cao et al. (2005)	0.0146	222.50	0.200	90	24.10	2.70	249.00	0.167	46.00
29	Cao et al. (2005)	0.0146	222.50	0.300	90	24.10	1.80	249.00	0.167	44.00
30	Cao et al. (2005)	0.0127	222.50	0.500	90	23.70	2.47	20.500	1.270	70.00
31	(Cao et al. 2005)	0.0127	222.50	0.250	90	23.70	2.47	20.500	1.270	55.00
32	(Cao et al. 2005)	0.0127	222.50	0.500	90	23.70	1.35	20.500	1.270	56.00
33	Cao et al. (2005)	0.0211	222.50	0.500	90	15.24	2.92	5.3000	1.200	40.00
34	Cao et al. (2005)	0.0211	222.50	0.250	90	15.24	2.92	5.3000	1.200	35.00
35	Cao et al. (2005)	0.0211	222.50	0.500	90	15.24	1.80	5.3000	1.200	47.00
36	Cao et al. (2005)	0.0211	222.50	0.250	90	15.24	1.80	5.3000	1.200	35.00
37	Khalifa and Nanni (2000)	0.0166	255.75	1.000	90	35.00	2.88	228.00	0.165	65.00
38*	Zhang et al. (2004)	0.0133	203.20	1.000	90	41.34	1.875	73.100	0.330	53.00
39	Zhang et al. (2004)	0.0133	203.20	1.000	90	41.34	1.25	73.100	0.330	40.05
40	Panda et al. (2013)	0.0121	165.00	1.000	90	33.60	3.26	13.180	0.360	19.50
41	Diagana et al. (2003)	0.0133	403.00	0.200	90	30.00	2.23	105.00	0.430	65.00
42	Diagana et al. (2003)	0.0133	403.00	0.160	90	30.00	2.23	105.00	0.430	49.00
43	Diagana et al. (2003)	0.0133	403.00	0.133	45	30.00	2.23	105.00	0.430	89.00
44	Diagana et al. (2003)	0.0133	403.00	0.114	45	30.00	2.23	105.00	0.430	80.00
45	Bousselham and Chaallal (2008)	0.0140	171.02	1.000	90	25.00	3.03	231.00	0.060	21.00
46	Bousselham and Chaallal (2008)	0.0140	171.02	1.000	90	25.00	3.03	231.00	0.107	39.00
47	Bousselham and Chaallal (2006)	0.0140	109.00	1.000	90	25.20	1.51	231.00	0.066	15.40
48	Bousselham and Chaallal (2006)	0.0140	109.00	1.000	90	25.20	1.51	231.00	0.132	17.00

Table 9 (continued)

No.	Refs.	ε_f	d_f (mm)	W_f/S_f	α	f_c' (MPa)	a/d	E_f (GPa)	t_f (mm)	V_f (kN)
49	Bousselham and Chaallal (2006)	0.0140	109.00	1.000	90	25.20	3.03	231.00	0.066	23.20
50*	Bousselham and Chaallal (2006)	0.0140	109.00	1.000	90	25.20	3.03	231.00	0.132	32.40
51	Li et al. (2002)	0.0111	116.00	1.000	45	30.00	3.01	42.400	1.500	12.00
52	Li et al. (2002)	0.0111	191.00	1.000	45	30.00	3.01	42.400	1.500	23.50
53	Li et al. (2002)	0.0111	116.00	1.000	45	30.00	3.01	42.400	1.500	10.50
54	Barros and Dias (2006)	0.0080	273.00	0.130	90	40.18	2.20	390.00	0.334	10.80
55	Barros and Dias (2006)	0.0080	273.00	0.260	90	40.18	2.20	390.00	0.334	31.50
56	Barros and Dias (2006)	0.0080	123.00	0.313	90	46.55	2.44	390.00	0.334	18.60
57	Barros and Dias (2006)	0.0080	123.00	0.630	90	46.55	2.44	390.00	0.334	33.70
58	Umezu (1997)	0.0370	272.00	0.500	90	43.00	2.94	73.000	0.044	26.00
59	Umezu (1997)	0.0370	272.00	1.000	90	44.80	2.94	73.000	0.088	49.40
60	Umezu (1997)	0.0175	257.00	1.000	90	40.50	2.96	244.00	0.111	109.8
61	Umezu (1997)	0.0175	257.00	0.500	90	40.50	3.15	244.00	0.111	54.80
62	Umezu (1997)	0.0175	272.00	0.500	90	44.80	3.15	244.00	0.111	51.40
63	Umezu (1997)	0.0370	253.00	1.000	90	41.90	3.20	73.000	0.044	62.00
64	Umezu (1997)	0.0370	253.00	1.000	90	41.90	3.20	73.000	0.088	92.80
65	Umezu (1997)	0.0370	253.00	1.000	90	42.70	3.20	73.000	0.144	122.0
66*	Umezu (1997)	0.0370	253.00	1.000	90	43.50	3.20	73.000	0.144	130.2
67	Umezu (1997)	0.0370	399.00	1.000	90	39.90	3.20	73.000	0.144	161.5
68	Miyauchi et al. (1998)	0.0151	165.00	0.200	90	35.10	3.00	230.00	0.111	18.75
69*	Miyauchi et al. (1998)	0.0151	165.00	0.500	90	32.40	3.00	230.00	0.111	29.50
70	Teng et al. (2009)	0.0150	260.00	0.400	90	40.48	2.50	266.00	0.110	67.80
71	Teng et al. (2009)	0.0150	260.00	0.400	90	36.32	2.50	266.00	0.110	64.00
72	Deniaud and Cheng (2001)	0.0150	390.00	0.500	90	35.55	2.74	230.00	0.110	85.25
73	Deniaud and Cheng (2001)	0.0060	390.00	1.000	90	35.55	2.74	17.700	1.800	109.9
74	Deniaud and Roger Cheng (2003)	0.0150	211.00	0.500	45	29.45	2.85	230.00	0.110	17.80
75	Deniaud and Roger Cheng (2003)	0.0060	211.00	1.000	90	30.00	2.85	17.700	1.800	48.65
76*	Park et al. (2001)	0.0142	204.00	1.000	90	17.10	2.50	240.00	0.160	38.60
77*	Beber and Campos Filho (2005)	0.0148	255.00	0.500	90	24.50	2.90	230.00	0.111	67.88
78	Beber and Campos Filho (2005)	0.0148	255.00	0.500	90	24.50	2.90	230.00	0.111	83.20
79*	Beber and Campos Filho (2005)	0.0148	255.00	0.350	45	24.50	2.90	230.00	0.111	61.50
80	Abdel-Jaber et al. (2003)	0.0148	165.00	1.000	90	47.67	2.42	230.00	0.270	41.86
81*	Adhikary et al. (2003)	0.0148	245.00	1.000	90	37.20	4.08	230.00	0.167	53.00
82	Adhikary and Mutsuyoshi (2004)	0.0180	245.00	1.000	90	39.60	4.08	120.00	0.286	43.00
83	Feng and Zhong Fan (2004)	0.0179	310.00	1.000	90	32.55	2.79	235.00	0.220	76.25
84	Feng and Zhong Fan (2004)	0.0179	310.00	0.250	90	32.55	2.79	235.00	0.220	53.25
85	Feng and Zhong Fan (2004)	0.0179	310.00	0.250	90	32.55	2.79	235.00	0.440	63.25
86*	Miyajima et al. (2005)	0.0191	375.00	0.333	90	29.82	2.93	253.00	0.110	81.30
87	Miyajima et al. (2005)	0.0191	375.00	0.500	90	29.82	2.93	253.00	0.110	122.0
88*	Miyajima et al. (2005)	0.0191	375.00	0.580	90	29.82	2.93	253.00	0.110	132.2
89	Miyajima et al. (2005)	0.0191	375.00	0.667	90	29.82	2.93	253.00	0.110	162.6

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