



A novel WASPAS approach for multi-criteria physician selection problem with intuitionistic fuzzy type-2 sets

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Abstract

Due to innovative and practical technology, the selection of a right physician is an important issue for patients. However, uncertainty and vagueness frequently arise during the process of selecting physicians. Intuitionistic fuzzy type-2 sets (IFT2Ss) (recently named as Pythagorean fuzzy sets) provide an important tool to handle the uncertainty arises in real-life decision-making problems. This paper presents an extended weighted aggregated sum product assessment (WASPAS) method based on novel information measures (entropy and divergence measures) and operators under IFT2Ss context. In the proposed WASPAS method, entropy and divergence measure-based formula is developed to find the criteria weights. For this, several intuitionistic fuzzy entropy and divergence measures are developed for IFT2Ss. To increase the stability of the proposed methodology, the criteria's weights are calculated in the form of objective and subjective weights. Further, to reveal the applicability and effectiveness of proposed method, an uncertain multi-criteria decision-making problem of physician selection is executed with intuitionistic fuzzy information of second type. Finally, the validity of the proposed method is implemented by comparison with existing methods and sensitivity analysis and also proves that the proposed method is valid and feasible in the physician selection processes with information given in intuitionistic fuzzy type-2 numbers (IFT2Ns).

Keywords Divergence measure · Entropy measure · Multi-criteria decision making · Intuitionistic fuzzy type-2 set · Pythagorean fuzzy set · WASPAS

1 Introduction

Multi-criteria decision making (MCDM), a part of decision theory, is one of the fastest growing research areas of operation research both from hypothetical and execution

points of view. In recent times, several new approaches have been developed to solve the real-life MCDM problems such as renewable energy selection, flood reservoir management, cloud service provider, and vehicle insurance service quality assessment. Recently, medical decision-making domain has widely been concentrated by various researchers. For instance, Harris (2003) scrutinized the process to find and choose the suitable physician for patients and investigated that there is a need for the development of decision support tools in healthcare systems. Madupu (2009) paid attention to select the best physician based on the types of information resources. Fhkam and Lam (2010) focused their study on those aspects which affect the selection of doctors for patients. Verhoef et al. (2014) centered their study on the rapid development of information technology that allows the patients to rate the physicians on the basis of their experience on online websites. In the recent time, only some researchers have focused on the development of new decision-making support tools for the selection of

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suitable physicians. Sun et al. (2017) developed an integrated approach based on TODIM and ELECTRE methods to solve the physician selection decision-making problem under single-valued neutrosophic fuzzy environment. Hu et al. (2017) extended the VIKOR method and established a projection-based VIKOR method under interval neutrosophic fuzzy doctrines for solving a doctor selection problem.

In MCDM problems, the decision makers desire to get more proficient solution, but it is not possible all the time due to inaccurate and uncertain information available for the given situation. Fuzzy set (FS) theory developed by Zadeh (1965) have gained more attention in the field of decision making as the FSs are more capable to deal with uncertain information. Since FS is only characterized by a membership degree, for that reason, it is unable to express the decision information in sustain and oppose concurrently. Atanassov (1986) pioneered the concept of intuitionistic fuzzy sets (IFSs) to overcome the shortcomings of FSs. As the extension of FSs, IFSs are characterized by the membership, non-membership and hesitation degrees and satisfy the condition that the sum of the degrees of membership and non-membership is less than or equal to 1. In recent times, IFSs have broadly been applied in the field of decision making, medical diagnosis, image processing, pattern recognition and many decision-making methods have been developed under intuitionistic fuzzy environment (Tanev 1995; Ansari et al. 2018; Mishra et al. 2016, 2017a, b, c; Rani and Jain 2017; Rani et al. 2018, Mishra et al. 2018a, b).

In 1989, Atanassov (1989) pioneered the concept of intuitionistic fuzzy type-2 set (IFT2S) with its geometrical interpretation. It is characterized by the membership and non-membership degrees satisfying the condition that the square sum of the membership and non-membership degrees is less and equal to 1. They also discussed the general form of IFS-2T, which is the intuitionistic fuzzy set of n-th type or p-th type (IFS-nT or IFS-pT) (Atanassov 1999; Atanassov et al. 2017; Atanassov and Vassilev 2018). After that, many authors (Vassilev 2006; Vassilev et al. 2008; Parvathi et al. 2012) have focused their study on IFT2Ss and IFTnSs. Later on, Yager (2013, 2014) studied the same concept of IFT2Ss with the new name Pythagorean fuzzy set (PFS). Some authors [Yager (2013, 2014), Yager and Abbasov (2013)] gave a figure which shows the regions and relation between PFSs and IFSs. Due to its capability and popularity of handling complicated decision-making problems, the PFSs have attracted much attention from decision makers in a very short duration of time. Yager and Abbasov (2013) discussed the concepts related to PFSs and showed a relationship between the PFNs and the complex numbers. In addition, they discussed a decision-making approach in

which the criteria values are given by complex numbers. Zhang and Xu (2014) defined some basic operations such as addition, multiplication, union, intersection for PFNs and then extended an MCDM method namely TOPSIS under Pythagorean fuzzy environment. Recently, several literatures show the rapid growth and potentiality of the PFSs (Gou et al. 2016; Peng and Yang 2015; Zeng et al. 2016; Peng and Yang 2016; Peng et al. 2017; Li and Zeng 2018).

Due to diverse nature of MCDM problems, several methods have been developed to cope with the MCDM problems under different fuzzy environments. The weighted aggregated sum product assessment (WASPAS), proposed by Zavadskas et al. (2014), is a novel utility theory-based approach that has been widely extended in different doctrines. The WASPAS method is an integration of weighted sum model (WSM) and weighted product model (WPM) and is more accurate than WSM and WPM. For instance, Zavadskas et al. (2013) focused their study on the verification of robustness of the MOORA (Multiple Objective Optimization on the basis of Ratio Analysis) and WASPAS approaches. Bagočius et al. (2014) developed a MCDM approach to select and rank the location areas of wind farms based on WASPAS and then assessed the types of wind turbines in the Baltic Sea offshore area. Bitarafan et al. (2014) presented the WASPAS- and SWARA-based approach to analyze the real-time intelligent sensors for structural health monitoring of bridges. Recently, Mardani et al. (2017) proposed an overview of two new MCDM utility-based approaches named as SWARA (stepwise weight assessment ratio analysis) and WASPAS, and their applications in different fuzzy doctrines. Peng and Dai (2017) extended the MABAC (multi-attributive border approximation area comparison), WASPAS and COPRAS (complex proportional assessment) approaches to solve hesitant fuzzy soft decision-making problem. Mishra et al. (2018a, b) developed the intuitionistic fuzzy WASPAS method to compare the performance of telephone service providers in Madhya Pradesh (India). Mishra and Rani (2018) introduced the WASPAS approach under interval-valued intuitionistic fuzzy environment and applied to solve the reservoir flood control management policy selection problem. As the IFT2S is an interesting concept to deal with uncertainty and vagueness, the present study focuses within the context of IFT2Ss. In medical domain, the patients want to select the best physician from many physicians influenced by several factors such as rating scores, quality care, conformability, available reviews and many other points; as a result, a physician selection problem is considered as an MCDM problem. In this paper, a novel approach is developed to handle the MCDM problem of physician selection under IFT2Ss environment. The outcomes of the present study are as follows:

1. Divergence and entropy measures are an important concern for researchers in the study of uncertainty. Thus, we have developed new entropy and divergence measures for IFT2Ss.
2. The classical WASPAS method is extended to handle the MCDM problems under IFT2Ss environment with objective and subjective criteria's weights.
3. In this method, the criteria's weights are calculated using proposed entropy and divergence measures.
4. A decision-making problem of selecting a proper physician among set of alternatives is applied to illustrate the applicability of the proposed WASPAS method.
5. Comparative study and sensitivity analysis reveal the validity of the results obtained by proposed method.

The remainder of this paper is organized as follows: In Sect. 2, some fundamental results are explained. In Sect. 3, firstly existing literatures are presented and then new entropy and divergence measures are developed. In Sect. 4, the classical WASPAS method is extended within IFT2Ss context with unknown criteria and decision-makers' weights, which is based on divergence and entropy measures. In Sect. 5, a numerical study is presented to illustrate the validity and applicability of the proposed WASPAS method. Comparative study with existing methods and sensitivity analysis are also discussed. Finally, the concluding remarks are given in Sect. 6.

2 Basic concepts

In this section, some fundamental concepts related to intuitionistic fuzzy sets (IFSs), intuitionistic fuzzy sets of second type (IFT2Ss) and information measures (entropy and divergence measures) of IFT2Ss are presented.

Definition 2.1 (Atanassov 1986) An IFS M in a finite universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ is defined as

$$M = \{ \langle x_i, \mu_M(x_i), \nu_M(x_i) \rangle \mid x_i \in X \},$$

where $\mu_M : X \rightarrow [0, 1]$ is the degree of membership and $\nu_M : X \rightarrow [0, 1]$ the degree of non-membership of the element $x_i \in X$ in M such that $0 \leq \mu_M(x_i) + \nu_M(x_i) \leq 1$. The hesitant degree of $x_i \in X$ in M is expressed by $\pi_M(x_i) = 1 - \mu_M(x_i) - \nu_M(x_i)$. For ease, the intuitionistic fuzzy number (IFN) is denoted by $\alpha = M(\mu_\alpha, \nu_\alpha)$ which satisfies $\mu_\alpha, \nu_\alpha \in [0, 1]$ and $0 \leq \mu_\alpha + \nu_\alpha \leq 1$.

Definition 2.2 (Atanassov 1989; Yager 2013) An IFT2S (or PFS) A in a finite universe of discourse X is defined as $A = \{ \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle \mid x_i \in X \}$,

where $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ denote the degrees of membership and non-membership of the element $x_i \in X$, respectively and for every $x_i \in X$,

$$0 \leq (\mu_A(x_i))^2 + (\nu_A(x_i))^2 \leq 1.$$

For each $x_i \in X$, the degree of indeterminacy or hesitation is given by $\pi_A(x_i) = \sqrt{1 - \mu_A^2(x_i) - \nu_A^2(x_i)}$. Zhang and Xu (2014) denoted the Pythagorean fuzzy number (PFN) by $\eta = A(\mu_\eta, \nu_\eta)$ which fulfills $\mu_\eta, \nu_\eta \in [0, 1]$ and $0 \leq \mu_\eta^2 + \nu_\eta^2 \leq 1$.

Definition 2.3 (Zhang and Xu 2014) Let $\eta = A(\mu_\eta, \nu_\eta)$ be a PFN. Then, the score function and the accuracy function of η are defined by

$$\mathbb{S}(\eta) = (\mu_\eta)^2 - (\nu_\eta)^2, \quad \mathbb{h}(\eta) = (\mu_\eta)^2 + (\nu_\eta)^2, \\ \text{where } \mathbb{S}(\eta) \in [-1, 1] \text{ and } \mathbb{h}(\eta) \in [0, 1].$$

Since $\mathbb{S}(\eta) \in [-1, 1]$, when several score functions are aggregated with linear weighted summation method and it may appear that positive score functions are offset by negative score functions. Therefore, a modified score function of PFNs is defined as follows:

Definition 2.4 Let $\eta = A(\mu_\eta, \nu_\eta)$ be PFN. Then, the normalized score and uncertainty functions of η is defined by

$$\mathbb{S}^*(\eta) = \frac{1}{2}(\mathbb{S}(\eta) + 1), \quad \mathbb{h}^\circ(\eta) = 1 - \mathbb{h}(\eta), \\ \text{such that } \mathbb{S}^*(\eta), \mathbb{h}^\circ(\eta) \in [0, 1].$$

For any two PFNs $\eta_1 = A(\mu_{\eta_1}, \nu_{\eta_1})$ and $\eta_2 = A(\mu_{\eta_2}, \nu_{\eta_2})$,

- (i) If $\mathbb{S}^*(\eta_1) > \mathbb{S}^*(\eta_2)$, then $\eta_1 > \eta_2$,
- (ii) If $\mathbb{S}^*(\eta_1) = \mathbb{S}^*(\eta_2)$, then
 - (a) if $\mathbb{h}^\circ(\eta_1) > \mathbb{h}^\circ(\eta_2)$, then $\eta_1 < \eta_2$;
 - (b) if $\mathbb{h}^\circ(\eta_1) = \mathbb{h}^\circ(\eta_2)$, then $\eta_1 = \eta_2$.

Definition 2.5 (Atanassov 1989; De et al. 2000; Yager 2013, 2014) Let $\eta = A(\mu_\eta, \nu_\eta)$, $\eta_1 = A(\mu_{\eta_1}, \nu_{\eta_1})$ and $\eta_2 = A(\mu_{\eta_2}, \nu_{\eta_2})$ are three PFNs. Then, the following operations are satisfied for PFSs:

- (i) $\eta^c = A(\nu_\eta, \mu_\eta)$;
- (ii) $\eta_1 \oplus \eta_2 = A\left(\sqrt{\mu_{\eta_1}^2 + \mu_{\eta_2}^2 - \mu_{\eta_1}^2 \mu_{\eta_2}^2}, \nu_{\eta_1} \nu_{\eta_2}\right)$;
- (iii) $\eta_1 \otimes \eta_2 = A\left(\mu_{\eta_1} \mu_{\eta_2}, \sqrt{\nu_{\eta_1}^2 + \nu_{\eta_2}^2 - \nu_{\eta_1}^2 \nu_{\eta_2}^2}\right)$;
- (iv) $\lambda \eta = A\left(\sqrt{1 - (1 - \mu_\eta^2)^\lambda}, (\nu_\eta)^\lambda\right), \quad \lambda > 0$;

$$(v) \quad \eta^\lambda = A \left((\mu_\eta)^\lambda, \sqrt{1 - (1 - v_\eta^2)^\lambda} \right), \quad \lambda > 0.$$

Definition 2.6 (Peng et al. 2017) An entropy measure $E : \text{PFS}(X) \rightarrow [0, 1]$ is a mapping which satisfies the following axioms:

- (A1) $0 \leq E(A) \leq 1$;
- (A2) $E(A) = 0$ if A is a crisp set;
- (A3) $E(A) = 1$ if $\mu_A(x_i) = v_A(x_i), \forall x_i \in X$;
- (A4) $E(A) = E(A^c)$;
- (A5) For each $x_i \in X, E(A) \leq E(B)$ if A is less than B , i.e., $\mu_A(x_i) \leq \mu_B(x_i) \leq v_B(x_i) \leq v_A(x_i)$ or $v_A(x_i) \leq v_B(x_i) \leq \mu_B(x_i) \leq \mu_A(x_i)$.

Definition 2.7 (Montes et al. 2015) A real-valued function $D : \text{PFSs}(X) \times \text{PFSs}(X) \rightarrow \mathbb{R}$ is said to be a divergence measure for PFSs if it satisfies the following requirements:

- (P1) $D(A, B) = D(B, A)$;
- (P2) $D(A, B) = 0$ if and only if $A = B$;
- (P3) $D(A \cap C, B \cap C) \leq D(A, B)$ for every $C \in \text{PFSs}(X)$;
- (P4) $D(A \cup C, B \cup C) \leq D(A, B)$ for every $C \in \text{PFSs}(X)$.

3 Information measures for IFT2Ss

Since entropy and divergence measures play a significant role in handling with uncertainty, therefore, in this section, some information measures (entropy and divergence measures) are proposed for IFT2Ss. However, very few entropy measures are developed for PFSs (as same as IFT2Ss) but no study is available in the literature regarding divergence measure for PFSs.

3.1 Entropy measures for IFSSs-2T

Corresponding to Mishra and Rani (2017), we have proposed a new entropy measure for IFT2S, which as

$$E_1(A) = \frac{1}{n(1 - e^{-1/2})} \sum_{i=1}^n \left[\left\{ 1 - e^{-\left(\frac{\mu_A^2(x_i) + 1 - v_A^2(x_i)}{2}\right)} \right\} I_{[\mu_A^2(x_i) \geq v_A^2(x_i)]} + \left\{ 1 - e^{-\left(\frac{\mu_A^2(x_i) + 1 - v_A^2(x_i)}{2}\right)} \right\} I_{[\mu_A^2(x_i) < v_A^2(x_i)]} \right]. \tag{1}$$

Theorem 3.1 The mapping $E_1(A)$, defined by (1), is an entropy measure for IFT2S (X).

Proof In order to prove this theorem, the function (1) must satisfy the axiomatic requirements (A1)–(A5) of Definition 2.6.

(A1) In PFSs, as we know that $0 \leq \mu_A^2(x_i) + v_A^2(x_i) \leq 1$, then it is clear from (1) that $0 \leq E_1(A) \leq 1$.

(A2) Let A be a crisp set, i.e., $\mu_A(x_i) = 1, v_A(x_i) = 0$ or $\mu_A(x_i) = 0, v_A(x_i) = 1$. Then, we obtain $E_1(A) = 0$.

On the other hand, suppose that $E_1(A) = 0$.

Also, let us assume that $\frac{\mu_A^2(x_i) + 1 - v_A^2(x_i)}{2} = \psi_A(x_i)$. In view of this assumption, (1) becomes

$$E_1(A) = \frac{1}{n(1 - e^{-1/2})} \sum_{i=1}^n \left[\left\{ 1 - e^{-(1 - \psi_A(x_i))} \right\} I_{[\psi_A(x_i) \geq \frac{1}{2}]} + \left\{ 1 - e^{-\psi_A(x_i)} \right\} I_{[\psi_A(x_i) < \frac{1}{2}]} \right]. \tag{2}$$

Corresponding to Mishra and Rani (2017), (2) becomes zero if and only if $\psi_A(x_i) = 0$ or 1, i.e.,

$$\frac{\mu_A^2(x_i) + 1 - v_A^2(x_i)}{2} = 0 \tag{3}$$

$$\text{or } \frac{\mu_A^2(x_i) + 1 - v_A^2(x_i)}{2} = 1. \tag{4}$$

From Definition 2.2, $0 \leq \mu_A^2(x_i) + v_A^2(x_i) \leq 1$. Now, solving (3) and (4), we have $\mu_A^2(x_i) = 0 = \mu_A(x_i), v_A^2(x_i) = 1 = v_A(x_i)$ or $\mu_A^2(x_i) = 1 = \mu_A(x_i), v_A^2(x_i) = 0 = v_A(x_i), \forall x_i \in X$, it implies that A is a crisp set.

(A3) It can easily be seen that if $\mu_A(x_i) = v_A(x_i), \forall x_i \in X$, then $E_1(A) = 1$.

On the contrary, if $E_1(A) = 1$, then it implies that

$$E_1(A) = \frac{1}{n(1 - e^{-1/2})} \sum_{i=1}^n \left[\left\{ 1 - e^{-(1 - \psi_A(x_i))} \right\} I_{[\psi_A(x_i) \geq \frac{1}{2}]} + \left\{ 1 - e^{-\psi_A(x_i)} \right\} I_{[\psi_A(x_i) < \frac{1}{2}]} \right] = 1, \Rightarrow \psi_A(x_i) = \frac{1}{2},$$

$$\forall x_i \in X, \Rightarrow \frac{\mu_A^2(x_i) + 1 - v_A^2(x_i)}{2} = \frac{1}{2},$$

$$\forall x_i \in X, \Rightarrow \mu_A^2(x_i) = v_A^2(x_i),$$

$$\forall x_i \in X, \Rightarrow \mu_A(x_i) = v_A(x_i), \quad \forall x_i \in X.$$

(A4) It is obvious from (1) that $E_1(A) = E_1(A^c)$.

(A5) Let

$$\psi(x, y) = \left[\left\{ 1 - e^{-\left(\frac{x+y}{2}\right)} \right\} I_{[x \geq y]} + \left\{ 1 - e^{-\left(\frac{x+y}{2}\right)} \right\} I_{[x < y]} \right], \tag{5}$$

where $x, y \in [0, 1]$. Differentiating (5) partially w.r.t. ‘ x ’ and ‘ y ’ respectively, we have

$$\frac{\partial \psi(x, y)}{\partial x} = \frac{1}{2} \left[- \left\{ e^{-\left(\frac{x+y}{2}\right)} \right\} I_{[x \geq y]} + \left\{ e^{-\left(\frac{x+y}{2}\right)} \right\} I_{[x < y]} \right], \tag{6}$$

$$\frac{\partial \psi(x, y)}{\partial y} = \frac{1}{2} \left[\left\{ e^{-\frac{(x+1-y)}{2}} \right\} I_{[x \geq y]} - \left\{ e^{-\frac{(x+1-y)}{2}} \right\} I_{[x < y]} \right]. \tag{7}$$

To obtain the critical point of $\psi(x, y)$, we equate $\frac{\partial \psi(x, y)}{\partial x} = 0$ and $\frac{\partial \psi(x, y)}{\partial y} = 0$. This gives $x_c = y$. From (6) and (7), we get

$$\frac{\partial \psi(x, y)}{\partial x} \geq 0 \text{ for } x \leq y \text{ and } \frac{\partial \psi(x, y)}{\partial x} \leq 0 \text{ for } x \geq y.$$

Thus, $\psi(x, y)$ is an increasing function w.r.t. x for $x \leq y$ and decreasing for $x \geq y$. In the same way, we can prove that $\frac{\partial \psi(x, y)}{\partial y} \leq 0$ for $x \leq y$ and $\frac{\partial \psi(x, y)}{\partial y} \geq 0$ for $x \geq y$.

If $A \subseteq B$ and also suppose that the universal set X is divided into two parts X_1 and X_2 such that $\phi = X_1 \cap X_2$ and $X = X_1 \cup X_2$. Furthermore, assume that $x_i \in X_1$ with $\mu_A(x_i) \leq \mu_B(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i)$ and $x_i \in X_2$ with $\mu_A(x_i) \geq \mu_B(x_i) \geq \nu_B(x_i) \geq \nu_A(x_i)$. Thus, by monotonicity of $\psi(x, y)$, we have $E_1(A) \leq E_1(B)$.

Theorem 3.2 For $A \in \text{IFT2S}(X)$, a real-valued function $E_2(A)$ defined by

$$E_2(A) = \frac{1}{n\sqrt{e}(\sqrt{e}-1)} \sum_{i=1}^n \left[e - \left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right) e^{\left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right)} - \left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right) e^{\left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right)} \right] \tag{8}$$

is an entropy measure for IFT2S(X).

Proof The proof is similar to Theorem 3.1.

Few entropy measures for IFT2Ss are generalized based on intuitionistic fuzzy entropy measures (Ansari et al. 2018; Mishra et al. 2017a; Mishra 2016), which are

$$E_3(A) = \frac{1}{n(e^{1-0.5^x} - 1)} \sum_{i=1}^n \left[1 - \left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right) e^{\left(1 - \left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right)^x \right)} - \left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right) e^{\left(1 - \left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right)^x \right)} \right]. \tag{9}$$

$$E_4(A) = \frac{1}{ne^{\beta/2}(e^{\beta/2} - 1)} \sum_{i=1}^n \left[e^\beta - \left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right) e^{\beta \left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right)} - \left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right) e^{\beta \left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right)} \right], \quad \beta \neq 1(\beta > 0). \tag{10}$$

$$E_5(A) = -\frac{2}{n} \sum_{i=1}^n \left[\ln \left\{ \left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right) e^{\left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right)} + \left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right) e^{\left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right)} \right\} - 1 \right]. \tag{11}$$

$$E_6(A) = \frac{2}{n} \sum_{i=1}^n \ln \left[\left\{ \left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right) e^{\left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right)} + \left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right) e^{\left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right)} \right\} \right]. \tag{12}$$

$$E_7(A) = \frac{1}{n} \sum_{i=1}^n \left[1 - \sin \left\{ \frac{(\mu_A^2(x_i) \sim \nu_A^2(x_i))}{2(1 + \pi_A^2(x_i))} \right\} \pi \right]. \tag{13}$$

$$E_8(A) = \frac{1}{2n} \sum_{i=1}^n \left[\sin \left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right) \pi + \sin \left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right) \pi \right]. \tag{14}$$

3.2 Divergence measure for IFT2Ss

Theorem 3.3 Let $A, B \in \text{IFT2Ss}(X)$. Corresponding to Mishra et al. (2017b), the Jensen divergence measure for IFT2Ss is defined as

$$D_1(A, B) = -\frac{1}{n\sqrt{e}(\sqrt{e}-1)} \sum_{i=1}^n \left[\left(\frac{(\mu_A^2(x_i) + \mu_B^2(x_i)) + 2 - (\nu_A^2(x_i) + \nu_B^2(x_i))}{4} \right) e^{\left(\frac{(\mu_A^2(x_i) + \mu_B^2(x_i)) + 2 - (\nu_A^2(x_i) + \nu_B^2(x_i))}{4} \right)} + \left(\frac{(\nu_A^2(x_i) + \nu_B^2(x_i)) + 2 - (\mu_A^2(x_i) + \mu_B^2(x_i))}{4} \right) e^{\left(\frac{(\nu_A^2(x_i) + \nu_B^2(x_i)) + 2 - (\mu_A^2(x_i) + \mu_B^2(x_i))}{4} \right)} - \frac{1}{2} \left\{ \left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right) e^{\left(\frac{\mu_A^2(x_i) + 1 - \nu_A^2(x_i)}{2} \right)} + \left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right) e^{\left(\frac{\nu_A^2(x_i) + 1 - \mu_A^2(x_i)}{2} \right)} + \left(\frac{\mu_B^2(x_i) + 1 - \nu_B^2(x_i)}{2} \right) e^{\left(\frac{\mu_B^2(x_i) + 1 - \nu_B^2(x_i)}{2} \right)} + \left(\frac{\nu_B^2(x_i) + 1 - \mu_B^2(x_i)}{2} \right) e^{\left(\frac{\nu_B^2(x_i) + 1 - \mu_B^2(x_i)}{2} \right)} \right\} \right] \tag{15}$$

is a divergence measure for IFT2Ss(X).

Proof To prove this theorem, the measure (15) must have to satisfy the conditions (A1)–(A4) of Definition 2.7. As, if $A = (1, 0, 0)$ and $B = (0, 1, 0)$ ($A = (0, 0, 1)$, $B = (0, 1, 0)$ or $A = (1, 0, 0)$, $B = (0, 0, 1)$), then $D_1(A, B)$ reaches its maximum value and thus, $D_1(A, B) \geq 0$.

(P1) It is obvious from (15) that $D_1(A, B) = D_1(B, A)$.

(P2) It is evident from (15).

(P3) For every $A, B, C \in \text{IFT2Ss}(X)$, we have

$$\begin{aligned}
 & \frac{(\min\{\mu_A^2(x_i), \mu_C^2(x_i)\})}{+2 - (\max\{v_A^2(x_i), v_C^2(x_i)\})} e^{\left(\frac{(\min\{\mu_A^2(x_i), \mu_C^2(x_i)\})}{+2 - (\max\{v_A^2(x_i), v_C^2(x_i)\})}\right)} \\
 & \Rightarrow \frac{(\max\{v_A^2(x_i), v_C^2(x_i)\})}{+2 - (\min\{\mu_A^2(x_i), \mu_C^2(x_i)\})} e^{\left(\frac{(\max\{v_A^2(x_i), v_C^2(x_i)\})}{+2 - (\min\{\mu_A^2(x_i), \mu_C^2(x_i)\})}\right)} \\
 & \leq \left(\frac{v_A^2(x_i) + v_B^2(x_i)}{+2 - (\mu_A^2(x_i) + \mu_B^2(x_i))}\right) e^{\left(\frac{v_A^2(x_i) + v_B^2(x_i)}{+2 - (\mu_A^2(x_i) + \mu_B^2(x_i))}\right)} \\
 & \Rightarrow \frac{(\min\{\mu_A^2(x_i), \mu_C^2(x_i)\})}{+1 - (\max\{v_A^2(x_i), v_C^2(x_i)\})} e^{\left(\frac{(\min\{\mu_A^2(x_i), \mu_C^2(x_i)\})}{+1 - (\max\{v_A^2(x_i), v_C^2(x_i)\})}\right)} \\
 & \leq \frac{(\mu_A^2(x_i) + 1 - v_A^2(x_i))}{2} e^{\frac{(\mu_A^2(x_i) + 1 - v_A^2(x_i))}{2}} \\
 & \Rightarrow \frac{(\max\{v_A^2(x_i), v_C^2(x_i)\})}{+1 - (\min\{\mu_A^2(x_i), \mu_C^2(x_i)\})} e^{\left(\frac{(\max\{v_A^2(x_i), v_C^2(x_i)\})}{+1 - (\min\{\mu_A^2(x_i), \mu_C^2(x_i)\})}\right)} \\
 & \leq \frac{(v_A^2(x_i) + 1 - \mu_A^2(x_i))}{2} e^{\frac{(v_A^2(x_i) + 1 - \mu_A^2(x_i))}{2}} \\
 & \Rightarrow \frac{(\min\{\mu_B^2(x_i), \mu_C^2(x_i)\})}{+1 - (\max\{v_B^2(x_i), v_C^2(x_i)\})} e^{\left(\frac{(\min\{\mu_B^2(x_i), \mu_C^2(x_i)\})}{+1 - (\max\{v_B^2(x_i), v_C^2(x_i)\})}\right)} \\
 & \leq \frac{(\mu_B^2(x_i) + 1 - v_B^2(x_i))}{2} e^{\frac{(\mu_B^2(x_i) + 1 - v_B^2(x_i))}{2}}
 \end{aligned}$$

and

$$\begin{aligned}
 & \frac{(\max\{v_B^2(x_i), v_C^2(x_i)\})}{+1 - (\min\{\mu_B^2(x_i), \mu_C^2(x_i)\})} e^{\left(\frac{(\max\{v_B^2(x_i), v_C^2(x_i)\})}{+1 - (\min\{\mu_B^2(x_i), \mu_C^2(x_i)\})}\right)} \\
 & \leq \frac{(v_B^2(x_i) + 1 - \mu_B^2(x_i))}{2} e^{\frac{(v_B^2(x_i) + 1 - \mu_B^2(x_i))}{2}}.
 \end{aligned}$$

This implies that $D_1(A \cap C, B \cap C) \leq D_1(A, B)$.
 (P4) Similar as proof (P3).

Theorem 3.4 *The mapping defined by*

$$\begin{aligned}
 D_2(A, B) = & -\frac{1}{2n(1 - e^{-1})} \sum_{i=1}^n \\
 & \left[\left\{ e^{-\left(\frac{(\mu_B^2(x_i) - \mu_A^2(x_i))}{4} + \left(\frac{v_A^2(x_i) - v_B^2(x_i)}{2}\right)\right)} + e^{-\left(\frac{(\mu_A^2(x_i) - \mu_B^2(x_i))}{4} + \left(\frac{v_B^2(x_i) - v_A^2(x_i)}{2}\right)\right)} - 2 \right\} I_{[\mu_A^2(x_i) \geq v_A^2(x_i)]} \right. \\
 & \left. + \left\{ e^{-\left(\frac{(\mu_A^2(x_i) - \mu_B^2(x_i))}{4} + \left(\frac{v_B^2(x_i) - v_A^2(x_i)}{2}\right)\right)} + e^{-\left(\frac{(\mu_B^2(x_i) - \mu_A^2(x_i))}{4} + \left(\frac{v_A^2(x_i) - v_B^2(x_i)}{2}\right)\right)} - 2 \right\} I_{[\mu_A^2(x_i) < v_A^2(x_i)]} \right] \tag{16}
 \end{aligned}$$

is a divergence measure for IFT2Ss.

Proof The proof is similar as Theorem 3.3.

4 Novel WASPAS approach for MCDM problems

This section extends the classical weighted aggregated sum product assessment (WASPAS) method within IFT2S context. The proposed method is developed to handle the MCDM problems with unknown criteria and decision-makers' weights. The algorithm for WASPAS approach is as follows:

Step 1 Formulate the MCDM problem and then construct the decision matrix.

In MCDM problem, let $R = \{R_1, R_2, \dots, R_m\}$ be a set of alternatives and $Q = \{Q_1, Q_2, \dots, Q_n\}$ be a set of criteria. A set of decision experts $\{B_1, B_2, \dots, B_l\}$ gives their decisions on each alternative R_i with respect to the criterion Q_j in terms of linguistic variables. Let $N = (g_{ij}^{(k)})$, $i = 1(1)m, j = 1(1)n$ be the linguistic matrix provided by the decision experts, where $h_{ij}^{(k)}$ denotes the evaluation of an alternative R_i with respect to criterion Q_j in terms of linguistic variables for k^{th} decision expert.

Step 2 Determine the weights of the decision experts.

During the process of decision making, it is an important concern to determine the weights of the decision experts. For this, suppose that the significances of the decision experts are expressed as linguistic variables given in terms of intuitionistic fuzzy numbers of second type (IFT2 Ns). For evaluation of k th decision expert, let $B_k = [\mu_k, v_k, \pi_k]$ be the IFT2N, then the weight of the k th decision expert is obtained by using the following formula (Boran et al. 2009):

$$\lambda_k = \frac{\left(\mu_k^2 + \pi_k^2 \times \left(\frac{\mu_k^2}{\mu_k^2 + v_k^2}\right)\right)}{\sum_{k=1}^{\ell} \left(\mu_k^2 + \pi_k^2 \times \left(\frac{\mu_k^2}{\mu_k^2 + v_k^2}\right)\right)}, \quad k = 1(1)\ell. \tag{17}$$

Clearly, $\lambda_k \geq 0$ and $\sum_{k=1}^{\ell} \lambda_k = 1$.

Step 3: Aggregate the decision matrix.

To construct the aggregated intuitionistic fuzzy decision matrix of second type, all individual matrices need to be

combined into one group based on decision experts' opinions. To facilitate that intuitionistic fuzzy weighted averaging operator of type-2 (IFT2WAO) or Pythagorean fuzzy weighted averaging operator (PFWAO) [Yager (2013)] is applied and then $\mathbb{N} = (\varepsilon_{ij})_{m \times n}$, where

$$\begin{aligned} \varepsilon_{ij} &= \text{PFWAO}_\lambda \left(h_{ij}^{(1)}, h_{ij}^{(2)}, \dots, h_{ij}^{(\ell)} \right) \\ &= \left(\sqrt[1-\prod_{k=1}^{\ell} (1-\mu_k^2)^{\lambda_k}] , \prod_{k=1}^{\ell} (v_k)^{\lambda_k} \right). \end{aligned} \tag{18}$$

Step 4: Calculate the weights of the criteria.

To determine the relative importance of each criterion, we have developed the following formula with the help of entropy and divergence measures:

All the criteria are not assumed to be equal importance. Let $w = (w_1, w_2, \dots, w_n)^T$ such that $\sum_{j=1}^n w_j = 1$, $w_j \in [0, 1]$ be an importance vector for the criterion set. Consecutively, to achieve w , we apply the following procedure:

(1) Calculate the objective weight w_j^o for each criterion using the divergence measure

$$w_j^o = \frac{\sum_{i=1}^m \left[\frac{1}{m-1} \sum_{k=1, k \neq i}^m D_1(\eta_{ij}, \eta_{ki}) + (1 - E_1(\eta_{ij})) \right]}{\sum_{j=1}^n \sum_{i=1}^m \left[\frac{1}{m-1} \sum_{k=1, k \neq i}^m D_1(\eta_{ij}, \eta_{ki}) + (1 - E_1(\eta_{ij})) \right]}, \tag{19}$$

$j = 1, 2, \dots, n.$

Here, $D_1(\eta_{ij}, \eta_{kj})$ denotes the divergence measure between η_{ij} and η_{kj} , and $E_1(\eta_{ij})$ denotes the entropy measure of η_{ij} .

(2) Calculate the subjective weight w_j^s for criterion

First, construct the subjective weighted matrix (w_j^s) for k th decision expert as follows:

$$w_j^s = \left[w_{j(k)}^s \right]_{n \times 1} = \begin{bmatrix} w_{1(k)}^s \\ w_{2(k)}^s \\ \vdots \\ w_{n(k)}^s \end{bmatrix}, \tag{20}$$

where $w_{j(k)}^s$ is the subjective weight of Q_j assigned by the k th decision expert, $j = 1(1)n, k = 1(1)\ell$. To calculate the subjective weight of the criterion, we have

$$w_j^s = \left(\left(w_{j(1)}^s \oplus w_{j(2)}^s \oplus \dots \oplus w_{j(\ell)}^s \right) / \ell \right). \tag{21}$$

Let $w_{j(k)}^s = (p_{ijk})$ be the decision weight of k th decision expert such that $p_{ijk} = \langle \mu_{ijk}, v_{ijk} \rangle$, $k = 1(1)\ell$ is an IFT2N or PFN. Next, we construct the combined subjective criterion weight value as follows:

$$w_j^s = \left\langle \left[\sqrt[1-\prod_{k=1}^{\ell} (1-\mu_{ijk}^2)^{1/\ell}] , \left[\prod_{k=1}^{\ell} (v_{ijk})^{1/\ell} \right] \right] \right\rangle. \tag{22}$$

Here, $w_j^s = \langle \mu_{ij*}, v_{ij*} \rangle$ is also a PFN.

(3) Compute the aggregate or combined weight by using Eqs. (19)–(22) is given by

$$w_j = \gamma w_j^s + (1 - \gamma) w_j^o, \tag{23}$$

where γ is the aggregating coefficient of decision procedure within the range of 0 to 1.

Step 5 Compute the measures of weighted sum model (WSM) $\mathbb{C}_i^{(1)}$ for each alternative using the formula

$$\mathbb{C}_i^{(1)} = \sum_{j=1}^n w_j \eta_{ij}. \tag{24}$$

Step 6 Compute the measures of weighted product model (WPM) $\mathbb{C}_i^{(2)}$ for each alternative by using the following formula

$$\mathbb{C}_i^{(2)} = \prod_{j=1}^n w_j \eta_{ij}. \tag{25}$$

Step 7 Calculate the aggregated measure of the WASPAS method for each alternative, which as

$$\mathbb{C}_i = \lambda \mathbb{C}_i^{(1)} + (1 - \lambda) \mathbb{C}_i^{(2)}, \tag{26}$$

where λ is the aggregating coefficient of decision precision. It is developed to estimate the accuracy of WASPAS based on initial criteria exactness and when $\lambda \in [0, 1]$ (when $\lambda = 0$, and $\lambda = 1$, WASPAS is transformed to the WPM and the WSM, respectively). It has been proven that the accuracy of the aggregating methods is higher than the accuracy of single ones.

Step 8 Rank the alternatives according to decreasing values (i.e., crisp score values) of \mathbb{C}_i .

Step 9 End.

5 Physician selection problem

In this section, we have applied the proposed WASPAS method to solve a decision-making physician selection problem, which verifies the applicability and effectiveness of the proposed method.

In India, www.practo.com is the one of the most commonly visited online physician rating website which collects the information about the facilities, the reviews given by the patients and ratings of every doctor. On this site, a patient may rate a physician in a five-star-marking system on the basis of their experience and post a comment/review

related to the rating score. For example, if a patient may review any particular physician that the diagnosis is perfect, the waiting time is too long and the medical staffs' behaviors are quite rude. Thus, these reviews suggest the patients to take a suitable decision in the selection of a doctor. In a selection of physician, the patients' decisions are widely influenced by the available reviews. For instance, if a patient who wants to select a physician, he/she may peruse the reviews of the physician according as location, hospitality, specialty and so many other factors. However, how to choose a suitable physician is still a challenge for the patients due to a bulky number of reviews accessible on online websites. In this section, the proposed PF-WASPAS approach is used to assess the most suitable physician for patients based on several factors.

Let us consider that a set of three decision makers (B_1, B_2, B_3) want to choose a physician for the patients based on available online reviews. Subsequent to initial screening, five physicians (R_1, R_2, R_3, R_4, R_5) are selected for further assessment. The ratings of the physician are assessed in a five-star marking system based on the seven criteria, which as (1) ease of appointment (Q_1), (2) punctuality (Q_2), (3) polite staff (Q_3), (4) accurate diagnosis (Q_4), (5) behavior of a physician in the presence of a patient (Q_5), (6) spends time with patients (Q_6) and (7) follows up after visit (Q_7). The decision makers afford their reviews about the physicians in terms of PFNs according as the above-mentioned seven criteria. Now, the process for implementation of proposed WASPAS method on the present application is as follows:

Step 1 Construct a decision matrix.

In this method, the decision makers firstly express their estimations concerning the ratings of physicians over the preferred factors. Table 1 presents the value of linguistic variables which evaluates the relative importance of the preferred criteria. Table 2 shows the value of linguistic variables that evaluates the importance of the decision experts. Table 3 presents the important degree of each

Table 1 Linguistic terms that denote the importance of the alternatives for physician selection

Linguistic terms	IFNs-2T
Very insignificant (VI)	$A(0.15, 0.95)$
Insignificant (<i>I</i>)	$A(0.20, 0.88)$
Slightly insignificant (SI)	$A(0.40, 0.70)$
Average (A)	$A(0.55, 0.60)$
Slightly significant (SS)	$A(0.70, 0.45)$
Significant (S)	$A(0.85, 0.25)$
Very significant (VS)	$A(0.95, 0.10)$

Table 2 Importance weights in linguistic terms for DEs'

Linguistic terms	IFNs-2T
Very very qualified (VVQ)	$A(0.95, 0.10)$
Very qualified (VQ)	$A(0.85, 0.25)$
Qualified (<i>Q</i>)	$A(0.55, 0.60)$
Less qualified (LS)	$A(0.40, 0.70)$
Very less qualified (VLQ)	$A(0.20, 0.88)$

Table 3 Decision experts' weights

DMs	Linguistic variables	PFNs	Weights
B_1	Very qualified (VQ)	$A(0.85, 0.25)$	0.3769
B_2	Qualified (<i>Q</i>)	$A(0.55, 0.60)$	0.1747
B_3	Very very qualified (VVQ)	$A(0.95, 0.10)$	0.4484

Table 4 Evaluation of physician selection alternative w.r.t. different criteria and decision experts

DMs	Alternatives	Criteria						
		Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
B_1	R_1	<i>I</i>	SI	VS	A	SS	VS	A
	R_2	<i>I</i>	VI	VS	S	VS	SS	S
	R_3	S	SS	A	SS	SI	VS	SS
	R_4	SS	VS	SS	<i>I</i>	VI	SI	A
	R_5	A	VS	A	SI	SS	A	SI
B_2	R_1	VI	<i>I</i>	S	SS	A	VS	A
	R_2	SI	VI	VS	S	VS	S	VS
	R_3	SS	A	SS	SS	A	S	A
	R_4	SS	SS	S	SI	SI	<i>I</i>	SI
	R_5	A	S	A	A	SS	SI	<i>I</i>
B_3	R_1	VI	A	S	SS	S	S	SS
	R_2	VI	VI	VS	S	VS	S	VS
	R_3	A	SS	A	A	A	SS	S
	R_4	A	VS	A	VI	<i>I</i>	<i>I</i>	SS
	R_5	SI	S	SS	SI	S	SI	A

decision expert (B_1, B_2, B_3), which is evaluated by (17). To evaluate the performance ratings of the alternatives with respect to each criterion, decision makers express the linguistic rating variables in Table 4.

With the use of (18), the aggregated intuitionistic fuzzy decision matrix of second type is estimated for physician selection based on the decision experts' opinions, and thus the result is shown in Table 5.

Table 5 Intuitionistic fuzzy aggregated decision matrix of second type for physician selection

	R_1	R_2	R_3	R_4	R_5
Q_1	A(0.1707, 0.9230)	A(0.2335, 0.8750)	A(0.7304, 0.4102)	A(0.6428, 0.5120)	A(0.4918, 0.6429)
Q_2	A(0.4571, 0.6799)	A(0.1500, 0.9500)	A(0.6792, 0.4732)	A(0.9326, 0.1301)	A(0.9016, 0.1770)
Q_3	A(0.9016, 0.1770)	A(0.9500, 0.1000)	A(0.5828, 0.5706)	A(0.6872, 0.4620)	A(0.6276, 0.5274)
Q_4	A(0.6528, 0.5015)	A(0.8500, 0.2500)	A(0.6428, 0.5120)	A(0.2335, 0.8750)	A(0.4322, 0.6814)
Q_5	A(0.7681, 0.3636)	A(0.9500, 0.1000)	A(0.7978, 0.6359)	A(0.2361, 0.8702)	A(0.7822, 0.3457)
Q_6	A(0.9188, 0.1508)	A(0.8068, 0.3120)	A(0.8684, 0.2304)	A(0.2951, 0.8073)	A(0.4657, 0.6605)
Q_7	A(0.6276, 0.5274)	A(0.9249, 0.1412)	A(0.7681, 0.3636)	A(0.6114, 0.5418)	A(0.4571, 0.6799)

Table 6 Weights of criteria demonstrated by decision experts

Decision expert	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
D_1	VS	SS	A	S	SS	VS	SS
D_2	VS	SS	SI	S	SS	S	A
D_3	S	A	A	SS	S	SS	A
Aggregated subjective weight (ϖ_j^s)	0.1915	0.1235	0.0893	0.1629	0.1435	0.1777	0.1116

Based on Table 5 and (19), the objective weight of each criterion based on entropy and divergence measures for IFSs-2T is computed as follows:

$$w_j^o = (0.1270, 0.2499, 0.0766, 0.1247, 0.1660, 0.1750, 0.0808)^T.$$

Corresponding to Table 1, the subjective weight matrix (W_j^s) given in Table 6 is constructed. By using (20) and (21), the aggregated subjective weight (ϖ_j^s) of criteria is computed.

Based on (23), the combined weight (ϖ_j) of each criterion (with $\gamma = 0.5$) is computed as follows:

$$w_j = (0.1593, 0.1867, 0.0829, 0.1438, 0.1548, 0.1764, 0.0962)^T.$$

Using Table 5 and (24) and (25), the WSM ($C_i^{(1)}$) and the WPM ($C_i^{(2)}$) measures are evaluated for each physician and given in Table 7. Based on (26), the WASPAS (C_i) measure and their score value $S^*(C_i)$ are given in Table 7 for each physician (with $\lambda = 0.5$). Then, the rank order for

the five physician selection is determined as $R_3 \succ R_2 \succ R_1 \succ R_5 \succ R_4$.

5.1 Sensitivity analysis and comparison

In this section, the outcomes of the developed method are investigated based on a comparison and a sensitivity analysis. Nowadays, due to popularity of PFSs, various MCDM approaches have been introduced in recent years within the framework of PFSs. Each of these methods has its own uniqueness and steps which distinguish it from the others. We have tried to choose methods for the comparison which have good efficiency in the literature and could be applicable in the considered MCDM problem. After a literature survey, the methods given by Zhang and Xu (2014), Ren et al. (2016) and Peng and Yang (2016) are selected for the comparative analysis. Here, the ranking result obtained by the proposed method is compared with the existing approaches in Table 8. The ranking order from the developed approach is same as various existing

Table 7 Computational assessment of proposed WASPAS method

Alternative	$C_i^{(1)}$	$C_i^{(2)}$	C_i	$S^*(C_i)$	Rank
R_1	A(0.7364, 0.4150)	A(0.5498, 0.6376)	A(0.6595, 0.5144)	0.5851	3
R_2	A(0.8213, 0.3100)	A(0.5133, 0.7086)	A(0.7144, 0.4687)	0.6454	2
R_3	A(0.7532, 0.4271)	A(0.7292, 0.4757)	A(0.7415, 0.4507)	0.6734	1
R_4	A(0.6703, 0.5021)	A(0.4450, 0.7160)	A(0.5792, 0.5996)	0.4880	5
R_5	A(0.6877, 0.4600)	A(0.5829, 0.5672)	A(0.6404, 0.5108)	0.5746	4

Table 8 Final ranking results of different methods

Method	Author(s)	Criterion weight	Benchmark	MCDM model	Ranking	Optimal rank
TOPSIS	Zhang and Xu (2014)	Assumed	Hamming distance measure	Compromising model	$R_3 \succ R_2 \succ R_1 \succ R_5 \succ R_4$	R_3
TODIM	Ren et al. (2016)	Assumed	Euclidean distance measure	Non-compensatory model based on prospect theory	$R_2 \succ R_1 \succ R_5 \succ R_3 \succ R_4$	R_2
MABAC	Peng and Yang (2016)	Assumed	Pythagorean fuzzy Choquet integral average (PFCIA)	Compromising model	$R_3 \succ R_2 \succ R_1 \succ R_5 \succ R_4$	R_3
Proposed WASPAS	Rani et al. (2018)	Computed (Divergence and entropy measures)	Intuitionistic fuzzy average and geometric operators of second type	Scoring model	$R_3 \succ R_2 \succ R_1 \succ R_5 \succ R_4$	R_3

Table 9 Simulated weights of criteria in different parameter values

γ	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
0.0	0.1270	0.2499	0.0766	0.1247	0.1660	0.1750	0.0808
0.1	0.1335	0.2373	0.0779	0.1285	0.1638	0.1753	0.0839
0.2	0.1399	0.2246	0.0791	0.1323	0.1615	0.1755	0.0870
0.3	0.1463	0.2120	0.0804	0.1362	0.1593	0.1758	0.0900
0.4	0.1528	0.1993	0.0817	0.1400	0.1570	0.1761	0.0931
0.5	0.1593	0.1867	0.0829	0.1438	0.1548	0.1764	0.0962
0.6	0.1657	0.1714	0.0842	0.1476	0.1525	0.1766	0.0993
0.7	0.1722	0.1614	0.0855	0.1514	0.1502	0.1769	0.1024
0.8	0.1786	0.1488	0.0868	0.1553	0.1480	0.1772	0.1054
0.9	0.1850	0.1361	0.0880	0.1591	0.1457	0.1774	0.1085
1.0	0.1915	0.1235	0.0893	0.1629	0.1435	0.1777	0.1116

approaches [except Ren et al. (2016) method], and we observe that R_3 is the best physician.

We also perform a sensitivity analysis based on ten sets of criteria weights and different values of precision parameter. Values of $\lambda(0.0, 0.2, 0.5, 0.8$ and $1.0)$ and values of $\gamma(0.0, 0.2, 0.5, 0.8$ and $1.0)$ are chosen for this analysis. Varying the values of λ can help us to assess the sensitivity of method to moving from weighted sum model to weighted product model. Moreover, changing the values of γ is performed to analyze the sensitivity of method to the quality of criteria weights (subjective and objective). Also, Table 9 represents seven sets which used in the sensitivity analysis. As can be seen in Table 9, one of the criteria has the highest weight in each set and the other criteria have lower weights than it. Using this pattern can help us to consider a wide extent of criteria weights for analyzing the sensitivity of method to changing the weights of criteria. The ranking results with different sets and different

Table 10 Ranking results with different sets of criteria weights and various values of parameter

γ	λ	R_1	R_2	R_3	R_4	R_5	Correlation coefficient (r_p)
$\gamma = 0.0$	0.0	3	5	1	4	2	0.60
	0.2	3	4	1	5	2	0.60
	0.5	4	2	1	5	3	0.90
	0.8	4	1	2	5	3	0.80
	1.0	3	1	2	5	4	0.90
$\gamma = 0.2$	0.0	3	4	1	5	2	0.60
	0.2	3	4	1	5	2	0.60
	0.5	4	2	1	5	3	0.90
	0.8	3	1	2	5	4	0.90
	1.0	3	1	2	5	4	0.90
$\gamma = 0.5$	0.0	3	4	1	5	2	0.60
	0.2	3	4	1	5	2	0.60
	0.5	3	2	1	5	4	1.00
	0.8	3	1	2	5	4	0.90
	1.0	3	1	2	5	4	0.90
$\gamma = 0.8$	0.0	2	4	1	5	3	0.70
	0.2	4	2	1	5	3	0.90
	0.5	3	2	1	5	4	1.00
	0.8	3	1	2	5	4	0.90
	1.0	3	1	2	5	4	0.90
$\gamma = 1.0$	0.0	3	4	1	5	2	0.60
	0.2	4	2	1	5	3	0.90
	0.5	3	2	1	5	4	1.00
	0.8	3	1	2	5	4	0.90
	1.0	3	1	2	5	4	0.90

parameters are shown in Table 10, and we represent the correlation between the results of using different sets in Table 10. According to these tables, all values of r_p are

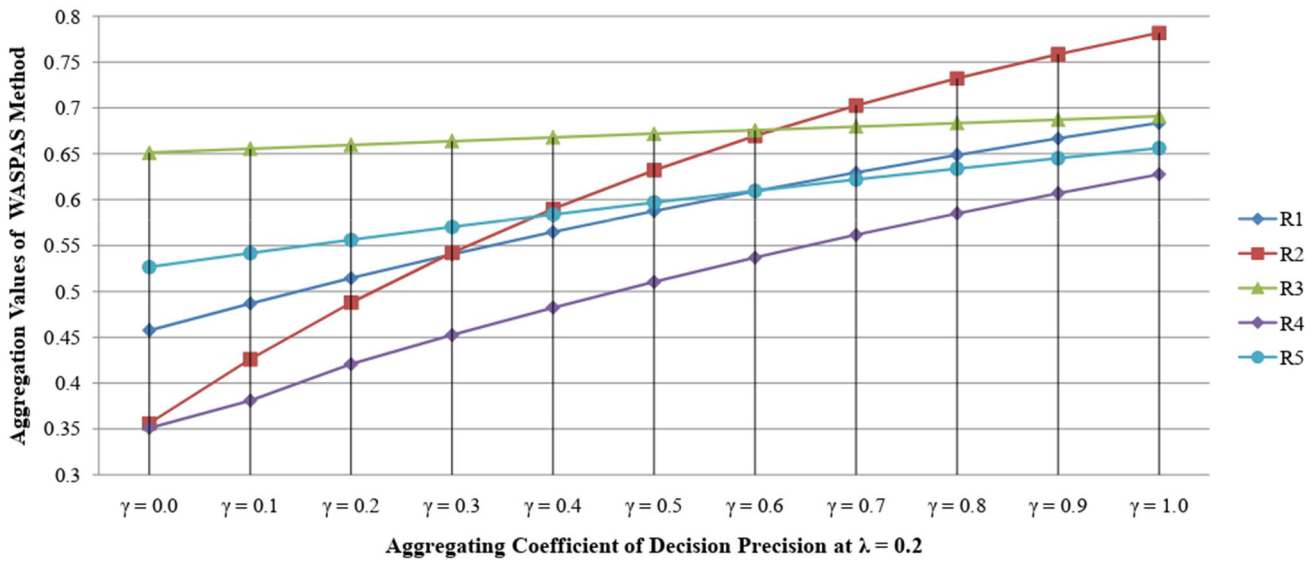


Fig. 1 Sensitivity results of the C_i values w.r.t. precision parameter for $\lambda = 0.2$

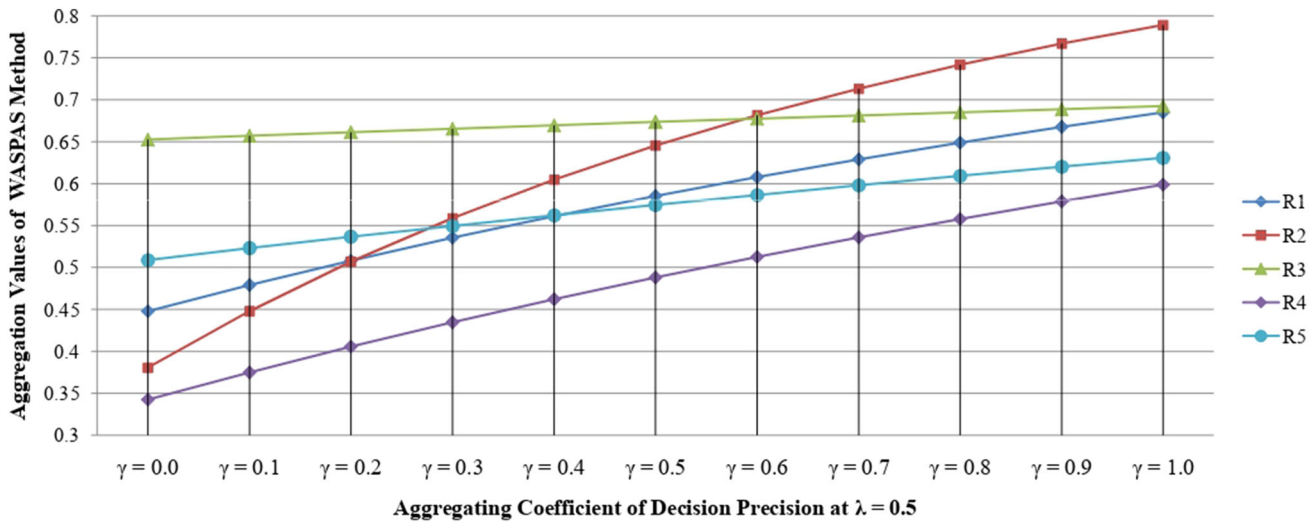


Fig. 2 Sensitivity results of the C_i values w.r.t. precision parameter for $\lambda = 0.5$

varying between 0.6 and 1.0. Next, Figs. 1, 2 and 3 represent five sets which are implemented in the sensitivity analysis. The ranking orders with the different weights and the parameters are depicted in Figs. 1, 2 and 3. According to these figures, for precision parameter $\gamma = 0.2, 0.5, 0.8$, the alternative R_3 has the highest ranking when $\lambda = 0.0$ to $\lambda = 0.5$, while R_2 has the highest rank when $\lambda = 0.6$ to $\lambda = 1.0$. Therefore, it can be said that the proposed approach has good stability with different weights and different values of parameters. Also, we can see that using more subjective weights (increasing the value of γ) can lead to increasing the sensitivity of the method. Based on this analysis, it can be concluded that using a combination

of subjective and objective weights of criteria could increase the stability of the proposed approach.

6 Conclusions

In this study, a modified WASPAS method is proposed to handle the MCDM problems within the context of IFSs-2T, in with the criteria and decision-makers' weights are completely unknown. To find the weights of the criteria and decision makers, new formulae are developed based on novel divergence and entropy measures for IFSs-2T. In the present method, the evaluation scores of the alternatives are expressed by linguistic variables and then given in

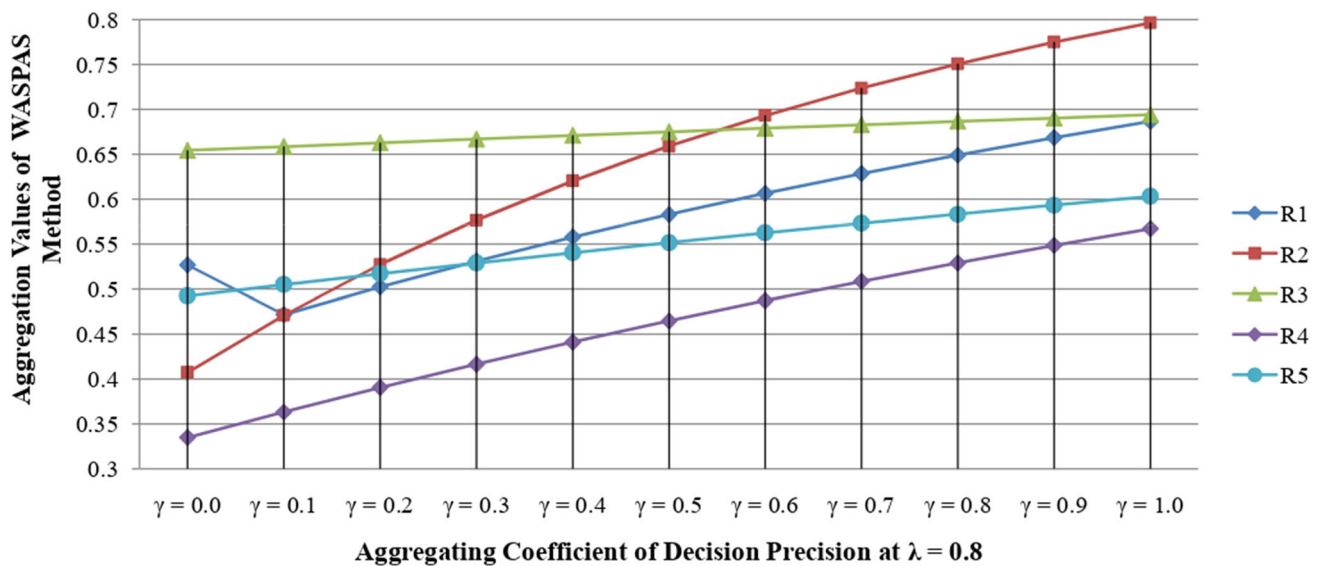


Fig. 3 Sensitivity results of the C_i values w.r.t. precision parameter for $\lambda = 0.8$

terms of IFNs-2T. The applicability and feasibility of the proposed WASPAS approach are demonstrated through a decision-making problem of physician selection with intuitionistic fuzzy information of second type.

Comparative study with different existing methods also reveals the validity of the results obtained by proposed method. Finally, sensitivity analysis is presented to see the impact of different weights, which also shows the feasibility of the method. On the basis of comparative and sensitivity analyses, it can be concluded that using a combination of subjective and objective weights of criteria could increase the stability of the proposed approach.

Compliance with ethical standards

Conflict of interest All authors declare that they have no conflict of interest.

Ethical approval The present article does not contain any studies with human participants or animals performed by any of the authors.

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