Generalised grey target decision method for mixed attributes based on the improved Gini–Simpson index

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Abstract

A new generalised grey target decision method for mixed attributes is presented. The proposed method makes a modification to the Gini–Simpson (G–S) index and adopts it as the basis of decision making. The improved version of G–S index could represent the difference (gap) between an alternative and its target centre as a whole and reflect the difference in various alternative indices and the similarity between alternative indices and target centre indices in detail. And the G–S index also has the advantage of combining the alternative index and target centre index in a simple and effective form. Besides, ranking alternatives easily or not by a given decision-making method is investigated, which arrives at the discrimination ability index. In decision making, the proposed method first transforms all mixed attribute indices into binary connection numbers and divides them into the deterministic terms and uncertain terms to constitute two-tuple (determinacy, uncertainty) numbers. Then the target centre indices of two-tuple (determinacy, uncertainty) number are determined. Following this, the improved comprehensive weighted Gini–Simpson index (CWGSI) of all alternatives can be obtained. Finally, the alternatives can be sorted with the CWGSI: the smaller the better. A case study exemplifies the proposed approach.

Keywords Generalised grey target decision method · Mixed attributes · Gini–Simpson index · Binary connection number · Two-tuple (determinacy, uncertainty) number

1 Introduction

The grey target decision method has been improved by many scholars, since it was proposed by Deng (2002). The indices of alternatives were extended from real numbers to mixed attribute values, with the in-depth study of the decision-making theory. Then the mixed attribute-based grey target decision method was studied, which enhances its application. For grey target decision method, a decision is made by the target centre distance. Distance-based methods, such as Euclidean distance and Mahalanobis distance, are mainly applied to determine the crisp number-based grey target decision making (Dang et al. 2004; Wang et al. 2009). There are some ways of obtaining the target

centre distance in the reported literature regarding the mixed attribute-based method: one way is by distance method, mainly relying on Euclidean distance (Luo and Wang 2012; Shen et al. 2010; Song et al. 2009); then, the equivalent method including cobweb area and incidence coefficient is proposed (Guan et al. 2015; Zeng et al. 2013);besides, the vector-based method and entropy-based method are also investigated, as they are named as generalised grey target decision method (Ma 2018a, b; Ma and Ji 2014). The generalised grey target decision method differs from the conventional one in the calculation process, but obeys the same principle as the conventional method (Ma 2014, 2018a, b; Ma and Ji 2014). The traditional distance-based method for mixed attribute-based grey target decision method has its deficiency such as complex calculation and information deformation in handling uncertain number. Fortunately, the vector-based method overcomes the drawback of distance-based method to a certain extent. However, the vector-based method considers equally the importance of the deterministic term and uncertain term of an uncertain number (usually denoted by a binary

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connection number). Compare to the vector-based method, the entropy-based method mainly depending on Kullback–Leibler (K–L) distance makes a decision from the perspective of measuring the uncertainty. Nevertheless, the K–L distance does not satisfy the symmetry of distance in the usual sense, which might lead to an unexpected consequence in decision making.

Given that uncertainty originated from the uncertain number, an effective tool is compulsory to measure it in the generalised grey target decision method. The Gini index (Gini coefficient) was put forward by Italy statistician Gini, which is a quantitative index to measure the difference in income distribution (Fabrizi and Trivisano 2016). Now, the Gini index has been studied and used in a wide range of fields (Parsa et al. 2018). There are over ten types of Gini indices; however, in recent years, one of its kinds that is the Gini–Simpson index has been paid high attention and employed in more and more different fields (Casquilho 2016; Guiasu and Guiasu 2012; Pesenti et al. 2017). At first, Gini–Simpson was applied to measure the biodiversity (Guiasu and Guiasu 2012; Pesenti et al. 2017) and detect the medical virus (Gregori et al. 2016). In addition, it was also used to measure the uncertainty of information (Eliazar and Sokolov 2010; Jiang et al. 2017; Petry et al. 2015). In view of the characteristics of Gini–Simpson index, it could be applied to the generalised grey target decision method involving uncertain numbers.

A sustained investigation is needed to improve the decision-making method involving uncertain information. Thus, the strategies and technical methods may affect the decision making should be considered as full as possible. Specifically, some ideas embedded in the optimisation methodologies are helpful to better the decision-making method. To solve the travelling salesman problem, a novel two-stage hybrid swarm intelligence optimisation algorithm is put forward, and the problem solving is divided into rough searching and detailed searching exploiting these hybrid methods' advantages and avoiding their shortcomings (Deng et al. 2012). A genetic and ant colony collaborative algorithm merging multiple strategies and methods is proposed to solve the complex optimisation problems, and the collaborative algorithm avoids the deficiencies of weak local search ability in genetic algorithm and slow global convergence speed in ant colony algorithm (Deng et al. 2017a, b, c). The adaptive method and multi-variant strategies are also employed to improve the heuristic algorithm, which reaches the desirable effect (Deng et al. 2015; Deng et al. 2017a, b, c). In practical use (for example in solving the black box problem), the data mining algorithms are often combined with other methods and functions, which makes the problem to be solved perfectly (Ali 2013; Al-Janabi and Alwan 2017). And in fault diagnosis, some hybrid optimal methods are also integrated to achieve the ideal effect compared with the traditional

methods (Deng et al. 2017a, b, c; Deng et al. 2018). Besides, it is necessary to seek for the suitable methods to solve the practical problems such as forest fires prediction and intrusion detection (Al Janabi 2018; Al Janabi et al. 2018; Al-Janabi 2017). Inspirited by the ideas that included in these articles, this study also applies strategies such as transforming and simplifying ideas to the mixed attribute-based generalised grey target decision method to fulfil its task perfectly.

The innovation of this method is as follows. Above all, the comprehensive weighted Gini–Simpson index is improved and regarded as the generalised target centre distance. And the modified version of CWGSI reflecting the difference between an alternative and its target centre is convenient to calculate in contrast to the traditional one. Second, the converting strategy is adopted to transform mixed attribute values into two-tuple (determinacy, uncertainty) numbers through the binary connection numbers; and the two-tuple numbers make a basis for building the connection of alternative indices and target centre indices through the CWGSI. In the end, a discrimination ability index of alternatives ranking is established to evaluate the performance of each decision-making method in sorting the alternatives easily or not.

The remainder of this paper is organised as follows: Sect. 2 introduces the basic theory, Sect. 3 discusses the proposed approach, Sect. 4 presents a case study, and Sect. 5 is the conclusion.

2 Preliminaries

2.1 Interval number and its extension

Definition 1 Let R be a real domain; if $\tilde{x}$ denotes an interval number, then we can write $[x^L, x^U]$, where x^L and x^U satisfy $0 < x^L < x^U \in R$ (Zhao 2008, 2010). Furthermore, if an interval number contains more than two parameters in its expression, then it can be termed the extension of the primary form, as it is also referred to as an n-parameter interval number (also called as multi-parameter interval number). The n-parameter interval number can be written as $[x^1, x^2, \ldots x^j, \ldots x^n]$, where $x^L = x^1, x^U = x^n$, and the parameters satisfy $0 < x^1 < \ldots \ldots < x^j < \ldots \ldots < x^n \in R$. For an n-parameter interval number, if $n = 3$ and 4, then it can also be known as three-parameter interval number and four-parameter interval number, respectively, expressed by $[x^L, x^M, x^U]$ and $[x^L, x^M, x^N, x^U]$, respectively, where x^L, x^M, x^N and x^U satisfy $0 < x^L < x^M < x^N < x^U \in R$.

Remark For an n-parameter interval number, it can be named differently in specific theoretical framework. If $n = 2, 3, 4$, it can be read as interval number, triangular

fuzzy number, or trapezoidal fuzzy number, respectively, in fuzzy theory, while it can be called interval grey number, triangular grey number, or trapezoidal grey number, respectively, in grey theory. In this research, the n-parameter interval number is spoken of as interval number, three-parameter interval number or four-parameter interval number for $n = 2, 3, 4$, respectively. Thus, the uncertain number in this research mainly refers to the interval number and its extension.

2.2 Binary connection number

Definition 2 Let $\mathbf{R}$ be a real domain; $A + Bi$ is called a binary connection number, where A denotes the deterministic term, B represents the uncertain term and i is a variable term unifying the determinacy and uncertainty of an uncertain number (interval number and its extension), and $A, B \in \mathbf{R}, i \in [-1, 1]$.

Definition 3 Let $\bar{x}$ and v be the mean value and deviation value of the n ($n \geq 2$) parameters of $\widetilde{x}$, respectively, then

$$u(\bar{x}, v) = A + Bi = \bar{x} + vi, \quad (i \in [-1, 1]) \tag{1}$$

is referred to as a mean value–deviation value connection number, where $\bar{x}$, ξ, φ and v are obtained by using Eqs. (2)–(5):

$$\bar{x} = \frac{1}{n} \sum_{j=1}^{n} x_j \tag{2}$$

$$\xi = \sqrt{\frac{1}{(n-1)} \sum_{j=1}^{n} (x_j - \bar{x})^2} \tag{3}$$

$$\varphi = \max \left\{ |x^L - \bar{x}|, |x^U - \bar{x}| \right\} \tag{4}$$

$$v = \min\{\xi, \varphi\} \tag{5}$$

where $x_j (j = 1, \cdots, n)$ is the jth parameter of the uncertain number (interval number and its extension) $\widetilde{x}$, $\bar{x}$ represents the mean value of the parameters, ξ denotes the standard deviation of the parameters, φ is the maximum deviation of the parameters, x^L and x^U are the uncertain number's lower limits and upper limits, respectively (Ma and Ji 2014; Ma et al. 2014).

Definition 4 The mutual interaction of the mean value $\bar{x}$ and the deviation value v (standard deviation or maximum deviation) of the micro-vector $(\bar{x}, v)$ originated from binary connection number $u(\bar{x}, v)$ can be mapped to the determinacy–uncertainty space (D–U space) (Zhao 2008, 2010).

Figure 1 is a D–U space. The U axis is representative of the relative uncertain measure, while the D axis is the relative deterministic measure. Figure 1 shows that $\bar{x}$ and v interact with each other, and the space reflection is the
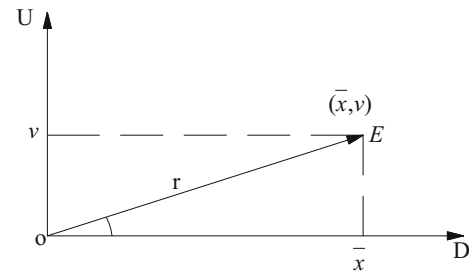

Fig. 1 Determinacy–uncertainty space (Ma et al. 2014)

vector $\overline{OE}$ from O to E, and the degree of interaction means the module of the vector $\overline{OE}$ denoted by r.

2.3 Gini–Simpson index

Definition 5 The Gini–Simpson index Let $P = (p_1, p_2, \ldots, p_m)$ be a probability distribution, the Gini–Simpson index is defined as (Casquilho 2016; Guiasu and Guiasu 2012):

$$G(P) = \sum_{j=1}^{m} p_j (1 - p_j). \tag{6}$$

In Eq. (6), where p_j is the probability of a variable and $G(P)$ is the Gini–Simpson index of P.

Definition 6 The improved comprehensive weighted Gini–Simpson index (CWGSI) Suppose that $S = ((x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m))^{\mathrm{T}}$ and $E = ((p_1, q_1), (p_2, q_2), \ldots, (p_m, q_m))^{\mathrm{T}}$ are the vector of two-tuple (determinacy, uncertainty) numbers of an alternative and that of a target centre, respectively, where $x_j, y_j, p_j, q_j \geq 0, j = 1, 2, \ldots, m$, (x_j, y_j) and (p_j, q_j) are the normalised two-tuple (determinacy, uncertainty) numbers under the same attribute in S and E, respectively; the weight vector is $W = (w_1, w_2, \cdots, w_m)^{\mathrm{T}}$; then, the improved comprehensive weighted Gini–Simpson index is calculated using Eq. (7).

$$\begin{aligned} G(S, E) &= \sum_{j=1}^{m} w_j x_j |x_j - p_j| + \sum_{j=1}^{m} w_j y_j |y_j - q_j| \\ &= \sum_{j=1}^{m} w_j (x_j |x_j - p_j| + y_j |y_j - q_j|). \end{aligned} \tag{7}$$

Equation (7) originated from Eq. (6) builds a bridge between the two-tuple (deterministic degree, uncertainty degree) numbers of each alternative and that of target centre. The CWGSI as a whole denotes the difference (gap) between an alternative and its target centre from the viewpoint of uncertainty measurements: the smaller the value the better the alternative. However, the CWGSI

observed from the microscopic perspective has a two-way characteristic: reflecting the difference in various alternative indices and the similarities between alternative indices and target centre indices. The terms x_j and y_j in the equation keep the alternative indices different from others, and the similarities between alternative indices and target centre indices are represented by the terms $\left| x_j - p_j \right|$ and $\left| y_j - q_j \right|$.

3 Generalised grey target decision method for mixed attributes based on the improved Gini–Simpson index

Let $F = \{F_1, F_2, \ldots, F_n\}$, $Z = \{Z_1, Z_2, \ldots, Z_m\}$ and $W = (w_1, w_2, \ldots, w_m)^{\mathrm{T}}$ be an alternative set, attribute set and weight vector of index attributes, respectively, then the index value of alternative F_s under attribute Z_t is $v_{st} (s = 1, 2, \ldots, n; t = 1, 2, \ldots, m)$.

3.1 Transformation of index values into binary connection numbers

Different types of index values can be converted into binary $A + Bi$ connection numbers using Eqs. (1)–(5). Note that the converted binary connection number for real number is of the form $A + 0i$, which means that the deterministic term is the real number itself and the uncertain term is $0i$. The transformed index number can be expressed as
$$V_{st} = A_{st} + B_{st} i \ (s = 1, 2, \ldots, n; t = 1, 2, \ldots, m).$$

3.2 Determination of the target centre indices

Having obtained the binary connection number $A_{st} + B_{st} i \ (s = 1, 2, \ldots, n; t = 1, 2, \ldots, m)$ converted from each index value, which can also be denoted by two-tuple number $U_{st} = (A_{st}, B_{st}) \ (s = 1, 2, \ldots, n; t = 1, 2, \ldots, m)$, and denoting the benefit-type index set and cost-type index set by J^+ and J^-, respectively, the target centre index of two-tuple (determinacy, uncertainty) can be obtained using Eq. (8).

$$C_t^0 = \begin{cases} (\max\{A_{st}\}, \min\{B_{st}\}), U_{st} \in J^+ \\ (\min\{A_{st}\}, \min\{B_{st}\}), U_{st} \in J^- \end{cases}, s = 1, 2, \cdots, n, t = 1, 2, \cdots, m. \tag{8}$$

Equation (8) indicates that the target centre of two-tuple (determinacy, uncertainty) number under attribute Z_t is such that the index vector corresponding to the maximum and minimum terms applies to benefit-type indices and that of the maximum term and minimum term to cost-type indices.

3.3 Normalisation of all alternative indices

The index vectors of all alternatives $U_{st} = (A_{st}, B_{st}) \ (s = 1, 2, \ldots, n; t = 1, 2, \ldots, m)$ and target centre index vectors $U_{ct} = (A_{st}, B_{st}) \ (c = n + 1; t = 1, 2, \ldots, m)$ can be expressed as vectors of two-tuple (deterministic degree, uncertainty degree) number:

$$a_{st} = \frac{A_{st}}{A_{st} + B_{st}}, \quad b_{st} = \frac{B_{st}}{A_{st} + B_{st}}, \quad s = 1, 2, \ldots, n; \ t = 1, 2, \ldots, m. \tag{9}$$

In Eq. (9), a_{st} and b_{st} denote, respectively, the deterministic degree and uncertain degree under the same attribute in the normalised binary connection numbers. Then an alternative vector of two-tuple (deterministic degree, uncertainty degree) number can be given as $((a_{s1}, b_{s1}), (a_{s2}, b_{s2}), \ldots, (a_{sm}, b_{sm}))^{\mathrm{T}}$. The values of a_{st} and b_{st} in a two-tuple (determinacy, uncertainty) number (a_{st}, b_{st}) should be normalised further for they are incomparable under different attributes. The normalisation equation is as follows.

$$na_{st} = \frac{a_{st}}{\sum_{s=1}^{n} a_{st}}, nb_{st} = \frac{b_{st}}{\sum_{s=1}^{n} b_{st}}, s = 1 \ldots n; t = 1 \ldots m. \tag{10}$$

In Eq. (10), na_{st} and nb_{st} are the normalised deterministic and uncertain terms of the two-tuple number, respectively.

3.4 Discrimination ability of alternatives ranking

In decision making, alternatives ranking is usually determined by the final values that the alternatives have scored. If the values corresponding to different alternatives have a larger gap with each other, then the alternatives ranking could be decided easily. Otherwise, if these values have a smaller gap with each other, then it is somewhat difficult to rank them. In order to estimate the discrimination ability of the same alternatives sorted by given decision-making method, we provide an index to perform the task.

Suppose that there are n alternatives denoted by F_s ($s = 1, \ldots, n$) and m decision-making methods represented by D_t ($t = 1, \ldots, m$), then the values for the alternatives obtained by different methods are $H_{st}(s = 1, \ldots, n; t = 1, \ldots, m)$. Consequently, the discrimination ability index Q_t could be built as follows.

$$\mu_t = \frac{1}{n}\sum_{s=1}^{n} H_{st}, \quad t = 1,\ldots,m \tag{11}$$

$$\lambda_t = \sum_{\substack{k \neq l \\ k,l=1,\cdots,n}} |H_{kt} - H_{lt}|, t = 1,\ldots,m \tag{12}$$

$$Q_t = \frac{\lambda_t}{\mu_t}, (t = 1,\ldots,m). \tag{13}$$

The index Q_t reflects the discrimination ability of alternatives F_s $(s = 1,\ldots,n)$ ranked by D_t. Only two or more methods when compared with each other may imply the importance of the index. That is to say, if $Q_j > Q_k$, then the discrimination ability of decision-making method D_j is better than that of D_k. In brief, the discrimination ability index provides a new way to evaluate the different decision-making methods.

3.5 Algorithm of the proposed method

The flowchart of the proposed approach is shown in Fig. 2, and the detailed steps are as follows.

Algorithm: Generalised grey target decision method based on the CWGSI.

Input: All index values $v_{st}(s = 1, 2, \cdots, n; t = 1, 2, \ldots, m)$.

Output: The comprehensive weighted Gini–Simpson indices I_{CWGSI} and alternatives ranking.

Step 1: All index values $v_{st}(s = 1, 2, \cdots, n; t = 1, 2, \ldots, m)$ are converted into binary connection numbers $A_{st} + B_{st}i$ $(s = 1, 2, \cdots, n; t = 1, 2, \ldots, m)$ and also translated into two-tuple (determinacy, uncertainty) numbers (A_{st}, B_{st}), $(s = 1, 2, \cdots, n; t = 1, 2, \ldots, m)$ using Eqs. (1) to (5).

Step 2: The target centre index of two-tuple (determinacy, uncertainty) number C_t^0 under attribute Z_t is determined using Eq. (8).

Step 3: The (A_{st}, B_{st}), $(s = 1, 2, \ldots, n; t = 1, 2, \ldots, m)$ are transformed into (deterministic degree, uncertainty degree) numbers (a_{st}, b_{st}), $(s = 1, 2, \ldots, n; t = 1, 2, \ldots, m)$ by using Eq. (9), and they are normalised in linear method as (na_{st}, nb_{st}), $(s = 1, 2, \cdots, n; t = 1, 2, \cdots, m)$ using Eq. (10).

Step 4: The weights of all index attributes $W = (w_1, w_2, \ldots, w_m)^{\mathrm{T}}$ are obtained.

Step 5: The improved CWGSIs of all alternatives can be calculated as I_{CWGSI} using Eq. (7).

Step 6: Output the comprehensive weighted Gini–Simpson indices I_{CWGSI} and alternatives ranking.

The decision making is achieved according to the Gini–Simpson value of each alternative: the smaller the better.

3.6 Hypothesis/questions/limitation of the proposed method

The grey target decision method makes a decision relying on the target centre distance. However, the generalised
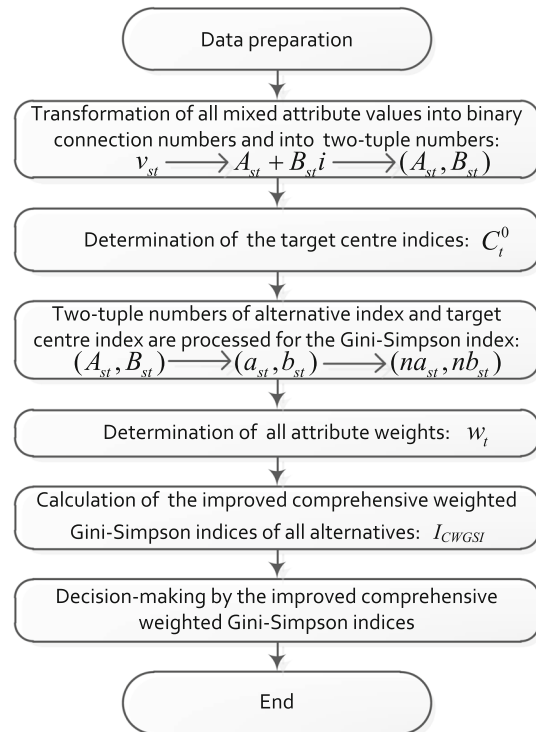

Fig. 2 Flowchart of improved G–S index-based generalised grey target decision method

grey target decision method extends the traditional target centre distance to generalised one in different ways such as proximity and Kullback–Leibler distance. This work adopts the modified version of G–S index as the generalised target centre distance, as it provides a novel way of handling the mixed attribute-based values in grey target decision method. Using the characteristics of the G–S index, the proposed method may discriminate the alternatives ranking easily. But the alternatives ranking determined by this method may remain unsteady with the number of alternatives increasing dramatically. In addition, the proposed approach considers the deterministic terms and uncertain terms of the uncertain numbers equally important, which may not comprise all the preferences of decision makers involving uncertain information.

4 Case study

4.1 Background and data

In multi-attribute decision making, the characteristics of the objective thing are often denoted by mixed attribute values (including real numbers and uncertain numbers) for various reasons. Different from the certain numbers, the mixed attribute values containing uncertain numbers are difficult to be addressed in a unified way to keep the loss of

Table 1 Index values of every alternative

F_i	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
F_1	2.0	500	[55 56]	[4.7 5.7]	[0.4 0.5 0.6]	[0.8 0.9 1.0]
F_2	2.5	540	[30 40]	[4.2 5.2]	[0.2 0.3 0.4]	[0.4 0.5 0.6]
F_3	1.8	480	[50 60]	[5 6]	[0.6 0.7 0.8]	[0.6 0.7 0.8]
F_4	2.2	520	[35 45]	[4.5 5.5]	[0.4 0.5 0.6]	[0.4 0.5 0.6]

Table 4 Two-tuple numbers transformed from index binary connection numbers

F_i	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
F_1	(2.0,0)	(500,0)	(55.5,0.5)	(5.2,0.5)	(0.5,0.1)	(0.9,0.1)
F_2	(2.5,0)	(540,0)	(35,5)	(4.7,0.5)	(0.3,0.1)	(0.5,0.1)
F_3	(1.8,0)	(480,0)	(55,5)	(5.5,0.5)	(0.7,0.1)	(0.7,0.1)
F_4	(2.2,0)	(520,0)	(40,5)	(5,0.5)	(0.5,0.1)	(0.5,0.1)

information less. Thus, the proposed approach is employed to the decision making considering the mixed attribute values mentioned above. We provide a case study to verify the proposed method as follows.

To evaluate the tactical missiles, six indices including hit accuracy(km), warhead payload (kg), mobility(km h^{-1}), price(10^6g), reliability and maintainability are considered denoted by Z_1 to Z_6 (Shen et al. 2010). For all the data type of the attributes, Z_1 and Z_2 are real numbers, Z_3 and Z_4 are interval numbers, and Z_5 and Z_6 are three-parameter interval numbers. Among these attributes Z_1 and Z_4 are cost-type indices and the others are benefit-type indices. There are four feasible alternatives denoted by F_1 to F_4. The data are shown in Table 1. And the attribute weights are also given by the experts as $W = (0.1818, 0.2017, 0.1004, 0.2124, 0.1618, 0.1419)$.

4.2 Decision-making process

(1) Calculation of the parameters of binary connection number for all alternatives

The parameters of binary connection number of all alternatives can be calculated by using the data in Table 1 using Eqs. (1)–(5): the results are shown in Table 2.

(2) Translation of all index values into binary connection numbers

All index values can be transformed into index binary connection numbers using Eqs. (1)–(5) based on the data shown in Table 2. Table 3 lists the index binary connection numbers as converted from all indices.

Then the two-tuple (determinacy, uncertainty) numbers shown in Table 4 are converted from the index binary connection numbers shown in Table 3.

(3) Determination of the two-tuple numbers of target centres

The vectors of two-tuple numbers of target centre are calculated as ((1.8,0), (540,0), (55.5,0.5), (4.7,0.5), (0.7,0.1), (0.9,0.1)) by using Eq. (8).

(4) Normalisation of the two-tuple (deterministic degree, uncertainty degree) numbers

Two-tuple (deterministic degree, uncertainty degree) numbers of alternative indices and target centre indices can be normalised using Eq. (10): the results are shown in Table 5.

In Table 5, the symbols na_{st} and nb_{st} that in (na_{st}, nb_{st}) represent, respectively, the deterministic term and uncertain term of the same index. If an index is a real number, then nb_{st} is zero; for example, the indices under attributes Z_1 and Z_2 are all real numbers. The symbol NC_P denotes the normalised two-tuple (deterministic degree, uncertainty degree) number of target centre.

Table 2 Average values, standard deviations and maximum deviations of all indices

F_i	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
F_1	2.0/0/0	500/0/0	55.5/0.7071/0.5	5.2/0.7071/0.5	0.5/0.1/0.1	0.9/0.1/0.1
F_2	2.5/0/0	540/0/0	35/7.0711/5	4.7/0.7071/0.5	0.3/0.1/0.1	0.5/0.1/0.1
F_3	1.8/0/0	480/0/0	55/7.0711/5	5.5/0.7071/0.5	0.7/0.1/0.1	0.7/0.1/0.1
F_4	2.2/0/0	520/0/0	40/7.0711/5	5/0.7071/0.5	0.5/0.1/0.1	0.5/0.1/0.1

"a/b/c" in Table 2 denotes "average value/standard deviation/maximum deviation"

Table 3 Index binary connection numbers transformed from index values

F_i	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
F_1	$2.0 + 0i$	$500 + 0i$	$55.5 + 0.5i$	$5.2 + 0.5i$	$0.5 + 0.1i$	$0.9 + 0.1i$
F_2	$2.5 + 0i$	$540 + 0i$	$35 + 5i$	$4.7 + 0.5i$	$0.3 + 0.1i$	$0.5 + 0.1i$
F_3	$1.8 + 0i$	$480 + 0i$	$55 + 5i$	$5.5 + 0.5i$	$0.7 + 0.1i$	$0.7 + 0.1i$
F_4	$2.2 + 0i$	$520 + 0i$	$40 + 5i$	$5 + 0.5i$	$0.5 + 0.1i$	$0.5 + 0.1i$

Table 5 Normalised two-tuple numbers of all alternatives and target centre indices

NF_i	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
NF_1	(0.2353, 0)	(0.2451, 0)	(0.2699,0.0272)	(0.2505,0.2449)	(0.2532,0.2353)	(0.2615, 0.1791)
NF_2	(0.2941, 0)	(0.2647, 0)	(0.2383,0.3807)	(0.2482,0.2685)	(0.2278,0.3529)	(0.2421, 0.2985)
NF_3	(0.2118, 0)	(0.2353, 0)	(0.2497,0.2538)	(0.2517,0.2327)	(0.2658,0.1765)	(0.2542, 0.2239)
NF_4	(0.2588, 0)	(0.2549, 0)	(0.2421,0.3384)	(0.2496,0.2539)	(0.2532,0.2353)	(0.2421, 0.2985)
NC_P	(0.2118, 0)	(0.2647, 0)	(0.2699,0.0272)	(0.2482,0.2685)	(0.2658,0.1765)	(0.2615, 0.1791)

(5) Determination of the improved comprehensive weighted Gini–Simpson indices

Given the weight vector W = (0.1818, 0.2017, 0.1004, 0.2124, 0.1618, 0.1419), the CWGSIs of all alternatives to their target centre indices are calculated as I_{CWGSI} = (0.0061, 0.0359, 0.0113, 0.0233)using Eq. (7). The decision making can be made in accordance with the CWGSI: the smaller the value the better alternative as: $F_1 \succ F_3 \succ F_4 \succ F_2$.

4.3 Discussion

4.3.1 Results comparison

In order to verify the proposed method, the vector-based method reported in Ma and Ji (2014) is used to make a comparison. The principle of the vector-based method is as follows. It first translates different mixed attribute-based numbers into index binary connection numbers and regards them as index vectors. Then the target centre indices are obtained. After that, the comprehensive weighted proximities (CWPs) of all alternatives to their target centre are calculated. In the end, the decision making is calculated by CWP: the smaller the value the better the alternative.

The calculation process of the vector-based method is introduced briefly as follows. Using the same data shown in Table 1, all mixed attribute values of alternatives are converted into binary connection numbers shown in Table 3. Then the target centre indices by the vector-based

method can be obtained as $(1.8 + 0i, 540 + 0i, 55 + 5i, 4.7 + 0.5i, 0.7 + 0.1i, 0.9 + 0.1i)$. The target centre indices by the vector-based method are somewhat different from the target centre indices determined by the proposed method for different mechanisms. And the two kinds of target centre indices (determined by G–S-based method and vector-based method denoted by C_G and C_V, respectively) are listed in Table 6. Seen from Table 6, the expressions of the two target centre indices are different, and specifically the values of deterministic term and uncertain term of target centre under attribute Z_3 are completely different. Next, the index proximities of all alternative indices to target centre indices are calculated and normalised to obtain the comprehensive weighted proximities. The same attribute weights vector W = (0.1818, 0.2017, 0.1004, 0.2124, 0.1618, 0.1419)is given; then, the final comprehensive proximity is: I_{CWP} = (0.2023, 0.2928, 0.2354, 0.2695). According to the rule the smaller the proximity the better the alternative, the alternatives are ranked as: $F_1 \succ F_3 \succ F_4 \succ F_2$. Table 7 gives the comparison of the proximity-based method and the proposed method. Figure 3 is the comparison of alternatives ranked by the two different methods. The left-hand side of the vertical axis denotes the alternatives ranking by CWGSI, while the right-hand side of the vertical axis represents the result of alternatives ordered by CWP.

Through the comparison of the data shown in Table 7 and the rectangular and triangular dots illustrated in Fig. 3,

Table 6 Comparison of target centre indices by the two different methods

C	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
C_G	(1.8,0)	(540,0)	(55.5,0.5)	(4.7,0.5)	(0.7,0.1)	(0.9,0.1)
C_V	$1.8 + 0i$	$540 + 0i$	$55 + 5i$	$4.7 + 0.5i$	$0.7 + 0.1i$	$0.9 + 0.1i$

Table 7 Comparison of results by the two decision-making methods

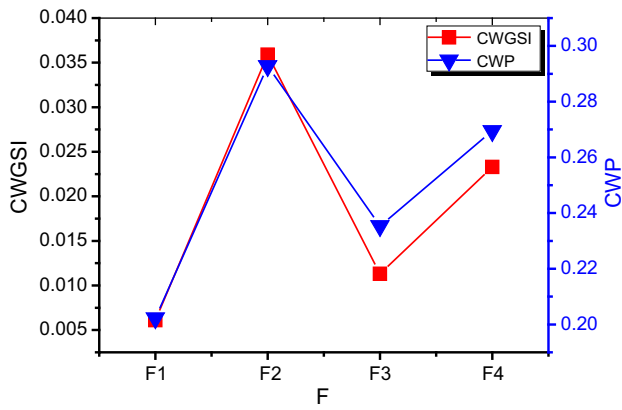|	Comprehensive weighted Gini–Simpson index		Comprehensive weighted proximity	
	Value	Rank	Value	Rank
F_1	0.0061	1	0.2023	1
F_2	0.0359	4	0.2928	4
F_3	0.0113	2	0.2354	2
F_4	0.0233	3	0.2695	3

Fig. 3 Results comparison by the two decision-making methods

the decision making by the proposed method is in agreement with that by the vector-based method.

4.3.2 Comparison of discrimination abilities of alternatives ranking

The proposed approach has the same result of decision making as the vector-based method even for the different final values. But it can be noted that the discrimination ability of alternatives ranking of the proposed method is superior to that of the vector-based method. It can be deduced intuitionally from the final values (CWGSIs and CWPs) shown in Table 7. Take F_1 and F_3 for example. A simple computation can be made to explain this inference. The final values of F_1 and F_3 for alternatives ranking are 0.0061 and 0.1113, respectively, by the proposed method. The difference of 0.0061 and 0.1113 is 0.0052. If 0.0052 is divided by 0.0061, the result is 0.8525. Similarly, the values of F_1 and F_3 achieved by the vector-based method are 0.2023 and 0.2054, respectively. The 0.0331 is obtained for 0.2054 minus 0.2023. And the result is 0.1636 when 0.0331 is divided by 0.2023. This shows that sorting F_1 and F_3 by the proposed method is easier than that by the vector-based method as 0.8525 is larger than 0.1636. But for the whole alternatives ranking, a discrimination ability index discussed in Sect. 3.4 could be used to fulfil this task. We can easily calculate the discrimination ability indices of the two methods with the data shown in Table 8 using Eqs. (11)–(13). Table 8 is the comparison of discrimination abilities of alternatives ranking, and it is also illustrated graphically in Fig. 4. Seen from Table 8 and Fig. 4, the discrimination ability index of the proposed method is 5.2532, while the index of the vector-based method is 1.2224. Thus, the final decision making for alternatives ranking by the proposed method is easier than that of the vector-based method.

Table 8 Comparison of discrimination abilities of alternatives ranking

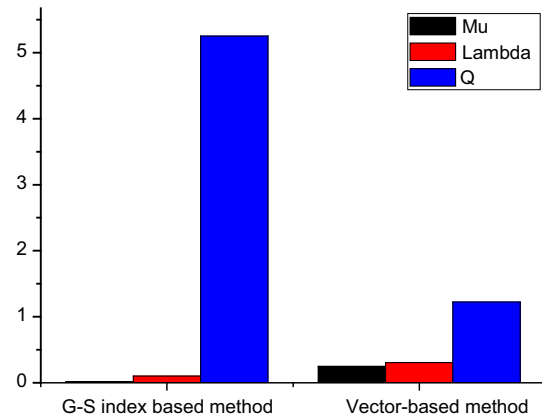| Item	Gini–Simpson index-based method	Vector-based method
Mu (μ_t)	0.01915	0.25
Lambda (λ_t)	0.1006	0.3056
Q (Q_t)	5.252	1.2224

Fig. 4 Discrimination abilities comparison by the two decision-making methods

4.3.3 Methodologies comparison

The proposed method differs from the vector-based method in the principle of its decision making. The similarities and differences in the two methods are analysed as follows. The two methods, the proposed method and the method reported in (Ma and Ji 2014), have the following similarity: all transform different types of data into the binary connection number which can be handled in the same way. In brief, the binary connection number is the main tool dealing with mixed attribute values. The difference between the two methods is that the proposed method adopts the improved CWGSI to determine the alternatives ranking, making the decision from the prospect of the uncertainty of alternative measurements; while the method in (Ma and Ji 2014) takes the comprehensive weighted proximity to determine the alternative's order, as is from the viewpoint of the similarities between vectors.

5 Conclusions

This paper proposes a novel generalised grey target decision method for mixed attributes. The decision making by the proposed method is based on the modified version of G–S index (regarded as the generalised target centre distance). The improved CWGSI macroscopically could be

representative of the difference between an alternative and its target centre from the viewpoint of measuring the uncertainty. The modified CWGSI has the advantage of combining the alternative index and target centre index in a simple and effective form, which could also reflect the difference in various alternative indices and the similarity between alternative indices and target centre indices from the microscopic perspective. The calculation result is in line with the reported vector-based method under the same condition, but the discrimination ability of alternatives ranking of the proposed method is better than that of the vector-based method. Different from the vector-based generalised grey target decision method, the proposed method makes a decision based on the improved G–S index, as is from the viewpoint of measuring the uncertainty.

Compliance with ethical standards

Conflict of interest The author declares that he has no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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