Soft Computing (2019) 23:13309–13320
https://doi.org/10.1007/s00500-019-03873-z

METHODOLOGIES AND APPLICATION

Random credibilitic portfolio selection problem with different convex transaction costs

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Published online: 4 March 2019

Abstract
Most of the portfolio optimization problems are devoted to either stochastic model or fuzzy one. However, practical portfolio selection problems often involve the mixture of the stochastic returns with fuzzy information. In this paper, we propose a new mean variance random credibilitic portfolio selection problem with different convex transaction costs, i.e., linear function, non-smooth convex function, smooth convex function. In this proposed model, we assume that the returns of assets obey the trapezoidal-type credibilitic distributions, and the risks obey the stochastic distributions. Based on the random credibilitic theories, these models are transformed into crisp convex programming problems. To find the optimal solution, we, respectively, present a pivoting algorithm, a branch-and-bound algorithm, and a sequence quadratic programming algorithm to solve these models. Furthermore, we offer numerical experiments of different forms of convex transaction costs to illustrate the effectiveness of the proposed model and approach.

Keywords Mean variance portfolio optimization model · Random credibilitic variable · Convex transaction costs · A pivoting algorithm · Sequence quadratic programming

1 Introduction

The traditional mean variance method, which was proposed by Markowitz (1952), is the foundation of the modern portfolio theory. After that, a number of methods were proposed to find efficient portfolio such as Gao and Li (2013), Macedo et al. (2017), Al Janabi et al. (2017), Liagkouras and Metaxiotis (2017)and so on. The basic assumption for using portfolio model within a probabilistic framework is that the situation of financial markets in future can be correctly reflected by security data in the past. If there are not enough data for the practical problem, these models will be invalid. Because of lack of data in an emerging financial market, the parameters or probability distributions of random returns are difficult to be accurately estimated.

However, these quantities can be provided by the experts based on their past information and subjective belief. In other words, security returns may be considered as fuzzy variables instead of random variables when there is lack of data. The fuzzy set is a powerful tool used to describe an uncertain financial environment where not only the financial markets but also the investment decision-makers are subject to vagueness, ambiguity or fuzziness. Possibility theory has been proposed in Zadeh (1978), where fuzzy variables are associated with possibility distributions. Recently, a number of researchers have investigated fuzzy portfolio selection problem, such as Zhang et al. (2009), Qin et al. (2009), Zhang and Zhang (2014), and Kar et al. (2018). But the possibility measure is not self-dual. Credibility theory has been newly proposed in Liu (2002a). As the average of a possibility measure and a necessity measure, the credibility measure is self-dual. In this respect, the credibility measure shares some properties with the probability measure. Recently, a number of researchers have investigated credibilitic portfolio selection problem, such as Zhang et al. (2010), Mehlawat and Gupta (2014), Jalota et al. (2017), and Deng et al. (2018).

Actually, the investors may encounter uncertainty of both randomness and fuzziness simultaneously when

Communicated by V. Loia.

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handling the practical portfolio selection problem. Random and credibilitic optimization models provide useful methods for investors to handle uncertainty. For example, the probability distributions of security returns may be partially known. Therefore, the uncertain parameters may be estimated by experts on the basis of available data, which implies that security returns may be characterized by random credibilitic variables. Random credibilitic variable was a recently introduced variable by Liu (2002b) to describe a random fuzzy phenomenon. After that, several authors have also applied random credibilitic variable to model portfolio optimization, such as Hasuike et al. (2009), Huang (2007a, b), Liu et al. (2011), Qin (2017),Wang et al. (2017), Dutta et al. (2018).

To the best of the author's knowledge, at present, there is little research in modeling and solving random credibilitic portfolio selection problem with different convex transaction costs. This stimulates the authors to employ the latest development of mathematics on uncertainty theory to study the portfolio selection problem in random credibilitic environment. The contribution of this work is as follows. We propose a new mean variance random credibilitic portfolio selection model with different convex transaction costs. Using the random credibilitic decision-making approach, the proposed model is transformed into a crisp convex programming problem. Three algorithms are designed to obtain the optimal portfolio strategy.

The rest of the paper is organized as follows. In Sect. 2, necessary knowledge about credibilitic variable, random credibilitic variable, and some properties will be briefly introduced. In Sect. 3, we propose a new mean variance random credibilitic portfolio selection model with three types of convex transaction costs functions, i.e., linear function, non-smooth convex function, and smooth function. Using the random credibilitic theory, the proposed model is turned into a crisp convex programming problem. A novel pivoting algorithm, a branch-and-bound algorithm, and a sequence quadratic programming are proposed to solve the proposed models with different convex transaction costs in Sect. 4. Numerical examples are presented in Sect. 5 to help understanding the modeling idea and the designed algorithm. Finally, some conclusions are given in Sect. 6.

2 Preliminaries

Some definitions, which are needed in the following section, will be introduced herein.

Definition 1 (Liu (2002a)). Let Θ be a nonempty set and P the power set of Θ. A set function Cr on P is called credibility measure if it satisfies:

(1) (Normality) $Cr\{\boldsymbol{\Theta}\} = 1$;
(2) (Monotonicity) $Cr\{A\} \le Cr\{B\}$ whenever $A \subset B$;
(3) (Self-duality) $Cr\{A\} + Cr\{A^c\} = 1$ for any $A \in P$;
(4) (Maximality) $Cr\{\cup_i A_i\} = \sup_i Cr\{A_i\}$ for any sequence $\{A_i\} \subset P$ with $\sup_i Cr\{A_i\} < 0.5$. The triplet $(\boldsymbol{\Theta}, P, Cr)$ is called a credibility space.

Let ξ be a fuzzy variable with membership function μ. For any $x \in \Re$, $\mu(x)$ represents the possibility that ξ takes value x. Hence, it is also called the possibility distribution. For any set B, the possibility measure and necessity measure of $\xi \in \mathrm{B}$ were, respectively, defined by Zadeh (1978) as

$$\mathrm{Pos}\{\xi \in B\} = \sup_{x \in B} \mu(x) \tag{1}$$

$$\mathrm{Nec}\{\xi \in B\} = 1 - \sup_{x \in B^c} \mu(x) \tag{2}$$

It is proved that both possibility measure and necessity measure satisfy the properties of normality, nonnegativity, and monotonicity. However, neither of them is self-dual. Since the self-duality is intuitive and important in real problems, Liu (2002a, b) defined a credibility measure as the average of possibility measure and necessity measure

$$Cr\{\xi \in B\} = \frac{1}{2}\left[\sup_{x \in B} \mu(x) + 1 - \sup_{x \in B^c} \mu(x)\right] \tag{3}$$

where Cr is self-dual measure and satisfies $Cr\{\xi \le r\} + Cr\{\xi \ge r\} = 1$.

Definition 2 (Liu (2002a)). Let ξ be a credibilitic variable. Then, the expected value is defined as

$$E[\xi] = \int_0^\infty Cr\{\xi \ge r\}\mathrm{d}r - \int_{-\infty}^0 Cr\{\xi \le r\}\mathrm{d}r \tag{4}$$

Based on Definition 2, Liu (2002a) deduced the following two theorems.

Theorem 1 (Liu (2002a)). *Let ξ be a credibilitic variable with finite expected value. Then, for any real numbers λ and μ, it holds that*

$$E(\lambda\xi + \mu) = \lambda E(\xi) + \mu \tag{5}$$

Theorem 2 (Liu (2002a)) *Let ξ and η be independent credibilitic variables with finite expected values. Then, for any real numbers λ and μ, it holds that*

$$E(\lambda\xi + \mu\eta) = \lambda E(\xi) + \mu E(\eta) \tag{6}$$

A popular credibilitic number is the trapezoidal credibilitic number $\xi = (a, b, \alpha, \beta)$ with membership function $\mu_\xi(x)$ in the following form

$$\mu_\xi(x) = \begin{cases} \dfrac{x-(a-\alpha)}{c}, & x \in [a-\alpha, a] \\ 1, & x \in [a,b] \\ \dfrac{b+\beta-x}{\beta}, & x \in [b, b+\beta] \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where α and β are positive numbers, i.e., α, $\beta > 0$.

From Eq. (3), the credibility of the event $\{\xi \le r\}$ is as follows:

$$Cr\{\xi \le r\} = \begin{cases} 1, & \text{if } r \ge b+\beta \\ \dfrac{r-b+\beta}{2\beta}, & \text{if} b \le r \le b+\beta \\ \dfrac{1}{2}, & \text{if } a \le r \le b \\ \dfrac{r-a+\alpha}{2\alpha}, & \text{if} a-\alpha \le r \le a \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

Theorem 3 *Let ξ be a the trapezoidal credibilitic number ξ = (a, b, α, β) with membership function $\mu_\xi(x)$. Then,*

$$E(\xi) = \frac{a+b}{2} + \frac{\beta-\alpha}{4} \tag{9}$$

Proof Because the credibility measure is self-dual measure, according to Eq. (8), we can obtain the following:

$$Cr\{\xi \ge r\} = \begin{cases} 0, & \text{if } r \ge b+\beta \\ \dfrac{b+\beta-r}{2\beta}, & \text{if } b \le r \le b+\beta \\ \dfrac{1}{2}, & \text{if } a \le r \le b \\ \dfrac{a+\alpha-r}{2\alpha}, & \text{if } a-\alpha \le r \le a \\ 1, & \text{otherwise} \end{cases} \tag{10}$$

According to Definition 2, we can obtain as follows:

$$\begin{aligned} E[\xi] &= \int_0^\infty Cr\{\xi \ge r\}dr - \int_{-\infty}^0 Cr\{\xi \le r\}dr \\ &= \int_0^{a-\alpha} 1dr + \int_{a-\alpha}^a \frac{a+\alpha-r}{2\alpha}dr + \int_a^b \frac{1}{2}dr \\ &\quad + \int_b^{b+\beta} \frac{b+\beta-r}{2\beta}dr + \int_{b+\beta}^{+\infty} 0dr \\ &= r|_0^{a-\alpha} + \frac{a+\alpha}{2\alpha} r|_{a-\alpha}^a - \frac{1}{2\alpha}\frac{r^2}{2}\Big|_{a-\alpha}^a + \frac{1}{2}r\Big|_a^b \\ &\quad + \frac{b+\beta}{2\beta} r\Big|_b^{b+\beta} - \frac{1}{2\beta}\frac{r^2}{2}\Big|_b^{b+\beta} + 0 \\ &= a-\alpha + \frac{a+\alpha}{2} - \frac{a}{2} + \frac{\alpha}{4} + \frac{b-a}{2} + \frac{b+\beta}{2} - \frac{b}{2} - \frac{\beta}{4} \\ &= \frac{a+b}{2} + \frac{\beta-\alpha}{4}. \end{aligned}$$

Thus, the proof of Theorem 3 is ended.

Until now, there are many studies of portfolio selection problems whose future returns are assumed to be random variables or fuzzy numbers. However, since few studies of them are treated as random credibilitic variables, we introduce a random credibilitic variables defined by Liu (2002b) as follows.

Definition 3 (Liu (2002b)). A random credibilitic variable is a function ξ from a credibility space(Θ, $Cr(\Theta)$, Cr) to collection of random variables R. An n-dimensional random credibilitic vector $\xi = (\xi_1, \xi_2,\ldots, \xi_n)$ is an n-tuple of random credibilitic variables $\xi_1, \xi_2,\ldots, \xi_n$.

That is, a random credibilitic variable is a fuzzy set defined on a universal set of random variables. Furthermore, the following random credibilitic definition is introduced.

Definition 4 (Liu (2002b)). Let $\xi_1, \xi_2,\ldots, \xi_n$ be random credibilitic variables, and f: $R_n \to R$ be a continuous function. Then, $\xi = f(\xi_1, \xi_2,\ldots, \xi_n)$ is a random credibilitic variable on the product credibility space(Θ, $Cr(\Theta)$, Cr), defined as

$$x(\theta_1, \theta_2, \ldots, \theta q_n) = f(\xi_1(\theta_1), \xi_2(\theta_2), \ldots, \xi_n(\theta_n))$$

for all($\theta_1, \theta_2,\ldots, \theta_n$) $\in\Theta$.

From these definitions, the following theorem is derived.

Theorem 4 (Liu (2002b)). *Let ξ_i be random credibilitic variables with membership functions μ_i, i = 1, 2,…, n, respectively, and f: $R^n \to R$ be a continuous function. Then, $\xi = f(\xi_1, \xi_2,\ldots, \xi_n)$ is a random credibilitic variable whose membership function is*

$$\mu(\eta) = \sup_{\eta_i \in R_i, 1 \le i \le n} \left\{ \min_{1 \le i \le n} \mu_i(\eta_i) | \eta = f(\eta_1, \eta_2, \cdots, \eta_n) \right\}$$

for all $\eta \in R$, where $R = \{f(\eta_1, \eta_2,\ldots, \eta_n) | \eta_i \in R_i, i = 1, 2,\ldots, n\}$.

Definition 5 Let ξ be a random credibilitic variable with finite mean value e. Then, the variance of ξ is defined by

$$\sigma^2[\xi] = E[(\xi - e)^2] \tag{11}$$

3 The random fuzzy portfolio selection model

Assume that there are n risky assets in financial market for trading. An investor wants to allocate his/her wealth among n assets. Then, an optimal portfolio should be the one with minimized variance for the given expected return level. Suppose that the return rates of the n risky assets at each period are denoted as trapezoidal credibilitic variables. Let w be the portfolio, where $w = (w_1,w_2,\ldots,w_n)'$; $\widetilde{R}_i$ be the random credibilitic return of risky asset i; r_{p} be the excepted return rate of the portfolio w; σ^2 be the variance of the portfolio w, where $\sigma^2 = (\sigma_{ij})_{n\times n}$, $\sigma_{ij} = \mathrm{Cov}(R_i,R_j)$; $c(w)$ be the transaction cost of the portfolio w; r_N be the net return rate of the portfolio w.

3.1 Return, risk, and transaction costs constraints

In this section, we employ the credibilitic mean value to measure the return of portfolio. The risk on the return rate of portfolio is quantified by the random variance. The return rate of security i, $\widetilde{R}_i = (a_i, b_i, \alpha_i, \beta_i)$, is trapezoidal random credibilitic variable for all $i = 1,\ldots,n$.

The credibilitic value of the portfolio $w = (w_1, w_2, \ldots, w_n)'$ can be expressed as

$$r_p = \sum_{i=1}^{n} \widetilde{R}_i w_i \tag{12}$$

Let $\widetilde{R}_i$ be trapezoidal credibilitic variable for all $i = 1,\ldots,n$, $\widetilde{R}_i = (a_i, b_i, \alpha_i, \beta_i)$. The mean value of the portfolio $w = (w_1, w_2, \ldots, w_n)'$ can be expressed as

$$r_p = \sum_{i=1}^{n} \overline{M}(\widetilde{R}_i) w_i = \sum_{i=1}^{n} \left(\frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{4} \right) w_i \tag{13}$$

Transaction cost is one of the main concerns for portfolio managers. Arnott and Wagner (1990) and Yoshimoto (1996) found that ignoring transaction costs would result in an inefficient portfolio. Bertsimas and Pachamanova (2008), Gülpınar et al. (2003), and Zhang (2016, 2017) incorporated transaction cost into the multiperiod portfolio selection problem. In this paper, the transaction cost for asset i can be expressed as $c_i(w_i)$. Hence, the total transaction costs of the portfolio $w = (w_1, w_2, \ldots, w_n)'$ can be represented as $C(w)$

$$C(w) = \sum_{i=1}^{n} c_i(w_i) \tag{14}$$

The net return rate of the portfolio w can be denoted as

$$r_N = \sum_{i=1}^{n} \left(\frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{4} \right) w_i - \sum_{i=1}^{n} c_i(w_i) \tag{15}$$

The variance of the portfolio w can be expressed as

$$V(w) = w' \sigma^2 w \tag{16}$$

where $w = (w_1, w_2, \ldots, w_n)'$, $\sigma^2 = (\sigma_{ij})_{n \times n}$, $\sigma_{ij} = \mathrm{Cov}(R_i, R_j)$.

The capital constraint of risky assets is

$$w_1 + w_2 + \cdots + w_n = 1 \tag{17}$$

3.2 The basic multiperiod portfolio optimization models

Let $\widetilde{R}_i$ be expressed with a fuzzy set and r_0 be a minimum target value of the total future profit. We formally introduce the following mean–variance model:

$$\min \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} w_i w_j$$
$$\text{s.t.} \begin{cases} \sum_{i=1}^{n} \widetilde{R}_i w_i - \sum_{i=1}^{n} c_i(w_i) \geq r_0 & \text{(a)} \\ w_1 + w_2 + \cdots + w_n = 1 & \text{(b)} \\ w_i \geq 0, \quad i = 1, \ldots, n & \text{(c)} \end{cases} \tag{18}$$

where constraint (a) denotes that the return of the portfolio is not less than the preset value r_0; constraint (b) indicates that the proportion of risk assets sums to one; and constraint (c) states the nonnegative constraints of w_i.

Model (18) is a fuzzy optimization problem. Let $\widetilde{R}_i = (a_i, b_i, \alpha_i, \beta_i)$. Using the random credibilitic theory, the proposed Model (18) can be turned into as

$$\min \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} w_i w_j$$
$$\text{s.t.} \begin{cases} \sum_{i=1}^{n} \left(\frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{4} \right) w_i - \sum_{i=1}^{n} c_i(w_i) \geq r_0 \\ w_1 + w_2 + \cdots + w_n = 1 \\ w_i \geq 0, i = 1, \ldots, n \end{cases} \tag{19}$$

If the covariance matrix σ^2 is semi-definite, and $C(w_1, w_2, \ldots, w_n)$ is linear function, Model (19) is a convex quadratic programming problem, which can be solved by Algorithm 1. If the covariance matrix σ^2 is semi-definite, and $C(w)$ is non-smooth or smooth convex function, Model (19) is a convex programming problem, which can be solved by Algorithm 2 or Algorithm 3.

4 Solution algorithm

In this section, the smallest and biggest value of r_0 can be obtained. Several methods will be proposed to solve the problems with the different transaction costs.

4.1 The smallest and biggest value of r_0

In Model (19), investors can choose r_0 between $r_0^{\min}$ and $r_0^{\max}$. $r_0^{\min}$ and $r_0^{\max}$ can be, respectively, obtained as follows:

The investor considers to maximize the expected return of the portfolio

$$\max \sum_{i=1}^{n} \left(\frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{4} \right) w_i - \sum_{i=1}^{n} c_i(w_i)$$
$$\text{s.t.} \begin{cases} w_1 + w_2 + \cdots + w_n = 1 \\ w_i \geq 0, \quad i = 1, \ldots, n \end{cases} \tag{20}$$

w^* (the optimal solution $w = (w_1, w_2, \ldots, w_n)'$) can be obtained solving Model (20) by the following algorithms.

The biggest of objective ($r_N^{\max} = \sum_{i=1}^{n}\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i^* - \sum_{i=1}^{n} c_i(w_i^*)$) can be obtained, i.e., $r_0^{\max} = r_N^{\max}$.

Let μ_1 and μ_2 be the Lagrange multiplier. The KKT conditions for Model (22) are as follows:

$$\begin{cases} \sigma_{i1}w_1 + \cdots + \sigma_{in}w_n - \mu_1\left(\dfrac{a_i+b_i}{2}+\dfrac{\beta_i-\alpha_i}{4}-k_i\right)+\mu_2 \geq 0\ i=1,\ldots,n \\ \left(\dfrac{a_1+b_1}{2}+\dfrac{\beta_1-\alpha_1}{4}-k_1\right)w_1+\cdots+\left(\dfrac{a_n+b_n}{2}+\dfrac{\beta_n-\alpha_n}{4}-k_n\right)w_n \geq r_0 \\ w_1+w_2+\cdots+w_n=1 \\ w_i \geq 0, i=1,\ldots,n, \mu_1 \geq 0 \\ \left[\left(\dfrac{a_1+b_1}{2}+\dfrac{\beta_1-\alpha_1}{4}-k_1\right)w_1+\cdots+\left(\dfrac{a_n+b_n}{2}+\dfrac{\beta_n-\alpha_n}{4}-k_n\right)w_n - r_0\right]\mu_1=0 \\ \left[\sigma_{i1}w_1+\cdots+\sigma_{in}w_n-\mu_1\left(\dfrac{a_i+b_i}{2}+\dfrac{\beta_i-\alpha_i}{4}-k_i\right)+\mu_2\right]w_i=0 \end{cases} \tag{23}$$

The smallest value of the $r_0^{\min}$ can be obtained as follows:

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_iw_j \quad \text{s.t.}\begin{cases} w_1+w_2+\cdots+w_n=1 \\ w_i \geq 0,\quad i=1,\ldots,n \end{cases} \tag{21}$$

w^* (the optimal solution $w = (w_1, w_2,\ldots, w_n)'$) can be obtained solving Model (21) by Algorithm 1. Simultaneously, $r_N^{\min}$ (the smallest of $r_N^{\min} = \sum_{i=1}^{n}\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i^* - \sum_{i=1}^{n} c_i(w_i^*)$) is obtained, i.e., $r_0^{\min} = r_N^{\min}$.

4.2 Several methods for the problems with different transaction costs

In this section, we will present several methods for the random credibilitic mean variance portfolio selection models with different types of transaction costs, i.e., linear function, non-smooth convex functions, and smooth functions

4.2.1 The transaction cost is linear function

In Model (19), let the transaction costs be linear functions, i.e., $c_i(w_i) = k_iw_i$, and Model (19) can be obtained as follows:

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_iw_j \quad \text{s.t.}\begin{cases} \sum_{i=1}^{n}\left(\dfrac{a_i+b_i}{2}+\dfrac{\beta_i-\alpha_i}{4}-k_i\right)w_i \geq r_0 \\ w_1+w_2+\cdots+w_n=1 \\ w_i \geq 0, i=1,\ldots,n \end{cases} \tag{22}$$

where the KKT conditions of Model (22) are a system of equalities where all the expressions are linear equalities or inequalities except for complementarity conditions, and there are $n + 2$ variables and $2n + 4$ linear equalities or inequalities in Eq. (23).

Algorithm 1. The pivoting algorithm for Eq. (23).

Step 1 Construct initial table.

Let w_i^3 0, $i = 1,2,\ldots,n$, μ_1^3 0, μ_2^3 0 be the initial basic inequality. The initial basic solution is $x^0 = (0,\ldots,0,0,0)'$. The initial basic vector is e_i, $i = 1, 2,\ldots, n + 2$, which is the ith row of the identity matrix of order $n + 2$. Other equalities and inequalities of Eq. (23) and their coefficient vectors are non-basic g_i.

Let $d_i = \frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}-k_i$, $q=(0,\ldots,0,r_0,1)'$; $g_i = (\sigma_{i1},\sigma_{i2},\ldots,\sigma_{in},-d_i,1)$, $i = 1,2,\ldots,n$; $g_{n+1} = (d_1,\ldots,d_n,0,0)$; $g_{n+2} = (1,\ldots,1,0,0)$

The deviation with respect to x^0 is $\Delta = gx^0 - q$. Thus, we have an initial table as shown in Table 1.

Step 2 Preprocessing.

Let I_3 be the index set of equalities, i.e., $I_3 = \{n + 2\}$, I_4 be the index set of inequalities, i.e., $I_4 = \{1,2,\ldots,n,n + 1\}$. Select a non-basic vector g_{n+2} to enter the basis, and the basic vector e_1 leaves the basis. Select a non-basic vector g_1 to enter the basis, and the basic vector e_{n+2} leaves the basis. Then, carry out two pivoting on the positive elements 1. Then, delete the column of g_{n+2} and the row of e_{n+2}. The calculating process is as follows:

$$g_1 = g_{11}e_1 + g_{12}e_2 + \cdots + g_{1n}e_n + g_{1n+1}e_{n+1} + g_{1n+2}e_{n+2}, i = 1,\ldots 3n+2 \tag{24}$$

According to Eq. (24), we can obtain Eq. (25).

Table 1 Initial table

	e_1	…	e_n	e_{n+1}	e_{n+2}	Δ_i
g_1	σ_{11}	…	σ_{1n}	$-(r_1-k_1)$	1	0
…	…	…	…	…	…	…
g_n	σ_{n1}	…	σ_{nn}	$-(r_n-k_n)$	1	0
g_{n+1}	r_1-k_1	…	r_n-k_n	0	0	$-r_0$
g_{n+2}	1	…	1	0	0	-1

$$e_{n+2}=\frac{1}{a_{rn+2}}g_1+\sum_{j=1,j\neq n+2}^{n+2}\left(-\frac{a_{1j}}{a_{1n+2}}\right)e_j \tag{25}$$

Substituted Eq. (25) into Eq. (24), we can get Eq. (26).

$$g_i=\frac{a_{in+2}}{a_{1n+2}}g_1+\sum_{j=1,j\neq n+2}^{n+2}\left(a_{ij}-\frac{a_{in+2}}{a_{1n+2}}a_{1j}\right)e_j,i=\mathbf{1},\ldots,n,n+\mathbf{1},n+\mathbf{2} \tag{26}$$

Using the same method, we obtain the following pivoting operation: $g_{n+2}\leftrightarrow e_1$.

Step 3 Main iterations.

(1) If all of the deviations of non-basic vectors are nonnegative, the current basic solution is a solution to Eq. (23).

(2) Otherwise, select a non-basic vector g_r with negative deviation to enter the basis. If there is no positive element in the row of entering vector that is corresponding to the basic inequality, Eq. (23) has no solution. If there is the biggest positive element g_{rs} in the row of entering vector, the basic vector of the sth column corresponding to g_{rs} leaves the basis. Then, carry out a pivoting operation: $g_r\leftrightarrow e_s$. After that, return to Step 3. (1).

4.2.2 The transaction cost is non-smooth convex functions

Let the transaction costs be non-smooth convex functions; the random credibilitic portfolio selection model is:

$$\min\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_iw_j$$
$$\text{s.t.}\begin{cases}\sum_{i=1}^{n}\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i-\sum_{i=1}^{n}c_i(w_i)\geq r_0\\ w_1+w_2+\cdots+w_n=1\\ 0\leq w_i\leq u_i,i=1,\ldots,n\end{cases} \tag{27}$$

Model (27) is a non-smooth convex quadratic programming problem.

If $c_i(w_i)$ is a non-smooth convex function, $\sum_{i=1}^{n}\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i-\sum_{i=1}^{n}c_i(w_i)$ is concave function. So, Model (27) is a non-smooth convex programming problem. We propose a branch-and-bound method and a pivoting algorithm to solve Model (27). Let the function of OB be $c_i(w_i)=k_iw_i$, where $k_i=c_i(u_i)/u_i$. When $c_i(w_i)$ is substituted by $k_i\ x_i$, Model (27) can be turned into as follows:

$$\min\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_iw_j$$
$$\text{s.t.}\begin{cases}\sum_{i=1}^{n}\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i-\sum_{i=1}^{n}k_iw_i\geq r_0\\ w_1+w_2+\cdots+w_n=1,0\leq w_i\leq u_i,i=1,\ldots,n\end{cases} \tag{28}$$

Theorem 5 *Let w^0 be the optimal solution of Model (28). Then, w^0 is the feasible solution of Model (27).*

Proof If x^0 is the optimal solution of Model (28), we can get the following equation:

$$\sum_{i=1}^{n}\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i^0-\sum_{i=1}^{n}k_iw_i^0\geq r_0 \tag{29}$$

Because $c_i(w_i)$ is a non-smooth convex function, we can obtain the following equation:

$$k_iw_i^0\geq c_i\left(w_i^0\right) \tag{30}$$

According to Eq. (29) and Eq. (30), we can get

$$\sum_{i=1}^{n}\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i^0-\sum_{i=1}^{n}c_i(w_i^0)\geq r_0$$

So, w_i^0 is the feasible solution of Model (27).

Thus, the proof of Theorem 5 is ended.

Theorem 6 *Let w^0 and $g(w^0)$, respectively, be the optimal solution and the optimal objective function value of Model (28), $w^*$and $f(w^*)$, respectively, be the optimal solution and the optimal objective function value of Model (27), $f(w^0)$ be the objective function value of the feasible solution w^0 of Model (27). Then,*

$$g\left(w^0\right)\leq f\left(w^*\right)\leq f\left(w^0\right) \tag{31}$$

Proof According to Theorem 5, we can obtain that w^0 is the feasible solution of Model (27). Because w^* is the optimal solution of Model (27), we can get that

$$f\left(w^*\right)\leq f\left(w^0\right) \tag{32}$$

According to Fig. 1, we can obtain that

$$k_iw_i\geq c_i(w_i) \tag{33}$$

According to Eq. (33), we can obtain that

$$\sum_{i=1}^{n}[r_iw_i-c_i(w_i)]\geq\sum_{i=1}^{n}(r_iw_i-k_iw_i)\geq r_0 \tag{34}$$

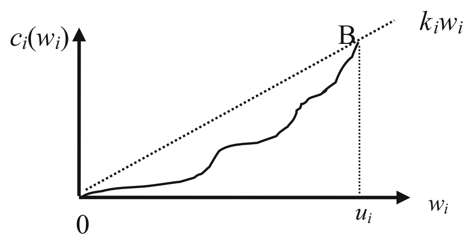

Fig. 1 Non-smooth convex function

According to Eq. (34), we can obtain that w^* is the feasible solution of Model (28), i.e.,

$$g(w^0) \le g(w^*). \tag{35}$$

Because the objective functions of Model (27) and Model (28) are same, we can get that

$$g(w^*) \le f(w^*). \tag{36}$$

According to Eqs. (32), (35), and (36), we can obtain that

$$g(w^0) \le f(w^*) \le f(w^0).$$

Thus, the proof of Theorem 6 is ended.

From Eq. (36), we can obtain that there are upper and lower bounds on the optimal objective function value of Model (28).

Definition 6 Let w^0 be the optimal solution of Model (28), and

$$\sum_{i=1}^{n} (k_i w_i^0 - c_i(w_i^0)) \le \varepsilon, e = 10^{-6}. \tag{37}$$

Then, w^0 is almost the optimal solution of Model (27).

If there is an optimal solution of Model (28) w^0, which cannot satisfy Eq. (37), then let

$$k_s w_s^0 - c_s(w_s^0) = \max\{k_i w_i^0 - c_i(w_i^0) | i = 1, \ldots, n\} \tag{38}$$

$$W^1 = \{x | 0 \le w_s \le u_s/2,\ 0 \le w_j \le u_j, j \ne s\} \tag{39}$$

$$W^2 = \{x | u_s/2 \le w_s \le u_s,\ 0 \le w_j \le u_j, j \ne s\}. \tag{40}$$

Model (27) can be turned into the following two models:

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_i w_j$$
$$\text{s.t.}\begin{cases}\sum_{i=1}^{n}\left[\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i - c_i(w_i)\right] \ge r_0 \\ e^T w = 1 \\ w \in X_1\end{cases} \tag{41}$$

and

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_i w_j$$
$$\text{s.t.}\begin{cases}\sum_{i=1}^{n}\left[\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i - c_i(w_i)\right] \ge r_0 \\ e^T w = 1 \\ w \in X_2\end{cases} \tag{42}$$

Let $c_s(w_s)$ be substituted by two piecewise linear function. The function of $c_s(x_s)$ can be denoted as in Fig. 2.

In Fig. 2, the linear OA is $k_{s1}(w_{s1})$, in which the slope of OA is $c_s(u_s/2)/(u_s/2)$, and the linear AB is $k_{s2}(w_{s2})$, in which the slope of AB is $k_{s2} = (c_s(a_s) - c_s(a_s/2))/(a_s/2)$.

Model (28) can be turned into the two following models:

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_i w_j$$
$$\text{s.t.}\begin{cases}\sum_{i=1, i\ne s}^{n}(r_i w_i - k_i w_i) + (r_s w_s - k_{s1}(w_{s1})) \ge r_0 \\ e^T w = 1 \\ w \in X_1\end{cases} \tag{43}$$

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_i w_j$$
$$\text{s.t.}\begin{cases}\sum_{i=1, i\ne s}^{n}(r_i w_i - k_i w_i) + (r_x w_s - k_{s2}(w_{s2})) \ge r_0 \\ e^T w = 1 \\ w \in X_2\end{cases} \tag{44}$$

Definition 7 Let w^1 and w^2 be, respectively, the optimal solution of Model (43) and Model (44). If $\sum_{i=1}^{n}(k_i w_i^1 - c_i(w_i^1)) \le \varepsilon$ or $\sum_{i=1}^{n}(k_i w_i^2 - c_i(w_i^2)) \le \varepsilon$, then x^1 or x^2 is the optimal solution of Model (27).

Algorithm 2. The branch-and-bound algorithm for Model (27)

Step 1 Let $k = 0$, $l_i^0 = 0$, $u_i^0 = u_i$, $w_i^0 = \{w_i | l_i^0 \le w_i \le u_i^0\}$.

Step 2 Let $k_i^j(w_i)$ be the approximate linear function of $c_i(w_i)$. We can obtain the optimal solution of the following model by Algorithm 1.

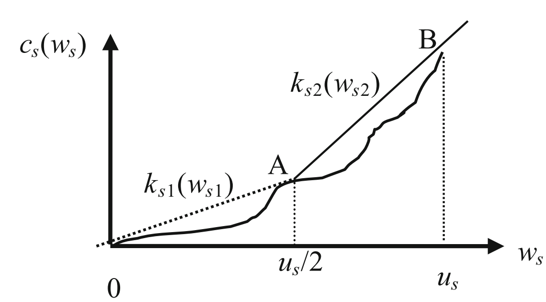

Fig. 2 $c_s(w_s)$ substituted by two piecewise linear functions

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_iw_j$$
$$\text{s.t.}\begin{cases}\sum_{i=1}^{n}\left[\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i-k_i^j(w_i)\right]\geq r_0\\ e^Tw=1\\ w\in W^j\end{cases} \tag{45}$$

where the optimal solution of Model (45) is $W^j = (w_1^j,\ldots,w_n^j)$.

Step 3 If $\sum_{i=1}^{n}(k_i^j(w_i^j)-c_i(w_i^j))\leq\varepsilon$, $W^j=\left(w_1^j,\ldots,w_n^j\right)$ is the optimal solution of Model (27). Otherwise,

$$k_s^j\left(w_s^j\right)-c_s^j\left(w_s^j\right)=\max\{k_i^j\left(w_i^j\right)-c_i^j\left(w_i^j\right)|i=1,\ldots,n\}$$

, $W^{j+1}=\{l_s^j\leq w_s\leq(u_s^j+l_s^{j)})/2, w\in W^j|w_i\neq w_s\}$, $W^{j+2}=\{(u_s^j+l_s^{j)})/2\leq w_s\leq u_s^j, x\in X^j|w_i\neq w_s\}$, turn Step 2.

4.2.3 The transaction cost is smooth convex functions

Let the transaction costs be smooth convex functions. The function can be denoted as in Fig. 3.

The random credibilitic portfolio selection with smooth convex transaction costs is as follows:

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_iw_j$$
$$\text{s.t.}\begin{cases}\sum_{i=1}^{n}\left[\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i-c_i(w_i)\right]\geq r_0\\ e^Tw=1, w_i\geq 0,\quad i=1,\ldots,n\end{cases} \tag{46}$$

If $c_i(x_i)$ is a convex function, $\sum_{i=1}^{n}[r_iw_i-c_i(w_i)]$ is a concave function. So, Model (46) is a smooth convex programming problem.

Algorithm 3. A sequence quadratic programming algorithm for Model (46)

We propose a sequence quadratic programming and a pivoting algorithm to solve Model (46).

Step 1 Let $w^0 = (1/n, \ldots, 1/n)^T$, $c_i(w_i)$ be approximated as $g_i(w_i) = c_i(w_i^0) + c_i'(w_i^0)(w_i - w_i^0)$. The subproblem of Model (46) can be obtained as follows:

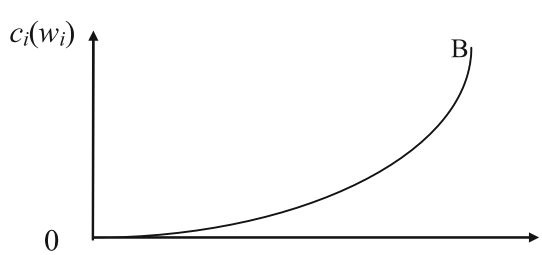

Fig. 3 Smooth convex function

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_iw_j$$
$$\text{s.t.}\begin{cases}\sum_{i=1}^{n}\left[\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i-\left(c_i(w_i^0)+c_i'(w_i^0)(w_i-w_i^0)\right)\right]\geq r_0\\ e^Tw=1\\ w_i\geq 0,\quad i=1,\cdots,n\end{cases} \tag{47}$$

The optimal solution of Model (47) w^* can be obtained by Algorithm 1. Let $w^1 = w^*$.

Step 2 Let $c_i(w_i)$ be approximated as $g_i(w_i) = c_i(w_i^k) + c_i'(w_i^k)(w_i - w_i^k)$. The subproblem of Model (46) can be obtained as follows:

$$\min \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}w_iw_j$$
$$\text{s.t.}\begin{cases}\sum_{i=1}^{n}\left[\left(\frac{a_i+b_i}{2}+\frac{\beta_i-\alpha_i}{4}\right)w_i-\left(c_i(w_i^k)+c_i'(w_i^k)(w_i-w_i^k)\right)\right]\geq r_0\\ e^Tw=1\\ w_i\geq 0, i=1,\cdots,n\end{cases} \tag{48}$$

The optimal solution of Model (48) w^*can be obtained by Algorithm 1. Let $w^{k+1} = w^*$.

Step 3 If $\left|w^{k+1}-w^k\right|\leq\varepsilon$, $\varepsilon=10^{-6}$, w^{k+1} is the optimal solution of Model (46). Otherwise, let $w^k := w^{k+1}$. Turn to Step 2.

5 Numerical examples

In this section, three numerical examples are given to express the idea of the proposed model. Assume that an investor chooses twenty stocks from Shanghai Stock Exchange for his investment. The stocks codes are, respectively, S_1 (600036), S_2 (600002), S_3 (600060), S_4 (600362), S_5 (600519), S_6 (601111), S_7 (601318), S_8 (600900), S_9 (600887), S_{10} (600690), S_{11} (6000970), S_{12} (600000), S_{13} (600009), S_{14} (600019), S_{15} (600029), S_{16} (600104), S_{17} (600315), S_{18} (600518), S_{19} (600570), and S_{20} (600880). He/she assumes that the returns and risk of the twenty stocks at each period are represented as trapezoidal fuzzy numbers. We collect historical data of them from April 2006 to June 2018 and set every three months as a period to handle the historical data. By using the simple estimation method in Vercher et al. (2007) to handle their historical data, the trapezoidal possibility distributions of the return rates of assets can be obtained as shown in Table 2. We use the above algorithms to obtain the optimal solution of Model (19) with the three types transaction costs. (1) When the transaction cost is linear function, we assumed that $c_i(w_i) = 0.003w_i$. (2) When the transaction

Table 2 Fuzzy return rates on assets

Asset 1	0.1115	0.1439	0.0719	0.0526	Asset 2	0.0756	0.0943	0.0463	0.0470
Asset 3	0.0956	0.1213	0.0617	0.0443	Asset 4	0.0974	0.1178	0.0594	0.0679
Asset 5	0.0681	0.0989	0.0541	0.0426	Asset 6	0.0887	0.1231	0.0470	0.0453
Asset 7	0.0687	0.0920	0.0439	0.0571	Asset 8	0.0981	0.1495	0.0558	0.0766
Asset 9	0.0513	0.0765	0.0396	0.0825	Asset 10	0.0310	0.0443	0.0258	0.0347
Asset 11	0.0510	0.0639	0.0338	0.0217	Asset 12	0.1048	0.1438	0.0975	0.0645
Asset 13	0.0778	0.1319	0.0573	0.0706	Asset 14	0.0508	0.0746	0.0489	0.0364
Asset 15	0.0422	0.1250	0.0334	0.0526	Asset 16	0.0603	0.0833	0.0503	0.0598
Asset 17	0.0832	0.1321	0.0608	0.0701	Asset 18	0.0648	0.1183	0.0612	0.4231
Asset 19	0.0760	0.1000	0.0540	0.0550	Asset 20	0.0700	0.1184	0.0578	0.0552

cost is non-smooth convex function, we assumed that $c_i(w_i) = \begin{cases} 0.003w_i, 0 \le w_i \le 0.5 \\ 0.004w_i - 0.0005, 0.5 \le w_i \le 1 \end{cases}$.

(3) When the transaction cost is smooth convex function, we assumed that $c_i(w_i) = 0.003w_i^2 + 0.003w_i$.

The trapezoidal possibility distributions $\widetilde{R}_i = (a_i, b_i, \alpha_i, \beta_i)$ can be obtained as shown in Table 2.

The covariance matrix $\sigma^2 = (\sigma_{ij})_{n\times n}$ can be obtained as follows:

$$\sigma_{ij} = \frac{1}{m}\sum_{k=1}^{m}\left(R_{ik} - \overline{M}(\widetilde{R}_i)\right)\left(R_{jk} - \overline{M}(\widetilde{R}_j)\right)$$

As illustrations, the following numerical examples are given to show the effectiveness of the proposed model and the algorithms. The algorithm was programmed by C++ language and run on a personal computer with Pentium Dual CPU, 4 GHz, and 8 GB RAM.

(1) When the transaction cost is linear function, we assumed that the coefficients of transaction costs are generated same for all assets, i.e., $c_i(w_i) = 0.003w_i$. By using the pivoting algorithm to solve Model (20) and Model (21), we can, respectively, obtain $r_0^{\max} = 0.179025$, $r_0^{\min} = 0.0964$.

By using the pivoting algorithm to solve Model (22), the computational results are summarized in Table 2.

If $r_0 = 0.1, 0.12, 0.13$, the optimal solution will be obtained as Table 3.

When $r_0 = 0.1$, the optimal investment strategy at period 1 is $w_1 = 0.1207$, $w_3 = 0.6370$, $w_6 = 0.1018$, $w_9 = 0.0984$, $w_{12} = 0.0004$, $w_{19} = 0.0324$, $w_{20} = 0.0093$ and being the rest of variables equal to zero, which means investor should allocate his initial wealth on asset 1, asset 3, asset 6, asset 9, asset 12, asset 19, asset 20 and otherwise asset by the proportions of 12.07%, 63.7%, 10.18%, 9.84%, 0.04%, 3.24%, 0.93% and being the rest of variables equal to zero among the twenty stocks, respectively. From Table 3, it can be seen that the investment proportion of asset 8 will increase when the minimum target value of the total future profit r_0 increases.

If $r_0 = 0.0964, 0.1, 0.105, 0.01,\ldots, 0.17, 0.175, 0.179025$, the variance and expected return will be obtained as given in Table 4.

In Table 4, it can be seen that the standard deviation will increase when the net expected return increases.

(2) When the transaction cost is non-smooth convex function, we assumed that

$c_i(w_i) = \begin{cases} 0.003w_i, 0 \le w_i \le 0.5 \\ 0.004w_i - 0.0005, 0.5 \le w_i \le 1 \end{cases}$. By using Algorithm 2 to solve Model (20) and Model (21), we can, respectively, obtain $r_0^{\max} = 0.178525$, $r_0^{\min} = 0.096203$.

By using Algorithm 2 to solve Model (27), the computational results are summarized in Table 2.

If $r_0 = 0.1, 0.12, 0.13$, the optimal solution will be obtained as given in Table 5.

From Table 5, it can be seen that the investment proportion of asset 8 will increase when the minimum target value of the total future profit r_0 increases.

If $r_0 = 0.096203, 0.1, 0.105, 0.01,\ldots, 0.17, 0.175, 0.178525$, the variance and expected return will be obtained as given in Table 6.

Table 3 Optimal solution when $r_0 = 0.1, 0.12, 0.13$

Asset i	Optimal investment proportions						
r_0							
0.1	Asset 1	Asset 3	Asset6	Asset 9	Asset 12	Asset 19	Asset 20
	0.1207	0.6370	0.1018	0.0984	0.0004	0.0324	0.0093
0.12	Asset 1	Asset 3	Asset6	Asset 8	Asset 12	Asset 18	Asset 19
	0.1272	0.1428	0.0231	0.6456	0.0214	0.0083	0.0316
0.13	Asset 1	Asset 8	Asset12	Asset 18			

Table 4 Variance and expected return when r_0 = 0, 0.01,…, 0.16, 0.1671

r_0	0.0964	0.1	0.105	0.11	0.115	0.12	0.125
σ^2 (10^{-4})	5.5764	5.9887	7.4112	9.0942	11.0435	13.4378	16.3564
σ (10^{-2})	2.3614	2.4472	2.7224	3.0157	3.3232	3.6658	4.0443
r_0	0.13	0.135	0.14	0.145	0.15	0.155	0.16
σ^2 (10^{-4})	20.4697	28.0242	40.8935	59.1103	82.6743	111.6139	146.9718
σ (10^{-2})	4.5243	5.2938	6.3948	7.6883	9.0925	10.5647	12.1232
r_0	0.165	0.17	0.175	0.179025			
σ^2 (10^{-4})	189.2751	238.5238	294.7178	345			
σ(10^{-2})	13.7577	15.4442	17.1673	18.5742			

Table 5 Optimal solution when r_0 = 0.1, 0.12, 0.13

Asset i	Optimal investment proportions						
r_0							
0.1	Asset 1	Asset 3	Asset6	Asset 9	Asset 12	Asset 19	Asset 20
	0.1246	0.6331	0.1032	0.0963	0.0006	0.0325	0.0096
0.12	Asset 1	Asset 3	Asset6	Asset 8	Asset 12	Asset 18	Asset 19
	0.1380	0.1550	0.0271	0.6146	0.0215	0.0162	0.0276
0.13	Asset 1	Asset 8	Asset12	Asset 18			

Table 6 Variance and expected return when r_0 = 0, 0.01, …, 0.175, 0.178525

r_0	0.096203	0.1	0.105	0.11	0.115	0.12	0.125
σ^2 (10^{-4})	5.5764	6.0188	7.4189	9.0942	11.0435	13.5074	16.5064
σ (10^{-2})	2.3614	2.4533	2.7238	3.0157	3.3232	3.6752	4.0628
r_0	0.13	0.135	0.14	0.145	0.15	0.155	0.16
σ^2 (10^{-4})	20.6988	28.1624	40.8935	59.1103	82.6743	112.2238	148.3853
σ (10^{-2})	4.5496	5.3068	6.3948	7.6883	9.0925	10.5936	12.1814
r_0	0.165	0.17	0.175	0.178525			
σ^2 (10^{-4})	191.7332	242.2674	299.9880	345			
σ (10^{-2})	13.8468	15.5649	17.3202	18.5742			

Table 7 Optimal solution when r_0 = 0.1, 0.12, 0.13

Asset i	Optimal investment proportions							
r_0								
0.1	Asset 1	Asset 3	Asset6	Asset 8	Asset 9	Asset 12	Asset 19	Asset 20
	0.1361	0.6069	0.1071	0.0203	0.0849	0.0027	0.0325	0.0095
0.12	Asset 1	Asset 3	Asset6	Asset 8	Asset 12	Asset 18	Asset 19	
	0.1578	0.1596	0.0280	0.5794	0.0218	0.0344	0.0190	
0.13	Asset 1	Asset 8	Asset 18					

In Table 6, it can be seen that the standard deviation will increase when the net expected return increases.

(3) When the transaction cost is smooth convex function, we assumed that $c_i(w_i) = 0.003w_i^2 + 0.003w_i$. By using Algorithm 3 to solve Model (20) and Model (21), we can, respectively, obtain

$$r_0^{\max} = 0.1760250, \quad r_0^{\min} = 0.094818.$$

By using Algorithm 3 to solve Model (16), the corresponding results can be obtained as follows.

If r_0 = 0.1, 0.12, 0.13, the optimal solution will be obtained as given in Table 7.

From Table 7, it can be seen that the investment proportion of asset 8 will increase when the minimum target value of the total future profit r_0 increases.

If r_0 = 0.094818, 0.1, 0.105, 0.01,…, 0.175, 0.1760250, the variance and expected return will be obtained as given in Table 8.

In Table 8, it can be seen that the standard deviation will increase when the net expected return increases.

Table 8 Variance and expected return when r_0 = 0.094818, 0.1, 0.105, 0.01,…, 0.175, 0.1760250

r_0	0.094818	0.1	0.105	0.11	0.115	0.12	0.125
σ^2 (10^{-4})	5.5764	6.3058	7.7062	9.3954	11.4760	14.1435	17.4504
σ (10^{-2})	2.3614	2.5111	2.7760	3.0652	3.3876	3.7608	4.1774
r_0	0.13	0.135	0.14	0.145	0.15	0.155	0.16
σ^2 (10^{-4})	22.2128	30.5465	44.1565	63.5580	89.2194	121.5794	161.0640
σ (10^{-2})	4.7130	5.5269	6.6450	7.9723	9.4456	11.0263	12.6911
r_0	0.165	0.17	0.175	0.178525			
σ^2 (10^{-4})	208.4321	264.6697	330.3242	345			
σ (10^{-2})	14.4372	16.2687	18.1748	18.5742			

The computational results for these three types of transaction costs are summarized in Tables 3, 5, and 7. In general, the optimal strategies under different transaction costs are different. The change of the transaction cost has great influence on the strategy making.

In the general type of convex transaction cost function $c_i(w_i)$ contains three types, i.e., linear function, non-smooth convex function, and smooth convex function. The above three types of transaction costs can be used to describe the practical situation for the transaction cost precisely.

6 Conclusions

A random credibilitic portfolio optimization model with different convex transaction costs is proposed, which can be deduced into any specific form by investors' estimation and practical situation. We present three algorithms especially for solving the different convex transaction costs function form of portfolio selection problem. Moreover, we define three general types of transaction cost and describe the practical situation for the transaction cost precisely. The computational experiments show that different types of transaction cost have great influence on strategy making.

Acknowledgements This research was supported by the National Natural Science Foundation of China (Nos. 71271161).

Compliance with ethical standards

Conflict of interest Peng Zhang declares that he/she has no conflict of interest.

Ethical approval This article does not contain any studies with human participants performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study.

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