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A comprehensive model for fuzzy multi-objective portfolio selection based on DEA cross-efficiency model

Wei Chen[1] · Si-Si Li[1] · Jun Zhang[1] · Mukesh Kumar Mehlawat[2]

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Abstract
In this paper, we discuss the fuzzy portfolio selection problems in multi-objective frameworks. A comprehensive model for multi-objective portfolio selection in fuzzy environment is proposed by incorporating mean-semivariance model and data envelopment analysis cross-efficiency model. In the proposed model, the cross-efficiency model is formulated within the framework of Sharpe ratio; bounds on holdings, and cardinality constraints are also considered. The nonlinear constrained multi-objective portfolio optimization problem cannot be efficiently solved by using traditional approaches. Thus, a multi-objective firefly algorithm is developed to solve the relevant model. Finally, an example verifies the validity of the proposed approaches.

Keywords Fuzzy portfolio selection · Data envelopment analysis · Cross-efficiency evaluation · Sharpe ratio · Multi-objective firefly algorithm

1 Introduction

The mean-variance (M–V) model proposed by Markowitz (1952) has made tremendous contribution to the modern portfolio selection theory, in which return is quantified as the mean and risk as the variance. Since then, many researchers have improved and expanded the M–V model based on different risk measurements, see for instance, mean-semivariance models (Markowitz 1959; Grootveld and Hallerbach 1999), mean absolute deviation (MAD) model (Konno and Yamazaki 1991), mean semiabsolute deviation models (Speranza 1993; Ogryczak and Ruszczynski 1999), mean absolute deviation skewness model (Konno et al. 1993), etc. All above researches are based on the probabilistic framework where the returns of securities are regarded as random variables with probability distributions. However, because the financial markets are complex, and we sometimes lack enough historical data, it is difficult to obtain the precise probability distributions of the security returns. With the help of fuzzy set theory proposed by Zadeh (1965), a number of scholars have studied portfolio selection problems in fuzzy environment, for instance, Carlsson et al. (2002), Gupta et al. (2008), Wang et al. (2011), Liu and Zhang (2013), Chen (2015), Chen et al. (2018), Vercher and Bermúdez (2015), Mehlawat (2016) and Liagkouras and Metaxiotis (2018).

Data Envelopment Analysis (DEA) approach proposed by Charnes et al. (1978) is a mathematical programming-based approach for measuring relative efficiency of decision-making units (DMUs) that have multiple inputs and outputs. Subsequently, it turns out that DEA is a worthy tool for evaluating performance in a wide range of fields, such as the interesting applications in health care (Sherman 1984), education (Avkiran 2001), environment (Fried et al. 2002), banking (Grigorian and Manole 2006), energy (Hu and Kao 2007). In addition to above applications, in recent years, DEA method has been applied to portfolio performance evaluation. Murthi et al. (1997) first applied DEA method to portfolio performance evaluation and concluded that the pro-

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✉ Wei Chen
chenwei@cueb.edu.cn

Si-Si Li
lisisi@cueb.edu.cn

Jun Zhang
zhangjun@cueb.edu.cn

Mukesh Kumar Mehlawat
mukesh0980@yahoo.com

1 School of Information, Capital University of Economics and Business, Beijing, China

2 Department of Operational Research, University of Delhi, Delhi, India

posed approach is consistent with traditional Sharpe index (Sharpe 1966) and Jesnen Index (Jensen 1968). Joro and Na (2006) developed a portfolio performance measure in a mean-variance-skewness framework by utilizing a non-parametric DEA method. Branda (2013) introduced new efficiency tests, in which deviation and return measures were regarded as the inputs and outputs, respectively. Lim et al. (2014) presented a DEA cross-efficiency method and proposed a new model called DEA M–V cross-efficiency model. Liu et al. (2015) evaluated the efficiency of portfolios by constructing DEA models and proved that the DEA frontiers can approximate the real frontier of portfolios with big enough sample size. More recently, Gouveia et al. (2017) used the value-based DEA method to assess the performance of Portuguese mutual fund portfolios. Zhou et al. (2018) proposed a DEA frontier improvement approach under the M–V framework. Later, Zhou et al. (2018) presented a segmented DEA approach based on data segment points, and proved that the approach was effective and practical in evaluating the cardinality constrained portfolio performance. Tarnaud and Leleu (2017) presented an idea that performance measurement of portfolios with DEA should not rely on a technology defined through a production process that assimilates risk to an input generating some return. Zhou et al. (2018) proposed a multi-objective evolutionary algorithm based on decomposition and DEA approach for portfolio optimization. It should be noted that above researches are based on the assumption that security returns are random variables instead of fuzzy variables. At present, few researchers have applied the DEA approach to fuzzy portfolio evaluation problems. Chen et al. (2018) presented three kinds of DEA-based fuzzy portfolio efficiency evaluation models in different risk measures.

Note that, all above-mentioned researches focus either on developing different portfolio selection models, or on presenting various portfolio performance evaluation models. But, no research has yet been carried out from both above aspects. Recently, Mashayekhi and Omrani (2016) proposed a fuzzy multi-objective portfolio selection model based on M–V model and DEA cross-efficiency models to simultaneously consider return, risk and the efficiency of the portfolio. To the best of the authors' knowledge, except for the above one work, there is few research on constructing fuzzy portfolio evaluation model by integrating M–V model and DEA cross-efficiency model. Especially, when return distributions of securities are asymmetric, using variance as risk measure leads to an unsatisfactory prediction of portfolio behavior. This lack of works has motivated this work. In this paper, we will develop a comprehensive model for fuzzy multi-objective portfolio selection by incorporating fuzzy mean-semivariance model and DEA cross-efficiency model. It should be noted that the cross-efficiency model is formulated within the framework of Sharpe ratio.

With the introduction of some practical constraints including cardinality constraint in multi-objective frameworks, the multi-objective portfolio selection problem has become popular; and the complexity of computation makes it be the NP-hard problems (Shaw et al. 2008). Several researchers have attempted to solve this problem by a variety of techniques, but exact solution methods may fail to obtain an optimal solution in reasonable time; and the computation time grows rapidly with the problem size. Using metaheuristics in this case is imperative. At present, several scholars have applied metaheuristic optimization techniques including evolutionary algorithms (EAs) for multi-objective portfolio optimization problem, such as Krink and Paterlini (2011), Anagnostopoulos and Mamanis (2011), Bermúdez et al. (2012), Lwin et al. (2014), Saborido et al. (2016) and Liagkouras and Metaxiotis (2018). In 2008, a new biologically inspired metaheuristic algorithm, known as the firefly algorithm (FA), was developed by Yang (2008). Since the introduction of FA, it has been successfully applied to various optimization problems, see the survey by Fister et al. (2013) and Yang and He (2013). However, to our knowledge, few researchers have applied FA for solving fuzzy multi-objective portfolio optimization problems with complex realistic constraints. In addition, the basic FA was developed for unconstrained issues and exhibits some deficiencies when solving the constrained multi-objective model. Therefore, a multi-objective FA is developed for the fuzzy multi-objective portfolio optimization model.

In summary, this paper discusses the fuzzy portfolio selection problem, in which return, risk, and the efficiency of the portfolio are considered simultaneously. The main contributions of this paper are as follows: (1) we propose a comprehensive model for fuzzy multi-objective portfolio selection by incorporating fuzzy mean-semivariance model and DEA cross-efficiency model. Especially, inspired by the ideas of Sharpe ratio (SR), the cross-efficiency model is formulated within the framework of SR; and (2) we develop a multi-objective firefly algorithm (MOFA) to solve the proposed multi-objective portfolio optimization model.

The rest of the paper is organized as follows. Section 2 presents the proposed fuzzy multi-objective portfolio comprehensive model. In Sect. 3, the multi-objective firefly algorithm is introduced. After that, an example is given to verify the validity of the proposed approaches in Sect. 4. Finally, the conclusion of the paper is summarized in Sect. 5.

2 Model formulation

2.1 Possibilistic mean-semivariance portfolio model

Similar to Carlsson et al. (2002) and Chen (2015), we assume that the returns of assets are trapezoidal fuzzy numbers. Let security return r_i be a trapezoidal fuzzy number with tolerance interval $[a_i, b_i]$, left width α_i and right width β_i, i.e., $r_i = (a_i, b_i, \alpha_i, \beta_i)$ with $\gamma-$ level sets $[r_i]^\gamma = [a_i - (1-\gamma)\alpha_i, b_i + (1-\gamma)\beta_i]$, $i = 1, 2, \ldots, n$.

Carlsson and Fullér (2001) introduced the notions of crisp possibilistic mean and crisp possibilistic variances of fuzzy numbers. Easily seen that if $r_i = (a_i, b_i, \alpha_i, \beta_i)$ is a trapezoidal fuzzy number then

$$E(r_i) = \frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6}, \tag{1}$$

and

$$\mathrm{Var}(r_i) = \left(\frac{b_i - a_i}{2} + \frac{\alpha_i + \beta_i}{6}\right)^2 + \frac{(\alpha_i + \beta_i)^2}{72}. \tag{2}$$

Furthermore, the possibilistic mean of the return associated with the portfolio $(w_1, w_2, \ldots, w_n)$ can be obtained as

$$E\left(\sum_{i=1}^{n} r_i w_i\right) = \sum_{i=1}^{n}\left(\frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6}\right) w_i, \tag{3}$$

and the possibilistic variance of return associated with the portfolio $(w_1, w_2, \ldots, w_n)$ as

$$\mathrm{Var}\left(\sum_{i=1}^{n} r_i w_i\right) = \left(\sum_{i=1}^{n} \frac{1}{2}\left[(b_i - a_i) + \frac{\alpha_i + \beta_i}{3}\right] w_i\right)^2 + \frac{1}{72}\left[\sum_{i=1}^{n} (\alpha_i + \beta_i) w_i\right]^2, \tag{4}$$

where w_i is the proportion of security i, $i = 1, 2, \ldots, n$.

Taking the possibilistic mean of portfolio return as return measure and the possibilistic variance as the risk measure, several researchers have proposed various types of fuzzy portfolio models in the mean-variance framework, such as Carlsson et al. (2002) and Liagkouras and Metaxiotis (2018). However, when return distributions of securities are asymmetric, using variance as risk measure leads to an unsatisfactory prediction of portfolio behavior. Therefore, some scholars employed semivariance as an alternative risk measure to qualify the portfolio risk, see for instance Markowitz (1959), Ballestero (2005), Zhang et al. (2012) and Liu and Zhang (2015). In this paper, we employ the lower possibilistic semivariance to measure the risk of portfolio. Based on Carlsson and Fullér (2001), and Saeidifar and Pasha (2009), Zhang et al. (2012) presented the definition of the lower possibilistic semivariances of fuzzy number A with $[A]^\gamma = [\underline{a}(\gamma), \bar{a}(\gamma)]$ $(\gamma \in [0, 1])$, as follows,

$$\mathrm{Var}^-(A) = \int_0^1 2\gamma (E(A) - \underline{a}(\gamma))^2 \mathrm{d}\gamma. \tag{5}$$

Besides, the lower possibilistic semivariance of return related with the portfolio $(w_1, w_2, \ldots, w_n)$ can be expressed by

$$\mathrm{Var}^-\left(\sum_{i=1}^{n} r_i w_i\right) = \left[\sum_{i=1}^{n} w_i \left(\frac{b_i - a_i}{2} + \frac{\alpha_i + \beta_i}{6}\right)\right]^2 + \frac{1}{18}\left(\sum_{i=1}^{n} w_i \alpha_i\right)^2. \tag{6}$$

In the following, we use the possibilistic mean of portfolio return as return measure and the lower possibilistic semivariance as the risk measure. Furthermore, the possibilistic mean-semivariance portfolio model can be formulated as the following bi-objective programming problem:

$$\begin{array}{lll}
\max & E\left(\sum_{i=1}^{n} r_i w_i\right) = \sum_{i=1}^{n}\left(\frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6}\right) w_i & \\
\min & \mathrm{Var}^-\left(\sum_{i=1}^{n} r_i w_i\right) = \left[\sum_{i=1}^{n} w_i \left(\frac{b_i - a_i}{2} + \frac{\alpha_i + \beta_i}{6}\right)\right]^2 & \\
 & \quad + \frac{1}{18}\left(\sum_{i=1}^{n} w_i \alpha_i\right)^2 & \\
\text{s.t.} & \sum_{i=1}^{n} w_i = 1, & (a) \\
 & \sum_{i=1}^{n} z_i = m, & (b) \\
 & \varepsilon_i z_i \le w_i \le \delta_i z_i,\ i = 1, 2, \ldots, n, & (c) \\
 & z_i \in \{0, 1\},\ i = 1, 2, \ldots, n, & (d) \\
 & w_i \ge 0,\ i = 1, 2, \ldots, n. & (e)
\end{array} \tag{7}$$

Constraint (7)(a) denotes the budget constraint, namely, all the money available should be invested. Constraint (7)(b) denotes the cardinality constraint which imposes a limit on the number of assets in the portfolio. Constraint (7)(c) ensures that if any of security i is held $(z_i = 1)$ its proportion w_i must lie no less than ε_i and no more than δ_i while if no security i is held $(z_i = 0)$, its ratio w_i is zero. Constraint (7)(d) is the integrality constraint. Constraint (7)(e) ensures that short selling is not allowed.

2.2 A comprehensive model for fuzzy multi-objective portfolio selection

Nowadays, the DEA cross-efficiency model, developed by Doyle and Green (1994), has applied to the fuzzy portfolio selection problems, see for example Ruiz and Sirvent (2017) and Mashayekhi and Omrani (2016). However, there are two shortcomings for cross-efficiency evaluation in portfolio selection. The first one is the lack of portfolio diversification and the second one is the 'ganging-together' phenomenon

(Tofallis 1996). To address this issue, Lim et al. (2014) proposed a DEA M–V cross-efficiency model by taking the cross-efficiency into the M–V framework. Mashayekhi and Omrani (2016) presented an integrated fuzzy multi-objective Markowitz-DEA cross-efficiency model. In this paper, we will propose a comprehensive model for fuzzy multi-objective portfolio selection by incorporating fuzzy mean-semivariance model and DEA cross-efficiency model. It should be noted that, in Mashayekhi and Omrani (2016), the integrated model was formulated within the framework of Markowtiz's mean-variance. However, in this paper, based on the Sharpe ratio (SR) (Sharpe 1966), the cross-efficiency model is formulated within the framework of SR. For a DMU (decision-making unit) l, returns and risks are replaced by the means and variances of the cross-efficiency scores, respectively. The proposed DEA cross-efficiency model is expressed as follows:

$$\begin{aligned} \max \quad & \theta^{\text{Sharpe}} = \frac{\sum_{i=1}^{n} w_i \overline{e}_i}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \operatorname{cov}(e_i, e_j)}} \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i = 1, \\ & w_i \geq 0,\ i = 1, 2, \ldots, n, \end{aligned} \tag{8}$$

where $\overline{e}_i$ is the cross-efficiency score of DMU i, $\operatorname{cov}(e_i, e_j)$ is the covariance between DMU i's cross-efficiencies (e_i) and DMU j's cross-efficiencies (e_j).

To solve the model (8), cross-efficiencies (e_j) should be first obtained. The basic steps of obtaining e_j are summarized as shown in below.

Step 1. Because of the existence of negative values in inputs and outputs, this paper uses the additive variable returns to scale (VRS) DEA model with a range-adjusted measure (RAM) of inefficiency. The additive model with a range-adjusted measure (RAM) of inefficiency is as follows:

$$\begin{aligned} \max \quad & \sum_{k=1}^{n}\sum_{r=1}^{s} p_{rk} y_{rk} - \sum_{k=1}^{n}\sum_{i=1}^{m} q_{ik} x_{ik} + \varepsilon_k \\ \text{s.t.} \quad & \sum_{r=1}^{s} p_{rk} y_{rj} - \sum_{i=1}^{m} q_{ik} x_{ij} + \varepsilon_k \leq 0, \quad \forall j, k, \\ & p_{rk} \geq \frac{1}{(m+s)R_r^{+}}, \quad \forall r, k, \\ & q_{ik} \geq \frac{1}{(m+s)R_r^{-}}, \quad \forall i, k, \\ & j = 1, 2, \ldots, n, \end{aligned} \tag{9}$$

where q_{ik} and p_{rk} represent the cost of input i and the price of output r for DMU k, respectively. n, m and s are the numbers of DMUs, inputs and outputs, respectively. x_{ij} and y_{rj} are the amount of the ith input and the rth output for the jth DMU, respectively. And ε_k is a positive infinitesimal value. The model (9) maximizes DMU's efficiency score and optimizes the weight for all DMUs simultaneously. The directional vectors R_i^{-} and R_r^{+} can be defined as:

$$R_i^{-} = \max_{j=1,2,\ldots,n}\{x_{ij}\} - \min_{j=1,2,\ldots,n}\{x_{ij}\},\ i = 1, 2, \ldots, m,$$
$$R_r^{+} = \max_{j=1,2,\ldots,n}\{y_{rj}\} - \min_{j=1,2,\ldots,n}\{y_{rj}\},\ r = 1, 2, \ldots, s.$$

Step 2. Let * represent the optimal solution of model (9). The efficiency score of other DMUs are obtained by using the weights that DMU k has chosen. The cross-efficiency of DMU l with the weights of DMU k (e_{kl}) can be expressed as follows:

$$e_{kl}^{*} = \sum_{r=1}^{s} p_{rk}^{*} y_{rl} - \sum_{i=1}^{m} q_{ik}^{*} x_{il} + \varepsilon_k.$$

Step 3. A matrix of cross-efficiencies are obtained as $E = (e_{kl}),\quad (k, l = 1, 2, \ldots, n)$, where e_{kl} is the cross-efficiency of DMU l evaluated by DMU k. The cross-efficiency score of DMU l can be calculated as the average of lth column:

$$\overline{e}_l = \frac{1}{n}\sum_{k=1}^{n} e_{kl}^{*}.$$

Based on above discussions, we incorporate fuzzy mean-semivariance model and SR-based DEA cross-efficiency model to construct a comprehensive model for fuzzy multi-objective portfolio selection, which is formulated as follows:

$$\begin{aligned} \max \quad & E\left(\sum_{i=1}^{n} r_i w_i\right) = \sum_{i=1}^{n}\left(\frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6}\right) w_i \\ \min \quad & \operatorname{Var}^{-}\left(\sum_{i=1}^{n} r_i w_i\right) = \left[\sum_{i=1}^{n} w_i \left(\frac{b_i - a_i}{2} + \frac{\alpha_i + \beta_i}{6}\right)\right]^2 \\ & + \frac{1}{18}\left(\sum_{i=1}^{n} w_i \alpha_i\right)^2 \\ \max \quad & \theta^{\text{Sharpe}} = \frac{\sum_{i=1}^{n} w_i \overline{e}_i}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \operatorname{cov}(e_i, e_j)}} \\ \text{s.t.} \quad & \sum_{i=1}^{n} w_i = 1, \\ & \sum_{i=1}^{n} z_i = m, \\ & \varepsilon_i z_i \leq w_i \leq \delta_i z_i,\ i = 1, 2, \ldots, n, \\ & z_i \in \{0, 1\},\ i = 1, 2, \ldots, n, \\ & w_i \geq 0,\ i = 1, 2, \ldots, n. \end{aligned} \tag{10}$$

3 Multi-objective firefly algorithm

3.1 The basic FA

The firefly algorithm (FA), which was inspired by the social and flashing activity of fireflies, was proposed by Yang (2008). The FA follows the three rules:

1. Fireflies are attractive to each other regardless of the sex.
2. Attractiveness is based on brightness. So a less bright firefly moves toward a brighter firefly. The attractiveness and brightness are inversely proportional to distance.
3. The landscape of the objective function value is the brightness of fireflies.

Let w_i be the ith firefly in the population, where $i = 1, 2, \ldots, SN$, and SN is the population size. The attractiveness between two fireflies w_i and w_j can be calculated as follows:

$$\beta(r_{ij}) = \beta_0 e^{-\gamma r_{ij}^2},$$
$$r_{ij} = |w_i - w_j| = \sqrt{\sum_{k=1}^{D}(w_{i,k} - w_{j,k})^2}, \tag{11}$$

where D is the dimension of the problem, r_{ij} is distance between w_i and w_j, and $w_{i,k}$ and $w_{j,k}$ are the kth component element of w_i and w_j, respectively. Further, the parameter β_0 denotes the attractiveness at the distance $r = 0$, and γ is the light absorption coefficient. By the suggestions of Yang (2008), γ is set to $1/\Gamma^2$, where Γ is the length scale for designed variables.

In the FA, the firefly with less brightness is attracted to the firefly with more brightness. The movement equation of firefly i moves to firefly j can be stated as:

$$w_i^{t+1} = w_i^t + \beta_0 e^{-\gamma r_{ij}^2}(w_j^t - w_i^t) + \alpha_t \in_i^t, \tag{12}$$

where α_t is randomization parameter, and $\in_i^t$ is a vector of random numbers from uniform distribution. Equation (12) consists of three terms. The first term is the current position of a firefly. The second term is the form of attractiveness function which is a monotonically decreasing function. The third term is the randomization.

3.2 The proposed MOFA

3.2.1 Initialization

At the initialization step, following Bacanin and Tuba (2014), FA generates SN random populations using

$$w_{i,j} = \varepsilon_j + \mathrm{rand}(0, 1)(\delta_j - \varepsilon_j), \tag{13}$$

where rand(0, 1) is a random number uniformly distributed in [0, 1].

3.2.2 Constraint handling

(1) Boundary constraint. If the initially generated value for the jth parameter of the ith firefly does not fit in the scope $[\varepsilon_j, \delta_j]$, it is being modified:

$$\begin{aligned} &\text{if} \quad w_{i,j} > \delta_j, \quad \text{then} \quad w_{i,j} = \delta_j, \\ &\text{if} \quad w_{i,j} < \varepsilon_j, \quad \text{then} \quad w_{i,j} = \varepsilon_j. \end{aligned} \tag{14}$$

(2) Cardinality constraint. Decision variables $z_{i,j}$ $(i = 1, 2, \ldots, SN,\ j = 1, 2, \ldots, n)$ are generated randomly by applying

$$z_{i,j} = \begin{cases} 1, & \text{if } \phi < 0.5, \\ 0, & \text{if } \phi \geq 0.5, \end{cases} \tag{15}$$

where ϕ is random real number between 0 and 1.

(3) Budget constraint. For the constraint $\sum_{i=1}^{n} w_i = 1$, we set $\psi = \sum_{i=1}^{n} w_{i,j}$ and put $w_{i,j} = w_{i,j}/\psi$ for all assets that satisfy $j = 1, 2, \ldots, n$. The same approach for satisfying this constraint was used in Cura (2009).

3.2.3 Firefly movement

(1) For a dominated firefly i, the movement of the firefly toward firefly j that dominates itself is calculated as in the original FA implementation (Yang 2008):

$$w_i^{t+1} = w_i^t + \beta_0 e^{-\gamma r_{ij}^2}(w_j^t - w_i^t) + \alpha_t \in_i^t. \tag{16}$$

The position of each individual can be updated sequentially, by computing the fitness of each particle and updating them during every iteration of the cycle.

(2) For a non-dominated firefly, each value of objectives is defined a weight vector to calculate the integrated best solution g_*^t. The g_*^t minimizes a combined objective via the weighted sum

$$\psi(\omega) = \sum_{k=1}^{3} \omega_k f_k, \quad \sum_{k=1}^{3} \omega_k = 1, \tag{17}$$

where ω_k is a random number uniformly distributed between 0 and 1. f_k is the kth objective. To ensure the sum of ω_k equal to 1, the weight is normalized that $\omega_k' = \omega_k / \sum_{k=1}^{3} \omega_k$. To maintain a diverse set of non-dominated solutions along the Pareto front, for each iteration, ω_k should be regenerated randomly.

Algorithm 1 The multi-objective firefly algorithm (MOFA).

```
Set objective functions f_k where k = (1, 2, 3)^T
Randomly generate SN fireflies w_i (i = 1, 2, ..., SN)
while (t < t_max)
    for i, j = 1 → n do
        if i ≠ j and the constraints criterion fulfilled then
            Evaluate P_i (Pareto front) and P_j
            if P_j dominates P_i then
                Update firefly w_i → w_j by Eq. (16)
                Generate a new firefly if the constraints criterion not fulfilled
            end if
            if non-dominated solution fulfilled then
                Define random weights vector ω_k (k = 1, 2, 3)
                Select g_*^t, the minimum of ψ by Eq. (17)
                Update non-dominated solution by Eq. (18)
            end if
    end for
    Rank and find the Pareto solutions
    Set t → t + 1
end while
```

Table 2 Inputs and outputs

Type	Parameter	Perspective
Inputs	Receivable turnover	Asset utilization
	Inventory turnover	
	Asset turnover	
	Current ratio	Liquidity
	Quick ratio	
	Debt to equity ratio	
	Leverage ratio	Leverage
	Solvency ratio-I	
	Solvency ratio-II	
Outputs	Return on equity	Profitability
	Return on assets	
	Net profit margin	
	Earnings per share (EPS)	
	Revenue growth rate	Growth
	Net income growth rate	
	Earnings per share growth rate	

Then, the firefly moves by

$$w_i^{t+1} = g_*^t + \alpha_t \in_i^t, \tag{18}$$

Table 1 Trapezoidal fuzzy returns of 52 stocks

No.	a_i	b_i	α_i	β_i	No.	a_i	b_i	α_i	β_i
1	0.127	0.159	0.208	0.432	2	– 0.012	0.036	0.118	0.177
3	– 0.126	– 0.036	0.620	0.229	4	– 0.030	0.009	0.377	0.250
5	– 0.002	0.035	0.157	0.207	6	0.038	0.052	0.159	0.084
7	0.071	0.135	0.162	0.291	8	0.003	0.022	0.164	0.257
9	0	0.039	0.344	0.322	10	– 0.014	0.041	0.368	0.267
11	0.014	0.026	0.142	0.161	12	– 0.050	– 0.003	0.618	0.136
13	0.022	0.055	0.233	0.079	14	0.006	0.069	0.159	0.354
15	0.068	0.106	0.181	0.400	16	– 0.012	0.058	0.112	0.423
17	0.067	0.111	0.133	0.226	18	0.051	0.082	0.208	0.179
19	0.072	0.082	0.138	0.393	20	0.009	0.056	0.213	0.325
21	– 0.014	0.072	0.247	0.268	22	0.056	0.068	0.110	0.143
23	0.043	0.074	0.196	0.322	24	0.025	0.030	0.162	0.405
25	– 0.002	0.029	0.256	0.089	26	– 0.005	0.005	0.080	0.024
27	0.059	0.096	0.128	0.220	28	– 0.136	0.093	0.540	1.671
29	0.010	0.088	0.208	0.374	30	0.031	0.047	0.175	0.171
31	0.149	0.188	0.371	0.463	32	0.089	0.102	0.309	0.441
33	0.042	0.099	0.176	0.629	34	0.014	0.020	0.317	0.266
35	0.094	0.188	0.094	0.938	36	0.011	0.080	0.267	0.324
37	0.043	0.090	0.144	0.113	38	0.011	0.078	0.155	0.448
39	0.190	0.212	0.445	0.617	40	0.191	0.206	0.382	0.720
41	0.040	0.047	0.276	0.454	42	0.002	0.089	0.368	0.639
43	– 0.025	0.049	0.102	0.223	44	– 0.104	0.063	0.598	0.339
45	0.104	0.150	0.154	0.276	46	0.061	0.093	0.305	0.277
47	0.101	0.109	0.298	0.258	48	0.128	0.168	0.283	0.467
49	0.073	0.107	0.224	0.465	50	0.059	0.117	0.059	0.585
51	– 0.007	0.028	0.143	0.354	52	0.040	0.123	0.234	0.921

where g_*^t is currently the best position achieved by the given set of ω_k. And we use

$$\alpha_t = \alpha_0 0.9^t, \tag{19}$$

where α_0 is the initial randomness factor.

Finally, the implementation procedure of MOFA is described as Algorithm 1.

4 Numerical experiments

We consider an example introduced by Mashayekhi and Omrani (2016). In this example, the data source is taken from 52 firms of the stock exchange market in Iran. The trapezoidal fuzzy return of 52 securities are shown in Table 1. In addition, the data required for inputs and outputs of DEA are obtained from the latest financial statements which are published by the firms (period 21 March 2013 to 21 December 2013). As in Mashayekhi and Omrani (2016), 16 financial input/output parameters are employed, which are presented in Table 2. Solving the model (9), the cross-efficiency scores of firms are presented in Table 3.

Table 3 Cross-efficiency score of firms

No.	e_i	No.	e_i	No.	e_i	No.	e_i
1	− 1.278	14	− 10.055	27	− 1.547	40	− 0.493
2	− 5.325	15	− 1.059	28	− 4.289	41	− 171.857
3	− 1.901	16	− 0.887	29	− 0.442	42	− 2.537
4	− 0.893	17	− 1.077	30	− 1.097	43	− 15.187
5	− 0.0785	18	− 0.184	31	− 1.568	44	− 0.427
6	− 48.893	19	− 0.367	32	− 0.054	45	− 0.888
7	− 0.250	20	− 0.744	33	− 0.755	46	− 0.296
8	− 0.237	21	− 0.246	34	− 2.389	47	− 0.045
9	− 0.348	22	− 0.441	35	− 0.431	48	− 1.019
10	− 0.529	23	− 11.388	36	− 0.605	49	− 0.072
11	− 0.426	24	− 7.558	37	− 0.232	50	− 1.995
12	− 0.316	25	− 15.631	38	− 0.467	51	− 0.533
13	− 1.215	26	− 0.585	39	− 0.993	52	− 0.989

4.1 Algorithm experiment

The parameters of the MOFA are set as follows: the max generation is set to 100, $SN = 50$, $\alpha_0 = 0.5$, $\beta_0 = 0.2$,

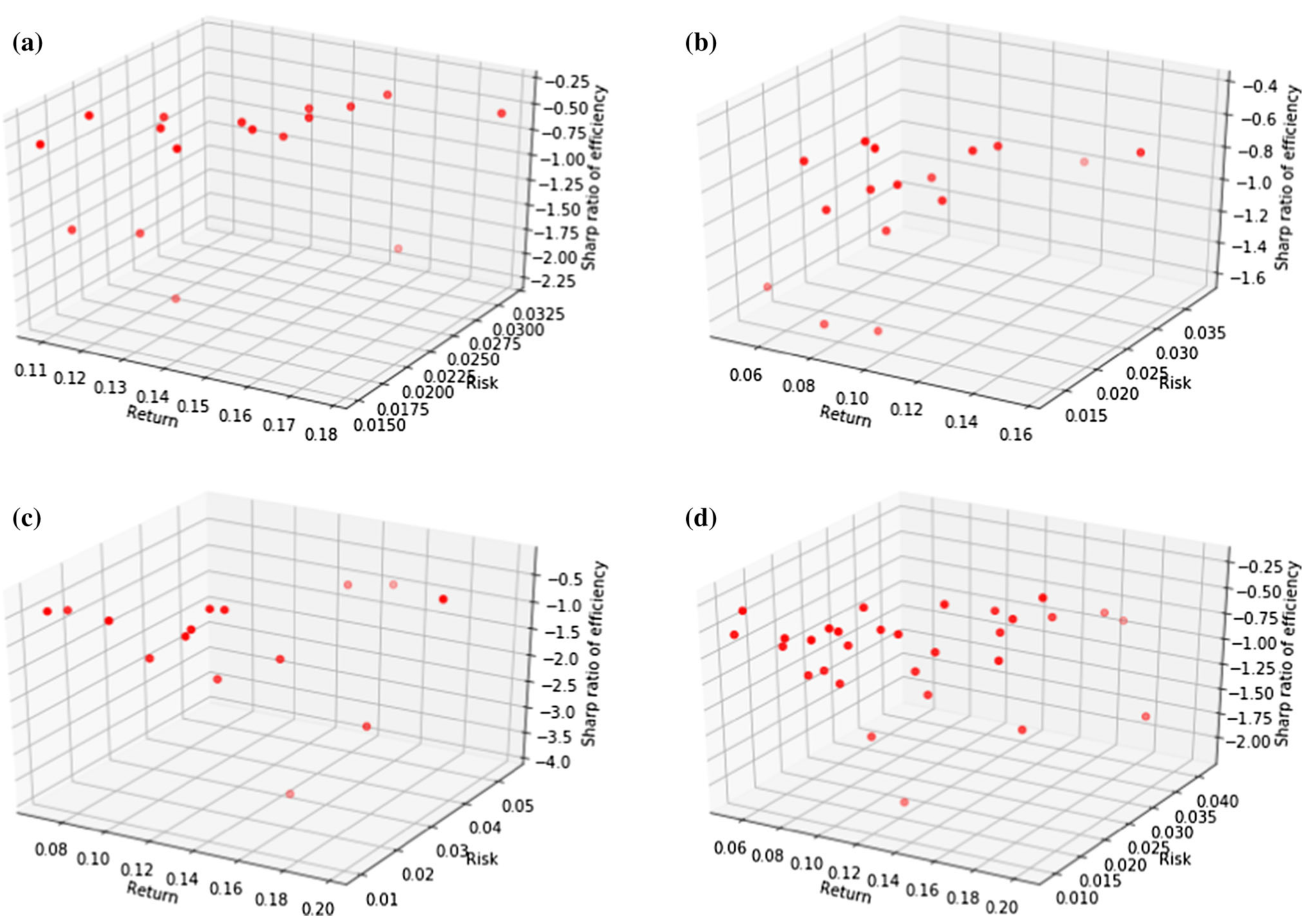

Fig. 1 Approximate efficient frontier in the case of $m = 8$. **a** GA, **b** PSO, **c** FA, **d** MOFA

Table 4 Performance comparisons of different algorithms when $m = 8$

		GA	PSO	FA	MOFA
Return	Max	0.1974	0.1717	0.1979	**0.2018**
	Min	**0.0755**	0.0274	0.0282	0.0415
	Mean	**0.1472**	0.1073	0.1125	0.1199
	SD	**0.0237**	0.0272	0.0334	0.0347
Risk	Max	0.0494	0.0497	0.0556	**0.0483**
	Min	0.0130	0.0078	0.00825	**0.0069**
	Mean	0.0243	0.0201	0.0202	**0.0186**
	SD	**0.0051**	0.0077	0.0077	0.0073
Sharpe ratio	Max	− 0.2463	− 0.2987	− 0.2376	− **0.2368**
	Min	− 3.5110	−4.0675	− 3.8631	− **3.4850**
	Mean	− 1.1357	− 0.9374	− **0.8898**	− 0.9326
	SD	0.6891	**0.5926**	0.6425	0.6633

Table 5 Some Pareto optimal solutions when $m = 8$

Portfolio	Return	Risk	Sharpe ratio
1	0.1789	0.0299	− 0.2965
2	0.1507	0.0245	− 0.3438
3	0.1439	0.0220	− 0.3704
4	0.1056	0.0140	− 0.2542
5	0.0956	0.0187	− 0.3183
6	0.0853	0.0111	− 0.3547
7	0.1145	0.0152	− 0.4208
8	0.1269	0.0184	− 0.3183
9	0.1649	0.0280	− 0.5427
10	0.1321	0.0169	− 0.5471

$\gamma = 1$. Moreover, ε_i and δ_i are set to 0.05 and 0.2, $i = 1, 2, \ldots, n$. Other values of control parameters employed for genetic algorithm (GA), particle swarm optimization (PSO) and basic FA are presented below.

GA settings: The crossover probability p_c and the mutation probability p_m are set to 0.9 and 0.1, respectively. The selection method is roulette wheel and the crossover method is one-point crossover.

PSO settings: The inertia weight factor ω is 0.8, the learning factors, c_1 and c_2 are both set to 1.5.

Basic FA settings: The values of parameters are the same as those of MOFA.

In addition, a total of 20 runs for each experimental setting are conducted.

Given the cardinality $m = 8$, the performance indicator parameters such as maximum, minimum, mean, and standard deviation (SD) of the three objectives using different algorithms are tabulated in Table 4. The best results are marked in bold. From Table 4, it can be easily observed that, in most cases, the minimum, maximum, and mean results obtained

Table 6 Some Pareto optimal solutions when $m = 10$

Portfolio	Return	Risk	Sharpe ratio
1	0.1775	0.0429	− 0.9279
2	0.1258	0.0181	− 0.5832
3	0.1453	0.0249	− 0.3096
4	0.1590	0.0239	− 0.5904
5	0.1089	0.0155	− 0.3332
6	0.1148	0.0161	− 0.3021
7	0.1346	0.0241	− 0.3039
8	0.0975	0.0153	− 0.2614
9	0.1210	0.0167	− 0.2637
10	0.0889	0.0140	− 0.2442

Table 7 Some Pareto optimal solutions when $m = 12$

Portfolio	Return	Risk	Sharpe ratio
1	0.1720	0.0362	− 0.5842
2	0.1127	0.0147	− 0.3004
3	0.1423	0.0190	− 0.4463
4	0.1236	0.0175	− 0.2785
5	0.1368	0.0184	− 0.2758
6	0.0846	0.0095	− 0.3411
7	0.1715	0.0358	− 0.2522
8	0.0912	0.0118	− 0.5726
9	0.1575	0.0331	− 0.3102
10	0.1047	0.0130	− 0.2602

by the MOFA are better than those listed for the other algorithms. That is, the proposed MOFA is more accurate solution than some of the other standard heuristic algorithms. Moreover, we find that the values of SD obtained by the MOFA is higher than those obtained by GA and PSO, indicating that the MOFA leads to the diversity of solution.

The approximate efficient frontiers produced at random by GA, PSO, FA and MOFA in the case of $m = 8$ are shown in Fig. 1. It is obvious that among the four algorithms, the distribution of the MOFA is the best, while that of the other three algorithms are more concentrates. Additionally, we can see that, in most cases, the solutions by MOFA have large return, small risk and better Sharpe ratio of efficiency.

4.2 Model experiment

For the proposed model (10), given $m = 8$, 10, 12 and 15 respectively, some Pareto solutions are presented in Tables 5, 6, 7 and 8. First, we present the diversity of portfolio regarding mentioned criteria. For example, in Table 5, there are portfolios with return ranging from 0.0853 to 0.1789, and the Sharpe ratio of efficiency is between −0.2542 and

Table 8 Some Pareto optimal solutions when $m = 15$

Portfolio	Return	Risk	Sharpe ratio
1	0.1552	0.0244	−0.4620
2	0.1176	0.0136	−0.2163
3	0.1214	0.0207	−0.2505
4	0.1303	0.0176	−0.3844
5	0.1477	0.0248	−0.2051
6	0.1042	0.0138	−0.3169
7	0.1530	0.0199	−0.7039
8	0.0993	0.0123	−0.2518
9	0.1280	0.0196	−0.7546
10	0.1093	0.0172	−0.4934

−0.5471 among the solutions of the proposed model. Moreover, decision makers can weigh their preferences between mentioned criteria in choosing portfolios from Pareto solutions which are calculated from the model (10). For instance, in Table 7, there are portfolios which have return (0.1720 → 0.1715), Sharpe ratio of efficiency (−0.5842 → −0.2522) and nearly risk (0.0362 → 0.0358). If the decision maker are more effect-oriented, he/she can choose the seventh portfolio, whereas if he/she wants higher returns, he/she can choose the first portfolio. Similarly, the fourth portfolio and the fifth one have nearly Sharpe ratio of efficiency (−0.2785 → −0.2758) with different return (0.1236 → 0.1368) and risk (0.0175 → 0.0184). Then, risk avoiders can choose the former portfolio, while risk suitors can choose the latter.

Moreover, approximate efficient frontiers are shown in Fig. 2 in the case of m = 8, 10, 12 and 15. From Fig. 2, it also can be found that the portfolios are well-diversified and the investor can choose the satisfying portfolio based on the preferences between three investment objectives.

Finally, in order to illustrate the effectiveness of the proposed model (10), given $m = 6$, under $\delta = 0.2$, 0.4 and 0.6, respectively, we compare the results with those obtained by the possibilistic mean-semivariance portfolio model, i.e., model (7). Three objective function values, i.e., return, risk and SR, are given in Table 9. From Table 9, it can be easily observed that, in most cases, the proposed model (10) increases the portfolio efficiency at nearly identical returns. In addition, we can find that the Sharpe ratio of efficiency

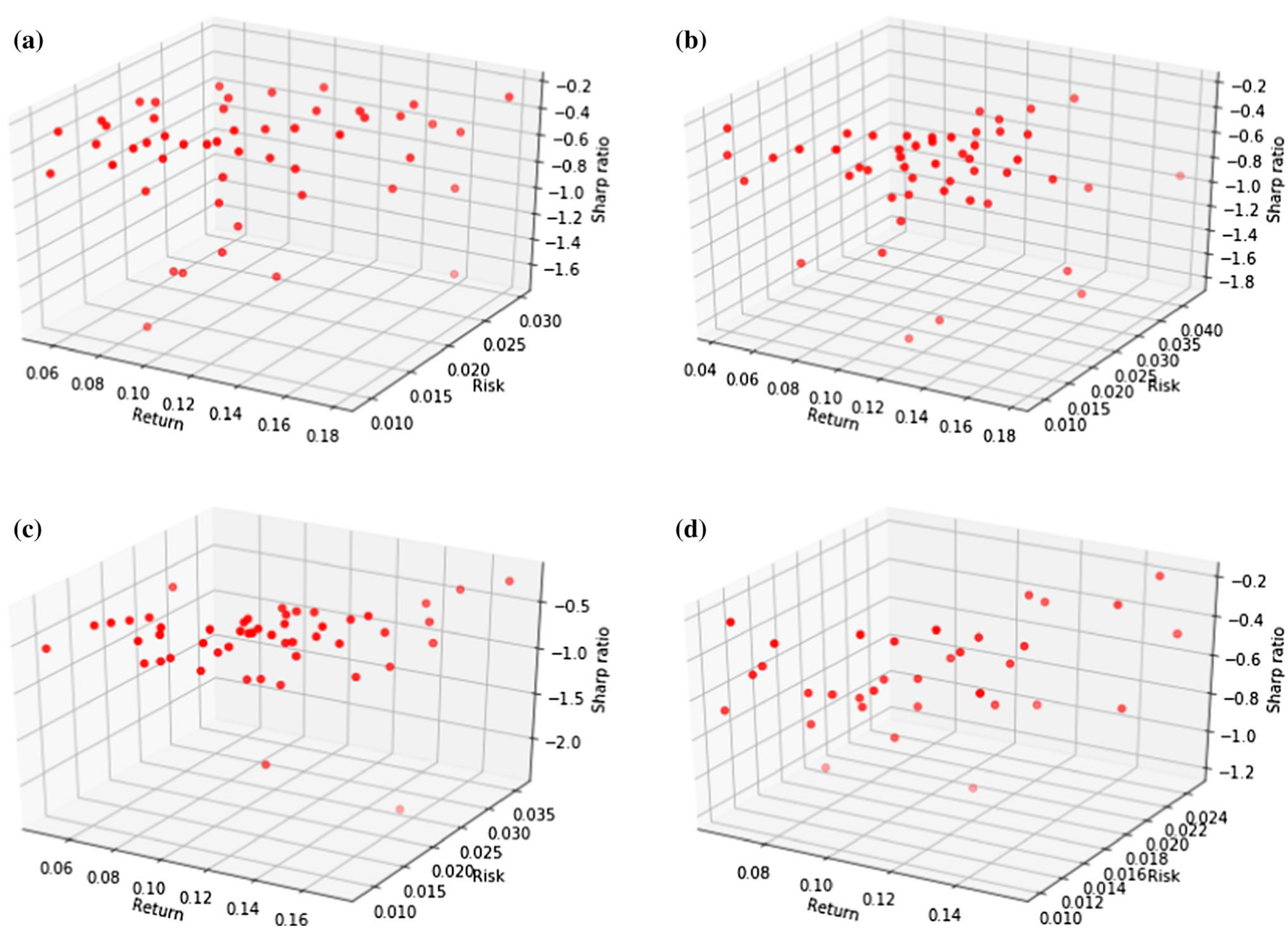

Fig. 2 Approximate efficient frontiers in the case of $m = 8$, 10, 12 and 15. **a** $m = 8$, **b** $m = 10$, **c** $m = 12$, **d** $m = 15$

Table 9 Some Pareto solutions obtained in the case of $m = 6$

	Portfolio	Model (7)			Model (10)		
		Return	Risk	SR	Return	Risk	SR
$\delta = 0.2$	1	0.2085	0.0327	− 2.2679	0.1981	0.0336	− 0.4364
	2	0.0971	0.0112	− 0.7136	0.0901	0.0144	− 0.2906
	3	0.1795	0.0265	− 0.8446	0.1778	0.0288	− 0.5413
	4	0.1654	0.0196	− 1.8538	0.1672	0.0293	− 0.3761
	5	0.1368	0.0161	− 2.6160	0.1349	0.0178	− 0.3975
	6	0.1162	0.0120	− 1.4702	0.1159	0.0143	− 0.6920
	7	0.1230	0.0154	− 1.6770	0.1207	0.0221	− 0.3428
	8	0.1512	0.0194	− 1.8946	0.1551	0.0191	− 0.4412
	9	0.0788	0.0079	− 0.8161	0.0778	0.0112	− 0.7092
	10	0.1716	0.0223	− 1.6799	0.1738	0.0390	− 0.3692
$\delta = 0.4$	1	0.2322	0.0399	− 1.8290	0.2289	0.0412	− 0.5840
	2	0.0933	0.0075	− 0.5237	0.0967	0.0149	− 0.4403
	3	0.1158	0.0095	− 1.9191	0.1157	0.0129	− 0.5890
	4	0.1228	0.0103	− 2.9177	0.1201	0.0176	− 0.7979
	5	0.1077	0.0094	− 2.1036	0.1007	0.0128	− 0.5541
	6	0.1772	0.0200	− 2.8817	0.1721	0.0287	− 0.2574
	7	0.2059	0.0285	− 2.0491	0.1966	0.0363	− 0.2573
	8	0.1569	0.0151	− 2.3290	0.1515	0.0230	− 0.4200
	9	0.0798	0.0061	− 2.9544	0.0863	0.0113	− 0.4895
	10	0.1794	0.0237	− 2.0332	0.1813	0.0390	− 0.3149
$\delta = 0.6$	1	0.2387	0.0390	− 3.0334	0.2209	0.0388	− 0.5637
	2	0.1655	0.0169	− 2.2683	0.1628	0.0279	− 0.8501
	3	0.0931	0.0074	− 1.9094	0.1065	0.0111	− 0.7641
	4	0.1870	0.0223	− 2.2626	0.1949	0.0286	− 0.2512
	5	0.2053	0.0269	− 2.7307	0.2003	0.0335	− 0.7004
	6	0.1495	0.0154	− 1.5205	0.1421	0.0216	− 0.9284
	7	0.1066	0.0083	− 0.8793	0.1080	0.0132	− 0.7460
	8	0.1597	0.0168	− 3.6335	0.1531	0.0232	− 0.4714
	9	0.1730	0.0200	− 2.6254	0.1756	0.0336	− 0.4550
	10	0.1341	0.0115	− 2.0666	0.1296	0.0143	− 0.3172

obtained by the proposed model is better than those obtained by the possibilistic mean-semivariance portfolio model.

5 Conclusion

This paper presented a comprehensive model for fuzzy multi-objective portfolio selection model based on mean-semivariance and DEA cross-efficiency models. Inspired by the ideas of Sharpe ratio, a novel cross-efficiency model was presented. Furthermore, we formulated a comprehensive model simultaneously considered return, risk, the efficiency of the portfolio, bounds on holdings, and cardinality. Moreover, the multi-objective firefly algorithm (MOFA) was developed to solve the proposed model. In order to illustrate the proposed approach, a case study involving 52 firms were considered. The numerical results showed that there are good diversity of objectives between Pareto solutions of the proposed model for investors to trade-off.

For future research, some other variant objectives or realistic constraints can be added to the proposed model (e.g., skewness, kurtosis, and liquidity). In addition, some widely adopted metrics, such as generation distance (GD), spacing (S), diversity metric (△), can be used to evaluate the performance of the MOFA.

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Compliance with ethical standards

Conflict of interest The authors declare no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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