Soft Computing (2018) 22:7445–7461
https://doi.org/10.1007/s00500-018-3346-z

FOCUS

A scientific decision-making framework for supplier outsourcing using hesitant fuzzy information

R. Krishankumar[1] · K. S. Ravichandran[1] · K. K. Murthy[1] · A. B. Saeid[2]

Published online: 28 June 2018

Abstract
Supply chain management (SCM) is an attractive area for research which has seen tremendous growth in the past decades. From the literature we observe that, supplier outsourcing (SO) is a highly explored research field in SCM which lacks significant scientific contribution. The major concern in SO is the decision makers' (DMs) viewpoint which are often vague and imprecise. To better handle such imprecision, in this paper, we propose a new two-stage decision-making framework called TSDMF, which uses hesitant fuzzy information as input. In the first stage, the DMs' preferences are aggregated using a newly proposed simple hesitant fuzzy-weighted geometry operator, which uses hesitant fuzzy weights for better understanding the importance of each DM. Following this, in the second stage, criteria weights are estimated using newly proposed hesitant fuzzy statistical variance method and finally, a new ranking method called three-way hesitant fuzzy VIKOR (TWHFV) is proposed by extending the VIKOR ranking method to hesitant fuzzy environment. This ranking method uses three categories viz., cost, benefit and neutral along with Euclid distance for its formulation. The practicality of the proposed TSDMF is verified by demonstrating a supplier outsourcing example in an automobile factory. The robustness of TWHFV is realized by using sensitivity analysis and other strengths of TSDMF are discussed by comparison with another framework.

Keywords Supplier outsourcing · Hesitant fuzzy · VIKOR · Standard variance · Aggregation · Decision making

1 Introduction

The success of an organization is directly governed by their stakeholders (Fülöp 2001; O'Haire et al. 2011). Suppliers are one such important stakeholder, who determines the key success in an organization (Zhaoxia et al. 2013). In the process of SCM, supplier outsourcing plays an inevitable role and it is given the highest priority in the chain (Choi et al. 2007). The proper assessment and evaluation of suppliers bring enormous gain to the organization. They promote customer satisfaction, improve economic status, mitigate risk and boost the relationship between clients and service providers (Yücel and Güneri 2011). Such claims motivated researchers in the past decade to propose a huge set of methodologies for SO. This becomes evident from the discussion made in Sect. 2. (Öztayşi and Sürer 2014; Xu and Liao 2015) made a clear argument that evaluation of suppliers based on their preference rating involved a great deal of imprecision and vagueness due to the intervention of humans in the decision-making process. They claimed that human aspect and cognition involved implicit vagueness and ambiguity which needs proper treatment for better decisions.

To address such vagueness better, scholars adopted different variants of fuzzy concepts. One such variant is the hesitant fuzzy set (HFS), proposed by Torra and Narukawa (2009). This is by far the best variant of fuzzy that can model human cognition and thought process better. The claim is evident from the nature of HFS, which gets n different preferences for the same instance at different time intervals. These n instances denote n different choices made by the DM for a single case and this eventually helps DMs to represent fuzziness better. This claim by Torra and Narukawa (2009), inspired many scholars to work with HFS concepts in decision making. Some prominent contributions of HFS-based decision making are (Liao et al. 2014b; Liao and Xu 2014; Xia and

Communicated by A. Genovese, G. Bruno.

✉ K. S. Ravichandran
ravichandran20962@gmail.com

[1] ICT, School of Computing, SASTRA University, Thanjavur, Tamil Nadu 613401, India

[2] Department of Pure Mathematics, Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Kerman, Iran

Xu 2011; Xu and Xia 2011a; Yu et al. 2016a; Zhu et al. 2016). Thus, motivated by the power of HFS, in this paper, we adopt hesitant fuzzy information as input for supplier evaluation.

In the field of decision making, group decision making (GDM) has always experienced great appetite from research communities, as they closely resemble the real case decision-making scenarios and involves a good amount of fuzziness to be addressed. Researchers often aggregate different DMs viewpoint into a single entity and then apply decision methods for evaluation. This eases the process of decision making and builds confidence among the group by giving equal chance to everyone for rating alternatives and by considering all their preferences equally for evaluation. Some of the popular aggregation operators in the hesitant fuzzy environment are discussed in (Thakur et al. 2014; Xia and Xu 2011; Zhou and Li 2014). From the analysis, we infer that all these aggregation operators lacked the ability to properly reflect the hesitancy involved in providing apt relative importance values for each DM and/or had complex formulation procedures. To circumvent the issue, in this paper, we adopted SHFWG operator which is an extended version of hesitant fuzzy-weighted geometry (HFWG) (Xia and Xu 2011) operator with hesitant fuzzy weight values for DMs. By considering DMs' relative importance (weights) in the form of hesitant fuzzy information, the major issue of proper representation and realization of DMs' relative importance is mitigated and a better understanding of each DM is obtained. Motivated by this challenge, we make such proposal.

Followed by the concept of aggregation, the research community has also paid significant attention toward criteria weight estimation. Many scholars have adopted either manual entry process or weight estimation through entropy methods (Jin et al. 2014; Wood 2016; Xia and Xu 2012; Zhao et al. 2016). (Liu et al. 2016) rightly pointed out that, such methods often lead to imprecision and unrealistic weight values. To alleviate the issue, (Liu et al. 2016) came up with an idea of using statistical variance (SV) for weight estimation. They claimed that, SV method was simple and had the property of considering all the points in its estimation which prevents loss of information and yields sensible weight values. This helped the SV method to produce realistic and sensible weight values. Thus, motivated by the ease and power of SV method, we proposed a new weight estimation method called HFSV method, which is an extension of SV method under hesitant fuzzy environment.

Finally, in this proposal, we set our hands on the ranking method. Many scholars have widely exploited this field by proposing different ranking methods under different fuzzy environments. Motivated by the power of compromise solution in yielding a near optimal result and from the deep survey conducted by Mardani et al. (2016), we identified VIKOR ranking as a potential candidate for the process. This method has the property of categorizing criteria into benefit and cost zones, which is lacking in other ranking methods. Though TOPSIS method is a close counterpart to VIKOR, we chose VIKOR for the process as it had optimistic style of ranking and supplier outsourcing problem closely resembled to this style of ranking. Also, VIKOR method chose optimal points based on their closeness to positive ideal solution. For further investigation on these two compromise solution methods, readers are encouraged to refer (Opricovic and Tzeng 2004). Motivated by the power of HFS in representing fuzziness and vagueness, we extend the VIKOR ranking method under hesitant fuzzy environment by proposing new formulation for estimation function with three categories viz., cost, benefit and neutral under the realm of Euclid distance. Though, (Liao and Xu 2013) have already proposed HF-VIKOR and adopted Manhattan distance for its formulation, they have not considered three categories for estimation in their formulation. In this proposal, we adopt cost, benefit and neutral categories for evaluation of estimation functions under the realm of Euclid distance. The main advantage of using these three categories for evaluation is that, it gives the DMs a clear understanding on the nature of each criterion and enables rational and better judgments. To clarify the scenario, consider an example of car purchase, criteria like, fuel consumption, cost must be low and so they are placed in cost category. Similarly, criteria like speed, mileage, safety must be high and hence, these are placed in benefit zone. Criteria like color and style are considered to be benefit in normal cases. But, paying close attention to these criteria will give us an intuition that, these are neither benefit nor cost and hence, there is an urge need for a new zone for evaluation. This challenge motivated us to propose a new three-way hesitant fuzzy VIKOR (TWHFV) under the realm of Euclid distance. The main motivation for choosing Euclid distance is that, scholars (Charulatha et al. 2013; Sinwar and Kaushik 2014) argue that, Euclid distance has a better rate of convergence and hence are suitable for decision-making problems involving imprecision and vagueness. Also, Euclid measure has a better chance for handling vagueness as they consider varying (unequal or biased) deviations with different values for weights. Hence, TWHFV method is proposed for outsourcing suitable supplier for the task.

Thus, to follow the context, the rest of the paper is organized as, related works in Sect. 2, followed by, basic knowledge in Sect. 3, proposed TSDMF framework in Sect. 4, illustrative example in Sect. 5, comparative analysis in Sect. 6 and conclusion in Sect. 7.

2 Related works

In this section, we conduct a survey to gain inference and to identify research lacuna. The procedure applied for surveying is a two-stage approach, where application and method are both concentrated. Here, the application is 'supplier outsourcing' and method is 'hesitant fuzzy decision-making method'. We set these terms as the keywords and perform a search to identify potential papers for investigation. We identified some potential papers that are closely related to the keywords. Table 1 conducts an investigation on these 15 papers. All the fields of Table 1 are easy to follow. So, let us conduct the review process.

(Chai et al. 2013) conducted a thorough survey on multi-criteria decision-making methods and its application in supplier selection and evaluation. They conducted the survey from 2008 to 2012 and inferred that compromise solution viz., VIKOR and TOPSIS and hierarchical method viz., analytical hierarchy process (AHP) are good candidates for suitable selection of suppliers. (Gul et al. 2016) conducted a thorough investigation on VIKOR method and its extension in fuzzy concepts. From the analysis, we infer that the extension of VIKOR under HFS context is highly attractive and needs good exploration for taking sensible and rational decision. Motivated by this inference, efforts are made in this paper to extend VIKOR method under HFS context. (Simić et al. 2017) conducted a deep investigation with fifty years of articles in the field of fuzzy-based decision models for supplier selection. From the investigation, it can be observed that fuzzy-based decision methods are an effective tool for decision making and the complex and uncertain application of supplier selection is very well handled by fuzzy methods. Also, (Liu and Liao 2017) performed a detailed investigation on various fuzzy-based decision methods from 1970 to 2015 and inferred that hesitant-based decision methods are attractive and powerful for handling uncertainty and vagueness in the decision process. Motivated by the investigation made in Table 1 and from these four surveys, concrete research lacunas can be identified.

From Table 1, we make the following inferences:

(1) Supplier selection/outsourcing is an attractive and hot area for research in SCM, which is earning high appetite from the research community.
(2) This problem is better handled using MCDM approaches and viewing of SO problem from the lens of HFS-based MCDM methods has just started and it needs wide exploration.
(3) Aggregation of preferences in such problem becomes a trivial aspect and hence, operators that make rational and logical aggregation are to be proposed.
(4) Scholars have paid only little attention towards the integration of VIKOR ranking method under HFS for solving SO problem. On the other hand, the use of AHP, TOPSIS and QUALIFLEX under HFS for SO has been greatly exploited.
(5) Criteria weight estimation for effective selection of supplier has been done either by expert advice or methodical way. Common methods include entropy measures which often yields unrealistic weights.

From these inferences, following research lacunas are identified:

(1) Since the number of research articles under 'hesitant fuzzy-based supplier selection' is less, there is an urge need for a new decision-making framework under HFS for SO. Thus, we propose a new framework called TSDMF for SO.
(2) Since aggregation is a trivial concept in SO and DMs want to have a sensible aggregation, we propose a new aggregation operator called, SHFWG, which considers DMs' weights (relative importance) not as single term but as hesitant fuzzy values.
(3) Since the criteria weight evaluation is often unrealistic and imprecise, we propose a new method for criteria weight estimation called HFSV, which is an extension to SV under HFS.
(4) Finally, from the inference, we observe that, the use of VIKOR under HFS for SO is only rarely addressed and hence, we set our hands on this lacuna, by proposing a new ranking method called, TWHFV, which is an extension to VIKOR under HFS that formulates estimation function (group utility and individual regret) using three categories viz., cost, benefit and neutral under the realm of Euclid distance.

Thus, the proposal of such decision-making framework for supplier outsourcing will surely benefit the SCM community and the flexibility of the framework will also benefit research communities in making rational and critical decisions with regard to other selection problems.

3 Background knowledge

Definition 1 (Torra and Narukawa 2009) Let A be a fixed set, then the hesitant fuzzy set on A is a function h, which when applied gives a subset of [0, 1]. Mathematically, it is represented as,

$$\bar{A} = \left\{a, h_{\bar{A}}(a) | a \in A\right\} \tag{1}$$

where $h_{\bar{A}}(a)$ is a subset of [0, 1] and it represents the possible membership degree for the element $a \in A$.

Definition 2 (Xia and Xu 2011) Some operational laws on hesitant fuzzy element (HFE) are as follows:Consider three HFEs h, h_1 and h_2 with λ being a positive real number, then,

$$h^{\lambda} = \cup_{\beta \in h}\left\{\beta^{\lambda}\right\} \tag{2}$$

Table 1 Survey on supplier selection

References	Year	Aggregation	Method(s)	Application	Discussion
Xue and Du (2017)	2017	Yes	Hesitant fuzzy preference relation (HFPR), regression model	Supplier selection	A new framework was proposed to test the consistency of HFPR using linear programming concept. A new regression model was also proposed to remove the unreasonable information. Applicability of the framework was tested using supplier selection problem
Li and Wang (2017)	2017	No	Probability hesitant fuzzy set (PHFS), QUALIFLEX	Supplier selection	An extension to Hausdorff distance was proposed for calculating distance between two elements. Using this, a new extension was presented for QUALIFLEX method under PHFS context and the practicality is tested using supplier selection example
Gitinavard et al. (2017)	2017	No	Interval valued hesitant fuzzy set (IVHFS), ELECTRE	Green supplier selection	The decision framework presented a new method for calculating DMs' relative importance using preference selection index and an extension was proposed for ELECTRE method under IVHFS context and the practicality was tested with supplier selection example
Zhang (2017)	2017	No	IVHFS, QUALIFLEX	Green supplier selection	Two new QUALIFLEX algorithms were proposed using likelihood-based comparison for the selection of suitable green supplier
Tang (2017)	2017	Yes	HFS	Green supplier selection	A new extension was proposed for Hamacher power weighted average operator under HFS context for selecting suitable green supplier
Ren et al. (2017)	2017	No	Dual Hesitant Fuzzy set (DHFS), VIKOR	Supplier selection	A new decision framework was proposed by extending VIKOR method under DHFS context. Also a new score function is defined for comparison of DHFS elements. The applicability of the framework is tested using supplier selection example
Liang et al. (2017)	2017	No	DHFS, decision theoretic rough set	Supplier selection	A new decision framework was presented using decision theoretic rough set under DHFS context for three-way decision making by reducing the loss function for effective estimation. The applicability of the proposal was verified by assessing the blood suppliers for medical aid
Dong et al. (2017)	2017	No	Linguistic hesitant fuzzy set (LHFS), VIKOR	Transport service provider	A new concept called LHFS was proposed and a new deviation method was proposed for criteria weight estimation and VIKOR method was extended under LHFS context for selection of suitable transport service provider
Tyagi (2016)	2016	No	Hesitant fuzzy triangular set, TOPSIS	Fertilizer supplier selection	A new decision-making model was proposed for effective selection of supplier from a fertilizer industry. Linguistic terms were used for rating that were converted to HFTS. Finally, TOPSIS method was used for ranking
Yu et al. (2016b)	2016	Yes	Dual hesitant fuzzy set (DHFS)	Supplier selection	Two new aggregation operators viz., DHFS-based Heronian mean and DHFS-based geometric Heronian mean were proposed and some of their properties were investigated. Applicability was tested using supplier selection problem
Hossein Gitinavard et al. (2016)	2016	No	IVHFS, TOPSIS	Supplier selection	A new decision-making model was proposed under IHFS for effective selection of supplier. Criteria and DMs' weights are estimated using deviation approach with the help of Hamming distance. Ranking was done using TOPSIS method
Zhang et al. (2016)	2016	No	HFS, Linear programming technique for multi-dimensional analysis of preferences (LINMAP)	Green supplier selection	A new extension to LINMAP was made under HFS and criteria weights were estimated using deviation method. Efficacy of the proposal was validated using comparison with other methods

Table 1 continued

References	Year	Aggregation	Method(s)	Application	Discussion
Wang and Xu (2016)	2016	Yes	Interval valued hesitant fuzzy set (IVHFS)	Supplier selection	A new framework for decision making was proposed with an aim of reducing the number of aggregation. For this purpose, three new algorithms were proposed and their efficacy were tested through comparison
Zhang (2016)	2016	Yes	IVHFS, QUALIFLEX	Green supplier selection	A new outranking method was proposed under IVHFS for better selection of supplier from the set of suppliers. Two concordance and discordance indices viz., weighted and comprehensive were estimated for the formulation of QUALIFLEX method
Fahmi et al. (2016)	2016	Yes	Hesitant linguistic term set (HFLTS), ELECTRE	Supplier selection	A new decision-making model was proposed which integrates HFLTS and ELECTRE method for better selection of supplier
Darabi and Heydari (2016)	2016	Yes	IVHFS, Utility-based ranking	Green supplier selection	A novel ranking method based on utility scheme was proposed under IHFS environment for effective selection of supplier. Criteria weights were estimated using entropy measures
Taciana and Gussen (2015)	2015	Yes	Hesitant fuzzy AHP	Supplier selection	A new model for supplier selection was proposed which integrates OWA operator and AHP under HFS
Chang (2015)	2015	Yes	Soft set and HFLTS	Supplier selection	A new ranking framework was proposed by integrating both soft sets and HFLTS for effective selection of supplier from a set of suppliers in the liquid crystal industry. Missing values were filled using weighted average approach
Zhang and Xu (2015)	2015	Yes	HFS, QUALIFLEX	Green supplier selection	A new outranking framework called QUALIFLEX was proposed under HFS for effective selection of supplier. The proposed method adopted new hesitancy index for formulating weighted and comprehensive concordance and discordance values
Gitinavard et al. (2015)	2015	Yes	HFS, Balance ranking (BR) method	Supplier selection	A new extension was proposed to BR method under hesitant fuzzy environment for an optimal selection of a supplier
Chai and Ngai (2015)	2015	Yes	IVHFS, Utility-based ranking	Supplier selection	A novel utility-based ranking model was proposed under IVHFS and the suitable supplier was selected for the task
Zhao et al. (2014)	2014	Yes	Hesitant triangular fuzzy set	Supplier selection	A new extension was presented for the Einstein aggregation operator under hesitant triangular fuzzy context and the usefulness of the method is demonstrated using a supplier selection example
Zhang et al. (2014)	2014	Yes	HFS, TOPSIS	Supplier selection	A new ranking framework was proposed under HFS with TOPSIS method and criteria weights were estimated using newly extended Shanon entropy for better supplier selection

$$\lambda h = \cup_{\beta \in h} \left\{1 - (1 - \beta)^{\lambda}\right\} \quad (3)$$

$$h_1 \oplus h_2 = \cup_{\beta_1 \in h_1, \beta_2 \in h_2} \{\beta_1 + \beta_2 - \beta_1 \beta_2\} \quad (4)$$

$$h_1 \otimes h_2 = \cup_{\beta_1 \in h_1, \beta_2 \in h_2} \{\beta_1 \beta_2\} \quad (5)$$

Definition 3 (Xia and Xu 2011) For a HFE h_1, the score function $s(h_1)$ is given by,

$$s(h_1) = \frac{1}{l_{h_1}} \sum_{\beta \in h_1} \beta \quad (6)$$

where l_{h_1} is the length of the HFE h_1.

Definition 4 (Liao et al. 2014) For a HFE h_1, the variance function $v(h_1)$ is given by,

$$v(h_1) = \frac{1}{l_{h_1}} \sqrt{\sum_{\beta_i, \beta_j \in h_1} (\beta_j - \beta_i)^2} \quad (7)$$

where β_i, β_j are the different membership values for the same instance.

Remark 1 To rank two HFEs h_1 and h_2, we apply Eqs. (6, 7) as follows:

- If $s(h_1) < s(h_2)$ then, $h_1 < h_2$
- If $s(h_1) = s(h_2)$ then, we follow,
  - If $v(h_1) < v(h_2)$ then, $h_1 > h_2$
  - If $v(h_1) = v(h_2)$ then, $h_1 = h_2$

Definition 5 (Xu and Xia 2011b) The Euclid distance measure for two HFEs, h_1 and h_2 is given by,

$$d(h_1, h_2) = \frac{1}{l}\sqrt{\sum_{i,j=1}^{l}\left(\beta_i - \beta_j\right)^2} \tag{8}$$

where, $\beta_i \in h_1$, $\beta_j \in h_2$ and l is the length of the HFE. The length of the HFEs must be equal. If they are not equal, then, the procedure suggested by Liao et al. (2014a) is adopted for making them equal.

4 Proposed two-stage decision-making framework (TSDMF)

4.1 Architecture of TSDMF

In this section, Fig. 1 depicts the architecture of the proposed TSDMF. The architecture is a two tier framework used for decision making. We demonstrate the applicability of TSDMF using supplier outsourcing problem. The framework is also flexible enough to handle any type of selection problem. The architecture is self-contained and straightforward depiction of the working process and hence, we confine our discussion on the architecture and elaborate upon each step of the decision-making process in the upcoming sections.

4.2 Proposed SHFWG operator

The aggregation is the process of combining DMs' preferences in a manner such that, every DMs viewpoint is addressed equally. Many scholars have worked on the concept of aggregation of preferences and have given significant insights. Based on the understanding, scholars infer that, aggregation of preferences must preserve originality and must pay equal attention to all DMs' viewpoint.

(Xia and Xu 2011) inspired by this idea, proposed HFWG operator which is an extension to simple intuitionistic fuzzy-weighted geometry (SIFWG) (Liao and Xu 2015) operator under HFS context. The HFWG operator has a unique property of yielding consistent fused decision matrix. They also proved that, this operator had the ability to yield complete consistent and acceptable consistent matrix based on the DMs' initial preference matrices. They also claimed that, other operators like ordered weighted arithmetic, ordered weighted geometry, symmetric weighted geometry, hybrid arithmetic, hybrid geometry etc. lacked this property. Though HFWG operator enjoys such strength, it does suffer from the problem of proper representation of DMs' relative importance values.

Motivated from the work of Xu (2014) and based on the investigation, we observe that, most of the operators in the hesitant fuzzy family use only single value to represent DMs' weight, which causes some problems in the representation of fuzziness and vagueness. The use of single value to determine DMs' relative importance causes fuzziness and imprecision in the decision-making process (Yue 2011). Motivated by this challenge, we make efforts to extend HFWG operator for better representation of DMs' relative importance using hesitant fuzzy values. This idea of associating multiple membership degrees as relative importance (weight) to the DM reflects the hesitancy in the process better. Thus, the definition of SHFWG operator is given below:

Definition 6 The aggregation is a mapping defined by $U^n \rightarrow U$ for aggregating decision matrices. The operator is given by,

$$\text{SHFWG} = \prod_{h=1}^{m} \beta_{ij}^{\lambda_h^*} \tag{9}$$

where λ^* is the normalized relative importance for each DM with $\sum_{h=1}^{m} \lambda_h^* = 1$, m is the total number of DMs and μ is the degree of preference rating.

Here, $\lambda_i = \{x, \mu_\lambda(x)|x \in X\}$. Now find the score for each of λ_i using Definition 3. Now normalize these score values using, $\lambda_i^* = \frac{\lambda_i}{\sum_{i=1}^{m} \lambda_i}$, where m is the total number of DMs.

Remark 2 The length of every instance taken for aggregation must be equal. If they are unequal, follow the procedure given by Xu (2014) to make them equal.

The SHFWG operator satisfies the following properties:

Property 1 (Idempotent) *If the value of a particular instance is equal in all the decision matrices, then, the resultant value in the fused matrix remains the same.*

$$SHFWG(\alpha_1, \alpha_2, \ldots, \alpha_m) = \alpha$$

Property 2 *(Boundedness) The aggregation using SHFWG operator yields values in the range,*

$$\alpha_i^- \leq SHFWG(\alpha_1, \alpha_2, \ldots, \alpha_m) \leq \alpha_i^+$$

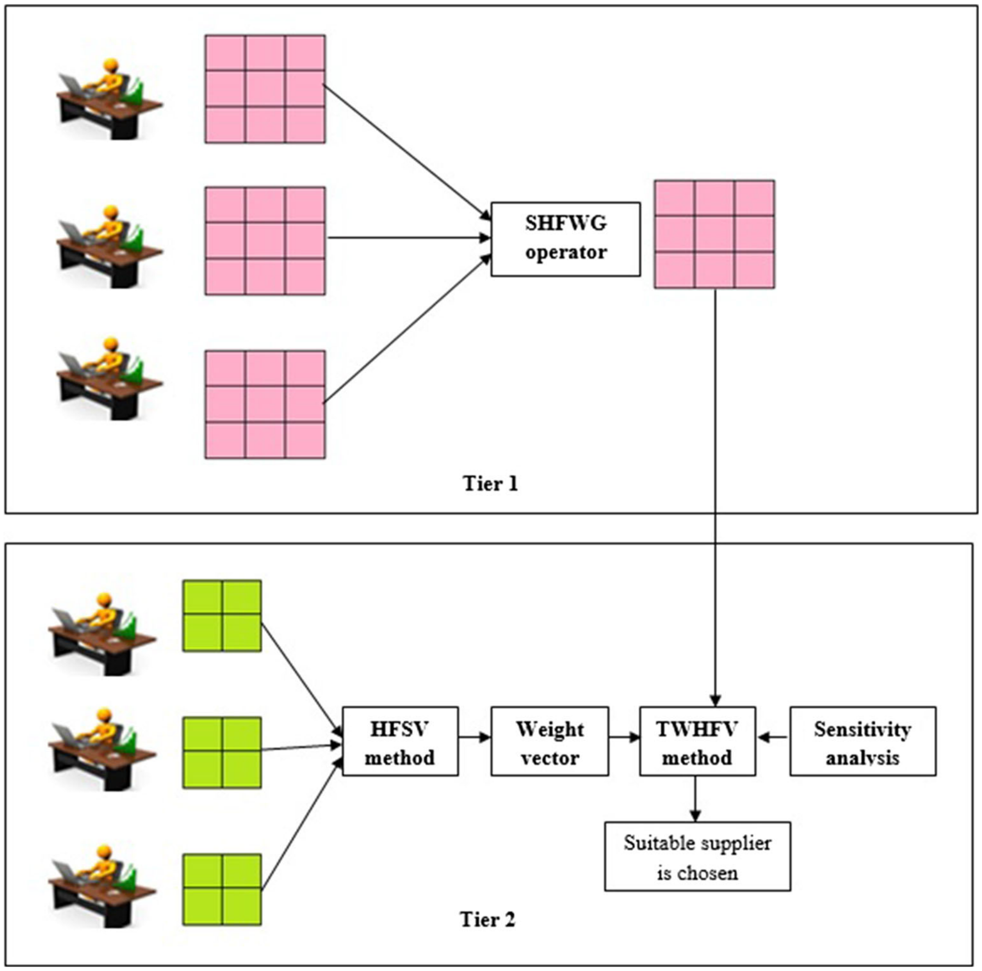

Fig. 1 Proposed TSDMF for supplier selection

where α_i^- is the minimum value of the instance and α_i^+ is the maximum value of the instance.

Property 3 (Monotonic) *If $\alpha^i < \alpha^j$ then,*

$$SHFWG\left(\alpha_1^i, \alpha_2^i, \ldots, \alpha_m^i\right) \leq SHFWG\left(\alpha_1^j, \alpha_2^j, \ldots, \alpha_m^j\right).$$

Theorem 1 *The aggregation of hesitant fuzzy decision matrices using SHFWG operator yields a matrix which is hesitant fuzzy in nature.*

Proof To prove that, the aggregated matrix is also hesitant fuzzy in nature, we have to show that, the instances of the aggregated matrix is in the range [0, 1] and the order of the aggregated matrix is the same as the individual matrices. From the basic definition of aggregation, the second half of the proof is trivial and hence our aim is to show the instance of the aggregated matrix is in the range [0, 1]. To follow the context;

Let us consider, $\mu_{ij}^* = \prod_{\mathrm{h}=1}^{\mathrm{m}} \mu_{\mathrm{ij}}^{\lambda_\mathrm{h}}$ as a particular instance of the aggregated matrix. We know that, $\prod_{h=1}^{m} x_i^{\lambda_h} \leq \sum_{h=1}^{m} \lambda_h x_i \forall x_i,\ i = 1, 2, \ldots, n$. Thus, applying this property, we get, $\prod_{h=1}^{m} \mu_{ij}^{\lambda_h} \leq \sum_{h=1}^{m} \lambda_h \mu_{ij} \forall \mu_{ij} \in [0, 1]$.

Now, we extend this idea to get $0 \leq \prod_{h=1}^{m} \mu_{ij}^{\lambda_h} \leq \sum_{h=1}^{m} \lambda_h \mu_{ij} \leq \sum_{h=1}^{m} \lambda_h = 1 \forall \mu_{ij} \in [0, 1] \text{and} \lambda \in [0, 1]$. Thus, the instance of the aggregated matrix is in the range [0, 1]. □

Example 1 Consider two DMs rating a particular alternative with respect to a particular criterion. The hesitant fuzzy elements $h_{11}^1 = (0.3, 0.4, 0.5)$ and $h_{11}^2 = (0.33, 0.48, 0.62)$. Let $\lambda^* = \left(\lambda_1^*, \lambda_2^*\right) = (0.5, 0.5)$. Calculate the aggregated instance h_{11}^*. $h_{11}^* = \left\{\left(0.3^{0.5} \times 0.33^{0.5}\right), \left(0.4^{0.5} \times 0.48^{0.5}\right), \left(0.5^{0.5} \times 0.62^{0.5}\right)\right\} = \{0.31, 0.44, 0.56\}$. Thus, the idea of Theorem 1 can be realized.

4.3 Proposed HFSV method

The process of assigning weights to criteria is an essential operation in decision making. This weight value infers the relative importance of each criterion during evaluation. The assignment of weight is done using two popular methods. The first method is the manual entry of weight values and the second is the procedural way of estimating weight values. The former method is often difficult as the DMs are not fully aware of each of the criterion and its importance in the evaluation. Thus, the second method can be a solution for the issue. The popular methods for estimating weight values are entropy based (Zhang et al. 2014), optimization based (Chýna et al. 2013), AHP based (Saaty 1980) etc. But, all these methods produce weight values that are often unrealistic and irrational. To circumvent the issue, statisticians developed the idea of variance estimation for weight assignment.

The main advantage of the variance method is that, (1) it is simple and rational; (2), unlike other statistical methods, the variance measure considers every data point for determining the distribution without ignoring any potential information. This preserves the viewpoint of all DMs and keeps the estimation sensible. Motivated by the strength of SV method, we make efforts to extend SV method to HFS context.

The procedure for HFSV method is given below:

Step 1: Construct a criteria rating matrix which involves different DMs rating each criterion using hesitant fuzzy values.

Step 2: Rearrange the matrix in some order and then calculate the variance using Definition 4 for each instance of the matrix obtained from Step 1.

Step 3: Calculate statistical variance between instances obtained from Step 2 using Eq. (10).

$$HFSV_l = \frac{1}{n} \sum\nolimits_{k=1}^{n} (v_{kl} - \bar{v}_{kl})^2 \tag{10}$$

where, $\bar{v}_{kl}$ is the mean value of each criterion (matrix taken from Step 2), l refers to the l^{th} criterion and n is the total number of DMs.

Step 4: Normalize the values obtained by Step 3 to estimate the actual weight values of each criterion. This is mathematically given by Eq. (11).

$$\omega_l^* = \frac{HFSV_l}{\sum_{l=1}^{m} HFSV_l} \tag{11}$$

where m is the total number of criteria and ω_l^* is the normalized weight value.

4.4 Proposed TWHFV method

The VIKOR method is a compromise ranking method which has the ability to categorize criteria as cost and benefit. The base idea for VIKOR is inspired from L_p-metric. The VIKOR method finds optimal points that are close to the positive ideal solution and provides ranking in an optimistic fashion. Since the application of supplier outsourcing in SCM closely resembles to the optimistic style of ranking, we gain motivation and use VIKOR ranking method in our framework. The optimistic style infers that selection is based on the domination of benefit criteria. Thus, in SO problem, suppliers with good service and quality are preferred more. Details on the procedure for VIKOR can be found in (Opricovic and Tzeng 2004).

Inspired by the strength of VIKOR, (Liao and Xu 2013) extended the method for HFS with Manhattan distance. With the view of tackling the weakness of traditional VIKOR ranking method, in this paper, we made rational and effective modification to this method by proposing TWHFV method which uses three categories (benefit, cost and neutral) for evaluation of estimation function viz., group utility (S) and individual regret (R) functions under the realm of Euclid distance rather than Manhattan distance. The motivation for this modification to HF-VIKOR (Liao and Xu 2013) is gained from intuition and statistical theories, which claims Euclid distance to be more effective in handling fuzziness than Manhattan distance (Charulatha et al. 2013; Xu and Xia 2011b). Also, our motivation is strengthened by understanding the concept of L_p-metric, which is the base for VIKOR method. (Liao and Xu 2013) made an argument that, the greater the value of p, the estimation moves from minimizing the sum of regrets to minimizing maximum regret. Thus, they stated that, for $p = 1$, deviations are equally weighted and for $p = 2$, the deviations are unequal and they are weighted based on the magnitude. Manhattan distance is analogous to $p = 1$ condition and hence, cannot offer rational inference. So, we make efforts to move to a distance measure which is analogous to $p = 2$, i.e., Euclid distance. Thus, the Euclid distance can produce sensible and rational inference by handling fuzziness effectively.

Based on these claims, we gained motivation to extend VIKOR method under hesitant fuzzy environment with three categories for evaluation of S and R under the realm of Euclid distance. The procedure for TWHFV is given below:

Step 1: Calculate positive ideal solution (PIS) and negative ideal solution (NIS) using Eqs. (12 and 13).

$$\text{PIS}\left(h^*\right) = \max_{\text{benefit}}\left(h_{ij}\right)\text{or } \min_{\text{cost}}\left(h_{ij}\right)\text{or} \left(\max_{\text{nuetral}}\left(h_{ij}\right) \oplus \min_{\text{neutral}}\left(h_{ij}\right)\right) \tag{12}$$

$$\text{NIS}\left(h^-\right) = \min_{\text{benefit}}\left(h_{ij}\right)\text{or } \max_{\text{cost}}\left(h_{ij}\right)\text{or} \left(\max_{\text{nuetral}}\left(h_{ij}\right) \otimes \min_{\text{neutral}}\left(h_{ij}\right)\right) \tag{13}$$

where $\oplus$ and $\otimes$ are operators from Definition 2, h_{ij} is that instance of the decision matrix whose score value is either maximum or minimum depending on the ideal solution considered. When score values are equal, consider variance value. The lesser the variance value, the higher the rating.

Step 2: Calculate group utility (S) and individual regret (R) using (14) and (15).

$$S_i = \sum_{k\in\text{benefit}} \omega_k\left(\frac{d(h_k^*, h_{ik})}{d(h_k^*, h_k^-)}\right) + \sum_{k\in\text{cost}} \omega_k\left(\frac{d(h_{ik}, h_k^*)}{d(h_k^*, h_k^-)}\right) + \sum_{k\in\text{neutral}} \omega_k\left(\frac{d(h_{ik}, h_k^-) + d(h_k^*, h_{ik})}{d(h_k^*, h_k^-)}\right) \tag{14}$$

$$R_i = \max_{k\in\text{benefit, cost, nuetral}}\left(\omega_k\left(\frac{d(h_k^*, h_{ik})}{d(h_k^*, h_k^-)}\right), \omega_k\left(\frac{d(h_{ik}, h_k^*)}{d(h_k^*, h_k^-)}\right), \omega_k\left(\frac{d(h_{ik}, h_k^-) + d(h_k^*, h_{ik})}{d(h_k^*, h_k^-)}\right)\right) \tag{15}$$

Step 3: Calculate the merit function (Q) using (16).

$$Q_i = v\left(\frac{S_i - S^*}{S^- - S^*}\right) + (1-v)\left(\frac{R_i - R^*}{R^- - R^*}\right) \tag{16}$$

where v is the strategy adopted by the DM which is in the range 0 to 1, $S^* = \min(S_i)$, $R^* = \min(R_i)$, $S^- = \max(S_i)$ and $R^- = \max(R_i)$.

Step 4: Determine the final ranking order by arranging merit functions in the ascending order. The alternative that has the minimum Q value is ranked first and then the process follows.

Before demonstrating the practicality of the proposed framework, it is worth to understand certain details of the method. The discussion is as follows:

(1) The process of aggregation and weight estimation are self-contained and straightforward. So for brevity, we confine our discussion on these two concepts and move on with the ranking of alternatives.
(2) In the step (1) of TWHFV, the PIS and NIS are calculated for each of the criterion based on its respective category. We introduce a new category called neutral for enhancing choice representation by DM. Since the idea of benefit and cost are similar to that of HF-VIKOR, we see neutral category. Suppose, two criteria are neutral, we find the maximum and minimum of these two criteria and perform operations from Definition 2. The result of these operations yields a set of value which is hesitant fuzzy in nature.
(3) In step 2 of TWHFV, the values of S and R are estimated using Euclid distance. For the neutral part, both the distances of an instance from the PIS as well as the NIS are taken in consideration. This ensures better rationality of the decision process from the DMs' point of view.
(4) The Step 3 of TWHFV is used to calculate the merit function (Q) which is similar to the HF-VIKOR method. Here, the strategy value (v) is explored for different types of strategy ($v = 0.1$ to 0.9) to verify the robustness of the method.

5 An illustrative example

In this section let us consider the supplier outsourcing example in SCM. An automobile industry needs to purchase auto parts for a specific job. The industry identified six suppliers and five criteria for evaluating those suppliers. The industry decides to have three DMs viz., senior technical officer (D_1), chief finance officer (D_2) and senior executive (D_3). Based on the initial screening, four potential suppliers were selected and they were evaluated based on the five competing criteria (see Appendix). The DMs adopted hesitant fuzzy information for rating suppliers. Let us now review the procedure involved in decision-making process.

Step 1: Construct the decision matrix of order (4×5) for all three DMs with hesitant fuzzy information as the source for rating alternatives.

Step 2: Rearrange the decision matrices in some order and then aggregate these matrices into a single decision matrix of order (4×5) using the aggregation operator defined in Sect. 4.2. Table 2 shows the matrix from both step 1 and step 2.

The relative importance of each DM is expressed in the form of hesitant fuzzy values, given by, $\lambda_1 = (0.3, 0.32, 0.35)$, $\lambda_2 = (0.35, 0.25, 0.38)$, $\lambda_3 = (0.36, 0.4, 0.30)$ and these values are converted to single valued weights based on the procedure given in Definition 6. The values are given by $\lambda_1 = 0.2941$, $\lambda_2 = 0.3390$ and $\lambda_3 = 0.3667$. Finally, these values are used by SHFWG operator for computing the aggregated matrix.

The score of the aggregated matrix is evaluated using (6) and if the score values are found to be equal between the alternatives, then variance is estimated for such alternatives using Eq. (7). Table 3 shows the score and variance value of each alternative with respect to each criterion.

Step 3: Construct a weight evaluation matrix for assigning weights to each of the criterion. Table 4 shows this matrix which is of order (3×5). The rating is in the form of hesitant fuzzy information. Apply HFSV method from Sect. 4.3, to estimate the weight values for each of the criterion.

Using the score values from Table 5, the weight values are estimated using HFSV method and are given by $\omega \approx (0.2, 0.2, 0.4, 0.1, 0.1)$.

Step 4: Determine the ranking order for all four suppliers using step 2, step 3 and TWHFV ranking method.

Using Eqs. (12, 13), PIS and NIS are calculated and it is given by.

Table 2 Decision matrix

DMs	Alternatives	Criteria				
		C_1	C_2	C_3	C_4	C_5
D_1	A_1	(0.52, 0.43, 0.45, 0.55)	(0.55, 0.52, 0.46, 0.57)	(0.56, 0.6 , 0.54, 0.48)	(0.65, 0.68, 0.52, 0.55)	(0.65, 0.67, 0.74, 0.72)
	A_2	(0.44, 0.55, 0.45, 0.53)	(0.47, 0.58, 0.64, 0.55)	(0.57, 0.55, 0.63, 0.6)	(0.53, 0.66, 0.62, 0.57)	(0.66, 0.68, 0.75, 0.78)
	A_3	(0.42, 0.53, 0.6, 0.57)	(0.45, 0.53, 0.55, 0.38)	(0.6, 0.64, 0.66, 0.58)	(0.66, 0.62, 0.57, 0.55)	(0.67, 0.74, 0.77, 0.8)
	A_4	(0.53, 0.56, 0.48, 0.6)	(0.56, 0.45, 0.58, 0.6)	(0.66, 0.64, 0.7, 0.62)	(0.64, 0.55, 0.52, 0.6)	(0.66, 0.64, 0.7, 0.68)
D_2	A_1	(0.6, 0.53, 0.56, 0.46)	(0.6, 0.58, 0.62, 0.55)	(0.7, 0.67, 0.65, 0.58)	(0.66, 0.54, 0.62, 0.57)	(0.58, 0.65, 0.72.0.7)
	A_2	(0.53, 0.62, 0.6, 0.56)	(0.62, 0.64, 0.58, 0.55)	(0.66, 0.63, 0.7, 0.68)	(0.64, 0.56, 0.5, 0.48)	(0.72, 0.68, 0.78, 0.82)
	A_3	(0.5, 0.4, 0.54, 0.46)	(0.52, 0.56 , 0.63, 0.67)	(0.57, 0.63, 0.65, 0.68)	(0.6, 0.55, 0.68, 0.7)	(0.57, 0.63, 0.68, 0.7)
	A_4	(0.62, 0.52, 0.43, 0.64)	(0.42, 0.47, 0.5, 0.56)	(0.6, 0.64, 0.58, 0.62)	(0.68, 0.66, 0.7, 0.62)	(0.67, 0.72, 0.75, 0.8)
D_3	A_1	(0.6, 0.62, 0.57, 0.52)	(0.56, 0.52, 0.6, 0.62)	(0.66, 0.62, 0.7, 0.6)	(0.57, 0.66, 0.68, 0.55)	(0.72, 0.74, 0.77, 0.82)
	A_2	(0.53, 0.64, 0.47, 0.5)	(0.5, 0.6, 0.66, 0.55)	(0.58, 0.64, 0.67, 0.72)	(0.54, 0.62, 0.58, 0.64)	(0.66, 0.62, 0.7, 0.73)
	A_3	(0.57, 0.56, 0.6, 0.64)	(0.53, 0.56, 0.45, 0.48)	(0.7, 0.6, 0.67, 0.72)	(0.54, 0.64, 0.58, 0.6)	(0.54, 0.63, 0.66, 0.7)
	A_4	(0.55, 0.52, 0.5, 0.6)	(0.52, 0.5, 0.57, 0.6)	(0.66, 0.63, 0.7, 0.65)	(0.55, 0.57, 0.6, 0.63)	(0.68, 0.74, 0.64, 0.76)
D_{123}	A_1	(0.47, 0.52, 0.56, 0.60)	(0.52, 0.56, 0.59, 0.61)	(0.56, 0.61, 0.63, 0.67)	(0.53, 0.56, 0.64, 0.67)	(0.65, 0.69, 0.73, 0.76)
	A_2	(0.48, 0.50, 0.55, 0.60)	(0.51, 0.56, 0.60, 0.65)	(0.59, 0.62, 0.66, 0.69)	(0.51, 0.54, 0.60, 0.65)	(0.65, 0.69, 0.74, 0.78)
	A_3	(0.46, 0.52, 0.56, 0.60)	(0.45, 0.50, 0.56, 0.59)	(0.59, 0.64, 0.66, 0.69)	(0.54, 0.59, 0.63, 0.67)	(0.59, 0.67, 0.70, 0.73)
	A_4	(0.45, 0.53, 0.58, 0.62)	(0.43, 0.49, 0.53, 0.57)	(0.60, 0.63, 0.65, 0.67)	(0.57, 0.60, 0.63, 0.66)	(0.66, 0.71, 0.75, 0.79)

Table 3 Score and variance evaluation

Score/variance	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	0.5362	0.5639	0.6154	0.6046	0.7074
A_2	0.5356	0.5788	0.6375	0.5774	0.7132
A_3	0.5316	0.5250	0.6431	0.6070	0.6681
A_4	0.5417	0.5054	0.6353	0.6165	0.7267

Since score values are all unique, variance is not calculated

Table 4 Rating of criteria

DMs	Criteria				
	C_1	C_2	C_3	C_4	C_5
D_1	(0.6, 0.5, 0.55)	(0.55, 0.52, 0.58)	(0.62, 0.66, 0.58)	(0.52, 0.55, 0.58)	(0.64, 0.67, 0.6)
D_2	(0.5, 0.57, 0.64)	(0.56, 0.62, 0.64)	(0.65, 0.68, 0.72)	(0.57, 0.54, 0.6)	(0.66, 0.62, 0.7)
D_3	(0.55, 0.58, 0.66)	(0.58, 0.62, 0.54)	(0.57, 0.64, 0.67)	(0.53, 0.5, 0.57)	(0.6, 0.64, 0.68)

Table 5 Score evaluation of criteria for weight calculation

DMs	Criteria				
	C_1	C_2	C_3	C_4	C_5
D_1	0.5500	0.5500	0.6200	0.5500	0.6366
D_2	0.5700	0.6066	0.6833	0.5700	0.6000
D_3	0.5966	0.5800	0.6266	0.5333	0.6400

Table 6 Evaluation of VIKOR parameters

Alternatives	Group utility (S)	Individual regret (R)	Merit function (Q)
A_1	0.4257	0.1863	0.8465
A_2	0.4188	0.2076	0.8897
A_3	0.0376	0.0236	0
A_4	0.4637	0.2316	1

Ranking order is $A_3 \succ A_1 \succ A_2 \succ A_4$

$$h^* = \begin{pmatrix} (0.45, 0.52, 0.58, 0.61), (0.43, 0.48, 0.53, 0.57), (0.58, 0.63, 0.66, 0.69), \\ (0.79, 0.82, 0.85, 0.88), (0.66, 0.71, 0.75, 0.78) \end{pmatrix} \text{ and}$$

$$h^- = \begin{pmatrix} (0.46, 0.52, 0.56, 0.59), (0.51, 0.56, 0.60, 0.65), (0.60, 0.63, 0.65, 0.68), \\ (0.30, 0.33, 0.38, 0.43), (0.59, 0.66, 0.70, 0.73) \end{pmatrix}$$

Table 6 shows the S, R and Q values for different alternatives which are calculated using Eqs. (14–16) for a strategy value of $v = 0.5$.

Step 5: Finally, sensitivity analysis is performed for TWHFV ranking method to validate its robustness of the method using Table 7. Also, in the next section, we perform a comparative investigation to verify other strengths of HFV ranking method with another ranking method.

6 Comparative study on proposed TWHFV and HF-VIKOR ranking methods

In this section, we demonstrate the power of TWHFV ranking method (belongs to TSDMF) by performing a comparative investigation with its close counterpart, HF-VIKOR method (belongs to Liao and Xu 2013 framework). We analyze these two methods via sensitivity analysis and realize the robustness of these two methods. Further, some strengths of TSDMF are also discussed by investigating their features with another framework. With a view to maintain homogeneity, we compare our proposed decision-making framework with the decision-making framework of Liao and Xu (2013). Table 8 shows the sensitivity analysis for HF-VIKOR method and Table 9 shows the comparative analysis of features for proposed TSDMF and (Liao and Xu 2013) framework.

We further understand the strength of the proposal by comparison with other methods under HFS context. With the view of maintaining homogeneity in the process of comparison, methods viz., HF-VIKOR (Liao and Xu 2013), HF-TOPSIS (Xu and Zhang 2013), HF-ELECTRE (Chen and Xu 2015) and HF-PROMETHEE (Mahmoudi et al. 2016) are taken for analysis with the proposed method.

From Table 9, it is clear that the proposed decision framework produces a unique ranking order with A_3 as the compromise solution. Moreover to understand the consistency of the proposed method, Spearman correlation (Spearman 1904) is applied over different ranking order. From Fig. 2, it can be inferred that the proposed decision framework is consistent with other methods.

Based on the investigation of Tables 7, 8, 9, 10, we make the following inferences about the novelty of the proposed decision framework:

(1) Tables 7 and 8 show the sensitivity analysis for two ranking methods viz., proposed TWHFV method and HF-VIKOR method. Nine unique values from 0.1 to 0.9 are considered for the evaluation and the final ranking order is given by

(2) $A_3 \succ A_1 \succcurlyeq A_2 \succ A_4$ (same ranking order with TWHFV and HF-VIKOR methods). From the sensitivity analysis, we observe that proposed TWHFV method shows a change in ranking order only at $v = 0.9$, while HF-VIKOR showed a change of order at $v = 0.1$. This signifies that proposed TWHFV method and HF-VIKOR method are less sensitive to vagueness and uncertainty. Also, the competition among suppliers A_1 and A_2 is clearly

(3) The main advantage of proposed TSDMF is that it allows only little intervention from the DM and has an automated setup for evaluation of alternatives. Such automation reduces the DMs' effort in the process of evaluation and reduces imprecision and vagueness to a certain extent.

(4) Another interesting feature of TSDMF is that it has a ranking method (TWHFV) which follows much sensible and rational procedure for ranking, which helps DMs to make clarified judgments. This method offers better flexibility to DMs by proposing three categories for classification of criteria, which drives proper and effective

Table 7 Sensitivity analysis for proposed TWHFV method

v values	Alternatives	Q values	Preference order
0.1	A_1	0.7952	$A_3 \succ A_1 \succ A_2 \succ A_4$
	A_2	0.8858	
	A_3	0	
	A_4	1	
0.2	A_1	0.8080	$A_3 \succ A_1 \succ A_2 \succ A_4$
	A_2	0.8868	
	A_3	0	
	A_4	1	
0.3	A_1	0.8208	$A_3 \succ A_1 \succ A_2 \succ A_4$
	A_2	0.8877	
	A_3	0	
	A_4	1	
0.4	A_1	0.8337	$A_3 \succ A_1 \succ A_2 \succ A_4$
	A_2	0.8887	
	A_3	0	
	A_4	1	
0.5	A_1	0.8465	$A_3 \succ A_1 \succ A_2 \succ A_4$
	A_2	0.8897	
	A_3	0	
	A_4	1	
0.6	A_1	0.8593	$A_3 \succ A_1 \succ A_2 \succ A_4$
	A_2	0.8906	
	A_3	0	
	A_4	1	
0.7	A_1	0.8722	$A_3 \succ A_1 \succ A_2 \succ A_4$
	A_2	0.8916	
	A_3	0	
	A_4	1	
0.8	A_1	0.8850	$A_3 \succ A_1 \succ A_2 \succ A_4$
	A_2	0.8926	
	A_3	0	
	A_4	1	
0.9	A_1	0.8978	$A_3 \succ A_2 \succ A_1 \succ A_4$
	A_2	0.8935	
	A_3	0	
	A_4	1	

Table 8 Sensitivity analysis of HF-VIKOR (Liao and Xu 2013) method

v values	Alternatives	Q values	Preference order
0.1	A_1	0.8941	$A_3 \succ A_1 \succ A_2 \succ A_4$
	A_2	0.8962	
	A_3	0	
	A_4	1	
0.2	A_1	0.8915	$A_3 \succ A_2 \succ A_1 \succ A_4$
	A_2	0.8829	
	A_3	0	
	A_4	1	
0.3	A_1	0.8889	$A_3 \succ A_2 \succ A_1 \succ A_4$
	A_2	0.8697	
	A_3	0	
	A_4	1	
0.4	A_1	0.8863	$A_3 \succ A_2 \succ A_1 \succ A_4$
	A_2	0.8564	
	A_3	0	
	A_4	1	
0.5	A_1	0.8837	$A_3 \succ A_2 \succ A_1 \succ A_4$
	A_2	0.8431	
	A_3	0	
	A_4	1	
0.6	A_1	0.8811	$A_3 \succ A_2 \succ A_1 \succ A_4$
	A_2	0.8298	
	A_3	0	
	A_4	1	
0.7	A_1	0.8785	$A_3 \succ A_2 \succ A_1 \succ A_4$
	A_2	0.8165	
	A_3	0	
	A_4	1	
0.8	A_1	0.8759	$A_3 \succ A_2 \succ A_1 \succ A_4$
	A_2	0.8032	
	A_3	0	
	A_4	1	
0.9	A_1	0.8734	$A_3 \succ A_2 \succ A_1 \succ A_4$
	A_2	0.7899	
	A_3	0	
	A_4	1	

Table 9 Ranking order of various ranking method(s)

Method(s)	Suppliers				Ranking order
	A_1	A_2	A_3	A_4	
TWHFV (proposed)	2	3	1	4	$A_3 \succ A_1 \succcurlyeq A_2 \succ A_4$
HF-VIKOR	3	2	1	4	$A_3 \succ A_2 \succcurlyeq A_1 \succ A_4$
HF-TOPSIS	3	2	1	4	$A_3 \succ A_2 \succcurlyeq A_1 \succ A_4$
HF-ELECTRE	3	2	1	4	$A_3 \succ A_2 \succ A_1 \succ A_4$
HF-PROMETHEE	2	3	1	4	$A_3 \succ A_1 \succ A_2 \succ A_4$

The ranking order of proposed TWHFV and HF-VIKOR method is presented after sensitivity analysis. In HF-ELECTRE method ELECTRE II is extended

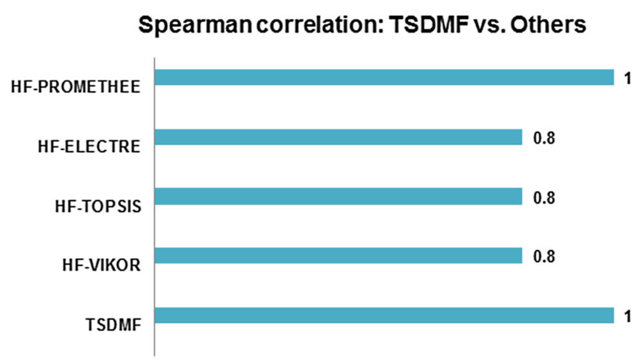

Fig. 2 Spearman correlation of different method(s)

understanding of the nature of each criterion and thereby alleviating the issue of imprecision and vagueness in the classification of criteria.

(5) Also, the proposed TWHFV method yields a broader rank value set compared to the HF-VIKOR method, which helps DMs to make sensible and rational decisions. Though, the rank value set appears close for both the methods, estimation of standard deviation clarifies the fact that, proposed TWHFV method is broader than HF-VIKOR method. The standard deviation values of TWHFV method and HF-VIKOR (Liao and Xu 2013) method for each set of v values (0.1–0.9) is given by:

TWHFV:(0.4546, 0.4560, 0.4574, 0.4590, 0.4606, 0.4623, 0.4640, 0.4660, 0.4678)

HFVIKOR:(0.4677, 0.4654, 4633, 0.4613, 0.4590, 0.4574, 0.4556, 0.4538, 0.4522)

From the values shown above, an instance of $v = 0.5$ is taken which clearly shows that the proposed TWHFV method (0.4606 at $v = 0.5$) produces broader rank value set than HF-VIKOR (0.4590 at $v = 0.5$) method (Liao and Xu 2013). The value 0.5 is taken specifically for analysis as this is the ideal strategy value adopted by DM for making rational decisions.

(6) The Spearman correlation is applied to different ranking order obtained from different methods to analyze the consistency of the proposed framework. From Fig. 2, it can be observed that the proposed framework is highly consistent with other methods.

(7) Further, a simulation study is conducted to understand the strength of the proposed framework. The study initially forms 300 decision matrices of order (4×5) with four objects and five criteria. In these datasets, three criteria are considered as benefit, one as cost and one as neutral. The constraints discussed in Definition 1 are followed for the creation of datasets and four instances of membership degrees are considered in each dataset. Using these datasets as input, the ranking method is adopted and standard deviation is calculated for the rank value set at $v = 0.5$. These standard deviation values are depicted in Fig. 3.
From the analysis, we infer that the proposed TSDMF has broader and sensible rank value set than its close counterpart (Liao and Xu 2013) method. Thus, the proposed framework can be used for proper and rational backup management under uncertain situations.

(8) Though the proposed framework enjoys such attractive strengths, it does suffer from some weakness. They are: (a) the framework needs trained DMs for its implementation and (b) the framework is computationally complex as it uses hesitant fuzzy values for evaluation. These weaknesses will be addressed in the future.

7 Conclusion

In this paper, a new two-stage automated scientific decision-making framework called TSDMF is proposed for solving supplier outsourcing problem. The framework uses hesitant fuzzy values as input and consists of aggregation stage, followed by weight estimation and ranking stage. The DMs' preferences are aggregated using newly proposed SHFWG operator, which considers DMs' weight as hesitant fuzzy values for sensible evaluation. Following this, the criteria weights are estimated using newly proposed HFSV method, which is an extension to SV method under HFS. This method yields a more reasonable and sensible weight value compared to other state of the art methods. Finally, the ranking is done using newly proposed TWHFV method, which is an extension to VIKOR method under HFS. The TWHFV method uses three categories viz., cost, benefit and neutral for clear classification of each criterion and Euclid distance in its formulation which is claimed to have better robustness than its close counterpart. Some significant contributions of proposed TSDMF are pointed out:

Table 10 Investigation of features: TSDMF versus Liao and Xu (2013)

Features	Proposed TSDMF	Liao and Xu (2013)
Type of input	Hesitant fuzzy values	Hesitant fuzzy values
Aggregation	Yes, SHFWG operator	No
Criteria weights	Estimated using HFSV method	Directly given by DMs
Distance measure	Euclid distance	Manhattan distance
Decision making	Group decision making	Single decision making
Handling of fuzziness	Handled better by having an automated setup with little human intervention during decision making. Also, ranking method uses a novel innovation, by introducing third category of classification for yielding better judgments	Handled normal by having more human intervention during decision making.
Total preoder	yes	yes
Convergence rate	Better convergence to optimal solution	Weak convergence to optimal solution
Nature of Q values	Broader and easily distinguishable	Narrow and difficult to distinguish
Human intervention	Low and hence, the rate of imprecision and vagueness in decision-making process is minimal.	High and hence, the rate of imprecision and vagueness in decision-making process is maximum.
Sensitivity to vagueness	Highly robust and less sensitive to uncertainty and vagueness	Less robust and more sensitive to uncertainty and vagueness
Adequacy test	Stable even after adequate changes are made to the criteria	Stable even after adequate changes are made to the criteria.
Scalability	Scalable and follows the principle of Saaty and Ozdemir (2003)	
Application tested	Supplier outsourcing	Flight service selection
Final ranking order (after sensitivity analysis)	Proposed TWHFV = $A_3 \succ A_1 \succcurlyeq A_2 \succ A_4$ HF-VIKOR = $A_3 \succ A_2 \succcurlyeq A_1 \succ A_4$	HF-VIKOR = $A_1 \succ A_3 \succ A_4 \succ A_2$ Proposed TWHFV = $A_1 \succ A_3 \succ A_4 \succ A_2$

(1) The proposed TSDMF is the first scientific framework under hesitant fuzzy environment to follow the combination of SHFWG operator for aggregation, HFSV method for weight estimation and TWHFV (new extension of VIKOR) which is a compromise ranking method for selecting a suitable supplier for the task.
(2) This framework complements the work done by Liao and Xu (2013), by taking full advantage of the hesitant fuzzy environment. Unlike the framework (Liao and Xu 2013), the TSDMF offers better scope for handling imprecision and vagueness by allowing only little intervention from humans. The framework by Liao and Xu (2013), uses hesitant fuzzy property only for ranking, while, criteria weights are directly given by DMs and there is no aggregation concept in this framework.
(3) Though, ranking by Liao and Xu (2013) uses HFS property, the difficulty in proper understanding of each criterion still remained unresolved. To better circumvent the issue and to help DMs in better understanding

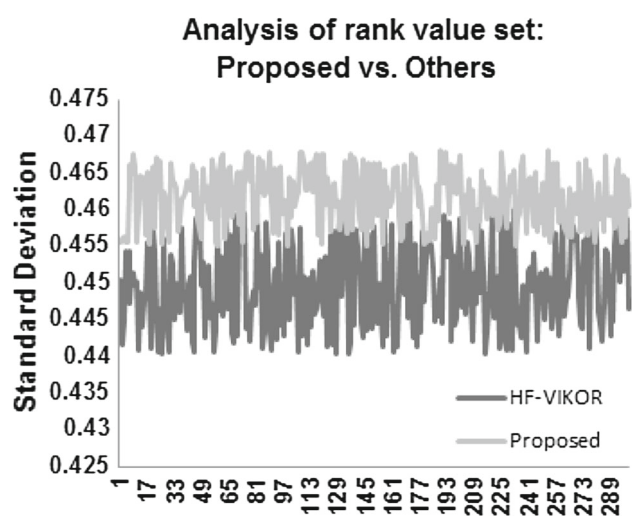

Fig. 3 Estimation of broadness of rank value set(s): Proposed versus Others

the nature of each criterion, we proposed a three-way classification setup involving cost, benefit and neutral zones. Also, the framework by Liao and Xu (2013), formulates the parameters S and R only for benefit zone and with the view of improving the formulation, proposed TWHFV method formulates S and R for all three classification zones viz., cost, benefit and neutral.

(4) Finally, the strength of the proposed TSDMF is realized using sensitivity analysis (Sect. 6) and inferences clarify the fact that, proposed TSDMF is robust and less sensitive to imprecision and vagueness. Also, we observe that, proposed TSDMF provides much broader rank value set which helps DMs to make rational decisions with ease and efficacy.

As a part of future work, we address the weakness of the framework and also make efforts to extend the framework to other decision-making applications like healthcare management, resource management etc. Also, we make efforts to propose new automated scientific decision-making frameworks under different fuzzy sets like soft sets, neutrosophic sets, shadow sets, m-polar fuzzy set etc.

Acknowledgements We the authors thank the funding agency, University Grants Commission (UGC) for their financial support through Rajiv Gandhi National Fellowship (RGNF) scheme under the award number: F./2015-17/RGNF-2015-17-TAM-83. We also thank the Department of Science and Technology (DST), India for their financial aid in setting up a cloud environment under the FIST programme (Award Number: SR/FST/ETI-349/2013). We also express our heartfelt thanks to SASTRA University for offering us an excellent infrastructure to carry out our research work. Finally, we express our sincere thanks to the editor and to the anonymous reviewers for their constructive comments.

Compliance with ethical standards

Conflict of interest All authors of this research paper declare that, there is no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Appendix

Let us analyze the five criteria taken for the purpose of evaluation. For choosing a suitable supplier, the committee decides 5 criteria, of which, two criteria belong to cost zone and 3 criteria belong to benefit zone. The details of these criteria are given below:

- On-time delivery rate (C_1): This defines the rate of delivery of the product on-time. This criterion must be maximized for better selection and hence, it belongs to the benefit zone.
- Cost (C_2): This defines the total cost incurred by each of the supplier. They include, product cost, freight cost and tarrif. Supplier with minimum cost is preferred more. Hence, it is placed in cost zone.
- Service (C_3): This defines the technical ability and managerial strength of the supplier. Preference is more for a supplier with maximum service rate. Hence, it belongs to benefit zone.
- Supplier profile (C_4): This defines the previous success stories, relationship ties with the organization, popularity of the supplier, risk involved (which is learned from previous history) etc. Clearly, this attribute poses a confusion to the DMs and hence, we recommend placing this attribute in the neutral zone. When the method by Liao and Xu (2013) is adopted, we place this attribute in the benefit zone.
- Quality (C_5): This defines the product reach and the market stability of the product from a particular supplier. This must be maximum for a supplier and so, it is placed in the benefit zone.

Let us now consider the abbreviation(s) and its expansion for easy understanding of the paper by using Table 11.

Table 11 Abbreviation and its expansion

Abbreviation(s)	Expansion
DM	Decision maker
HFS	Hesitant fuzzy set
HFE	Hesitant fuzzy element
SHFWG	Simple hesitant fuzzy-weighted geometry
HFSV	Hesitant fuzzy standard variance
VIKOR	VlseKriterijumskaOptimizacijaKompromisnoResenje
TWHFV	Three-way hesitant fuzzy VIKOR
ELECTRE	ELimination Et ChoixTraduisant la REalité
AHP	Analytical hierarchy process
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
QUALIFLEX	Qualitative flexible multi-criteria method
MCDM	Multi-criteria decision making
SV	Standard variance
SO	Supplier outsourcing
TSDMF	Two-stage decision-making framework

References

Chai J, Ngai EWT (2015) Multi-perspective strategic supplier selection in uncertain environments. Int J Prod Econ 166:215–225. https://doi.org/10.1016/j.ijpe.2014.09.035

Chai J, Liu JNK, Ngai EWT (2013) Application of decision-making techniques in supplier selection: a systematic review of literature. Expert Syst Appl 40(10):3872–3885. https://doi.org/10.1016/j.eswa.2012.12.040

Chang KH (2015) Enhanced assessment of a supplier selection problem by integration of soft sets and hesitant fuzzy linguistic term set. Proc Inst Mech Eng Part B J Eng Manuf 229(9):1635–1644. https://doi.org/10.1177/0954405414539498
Charulatha B, Rodrigues P, Chitralekha T (2013) A comparative study of different distance metrics that can be used in Fuzzy Clustering Algorithms. IJETTCS. http://www.ijettcs.org/NCASG-2013/NCASG38.pdf
Chen N, Xu Z (2015) Hesitant fuzzy ELECTRE II approach: a new way to handle multi-criteria decision making problems. Inf Sci 292:175–197. https://doi.org/10.1016/j.ins.2014.08.054
Choi J, Bai SX, Geunes J, Edwin Romeijn H (2007) Manufacturing delivery performance for supply chain management. Math Comput Model 45(1–2):11–20. https://doi.org/10.1016/j.mcm.2005.07.008
Chýna V, Kuncová M, Sekni J (2013) Estimation of weights in multi-criteria decision-making optimization models. In Proceedings of 30th international conference mathematical methods in economics, pp. 355–360
Darabi S, Heydari J (2016) An interval- valued hesitant fuzzy ranking method based on group decision analysis for green supplier selection. IFAC-PapersOnLine 49(2):12–17. https://doi.org/10.1016/j.ifacol.2016.03.003
Dong JY, Yuan FF, Wan SP (2017) Extended VIKOR method for multiple criteria decision-making with linguistic hesitant fuzzy information. Comput Ind Eng 112:305–319. https://doi.org/10.1016/j.cie.2017.07.025
Fahmi A, Kahraman C, Bilen Ü (2016) ELECTRE I method using hesitant linguistic term sets: an application to supplier selection. Int J Comput Intell Syst 9(1):153–167. https://doi.org/10.1080/18756891.2016.1146532
Fülöp J (2001) Introduction to decision making methods. Oper Res
Gitinavard H, Mousavi SM, Vahdani B (2015) A balancing and ranking method based on hesitant fuzzy sets for solving decision-making problems under uncertainty. Int J Eng 28(2):214–223
Gitinavard H, Mousavi SM, Vahdani B (2016) A new multi-criteria weighting and ranking model for group decision-making analysis based on interval-valued hesitant fuzzy sets to selection problems. Neural Comput Appl 27(6):1593–1605. https://doi.org/10.1007/s00521-015-1958-0
Gitinavard H, Ghaderi H, Pishvaee MS (2017) Green supplier evaluation in manufacturing systems: a novel interval-valued hesitant fuzzy group outranking approach. Soft Comput. https://doi.org/10.1007/s00500-017-2697-1
Gul M, Celik E, Aydin N, Gumas A, Guneri A (2016) A state of the art literature review of VIKOR and its fuzzy extensions on applications. Appl Soft Comput 46:60–89. https://doi.org/10.1016/j.asoc.2016.04.040
Jin F, Pei L, Chen H, Zhou L (2014) Interval-valued intuitionistic fuzzy continuous weighted entropy and its application to multi-criteria fuzzy group decision making. Knowl-Based Syst 59:132–141. https://doi.org/10.1016/j.knosys.2014.01.014
Li J, Wang J (2017) An extended QUALIFLEX method under probability hesitant fuzzy environment for selecting green suppliers. Int J Fuzzy Syst 19:40815. https://doi.org/10.1007/s40815-017-0310-5
Liang D, Xu Z, Liu D (2017) Three-way decisions based on decision-theoretic rough sets with dual hesitant fuzzy information. Inf Sci 396:127–143. https://doi.org/10.1016/j.ins.2017.02.038
Liao H, Xu Z (2013) A VIKOR-based method for hesitant fuzzy multi-criteria decision making. Fuzzy Optim Decis Mak 12(4):373–392. https://doi.org/10.1007/s10700-013-9162-0
Liao H, Xu Z (2014) Some new hybrid weighted aggregation operators under hesitant fuzzy multi-criteria decision making environment. J Intell Fuzzy Syst 26(4):1601–1617. https://doi.org/10.3233/IFS-130841
Liao H, Xu Z (2015) Consistency of the fused intuitionistic fuzzy preference relation in group intuitionistic fuzzy analytic hierarchy process. Appl Soft Comput 35:812–826. https://doi.org/10.1016/j.asoc.2015.04.015
Liao H, Xu Z, Xia M (2014a) Multiplicative consistency of hesitant fuzzy preference relation and its application in group decision making. Int J Inf Technol Decis Mak 13(1):47–76. https://doi.org/10.1142/S0219622014500035
Liao H, Xu Z, Zeng XJ (2014b) Distance and similarity measures for hesitant fuzzy linguistic term sets and their application in multi-criteria decision making. Inf Sci 271:125–142. https://doi.org/10.1016/j.ins.2014.02.125
Liao H, Xu Z, Zeng X-J (2014c) Hesitant fuzzy linguistic VIKOR method and its application in qualitative multiple criteria decision making. IEEE Trans Fuzzy Syst. https://doi.org/10.1109/TFUZZ.2014.2360556
Liu W, Liao H (2017) A bibliometric analysis of fuzzy decision research during 1970–2015. Int J Fuzzy Syst. https://doi.org/10.1007/s40815-016-0272-z
Liu S, Chan FTS, Ran W (2016) Decision making for the selection of cloud vendor: an improved approach under group decision-making with integrated weights and objective/subjective attributes. Expert Syst Appl 55:37–47. https://doi.org/10.1016/j.eswa.2016.01.059
Mahmoudi A, Sadi-Nezhad S, Makui A, Vakili MR (2016) An extension on PROMETHEE based on the typical hesitant fuzzy sets to solve multi-attribute decision-making problem. Kybernetes 45(8):1213–1231. https://doi.org/10.1108/K-10-2015-0271
Mardani A, Zavadskas EK, Govindan K, Senin AA, Jusoh A (2016) VIKOR technique: a systematic review of the state of the art literature on methodologies and applications. Sustainability 8(1):1–38. https://doi.org/10.3390/su8010037
O'Haire C, McPheeters M, Nakamoto E, LaBrant L, Most C, Lee K, Oregon Evidence-based Practice Center and the Vanderbilt Evidence-based Practice Center (2011) Engaging stakeholders to identify and prioritize future research needs. Methods future research needs report no. 4, (11-EHC044-EF). www.effectivehealthcare.ahrq.gov/reports/final.cfm
Opricovic S, Tzeng GH (2004) Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. Eur J Oper Res 156(2):445–455. https://doi.org/10.1016/S0377-2217(03)00020-1
Öztayşi B, Sürer Ö (2014) Supply chain management under fuzziness. Supply Chain Manag Under Fuzziness 313:199–224. https://doi.org/10.1007/978-3-642-53939-8
Ren Z, Xu Z, Wang H (2017) Dual hesitant fuzzy VIKOR method for multi-criteria group decision making based on fuzzy measure and new comparison method. Inf Sci 388–389:1–16. https://doi.org/10.1016/j.ins.2017.01.024
Saaty TL (1980) The analytic hierarchy process. McGraw-Hill, New York, pp 579–606
Saaty TL, Ozdemir MS (2003) Why the magic number seven plus or minus two. Math Comput Model 38(3):233–244. https://doi.org/10.1016/S0895-7177(03)90083-5
Simić D, Kovačević I, Svirčević V, Simić S (2017) 50 years of fuzzy set theory and models for supplier assessment and selection: a literature review. J Appl Log 24:85–96. https://doi.org/10.1016/j.jal.2016.11.016
Sinwar D, Kaushik R (2014) Study of Euclidean and Manhattan distance metrics using simple k-means clustering. Int J Res Appl Sci Eng Technol (IJRASET) 2(5):270–274
Spearman C (1904) The proof and measurement of association between two things. Am J Psychol 15(1):72–101
Taciana C, Gussen G (2015) Hesitant fuzzy analytic hierarchy process. In: IEEE international conference on fuzzy system, pp 1–7

Tang S-L (2017) Green supplier selection model with hesitant fuzzy information. J Intell Fuzzy Syst 32(1):189–195. https://doi.org/10.3233/JIFS-151377
Thakur GS, Thakur R, Singh R (2014) New hesitant fuzzy operators. Fuzzy Inf Eng 6(3):379–392
Torra V, Narukawa Y (2009) On hesitant fuzzy sets and decision. IEEE International Conference on Fuzzy Systems. https://doi.org/10.1109/FUZZY.2009.5276884
Tyagi SK (2016) Multiple attribute decision making using hesitant triangular fuzzy sets. In: International conference on electrical, electronics, and optimization techniques (ICEEOT), pp 1502–1510. http://dx.doi.org/10.1109/iceeot.2016.7754934
Wang H, Xu Z (2016) Multi-groups decision making using intuitionistic-valued hesitant fuzzy information. Int J Comput Intell Syst 9(3):468–482. https://doi.org/10.1080/18756891.2016.1175812
Wood DA (2016) Supplier selection for development of petroleum industry facilities, applying multi-criteria decision making techniques including fuzzy and intuitionistic fuzzy TOPSIS with flexible entropy weighting. J Nat Gas Sci Eng 28(December):594–612. https://doi.org/10.1016/j.jngse.2015.12.021
Xia M, Xu Z (2011) Hesitant fuzzy information aggregation in decision making. Int J Approx Reason 52(3):395–407. https://doi.org/10.1016/j.ijar.2010.09.002
Xia M, Xu Z (2012) Entropy/cross entropy-based group decision making under intuitionistic fuzzy environment. Inf Fusion 13(1):31–47. https://doi.org/10.1016/j.inffus.2010.12.001
Xu, Z. (2014). Hesitant fuzzy sets theory. In: Studies in fuzziness and soft computing, vol 314. http://doi.org/10.1007/978-3-319-04711-9
Xu Z, Liao H (2015) Intuitionistic fuzzy analytic hierarchy process. IEEE Trans Fuzzy Syst. https://doi.org/10.1109/TFUZZ.2013.2272585
Xu Z, Xia M (2011a) Distance and similarity measures for hesitant fuzzy sets. Inf Sci 181(11):2128–2138. https://doi.org/10.1016/j.ins.2011.01.028
Xu Z, Xia M (2011b) Distance and similarity measures for hesitant fuzzy sets. Inf Sci 181(11):2128–2138. https://doi.org/10.1016/j.ins.2011.01.028
Xu Z, Zhang X (2013) Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information. Knowl-Based Syst 52:53–64. https://doi.org/10.1016/j.knosys.2013.05.011
Xue M, Du Y (2017) A group decision-making model based on regression method with hesitant fuzzy preference relations. Math Problems Eng 2017:1–8
Yu D, Li D-F, Merigó JM (2016a) Dual hesitant fuzzy group decision making method and its application to supplier selection. Int J Mach Learn Cybern 7:819–831. https://doi.org/10.1007/s13042-015-0400-3
Yu D, Li D-F, Merigó JM (2016b) Dual hesitant fuzzy group decision making method and its application to supplier selection. Int J Mach Learn Cybernet 7:819–831. https://doi.org/10.1007/s13042-015-0400-3
Yücel A, Güneri AF (2011) A weighted additive fuzzy programming approach for multi-criteria supplier selection. Expert Syst Appl 38(5):6281–6286. https://doi.org/10.1016/j.eswa.2010.11.086
Yue Z (2011) A method for group decision-making based on determining weights of decision makers using TOPSIS. Appl Math Model 35(4):1926–1936. https://doi.org/10.1016/j.apm.2010.11.001
Zhang Z (2016) Multi-criteria decision-making using interval-valued hesitant fuzzy QUALIFLEX methods based on a likelihood-based comparison approach. Neural Comput Appl. https://doi.org/10.1007/s00521-015-2156-9
Zhang Z (2017) Multi-criteria decision-making using interval-valued hesitant fuzzy QUALIFLEX methods based on a likelihood-based comparison approach. Neural Comput Appl 28(7):1835–1854. https://doi.org/10.1007/s00521-015-2156-9
Zhang X, Xu Z (2015) Hesitant fuzzy QUALIFLEX approach with a signed distance-based comparison method for multiple criteria decision analysis. Expert Syst Appl 42(2):873–884. https://doi.org/10.1016/j.eswa.2014.08.056
Zhang Y, Wang Y, Wang J (2014) Objective attributes weights determining based on shannon information entropy in hesitant fuzzy multiple attribute decision making. Math Probl Eng. https://doi.org/10.1155/2014/463930
Zhang X, Xu Z, Xing X (2016) Hesitant fuzzy programming technique for multidimensional analysis of hesitant fuzzy preferences. OR Spectr 38:789–817. https://doi.org/10.1007/s00291-015-0420-0
Zhao X, Lin R, Wei G (2014) Hesitant triangular fuzzy information aggregation based on Einstein operations and their application to multiple attribute decision making. Expert Syst Appl 41(1):1086–1094. https://doi.org/10.1016/j.eswa.2013.07.104
Zhao H, You JX, Liu HC (2016) Failure mode and effect analysis using MULTIMOORA method with continuous weighted entropy under interval-valued intuitionistic fuzzy environment. Soft Comput. https://doi.org/10.1007/s00500-016-2118-x
Zhaoxia Z, De X, Jiang G (2013) A strategic decision framework for sustainability in supply chain. In: Proceedings of the 5th international conference on intelligent human-machine systems and cybernetics, IHMSC 2013. 1:7–10. http://doi.org/10.1109/IHMSC.2013.8
Zhou X, Li Q (2014) Multiple attribute decision making based on hesitant fuzzy Einstein geometric aggregation operators. J Appl Math. https://doi.org/10.1155/2014/745617
Zhu B, Xu Z, Zhang R, Hong M (2016) Hesitant analytic hierarchy process. Eur J Oper Res 250(2):602–614. https://doi.org/10.1016/j.ejor.2015.09.063