



# Application potential of combined fuzzy-TOPSIS approach in minimization of surface roughness, cutting force and tool wear during machining of CP-Ti grade II

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## Abstract

Titanium and its alloys possess numerous inherent qualities such as high corrosion resistance, temperature resistance, biocompatibility and high strength-to-weight ratio. These alloys are extensively used in widespread areas viz. spacecraft, aerospace, marine, medical, oil and gas, chemical processing industries, etc. around the globe. In spite of the aforesaid popularity, machining of titanium alloys is very costly and difficult due to their poor thermal conductivity and high chemical affinity. Low thermal conductivity causes rapid tool wear owing to excessive machining zone temperature. Tool failure at its pre-mature stage considerably curtails the surface quality of the end product. This situation necessitates an appropriate set of machining variables so that one can achieve high productivity without compromising the quality. Keeping these facts in mind, the present paper proposes fuzzy coupled with TOPSIS method to identify an optimal combination of process variables during turning of commercially pure titanium. Spindle speed, feed and depth of cut were selected as three input parameters, whereas surface roughness ( $R_a$ ), cutting force ( $F_c$ ) and flank wear ( $VB_c$ ) were the major responses. Experiments were performed according to Taguchi's  $L_{27}$  orthogonal array. Analysis of variance (ANOVA) test was performed to identify the most significant machining parameter and to verify the potential application of the proposed methodology. The results indicated that the fuzzy-TOPSIS method is capable of deal with both qualitative and quantitative criteria to reach at the best parametric combination during turning operation.

**Keywords** ANOVA · Surface roughness · Cutting force · Flank wear · Fuzzy-TOPSIS

## 1 Introduction

Machining operation, particularly turning process, has been the core of the production industry since the industrial revolution (Rao 2006). It is most widely used in aircraft, aerospace, automotive and medical industries. Good surface finish, higher tool life and lower cutting forces are primarily acknowledged as the basic manufacturing desires for turning operation (Davim and Antonio 2001). It is also observed that lower cutting force essentially results in an enhanced tool

life and surface quality. These three responses are signified as the leading quality attributes of a turning process. Lower cutting force, good surface finish and appreciable tool performance are crucial to achieve due to increased demand of low-cost products along with virtuous dimensional accuracy. The aforesaid objectives can be accomplished by adopting an optimization technique or process modeling. In general, turning operation comprises of different interactive process variables which strongly affects the responses discussed above. Therefore, modeling of these responses viz. surface roughness ( $R_a$ ), cutting force ( $F_c$ ) and tool wear ( $VB_c$ ) is quite difficult. On the other hand, several statistical, simulation and experimental studies using design of experiment (DOE)-based layout have been reported for the modeling of afore-discussed turning responses while machining a wide range of materials. Asiltürk and Neşeli (2012) determined optimal parametric combination of process variables when turning AISI 304 austenitic stainless steel with coated carbide inserts. Further, they extended the investigation by develop-

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ing a mathematical quadratic model for the estimation of different surface roughness characteristics, i.e., arithmetic mean roughness ( $R_a$ ) and maximum peak-to-valley height ( $R_z$ ) using response surface methodology (RSM). The results indicated that the proposed model was capable of producing satisfactory results and can be implemented to other metal machining operation. Lalwani et al. (2008) investigated the effects of cutting variables on cutting forces and surface roughness during turning of MDN 250 steel using coated ceramic inserts. They concluded that the studied performance measures largely affected by varied nature of cutting variables. In a different study conducted by Aouici et al. (2012), optimal cutting condition was obtained to get minimum surface roughness and cutting force while hard turning of AISI H11 steel with cubic boron nitride (CBN) tools. They also developed a mathematical model using RSM approach to predict the aforesaid turning responses. The results indicated that the estimated values were in good agreement with experimentally measured values. Rao et al. (2013) also investigated the influence of various machining parameters on  $R_a$  and  $F_c$ . In this study, feed rate was observed as the most prominent machining variable affecting both the responses. Hashmi et al. (2016) developed a quadratic model for the prediction of surface roughness using response surface analysis method. The suggested model showed that the surface quality of a turned part mainly depended on selected depth of cut. On the contrary, feed rate and spindle speed were found to be insignificant parameters. However, the observations might be limited to the selected range of cutting variables and cutting conditions. Tebassi et al. (2017) in their experimental investigation proposed two distinct prediction model for the estimation of surface roughness and cutting force while turning of Inconel 718. They developed and compared RSM-based quadratic model and artificial neural network (ANN)-based model in terms of their prediction efficiency. The results indicated that the ANN model was more precise by 10.1 and 24.83% in predicting  $F_c$  and  $R_a$ , respectively, than that of quadratic model.

It is evidently observed from the literature than an extensive research work has already been dedicated on statistical modeling of turning responses. However, the vague nature of the cutting variables and turning responses were not addressed adequately so far. This might be a reason behind a complex and unclear solution of several optimization problems dealing with multiple attributes in a real-time manufacturing system. In this situation, fuzzy set theory plays a key role in attaining the most suitable solution. Fuzzy linguistic variables allow the conversion of verbal expression into numerical quantities. In this way, the relationship between process variables and output responses can be described using fuzzy rules which consists of fuzzy linguistic variables rather than a complex mathematic model. Keeping all the above-discussed points in mind, the present paper adopts

three input variables viz. cutting speed, feed and depth of cut with three distinct levels (low, medium and high) to optimize a turning process having multiple attributes like surface roughness, cutting force and tool wear. Experiments were performed according to Taguchi's  $L_{27}$  orthogonal array. The fuzzy control rules were developed for each of the aforesaid attribute considering seven linguistic grades. An amalgamation of fuzzy with TOPSIS is introduced to obtain an optimal parametric setting during turning of CP-Ti grade II using uncoated carbide inserts in dry cutting environment.

## 2 Materials and methods

### 2.1 Design of experiment

Selection of an appropriate parametric combination has been identified as a complex issue in real-time manufacturing systems due to the interaction effects among various cutting variables. In this situation, optimization method plays key role in order to confirm the quality of the product in combination with production efficiency. An orthogonal array design (OAD) proposed by Taguchi has been widely implemented and suggested by many researchers around the globe. It helps in optimizing the quality characteristics through offering optimal parametric combinations (Huh et al. 2003; Anastasiou 2002; Dhavlikar et al. 2003; Kim et al. 2003). Moreover, Taguchi's OA investigates the influence of input variables on the specified performance measures in order to attain their best or optimal settings. In addition to that these arrays provide the best possible solution with reduced number of experimental trials without compromising the quality.

In the present investigation, three controllable factors, i.e., cutting speed, feed rate and depth of cut with three varied levels were selected to construct an orthogonal array as listed in Table 1.

### 2.2 Workpiece and cutting tool material

The work material selected in the present investigation was a cylindrical bar of commercially pure titanium (CP-Ti) grade II. The chemical composition of this material is 0.08% C, 0.03% N, 0.25% O, 0.30% Fe, 0.015% H and 99.3% Ti.

**Table 1** Domain of the present investigation

S. no.	Parameter	Unit	Level		
			Low	Medium	High
1.	Cutting speed ( $v$ )	m/min	35	70	105
2.	Feed rate ( $f$ )	mm/rev	0.05	0.10	0.15
3.	Depth of cut ( $d$ )	mm	0.1	0.2	0.3



Fig. 1 Photographic view of workpiece, cutting insert and tool holder

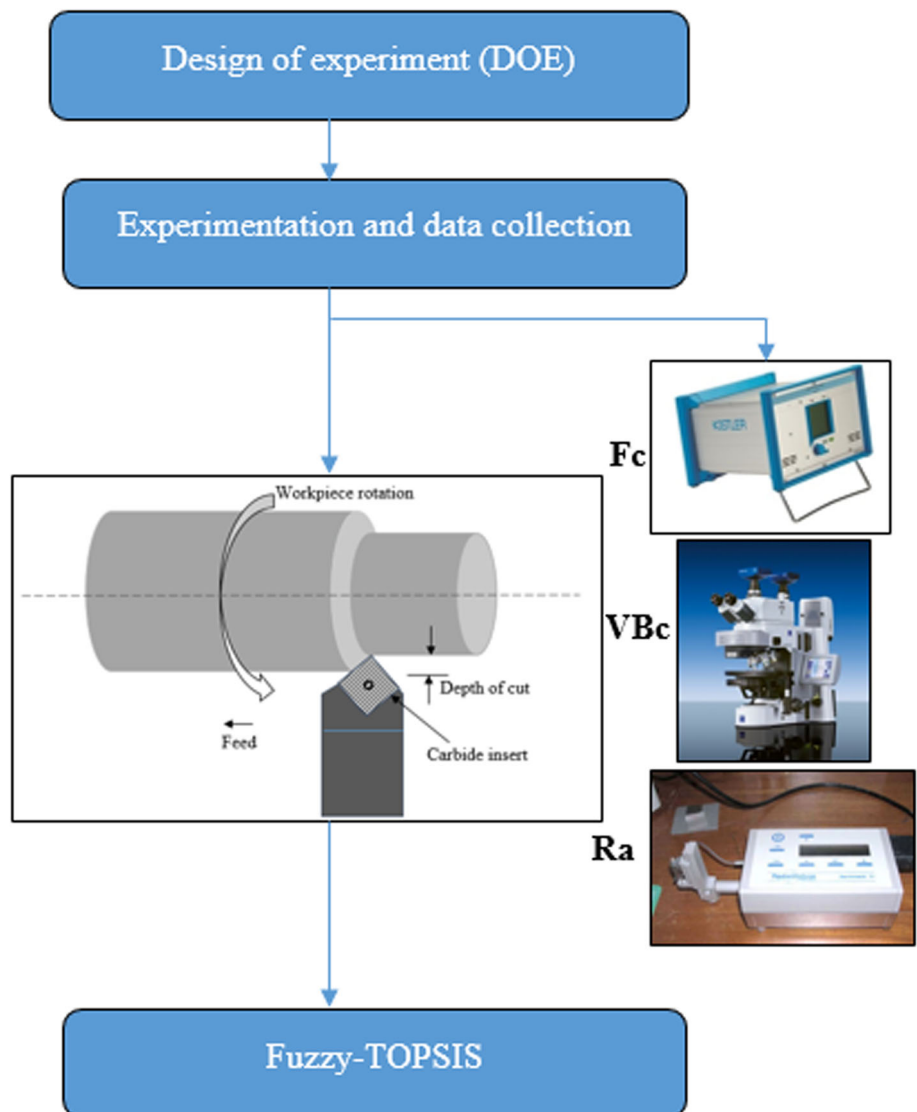
Square-shaped carbide inserts were used during machining of the work part. The selected cutting tools were manufactured by Kennametal having ISO designation: SNMG120408 (Grade: K313). These inserts were rigidly mounted on a right-handed tool holder: PSBNR 2020K12. Figure 1 depicts the

photographic view of the workpiece, tool holder and cutting tool material.

### 2.3 Experimental procedure

A round bar of CP-Ti grade II having diameter 50 mm and length 500 mm was turned on a heavy duty lathe manufactured by Hindustan Machine Tools (HMT), India. A series of experiments were conducted as per Taguchi's 3-factor-3-level  $L_{27}$  orthogonal array (OA). The machining length for each trial was fixed as 250 mm. Three distinct turning responses viz. cutting force ( $F_c$ ), surface roughness ( $R_a$ ) and flank wear ( $VB_c$ ) were investigated after each experimental run. The schematic view of the current investigation is shown in Fig. 2. The main cutting force was measured using a three-dimensional piezoelectric dynamometer manufactured by Kistler Instrument Corporation. In order to confirm the measurement accuracy, the force value was noted at three

Fig. 2 Schematic view of the present investigation



**Table 2** Experimental results

Run	Input parameters			Responses		
	$v$ (m/min)	$f$ (mm/rev)	$d$ (mm)	$F_c$ (N)	VBc (mm)	$R_a$ ( $\mu\text{m}$ )
1	35	0.05	0.1	64.307	0.081	0.448
2	35	0.05	0.2	112.433	0.099	0.658
3	35	0.05	0.3	96.482	0.222	0.886
4	35	0.10	0.1	132.488	0.103	0.728
5	35	0.10	0.2	107.529	0.086	0.628
6	35	0.10	0.3	75.353	0.231	0.930
7	35	0.15	0.1	123.480	0.098	1.303
8	35	0.15	0.2	75.103	0.098	1.028
9	35	0.15	0.3	93.527	0.165	1.375
10	70	0.05	0.1	73.654	0.193	0.648
11	70	0.05	0.2	113.966	0.240	0.673
12	70	0.05	0.3	98.077	0.224	0.761
13	70	0.10	0.1	135.287	0.156	0.982
14	70	0.10	0.2	109.733	0.092	0.950
15	70	0.10	0.3	65.606	0.229	0.846
16	70	0.15	0.1	126.140	0.145	1.030
17	70	0.15	0.2	76.727	0.110	1.428
18	70	0.15	0.3	94.791	0.195	1.154
19	105	0.05	0.1	74.663	0.109	0.770
20	105	0.05	0.2	115.807	0.123	0.835
21	105	0.05	0.3	99.532	0.281	0.828
22	105	0.10	0.1	136.277	0.141	1.129
23	105	0.10	0.2	110.800	0.216	0.771
24	105	0.10	0.3	67.400	0.230	0.547
25	105	0.15	0.1	126.932	0.224	0.999
26	105	0.15	0.2	81.371	0.238	1.073
27	105	0.15	0.3	96.908	0.252	0.942

different locations along the machining length and the average value was considered as final  $F_c$  value. On the other hand, a roughness testing device namely Taylor Hobson (Model: Surtronic 3+) was used to measure the value of arithmetic surface roughness of the machined part.  $R_a$  measurements were taken with a cut-off length and sampling length of 0.8 mm and 5 mm, respectively, and were recorded at eight different locations (roughly  $45^\circ$  apart) along the circumference of the turned part. Flank wear of the selected inserts was measured and examined under a stereo zoom microscope manufactured by Carl Zeiss (Model: Axio Cam ER<sub>c</sub> 5 s). Table 2 shows the domain adopted during this investigation along with the corresponding outcomes.

## 2.4 Fuzzy set theory: fuzzy interface system and fuzzy numbers

Fuzzy interface system (FIS) has been recognized as an efficient tool to deal with both numerical data and linguistic

knowledge, simultaneously. In fact, FIS is an effective routine in which the mapping from a given input to an output is performed with the help of certain fuzzy logics. In general, a fuzzy inference system comprises of four distinct components namely fuzzifier, inference engine, knowledge base and defuzzifier. The membership functions are primarily used by the fuzzifier to convert the crisp input to a linguistic variable, stored in the fuzzy knowledge base. Secondly, these fuzzy inputs are further converted to fuzzy output by performing fuzzy reasoning into the inference engine. Finally, the defuzzifier converts these fuzzy outputs attained from the aforesaid inference system, to a crisp value, utilizing the membership functions analogous to the ones used by the fuzzifier. Figure 3 depicts the schematic view of a fuzzy inference system.

In addition, selection of an appropriate membership function is of paramount importance in the fuzzification process. As already described in the above paragraph that fuzzification is a process of converting crisp values into fuzzy numbers.

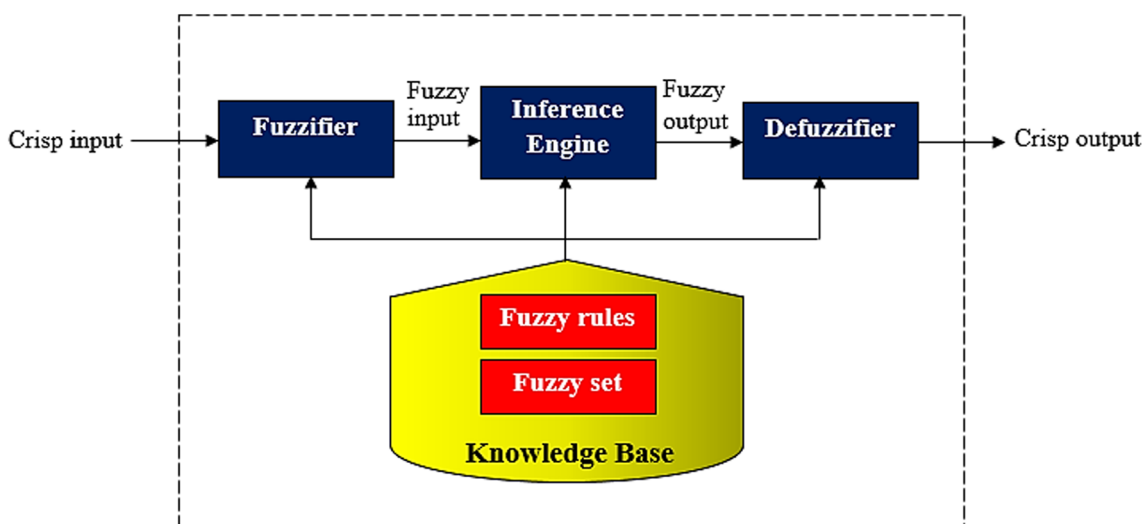


Fig. 3 Schematic view of fuzzy interface system

The conversion of fuzzy numbers is denoted with the help of membership function. A membership function is a curve which describes the membership value (which lies between 0 and 1) of each point, mapped in the input space. The aforesaid membership functions are usually constructed using plenty of basic functions viz. linear, quadratic and cubic polynomial curves, sigmoid curve, Gaussian distribution function, etc. On the other hand, the simplest membership function is constructed using straight lines. The triangular membership function is identified as the modest among all and portrayed by a center-based triplet tactic. In this approach, the triangles are formed by keeping an equal allowance between the lowest and highest points attached to the adjacent center. In this way, each input value belongs to not more than two fuzzy sets, and the summation of their membership degrees always remains equal to one. Thus, the above-discussed approach is also categorized as a simple, easily understandable, time-saving along with very less computational work, which in turn makes it an appropriate technique for wide application in real-time manufacturing system. In addition to that it has also been recognized as an efficient and capable approach for solving various multi-criteria decision-making problems. Figure 4 exemplifies a triangular fuzzy membership function. Some basic definitions of fuzzy numbers and fuzzy sets are described below.

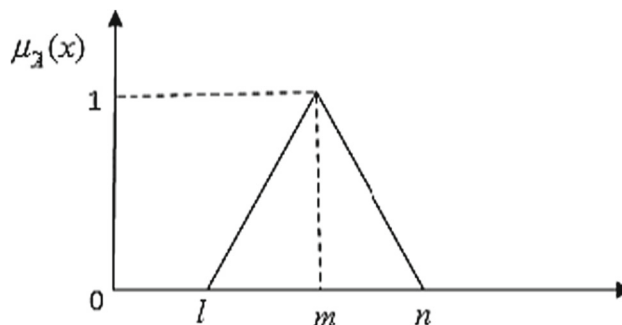


Fig. 4 A triangular fuzzy membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < l, \\ \frac{x-l}{m-l} & l \leq x \leq m, \\ \frac{n-x}{n-m} & m \leq x \leq n, \\ 0 & x > n \end{cases} \quad (1)$$

**Definition 3** Further, the addition, subtraction and division of any two triangular fuzzy numbers are also triangular fuzzy numbers. Let two triangular fuzzy numbers  $\tilde{A}_1 = (l_1, m_1, n_1)$  and  $\tilde{A}_2 = (l_2, m_2, n_2)$ , then the arithmetic operation between these two TFNs are defined as follows:

(1) Addition of fuzzy numbers

$$\tilde{A}_1(+)\tilde{A}_2 = (l_1 + l_2, m_1 + m_2, n_1 + n_2) \quad (2)$$

(2) Subtraction of fuzzy numbers

$$\tilde{A}_1(-)\tilde{A}_2 = (l_1 - l_2, m_1 - m_2, n_1 - n_2) \quad (3)$$

**Definition 1** A fuzzy set  $\tilde{A}$  in a universe of discourse  $X$  is characterized by a membership function  $\mu_{\tilde{A}}(x)$  which is termed as the grade of membership of  $x$  in  $\tilde{A}$ .

**Definition 2** The triangular fuzzy numbers (TFNs) can be represented as  $\tilde{A}=(l, m, n)$ , and the membership function of the fuzzy number  $\tilde{A}$  can be described as below (Eq. 1):

(3) *Multiplication of fuzzy numbers*

$$\tilde{A}_1(\times)\tilde{A}_2 = (l_1l_2, m_1m_2, n_1n_2) \tag{4}$$

(4) *Division of fuzzy numbers*

$$\tilde{A}_1(/)\tilde{A}_2 = (l_1/l_2, m_1/m_2, n_1/n_2) \tag{5}$$

(5) *Multiplication by a scalar number r*

$$\tilde{A}_1(\times)r = (l_1r, m_1r, n_1r) \tag{6}$$

**Definition 4** Let a triangular fuzzy number  $\tilde{A} = (l, m, n)$ , then the defuzzified value  $m(\tilde{A})$  can be evaluated by the equation given below (Eq. 7):

$$m(\tilde{A}) = \frac{l + m + n}{3} \tag{7}$$

**Definition 5** Let two triangular fuzzy numbers are  $\tilde{A}_1 = (l_1, m_1, n_1)$  and  $\tilde{A}_2 = (l_2, m_2, n_2)$ , then the distance between these two TFNs can be computed as follows:

$$d(\tilde{A}_1, \tilde{A}_2) = \sqrt{\frac{1}{3}(l_1 - l_2)^2 + (m_1 - m_2)^2 + (n_1 - n_2)^2} \tag{8}$$

### 2.5 Fuzzy-TOPSIS method

Fuzzy embedded with a multi-criteria decision-making (MCDM) technique has attracted many researchers toward decision science community. The effectiveness of fuzzy interface system (FIS) has also helped in the development of several MCDM-based techniques. Technique for order performance by similarity to ideal solution (TOPSIS) has been identified as one of the most appropriate approaches to deal with complex MCDM-based problems. TOPSIS method was initially introduced by Tzeng and Huang (2011) and is based on the assumption that the selected alternative should be the nearest to the positive ideal solution and farthest from the negative ideal solution (Tzeng and Huang 2011; Chen and Hwang 1992). In addition to that the concepts, computations and mathematical form of this method are simple and easily understandable (Byun and Lee 2005; Yang and Hung 2007). Thus, in the current investigation an amalgamation of fuzzy-TOPSIS is reported to exhibit the application potential of the approach in solving multi-attribute problems like turning. The proposed methodology utilizes a set of linguistic data to express the opinions of a decision maker. These linguistic data sets were further exploited for the construction of fuzzy decision matrix as well as normalized fuzzy decision matrix. In the next step, fuzzy positive and fuzzy negative ideal solutions were calculated by adopting a suitable weightage for

each output criteria. Finally, the distances of each alternative from the fuzzy positive and fuzzy negative ideal solution were calculated and the preference order of the alternatives is obtained (Zimmermann 2011). The steps involved in the proposed methodology are summarized below (Gok 2015; Khan and Maity 2016; Khan et al. 2017):

*Step 1* Formation of a fuzzy decision matrix in which each row represents one alternative and each column represents one attribute.

$$\tilde{D} = \begin{matrix} A_1 & \left[ \begin{matrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1j} & \dots & \tilde{x}_{1n} \\ A_2 & \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2j} & \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ A_i & \tilde{x}_{i1} & \tilde{x}_{i2} & \dots & \tilde{x}_{ij} & \dots & \tilde{x}_{in} \\ \vdots & \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ A_m & \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mj} & \dots & \tilde{x}_{mn} \end{matrix} \right. \end{matrix} \tag{9}$$

where each horizontal row ( $A_i$ ) denotes the possible alternatives ( $i=1, 2, \dots, m$ ) and each column represents the attributes ( $j=1, 2, \dots, n$ ). Similarly, the performance of alternative  $A_i$  and attribute  $X_j$  is represented by  $x_{ij}$ .

*Step 2* Obtain the normalized fuzzy decision matrix  $\tilde{R}$ . This can be represented as:

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n} \quad (i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n) \tag{10}$$

for the beneficial criteria, the value of  $\tilde{r}_{ij}$  can be calculated as

$$\tilde{r}_{ij} = \left( \frac{l_{ij}}{n_j^+}, \frac{m_{ij}}{n_j^+}, \frac{n_{ij}}{n_j^+} \right) \tag{11}$$

Similarly, the normalized value  $\tilde{r}_{ij}$  for a non-beneficial criteria can be calculated as

$$\tilde{r}_{ij} = \left( \frac{l_j^-}{n_{ij}^-}, \frac{l_j^-}{m_{ij}^-}, \frac{l_j^-}{l_{ij}^-} \right) \tag{12}$$

where  $n_j^+ = \max n_{ij}$  and  $l_j^- = \min l_{ij}$ .

*Step 3* Identify the weighted normalized fuzzy decision matrix  $\tilde{V}$ . This can be done by considering a user-defined weight for each criterion and multiplying them with the corresponding normalized fuzzy value.

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n} \quad (i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n) \tag{13}$$

$$\tilde{v}_{ij} = \tilde{r}_{ij} \times \tilde{w}_j \tag{14}$$



Step 4 Calculate the best fuzzy (positive ideal) and worst fuzzy (negative ideal) solution using Eqs. (15) and (16).

$$A^B = \{ (\max \tilde{n}_{ij} | j \in J); (\min \tilde{n}_{ij} | j \in J^c) \} \text{ Or}$$

$$A^B = \{ \tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_n^+ \} \tag{15}$$

$$A^W = \{ (\min \tilde{n}_{ij} | j \in J); (\max \tilde{n}_{ij} | j \in J^c) \} \text{ Or}$$

$$A^W = \{ \tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^- \} \tag{16}$$

where  $\tilde{v}_j^+ = (1, 1, 1)$ ,  $\tilde{v}_j^- = (0, 0, 0)$ ;  $j=1, 2, \dots, n$ ,  $J$  is the set of positive attributes and  $J^c$  is the set of negative attributes.

Step 5 Compute the separation measure corresponding to each alternative using Eqs. (17) and (18)

$$S_i^B = \sum_{j=1}^n d(\tilde{v}_{ij} - \tilde{v}_j^+) \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n \tag{17}$$

$$S_i^W = \sum_{j=1}^n d(\tilde{v}_{ij} - \tilde{v}_j^-) \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n \tag{18}$$

where  $S_i^B$  and  $S_i^W$  are the distances of each alternative from the best and worst fuzzy solution, respectively.

Step 6 Obtain the relative closeness to the ideal solution using Eq. (19)

$$C_i^+ = \frac{S_i^W}{S_i^B + S_i^W} \quad i = 1, 2, \dots, m; \quad 0 \leq C_i^+ \leq 1 \tag{19}$$

Step 7 Prepare a set of alternative by indicating the preference order according to the value of relative closeness in descending order. An alternative with maximum  $C_i^+$  is the most preferred and vice versa.

### 3 Results and discussion

#### 3.1 The fuzzy-TOPSIS analysis

In the present study, fuzzy technique for order performance by similarity to ideal solution (fuzzy-TOPSIS) method was employed to attain the most appropriate parametric setting with an aim to achieve improved productivity without compromising the quality. The major attention was given to minimize surface roughness, cutting force and tool wear during machining of CP-Ti grade II using uncoated carbide inserts. It is quite difficult to recommend the best parametric combination while turning the work material, due to the interactive effects between process variables and vague information about the performance measures. The aforementioned performance measures are designated as the most vital

**Table 3** Linguistic variables used for each criterion

Linguistic variable	Triangular fuzzy numbers (TFNs)
Very very low (VVL)	(0, 0, 0.1)
Very low (VL)	(0, 0.1, 0.3)
Low (L)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
High (H)	(0.5, 0.7, 0.9)
Very high (VH)	(0.7, 0.9, 1.0)
Very very high (VVH)	(0.9, 1.0, 1.0)

**Table 4** Relative weights of each criterion

Criteria	Decision maker	Fuzzy numbers
Surface roughness ( $R_a$ )	VH	(0.7, 0.9, 1.0)
Cutting force ( $F_c$ )	VH	(0.7, 0.9, 1.0)
Tool wear ( $VB_c$ )	VVH	(0.9, 1.0, 1.0)

**Table 5** Linguistic variables used for each alternative

Linguistic variable	Triangular fuzzy numbers (TFNs)
Very very poor (VVP)	(0, 0, 1)
Very poor (VP)	(0, 1, 3)
Poor (P)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Good (G)	(5, 7, 9)
Very good (VG)	(7, 9, 10)
Very very good (VVG)	(9, 10, 10)

qualitative parameters of turning operation which strongly influences the cost and productivity. In such situations, the decision maker uses linguistic terms viz. poor, good, very good, excellent, etc., to assess aforesaid quality measures in a real-time manufacturing system. In addition to that, the relative weights of various performance measures are also often described with the help of such linguistic values.

Thus, each alternative experiment was initially converted into the linguistic variables as listed in Table 3. This was done to identify the relative weights of all the reported criterion, i.e.,  $R_a$ ,  $F_c$  and  $VB_c$ , respectively (Table 4).

In the next step, an assessment was performed of all the alternatives according to the linguistic variables shown in Table 5. Seven different linguistic variables such as very very poor, very poor, good, very good, very very good were used. The results of the assessment process are portrayed in Table 6.

**Table 6** Results of the assessment

Alternative	Responses			Fuzzy linguistic variables		
	$F_c$ (N)	VBc (mm)	$R_a$ ( $\mu\text{m}$ )	$F_c$	VBc	$R_a$
1	64.307	0.081	0.448	VVG	VVG	VVG
2	112.433	0.099	0.658	P	VVG	VG
3	96.482	0.222	0.886	F	P	F
4	132.488	0.103	0.728	VVP	VVG	VG
5	107.529	0.086	0.628	P	VVG	VG
6	75.353	0.231	0.930	VG	VP	F
7	123.480	0.098	1.303	VP	VVG	VVP
8	75.103	0.098	1.028	VG	VVG	P
9	93.527	0.165	1.375	G	G	VVP
10	73.654	0.193	0.648	VVG	F	VG
11	113.966	0.240	0.673	P	VP	VG
12	98.077	0.224	0.761	F	P	G
13	135.287	0.156	0.982	VVP	G	F
14	109.733	0.092	0.950	P	VVG	F
15	65.606	0.229	0.846	VVG	VP	G
16	126.140	0.145	1.030	VVP	G	P
17	76.727	0.110	1.428	VG	VG	VVP
18	94.791	0.195	1.154	F	F	VP
19	74.663	0.109	0.770	VG	VG	G
20	115.807	0.123	0.835	VP	VG	G
21	99.532	0.281	0.828	F	VVP	G
22	136.277	0.141	1.129	VVP	G	P
23	110.800	0.216	0.771	P	P	G
24	67.400	0.230	0.547	VVG	VP	VVG
25	126.932	0.224	0.999	VVP	VP	F
26	81.371	0.238	1.073	VG	VP	P
27	96.908	0.252	0.942	F	VVP	F

Further, these assessment results were used to construct the fuzzy decision matrix by converting them into an appropriate fuzzy triangular number. Table 7 depicts the fuzzy decision matrix achieved after the aforesaid conversion process.

Fuzzy decision matrix, as illustrated in Table 7, was normalized using Eqs. (2–4) and the results are listed in Table 8. In the next step, the relevant weights of every performance criterion were multiplied with their corresponding values to obtain weighted normalized fuzzy decision matrix (Table 9).

The separation measures were calculated using Eqs. (9–10). These separation measures, i.e.,  $S_i^B$  and  $S_i^W$  represent the distance of each alternative from fuzzy positive ideal solution ( $A^B$ ) and fuzzy negative ideal solution ( $A^W$ ), respectively. The separation matrix consisting of separation measure of each of the alternative experiment is shown in Table 10. Afterward, the relative closeness

was evaluated using Eq. (11). Finally, preference ranking was assigned to each alternative by arranging the  $C_i^+$  value in descending order. From the aforesaid table, it can be visualized that, trial number 1 is the best alternative to attain minimum surface roughness, cutting force and tool wear. On the contrary, trial number 25 can be said as the worst alternative among 27 selected alternatives of this investigation. Thus, the lowest value of cutting speed (35 m/min), feed rate (0.05 mm/rev) and depth of cut (0.1 mm) offer the best parametric combination according to the proposed fuzzy-TOPSIS method. However, this might be limited to the studied range of cutting parameters.

### 3.2 Analysis of variance (ANOVA)

A statistical test also known as analysis of variance was conducted to identify which cutting variables were signif-



**Table 7** Fuzzy decision matrix

Alternative	Responses		
	$F_c$	VBc	$R_a$
1	9, 10, 10	9, 10, 10	9, 10, 10
2	1, 3, 5	9, 10, 10	7, 9, 10
3	3, 5, 7	1, 3, 5	3, 5, 7
4	0, 0, 1	9, 10, 10	7, 9, 10
5	1, 3, 5	9, 10, 10	7, 9, 10
6	7, 9, 10	0, 1, 3	3, 5, 7
7	0, 1, 3	9, 10, 10	0, 0, 1
8	7, 9, 10	9, 10, 10	1, 3, 5
9	5, 7, 9	5, 7, 9	0, 0, 1
10	9, 10, 10	3, 5, 7	7, 9, 10
11	1, 3, 5	0, 1, 3	7, 9, 10
12	3, 5, 7	1, 3, 5	5, 7, 9
13	0, 0, 1	5, 7, 9	3, 5, 7
14	1, 3, 5	9, 10, 10	3, 5, 7
15	9, 10, 10	0, 1, 3	5, 7, 9
16	0, 0, 1	5, 7, 9	1, 3, 5
17	7, 9, 10	7, 9, 10	0, 0, 1
18	3, 5, 7	3, 5, 7	0, 1, 3
19	7, 9, 10	7, 9, 10	5, 7, 9
20	0, 1, 3	7, 9, 10	5, 7, 9
21	3, 5, 7	0, 0, 1	5, 7, 9
22	0, 0, 1	5, 7, 9	1, 3, 5
23	1, 3, 5	1, 3, 5	5, 7, 9
24	9, 10, 10	0, 1, 3	9, 10, 10
25	0, 0, 1	0, 1, 3	3, 5, 7
26	7, 9, 10	0, 1, 3	1, 3, 5
27	3, 5, 7	0, 0, 1	3, 5, 7

**Table 8** Normalized fuzzy decision matrix

Alternative	Responses		
	$F_c$	VBc	$R_a$
1	0.9, 1.0, 1.0	0.9, 1.0, 1.0	0.9, 1.0, 1.0
2	0.1, 0.3, 0.5	0.9, 1.0, 1.0	0.7, 0.9, 1.0
3	0.3, 0.5, 0.7	0.1, 0.3, 0.5	0.3, 0.5, 0.7
4	0, 0, 0.1	0.9, 1.0, 1.0	0.7, 0.9, 1.0
5	0.1, 0.3, 0.5	0.9, 1.0, 1.0	0.7, 0.9, 1.0
6	0.7, 0.9, 1.0	0, 0.1, 0.3	0.3, 0.5, 0.7
7	0, 0.1, 0.3	0.9, 1.0, 1.0	0, 0, 0.1
8	0.7, 0.9, 1.0	0.9, 1.0, 1.0	0.1, 0.3, 0.5
9	0.5, 0.7, 0.9	0.5, 0.7, 0.9	0, 0, 0.1
10	0.9, 1.0, 1.0	0.3, 0.5, 0.7	0.7, 0.9, 1.0
11	0.1, 0.3, 0.5	0, 0.1, 0.3	0.7, 0.9, 1.0
12	0.3, 0.5, 0.7	0.1, 0.3, 0.5	0.5, 0.7, 0.9
13	0, 0, 0.1	0.5, 0.7, 0.9	0.3, 0.5, 0.7
14	0.1, 0.3, 0.5	0.9, 1.0, 1.0	0.3, 0.5, 0.7
15	0.9, 1.0, 1.0	0, 0.1, 0.3	0.5, 0.7, 0.9
16	0, 0, 0.1	0.5, 0.7, 0.9	0.1, 0.3, 0.5
17	0.7, 0.9, 1.0	0.7, 0.9, 1.0	0, 0, 0.1
18	0.3, 0.5, 0.7	0.3, 0.5, 0.7	0, 0.1, 0.3
19	0.7, 0.9, 1.0	0.7, 0.9, 1.0	0.5, 0.7, 0.9
20	0, 0.1, 0.3	0.7, 0.9, 1.0	0.5, 0.7, 0.9
21	0.3, 0.5, 0.7	0, 0, 0.1	0.5, 0.7, 0.9
22	0, 0, 0.1	0.5, 0.7, 0.9	0.1, 0.3, 0.5
23	0.1, 0.3, 0.5	0.1, 0.3, 0.5	0.5, 0.7, 0.9
24	0.9, 1.0, 1.0	0, 0.1, 0.3	0.9, 1.0, 1.0
25	0, 0, 0.1	0, 0.1, 0.3	0.3, 0.5, 0.7
26	0.7, 0.9, 1.0	0, 0.1, 0.3	0.1, 0.3, 0.5
27	0.3, 0.5, 0.7	0, 0, 0.1	0.3, 0.5, 0.7

icantly affecting the performance characteristics. The test was performed for a significance level of 95%. Table 11 describes the  $P$  values (Probability of significance) acquired during the aforesaid test for all cutting variables. In general, the model is said to be statistically significant, if the  $P$  value of a term seems  $\leq 0.05$  (for a significance level of 95%). From the ANOVA table, it is evident that depth of cut is the most influencing cutting variable on cutting force and tool wear, contributing 17.48 and 40.89%, respectively. However, the combined effect of feed rate and depth of cut was identified as the most prominent factor affecting the main cutting force with a percentage contribution of 77.77%. Similarly, feed rate was observed as the most effective machining parameter for attaining good surface finish due to its greater contribution (56.74%) in comparison with the other two factors, i.e., cutting speed and depth of cut. The ANOVA test was further extended to examine which cutting parameters were significantly influencing the multiple

performance measure, i.e., relative closeness value. Results of test (Table 11) indicate that feed rate is the most prominent factor followed by depth of cut and cutting speed. On the other hand, interactive effect of feed rate and depth of cut ( $f^*d$ ) was noticed to be more effective when compared to individual factor ( $v$ ,  $f$ , and  $d$ ) counterpart. Furthermore, high value of determination coefficient ( $R^2$ ) also explains the goodness of fit for the model at selected confidence level.

## 4 Conclusions

The present paper aimed at investigating the feasibility of fuzzy embedded TOPSIS method in attaining an optimal parametric combination while turning CP-Ti grade II using uncoated carbide inserts in dry cutting environment. Three distinct qualitative machinability aspects viz. cut-

**Table 9** Weighted normalized fuzzy decision matrix

Alternative	Responses		
	$F_c$	VB <sub>c</sub>	$R_a$
1	0.63, 0.9, 1.0	0.81, 1.0, 1.0	0.63, 0.9, 1.0
2	0.7, 0.27, 0.5	0.81, 1.0, 1.0	0.49, 0.81, 1.0
3	0.21, 0.45, 0.7	0.9, 0.3, 0.5	0.21, 0.45, 0.7
4	0, 0, 0.1	0.81, 1.0, 1.0	0.49, 0.81, 1.0
5	0.7, 0.27, 0.5	0.81, 1.0, 1.0	0.49, 0.81, 1.0
6	0.49, 0.81, 1.0	0, 0.1, 0.3	0.21, 0.45, 0.7
7	0, 0.9, 0.3	0.81, 1.0, 1.0	0, 0, 0.1
8	0.49, 0.81, 1.0	0.81, 1.0, 1.0	0.7, 0.27, 0.5
9	0.35, 0.63, 0.9	0.45, 0.7, 0.9	0, 0, 0.1
10	0.63, 0.9, 1.0	0.27, 0.5, 0.7	0.49, 0.81, 1.0
11	0.7, 0.27, 0.5	0, 0.1, 0.3	0.49, 0.81, 1.0
12	0.21, 0.45, 0.7	0.9, 0.3, 0.5	0.35, 0.63, 0.9
13	0, 0, 0.1	0.45, 0.7, 0.9	0.21, 0.45, 0.7
14	0.7, 0.27, 0.5	0.81, 1.0, 1.0	0.21, 0.45, 0.7
15	0.63, 0.9, 1.0	0, 0.1, 0.3	0.35, 0.63, 0.9
16	0, 0, 0.1	0.45, 0.7, 0.9	0.7, 0.27, 0.5
17	0.49, 0.81, 1.0	0.63, 0.9, 1.0	0, 0, 0.1
18	0.21, 0.45, 0.7	0.27, 0.5, 0.7	0, 0.9, 0.3
19	0.49, 0.81, 1.0	0.63, 0.9, 1.0	0.35, 0.63, 0.9
20	0, 0.9, 0.3	0.63, 0.9, 1.0	0.35, 0.63, 0.9
21	0.21, 0.45, 0.7	0, 0, 0.1	0.35, 0.63, 0.9
22	0, 0, 0.1	0.45, 0.7, 0.9	0.7, 0.27, 0.5
23	0.7, 0.27, 0.5	0.9, 0.3, 0.5	0.35, 0.63, 0.9
24	0.63, 0.9, 1.0	0, 0.1, 0.3	0.63, 0.9, 1.0
25	0, 0, 0.1	0, 0.1, 0.3	0.21, 0.45, 0.7
26	0.49, 0.81, 1.0	0, 0.1, 0.3	0.7, 0.27, 0.5
27	0.21, 0.45, 0.7	0, 0, 0.1	0.21, 0.45, 0.7

**Table 10** Separation matrix and relative closeness

Alternative	$S_i^B$	$S_i^W$	$C_i^+$	Remarks
1	0.552	2.656	0.828	Best alternative
2	0.963	2.257	0.701	
3	1.664	1.610	0.492	
4	1.392	1.794	0.563	
5	0.963	2.257	0.701	
6	1.772	1.473	0.454	
7	1.785	1.546	0.464	
8	0.963	2.257	0.701	
9	1.770	1.431	0.447	
10	1.075	2.173	0.669	
11	1.729	1.498	0.464	
12	1.518	1.780	0.540	
13	1.916	1.261	0.397	
14	1.231	1.957	0.614	
15	1.533	1.706	0.527	
16	1.874	1.286	0.407	
17	1.503	1.710	0.532	
18	1.829	1.564	0.461	
19	0.971	2.318	0.705	
20	1.364	2.071	0.603	
21	1.986	1.219	0.380	
22	1.874	1.286	0.407	
23	1.475	1.805	0.550	
24	1.318	1.898	0.590	
25	2.426	0.736	0.233	Worst alternative
26	1.729	1.498	0.464	
27	2.132	1.049	0.330	

ting force, tool wear and surface roughness were the major attention. The following conclusions may be drawn after successful completion of afore-discussed machining operation.

1. ANOVA test was applied to  $F_c$ ,  $R_a$ , VB<sub>c</sub> and  $C_i^+$ , to discover the individual and interaction effects of cutting variables. The results of the test indicated that depth of cut was the most prominent individual factor affecting  $F_c$  and VB<sub>c</sub>, whereas feed rate was found to be a significant factor affecting  $R_a$  and  $C_i^+$ . However, the interaction effect of feed and depth of cut ( $f^*d$ ) was observed more significant than that of any other individual factor for  $F_c$  and  $C_i^+$ .
2. The optimal combination of machining parameters was perceived at cutting speed of 35 m/min, feed rate of 0.05 mm/rev and depth of cut of 0.1 mm, which appeared in trial number 1. Thus, the above combination offered

minimum  $F_c$ ,  $R_a$  and VB<sub>c</sub> according to the proposed methodology. However, this might be limited to the studied range of machining parameters.

3. The proposed methodology, i.e., fuzzy coupled with TOPSIS approach, was found to be effective, modest and certainly comprehensible to solve the problems having multiple criteria.
4. The determination coefficients during the statistical analysis were acquired as 0.99, 0.87, 0.91 and 0.93 for  $F_c$ , VB<sub>c</sub>,  $R_a$  and  $C_i^+$ , respectively. Higher values of the coefficients (i.e., close to unity) indicated the fitness for good of the suggested model.
5. The combination of fuzzy-TOPSIS was found to be an efficient and satisfactory effort in achieving an optimal parametric setting within a specified cutting conditions. This may be useful to resolve other MCDM-based problems associated with different academic and industrial sectors.

**Table 11** Results of ANOVA test

Source	DOF	SS	MS	F value	P value	Contribution (%)
<i>F<sub>c</sub></i>						
<i>v</i>	2	46.8	23.39	4.36	0.050	0.34
<i>f</i>	2	465.6	232.82	43.42	0.000	3.44
<i>d</i>	2	2359.9	1179.95	220.05	0.000	17.48
<i>v*f</i>	4	36.9	9.23	1.72	0.238	0.27
<i>v*d</i>	4	49.6	12.41	2.31	0.145	0.36
<i>f*d</i>	4	10,500.0	2625.00	489.55	0.000	77.77
Error	8	42.9	5.36			0.32
Total	26	13,501.8			<i>R</i> <sup>2</sup> = 99.7%	100.00
<i>VB<sub>c</sub></i>						
<i>v</i>	2	0.0225	0.0113	6.55	0.021	21.80
<i>f</i>	2	0.0004	0.0002	0.13	0.884	0.39
<i>d</i>	2	0.0422	0.0211	12.24	0.004	40.89
<i>v*f</i>	4	0.0155	0.0038	2.25	0.153	15.02
<i>v*d</i>	4	0.0047	0.0011	0.69	0.621	4.55
<i>f*d</i>	4	0.0040	0.0010	0.58	0.684	3.88
Error	8	0.0138	0.0017			13.37
Total	26	0.1032			<i>R</i> <sup>2</sup> = 86.9%	100.00
<i>R<sub>a</sub></i>						
<i>v</i>	2	0.021	0.0107	0.60	0.572	1.36
<i>f</i>	2	0.8738	0.4369	24.36	0.000	56.74
<i>d</i>	2	0.0039	0.0019	0.11	0.899	0.25
<i>v*f</i>	4	0.1504	0.0376	2.10	0.173	9.77
<i>v*d</i>	4	0.2259	0.0564	3.15	0.047	14.67
<i>f*d</i>	4	0.1211	0.0303	1.69	0.245	7.86
Error	8	0.1434	0.0179			9.31
Total	26	1.5401			<i>R</i> <sup>2</sup> = 90.7%	
<i>C<sub>i</sub><sup>+</sup></i>						
<i>v</i>	2	0.0687	0.0344	8.71	0.010	14.48
<i>f</i>	2	0.1008	0.0504	12.78	0.003	21.25
<i>d</i>	2	0.0691	0.0346	8.76	0.010	14.57
<i>v*f</i>	4	0.0224	0.0056	1.42	0.311	4.72
<i>v*d</i>	4	0.0402	0.01006	2.55	0.121	8.47
<i>f*d</i>	4	0.1415	0.0354	8.97	0.005	29.83
Error	8	0.0316	0.0039			6.66
Total	26	0.4744			<i>R</i> <sup>2</sup> = 93.3%	100

**Compliance with ethical standards**

**Conflict of interest** All the authors declare that they have no conflict of interest.

**Ethical approval** This article does not contain any studies with human participants or animals performed by any of the authors.

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