Soft Computing (2018) 22:5007–5020
https://doi.org/10.1007/s00500-018-3234-6

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Pythagorean fuzzy engineering economic analysis of solar power plants

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Published online: 14 May 2018
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Abstract

The total world energy consumption is rising, and the alternative energy sources are sought to meet this demand. Renewable energy sources have distinctive features that make these sources environmental friendly and increase their share in total energy supply. Renewable energies, which are inexhaustible and renew themselves, are predicted to be the primary energy source for the future. The sun, which is the most important renewable energy source and the source of other energies, is also used for direct and indirect energy generation. In order to realize investments in solar energy systems that require high initial investment, their economic suitability must be assessed appropriately. Life cycle cost (LCC) and levelized cost of energy (LCOE) methods are widely used in economic evaluation and comparison of the large-scale solar energy system. Yet, solar energy investment decisions involve uncertainty and imprecision due to the vagueness in production levels and energy prices. An ample economic analysis should be able to evaluate the uncertainty and consider the dynamic costs and benefits. Pythagorean fuzzy sets are excellent tools for dealing with uncertainty and imprecision inherent in a system. In this study, the Pythagorean fuzzy set theory is applied so that the uncertainties and the opinions of the decision makers are more realistically incorporated into the economic analysis. The proposed Pythagorean LCC and LCOE methods enable dealing with the solar energy investments with fuzzy parameters. Alternative energy systems with different technological features and economic conditions can be more accurately compared using the proposed method.

1 Introduction

The technological developments and the increase in the population rise the need for energy. The fossil fuels, primary sources of energy, are currently cost-effective, but they are limited. Carbon-based fuels produce carbon dioxide and other greenhouse gases, and the rise in the greenhouse gases weakens the ozone layer and causes harmful radiation that endangers the food chain and ecological order (Kalogirou 2013). The ecological problems and need for new energy resources have enhanced the interest on alternative energy sources. Hydraulics, wind, solar, geothermal and biomass are the non-fossil energy sources and defined as renewable energy sources (ENRM 2017b). There is a growing interest in renewable energy sources since they create environmental and social benefits (USEIA 2017a).

Sun is one of the leading energy providers for most of the renewable and conventional generated energy (except nuclear and tidal energies) (Çoban and Onar 2017). Additional to its secondary usage, solar energy is directly used for generating energy. The newly developed technologies enhance solar energy usage. Solar energy investments as costly, especially the initial investment costs, are very high, which limits the usage of solar energy. Evaluating both the costs and benefits of solar energy can reveal the economic value of solar energy investments. Such an analysis enables selecting the most suitable locations and conditions for solar energy investments. The economic analyses and evaluations to be made are of critical importance at this stage. On the other hand, predicting both the revenue generated from solar energy and the costs initiated by the solar energy generation is hard since both the generated energy and energy prices are highly uncertain (Zatzman 2012). Fuzzy sets are excellent tools for modeling uncertainty.

Pythagorean fuzzy sets (Yager and Abbasov 2013) initially developed by Atanassov in 1986 (Atanassov 1989)

Communicated by C. Kahraman.

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as second-type intuitionistic fuzzy sets are the extensions of intuitionistic fuzzy sets and useful for representing the uncertainty inherent in a system. Similar to the intuitionistic fuzzy sets they enable defining both the membership and non-membership values for an element, but unlike intuitionistic fuzzy sets, the square sum of the membership and non-membership degrees is maximum one. This advantage of Pythagorean fuzzy sets provides a stronger representation of uncertainty than intuitionistic fuzzy sets.

In this study, Pythagorean fuzzy sets are used for engineering analyses. To the best of our knowledge, this is the first study that uses Pythagorean fuzzy sets in engineering economic analyses. This method is applied to a solar energy investment decision. More realistic results are achieved due to a better representation of uncertainties.

The rest of the paper is organized as follows: The following section refers to renewable energy, solar energy and economic bases of solar energy. In Sect. 3, intuitionistic fuzzy sets, engineering economic and basic Pythagorean fuzzy operations are discussed as preliminaries of Pythagorean fuzzy sets. In Sect. 4, Pythagorean fuzzy engineering economic methods and their parameter explained for large-scale solar energy systems. In Sect. 5, the developed models are applied on a sample solar energy system, and general evaluation and future studies are concluded in conclusion section.

2 Solar energy and economic bases

2.1 Renewable energy and solar energy

The United Nations estimate that the world population, which is about 8 billion in 2017, will rise to about 10 billion in 2050 and 16 billion in 2100 (UN 2017) (Fig. 1). The growing world population and the economy are expected to increase the global energy demand by about 37% by 2040. Fossil-based energy sources are used as primary energy source to meet the growing energy need. In 2040, the world energy supply is expected to be met by oil, gas, coal and low carbon energy sources (International Energy Agency (IEA) 2017) (Fig. 1). The use of fossil-based resources in the energy production process has serious problems: carbon dioxide emission, pollution, greenhouse effect, global warming, sulfur dioxide emission that causes acid rain. The use of fossil-based resources to meet energy demands causes economic and social problems as well as these environmental-based problems. The development of alternative energy sources against the depleting fossil energy sources is critical to meet the demand for energy for the future (BP 2017).

Renewable energy (solar, wind, geothermal, hydropower, biomass and hydroenergy) derived from existing energy flows in the natural processes emerges as the most important alternative energy source (Fig. 2). Renewable energy technologies constitute a significant part of the efforts to meet the global energy supply from low carbon energy sources. Global subsidies of $120 billion were provided in 2013 to improve renewable energies (International Energy Agency (IEA) 2017). Renewable energy sources (wind, biomass, solar energy, geothermal) planned to take the place of fossil-based (energy, oil and natural gas) energy resources have disadvantages of being expensive and less reliable (Conkling 2011).

The sun, which is the primary source of all fossil and non-fossil energy in the world (except nuclear, geothermal and tidal energies), is the most important renewable energy source with its environmental protection and low operational costs. The main advantages of solar energy are easy accessibility, high supply potential, inexhaustible, no release of greenhouse and other harmful gases, silence and the source of all renewable energies (except tidal and geothermal) (ENRM 2017a). The main disadvantages of solar energy can be listed as follows: intermittency, seasonal variation of energy potential, unpredictability, geographical obstacles, lack solar energy and storage. The sun rays reaching the earth are uti-

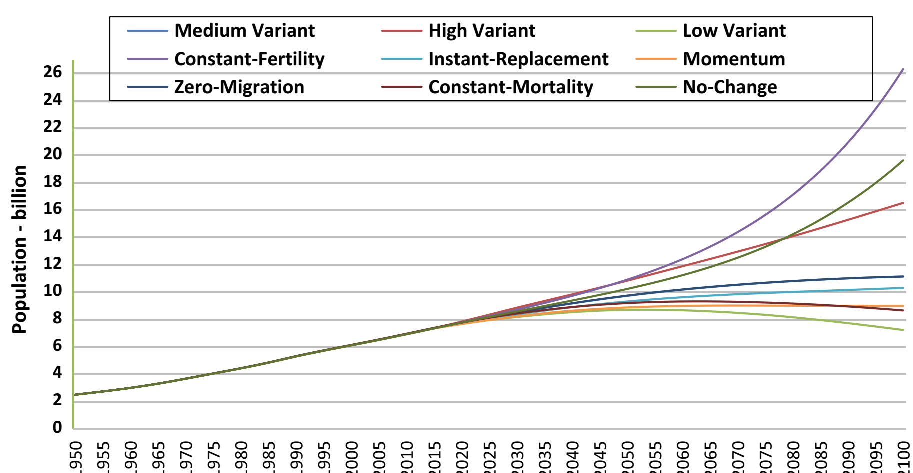


Fig. 1 Trend in world total population (UN 2017)

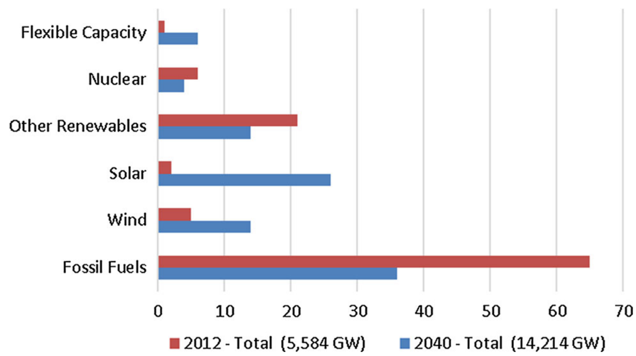


Fig. 2 Annual capacity additions for renewable energy (Finance 2015)

lized in the energy production directly or indirectly methods (Kalogirou 2013; ENRM 2017a). Solar energy technologies that differ according to the method, material and technological characteristics can be gathered in two main categories: photovoltaic (PV) and thermal solar technologies. Photovoltaic solar technology is a method of converting solar light directly into electricity in photovoltaic cells composed of semiconducting materials. Thermal solar technology is the direct use of thermal energy from solar energy or conversion of thermal liquid to electrical energy (ENRM 2017b).

PV technology works by converting photovoltaic energy into solar electricity (photovoltaic principle). The use of different materials (crystal silicon, cadmium telluride (CdTe) gallium arsenide (GaAs), amorphous silicon, optical concentrating cells) in the production of photovoltaic cells changes the usage and properties of the PV system (ENRM 2017a). The change in the physical properties of the system equipment also changes the life cycle (25–40 years), installation, operation and maintenance costs of the PV solar system (NREL 2017) (Fig. 3).

In addition to material properties, solar radiation collection and energy storage are other important factors affecting the efficiency of PV systems. PV systems are the most common solar energy generation method for small and large scale (Bolinger et al. 2017). The amount of sunlight reaching photovoltaic cells is increased with concentrated systems and concentrated photovoltaic (CPV) systems, which allows more energy production, and emerges as a new technology. Achieving 46% energy efficiency in CPV systems has increased expectations for the development of more efficient solar energy technologies (World Energy Council (WEC) 2016).

Thermal solar technology is divided into low- and high-temperature systems according to the reached thermal value. The technology used in low-temperature systems such as planetary solar collectors, solar vacuum collectors, solar pools, solar chimneys, water treatment systems, product drying and greenhouses, solar furnaces is more straightforward, and the solar energy obtained from the system is relatively low. High-temperature systems have high technological properties, and the system reaches very high thermal values. In these systems, high thermal values are obtained by using concentrator constructions which focus solar rays to a certain point (parabolic trough collectors, parabolic dish systems, central receiver systems, fresnel trough technology) (ENRM 2017a). The system fluid, which is heated to high temperatures by the concentrating system, allows the water to evaporate through the channels. The evaporated water activates the system turbines and provides electrical energy that is defined as concentrating solar power (CSP). PV and CSP systems are the most common methods for individual and commercial use regarding technological development and economic superiority (Fig. 4).

2.2 Economic requirements of solar energy systems

Solar energy systems similar to the other renewable energy investments initiate high initial costs but low operating costs. The economic analysis of solar investments is done by comparing the calculated initial investment cost that includes the purchase and installation of solar energy equipment with estimated future operating costs. The investment cost of solar energy is compared with the future equivalent fuel bill or the gain from the sale of the energy generated (Timilsina et al. 2012). The investment made for solar energy systems is aimed to reduce the future fuel consumption.

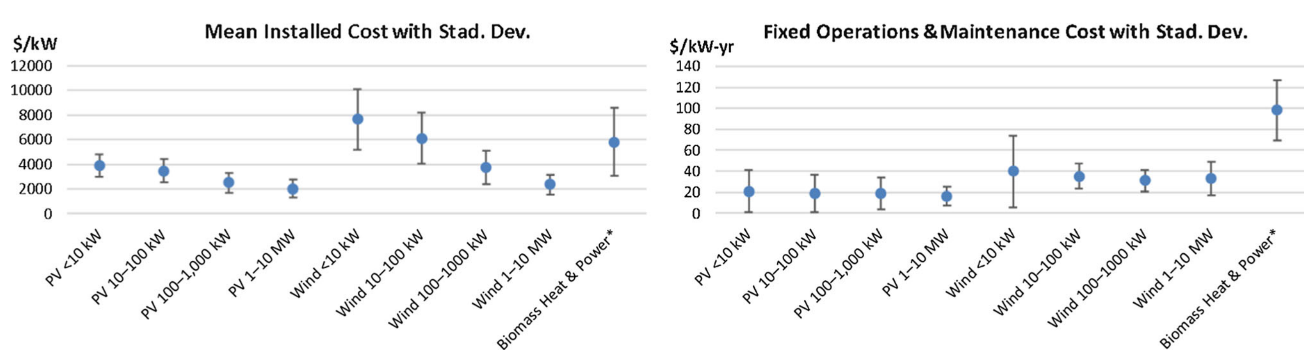


Fig. 3 Cost variation ranges according to PV system specifications (NREL 2017)

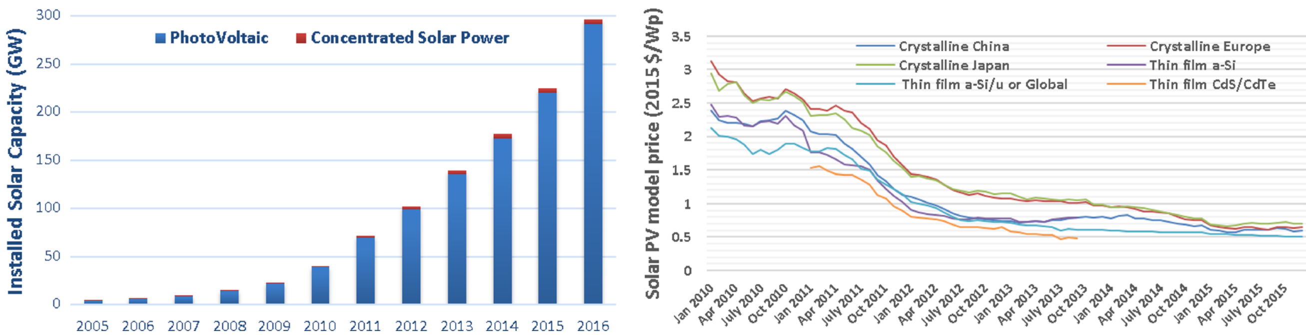


Fig. 4 Global installed solar capacity, 2005–2016 (kW) and trend in solar PV module prices (World Energy Council (WEC) 2016)

The overall cost of solar energy systems includes equipment costs, the installation costs, hardware costs, labor costs and operating costs. It is essential to consider the interest on borrowing money, maintenance, taxes (income and property), resale of equipment, insurance and other operating expenses in investment calculations (Crawley 2016). Hence, life cycle saving methods, which allow the time value of money, must be considered and the cost range should be examined in detail.

Usually, solar energy economic analyses focus on finding the lowest cost method that meets the energy need and these methods consider only the solar energy production (Timilsina et al. 2012). The most important problem encountered in the economic analysis of solar energy systems is to determine the size of the most suitable solar energy system that meets the lowest cost of solar energy production and the storage of the energy obtained.

The most significant disadvantage of renewable energy technologies that are developed as an alternative to fossil fuels is a high initial investment cost. This disadvantage is tackled with national and international political and economic support (Dahl 2015). Although economic supports encourage investors to invest in renewable energies, general economic evaluations should be done for investments to be successful in the long run. Solar energy systems similar to the other renewable energy investments initiate high initial costs but low operating costs. As a result of this characteristic, the economic analysis of solar investments is done by comparing the calculated initial investment cost that includes the purchase and installation of solar energy equipment with estimated future operating costs (Crawley 2016). The overall cost of solar energy systems includes equipment costs, the installation costs, hardware costs, labor costs and operating costs. Also, it is essential to consider the interest on borrowing money, maintenance, taxes (income and property), resale of equipment, insurance and other operating expenses in investment calculations. However, the most important economic problem of solar energy systems is to determine the size of the most suitable solar energy system that meets the lowest cost of solar energy production and the storage of the energy obtained.

The investment cost of solar energy is compared with the future equivalent fuel bill or the gain from the sale of the energy generated. The investment made for small-scale systems is aimed to reduce the future fuel consumption (Timilsina et al. 2012). High installation and total operating costs should be balanced against the gain from solar energy generation. Usually, solar energy economic analyses focus on finding the lowest cost method that meets the energy need and these methods consider only the solar energy production (ENRM 2017a; Timilsina et al. 2012). Hence, life cycle saving methods, which allow the time value of money, must be taken into account and the cost range should be examined in detail.

Large-scale solar energy systems (on-grid or off-grid) are actively involved in meeting rising energy demands. Large-scale systems whose processing principles resemble small-scale systems have more initial and annual costs than small systems because of the size difference. Improvements in solar energy technology and financial and technical support provided by national/international institutions also encourage the installation of large-scale solar energy facilities (e.g., Tengger Desert Solar Park—1500 MW—China, Kurnool Ultra Mega Solar Park—900 MW—India, Insure 2017). The realization of large-scale projects requires economic analysis methods that ensure the proper planning of financial resources. The economic analysis aims to realize solar projects with the lowest risk by accurately anticipating the vague and uncertain future expectations. Economic calculations made through the project life cycle also allow comparative assessment of alternative conditions. In general, large-scale projects (e.g., PV-Si, CPV, CSP, thin film) are preferred in solar energy generation due to their financial advantages (ENRM 2017a; Bolinger et al. 2017). When PV and CSP facilities are compared at the same nominal power and environmental conditions, the economic outcome of the CSP is more significant, and the occupied area is less than that of PV systems. However, the initial investment and regular maintenance costs for CSP facilities were found to be very

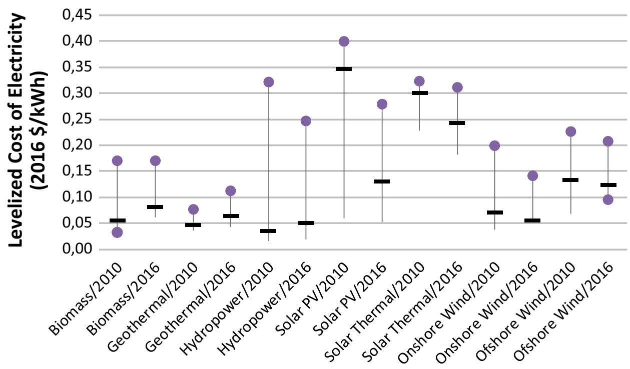


Fig. 5 Trends in LCOE of electricity in the period 2010–2016 (World Energy Council (WEC) 2016)

high compared to PV systems. It is hard to define priorities since environmental and technical conditions are different in the investment decisions (Vergura and Lameira 2011).

Life cycle cost (LCC) analysis and levelized cost of energy (LCOE) values are critical parameters used in the evaluation and comparison of large-scale solar energy projects. LCC analysis is based on the assessment of all costs incurred at the beginning and during the project lifetime (Kahraman et al. 2015). The LCC assessment is based on the current value of total costs for each alternative solar project (Crawley 2016). LCOE is defined as the average cost per kWh of the useful electricity generated by the solar energy system. The LCOE value calculated for the project is compared with the market price to determine the suitability and acceptance of the project (USEIA 2017a; Energy HOMER 2017) (Fig. 5). If the LCOE value is higher than the market energy price, unit margin becomes negative, and the project is rejected; otherwise, the project is accepted (USEIA 2017a).

3 Pythagorean fuzzy sets (PFS)

3.1 Preliminaries

3.1.1 Intuitionistic fuzzy sets

Intuitionistic fuzzy sets (IFS) introduced in 1986 by Atanassov are the generalization of the fuzzy set (FS) concept (Atanassov 1986). The membership and non-membership degrees are real numbers between 0 and one as in fuzzy sets. Intuitionistic fuzzy sets (IFSs) in the X fixed crisp set are defined as an objective of the following form (Atanassov 1999).

$$\tilde{A} = \left\{\left(x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\right) | x \in X\right\} \tag{1}$$

$\mu_{\tilde{A}}(x)$ is the degree of membership of x in $\tilde{A}$ and $v_{\tilde{A}}(x)$ is the degree of non-membership of x in $\tilde{A}$ in the [0, 1] interval, and the following condition is satisfied as

$$0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1, \forall x \in X \tag{2}$$

$$\mu_{\tilde{A}}: X \rightarrow [0, 1], v_{\tilde{A}}: X \rightarrow [0, 1] \tag{3}$$

$\tilde{A}$ and $\tilde{B}$ are two IFSs on a universe X, and some basic relations and operations are defined as follows

$$\tilde{A} \cup \tilde{B} = \left\{\left(x, \max\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right), \min\left(v_{\tilde{A}}(x), v_{\tilde{B}}(x)\right)\right) | x \in X\right\} \tag{4}$$

$$\tilde{A} \cap \tilde{B} = \left\{\left(x, \min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right), \max\left(v_{\tilde{A}}(x), v_{\tilde{B}}(x)\right)\right) | x \in X\right\} \tag{5}$$

$$\tilde{A}' = \left\{\left(x, v_{\tilde{A}}(x), \mu_{\tilde{A}}(x)\right) | x \in X\right\} \text{ (negation, complement)} \tag{6}$$

where Type-I fuzzy sets can be defined as

$$\left\{\left(x, \mu_{\tilde{A}}(x), 1 - \mu_{\tilde{A}}(x)\right) | x \in X\right\} \tag{7}$$

$$\tilde{A} + \tilde{B} = \left\{\left(x, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - \mu_{\tilde{A}}(x)\mu_{\tilde{B}}(x), v_{\tilde{A}}(x) v_{\tilde{B}}(x)\right) | x \in X\right\} \tag{8}$$

$$\tilde{A}.\tilde{B} = \left\{\left(x, \mu_{\tilde{A}}(x)\mu_{\tilde{B}}(x), v_{\tilde{A}}(x) + v_{\tilde{B}}(x) - v_{\tilde{A}}(x) v_{\tilde{B}}(x)\right) | x \in X\right\} \tag{9}$$

Subtraction and division operations can be defined for given IFSs A and B as follows (Atanassov and Riecan 2006):

$$\tilde{A} : \tilde{B} = \left\{\left(x, \mu_{\tilde{A}:\tilde{B}}(x), v_{\tilde{A}:\tilde{B}}(x)\right) | x \in X\right\} \tag{10}$$

where

$$\mu_{\tilde{A}:\tilde{B}}(x) = \begin{cases} \frac{\mu_{\tilde{A}}(x)}{\mu_{\tilde{B}}(x)}, & \text{if } \mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x) \text{ and } v_{\tilde{A}}(x) \geq v_{\tilde{B}}(x) \text{ and } \mu_{\tilde{B}}(x) > 0 \text{ and } \mu_{\tilde{A}}(x)\pi_{\tilde{B}}(x) \leq \mu_{\tilde{B}}(x)\pi_{\tilde{A}}(x) \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

$$v_{\tilde{A}:\tilde{B}}(x) = \begin{cases} \frac{v_{\tilde{A}}(x) - v_{\tilde{B}}(x)}{1 - v_{\tilde{B}}(x)}, & \text{if } \mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x) \text{ and } v_{\tilde{A}}(x) \geq v_{\tilde{B}}(x) \text{ and } \mu_{\tilde{B}}(x) > 0 \text{ and } \mu_{\tilde{A}}(x)\pi_{\tilde{B}}(x) \leq \mu_{\tilde{B}}(x)\pi_{\tilde{A}}(x) \\ 1, & \text{otherwise} \end{cases} \tag{12}$$

$$\tilde{A} - \tilde{B} = \left\{\left(x, \mu_{\tilde{A}-\tilde{B}}(x), v_{\tilde{A}-\tilde{B}}(x)\right) | x \in X\right\} \tag{13}$$

where

$$\mu_{\tilde{A}-\tilde{B}}(x) = \begin{cases} \frac{\mu_{\tilde{A}}(x)-\mu_{\tilde{B}}(x)}{1-\mu_{\tilde{B}}(x)}, & \text{if } \mu_{\tilde{A}}(x) \geq \mu_{\tilde{B}}(x) \text{ and } v_{\tilde{A}}(x) \leq v_{\tilde{B}}(x) \text{ and } v_{\tilde{B}}(x) > 0 \text{ and } v_{\tilde{A}}(x)\pi_{\tilde{B}}(x) \leq v_{\tilde{B}}(x)\pi_{\tilde{A}}(x) \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

$$v_{\tilde{A}-\tilde{B}}(x) = \begin{cases} \frac{v_{\tilde{A}}(x)}{v_{\tilde{B}}(x)}, & \text{if } \mu_{\tilde{A}}(x) \geq \mu_{\tilde{B}}(x) \text{ and } v_{\tilde{A}}(x) \leq v_{\tilde{B}}(x) \text{ and } v_{\tilde{B}}(x) > 0 \text{ and } v_{\tilde{A}}(x)\pi_{\tilde{B}}(x) \leq v_{\tilde{B}}(x)\pi_{\tilde{A}}(x) \\ 1, & \text{otherwise} \end{cases} \tag{15}$$

$$\tilde{A} * \tilde{B} = \left\{\left(x, \frac{\mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x)}{2\left(\mu_{\tilde{A}}(x)\mu_{\tilde{B}}(x) + 1\right)}, \frac{v_{\tilde{A}}(x) + v_{\tilde{B}}(x)}{2\left(v_{\tilde{A}}(x)v_{\tilde{B}}(x) + 1\right)}\right) \middle| x \in X\right\} \tag{16}$$

$\pi_{\tilde{A}}(x)$ represents the degree of non-determinacy (i.e., uncertainty) of the element $x \in X$ to the intuitionistic fuzzy set $\tilde{A}$ and its value is defined as:

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - v_{\tilde{A}}(x) \tag{17}$$

The degree of non-determinacy is described as zero in the ordinary fuzzy set for every $x \in X; \pi_A(x) = 0$. In this context, every ordinary fuzzy set is defined as follows

$$\left\{\left(x, \mu_{\tilde{A}}(x), 1 - \mu_{\tilde{A}}(x)\right) | x \in X\right\} \tag{18}$$

Multiplication of an IFS with a natural number n and nth power of an IFS is defined as (De et al. 2000):

$$n.\tilde{A} = \left\{\left(x, 1 - (1 - \mu_A(x))^n, (v_A(x))^n\right) | x \in X\right\} \tag{19}$$

$$\tilde{A}^n = \left\{\left(x, (\mu_A(x))^n, 1 - (1 - v_A(x))^n\right) | x \in X\right\} \tag{20}$$

3.1.2 Definitions and operations of Pythagorean fuzzy sets

Non-standard second-order fuzzy clusters such as intuitionistic fuzzy (Atanassov 1986, 2012) and interval-valued fuzzy (Mendel and John 2002; Mendel and Wu 2010) are referenced for more accurate acquisition and modeling of user-defined membership grades that are critical to the use of fuzzy clusters (Yager 2013). Pythagoras fuzzy sets developed by Yager as a new class of non-standard fuzzy subsets allow uncertainty in the specification of membership levels (Yager and Abbasov 2013). Nevertheless, Pythagorean fuzzy set (PFS) statement was first expressed and graphically defined as “second-type IFS” (2-IFS) by Atanassov in 1989 (Atanassov 1989).

Proposed PFS provides a condition that the sum of the squares of membership grade and non-membership is less than or equal to 1. Pythagorean membership grades allow for non-standard membership grades to be represented larger than intuitionistic membership grades (Yager 2013). The PFS allows the user to determine uncertainties in the real world better and more accurately model these uncertainties than IFS (Peng et al. 2017).

The main reason for the difference between the PFN and IFN is that the conditional constraints are different (Fig. 5). Each IFN is a PFN and that each PFN is not an IFN.

PFS $\tilde{A}$ in X that is non-empty set is defined as the following form:

$$\tilde{A} = \left\{\left(x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\right) | x \in X\right\} \tag{21}$$

where membership function and non-membership function of $\tilde{A}$ are denoted as $\mu_{\tilde{A}}(x) : X \to [0, 1]\, v_{\tilde{A}}(x) : X \to [0, 1]$ for each $x \in X$, respectively, and provide the following condition.

$$0 \leq \left(\mu_{\tilde{A}}(x)\right)^2 + \left(v_{\tilde{A}}(x)\right)^2 \leq 1 \tag{22}$$

It is clear that for all real numbers a, b in the $[0, 1]$ interval, if $0 \leq a+b \leq 1$, then $0 \leq a^2+b^2 \leq 1$. Therefore, it can be said that all intuitionistic fuzzy sets (IFSs) are Pythagorean fuzzy sets (PFS). The redefinition of second-type IFS by Yager as “Pythagorean fuzzy set” (Yager 2013) allows to generate the creation of new models and operation.

The degree of indeterminacy (uncertainty) of an element $x \in X$ to the PFS $\tilde{A}$ is defined as

$$\pi_{\tilde{A}}(x) = \sqrt{1 - \mu_{\tilde{A}}(x)^2 - v_{\tilde{A}}(x)^2} \tag{23}$$

3.1.3 Basic operations of PFSs

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)) | x \in X\}$ and $\tilde{B} = \{(x, \mu_{\tilde{B}}(x), v_{\tilde{B}}(x)) | x \in X\}$ are Pythagorean fuzzy sets (PFSs) and $\lambda > 0$

- Distance between $\tilde{A}$ and $\tilde{B}$, $d(\tilde{A}, \tilde{B})$ (Zhang and Xu 2014):

$$d\left(\tilde{A},\tilde{B}\right)=1/2\left(\left|(\mu_{\tilde{A}})^2-(\mu_{\tilde{B}})^2\right|+\left|(v_{\tilde{A}})^2-(v_{\tilde{B}})^2\right|+\left|(\pi_{\tilde{A}})^2-(\pi_{\tilde{B}})^2\right|\right) \tag{24}$$

- Score function of $\tilde{A}$, $s(\tilde{A})\in[-1,1]$ (Zhang and Xu 2014):

$$s\left(\tilde{A}\right)=(\mu_{\tilde{A}})^2-(v_{\tilde{A}})^2 \tag{25}$$

- Accuracy function of $\tilde{A}$, $a(\tilde{A})\in[-1,1]$ (Zhang and Xu 2014):

$$a\left(\tilde{A}\right)=(\mu_{\tilde{A}})^2+(v_{\tilde{A}})^2 \tag{26}$$

- Arithmetic operations (Zhang and Xu 2014):

$$\tilde{A}\oplus\tilde{B}=\left(\sqrt{(\mu_{\tilde{A}}(x)^2+\mu_{\tilde{B}}(x)^2-\mu_{\tilde{A}}(x)^2\mu_{\tilde{B}}(x)^2},v_{\tilde{A}}(x)\,v_{\tilde{B}}(x)\right) \tag{27}$$

$$\tilde{A}\otimes\tilde{B}=\left(\mu_{\tilde{A}}(x)\,\mu_{\tilde{B}}(x),\sqrt{v_{\tilde{A}}(x)^2+v_{\tilde{B}}(x)^2-v_{\tilde{A}}(x)^2\,v_{\tilde{B}}(x)^2}\right) \tag{28}$$

$$\lambda\tilde{A}=\left(\sqrt{1-(1-\mu_{\tilde{A}}(x)^2)^{\lambda}},v_{\tilde{A}}(x)^{\lambda}\right) \tag{29}$$

$$\tilde{A}^{\lambda}=\left(\mu_{\tilde{A}}(x)^{\lambda},\sqrt{1-(1-v_{\tilde{A}}(x)^2)^{\lambda}}\right) \tag{30}$$

Subtraction and division operations are generated by Peng and Yang (2015):

$$\tilde{A}\ominus\tilde{B}=\left(\sqrt{\frac{\mu_{\tilde{A}}(x)^2-\mu_{\tilde{B}}(x)^2}{1-\mu_{\tilde{B}}(x)^2}},\frac{v_{\tilde{A}}(x)}{v_{\tilde{B}}(x)}\right),$$
$$\text{if } \mu_{\tilde{A}}(x)\geq\mu_{\tilde{B}}(x),v_{\tilde{A}}(x)\ \min\left\{v_{\tilde{B}}(x),\frac{v_{\tilde{B}}(x)\,\pi_{\tilde{A}}(x)}{\pi_{\tilde{B}}(x)}\right\} \tag{31}$$

$$\tilde{A}\oslash\tilde{B}=\left(\frac{\mu_{\tilde{A}}(x)}{\mu_{\tilde{B}}(x)},\sqrt{\frac{v_{\tilde{A}}(x)^2-v_{\tilde{B}}(x)^2}{1-v_{\tilde{B}}(x)^2}}\right),$$
$$\text{if } \mu_{\tilde{A}}(x)\leq\min\left\{\mu_{\tilde{B}}(x),\frac{\mu_{\tilde{B}}(x)\,\pi_{\tilde{A}}(x)}{\pi_{\tilde{B}}(x)}\right\},v_{\tilde{A}}(x)\geq v_{\tilde{B}}(x) \tag{32}$$

Yager (2013) proposes aggregation operators as follows (Yager 2014):

- *Pythagorean fuzzy weighted geometric average (PFWGA) operator*
$\tilde{A}_i=(x,\mu_{\tilde{A}_i}(x),v_{\tilde{A}_i}(x))$ for $i=1,2,\ldots,n$ are a series of PFWGA, and $w=(w_1,w_2,\ldots,w_n)^T$ is the weight vector of $\tilde{A}_i(i=1,2,\ldots,n)$ with $\sum_{i=1}^{n}w_i=1$

$$\text{PFWGA}\left(\tilde{A}_i\right)=\left(\prod_{i=1}^{n}\mu_{\tilde{A}_i}(x)^{w_i},\prod_{i=1}^{n}v_{\tilde{A}_i}(x)^{w_i}\right) \tag{33}$$

- *Pythagorean fuzzy weighted power average (PFWPA) operator*
$\tilde{A}_i=(x,\mu_{\tilde{A}_i}(x),v_{\tilde{A}_i}(x))$ for $i=1,2,\ldots,n$ are a series of PFWPA, and $w=(w_1,w_2,\ldots,w_n)^T$ is the weight vector of $\tilde{A}_i(i=1,2,\ldots,n)$ with $\sum_{i=1}^{n}w_i=1$

$$\text{PFWPA}\left(\tilde{A}_i\right)=\left(\left(\sum_{i=1}^{n}w_i\mu_{\tilde{A}_i}(x)^2\right)^{\frac{1}{2}},\left(\sum_{i=1}^{n}w_iv_{\tilde{A}_i}(x)^2\right)^{\frac{1}{2}}\right) \tag{34}$$

- *Pythagorean fuzzy weighted power geometric average (PFWPGA) operator*
$\tilde{A}_i=(x,\mu_{\tilde{A}_i}(x),v_{\tilde{A}_i}(x))$ for $i=1,2,\ldots,n$ are a series of PFWPGA, and $w=(w_1,w_2,\ldots,w_n)^T$ is the weight vector of $\tilde{A}_i(i=1,2,\ldots,n)$ with $\sum_{i=1}^{n}w_i=1$

$$\text{PFWPGA}\left(\tilde{A}_i\right)=\left(1-\prod_{i=1}^{n}\left(1-\mu_{\tilde{A}_i}(x)^2\right)^{w_i}\right)^{\frac{1}{2}},\left(1-\prod_{i=1}^{n}\left(1-v_{\tilde{A}_i}(x)^2\right)^{w_i}\right)^{\frac{1}{2}} \tag{35}$$

- *Pythagorean fuzzy weighted average (PFWA) operator*
$\tilde{A}_i=(x,\mu_{\tilde{A}_i}(x),v_{\tilde{A}_i}(x))$ for $i=1,2,\ldots,n$ are a series of STIFNs, and $w=(w_1,w_2,\ldots,w_n)^T$ is the weight vector of $\tilde{A}_i(i=1,2,\ldots,n)$ with $\sum_{i=1}^{n}w_i=1$ (Yager 2014):

$$\text{PFWA}\left(\tilde{A}_i\right)=\left(\sum_{i=1}^{n}w_i\mu_{\tilde{A}_i}(x),\sum_{i=1}^{n}w_iv_{\tilde{A}_i}(x)\right) \tag{36}$$

3.1.4 Defuzzification of Pythagorean fuzzy

Comparing and sorting of the Pythagorean fuzzy numbers is an essential operation step. The Roubens sorting function (Roubens 1990), which is suitable for intuitionistic fuzzy numbers in the sequence, is also appropriate for the ordering of Pythagorean fuzzy numbers. $\tilde{A}$ is a PFN with $\mu_{\tilde{A}}$ membership and $v_{\tilde{A}}$ non-membership; the rank is defined as follows:

$$R\left(\tilde{A}\right)=\frac{R_r\left(\mu_{\tilde{A}}\right)+R_r\left(v_{\tilde{A}}\right)}{2} \tag{37}$$

where R_r is a Roubens sorting function. The Nayagam and Sivaraman's defuzzification method is transformed and applied to the membership and non-membership values for ranking of PFNs (Nayagam and Sivaraman 2011). The defuzzified value of a PFN $\tilde{A}=(\mu_{\tilde{A}},v_{\tilde{A}})$ can be defined as:

$$P_{\text{deff}}\left(\tilde{A}_{\mu v}\right) = \frac{\mu_{\tilde{A}}^2 (1-\gamma) + \gamma\left(1 - v_{\tilde{A}}^2\right)}{2} \tag{38}$$

where γ is a weight of member and non-membership values defined by decision maker. Defuzzification of $\tilde{A}$ PFN can be obtained by an extension of the Roubens sorting function.

$$P_{\text{deff}}\left(\tilde{A}\right) = \frac{\sum_{i=1}^{k} C_i \left(P_{\text{deff}}\left(\tilde{A}_{\mu v_i}\right)\right)^2}{\sum_{i=1}^{k}\left(P_{\text{deff}}\left(\tilde{A}_{\mu v_i}\right)\right)^2} \tag{39}$$

3.1.5 Fuzzy engineering economics

Failure to determine the future economic situation based on the estimation of past economic values leads to uncertainty in the identification and calculation of future economic values (Sullivan et al. 2009). The fuzzy set theory is utilized in the development of economy calculations involving uncertain conditions and values of the future. Thus, the primary concepts in the engineering economy (interest rates, time of money, worth factors, future value, present value, regular annuities capital recovery and sinking fund factors) can be fuzzy and should be defined in fuzzified forms (Kahraman et al. 2015). The economic calculations are made more realistic and obtained more accurate results for real-life problems.

Future value, current value and regular annual value calculations are the basic concepts of the engineering economy. The cash amount, interest rate and period parameters included in these basic economic calculation methods are defined as fuzzy (Kahraman et al. 2015; Kahraman 2008).

The value reached at the end of the uncertain investment becomes uncertain by the uncertain investment value and interest rate. The "future value" economic method calculates the $\widetilde{\text{FV}}$ future value of the $\widetilde{\text{PV}}$ (≥ 0) amout invested today at the end of $\tilde{n}$ periods. The interest rate applied for each period is $\tilde{r}$ in [0, 1] interval (Kahraman 2008; Buckley et al. 2013).

$$\widetilde{\text{FV}} = \widetilde{\text{PV}}\,(1+\tilde{r})^{\tilde{n}} \tag{40}$$

The $\widetilde{\text{PV}}$ of $\widetilde{\text{FV}}$ at a future time for the $\tilde{n}$ period and $\tilde{r}$ the interest rate is calculated by the "present value" economic method (Kahraman 2008; Buckley et al. 2013).

$$\widetilde{\text{PV}} = \widetilde{\text{FV}}\,(1+\tilde{r})^{-\tilde{n}} \tag{41}$$

The amount of $\widetilde{\text{FV}}$ obtained in the future from the $\tilde{A}_{\text{nn}}$ value which is regularly deposited in the $\tilde{n}$ period is calculated by the "future value of annuities" economic method. The uncertain interest rate is defined as $\tilde{r}$ (Kahraman 2008; Buckley et al. 2013).

$$\widetilde{\text{FV}} = \tilde{A}_{\text{nn}} q\,(\tilde{n}, \tilde{r}) \text{ where } q\,(\tilde{n}, \tilde{r}) = \frac{(1+\tilde{r})^{\tilde{n}} - 1}{\tilde{r}} \tag{42}$$

The present $\widetilde{\text{PV}}$ obtained from the $\tilde{A}_{\text{nn}}$ value deposited regularly in the $\tilde{n}$ period is calculated by the "present value of annuities" economic method. The uncertain interest rate is defined as $\tilde{r}$ (Kahraman 2008; Buckley et al. 2013).

$$\widetilde{\text{PV}} = \tilde{A}_{\text{nn}} \beta\,(\tilde{n}, \tilde{r}) \text{ where } \beta\,(\tilde{n}, \tilde{r}) = \frac{1 - (1+\tilde{r})^{-\tilde{n}}}{\tilde{r}} \tag{43}$$

The net present worth model is the most common method used to evaluate economic investments. The net present worth analysis collects the equivalent amount of all cash flows at the present time and evaluates alternative investments at a common point. Because the inputs used in the analysis contain fuzzy data, the net present value calculation is defined as fuzzy. The sum of the present values of the initial investment $\tilde{A}_0$ and the periodic cash flows $(\tilde{A}_i)$ gives the net present value $\widetilde{\text{NPV}}(\tilde{A}, n)$ (Kahraman 2008; Buckley et al. 2013).

$$\widetilde{\text{NPV}}\left(\tilde{A}, n\right) = \tilde{A}_0 + \sum_{i=1}^{n} \tilde{A}_i\,(1+\tilde{r})^{-i} \tag{44}$$

The fuzzy cash flows defined by the fuzzy interest rate represent the capital cost of the firm. While the initial investment value $(\tilde{A}_0)$ is expressed as a negative fuzzy number, other cash flows $(\tilde{A}_i)$ can be expressed as either positive or negative fuzzy numbers.

If alternative projects have different lifetimes, the net present value is calculated by taking the common multiple of the lifetimes of the alternatives. In this case, if the life of one of the alternative projects is infinite, the calculation period is taken as infinite. The evaluation of alternative projects by Pythagorean fuzzy present value analysis is defined as follows (Kahraman et al. 2017).

$$\begin{aligned}
\widetilde{\text{PV}} &= \widetilde{\text{NCF}}\left(\frac{P}{A}, \tilde{i}\%, n\right) - \text{FC} \\
&= (\text{NCF}; (\mu_{\text{NCF}}, v_{\text{NCF}}))\left(\frac{P}{A}, (i; (\mu_i, v_i)), n\right) \\
&\quad - (\text{FC}; (\mu_{\text{FC}}, v_{\text{FC}})) \\
&= \langle \text{NCF}\frac{(1+i)^n - 1}{i\,(1+i)^n}, (\min(\mu_{\text{NCF}}, \mu_i, \mu_{\text{FC}}), \\
&\quad \max(v_{\text{NCF}}, v_i, v_{\text{FC}}))\rangle
\end{aligned} \tag{45}$$

where $\widetilde{\text{NCF}}$ is net cash flow, $\tilde{i}$ is annual interest rate, n is life time period, and FC represents the initial cost of project. Present values for each alternative project are calculated, and the project with the largest $\widetilde{\text{PW}}$ value is selected. The following defuzzification method is applied for comparison of alternative projects (Kahraman et al. 2017).

$$\mathrm{deff}\tilde{A} = \frac{\sqrt{\mu} - v^2}{2} \tag{46}$$

4 Pythagorean fuzzy engineering economic analyses for solar systems

The objective of economic analysis of the solar energy system is to find the lowest cost system on uncertain conditions. Therefore, the economic analysis is based on a long-term comparison of the gain of energy generated from solar radiation with the high initial investment. There are various economic calculation methods for economic evaluation of solar energy systems, which vary according to the system type and dynamic country conditions (Duffie and Beckman 2013).

The life cycle cost (LCC) method is the most appropriate method for economic evaluation of the installation and operation period of the solar energy system (Crawley 2016; Duffie and Beckman 2013). The LCC method, which requires long-term computations and contains future uncertainties, has been restructured by fuzzy logic to make more realistic calculations as Pythagorean life cycle cost method (PLCC). The PLCC method allows a timely assessment of the investment with the technique of reducing future cash flow to the present worth. The time value of the money causes the PLCC method to be selected as an economic evaluation tool for solar energy systems.

The general PLCC of the solar power generation system is the sum of the initial investment ($\widetilde{\mathrm{II}}$), and the current values of the annual operating and maintenance costs incurred throughout the lifetime of the system (Talavera et al. 2013). Initial investment is financed directly by existing sources (own capital, $\widetilde{\mathrm{PW}_{\mathrm{OC}}}$) or financed by borrowing (external capital, $\widetilde{\mathrm{PW}_{\mathrm{EC}}}$) over a variable time period (t_l) at an annual loan interest rate (i_l). The initial investment financed by debt will result in an annual interest cost over the specified period.

The operating and maintenance ($\widetilde{\mathrm{OM}}$) costs foreseen for the life cycle of the solar power system are included in the total current value by withdrawing the investment turnover. Local economic expectations, technological developments and similar factors are taken into account in setting annual operating and maintenance costs. The nominal discount rate ($\tilde{r}$) is used to calculate the life cycle cost since the present value of the investment will be determined (Crawley 2016).

$$\widetilde{\mathrm{PLCC}} = \widetilde{\mathrm{PW}_{\mathrm{II}}} + \widetilde{\mathrm{PW}_{\mathrm{OM}}}(t) \tag{47}$$

$$\widetilde{\mathrm{PW}_{\mathrm{II}}} = \widetilde{\mathrm{PW}_{\mathrm{OC}}} + \widetilde{\mathrm{PW}_{\mathrm{EC}}} \tag{48}$$

If the investment is financed from own capital, the present value of the own portion of the investment cost ($\widetilde{\mathrm{PW}_{\mathrm{OC}}}$) is calculated under the assumption that the annual dividend rate ($\widetilde{d_i}$) is on the capital and the system investment will be depreciated at the end of the system life ($\tilde{t}$). The present worth of own capital use in the investment is as follows:

$$\widetilde{\mathrm{PW}_{\mathrm{OC}}} = \widetilde{\mathrm{OC}} \left[\tilde{d_i} \frac{\tilde{q}\left(1 - \tilde{q}^{\tilde{t}}\right)}{(1 - \tilde{q})} + \tilde{q}^{\tilde{t}} \right] \tag{49}$$

where $\tilde{q} = 1/(1 + \tilde{r})$. The present worth of the external capital portion of the total initial investment ($\widetilde{\mathrm{PW}_{\mathrm{EC}}}$) is calculated as:

$$\widetilde{\mathrm{PW}_{\mathrm{EC}}} = \left[\widetilde{\mathrm{EC}} * i_l \frac{(1 + i_l)^{t_l}}{(1 + i_l)^{t_l} - 1} \frac{\tilde{q}\left(1 - \tilde{q}^{t_l}\right)}{(1 - \tilde{q})} \right] \tag{50}$$

where $\widetilde{\mathrm{EC}}$ refers to the portion of the initial investment remaining from the own capital and is represented as $\widetilde{\mathrm{EC}} = \widetilde{\mathrm{II}} - \widetilde{\mathrm{OC}}$.

The present worth of the operating and maintenance costs is calculated after calculating the present worth of the initial investment. It is assumed that the initial investment value of the solar energy system reflects the system greatness. Therefore, it is predicted that the annual operating costs and maintenance costs ($\widetilde{\mathrm{OM}_{\mathrm{Ann}}}$) are related to the initial investment cost. In addition, the annual O/M increase rate ($\widetilde{ri_{\mathrm{OM}}}$) is determined for the operating and maintenance costs to be calculated for future periods, within the lifetime of the system. Thus, the present value of the operating and maintenance cost is calculated as:

$$\widetilde{\mathrm{PW}_{\mathrm{OM}}}(t) = \widetilde{\mathrm{OM}_{\mathrm{Ann}}} \frac{\left(\frac{1+\widetilde{ri_{\mathrm{OM}}}}{1+\tilde{r}}\right)\left(1 - \left(\frac{1+\widetilde{ri_{\mathrm{OM}}}}{1+\tilde{r}}\right)^{\tilde{t}}\right)}{1 - \left(\frac{1+\widetilde{ri_{\mathrm{OM}}}}{1+\tilde{r}}\right)} \tag{51}$$

Levelized cost of electricity (LCOE) defines the unit cost of electricity (¤/kWh) generated annually by the solar system over its lifetime. The cost expressed in terms of the current monetary value is levelized for each year the system plans to generate electricity. The LCOE, which is calculated based on the cost of life cycle, must account for temporal and environmental uncertainties. Therefore, the LCOE calculations are redeveloped by the Pythagorean fuzzy method and are defined as Pythagorean levelized cost of electricity (PLCOE) (Fig. 6).

Since the value obtained in the PLCOE analysis does not include the cost of transporting and maintaining the grid, it can be regarded as grid parity. Grid parity occurs when the unit cost of the electricity generated from the solar power plant is equal to the purchase price of the electricity from the electricity grid (Short et al. 1995). The PLCOE value is calculated by dividing the estimated total cost of the solar energy system during the life cycle (LCC) by the price of the electricity generated by the system.

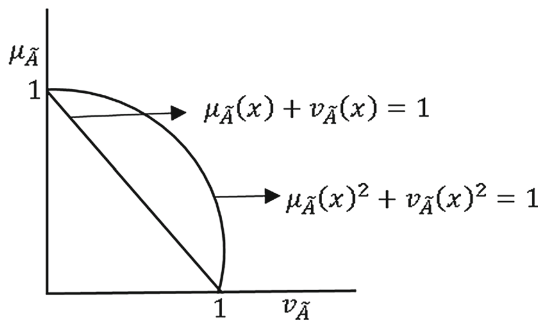


Fig. 6 Difference between PFS and IFS spaces (Yager 2013)

$$\widetilde{\mathrm{PLCOE}} = \frac{\widetilde{\mathrm{LCC}}}{\sum_{i=1}^{t} \frac{\widetilde{E_{\mathrm{Ann}}}\left(1-\widetilde{\mathrm{dr}_{\mathrm{pw}}}\right)^{i}}{(1+\tilde{r})^{i}}} \tag{52}$$

where $\widetilde{E_{\mathrm{Ann}}}$ represents the annual electricity yield (kWh/year) and $\widetilde{\mathrm{dr}_{\mathrm{pw}}}$ represents an annual decrease rate in producing power. The PLCOE value is an important parameter in the economic evaluation and comparison of the systems planned to produce electricity with solar energy (Campbell et al. 2009) (Fig. 7).

The LCOE formula can be expanded by including subsidies, taxes, depreciation, interest payment, debt payment and other unanticipated costs in the LCC account of the solar power system (Darling et al. 2011). PFS-based calculations are developed to eliminate the uncertainties in these parameters which are based on assumptions and predictions. The periodic cash flows in the equation are expressed in Pythagorean fuzzy form, and the expanded $\widetilde{\mathrm{PLCOE}}$ is redefined to include all cash flows.

$$\widetilde{\mathrm{PLCOE}} = \frac{\left(\begin{array}{l} \widetilde{\mathrm{PW}_{\mathrm{II}}} - \sum_{i=1}^{t} \frac{\widetilde{\mathrm{DEP}*\widetilde{\mathrm{TR}}}}{(1+\tilde{r})^{i}} - \sum_{i=1}^{t} \frac{\widetilde{\mathrm{INT}*\widetilde{\mathrm{TR}}}}{(1+\tilde{r})^{i}} + \sum_{i=1}^{t} \frac{\widetilde{\mathrm{LP}}}{(1+\tilde{r})^{i}} \\ + \sum_{i=1}^{t} \frac{\widetilde{\mathrm{AO}}*\left(1-\widetilde{\mathrm{TR}}\right)-\widetilde{\mathrm{RV}}}{(1+\tilde{r})^{i}} - \sum_{i=1}^{t} \frac{\widetilde{\mathrm{SS}}}{(1+\tilde{r})^{i}} + \sum_{i=1}^{t} \frac{\widetilde{SC}}{(1+\tilde{r})^{i}} \end{array}\right)}{\sum_{i=1}^{t} \frac{\widetilde{E_{\mathrm{Ann}}}\left(1-\widetilde{\mathrm{dr}_{\mathrm{pw}}}\right)^{i}}{(1+\tilde{r})^{i}}} \tag{53}$$

where DEP is depreciation, INT is interest payment, LP is debt payment, AO is periodic operating costs, RV is periodic residual values, SS is periodic subsidies, and SC is safety cost.

5 Application on the solar energy system

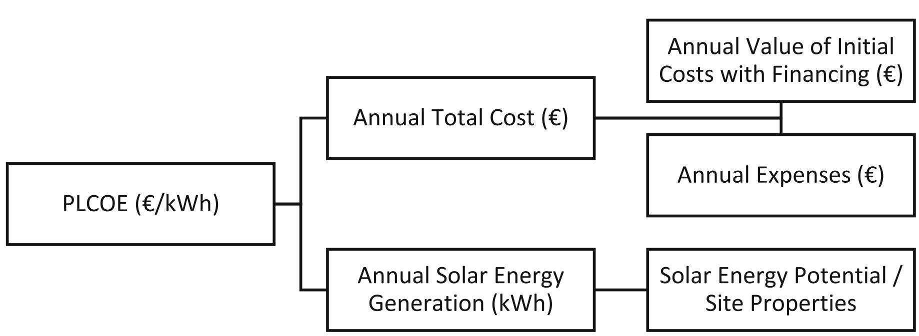
In the application section, the economic feasibility of the installation of the solar power plant from the Si-x PV modules is evaluated by PLCC and PLCOE methods under standard economic conditions. Expert opinions are taken to make the calculations more objective. Pythagorean fuzzy sets are used to evaluate parameters and experts. The parameters used for economic analysis and their possible values (valid for 2014, Crawley 2016) are shown in Table 1.

The weight values of the decision makers are converted to the usable form before calculating the parameter values. The calculation steps and the decision weights obtained for the first expert are calculated as follows (Table 1):

$$P_{\mathrm{deff}}\left(\tilde{E}_{11_{\mu v}}\right) = \left(0.5^2 * (1-0.62) + \left(1-0.7^2\right) * 0.62\right)/2 = 0.0754$$
$$P_{\mathrm{deff}}\left(\tilde{E}_{12_{\mu v}}\right) = \left(0.8^2 * (1-0.62) + \left(1-0.2^2\right) * 0.62\right)/2 = 0.320$$
$$P_{\mathrm{deff}}\left(\tilde{E}_{13_{\mu v}}\right) = \left(0.5^2 * (1-0.62) + \left(1-0.5^2\right) * 0.62\right)/2 = 0.125$$
$$P_{\mathrm{deff}}\left(\tilde{E}_{1}\right) = \frac{0.1 * 0.052^2 + 0.2 * 0.32^2 + 0.3 * 0.109^2}{0.052^2 + 0.32^2 + 0.109^2} = 0.208$$

where γ is selected as 0.62 for this step and other weights are $P_{\mathrm{deff}}(\tilde{E}_2) = 0.298$, $P_{\mathrm{deff}}(\tilde{E}_3) = 0.494$. The aggregation process is performed according to the experts' opinion weights for the solar economic analysis parameters. Pythagorean fuzzy weighted power geometric average (PFWPGA) operator is used for each parameter. Expert weights defuzzified in the previous step are considered as weight values (w_i) for PFWPGA operation, and it is seen that the sum of weight values is equal to 1 ($\sum_{i=1}^{3} w_i = 0.208 + 0.298 + 0.494 = 1$). The sample aggregation for

Fig. 7 PLCOE flowchart of sample solar energy system. . (Adapted from USDE 2015)



possible 1200 kWh/kWp/year value of $\overline{E_{\text{Ann}}}$ Pythagorean parameter is calculated below (Table 1).

$$\begin{aligned}&\text{PFWPGA}\left(\widetilde{E_{\text{Ann}\,1200}}\right)\\&=\left(\left(1-(1-0.3^2)^{0.208}*(1-0.9^2)^{0.298}*(1-0.5^2)^{0.494}\right)^{\frac{1}{2}},\right.\\&\left.\left(1-(1-0.5^2)^{0.208}*(1-0.0^2)^{0.298}*(1-0.6^2)^{0.494}\right)^{\frac{1}{2}}\right)\\&=(0.6938,0.4944)\end{aligned}$$

In the next step, the defuzzification operation is performed on the possible values of the Pythagorean annual electricity yield ($\widetilde{E_{\text{Ann}}}$) parameters. The sample defuzzification operations for $\gamma = 0.8$ are calculated as follows:

$$\begin{aligned}&P_{\text{deff}}\left(\widetilde{E_{\text{Ann}\,1200_{\mu v}}}\right)\\&=\left(0.6938^2*(1-0.8)+\left(1-0.4944^2\right)*0.8\right)/2\\&=0.3504\\&P_{\text{deff}}\left(\widetilde{E_{\text{Ann}\,1350_{\mu v}}}\right)\\&=\left(0.3681^2*(1-0.8)+\left(1-0.201^2\right)*0.8\right)/2\\&=0.3974\\&P_{\text{deff}}\left(\widetilde{E_{\text{Ann}\,1500_{\mu v}}}\right)\\&=\left(0.3186^2*(1-0.8)+\left(1-0.692^2\right)*0.8\right)/2\\&=0.2186\\&P_{\text{deff}}\left(\widetilde{E_{\text{Ann}}}\right)\\&=\frac{1200*0.3504^2+1350*0.3974^2+1500*0.2186^2}{0.3504^2+0.3974^2+0.2186^2}\\&=1315.465\end{aligned}$$

The obtained defuzzified Pythagorean parameters (Table 1) are used in the PLCC and PLCOE economic analyses to check the suitability of the proposed solar energy system.

Investment can be financed by own capital or external capital (Eq. 48). The present value of the own and external part of the investment costs is calculated under the annual dividend rate ($\widetilde{d_i}$) and limited system life ($\tilde{t}$) assumptions. The present worth of own and external capital use in the investment is as follows (Eq. 49):

$$\begin{aligned}\widetilde{\text{PW}_{\text{OC}}} &= 1315.765\left[2.918\frac{0.206\left(1-0.206^{21.924}\right)}{(1-0.206)}+0.206^{21.924}\right]\\&=320.765\,¤\end{aligned}$$

where $\tilde{q} = \frac{1}{1+\tilde{r}} = 0.206$. The present worth of the external capital is calculated as (Eq. 50):

$$\begin{aligned}\widetilde{\text{PW}_{\text{EC}}} &= 1522.401*0.0413\frac{(1+0.0413)^{20}}{(1+0.0413)^{20}-1}\\&\times\frac{0.206\left(1-0.206^{20}\right)}{(1-0.206)}\\&=1560.205\,¤\end{aligned}$$

The present worth of the total investment is calculated as follows (Eq. 48):

$$\widetilde{\text{PW}_{\text{II}}} = 320.765+1560.205 = 1880.97\,¤$$

The present worth of the operating and maintenance costs is calculated after calculating the present worth of the initial investment. It is assumed that the initial investment value of the solar energy system reflects the system greatness. The present worth of the annual operating and maintenance costs ($\widetilde{\text{OM}_{\text{Ann}}}$) with an annual O/M increase rate ($\widetilde{ri_{\text{OM}}}$) is calculated as follows (Eq. 51):

$$\widetilde{\text{PW}_{\text{OM}}}(t) = 28.916\frac{0.989\left(1-0.989^{21.924}\right)}{1-0.989} = 565.442\,¤$$

where $\tilde{p} = \frac{1+\widetilde{ri_{\text{OM}}}}{1+\tilde{r}} = 0.989$. The total current value of initial investment and annual maintenance, repair and operation costs is as follows (Eq. 47):

$$\widetilde{\text{PLCC}} = 1880.97+565.442 = 2446.412\,¤$$

Levelized cost of electricity (LCOE) generated based on the Pythagorean fuzzy numbers is calculated with the defuzzified parameters as follows (Eq. 52):

$$\widetilde{\text{PLCOE}} = \frac{2446.412}{17,679.84} = 0.1384\,¤/\text{kWh}$$

The 0.135 ¤/kWh LCOE value obtained from the source evaluation of the sample application (Crawley 2016) is reasonably consistent with the 13.84 values obtained from the Pythagorean basis.

Values obtained from the LCC and LCOE methods using Pythagorean fuzzy numbers reflect economic conditions more realistically. Alternative solar energy systems with different technological features and economic conditions can be compared to the values obtained from these methods. Alternative solar energy systems evaluated on achieving the same energy capacity are ranked according to the LCC value, and the system with the lowest value is preferred.

The LCOE model is used to compare the planned solar power projects with the current market prices. If the LCOE value is higher than the market price, the unit margin of the produced energy becomes negative, and the project is seen as unprofitable (PDEME 2017). Therefore, it would not be sufficient to compare alternative energy systems solely by

Table 1 Parameters' possible values, Pythagorean fuzzy membership values, aggregated and defuzzified result values

Parameters	Possible values	Experts' weights									Aggregated values	Defuzzified values
		E1			E2			E3				
		0.1 (0.5, 0.7)	0.2 (0.8, 0.2)	0.3 (0.5, 0.5)	0.4 (0.47, 0.6)	0.3 (0.85, 0.2)	0.5 (0.5, 0.7)	0.3 (0.3, 0.5)	0.5 (0.85, 0.3)	0.7 (0.2, 0.6)		
$\widetilde{E_{\mathrm{Ann}}}$(kWh/kWp/year)	1200	(0.3, 0.5)			(0.9, 0.0)			(0.5, 0.6)			(0.6938, 0.4944)	1315.765
	1350	(0.5, 0.4)			(0.5, 0.0)			(0.0, 0.1)			(0.3681, 0.2010)	
	1500	(0.2, 0.9)			(0.4, 0.5)			(0.3, 0.6)			(0.3186, 0.6920)	
$\widetilde{\mathrm{dr}_{\mathrm{pw}}}$(annual%)	0.3	(0.4, 0.5)			(0.2, 0.9)			(0.3, 0.2)			(0.3011, 0.6612)	0.4984
	0.5	(0.7, 0.4)			(0.3, 0.4)			(0.7, 0.3)			(0.6277, 0.3551)	
	0.6	(0.3, 0.7)			(0.3, 0.0)			(0.0, 0.4)			(0.2159, 0.4499)	
$\widetilde{\mathrm{EC}}$(¤)	1440	(0.4, 0.4)			(0.5, 0.2)			(0.6, 0.8)			(0.5385, 0.6518)	1522.401
	1520	(0.7, 0.4)			(0.8, 0.0)			(0.9, 0.4)			(0.8472, 0.3394)	
	1600	(0.5, 0.5)			(0.7, 0.5)			(0.4, 0.7)			(0.5412, 0.6165)	
$\widetilde{\mathrm{OC}}$(¤)	340	(0.6, 0.6)			(0.9, 0.1)			(0.7, 0.3)			(0.7756, 0.3643)	371.577
	380	(0.3, 0.7)			(0.8, 0.4)			(0.7, 0.3)			(0.6939, 0.4607)	
	420	(0.6, 0.6)			(0.9, 0.5)			(0.5, 0.7)			(0.7197, 0.6326)	
$\widetilde{\mathrm{OM}_{\mathrm{Ann}}}$(¤)	27.5	(0.4, 0.5)			(0.7, 0.5)			(0.7, 0.7)			(0.6590, 0.6165)	28.916
	28.5	(0.6, 0.3)			(0.9, 0.2)			(0.6, 0.9)			(0.7445, 0.7573)	
	30.0	(0.5, 0.5)			(0.5, 0.6)			(0.7, 0.2)			(0.6165, 0.4380)	
$\widetilde{ri_{\mathrm{OM}}}$(annual%)	2.4	(0.7, 0.6)			(0.1, 0.0)			(0.7, 0.5)			(0.6153, 0.4576)	2.8035
	2.8	(0.7, 0.5)			(0.2, 0.1)			(0.8, 0.5)			(0.6939, 0.4305)	
	3.2	(0.7, 0.6)			(0.5, 0.3)			(0.1, 0.3)			(0.4540, 0.3927)	
$\tilde{d}_i$(annual%)	2.5	(0.4, 0.5)			(0.5, 0.4)			(0.4, 0.6)			(0.4335, 0.5317)	2.9180
	3	(0.7, 0.3)			(0.3, 0.0)			(0.6, 0.7)			(0.5675, 0.5449)	
	3.5	(0.4, 0.5)			(0.1, 0.9)			(0.2, 0.2)			(0.2402, 0.6612)	
$\tilde{r}$(annual%)	3.6	(0.7, 0.2)			(0.9, 0.1)			(0.9, 0.3)			(0.8756, 0.2375)	3.8531
	3.9	(0.5, 0.2)			(0.5, 0.4)			(0.3, 0.4)			(0.4181, 0.3692)	
	4.2	(0.6, 0.4)			(0.7, 0.7)			(0.8, 0.2)			(0.7415, 0.4761)	
$\tilde{t}$(year)	15	(0.3, 0.1)			(0.2, 0.9)			(0.2, 0.4)			(0.2250, 0.6647)	21.924
	20	(0.5, 0.2)			(0.5, 0.6)			(0.6, 0.5)			(0.5536, 0.4969)	
	30	(0.5, 0.3)			(0.3, 0.8)			(0.7, 0.6)			(0.5859, 0.6480)	

LCC while developing a grid-connected solar energy system. The solar power system with the lowest LCOE value, which is below the market price, is preferred among the alternatives.

6 Conclusion

Increasing population and changing consumption habits stimulate to increase energy demand. Fossil fuels, which are used as the primary energy source in meeting the increasing energy demand, cause serious harm to the environment, economy and public health. Therefore, countries have turned to alternative energy sources such renewable, nuclear, tidal and geothermal. The sun has come to the forefront with its features among the renewable energy sources that are widely accepted by the countries in the world. Investment evaluation of solar energy systems, which has a high initial cost and a low operating cost, is made according to the results of the economic analysis. The accepted and widely used methods for evaluating and comparing solar energy systems are LCC and LCOE.

LCC and LCOE values are calculated based on future operation, maintenance costs and energy production expectations. Because the interest rates and cash flow values used in the calculations are dependent on local and global unpredictable political, economic and social variables, the cost and comparison calculations within LCC and LCOE are made incorrect and unrealistic. Also, the dependence of solar radiation values on extraterrestrial, atmospheric and terrestrial different and indefinite variabilities prevents precise calculation of the energy potential obtained from the solar energy system. Therefore, Pythagorean fuzzy numbers are included in the method to incorporate the future uncertainties and decision makers' views into the calculations. Pythagorean fuzzy sets are the enlarged states of intuitionistic fuzzy sets, which means that the sum of the squares of the membership and non-membership values of the fuzzy number is equal to or less than one. The LCC and LCOE calculation parameters defined according to the Pythagorean membership grades are exemplified by an application that includes the evaluators of the decision makers. It can be seen that the LCC and LCEO values obtained with Pythagorean-based calculations are acceptable when the results are compared with the actual application results.

In the future, it is planned to develop LCC and LCEO calculations by Pythagorean fuzzy set theory by elaborating the cost and gain titles of solar energy system installation. Also, the Pythagorean fuzzy calculation method can be applied in other economic analysis applications and other renewable energy technologies to make comparative evaluations.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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