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# A novel interval-valued neutrosophic AHP with cosine similarity measure

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**Abstract**

Neutrosophic Logic (Smarandache in Neutrosophy neutrosophic probability: set, and logic, American Research Press, Rehoboth, 1998) has been applied to many multicriteria decision-making methods such as Technique for Order Preference by Similarity to an Ideal Solution, Višekriterijumsko kompromisno rangiranje Resenje, and Evaluation based on Distance from Average Solution. Interval-valued neutrosophic sets are subclass of neutrosophic sets. Interval numbers can be used for their truth-membership, indeterminacy-membership, and falsity-membership degrees. The angle between the vector representations of two neutrosophic sets is defined cosine similarity measure. In this paper, we introduce a new Analytic Hierarchy Process (AHP) method with interval-valued neutrosophic sets. We also propose an interval-valued neutrosophic AHP (IVN-AHP) based on cosine similarity measures. The proposed method with cosine similarity provides an objective scoring procedure for pairwise comparison matrices under neutrosophic uncertainty. Finally, an application is given in energy alternative selection to illustrate the developed approaches.

**Keywords** Neutrosophic sets · AHP · Multi criteria decision making · Interval-valued neutrosophic sets · Cosine similarity measures

## 1 Introduction

The neutrosophic sets developed by Florantin Smarandache (1998) extend the concept of intuitionistic fuzzy sets (IFSs) introduced by Atanassov (1983) for a new point of view to uncertainty, impreciseness, inconsistency and vagueness. Smarandache (1998) introduced the degree of indeterminacy/neutrality as a new and independent component of fuzzy sets and defined a neutrosophic set by three components: truth membership, indeterminacy membership, and falsity membership. Since indeterminacy parameter helps a more detailed definition of membership functions, the usage of neutrosophic sets in decision making can produce better results. On the other hand, a neutrosophic set is more complex to apply in real scientific and engineering fields (Sahin and Yigider 2016). Neutrosophic logic is very useful to distinguish between absolute truth and relative truth or between absolute falsehood and relative falsehood in logics, and, respectively, between absolute membership and relative membership or absolute non-membership and relative non-membership (Smarandache 1998). When neutrosophic sets are preferred, a decision maker does not need to satisfy that the sum of elements in a membership function for a certain event should be at most equal to 1. If those elements are independent, the sum may increase up to 3.

In the recent years, several types of neutrosophic sets have been proposed by various researchers. Intuitionistic neutrosophic sets (Bhowmik and Pal 2010), single-valued neutrosophic sets (Liu 2016a, b; Wang et al. 2016), interval-valued neutrosophic sets (Ma et al. 2016), multi-valued neutrosophic sets (Liu et al. 2016), and trapezoidal neutrosophic sets (Ye 2015a, b) are some types of these neutrosophic sets.

One of the most popular MCDM methods is AHP introduced by Saaty (1977). It systematically allows researchers to calculate the weights of the criteria and alternatives. Because of incomplete information and uncertainty, classical AHP method has been extended to several fuzzy versions. These extensions are ordinary fuzzy AHP with type-1 fuzzy sets (Tan et al. 2015), fuzzy AHP with type 2 fuzzy sets (Oztaysi et al. 2018), fuzzy AHP with intuitionistic fuzzy sets (Wu

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et al. 2013), fuzzy AHP with hesitant fuzzy sets (Oztaysi et al. 2015), and fuzzy AHP with interval-valued intuitionistic sets (Tooranloo and Iranpour 2017).

Neutrosophic sets have been extensively used in decision-making processes since 2013. To the best of our knowledge, there are two neutrosophic AHP papers published by Radwan et al. (2016) and Abdel-Basset et al. (2017). Radwan et al. (2016) developed a novel hybrid neutrosophic AHP approach in learning management systems in decision making to handle indeterminacy of information. Abdel-Basset et al. (2017) developed the integration of AHP into Delphi framework under neutrosophic environment and introduced a new technique for checking consistency and calculating consensus degree of expert's opinions.

In this paper, we present two methods which are IVN-AHP alone and IVN-AHP with cosine similarity (IVNAHP-CS) measure. Cosine similarity provides an objective scoring procedure in pairwise comparisons instead of subjective scoring. To the best our knowledge, AHP method with interval-valued neutrosophic sets have not been yet proposed. Neutrosophic AHP enables decision makers to take into account their hesitancy in defining a membership function. Neutrosophic logic is a generalization of all other logics. Its definition needs more parameters, and it presents more information about the considered problem with its T, I, and F elements.

The rest of the paper is constructed as follows: A literature review on neutrosophic sets in MCDM is presented in Sect. 2. The fuzzy extensions of AHP are summarized in Sect. 3. In Sect. 4, the preliminaries of neutrosophic sets are given. In Sect. 5, the concept of cosine similarity is introduced. Our proposed methods, IVN-AHP and IVNAHP-CS, are presented in Sect. 6. In order to show their effectiveness, these methods are used in the selection of the best renewable energy alternative. Finally, the conclusion is given in Sect. 7.

## 2 Literature review

In this section, the papers on neutrosophic MCDM methods are reviewed. Neutrosophic sets have been used very rapidly in MCDM since they first time appeared in 1998. The frequencies of the neutrosophic publications on MCDM have significantly increased since 2013.

Neutrosophic MCDM papers in the literature are summarized in Table 12 in Appendix. These papers have been classified with respect to the type of neutrosophic sets (single-valued neutrosophic sets, trapezoidal neutrosophic sets, multi-valued neutrosophic numbers, interval neutrosophic sets, etc.). The most used methods among them are single-valued neutrosophic sets and interval-valued neutrosophic sets.

In Table 1, the papers using neutrosophic MCDM methods are summarized. It can be seen that EDAS, Multi Attribute Market Value Assessment (MAMVA), Multi-Attributive Border Approximation area Comparison (MABAC), TOPSIS, TODIM (an acronym in Portuguese for iterative multicriteria decision making), Elimination and Choice Translating Reality English (ELECTRE), Weighted Aggregated Sum Product Assessment (WASPAS), QUALItative FLEXible (QUALIFLEX), COmplex PRoportional Assessment (COPRAS), and MOORA plus full multiplicative form (MULTIMOORA) methods have been used in neutrosophic papers.

Table 1 summarizes neutrosophic MCDM papers with respect to the methods used in them. The most used method is neutrosophic TOPSIS.

Figure 1 illustrates the distribution of all the MCDM papers existing in the literature based on their publication years. The largest percentage on neutrosophic MCDM belongs to the year 2016 with the rate of 41%.

Figure 2 shows the researchers who most published the neutrosophic MCDM papers.

## 3 Fuzzy extensions of AHP

### 3.1 Intuitionistic fuzzy AHP

Atanassov's (1983) intuitionistic fuzzy sets which incorporate the degree of hesitation to the definition of a membership function have been frequently used in MCDM problems. Buyukozkan et al. (2016) developed a framework that integrates the intuitionistic fuzzy AHP and intuitionistic fuzzy VIKOR. Deepika and Kannan (2016) proposed an intuitionistic fuzzy AHP method in order to check the consistency for automatic repairing procedure.

### 3.2 Hesitant AHP

Hesitant Fuzzy Sets proposed by Torra (2010) have been used in the literature. Oztaysi et al. (2015) developed a hesitant fuzzy AHP method involving multi-experts' linguistic evaluations aggregated by ordered weighted averaging operator. The developed method is applied to a multicriteria supplier selection problem. Kahraman et al. (2016) presented a new hesitant fuzzy AHP method in order to solve a warehouse location selection problem for a Turkish humanitarian relief organization by using hesitant fuzzy preference information. The aim of this study is to eliminate the decision makers' hesitancy in the evaluation. Zhu et al. (2016) proposed hesitant AHP which can consider the hesitancy experienced by the decision makers. Senvar (2018) proposed a systematic approach based on hesitant fuzzy AHP to deal with incomplete information in complex customer-oriented MCDM problems.

**Table 1** MCDM methods with neutrosophic sets

| Author (year) | Multi criteria decision-making methods | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AHP | EDAS | MAMVA | MABAC | TOPSIS | TODIM | ELECTRE | WASPAS | QUALIFLEX | COPRAS | MULTIMOORA | VIKOR |
| Abdel-Basset et al. (2017) | x | | | | | | | | | | | |
| Bausys et al. (2015) | | | | | | | | | | x | | |
| Baušys and Juodagalvienė (2017) | | | | | | | | x | | | | |
| Bausys and Zavadskas (2015) | | | | | | | | | | | | x |
| Bausys et al. (2015) | | | | | | | | | | x | | |
| Biswas et al. (2016) | | | | | x | | | | | | | |
| Elhassouny and Smarandache (2016) | | | | | x | | | | | | | |
| Garg and Nancy (2017) | | | | | x | | | | | | | |
| Hu et al. (2017) | | | | | | | | | | | | x |
| Ji et al (2016) | | | | | | x | | | | | | |
| Ji et al. (2017) | | | | | | x | | | x | | | |
| Karasan and Kahraman (2018) | | x | | | | | | | | | | |
| Liang et al. (2017a, b) | | x | | | | | | | | | | |
| Liu (2016a, b) | | | | | x | | | | | | | |
| Ma et al. (2017a, b) | | | | | | | x | | | | | |
| NAdAban and Dzitac (2016) | | | | | x | | | | | | | |
| Otay and Kahraman (2018) | | | | | x | | | | | | | |
| Peng and Dai (2016) | | | | x | x | | | | | | | |
| Peng et al. (2016a, b, c) | | | | | | | x | | | | | |
| Peng et al. (2016a, b, c) | | | | | | | x | | | | | |
| Peng et al. (2017a, b) | | | | | | | | | x | | | |
| Radwan et al. (2016) | x | | | | | | | | | | | |
| Stanujkic et al. (2017) | | | | | | | | | | | x | |
| Wang and Li (2015) | | | | | | x | | | | | | |
| Zavadskas et al. (2015) | | | | | | | | x | | | | |
| Zavadskas et al. (2016) | | | | | | | | x | | | | |
| Zavadskas et al. (2017) | | | x | | | | | | | | | |
| Zhang et al. (2016a, b, c) | | | | | | x | | | | | | |

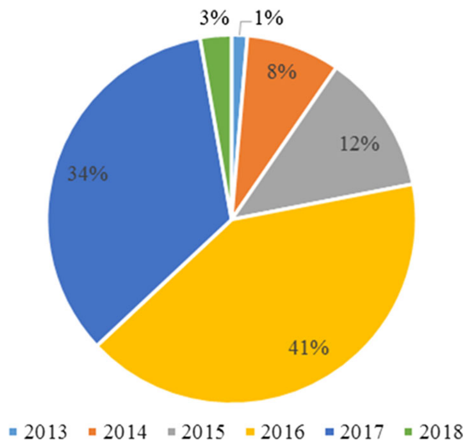


**Fig. 1** Distribution of neutrosophic MCDM papers with respect to publication years

### 3.3 Type-2 Fuzzy AHP

Type-2 fuzzy sets are the extension of type-1 fuzzy sets. Kahraman et al. (2014) introduced an interval type-2 fuzzy AHP method together with a new ranking method for type-2 fuzzy sets and applied it to a supplier selection problem. Abdullah and Najib (2016) studied on a version of interval type-2 fuzzy AHP and realized its implication to the computational procedure. Erdogan and Kaya (2016) proposed a MCDM methodology consisting of three techniques which are Delphi methodology, type-2 fuzzy AHP and type-2 fuzzy TOPSIS and applied it to a real case work in order to take attention for exhaust gases from the increasing number of motor vehicles as the major factor of air pollution in Istanbul. There are more than these papers in AHP with type-2 fuzzy sets.

### 3.4 Pythagorean AHP

There is only one paper on Pythagorean AHP method developed by Cebi et al. (2018) in the literature. It presents a novel approach to risk assessment for occupational health and safety using pythagorean fuzzy AHP and a fuzzy inference system.

### 3.5 Neutrosophic AHP

Radwan et al. (2016) proposed a neutrosophic AHP method and applied it to the selection of the best learning management system. Another paper which is related to neutrosophic AHP is published by Abdel-Basset et al. (2017). They developed a neutrosophic AHP-Delphi group decision-making model based on trapezoidal neutrosophic numbers in order to handle experts' non-deterministic evaluation values.

## 4 Preliminaries

In this section, we give basic notions and operations on interval-valued neutrosophic set.

### 4.1 Neutrosophic sets

In neutrosophic sets literature, a common specific symbol for a neutrosophic set has not been used up to now. We propose the symbol $\tilde{\dddot{A}}$ for the neutrosophic set $A$, that the three dots represent the elements of a neutrosophic set; $T$, $I$, $F$ and tilde represents that it is also a fuzzy set.

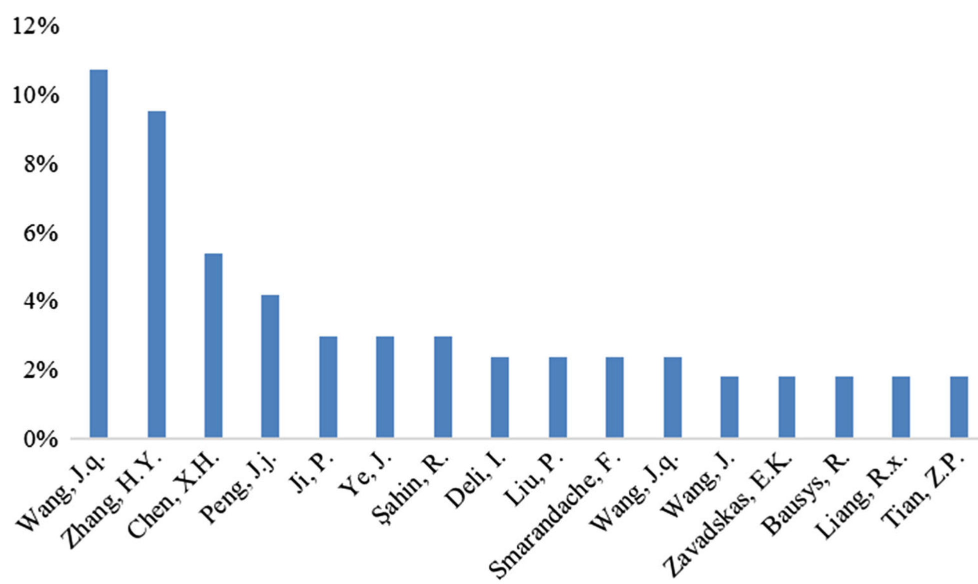


**Fig. 2** Researchers publishing neutrosophic MCDM papers

**Definition 1** (Smarandache 1998) Let $E$ be a universe. A neutrosophic set $\tilde{\tilde{A}}$ in $E$ is characterized by a truth-membership function $\mathrm{T_A}$, a indeterminacy-membership function $\mathrm{I_A}$, and a falsity-membership function $\mathrm{F_A}$.

$\mathrm{T_A}\ (x)$, $\mathrm{I_A}\ (x)$ and $\mathrm{F_A}\ (x)$ are real standard elements of [0,1]. A neutrosophic set $\tilde{\tilde{A}}$ can be given by Eq. (1):

$$\tilde{\tilde{A}} = \{< x, (T_A\ (x), I_A\ (x), F_A\ (x)) >: x \in E,\ (T_A\ (x), I_A\ (x), F_A\ (x) \in ]^{-}0, 1[^{+})\}. \tag{1}$$

There is no restriction on the sum of $\mathrm{T_A}\ (x)$, $\mathrm{I_A}\ (x)$ and $\mathrm{F_A}\ (x)$, so that $0^{-} \leq T_A\ (x) + I_A\ (x) + F_A\ (x) \leq 3^{+}$.

**Definition 2** (Biswas et al. 2014) Let E be a universe. A single-valued neutrosophic sets A in E are characterized by a truth-membership function $T_A$, a indeterminacy-membership function $I_A$ and a falsity-membership function $F_A$. $T_A$; $I_A$ and $F_A$ are real standard elements of [0,1]. It can be written as

$$\tilde{\tilde{A}} = \{< x, (T_A\ (x), I_A\ (x), F_A\ (x)) >: x \in E,\ T_A\ (x), I_A\ (x), F_A\ (x) \in [0, 1]\}. \tag{2}$$

There is no restriction on the sum of $\mathrm{T_A}\ (x)$; $\mathrm{I_A}\ (x)$ and $\mathrm{F_A}\ (x)$, so $0^{-} \leq T_A\ (x) + I_A\ (x) + F_A\ (x) \leq 3^{+}$.

**Definition 3** (Wang et al. 2010) Let X be a space of points (objects) with a generic element in X denoted by x. A neutrosophic set $\tilde{\tilde{A}}$ in X characterized by a truth-membership function $T_A$, an indeterminacy-membership function $I_A$ and a falsity-membership function $F_A$. $T_A$, $I_A$ and $F_A$ are real standard or non-standard subsets of $]0^{-}, 1^{+}[$. These are

$$T_A:\ \mathrm{X} \rightarrow ]0^{-}, 1^{+}[. \tag{3}$$
$$I_A:\ \mathrm{X} \rightarrow ]0^{-}, 1^{+}[. \tag{4}$$
$$F_A:\ \mathrm{X} \rightarrow ]0^{-}, 1^{+}[. \tag{5}$$

## 4.2 Interval-valued neutrosophic sets

**Definition 4** $\tilde{\tilde{\mathrm{x}}}_j = \left\langle \left[\mathrm{T_j^L}, \mathrm{T_j^U}\right], \left[\mathrm{I_j^L}, \mathrm{I_j^U}\right], \left[\mathrm{F_j^L}, \mathrm{F_j^U}\right] \right\rangle$ is a collection of interval-valued neutrosophic numbers where $j = 1, 2, \ldots, n$ and n is the number of decision makers.

**Definition 5** Deneutrosophication: We propose a new deneutrosophication function of an interval-valued neutrosophic number which is given below:

$$\mathfrak{D}\ (\mathrm{x}) = \left(\frac{(\mathrm{T_x^L} + \mathrm{T_x^U})}{2} + \left(1 - \frac{(\mathrm{I_x^L} + \mathrm{I_x^U})}{2}\right) * \left(\mathrm{I_x^U}\right) - \left(\frac{\mathrm{F_x^L} + \mathrm{F_x^U}}{2}\right) * \left(1 - \mathrm{F_x^U}\right)\right) \tag{6}$$

where $\tilde{\tilde{\mathrm{x}}}_j = \langle [\mathrm{T_x^L}, \mathrm{T_x^U}], [\mathrm{I_x^L}, \mathrm{I_x^U}], [\mathrm{F_x^L}, \mathrm{F_x^U}] \rangle$.

**Definition 6** (Li et al. 2016) $X$ be a universe of discourse. An interval-valued neutrosophic set $\tilde{\tilde{N}}$ in $X$ is independently defined by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$ for each $x \in X$, where $T_N(x) = \left[T^L_{N(x)}, T^U_{N(x)}\right] \subseteq [0, 1]$, $I_N(x) = \left[I^L_{N(x)}, I^U_{N(x)}\right] \subseteq [0, 1]$, and $F_N(x) = \left[F^L_{N(x)}, F^U_{N(x)}\right] \subseteq [0, 1]$. They also meet the condition $0 \leq T^L_N(x) + I^L_N(x) + F^L_N(x) \leq 3$. So, the interval-valued neutrosophic set $\tilde{\tilde{\mathrm{N}}}$ can be given by Eq. (7):

$$\tilde{\tilde{\mathrm{N}}} = \left\{\left\langle \mathrm{x}, \left[\mathrm{T_N^L(x)}, \mathrm{T_N^U(x)}\right], \left[\mathrm{I_N^L(x)}, \mathrm{I_N^U(x)}\right], \left[\mathrm{F_N^L(x)}, \mathrm{F_N^U(x)}\right] \right\rangle |\mathrm{x} \in \mathrm{X}\right\}. \tag{7}$$

**Definition 7** (Zhang et al. 2014) Let $\tilde{\tilde{a}} = \left\langle \left[T_a^L, T_a^U\right], \left[I_a^L, I_a^U\right], \left[F_a^L, F_a^U\right] \right\rangle$ and $\tilde{\tilde{b}} = \left\langle \left[T_b^L, T_b^U\right], \left[I_b^L, I_b^U\right], \left[F_b^L, F_b^U\right] \right\rangle$ be two interval-valued neutrosophic numbers. Their relations and arithmetic operations are given by Eqs. (8)–(11):

1. $$\tilde{\tilde{a}}^{c} = \left\langle \left[\mathrm{F_a^L}, \mathrm{F_a^U}\right], \left[1 - \mathrm{I_a^U}, 1 - \mathrm{I_a^L}\right], \left[\mathrm{T_a^L}, \mathrm{T_a^U}\right] \right\rangle \tag{8}$$
2. $$\tilde{\tilde{a}} \subseteq \tilde{\tilde{b}} \text{ if and only if } \mathrm{T_a^L} \leq \mathrm{T_b^L};\ \mathrm{T_a^U} \leq \mathrm{T_b^U};\ \mathrm{I_a^L} \geq \mathrm{I_b^L},\ \mathrm{I_a^U} \geq \mathrm{I_b^U};\ \mathrm{F_a^L} \geq \mathrm{F_b^L},\ \mathrm{F_a^U} \geq \mathrm{F_b^U} \tag{9}$$
3. $\tilde{\tilde{a}} = \tilde{\tilde{b}}$ if and only if $\tilde{\tilde{a}} \subseteq \tilde{\tilde{b}}$ and $\tilde{\tilde{b}} \subseteq \tilde{\tilde{a}}$.
4. $$\tilde{\tilde{a}} \oplus \tilde{\tilde{b}} = \left\langle \left[\mathrm{T_a^L} + \mathrm{T_b^L} - \mathrm{T_a^L T_b^L}, \mathrm{T_a^U} + \mathrm{T_b^U} - \mathrm{T_a^U T_b^U}\right], \times \left[\mathrm{I_a^L I_b^L}, \mathrm{I_a^U I_b^U}\right], \left[\mathrm{F_a^L F_b^L}, \mathrm{F_a^U F_b^U}\right] \right\rangle \tag{10}$$
5. $$\tilde{\tilde{a}} \oplus \tilde{\tilde{b}} = \left\langle \left[\mathrm{T_a^L T_b^L}, \mathrm{T_a^U T_b^U}\right] \left[\mathrm{I_a^L} + \mathrm{I_b^L} - \mathrm{I_a^L I_b^L}, \mathrm{I_a^U} + \mathrm{I_b^U} - \mathrm{I_a^U I_b^U}\right], \times \left[\mathrm{F_a^L} + \mathrm{F_b^L} - \mathrm{F_a^L F_b^L}, \mathrm{F_a^U} + \mathrm{F_b^U} - \mathrm{F_a^U F_b^U}\right] \right\rangle \tag{11}$$

**Definition 8** Subtraction operation of two interval-valued neutrosophic sets is given as below (Karasan and Kahraman 2018):

$$\tilde{\tilde{\mathrm{x}}} \ominus \tilde{\tilde{\mathrm{y}}} = \left\langle \left[\mathrm{T_x^L} - \mathrm{F_y^U}, \mathrm{T_x^U} - \mathrm{F_y^L}\right], \left[\mathrm{Max}\left(\mathrm{I_x^L}, \mathrm{I_y^L}\right), \mathrm{Max}\left(\mathrm{I_x^U}, \mathrm{I_y^U}\right)\right], \times \left[\mathrm{F_x^L} - \mathrm{T_y^U}, \mathrm{F_x^U} - \mathrm{T_y^L}\right] \right\rangle \tag{12}$$

where $\tilde{\tilde{\mathrm{x}}} = \langle [\mathrm{T_x^L}, \mathrm{T_x^U}], [\mathrm{I_x^L}, \mathrm{I_x^U}], [\mathrm{F_x^L}, \mathrm{F_x^U}] \rangle$ and $\tilde{\tilde{\mathrm{y}}} = \left\langle \left[\mathrm{T_y^L}, \mathrm{T_y^U}\right], \left[\mathrm{I_y^L}, \mathrm{I_y^U}\right], \left[\mathrm{F_y^L}, \mathrm{F_y^U}\right] \right\rangle$.

# 5 Cosine similarity

Cosine similarity is a fundamental angle-based measure of similarity between two vectors of $n$ dimensions using the cosine of the angle between them (Candan and Sapino 2010). It measures the similarity between two vectors based only on

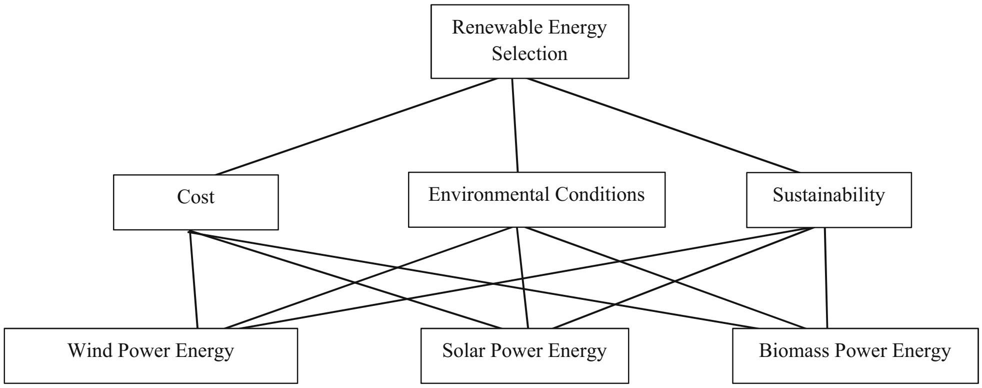


**Fig. 3** Hierarchical structure of application

the direction, ignoring the impact of the distance between them.

A cosine similarity measure based on Bhattacharya's distance in a vector space between two fuzzy sets $\mu_A(x_i)$ and $\mu_B(x_i)$ is shown as follows:

$$C_F(A, B) = \frac{\sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i)}{\sqrt{\sum_{i=1}^{n} \mu_A(x_i)^2}\sqrt{\sum_{i=1}^{n} \mu_B(x_i)^2}} \tag{13}$$

Broumi and Smarandache (2014) proposed a different cosine similarity between two interval-valued neutrosophic sets based on Bhattacharya's distance. Assume that $A$ and $B$ are two interval-valued neutrosophic sets in X= $\{x_1, x_2, \ldots, x_n$. A cosine similarity measure between interval-valued neutrosophic sets $A$ and $B$ is defined by Eq. (14).

$$C_N(A, B) = \frac{1}{n}\sum_{i=1}^{n} \frac{(T_A^L(x_i) + T_A^U(x_i))(T_B^L(x_i) + T_B^U(x_i)) + (I_A^L(x_i) + I_A^U(x_i))(I_B^L(x_i) + I_B^U(x_i)) + (F_A^L(x_i) + F_A^U(x_i))(F_B^L(x_i) + F_B^U(x_i))}{\sqrt{(T_A^L(x_i) + T_A^U(x_i))^2 + (I_A^L(x_i) + I_A^U(x_i))^2 + (F_A^L(x_i) + F_A^U(x_i))^2}\sqrt{(T_B^L(x_i) + T_B^U(x_i))^2 + (I_B^L(x_i) + I_B^U(x_i))^2 + (F_B^L(x_i) + F_B^U(x_i))^2}} \tag{14}$$

# 6 IVN-AHP proposals

## 6.1 The steps of IVN-AHP method

The advantages of this method are its similarity to classical AHP method and simplicity in calculation steps. However, it needs a deneutrosophication formula in its late steps. In the following, we present the steps of the IVN-AHP Method.

**Step 1** Determine the interval-valued neutrosophic evaluation scale (see Table 3).
**Step 2** Decompose the problem into a hierarchy of goal, criteria, sub-criteria and alternatives (see Fig. 3).
**Step 3** Construct the pairwise comparison matrices ($\tilde{\dot{\mathrm{P}}}$) by using interval-valued neutrosophic sets. Consistency of the pairwise comparison matrices have been measured

by using deneutrosophication equation given in Eq. (6). If the deneutrosophicated pairwise comparison matrix is consistent, the neutrosophic pairwise comparison matrix is also consistent.
Pairwise comparison matrices for criteria with respect to the goal are given by Eq. (15).

**Step 4.2** Select the upper value for each parameter in Eq. (17) and divide each term by its corresponding element to obtain $\tilde{\dddot{N}}_{ij}$ in Eq. (18).

$$\tilde{\dddot{P}}_C = \begin{array}{c} \\ C_1 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{bmatrix} \langle [\mathrm{T}^{\mathrm{L}}_{11}, \mathrm{T}^{\mathrm{U}}_{11}], [\mathrm{I}^{\mathrm{L}}_{11}, \mathrm{I}^{\mathrm{U}}_{11}], [\mathrm{F}^{\mathrm{L}}_{11}, \mathrm{F}^{\mathrm{U}}_{11}] \rangle & \cdots & \langle [\mathrm{T}^{\mathrm{L}}_{1n}, \mathrm{T}^{\mathrm{U}}_{1n}], [\mathrm{I}^{\mathrm{L}}_{1n}, \mathrm{I}^{\mathrm{U}}_{1n}], [\mathrm{F}^{\mathrm{L}}_{1n}, \mathrm{F}^{\mathrm{U}}_{1n}] \rangle \\ \vdots & \ddots & \vdots \\ \langle [\mathrm{T}^{\mathrm{L}}_{n1}, \mathrm{T}^{\mathrm{U}}_{n1}], [\mathrm{I}^{\mathrm{L}}_{n1}, \mathrm{I}^{\mathrm{U}}_{n1}], [\mathrm{F}^{\mathrm{L}}_{n1}, \mathrm{F}^{\mathrm{U}}_{n1}] \rangle & \cdots & \langle [\mathrm{T}^{\mathrm{L}}_{nn}, \mathrm{T}^{\mathrm{U}}_{nn}], [\mathrm{I}^{\mathrm{L}}_{nn}, \mathrm{I}^{\mathrm{U}}_{nn}], [\mathrm{F}^{\mathrm{L}}_{nn}, \mathrm{F}^{\mathrm{U}}_{nn}] \rangle \end{bmatrix} \end{array} \tag{15}$$

Pairwise comparison matrices for alternatives with respect to the criteria are given by Eq. (16).

$$\tilde{\dddot{P}}_A = \begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & \cdots & A_m \end{array} \\ \begin{bmatrix} \langle [\mathrm{T}^{\mathrm{L}}_{11}, \mathrm{T}^{\mathrm{U}}_{11}], [\mathrm{I}^{\mathrm{L}}_{11}, \mathrm{I}^{\mathrm{U}}_{11}], [\mathrm{F}^{\mathrm{L}}_{11}, \mathrm{F}^{\mathrm{U}}_{11}] \rangle & \cdots & \langle [\mathrm{T}^{\mathrm{L}}_{1m}, \mathrm{T}^{\mathrm{U}}_{1m}], [\mathrm{I}^{\mathrm{L}}_{1m}, \mathrm{I}^{\mathrm{U}}_{1m}], [\mathrm{F}^{\mathrm{L}}_{1m}, \mathrm{F}^{\mathrm{U}}_{1m}] \rangle \\ \vdots & \ddots & \vdots \\ \langle [\mathrm{T}^{\mathrm{L}}_{m1}, \mathrm{T}^{\mathrm{U}}_{m1}], [\mathrm{I}^{\mathrm{L}}_{m1}, \mathrm{I}^{\mathrm{U}}_{m1}], [\mathrm{F}^{\mathrm{L}}_{m1}, \mathrm{F}^{\mathrm{U}}_{m1}] \rangle & \cdots & \langle [\mathrm{T}^{\mathrm{L}}_{mm}, \mathrm{T}^{\mathrm{U}}_{mm}], [\mathrm{I}^{\mathrm{L}}_{mm}, \mathrm{I}^{\mathrm{U}}_{mm}], [\mathrm{F}^{\mathrm{L}}_{mm}, \mathrm{F}^{\mathrm{U}}_{mm}] \rangle \end{bmatrix} \end{array} \tag{16}$$

**Step 4** Calculate the normalized weights of criteria by using the proposed interval-valued neutrosophic evaluation scale. We show the steps of the proposed neutrosophic AHP based on the matrix for alternatives, $\tilde{\dddot{P}}_A$ with respect to a certain criterion.

$$\tilde{\dddot{N}}_{kj} = \left\langle \left[ \frac{\mathrm{T}^{\mathrm{L}}_{\mathrm{kj}}}{\sum_{k=1}^{m} T_{kj}^{U}}, \frac{\mathrm{T}^{\mathrm{U}}_{kj}}{\sum_{k=1}^{m} T_{kj}^{U}} \right], \left[ \frac{I^{\mathrm{L}}_{\mathrm{kj}}}{\sum_{k=1}^{m} I_{kj}^{U}}, \frac{\mathrm{I}^{\mathrm{U}}_{kj}}{\sum_{k=1}^{m} I_{kj}^{U}} \right], \times \left[ \frac{\mathrm{F}^{\mathrm{L}}_{\mathrm{kj}}}{\sum_{k=1}^{m} F_{kj}^{U}}, \frac{\mathrm{F}^{\mathrm{U}}_{kj}}{\sum_{k=1}^{m} F_{kj}^{U}} \right] \right\rangle \tag{18}$$

This results in the matrix in Eq. (19).

$$\tilde{\dddot{P}} = \begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & \cdots & A_m \end{array} \\ \begin{bmatrix} \left\langle \left[ \frac{\mathrm{T}^{\mathrm{L}}_{11}}{\sum_{k=1}^{m} T_{kj}^{U}}, \frac{\mathrm{T}^{\mathrm{U}}_{11}}{\sum_{k=1}^{m} T_{kj}^{U}} \right], \left[ \frac{\mathrm{I}^{\mathrm{L}}_{11}}{\sum_{k=1}^{m} I_{kj}^{U}}, \frac{\mathrm{I}^{\mathrm{U}}_{11}}{\sum_{k=1}^{m} I_{kj}^{U}} \right], \left[ \frac{\mathrm{F}^{\mathrm{L}}_{11}}{\sum_{k=1}^{m} F_{kj}^{U}}, \frac{\mathrm{F}^{\mathrm{U}}_{11}}{\sum_{k=1}^{m} F_{kj}^{U}} \right] \right\rangle & \cdots & \left\langle \left[ \frac{\mathrm{T}^{\mathrm{L}}_{1m}}{\sum_{k=1}^{m} T_{kj}^{U}}, \frac{\mathrm{T}^{\mathrm{U}}_{1m}}{\sum_{k=1}^{m} T_{kj}^{U}} \right], \left[ \frac{\mathrm{I}^{\mathrm{L}}_{1m}}{\sum_{k=1}^{m} I_{kj}^{U}}, \frac{\mathrm{I}^{\mathrm{U}}_{1m}}{\sum_{k=1}^{m} I_{kj}^{U}} \right], \left[ \frac{\mathrm{F}^{\mathrm{L}}_{1m}}{\sum_{k=1}^{m} F_{kj}^{U}}, \frac{\mathrm{F}^{\mathrm{U}}_{1m}}{\sum_{k=1}^{m} F_{kj}^{U}} \right] \right\rangle \\ \vdots & \ddots & \vdots \\ \left\langle \left[ \frac{\mathrm{T}^{\mathrm{L}}_{m1}}{\sum_{k=1}^{m} T_{ij}^{U}}, \frac{\mathrm{T}^{\mathrm{U}}_{m1}}{\sum_{k=1}^{m} T_{ij}^{U}} \right], \left[ \frac{\mathrm{I}^{\mathrm{L}}_{m1}}{\sum_{k=1}^{m} I_{kj}^{U}}, \frac{\mathrm{I}^{\mathrm{U}}_{m1}}{\sum_{k=1}^{m} I_{kj}^{U}} \right], \left[ \frac{\mathrm{F}^{\mathrm{L}}_{m1}}{\sum_{k=1}^{m} F_{kj}^{U}}, \frac{\mathrm{F}^{\mathrm{U}}_{m1}}{\sum_{k=1}^{m} F_{kj}^{U}} \right] \right\rangle & \cdots & \left\langle \left[ \frac{\mathrm{T}^{\mathrm{L}}_{mm}}{\sum_{k=1}^{m} T_{kj}^{U}}, \frac{\mathrm{T}^{\mathrm{U}}_{mm}}{\sum_{k=1}^{m} T_{kj}^{U}} \right], \left[ \frac{\mathrm{I}^{\mathrm{L}}_{mm}}{\sum_{k=1}^{m} I_{kj}^{U}}, \frac{\mathrm{I}^{\mathrm{U}}_{mm}}{\sum_{k=1}^{m} I_{kj}^{U}} \right], \left[ \frac{\mathrm{F}^{\mathrm{L}}_{mm}}{\sum_{k=1}^{m} F_{kj}^{U}}, \frac{\mathrm{F}^{\mathrm{U}}_{mm}}{\sum_{k=1}^{m} F_{kj}^{U}} \right] \right\rangle \end{bmatrix} \end{array} \tag{19}$$

**Step 4.1** Sum the values in each column as in Eq. (17):

$$\tilde{\dddot{S}}_{ij} = \left\langle \left[ \sum_{k=1}^{m} T_{kj}^{L}, \sum_{k=1}^{m} T_{kj}^{U} \right], \left[ \sum_{k=1}^{m} I_{kj}^{L}, \sum_{k=1}^{m} I_{kj}^{U} \right], \times \left[ \sum_{k=1}^{m} F_{kj}^{L}, \sum_{k=1}^{m} F_{kj}^{U} \right] \right\rangle \tag{17}$$

where j = 1, …, m.

**Step 4.3** Calculate the average of each row to obtain the neutrosophic priority vector of the alternatives as in Eq. (20).

$$\ddot{\tilde{W}}_A = \left( \frac{\left[\sum_{k=1}^{m} \frac{\mathrm{T}_{1j}^{\mathrm{L}}}{\sum_{k=1}^{m} T_{kj}^{U}}, \sum_{k=1}^{m} \frac{\mathrm{T}_{1j}^{\mathrm{U}}}{\sum_{k=1}^{m} T_{kj}^{U}}\right]}{m}, \frac{\left[\sum_{k=1}^{m} \frac{\mathrm{I}_{1j}^{\mathrm{L}}}{\sum_{k=1}^{m} I_{kj}^{U}}, \sum_{k=1}^{m} \frac{\mathrm{I}_{1j}^{\mathrm{U}}}{\sum_{k=1}^{m} I_{kj}^{U}}\right]}{m}, \times \frac{\left[\sum_{k=1}^{m} \frac{\mathrm{F}_{1j}^{\mathrm{L}}}{\sum_{k=1}^{m} F_{kj}^{U}}, \sum_{k=1}^{m} \frac{\mathrm{F}_{1j}^{\mathrm{U}}}{\sum_{k=1}^{m} F_{kj}^{U}}\right]}{m} \right), \mathrm{j} = 1, \ldots, \mathrm{m}. \tag{20}$$

**Step 4.4** Repeat the above steps with respect to each criterion and obtain neutrosophic weights vectors for all of the alternatives. The same process is repeated in order to obtain the priority weights of the criteria.

**Step 5** Construct the matrix $\ddot{\tilde{\Psi}}$ as in Eq. (21) in order to obtain the final combined priority weights

## 6.2 Steps of IVNAHP with CS measures

The advantages of this method are its similarity to classical AHP method and simplicity in its calculations up to Step 4.4. Cosine similarity measure provides a more informative approach. However, the method needs a deneutrosophication and cosine similarity formulas which make the usage of this method somewhat more complex. In the following, we present the steps of the IVNAHP with CS Measures.

**Step 1** Determine the interval-valued neutrosophic evaluation scale (see Table 1).

**Step 2** Decompose the problem into a hierarchy of goal, criteria, sub-criteria and alternatives (see Fig. 3).

**Step 3** Construct the pairwise comparison matrices ($\ddot{\tilde{P}}$) for alternatives and criteria by using interval-valued neutrosophic sets as in Eq. (16).

$\ddot{\tilde{\Psi}} =$

| Alternatives and Their Weights | Criteria and Their Weights | | | |
|---|---|---|---|---|
| | $\ddot{\tilde{W}}_{C_1}$ $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_1}}, \mathrm{T}^{\mathrm{U}}_{W_{C_1}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_1}}, \mathrm{I}^{\mathrm{U}}_{W_{C_1}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_1}}, \mathrm{F}^{\mathrm{U}}_{W_{C_1}}]\rangle$ | $\ddot{\tilde{W}}_{C_2}$ $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_2}}, \mathrm{T}^{\mathrm{U}}_{W_{C_2}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_2}}, \mathrm{I}^{\mathrm{U}}_{W_{C_2}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_2}}, \mathrm{F}^{\mathrm{U}}_{W_{C_2}}]\rangle$ | ... | $\ddot{\tilde{W}}_{C_n}$ $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_n}}, \mathrm{T}^{\mathrm{U}}_{W_{C_n}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_n}}, \mathrm{I}^{\mathrm{U}}_{W_{C_n}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_n}}, \mathrm{F}^{\mathrm{U}}_{W_{C_n}}]\rangle$ |
| $\ddot{\tilde{W}}_{A_1}$ | $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_1A_1}}, \mathrm{T}^{\mathrm{U}}_{W_{C_1A_1}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_1A_1}}, \mathrm{I}^{\mathrm{U}}_{W_{C_1A_1}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_1A_1}}, \mathrm{F}^{\mathrm{U}}_{W_{C_1A_1}}]\rangle$ | $\langle[\mathrm{T}^{\mathrm{L}}_{C_2,A_1}, \mathrm{T}^{\mathrm{U}}_{C_2W_{A_1}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_2A_1}}, \mathrm{I}^{\mathrm{U}}_{W_{A_1}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_2A_1}}, \mathrm{F}^{\mathrm{U}}_{W_{C_2A_1}}]\rangle$ | ... | $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_3A_1}}, \mathrm{T}^{\mathrm{U}}_{W_{C_3A_1}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_3A_1}}, \mathrm{I}^{\mathrm{U}}_{W_{C_3A_1}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_3A_1}}, \mathrm{F}^{\mathrm{U}}_{W_{C_3A_1}}]\rangle$ |
| $\ddot{\tilde{W}}_{A_2}$ | $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_1A_2}}, \mathrm{T}^{\mathrm{U}}_{W_{C_1A_2}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_1A_2}}, \mathrm{I}^{\mathrm{U}}_{W_{C_1A_2}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_1A_2}}, \mathrm{F}^{\mathrm{U}}_{W_{C_1A_2}}]\rangle$ | $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_2A_2}}, \mathrm{T}^{\mathrm{U}}_{W_{C_2A_2}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_2A_2}}, \mathrm{I}^{\mathrm{U}}_{W_{C_2A_2}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_2A_2}}, \mathrm{F}^{\mathrm{U}}_{W_{C_2A_2}}]\rangle$ | ... | $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_3A_2}}, \mathrm{T}^{\mathrm{U}}_{W_{C_3A_2}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_3A_2}}, \mathrm{I}^{\mathrm{U}}_{W_{C_3A_2}}], [\mathrm{F}^{\mathrm{L}}_{W_{A_2}}, \mathrm{F}^{\mathrm{U}}_{W_{C_3A_2}}]\rangle$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $\ddot{\tilde{W}}_{A_m}$ | $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_1A_m}}, \mathrm{T}^{\mathrm{U}}_{W_{C_1A_m}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_1A_m}}, \mathrm{I}^{\mathrm{U}}_{W_{C_1A_m}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_1A_m}}, \mathrm{F}^{\mathrm{U}}_{W_{C_1A_m}}]\rangle$ | $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_2A_m}}, \mathrm{T}^{\mathrm{U}}_{W_{C_2A_m}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_2A_m}}, \mathrm{I}^{\mathrm{U}}_{W_{C_2A_m}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_2A_m}}, \mathrm{F}^{\mathrm{U}}_{W_{C_2A_m}}]\rangle$ | ... | $\langle[\mathrm{T}^{\mathrm{L}}_{W_{C_3A_m}}, \mathrm{T}^{\mathrm{U}}_{W_{C_3A_m}}], [\mathrm{I}^{\mathrm{L}}_{W_{C_3A_m}}, \mathrm{I}^{\mathrm{U}}_{W_{C_3A_m}}], [\mathrm{F}^{\mathrm{L}}_{W_{C_3A_m}}, \mathrm{F}^{\mathrm{U}}_{W_{C_3A_m}}]\rangle$ |

(21)

**Step 6** Obtain the final combined interval-valued neutrosophic weights of alternatives by using Eq. (22).

$$\begin{aligned} \ddot{\tilde{\Omega}}_{A_j} = {} & \left\langle\left[\mathrm{T}^{\mathrm{L}}_{W_{C_1}}, \mathrm{T}^{\mathrm{U}}_{W_{C_1}}\right], \left[\mathrm{I}^{\mathrm{L}}_{W_{C_1}}, \mathrm{I}^{\mathrm{U}}_{W_{C_1}}\right], \left[\mathrm{F}^{\mathrm{L}}_{W_{C_1}}, \mathrm{F}^{\mathrm{U}}_{W_{C_1}}\right]\right\rangle \left\langle\left[\mathrm{T}^{\mathrm{L}}_{W_{C_1A_1}}, \mathrm{T}^{\mathrm{U}}_{W_{C_1A_1}}\right], \left[\mathrm{I}^{\mathrm{L}}_{W_{C_1A_1}}, \mathrm{I}^{\mathrm{U}}_{W_{C_1A_1}}\right], \left[\mathrm{F}^{\mathrm{L}}_{W_{C_1A_1}}, \mathrm{F}^{\mathrm{U}}_{W_{C_1A_1}}\right]\right\rangle \\ & + \left\langle\left[\mathrm{T}^{\mathrm{L}}_{W_{C_2}}, \mathrm{T}^{\mathrm{U}}_{W_{C_2}}\right], \left[\mathrm{I}^{\mathrm{L}}_{W_{C_2}}, \mathrm{I}^{\mathrm{U}}_{W_{C_2}}\right], \left[\mathrm{F}^{\mathrm{L}}_{W_{C_2}}, \mathrm{F}^{\mathrm{U}}_{W_{C_2}}\right]\right\rangle \left\langle\left[\mathrm{T}^{\mathrm{L}}_{W_{C_2A_2}}, \mathrm{T}^{\mathrm{U}}_{W_{C_2A_2}}\right], \left[\mathrm{I}^{\mathrm{L}}_{W_{C_2A_2}}, \mathrm{I}^{\mathrm{U}}_{W_{C_2A_2}}\right], \left[\mathrm{F}^{\mathrm{L}}_{W_{C_2A_2}}, \mathrm{F}^{\mathrm{U}}_{W_{C_2A_2}}\right]\right\rangle \\ & + \cdots + \left\langle\left[\mathrm{T}^{\mathrm{L}}_{W_{C_n}}, \mathrm{T}^{\mathrm{U}}_{W_{C_n}}\right], \left[\mathrm{I}^{\mathrm{L}}_{W_{C_n}}, \mathrm{I}^{\mathrm{U}}_{W_{C_n}}\right], \left[\mathrm{F}^{\mathrm{L}}_{W_{C_n}}, \mathrm{F}^{\mathrm{U}}_{W_{C_n}}\right]\right\rangle \left\langle\left[\mathrm{T}^{\mathrm{L}}_{W_{A_m}}, \mathrm{T}^{\mathrm{U}}_{W_m}\right], \left[\mathrm{I}^{\mathrm{L}}_{W_{A_m}}, \mathrm{I}^{\mathrm{U}}_{W_{A_m}}\right], \left[\mathrm{F}^{\mathrm{L}}_{W_{A_m}}, \mathrm{F}^{\mathrm{U}}_{W_{A_m}}\right]\right\rangle \end{aligned} \tag{22}$$

**Step 7** Apply the deneutrosophication formula in Eq. (6) in order to obtain the crisp weights of alternatives as given in Eq. (23).

$$\Omega_{A_j} = \left(w_{A_1}, w_{A_2}, w_{A_3}\right) \tag{23}$$

**Step 8** Normalize the crisp weights of alternatives.

**Step 9** Rank the alternatives and select the alternative with the largest weight.

**Step 4** Calculate the normalized weights of criteria by using the proposed interval-valued neutrosophic evaluation scale. We show the steps of the proposed neutrosophic AHP based on the matrix for alternatives, $\ddot{\tilde{P}}_A$ with respect to a certain criterion.

**Step 4.1.** Sum the values in each column as in Eq. (17).

**Step 4.2.** Select the upper value for each parameter in Eq. (17) and divide each term by its corresponding element to obtain $\ddot{\tilde{N}}_{ij}$ in Eq. (18) and this results in the matrix in Eq. (19).

**Table 2** Linguistic terms and neutrosophicated importance weights

| Linguistic term | Neutrosophic sets |
|---|---|
| Equal importance | $\langle[0.5, 0.5], [0.5, 0.5], [0.5, 0.5]\rangle$ |
| Weakly more importance | $\langle[0.50, 0.60], [0.35, 0.45], [0.40, 0.50]\rangle$ |
| Moderate importance | $\langle[0.55, 0.65], [0.30, 0.40], [0.35, 0.45]\rangle$ |
| Moderately more importance | $\langle[0.60, 0.70], [0.25, 0.35], [0.30, 0.40]\rangle$ |
| Strong importance | $\langle[0.65, 0.75], [0.20, 0.30], [0.25, 0.35]\rangle$ |
| Strongly more importance | $\langle[0.70, 0.80], [0.15, 0.25], [0.20, 0.30]\rangle$ |
| Very strong importance | $\langle[0.75, 0.85], [0.10, 0.20], [0.15, 0.25]\rangle$ |
| Very strongly more importance | $\langle[0.80, 0.90], [0.05, 0.10], [0.10, 0.20]\rangle$ |
| Extreme importance | $\langle[0.90, 0.95], [0, 0.05], [0.05, 0.15]\rangle$ |
| Extremely high importance | $\langle[0.95, 1.0], [0.0, 0.0], [0.0, 0.10]\rangle$ |
| Absolutely more importance | $\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.0]\rangle$ |

**Table 3** Pairwise comparison matrix

| | Cost | Environmental conditions | Sustainability |
|---|---|---|---|
| Cost | $\langle[0.5, 0.5], [0.5, 0.5], [0.5, 0.5]\rangle$ | $\langle[0.65, 0.75], [0.20, 0.30], [0.25, 0.35]\rangle$ | $\langle[0.15, 0.25], [0.10, 0.20], [0.75, 0.85]\rangle$ |
| Environmental conditions | $\langle[0.25, 0.35] [0.20, 0.30], [0.65, 0.75]\rangle$ | $\langle[0.5, 0.5], [0.5, 0.5], [0.5, 0.5]\rangle$ | $\langle[0.0, 0.10], [0.0, 0.0], [0.95, 1.0]\rangle$ |
| Sustainability | $\langle[0.75, 0.85], [0.10, 0.20], [0.15, 0.25]\rangle$ | $\langle[0.95, 1.0], [0.0, 0.0], [0.0, 0.10]\rangle$ | $\langle[0.5, 0.5], [0.5, 0.5], [0.5, 0.5]\rangle$ |

**Step 4.3.** Calculate the average of each row to obtain the neutrosophic priority vector of the alternatives as in Eq. (20).

**Step 4.4.** Repeat the above steps with respect to each criterion and obtain neutrosophic weights vectors for all of the alternatives. The same process is repeated in order to obtain the priority weights of the criteria.

**Step 5** Apply the cosine similarity measure between each alternative pair based on Eq. (14).

**Step 6** Assign the corresponding AHP score using the linear regression function given in Eq. (24) and obtain AHP scores. This function transforms the cosine similarities to pairwise comparison scores between 1 and 9.

$$x = \frac{(a - y)}{b} \tag{24}$$

**Step 7** Obtain the alternative weights based on the steps of classical AHP.

**Step 8** Rank the alternatives and select the alternative with the largest weight.

## 7 Application: selection among alternative renewable energy sources

An investor wants to invest in a renewable energy production system where three renewable energy alternatives exist: *wind power energy, solar power energy,* and *biomass power energy*. Three criteria will be considered in this decision-making process: *cost, environmental conditions* and *sustainability*. A group of energy experts make compromised assessments instead of independent assessments. The experts prefer using linguistic assessments rather than direct numerical assessments. Table 2 shows the linguistic scale and their corresponding neutrosophic numbers that will be used in the pairwise comparison matrices. The goal is to select the best renewable energy alternative. The hierarchy of the problem is illustrated in Fig. 3.

Table 3 presents the pairwise comparison matrix for the criteria with respect to the goal. It can be seen that *sustainability* criterion is more important than *cost* criterion and *environmental conditions* criterion. The *cost* criterion is more important than *environmental conditions* criterion.

In the following, we present the solutions of the defined problem through the proposed two neutrosophic AHP methods.

### 7.1 Application of IVN-AHP method

**Step 1** Determine the interval-valued neutrosophic evaluation scale (see Table 3).

**Step 2** Decompose the problem into a hierarchy of goal, criteria and alternatives (see Fig. 3).

**Step 3** Construct the pairwise comparison matrices ($\tilde{\ddot{P}}$) by using interval-valued neutrosophic sets.

**Step 4** Calculate the normalized weights of criteria by using the proposed interval-valued neutrosophic evaluation scale.

**Table 4** The column sums of the pairwise comparison matrix

| | Cost | | | | | | Environmental conditions | | | | | | Sustainability | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Tl | Tu | Il | Iu | Fl | Fu | Tl | Tu | Il | Iu | Fl | Fu | Tl | Tu | Il | Iu | Fl | Fu |
| Cost | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.65 | 0.75 | 0.2 | 0.3 | 0.25 | 0.35 | 0.15 | 0.25 | 0.1 | 0.2 | 0.75 | 0.85 |
| Environmental conditions | 0.25 | 0.35 | 0.2 | 0.3 | 0.65 | 0.75 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0 | 0.1 | 0 | 0 | 0.95 | 1 |
| Sustainability | 0.75 | 0.85 | 0.1 | 0.2 | 0.15 | 0.25 | 0.95 | 1 | 0 | 0 | 0 | 1 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| Sum | 1.5 | 1.7 | 0.8 | 1 | 1.3 | 1.5 | 2.1 | 2.25 | 0.7 | 0.8 | 0.75 | 1.85 | 0.65 | 0.85 | 0.6 | 0.7 | 2.2 | 2.35 |

**Table 5** The normalized values of the pairwise comparison matrix

| | Cost | | | | | | Environmental conditions | | | | | | Sustainability | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Tl | Tu | Il | Iu | Fl | Fu | Tl | Tu | Il | Iu | Fl | Fu | Tl | Tu | Il | Iu | Fl | Fu |
| Cost | 0.294 | 0.294 | 0.500 | 0.500 | 0.333 | 0.333 | 0.289 | 0.333 | 0.250 | 0.375 | 0.135 | 0.189 | 0.176 | 0.294 | 0.143 | 0.286 | 0.319 | 0.362 |
| Environmental conditions | 0.147 | 0.206 | 0.200 | 0.300 | 0.433 | 0.500 | 0.222 | 0.222 | 0.625 | 0.625 | 0.270 | 0.270 | 0 | 0.118 | 0 | 0 | 0.404 | 0.426 |
| Sustainability | 0.441 | 0.500 | 0.100 | 0.200 | 0.100 | 0.167 | 0.422 | 0.444 | 0 | 0 | 0 | 0.541 | 0.588 | 0.588 | 0.714 | 0.714 | 0.213 | 0.213 |

**Table 6** The weights of criteria

| | Weights | | | | | |
|---|---|---|---|---|---|---|
| | Tl | Tu | Il | Iu | Fl | Fu |
| Cost | 0.253 | 0.307 | 0.298 | 0.387 | 0.263 | 0.295 |
| Environmental conditions | 0.123 | 0.182 | 0.275 | 0.308 | 0.369 | 0.399 |
| Sustainability | 0.484 | 0.511 | 0.271 | 0.305 | 0.104 | 0.307 |

**Table 7** Pairwise comparison matrices for alternatives with respect to the criteria

| | Cost | | | | | | Environmental conditions | | | | | | Sustainability | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Tl | Tu | Il | Iu | Fl | Fu | Tl | Tu | Il | Iu | Fl | Fu | Tl | Tu | Il | Iu | Fl | Fu |
| Wind power energy | 0.153 | 0.296 | 0.118 | 0.387 | 0.850 | 0.950 | 0.153 | 0.277 | 0.298 | 0.335 | 0.963 | 0.975 | 0.103 | 0.157 | 0.289 | 0.359 | 0.863 | 0.895 |
| Solar power energy | 0.353 | 0.407 | 0.170 | 0.309 | 0.263 | 0.295 | 0.253 | 0.300 | 0.319 | 0.387 | 0.163 | 0.195 | 0.303 | 0.397 | 0.298 | 0.307 | 0.254 | 0.277 |
| Biomass power energy | 0.563 | 0.807 | 0.298 | 0.399 | 0.113 | 0.119 | 0.653 | 0.890 | 0.308 | 0.397 | 0.069 | 0.100 | 0.755 | 0.771 | 0.298 | 0.387 | 0.096 | 0.107 |

**Table 8** Cosine similarity measures

| Similarity between | Cost | Environmental conditions | Sustainability | Overall cosine similarities |
|---|---|---|---|---|
| Wind power energy (WPE) & Solar power energy (SPE) | 0.780 (SPE > WPE) | 0.685 (WPE > SPE) | 0.747 (SPE > WPE) | 0.738 |
| Wind power energy (WPE) & Biomass power energy (BPE) | 0.463 (BPE > WPE) | 0.404 (WPE > BPE) | 0.376 (WPE > BPE) | 0.415 |
| Biomass power energy (BPE) & Solar power energy (SPE) | 0.915 (SPE > BPE) | 0.858 (SPE > BPE) | 0.885 (SPE > BPE) | 0.886 |

**Step 4.1** Sum the values in each column as in Eq. (17). The summed values for Table 3 are given in Table 4.

**Step 4.2** Obtain $\tilde{\ddot{N}}_{ij}$ in Eq. (9). The normalized values are given in Table 5.

Table 6 gives the weights of the criteria by averaging the elements in the rows.

**Step 5** Obtain the final combined interval-valued neutrosophic weights of the alternatives by using Eq. (22). Table 7 shows the priority results of pairwise comparison matrices for the alternatives with respect to the criteria.

**Table 9** Pairwise comparisons with cosine similarities using the linear regression

| Alternatives | Cost | | | Environmental conditions | | | Sustainability | | |
|---|---|---|---|---|---|---|---|---|---|
| | Wind power energy | Solar power energy | Biomass power energy | Wind power energy | Solar power energy | Biomass power energy | Wind power energy | Solar power energy | Biomass power energy |
| Wind power energy | 1.000 | 0.363 | 0.189 | 1.000 | 0.284 | 0.173 | 1.000 | 0.331 | 5.990 |
| Solar power energy | 2.757 | 1.000 | 1.676 | 3.522 | 1.000 | 2.139 | 3.021 | 1.000 | 1.920 |
| Biomass power energy | 5.294 | 0.596 | 1.000 | 5.765 | 0.467 | 1.000 | 0.167 | 0.521 | 1.000 |

**Step 6** Apply the deneutrosophication formula in Eq. (6) in order to obtain the crisp weights of criteria as given in Eq. (23).

$$\Omega_{C_j} = (0.656, 0.515, 0.790) \tag{25}$$

$$\Omega^N_{C_j} = (0.335, 0.263, 0.403) \tag{26}$$

**Step 8** Normalize the crisp weights of alternatives.

$$\Omega_{A_j} = (0.070, 0.577, 0.936) \tag{27}$$

$$\Omega^N_{A_j} = (0.044, 0.364, 0.591) \tag{28}$$

**Step 9** Rank the alternatives and select the best alternative with the largest weight.

In our application, the biomass power energy is the most important alternative.

## 7.2 Application of IVNAHP-CS

**Step 1** Determine the interval-valued neutrosophic evaluation scale (see Table 2).

**Step 2** Decompose the problem into a hierarchy of goal, criteria and alternatives (see Fig. 3).

**Step 3** Construct the pairwise comparison matrices ($\ddot{\tilde{P}}$) for alternatives and criteria by using interval-valued neutrosophic sets as in Eqs. (15) and (16), respectively.

**Step 4** Calculate the normalized weights of criteria by using the proposed interval-valued neutrosophic evaluation scale. In the following, we give the steps of the proposed neutrosophic AHP based on the matrix for alternatives, $\ddot{\tilde{P}}_A$ with respect to a certain criterion.

**Step 4.1** Sum the values in each column as in Eq. (17).

**Step 4.2** Select the upper value for each parameter in Eq. (17) and divide each term by its corresponding element to obtain $\ddot{\tilde{N}}_{ij}$ in Eq. (18) and this results in the matrix in Eq. (19).

**Step 4.3** Calculate the average of each row to obtain the neutrosophic priority vector of the alternatives as in Eq. (20).

**Step 4.4** Repeat the above steps with respect to each criterion and obtain neutrosophic weights vectors for all of the alternatives. The same process is repeated in order to obtain the priority weights of the criteria.

**Step 5** Apply the cosine similarity measure between each alternative pair based on Eq. (14). The cosine similarity measures and superiorities between energy alternatives are given in Table 8. For instance, SPE is better than WPE with respect to Cost criterion and the cosine similarity between these alternatives with respect to Cost criterion is 0.780.

**Table 10** Obtained alternative weights based on the steps of classical AHP

| Weights based on cosine similarity measures | | | | |
|---|---|---|---|---|
| | Cost 0.335 | Environmental conditions 0.262 | Sustainability 0.403 | Result |
| Wind power energy | 0.121 | 0.104 | 0.363 | 0.214 |
| Solar power energy | 0.467 | 0.520 | 0.492 | 0.491 |
| Biomass power energy | 0.413 | 0.376 | 0.144 | 0.295 |

**Table 11** Comparative results of IVN-AHP and IVIF-AHP methods

| Criteria | IVN-AHP | IVIF-AHP |
|---|---|---|
| Cost | 0.338 | 0.330 |
| Environmental conditions | 0.140 | 0.110 |
| Sustainability | 0.572 | 0.560 |

**Step 6** Assign the corresponding AHP score using the linear regression function given in Eqs. (24) and (29) and obtain AHP scores. See Table 9.

$$x = \frac{(1.125 - y)}{0.125} \tag{29}$$

**Step 7** Obtain the alternative weights based on the steps of classical AHP. See Table 10.
**Step 8** Rank the alternatives and select the alternative with the largest weight.

From the results, it can be seen that the best alternative is solar power energy depending on cosine similarity measures. The following alternatives are biomass power energy and wind power energy.

### 7.3 Comparative analyses

For the validation, we compare the weights of the criteria calculated by our proposed IVN-AHP method with the ones obtained by the interval-valued intuitionistic fuzzy AHP (IVIF-AHP) method proposed by Wu et al. (2013). An interval-valued neutrosophic pairwise comparison matrix is transformed into interval-valued intuitionistic pairwise comparison matrix by removing the indeterminacy intervals. We obtain the rankings in Table 11.

The results in Table 11 indicate that the obtained ranking is same. However, it is observed that the priority weights of the criteria increase with the incorporation of indeterminacy, which means that indeterminacy parameter caused an increase in the criteria weights.

## 8 Conclusion

In this study, the neutrosophic MCDM papers are reviewed and summarized. It can be seen that there are several neutrosophic MCDM papers such as EDAS, TOPSIS, TODIM, and QUALIFLEX. AHP has been used in MCDM problems in the literature extensively. To the best of our knowledge, the AHP method firstly used with interval-valued neutrosophic sets. The IVN-AHP and IVNAHP-CS methods are proposed and the steps of two methods are detailed. An application in renewable energy management is presented, and these two methods are applied to this problem. In conclusion, both methods give the same results: biomass power energy is the best alternative. For further research, another similarity measure can be used in IVN-AHP or other extensions of AHP such as hesitant fuzzy AHP or Pythagorean fuzzy AHP can be used together with cosine similarity or any other similarity measure.

### Compliance with ethical standards

**Conflict of interest** The authors declare that they have no conflict of interest.

**Human and animal rights** This article does not contain any studies with human participants or animals performed by any of the authors.

## Appendix

See Table 12

**Table 12** Types of neutrosophic sets in MCDM papers

| Author (year) | Neutrosophic sets | Single-valued neutrosophic sets | Interval neutrosophic sets | Trapezoidal neutrosophic sets | Multi-valued neutrosophic sets | Single-valued trapezoidal neutrosophic set | Bipolar neutrosophic sets | Interval-valued bipolar fuzzy weighted neutrosophic sets |
|---|---|---|---|---|---|---|---|---|
| Baušys and Juodagalvienė (2017) | | x | | | | | | |
| Bausys and Zavadskas (2015) | | | x | | | | | |
| Bausys et al. (2015) | x | | | | | | | |
| Biswas et al. (2016) | | x | | | | | | |
| Broumi et al. (2017) | | | x | | | | | |
| Deli and Subas (2017) | | | | | | x | | |
| Deli et al. (2015) | | | | | | | x | |
| Deli et al. (2016) | | | | | | | | x |
| Elhassouny and Smarandache (2016) | | x | | | | | | |
| Garg and Nancy (2017) | | | x | | | | | |
| Hu et al. (2017) | | | x | | | | | |
| Huang (2016) | | x | | | | | | |
| Ji and Zhang (2016) | | | x | | | | | |
| Ji et al (2016) | | | | | x | | | |
| Ji et al. (2017) | | | | | | x | | |
| Karaaslan (2017) | | x | x | | | | | |
| Karasan and Kahraman (2018) | | | x | | | | | |
| Kharal (2014) | x | | | | | | | |
| Liang et al. (2016) | | | | | | x | | |
| Liang et al. (2017a, b) | | | | | | x | | |
| Liang et al. (2017a, b) | | | | | | x | | |
| Liu (2016a, b) | | x | | | | | | |
| Liu (2016a, b) | | | x | | | | | |
| Liu and Wang (2014) | | x | | | | | | |
| Liu and Tang (2016) | | | x | | | | | |
| Liu and Zhang (2017) | x | | | | | | | |
| Liu et al. (2016) | | | | | x | | | |
| Ma et al. (2016) | | | x | | | | | |
| Ma et al. (2017a, b) | | | x | | | | | |
| Ma et al. (2017a, b) | | | x | | | | | |

**Table 12** continued

| Author (year) | Neutrosophic sets | Single-valued neutrosophic sets | Interval neutrosophic sets | Trapezoidal neutrosophic sets | Multi-valued neutrosophic sets | Single-valued trapezoidal neutrosophic set | Bipolar neutrosophic sets | Interval-valued bipolar fuzzy weighted neutrosophic sets |
|---|---|---|---|---|---|---|---|---|
| NAdAban and Dzitac (2016) | x | | | | | | | |
| Otay and Kahraman (2018) | | | x | | | | | |
| Peng and Dai (2016) | | x | | | | | | |
| Peng et al. (2014) | | x | | | | | | |
| Peng et al. (2015) | x | | | | | | | |
| Peng et al. (2016a, b, c) | | | | | x | | | |
| Peng et al. (2016a, b, c) | | x | | | | | | |
| Peng et al. (2016a, b, c) | | | | | x | | | |
| Peng et al. (2017a, b) | | | | | x | | | |
| Peng et al. (2017a, b) | | x | | | | | | |
| Radwan et al. (2016) | x | | | | | | | |
| Ren (2017) | | x | | | | | | |
| Sahin (2017a) | x | | | | | | | |
| Sahin (2017b) | | | x | | | | | |
| Sahin and Kucuk (2015) | | x | | | | | | |
| Sahin and Liu (2017a) | | x | | | | | | |
| Sahin and Liu (2017b) | | x | | | | | | |
| Stanujkic et al. (2017) | | x | | | | | | |
| Sun and Sun (2016) | | x | | | | | | |
| Sun et al. (2015) | | | x | | | | | |
| Tian et al. (2016a, b) | | | x | | | | | |
| Tian et al. (2016a, b) | | x | | | | | | |
| Tian et al. (2017) | | x | | | | | | |
| Ulucay et al. (2016) | | | | | | | x | |
| Wang et al. (2016) | | x | | | | x | | |
| Wang (2016) | | | x | | | | | |
| Wang and Li (2015) | | | | | x | | | |
| Wang and Zhang (2017) | | | | | x | | | |
| Wu et al. (2016) | | x | | | | | | |
| Ye (2013) | | x | | | | | | |

**Table 12** continued

| Author (year) | Neutrosophic sets | Single-valued neutrosophic sets | Interval neutrosophic sets | Trapezoidal neutrosophic sets | Multi-valued neutrosophic sets | Single-valued trapezoidal neutrosophic set | Bipolar neutrosophic sets | Interval-valued bipolar fuzzy weighted neutrosophic sets |
|---|---|---|---|---|---|---|---|---|
| Ye (2014a, b, c) | | | x | | | | | |
| Ye (2014a, b, c) | | x | | | | | | |
| Ye (2014a, b, c) | | x | | | | | | |
| Ye (2015a, b) | | x | x | | | | | |
| Ye (2015a, b) | | | | x | | | | |
| Ye (2017) | x | | | | | | | |
| Zavadskas et al. (2015) | | x | | | | | | |
| Zavadskas et al. (2016) | | x | | | | | | |
| Zavadskas et al. (2017) | | x | | | | | | |
| Zhang et al. (2015) | | | x | | | | | |
| Zhang et al. (2016a, b, c) | x | | | | | | | |
| Zhang et al. (2016a, b, c) | | | x | | | | | |
| Zhang et al. (2016a, b, c) | x | | | | | | | |

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