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METHODOLOGIES AND APPLICATION

Multi-attribute group decision making based on power generalized Heronian mean operator under hesitant fuzzy linguistic environment

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Abstract

Generalized Heronian mean (GHM) is a useful aggregation operator with the characteristic of capturing the interrelationship of evaluation information. In this paper, we propose some new operators by combining the power average operator and the GHM operator under hesitant fuzzy linguistic environment, such as the hesitant fuzzy linguistic power generalized Heronian mean (HFLPGHM) operator, the hesitant fuzzy linguistic power generalized geometric Heronian mean (HFLPGGHM) operator, the hesitant fuzzy linguistic weighted power generalized Heronian mean (HFLWPGHM) operator and the hesitant fuzzy linguistic weighted power generalized geometric Heronian mean (HFLWPGGHM) operator. Then, some special cases of the proposed HFLPGHM and HFLPGGHM operators are discussed in detail. Furthermore, based on the proposed operators, we develop a novel method to solve multi-attribute group decision-making problem under hesitant fuzzy linguistic environment. Finally, a numerical example is given to illustrate the application of the developed method and a comparison analysis is also conducted, which further demonstrates the effectiveness and feasibility of the proposed method.

Keywords Multi-attribute group decision making (MAGDM) · Hesitant fuzzy linguistic set · Power generalized Heronian mean · Hesitant fuzzy linguistic power generalized Heronian mean (HFLPGHM) operator · Hesitant fuzzy linguistic power generalized geometric Heronian mean (HFLPGGHM) operator

1 Introduction

Multi-attribute group decision making (MAGDM) is an important research branch of modern decision science, which is to select the most desirable alternative(s) under multiple attributes based on the evaluation information given by many decision makers (He et al. 2015, 2016; Ju and Yang 2015; Ju et al. 2016a; Liu et al. 2014b; Merigó et al. 2016; Yue 2011). Since MAGDM problems often face complex and changeable environment, evaluation information is more suitable to be expressed in fuzzy form, such as interval fuzzy number (Yue 2011), fuzzy sets (Zadeh 1965), intuitionistic fuzzy sets (Atanassov 1986), type 2 fuzzy sets (Dubois and Prade 1980), fuzzy multi-sets (Yager 1986), etc. Among various forms of fuzzy information, Torra et al. (2009) proposed hesitant fuzzy sets (HFSs), which permit the membership degree of an element to have several different values. Recently, the hesitant fuzzy sets have received extensive concern of scholars (Chen et al. 2016; Dong et al. 2015; Gitinavard et al. 2017; Jin et al. 2016; Tong and Yu 2016; Wei 2012; Xia and Xu 2011; Zhang et al. 2016; Zhao et al. 2017).

In the practical decision-making process, due to the increasing complexity of socioeconomic environment and the vagueness of inherent subjective nature of human thinking, it is more suitable for decision makers to provide their preferences by means of linguistic variables, such as ‘very poor,’ ‘poor,’ ‘fair,’ ‘good’ and ‘very good.’ Based on the linguistic variables initially proposed by Zadeh (1975), Rodríguez et al. (2012) first proposed the concept of hesitant fuzzy linguistic term sets (HFLTSs) and a transformation function to obtain HFLTSs from the comparative linguistic expressions generated by a context-free grammar. Lin et al.

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(2014) proposed another type of HFLTSs, which permits the membership have a set of possible hesitant fuzzy linguistic values. Up to now, a lot of research work has been done on HFLTSs, such as the hesitant fuzzy linguistic measures (Farhadinia 2016; Hesamian and Shams 2015; Liao and Xu 2015; Liao et al. 2014), the hesitant fuzzy linguistic preference relations (Liu et al. 2014a; Zhang and Wu 2014; Zhu and Xu 2014), and the hesitant fuzzy linguistic decision-making methods (Beg and Rashid 2013; Chen and Hong 2014; Kahraman et al. 2016; Lee and Chen 2015a, b; Liao et al. 2015; Wang et al. 2016; Wei et al. 2015), etc.

For multi-attribute group decision-making problem, aggregation operator plays an important role in information fusion. Recently, some operators are proposed to aggregate hesitant fuzzy linguistic information, such as the hesitant fuzzy linguistic weighted average operator and the hesitant fuzzy linguistic ordered weighted average operator (Wei et al. 2014); the hesitant fuzzy linguistic weighted average (HFLWA) operator, the hesitant fuzzy linguistic weighted geometric (HFLWG) operator, the hesitant fuzzy linguistic ordered weighted average (HFLOWA) operator and the hesitant fuzzy linguistic ordered weighted geometric (HFLOWG) operator (Lee and Chen 2015a); the hesitant fuzzy linguistic Bonferroni mean operator and the weighted hesitant fuzzy linguistic Bonferroni mean operator (Gou et al. 2017); the uncertain hesitant fuzzy linguistic ordered weighted averaging operator and the uncertain hesitant fuzzy linguistic hybrid aggregation operator (Zhang and Qi 2013). However, most of the existing hesitant fuzzy linguistic aggregation operators do not take the information about the relationship between the values being combined into account. Power average (PA) operator, initially proposed by Yager (2001), is a new tool to aggregate input arguments by considering the relationship between the values being aggregated. The weight vector in the PA operator depends upon the input arguments being aggregated. It allows values being aggregated to support and reinforce each other. To consider the relationship between the input arguments, Beliakov et al. (2007) developed the Heronian mean (HM) operator. Based on the HM operator, Skora (2009) further proposed the generalized Heronian mean (GHM) operator. The main difference between the PA operator and the GHM operator is that the former reflects the objective characteristics of the input arguments being aggregated, while the latter reflects the subjective characteristics of the input arguments being aggregated. To date, there is no aggregation operator that combines PA operator and GHM operator to reflect the relationship between the input arguments being aggregated. Therefore, it is vital to address this issue.

Motivated by He et al. (2015, 2016), we focus our attention on proposing new aggregation operators for hesitant fuzzy linguistic information by combining the PA operator and the GHM operator in this paper. Based on the proposed operators, a novel approach is developed to solve MAGDM problems under hesitant fuzzy linguistic environment. To do so, the remainder of this paper is organized as follows. In Sect. 2, some basic concepts are briefly reviewed, such as hesitant fuzzy linguistic set, power average operator, generalized Heronian mean operator as well as generalized geometric Heronian mean operator. In Sect. 3, some novel operators are proposed, such as hesitant fuzzy linguistic power generalized Heronian mean (HFLPGHM) operator, hesitant fuzzy linguistic power generalized geometric Heronian mean (HFLPGGHM) operator, hesitant fuzzy linguistic weighted power generalized Heronian mean (HFLWPGHM) operator and hesitant fuzzy linguistic weighted power generalized geometric Heronian mean (HFLWPGGHM) operator. In Sect. 4, a novel method is proposed to solve MAGDM problems under hesitant fuzzy linguistic environment. In Sect. 5, a numerical example is provided to illustrate the application of the developed method. The paper is concluded in Sect. 6.

2 Preliminaries

This section reviews some basic concepts which will be used in the rest of this paper, such as the hesitant fuzzy set (HFS), hesitant fuzzy linguistic set (HFLS), power average (PA) operator, generalized Heronian mean (GHM) operator as well as generalized geometric Heronian mean (GGHM) operator.

Definition 1 (Xia and Xu 2011) Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set, a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0, 1], which can be represented as the following symbol:

$$E = \{\langle x, h_E(x)\rangle \,|x \in X\}, \tag{1}$$

where $h_E(x)$ is a set of values in [0,1], denoting possible membership degrees of the element $x \in X$ to the set E.

Definition 2 (Lin et al. 2014) Let X be a fixed set, a hesitant fuzzy linguistic set (HFLS) on X is represented as

$$E = \left\{\left\langle x, s_{\theta(x)}, h_E(x)\right\rangle |x \in X\right\}, \tag{2}$$

where $s_{\theta(x)} \in S$, $S = \{s_0, s_1, s_2, \ldots, s_g\}$ is a linguistic term set, and $h(x)= \cup_{r\in h(x)} \{r\}$ is a set of crisp values in [0,1], denoting the possible membership degrees of the element $x \in X$ to the set E. For computational convenience, $a = \left\langle s_{\theta(a)}, h(a)\right\rangle$ is called a hesitant fuzzy linguistic number (HFLN).

Definition 3 (Lin et al. 2014) Let $a = \left\langle s_{\theta(a)}, h(a)\right\rangle$,$a_1 = \left\langle s_{\theta(a_1)}, h(a_1)\right\rangle$ and $a_2 = \left\langle s_{\theta(a_2)}, h(a_2)\right\rangle$ be three hesitant fuzzy linguistic numbers (HFLNs), and then, the operational laws are defined as:

(1) $a_1 \oplus a_2 = \left\langle S_{\theta(a_1)+\theta(a_2)}, \cup_{r_1 \in h(a_1), r_2 \in h(a_2)} \{r_1 + r_2 - r_1 r_2\} \right\rangle$.
(2) $a_1 \otimes a_2 = \left\langle S_{\theta(a_1)\otimes\theta(a_2)}, \cup_{r_1 \in h(a_1), r_2 \in h(a_2)} \{r_1 r_2\} \right\rangle$.
(3) $\lambda a = \left\langle S_{\lambda\theta(a)}, \cup_{r \in h(a)} \{1 - (1-r)^{\lambda}\} \right\rangle, \lambda > 0$.
(4) $a^{\lambda} = \left\langle S_{\theta(a)^{\lambda}}, \cup_{r \in h(a)} \{r^{\lambda}\} \right\rangle, \lambda > 0$.

Definition 4 (Lin et al. 2014) Let $a = \langle s_{\theta(a)}, h(a) \rangle$ be a HFLN, and then, the score function $S(a)$ is defined as follows:

$$S(a) = \frac{\theta(a)}{\#h(a)} \sum_{r \in h(a)} r, \tag{3}$$

where $\#h(a)$ is the number of the elements in $h(a)$.

The ranking of two HFLNs can be compared according to the values of their score functions: for two HFLNs a_1 and a_2, if $S(a_1) > S(a_2)$, then $a_1 > a_2$; if $S(a_1) = S(a_2)$, then $a_1 = a_2$.

Definition 5 (Wang et al. 2016) The distance between two HFLNs $a_1 = \langle s_{\theta(a_1)}, h(a_1) \rangle$ and $a_2 = \langle s_{\theta(a_2)}, h(a_2) \rangle$ is defined as follows:

$$d(a_1, a_2) = \max \left\{ d^*(a_1, a_2), d^*(a_2, a_1) \right\}, \tag{4}$$

where $d^*(a_1, a_2) = \left| \max_{r_1 \in h(a_1)} \{r_1\} \times \frac{1}{g} \times \theta(a_1) - \min_{r_2 \in h(a_2)} \{r_2\} \times \frac{1}{g} \times \theta(a_2) \right|$ is the Hausdorff distance between a_1 and a_2, $d^*(a_2, a_1) = \left| \max_{r_2 \in h(a_2)} \{r_2\} \times \frac{1}{g} \times \theta(a_2) - \min_{r_1 \in h(a_1)} \{r_1\} \times \frac{1}{g} \times \theta(a_1) \right|$ is the Hausdorff distance between a_2 and a_1, and $g+1$ is the cardinality of the linguistic term set $S = \{s_0, s_1, s_2, \ldots, s_g\}$.

Power average (PA) operator, initially proposed by Yager (2001), is a nonlinear weighted average aggregation tool, and it takes the information about the relationship between the values being aggregated into account.

Definition 6 (Yager 2001) Let $a_1, a_2, \ldots, a_n$ be the aggregated variables, and the power average (PA) operator is defined as follows:

$$\mathrm{PA}(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} \frac{(1 + T(a_i))\, a_i}{\sum_{t=1}^{n} (1 + T(a_t))}, \tag{5}$$

where $T(a_i) = \sum_{j=1, j \neq i}^{n} \mathrm{Sup}(a_i, a_j)$, $i = 1, 2, \ldots, n$, and $\mathrm{Sup}(a_i, a_j)$ is considered to be the support for a_i from a_j, which satisfies the following properties:

(1) $\mathrm{Sup}(a_i, a_j) \in [0, 1]$;
(2) $\mathrm{Sup}(a_i, a_j) = \mathrm{Sup}(a_j, a_i)$;
(3) $\mathrm{Sup}(a_i, a_j) \geq \mathrm{Sup}(a_k, a_l)$, if $|a_i - a_j| \leq |a_k - a_l|$.

Definition 7 (Sykora 2009) Let $I = [0, 1]$, $p, q \geq 0$, $H^{p,q}: I^n \to I$, and then, the generalized Heronian mean (GHM) operator is defined as follows:

$$\mathrm{GHM}^{p,q}(a_1, a_2, \ldots, a_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} a_i^p a_j^q \right)^{1/p+q}. \tag{6}$$

If $p = q = \frac{1}{2}$, then the GHM operator reduces to the basic Heronian mean (BHM) operator (Beliakov et al. 2007).

$$\mathrm{BHM}^{1/2,1/2}(a_1, a_2, \ldots, a_n) = \frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} (a_i a_j)^{1/2}. \tag{7}$$

Definition 8 (Yu 2013) Let $I = [0, 1]$, $p, q \geq 0$, $H^{p,q}: I^n \to I$, and then, the generalized geometric Heronian mean (GGHM) operator is defined as follows:

$$\mathrm{GGHM}^{p,q}(a_1, a_2, \ldots, a_n) = \frac{1}{p+q} \prod_{i=1}^{n} \prod_{j=i}^{n} \left(p a_i + q a_j \right)^{2/n(n+1)}. \tag{8}$$

3 Hesitant fuzzy linguistic power generalized Heronian mean operators

In this section, we develop some new operators under hesitant fuzzy linguistic environment by combining the power average operator with the GHM operator as well as the GGHM operator, respectively.

3.1 HFLPGHM and HFLPGGHM operators

Definition 9 Let $a_i (i = 1, 2, \ldots, n)$ be a collection of HFLNs, and then, the hesitant fuzzy linguistic power generalized Heronian mean (HFLPGHM) operator is defined as follows:

$$\mathrm{HFLPGHM}^{p,q}(a_1, a_2, \ldots, a_n) = \left(\frac{2}{n(n+1)} \bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} \left(\frac{n(1 + T(a_i))}{\sum_{t=1}^{n} (1 + T(a_t))} a_i \right)^p \otimes \left(\frac{n(1 + T(a_j))}{\sum_{t=1}^{n} (1 + T(a_t))} a_j \right)^q \right)^{\frac{1}{p+q}}, \tag{9}$$

where $p, q \geq 0$, $T(a_i) = \sum_{j=1, j \neq i}^{n} \mathrm{Sup}(a_i, a_j)$, $\mathrm{Sup}(a_i, a_j) = 1\text{-}d(a_i, a_j)$, and $d(a_i, a_j)$ can be calculated by Eq. (4).

Based on the operational laws of HFLNs described in Definition 3, we can derive the following results.

Theorem 1 *Let $p \geq 0, q \geq 0$, and p, q do not take the value 0 simultaneously, $a_i = \langle s_{\theta(a_i)}, h(a_i) \rangle$ be a collection of HFLNs, and then, the aggregated value by using the HFLPGHM operator is also a HFLN, and*

$$\mathrm{HFLPGHM}^{p,q}(a_1, a_2, \ldots, a_n) = \left\langle S_{\left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))} \theta(a_i) \right)^{p} \left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))} \theta(a_j) \right)^{q} \right)^{\frac{1}{p+q}}}, \right.$$
$$\left. \cup_{r_{a_i} \in h(a_i), r_{a_j} \in h(a_j)} \left(1 - \left(\prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - (1 - r_{a_i})^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}} \right)^{p} \left(1 - (1 - r_{a_j})^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}} \right)^{q} \right) \right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}} \right\rangle. \tag{10}$$

Proof According to Definition 3, we have

$$\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))} a_i \right)^{p} = \left\langle s_{\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))} \theta(a_i) \right)^{p}}, \cup_{r_{a_i} \in h(a_i)} \left(1 - (1 - r_{a_i})^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}} \right)^{p} \right\rangle,$$
$$\left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))} a_j \right)^{q} = \left\langle s_{\left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))} \theta(a_j) \right)^{q}}, \cup_{r_{a_j} \in h(a_j)} \left(1 - (1 - r_{a_j})^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}} \right)^{q} \right\rangle,$$

and

$$\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))} a_i \right)^{p} \otimes \left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))} a_j \right)^{q} = \left\langle s_{\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))} \theta(a_i) \right)^{p} \left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))} \theta(a_j) \right)^{q}}, \right.$$
$$\left. \cup_{r_{a_i} \in h(a_i), r_{a_j} \in h(a_j)} \left(1 - (1 - r_{a_i})^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}} \right)^{p} \left(1 - (1 - r_{a_j})^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}} \right)^{q} \right\rangle.$$

Further, we have

$$\bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} \left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))} a_i \right)^{p} \otimes \left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))} a_j \right)^{q} = \left\langle s_{\sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_j))} \theta(a_i) \right)^{p} \left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_j))} \theta(a_j) \right)^{q}}, \right.$$
$$\left. \cup_{r_{a_i} \in h(a_i), r_{a_j} \in h(a_j)} \left(1 - \prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - (1 - r_{a_i})^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}} \right)^{p} \left(1 - (1 - r_{a_j})^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}} \right)^{q} \right) \right) \right\rangle$$

and

$$\frac{2}{n(n+1)} \left(\bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} \left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))} a_i \right)^{p} \otimes \left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))} a_j \right)^{q} \right)$$
$$= \left\langle s_{\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_j))} \theta(a_i) \right)^{p} \left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_j))} \theta(a_j) \right)^{q}}, \right.$$
$$\left. \cup_{r_{a_i} \in h(a_i), r_{a_j} \in h(a_j)} \left(1 - \left(\prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - \left(1 - (1 - r_{a_i})^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}} \right)^{p} \left(1 - (1 - r_{a_j})^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}} \right)^{q} \right) \right)^{\frac{2}{n(n+1)}} \right) \right\rangle.$$

Thus, we have

$$\mathrm{HFLPGHM}^{p,q}(a_1,a_2,\ldots,a_n)=\left\langle S_{\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_j))}\theta(a_i)\right)^{p}\left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_j))}\theta(a_j)\right)^{q}\right)^{\frac{1}{p+q}}},\right.$$
$$\left.\cup_{r_{a_i}\in h(a_i),r_{a_j}\in h(a_j)}\left(1-\left(\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(1-(1-r_{a_i})^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)^{p}\left(1-(1-r_{a_j})^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)^{q}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\rangle.$$

Finally, the aggregated values of $a_i(i=1,2,3)$ by the HFLPGHM operator can be obtained (Suppose $p=q=1$):

$$\mathrm{HFLPGHM}^{1,1}(a_1,a_2,a_3)=\left\langle S_{\left(\frac{2}{3(3+1)}\sum_{i=1}^{3}\sum_{j=i}^{3}\left(\frac{3(1+T(a_i))}{\sum_{t=1}^{3}(1+T(a_t))}\theta(a_i)\right)^{1}\left(\frac{3(1+T(a_j))}{\sum_{t=1}^{3}(1+T(a_t))}\theta(a_j)\right)^{1}\right)^{\frac{1}{2}}},\right.$$
$$\left.\cup_{r_{a_i}\in h(a_i),r_{a_j}\in h(a_j)}\left(1-\left(\prod_{i=1}^{3}\prod_{j=i}^{3}\left(1-\left(1-(1-r_{a_i})^{\frac{3(1+T(a_i))}{\sum_{t=1}^{3}(1+T(a_t))}}\right)^{1}\left(1-(1-r_{a_j})^{\frac{3(1+T(a_j))}{\sum_{t=1}^{3}(1+T(a_t))}}\right)^{1}\right)\right)^{\frac{2}{3(3+1)}}\right)^{\frac{1}{2}}\right\rangle$$
$$=<S_{2.671},\{0.377,0.405,0.469,0.451,0.476,0.535\}>.$$

Example 1 Let $a_1=\langle s_3,\{0.2,0.3,0.5\}\rangle$, $a_2=\langle s_2,\{0.5\}\rangle$ and $a_3=\langle s_3,\{0.4,0.6\}\rangle$ be three HFLNs, then we can use the HFLPGHM operator to aggregate them. By Definition 5, we can determine the support for a_i from a_j:

$$\mathrm{Sup}(a_1,a_2)=\mathrm{Sup}(a_2,a_1)=0.938,$$
$$\mathrm{Sup}(a_1,a_3)=\mathrm{Sup}(a_3,a_1)=0.850,$$
$$\mathrm{Sup}(a_2,a_3)=\mathrm{Sup}(a_3,a_2)=0.900.$$

Then we can calculate the total support for a_i:

$$T(a_1)=\sum_{j=1,j\neq1}^{3}\mathrm{Sup}(a_1,a_j)=1.788,$$
$$T(a_2)=\sum_{j=1,j\neq2}^{3}\mathrm{Sup}(a_2,a_j)=1.838,$$
$$T(a_3)=\sum_{j=1,j\neq3}^{3}\mathrm{Sup}(a_3,a_j)=1.750.$$

Further, we can get the weights of the HFLPGHM operator in Theorem 1:

$$\frac{(1+T(a_1))}{\sum_{t=1}^{3}(1+T(a_t))}=0.3326,\quad\frac{(1+T(a_2))}{\sum_{t=1}^{3}(1+T(a_t))}=0.3419,\quad\frac{(1+T(a_3))}{\sum_{t=1}^{3}(1+T(a_t))}=0.3255.$$

From Example 1, we can see that the aggregated value of $a_i(i=1,2,3)$ is also a HFLN, and it includes two parts: $s_{\theta(x)}=s_{2.671}$ and $h_E(x)=\{0.377,0.405,0.469,0.451,0.476,0.535\}$, where $h_E(x)$ denotes the possible membership degree of the element $x\in X$ to the linguistic term $s_{\theta(x)}$.

If $\mathrm{Sup}(a_i,a_j)=c(c\in[0,1])$ for all $i\neq j$, then we have $T(a_i)=(n-1)c$. Further, we have $\frac{(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}=\frac{[1+(n-1)c]}{\sum_{t=1}^{n}[1+(n-1)c]}=\frac{[1+(n-1)c]}{n[1+(n-1)c]}=\frac{1}{n}$.

By assigning different values of the parameters of p and q, some special cases of the HFLPGHM operator can be derived, which are shown as follows.

Case 1. If $q\to0$, then the HFLPGHM operator reduces to the hesitant fuzzy linguistic descending power average (HFLDPA) operator, which is shown as follows:

$$\begin{aligned}&\mathrm{HFLPGHM}^{p,0}(a_1,a_2,\ldots,a_n)\\&=\lim_{q\to0}\left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}\mathop{\oplus}_{j=i}^{n}\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}a_i\right)^{p}\right.\\&\quad\left.\otimes\left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}a_j\right)^{q}\right)^{\frac{1}{p+q}}\\&=\left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}\left((n+1-i)\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}a_i\right)^{p}\right)\right)^{\frac{1}{p}}.\end{aligned}\quad(11)$$

Case 2. If $p\to0$, then the HFLPGHM operator reduces to the hesitant fuzzy linguistic ascending power average (HFLAPA) operator, which is shown as follows:

$$\begin{aligned}&\mathrm{HFLPGHM}^{0,q}(a_1,a_2,\ldots,a_n)\\&=\lim_{p\to 0}\left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}\mathop{\oplus}_{j=i}^{n}\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}a_i\right)^p\right.\\&\left.\otimes\left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}a_j\right)^q\right)^{\frac{1}{p+q}}\\&=\left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}\left(i\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}a_i\right)^q\right)\right)^{\frac{1}{q}}.\end{aligned}\tag{12}$$

Case 3. If $q\to 0$, and $\mathrm{Sup}(a_i,a_j)=c(c\in[0,1])$ for all $i\neq j$, then the HFLPGHM operator reduces to the hesitant fuzzy linguistic linear descending weighted average (HFLLDWA) operator, which is shown as follows:

$$\begin{aligned}&\mathrm{HFLPGHM}^{p,0}(a_1,a_2,\ldots,a_n)\\&=\lim_{q\to 0}\left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}\mathop{\oplus}_{j=i}^{n}\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}a_i\right)^p\right.\\&\left.\otimes\left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}a_j\right)^q\right)^{\frac{1}{p+q}}\\&=\left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}((n+1-i)a_i^p)\right)^{\frac{1}{p}}.\end{aligned}\tag{13}$$

Obviously, the weight vector of $a_i^p(i=1,2,\ldots,n)$ is $(n,n-1,\ldots,1)$.

Case 4. If $p\to 0$, and $\mathrm{Sup}(a_i,a_j)=c(c\in[0,1])$ for all $i\neq j$, then the HFLPGHM operator reduces to the hesitant fuzzy linguistic linear ascending weighted average (HFLLAWA) operator as follows:

$$\begin{aligned}&\mathrm{HFLPGHM}^{0,q}(a_1,a_2,\ldots,a_n)\\&=\lim_{p\to 0}\left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}\mathop{\oplus}_{j=i}^{n}\left(\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}a_i\right)^p\right.\\&\left.\otimes\left(\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}a_j\right)^q\right)^{\frac{1}{p+q}}\\&=\left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}(ia_i^q)\right)^{\frac{1}{q}}.\end{aligned}\tag{14}$$

Obviously, the weight vector of $a_i^q(i=1,2,\ldots,n)$ is $(1,2,\ldots,n)$.

Case 5. If $p=q=1/2$, and $\mathrm{Sup}(a_i,a_j)=c(c\in[0,1])$ for all $i\neq j$, then the HFLPGHM operator reduces to the hesitant fuzzy linguistic basic Heronian mean (HFLBHM) operator:

$$\begin{aligned}&\mathrm{HFLPGHM}^{\frac{1}{2},\frac{1}{2}}(a_1,a_2,\ldots,a_n)\\&=\frac{2}{n(n+1)}\left(\mathop{\oplus}_{i=1}^{n}\mathop{\oplus}_{j=i}^{n}\left((a_i)^{1/2}\otimes(a_j)^{1/2}\right)\right).\end{aligned}\tag{15}$$

Case 6. If $p=q=1$, and $\mathrm{Sup}(a_i,a_j)=c(c\in[0,1])$ for all $i\neq j$, then the HFLPGHM operator reduces to the hesitant fuzzy linguistic linear Heronian mean (HFLLHM) operator:

$$\begin{aligned}&\mathrm{HFLPGHM}^{1,1}(a_1,a_2,\ldots,a_n)\\&=\left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}\mathop{\oplus}_{j=i}^{n}(a_i\otimes a_j)\right)^{\frac{1}{2}}.\end{aligned}\tag{16}$$

Definition 10 Let $a_i(i=1,2,\ldots,n)$ be a collection of HFLNs, and the hesitant fuzzy linguistic power generalized geometric Heronian mean (HFLPGGHM) operator is defined as follows:

$$\begin{aligned}&\mathrm{HFLPGGHM}^{p,q}(a_1,a_2,\ldots,a_n)\\&=\frac{1}{p+q}\left(\mathop{\otimes}_{i=1}^{n}\mathop{\otimes}_{j=i}^{n}\left(pa_i^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}}\oplus qa_j^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)\right)^{\frac{2}{n(n+1)}},\end{aligned}\tag{17}$$

where $p,q\geq 0, T(a_i)=\sum_{j=1,j\neq i}^{n}\mathrm{Sup}(a_i,a_j), \mathrm{Sup}(a_i,a_j)=1-d(a_i,a_j)$, and $d(a_i,a_j)$ can be calculated by Eq. (4).

Based on the operational laws of the hesitant fuzzy linguistic described in Definition 3, we can derive the following results:

Theorem 2 *Let $p\geq 0, q\geq 0$, and p,q do not take the value 0 simultaneously, $a_i=\langle s_{\theta(a_i)},h(a_i)\rangle(i=1,2,\ldots,n)$ be a collection of HFLNs, then the aggregated value by the HFLPGGHM operator is also a HFLN, and*

$$\begin{aligned}&\mathrm{HFLPGGHM}^{p,q}(a_1,a_2,\ldots,a_n)=\left\langle S_{\frac{1}{p+q}\left(\prod_{i=1}^{n}\prod_{j=i}^{n}\left(p\theta(a_i)^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}}+q\theta(a_j)^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)\right)^{\frac{2}{n(n+1)}}},\right.\\&\left.\cup_{r_{a_i}\in h(a_i),r_{a_j}\in h(a_j)}\left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(1-(r_{a_i})^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)^p\left(1-(r_{a_j})^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)^q\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right)\right\rangle.\end{aligned}\tag{18}$$

The proof of this theorem is similar to that of Theorem 1.

Example 2 For the three HFLNs in Example 1, we can use the HFLPGGHM operator to aggregate them. Due to the support among a_i and a_j is unchanged, the weights used in the HFLPGGHM operator are the same as that in Example 1. Then, the aggregated value of the HFLPGGHM operator can be obtained (Suppose $p = q = 1$):

$$\begin{aligned}
&\text{HFLPGGHM}^{1,1}(a_1, a_2, a_3)\\
&= \left\langle S_{\frac{1}{1+1}\left(\prod_{i=1}^{3}\prod_{j=i}^{3}\left(p\theta(a_i)^{\frac{3(1+T(a_i))}{\sum_{t=1}^{3}(1+T(a_t))}} + q\theta(a_j)^{\frac{3(1+T(a_j))}{\sum_{t=1}^{3}(1+T(a_t))}}\right)\right)^{\frac{2}{3(3+1)}}},\right.\\
&\quad \left. \cup_{r_{a_i}\in h(a_i), r_{a_j}\in h(a_j)}\left(1-\left(1-\left(\prod_{i=1}^{3}\prod_{j=i}^{3}\left(1-\left(1-(r_{a_i})^{\frac{3(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)^{1}\left(1-(r_{a_j})^{\frac{3(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)^{1}\right)\right)^{\frac{2}{3(3+1)}}\right)^{\frac{1}{1+1}}\right)\right\rangle\\
&= < S_{2.631}, \{0.352, 0.395, 0.466, 0.407, 0.454, 0.532\} >
\end{aligned}$$

By assigning different values of the parameters of p and q, some special cases of the HFLPGGHM operator can be obtained, which are shown as follows.

Case 7. If $q \to 0$, then the HFLPGGHM operator reduces to the hesitant fuzzy linguistic descending power geometric average (HFLDPGA) operator, which is shown as follows:

$$\begin{aligned}
&\text{HFLPGGHM}^{p,0}(a_1, a_2, \ldots, a_n)\\
&= \lim_{q\to 0}\frac{1}{p+q}\left(\underset{i=1}{\overset{n}{\otimes}}\underset{j=i}{\overset{n}{\otimes}}\left(pa_i^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}} \oplus qa_j^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)\right)^{\frac{2}{n(n+1)}}\\
&= \frac{1}{p}\left(\underset{i=1}{\overset{n}{\otimes}}\left(pa_i^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)^{(n+1-i)}\right)^{\frac{2}{n(n+1)}}.
\end{aligned}\tag{19}$$

Case 8. If $p \to 0$, then the HFLPGGHM operator reduces to the hesitant fuzzy linguistic ascending power geometric average (HFLAPGA) operator, which is shown as follows:

$$\begin{aligned}
&\text{HFLPGGHM}^{0,q}(a_1, a_2, \ldots, a_n)\\
&= \lim_{p\to 0}\frac{1}{p+q}\left(\underset{i=1}{\overset{n}{\otimes}}\underset{j=i}{\overset{n}{\otimes}}\left(pa_i^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}} \oplus qa_j^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)\right)^{\frac{2}{n(n+1)}}\\
&= \frac{1}{q}\left(\underset{i=1}{\overset{n}{\otimes}}\left(qa_i^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)^{i}\right)^{\frac{2}{n(n+1)}}.
\end{aligned}\tag{20}$$

Case 9. If $q \to 0$, and $\text{Sup}(a_i, a_j) = c(c \in [0, 1])$ for all $i \neq j$, then the HFLPGGHM operator reduces to the hesitant fuzzy linguistic descending geometric average (HFLDGA) operator, which is shown as follows:

$$\begin{aligned}
&\text{HFLPGGHM}^{p,0}(a_1, a_2, \ldots, a_n)\\
&= \lim_{q\to 0}\frac{1}{p+q}\underset{i=1}{\overset{n}{\otimes}}\underset{j=i}{\overset{n}{\otimes}}\left(\left(pa_i^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)\right.\\
&\quad \left.\oplus\left(qa_j^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)\right)^{\frac{2}{n(n+1)}}\\
&= \frac{1}{p}\left(\underset{i=1}{\overset{n}{\otimes}}(pa_i)^{(n+1-i)}\right)^{\frac{2}{n(n+1)}}.
\end{aligned}\tag{21}$$

Case 10. If $p \to 0$, and $\text{Sup}(a_i, a_j) = c(c \in [0, 1])$ for all $i \neq j$, then the HFLPGGHM operator reduces to the hesitant fuzzy linguistic ascending geometric average (HFLAGA) operator, which is shown as follows:

$$\begin{aligned}
&\text{HFLPGGHM}^{0,q}(a_1, a_2, \ldots, a_n)\\
&= \lim_{p\to 0}\left\langle\frac{1}{p+q}\underset{i=1}{\overset{n}{\otimes}}\underset{j=i}{\overset{n}{\otimes}}\left(\left(pa_i^{\frac{n(1+T(a_i))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)\right.\right.\\
&\quad \left.\left.\oplus\left(qa_j^{\frac{n(1+T(a_j))}{\sum_{t=1}^{n}(1+T(a_t))}}\right)\right)^{\frac{2}{n(n+1)}}\right\rangle\\
&= \frac{1}{q}\left(\underset{i=1}{\overset{n}{\otimes}}(qa_i)^{i}\right)^{\frac{2}{n(n+1)}}.
\end{aligned}\tag{22}$$

Case 11. If $p = q = 1/2$, and $\text{Sup}(a_i, a_j) = c(c \in [0, 1])$ for all $i \neq j$, then HFLPGGHM operator reduces to the hesitant fuzzy linguistic geometric Heronian mean (HFLGHM) operator:

$$\text{HFLPGGHM}^{\frac{1}{2},\frac{1}{2}}(a_1, a_2, \ldots, a_n) = \left(\frac{1}{2}\mathop{\otimes}_{i=1}^{n}\mathop{\otimes}_{j=i}^{n}(a_i \oplus a_j)\right)^{\frac{2}{n(n+1)}}. \tag{23}$$

Case 12. If $p = q = 1$, and $\text{Sup}(a_i, a_j) = c(c \in [0, 1])$ for all $i \neq j$, then the HFLPGGHM operator reduces to the hesitant fuzzy linguistic basic geometric Heronian mean (HFLBGHM) operator.

$$\text{HFLPGGHM}^{1,1}(a_1, a_2, \ldots, a_n) = \frac{1}{2}\left(\mathop{\otimes}_{i=1}^{n}\mathop{\otimes}_{j=i}^{n}(a_i \oplus a_j)\right)^{\frac{2}{n(n+1)}}. \tag{24}$$

3.2 HFLWPGHM and HFLWPGGHM operators

In what follows, we propose the hesitant fuzzy linguistic weighted power generalized Heronian mean (HFLWPGHM) operator and the hesitant fuzzy linguistic weighted power generalized geometric Heronian mean (HFLWPGGHM) operator by considering the importance of attributes.

Definition 11 Let $a_i (i = 1, 2, \ldots, n)$ be a collection of HFLNs, $p, q \geq 0$, $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of $a_i (i = 1, 2, \ldots, n)$, where $w_i \geq 0$, and $\sum_{i=1}^{n} w_i = 1$. The hesitant fuzzy linguistic weighted power generalized Heronian mean (HFLWPGHM) operator is defined as:

$$\text{HFLWPGHM}^{p,q}(a_1, a_2, \ldots, a_n) = \left(\frac{2}{n(n+1)}\mathop{\oplus}_{i=1}^{n}\mathop{\oplus}_{j=i}^{n}\left(\frac{nw_i(1+T(a_i))}{\sum_{t=1}^{n} w_t(1+T(a_t))}a_i\right)^p \otimes \left(\frac{nw_j(1+T(a_i))}{\sum_{t=1}^{n} w_t(1+T(a_t))}a_j\right)^q\right)^{\frac{1}{p+q}}, \tag{25}$$

where $T(a_i) = \sum_{j=1, j\neq i}^{n} \text{Sup}(a_i, a_j)$, $\text{Sup}(a_i, a_j) = 1 - d(a_i, a_j)$, and $d(a_i, a_j)$ can be calculated by Eq. (4).

Based on the operational laws of the hesitant fuzzy linguistic described in Definition 3, we can derive the following results:

Theorem 3 *Let $p \geq 0, q \geq 0$, and p, q do not take the value 0 simultaneously, $a_i = \langle s_{\theta(a_i)}, h(a_i)\rangle$ be a collection of HFLNs, then the aggregated value by using the HFLWPGHM operator is also a HFLN, and*

$$\text{HFLWPGHM}^{p,q}(a_1, a_2, \ldots, a_n) = \left\langle S_{\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{nw_i(1+T(a_i))}{\sum_{t=1}^{n} w_t(1+T(a_t))}\theta(a_i)\right)^p\left(\frac{nw_j(1+T(a_i))}{\sum_{t=1}^{n} w_t(1+T(a_t))}\theta(a_j)\right)^q\right)^{\frac{1}{p+q}}}, \right.$$
$$\left. \cup_{r_{a_i}\in h(a_i), r_{a_j}\in h(a_j)}\left(1-\left(\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(1-(1-r_{a_i})^{\frac{nw_i(1+T(a_i))}{\sum_{t=1}^{n} w_t(1+T(a_t))}}\right)^p\left(1-(1-r_{a_j})^{\frac{nw_j(1+T(a_j))}{\sum_{t=1}^{n} w_t(1+T(a_t))}}\right)^q\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\rangle. \tag{26}$$

The proof of this theorem is similar to that of Theorem 1.

Definition 12 Let $a_i (i = 1, 2, \ldots, n)$ be a collection of HFLNs, $p, q \geq 0$, $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of $a_i (i = 1, 2, \ldots, n)$, where $w_i \geq 0$, and $\sum_{i=1}^{n} w_i = 1$. The hesitant fuzzy linguistic weighted power generalized geometric Heronian mean (HFLWPGGHM) operator is defined as:

$$\text{HFLWPGGHM}^{p,q}(a_1, a_2, \ldots, a_n) = \frac{1}{p+q}\left(\mathop{\otimes}_{i=1}^{n}\mathop{\otimes}_{j=i}^{n}\left(pa_i^{\frac{nw_i(1+T(a_i))}{\sum_{t=1}^{n} w_t(1+T(a_t))}} \oplus qa_j^{\frac{nw_j(1+T(a_j))}{\sum_{t=1}^{n} w_t(1+T(a_t))}}\right)\right)^{\frac{2}{n(n+1)}}, \tag{27}$$

where $T(a_i) = \sum_{j=1, j\neq i}^{n} \text{Sup}(a_i, a_j)$, $\text{Sup}(a_i, a_j) = 1 - d(a_i, a_j)$, and $d(a_i, a_j)$ can be calculated by Eq. (4).

Based on the operational laws of the hesitant fuzzy linguistic described in Definition 3, we can derive the following results:

Theorem 4 *Let $p \geq 0, q \geq 0$, and p, q do not take the value 0 simultaneously, $a_i = \langle s_{\theta(a_i)}, h(a_i)\rangle$ be a collection of HFLNs, then the aggregated value by using the HFLWPGGHM is also a HFLN, and*

$$\mathrm{HFLWPGGHM}^{p,q}(a_1,a_2,\ldots,a_n) = \left\langle S_{\frac{1}{p+q}\left(\prod_{i=1}^{n}\prod_{j=i}^{n}\left(p\theta(a_i)^{\frac{nw_i(1+T(a_i))}{\sum_{t=1}^{n}w_t(1+T(a_t))}}+q\theta(a_j)^{\frac{nw_j(1+T(a_j))}{\sum_{t=1}^{n}w_t(1+T(a_t))}}\right)\right)^{\frac{2}{n(n+1)}}}, \cup_{r_{a_i}\in h(a_i),r_{a_j}\in h(a_j)}\left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-\left(1-(r_{a_i})^{\frac{nw_i(1+T(a_i))}{\sum_{t=1}^{n}w_t(1+T(a_t))}}\right)^{p}\left(1-(r_{a_j})^{\frac{nw_j(1+T(a_j))}{\sum_{t=1}^{n}w_t(1+T(a_t))}}\right)^{q}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right)\right\rangle. \tag{28}$$

The proof of this theorem is similar to that of Theorem 1.

Example 3 For the three HFLNs $a_i(i=1,2,3)$ in Example 1, let $w=(0.25,0.35,0.40)^T$ be the weight vector of them, we can use the HFLWPGHM operator to aggregate them. Based on the supports among a_i and a_j in Example 1, the comprehensive weights used in the HFLWPGHM operator can be calculated as follows:

$$\frac{w_1(1+T(a_1))}{\sum_{t=1}^{3}w_t(1+T(a_t))}=0.250,$$
$$\frac{w_2(1+T(a_2))}{\sum_{t=1}^{3}w_t(1+T(a_t))}=0.356,$$
$$\frac{w_3(1+T(a_3))}{\sum_{t=1}^{3}w_t(1+T(a_t))}=0.394.$$

Further, we can get the aggregated value of $a_i(i=1,2,3)$ by the HFLWPGHM operator (Suppose $p=q=1$):

$$\mathrm{HFLWPGGHM}^{1,1}(a_1,a_2,a_3) = \left\langle S_{\left(\frac{2}{3(3+1)}\sum_{i=1}^{3}\sum_{j=i}^{3}\left(\frac{3w_i(1+T(a_i))}{\sum_{t=1}^{3}w_t(1+T(a_t))}\theta(a_i)\right)^1\left(\frac{3w_j(1+T(a_j))}{\sum_{t=1}^{3}w_t(1+T(a_t))}\theta(a_j)\right)^1\right)^{\frac{1}{1+1}}}, \cup_{r_{a_i}\in h(a_i),r_{a_j}\in h(a_j)}\left(1-\left(\prod_{i=1}^{3}\prod_{j=i}^{3}\left(1-\left(1-(1-r_{a_i})^{\frac{3w_i(1+T(a_i))}{\sum_{t=1}^{n}w_t(1+T(a_t))}}\right)^1\left(1-(1-r_{a_j})^{\frac{3w_j(1+T(a_j))}{\sum_{t=1}^{n}w_t(1+T(a_t))}}\right)^1\right)\right)^{\frac{2}{3(3+1)}}\right)^{\frac{1}{1+1}}\right\rangle$$
$$=< S_{2.663}, \{0.392, 0.413, 0.462, 0.475, 0.494, 0.538\} > .$$

Similarly, we can get aggregated value of $a_i (i = 1, 2, 3)$ by the HFLWPGGHM operator (Suppose $p = q = 1$):

$$\begin{aligned}
&\mathrm{HFLWPGGHM}^{1,1}(a_1, a_2, a_3)\\
&= \Bigg\langle S_{\frac{1}{1+1}\left(\prod_{i=1}^{3}\prod_{j=i}^{3}\left(1\times\theta(a_i)^{\frac{3w_i(1+T(a_i))}{\sum_{t=1}^{n} w_t(1+T(a_t))}}+1\times\theta(a_j)^{\frac{3w_j(1+T(a_j))}{\sum_{t=1}^{n} w_t(1+T(a_t))}}\right)\right)^{\frac{2}{3(3+1)}}},\\
&\cup_{r_{a_i}\in h(a_i), r_{a_j}\in h(a_j)}\left(1-\left(1-\left(\prod_{i=1}^{3}\prod_{j=i}^{3}\left(1-\left(1-(r_{a_i})^{\frac{3w_i(1+T(a_i))}{\sum_{t=1}^{n} w_t(1+T(a_t))}}\right)^{1}\left(1-(r_{a_j})^{\frac{3w_j(1+T(a_j))}{\sum_{t=1}^{n} w_t(1+T(a_t))}}\right)^{1}\right)\right)^{\frac{2}{3(3+1)}}\right)^{\frac{1}{1+1}}\right)\Bigg\rangle\\
&=< S_{2.626}, \{0.367,0.405,0.462,0.432,0.474,0.538\} > .
\end{aligned}$$

4 The approach to solve multi-attribute group decision-making problem

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a finite set of m alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be the finite set of n attributes, whose weight vector is $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Let $D = \{D_1, D_2, \ldots, D_t\}$ be the set of decision makers. Suppose that $B^k = (b_{ij}^k)_{m\times n}$ is a hesitant fuzzy linguistic decision matrix provided by the decision maker D_k, where $b_{ij}^k = \left\langle s_{\theta(b_{ij}^k)}, h(b_{ij}^k)\right\rangle$ is in the form of HFLN given for the alternative $A_i (i = 1, 2, \ldots, m)$ with respect to the attribute $C_j (j = 1, 2, \ldots, n)$.

To solve the multi-attribute group decision-making problem under hesitant fuzzy linguistic environment, we develop a resolution process as shown in Fig. 1. In the process, first, the individual hesitant fuzzy linguistic decision matrices are constructed. Next, the collective hesitant fuzzy linguistic matrix is determined using the HFLPGHM or HFLPGGHM operator. Then, the overall assessment value of each alternative is calculated using the HFLWPGHM or HFLWPGGHM operator. Furthermore, the desirable alternative is selected based on the score value. In the following, a formal procedure of the proposed method is presented based on Fig. 1.

Step 1. Transform the hesitant fuzzy linguistic decision matrix $B^k = (b_{ij}^k)_{m\times n}$ into the normalized hesitant fuzzy linguistic decision matrix $\tilde{B}^k = (\tilde{b}_{ij}^k)_{m\times n}$. The normalized hesitant fuzzy linguistic assessment value $\tilde{b}_{ij}^k = \left\langle s_{\theta(\tilde{b}_{ij}^k)}, h(\tilde{b}_{ij}^k)\right\rangle$ of the alternative A_i with respect to the attribute C_j can be determined by Eq. (29).

$$\tilde{b}_{ij}^k = \left\langle s_{\theta(\tilde{b}_{ij}^k)}, h(\tilde{b}_{ij}^k)\right\rangle = \begin{cases} \left\langle s_{\theta(b_{ij}^k)}, h(b_{ij}^k)\right\rangle, \ j \in \Omega_B \\ \left\langle s_{(g-\theta(b_{ij}^k))}, h(b_{ij}^k)\right\rangle, \ j \in \Omega_C \end{cases} \tag{29}$$

where $g + 1$ is the cardinality of the linguistic term set $S = \{s_0, s_1, s_2, \ldots, s_g\}$, Ω_B and Ω_C are the sets of benefit attribute and cost attribute, respectively.

Step 2. Calculate the supports of evaluation values among different decision makers with respect to the evaluation value.

$$\mathrm{Sup}(\tilde{b}_{ij}^k, \tilde{b}_{ij}^l) = 1 - d(\tilde{b}_{ij}^k, \tilde{b}_{ij}^l), \quad k, l = 1, \ldots, t; \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n, \tag{30}$$

where $d(\tilde{b}_{ij}^k, \tilde{b}_{ij}^l)$ can be calculated by Eq. (4).

Step 3. Calculate the weights $\pi_{ij}^k (k = 1, 2, \ldots, t)$ associated with the decision maker D_k by Eq. (31).

$$\pi_{ij}^k = (1 + T(\tilde{b}_{ij}^k))/\sum_{k=1}^{t}(1 + T(\tilde{b}_{ij}^k)), \tag{31}$$

where t is the total number of decision makers, and $T(\tilde{b}_{ij}^k)$ can be calculated by Eq. (32).

$$T(\tilde{b}_{ij}^k) = \sum_{l=1, l\neq k}^{t} \mathrm{Sup}(\tilde{b}_{ij}^k, \tilde{b}_{ij}^l). \tag{32}$$

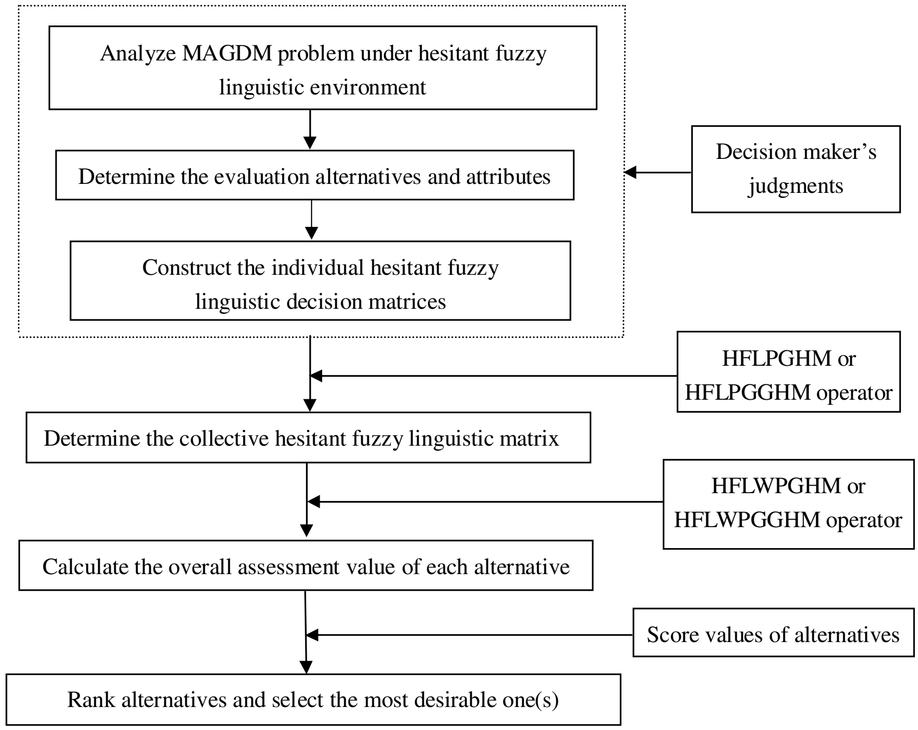

Fig. 1 The framework of MAGDM under hesitant fuzzy linguistic environment

Step 4. Aggregate all the individual hesitant fuzzy linguistic decision matrices $\tilde{B}^k = (\tilde{b}^k_{ij})_{m\times n} (k = 1, 2, \dots, t)$ into the collective one $\tilde{B} = (\tilde{b}_{ij})_{m\times n}$ by the HFLPGHM (or HFLPGGHM) operator, i.e.,

$$\begin{aligned}\tilde{b}_{ij} &= \left\langle s_{\theta(\tilde{b}_{ij})}, h(\tilde{b}_{ij})\right\rangle \\ &= \mathrm{HFLPGHM}^{p,q}(\tilde{b}^1_{ij}, \tilde{b}^2_{ij}, \dots, \tilde{b}^t_{ij}) \\ &= \left\langle S_{\left(\frac{2}{t(t+1)}\sum_{k=1}^{t}\sum_{l=i}^{t}\left(\pi^k_{ij}\theta(\tilde{b}^k_{ij})\right)^p\left(\pi^l_{ij}\theta(\tilde{b}^l_{ij})\right)^q\right)^{\frac{1}{p+q}}}, \right. \\ &\quad \left. \cup_{r^k_{ij}\in h(\tilde{b}^k_{ij}), r^l_{ij}\in h(\tilde{b}^l_{ij})} \left(1-\left(\prod_{k=1}^{t}\prod_{l=k}^{t}\left(1-\left(1-\left(1-r^k_{ij}\right)^{\pi^k_{ij}}\right)^p\left(1-\left(1-r^l_{ij}\right)^{\pi^l_{ij}}\right)^q\right)\right)^{\frac{2}{t(t+1)}}\right)^{\frac{1}{p+q}}\right\rangle \end{aligned} \tag{33}$$

or

$$\begin{aligned}\tilde{b}_{ij} &= \left\langle s_{\theta(\tilde{b}_{ij})}, h(\tilde{b}_{ij})\right\rangle \\ &= HFLPGGHM^{p,q}(\tilde{b}^1_{ij}, \tilde{b}^2_{ij}, \dots, \tilde{b}^t_{ij}) \\ &= \left\langle S_{\frac{1}{p+q}\left(\prod_{k=1}^{t}\prod_{l=k}^{t}\left(p\theta(\tilde{b}^k_{ij})^{\pi^k_{ij}}+q\theta(\tilde{b}^l_{ij})^{\pi^l_{ij}}\right)\right)^{\frac{2}{n(n+1)}}}, \right. \\ &\quad \left. \cup_{\tilde{r}^k_{ij}\in h(\tilde{b}^k_{ij}), \tilde{r}^l_{ij}\in h(\tilde{b}^l_{ij})} \left(1-\left(1-\left(\prod_{k=1}^{t}\prod_{l=k}^{t}\left(1-\left(1-(\tilde{r}^k_{ij})^{\pi^k_{ij}}\right)^p\left(1-(\tilde{r}^l_{ij})^{\pi^l_{ij}}\right)^q\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right)\right\rangle. \end{aligned} \tag{34}$$

Step 5. Calculate the supports of evaluation values among different attributes.

$$\mathrm{Sup}(\tilde{b}_{ij}, \tilde{b}_{iv}) = 1 - d(\tilde{b}_{ij}, \tilde{b}_{iv}), \quad i = 1, \ldots, m; \quad j, v = 1, 2, \ldots, n; \quad j \neq v, \tag{35}$$

where $d(\tilde{b}_{ij}, \tilde{b}_{iv})$ can be calculated by Eq. (4).

Step 6. Calculate the weights λ_{ij} associated with the evaluation value $\tilde{b}_{ij}$ by Eq. (36).

$$\lambda_{ij} = w_j(1 + T(\tilde{b}_{ij}))/\sum_{j=1}^{n}(w_j(1 + T(\tilde{b}_{ij}))), \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n, \tag{36}$$

where n is the total number of attributes, and $T(\tilde{b}_{ij})$ can be calculated by Eq. (37).

$$T(\tilde{b}_{ij}) = \sum_{v=1, v\neq j}^{n} \mathrm{Sup}(\tilde{b}_{ij}, \tilde{b}_{iv}), \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n. \tag{37}$$

Step 7. Aggregate all the hesitant fuzzy linguistic assessment values $\tilde{b}_{ij} = \left\langle s_{\theta(\tilde{b}_{ij})}, h(\tilde{b}_{ij})\right\rangle$ of the alternative $A_i (i = 1, 2, \ldots, m)$ on all attributes $C_j (j = 1, 2, \ldots, n)$ into the overall assessment values $\tilde{b}_i = \left\langle s_{\theta(\tilde{b}_i)}, h(\tilde{b}_i)\right\rangle (i = 1, 2, \ldots, m)$ by the HFLWPGHM (or HFLWPGGHM) operator, i.e.,

$$\begin{aligned} \tilde{b}_i &= \left\langle s_{\theta(\tilde{b}_i)}, h(\tilde{b}_i)\right\rangle \\ &= \mathrm{HFLWPGHM}^{p,q}(\tilde{b}_{i1}, \tilde{b}_{i2}, \ldots, \tilde{b}_{in}) \\ &= \Bigg\langle S_{\left(\frac{2}{n(n+1)}\sum_{j=1}^{n}\sum_{v=j}^{n}\left(\lambda_{ij}\theta(\tilde{b}_{ij})\right)^{p}\left(\lambda_{iv}\theta(\tilde{b}_{iv})\right)^{q}\right)^{\frac{1}{p+q}}}, \\ &\quad \cup_{r_{ij}\in h(\tilde{b}_{ij}), r_{iv}\in h(\tilde{b}_{iv})}\left(1 - \left(\prod_{j=1}^{n}\prod_{v=j}^{n}\left(1 - \left(1 - (1 - r_{ij}))^{\lambda_{ij}}\right)^{p}(1 - (1 - r_{iv}))^{\lambda_{iv}}\right)^{q}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\Bigg\rangle \end{aligned} \tag{38}$$

or

$$\begin{aligned} \tilde{b}_i &= \left\langle s_{\theta(\tilde{b}_i)}, h(\tilde{b}_i)\right\rangle \\ &= \mathrm{HFLWPGGHM}^{p,q}(\tilde{b}_{i1}, \tilde{b}_{i2}, \ldots, \tilde{b}_{in}) \\ &= \Bigg\langle S_{\frac{1}{p+q}\left(\prod_{j=1}^{n}\prod_{v=j}^{n}\left(p\theta(\tilde{b}_{ij})^{\lambda_{ij}} + q\theta(\tilde{b}_{iv})^{\lambda_{iv}}\right)\right)^{\frac{2}{n(n+1)}}}, \\ &\quad \cup_{r_{ij}\in h(\tilde{b}_{ij}), r_{iv}\in h(\tilde{b}_{iv})}\left(1 - \left(1 - \left(\prod_{j=1}^{n}\prod_{v=j}^{n}\left(1 - \left(1 - r_{ij}^{\lambda_{ij}}\right)^{p}\left(1 - r_{iv}^{\lambda_{iv}}\right)^{q}\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right)\Bigg\rangle. \end{aligned} \tag{39}$$

Step 8. Calculate the score values $S(\tilde{b}_i)$ of overall assessment values $\tilde{b}_i (i = 1, 2, \ldots, m)$ by Eq. (40).

$$S(\tilde{b}_i) = \frac{\theta(\tilde{b}_i)}{\#h(\tilde{b}_i)} \sum_{r_i \in h(\tilde{b}_i)} r_i, \tag{40}$$

where $\#h(\tilde{b}_i)$ is the number of the elements in $h(\tilde{b}_i)$.

Step 9. Rank all alternatives $A_i (i = 1, 2, \ldots, \mathrm{m})$ according to the score values in Eq. (40) and select the most desirable one(s).

Step 10. End.

5 Numerical example and comparative analysis

5.1 Numerical example

In this section, a MAGDM problem adapted from Ju et al. (2016b) is used to illustrate the application of the MAGDM method proposed in this paper, and to demonstrate its feasibility and effectiveness in a realistic scenario. An emergency management department wants to select the most desirable alternative(s) from five emergency alternatives, which are denoted by $A_i (i = 1, 2, 3, 4, 5)$, according to the following four attributes: emergency process capability (C_1), emergency forecasting capacity (C_2), emergency support capacity

Table 1 The normalized hesitant fuzzy linguistic decision matrix $\tilde{B}_1$ given by the decision maker D_1

Alternatives	C_1	C_2	C_3	C_4
A_1	$< S_2, \{0.6, 0.7\} >$	$< S_3, \{0.5\} >$	$< S_4, \{0.6\} >$	$< S_5, \{0.4, 0.5\} >$
A_2	$< S_3, \{0.3\} >$	$< S_5, \{0.4, 0.5, 0.6\} >$	$< S_2, \{0.8\} >$	$< S_7, \{0.6\} >$
A_3	$< S_3, \{0.5\} >$	$< S_5, \{0.6\} >$	$< S_4, \{0.3, 0.5\} >$	$< S_6, \{0.4, 0.5\} >$
A_4	$< S_3, \{0.3, 0.4, 0.5\} >$	$< S_2, \{0.7\} >$	$< S_5, \{0.6\} >$	$< S_6, \{0.4\} >$
A_5	$< S_2, \{0.6\} >$	$< S_2, \{0.5\} >$	$< S_5, \{0.5\} >$	$< S_6, \{0.6\} >$

Table 2 The normalized hesitant fuzzy linguistic decision matrix $\tilde{B}_2$ given by the decision maker D_2

Alternatives	C_1	C_2	C_3	C_4
A_1	$< S_2, \{0.6\} >$	$< S_4, \{0.7\} >$	$< S_1, \{0.6\} >$	$< S_6, \{0.3, 0.5\} >$
A_2	$< S_2, \{0.3, 0.4\} >$	$< S_5, \{0.6\} >$	$< S_3, \{0.7\} >$	$< S_7, \{0.4, 0.6\} >$
A_3	$< S_3, \{0.4\} >$	$< S_2, \{0.5, 0.7\} >$	$< S_6, \{0.3, 0.4\} >$	$< S_8, \{0.6\} >$
A_4	$< S_2, \{0.5\} >$	$< S_5, \{0.7\} >$	$< S_3, \{0.7\} >$	$< S_6, \{0.5\} >$
A_5	$< S_2, \{0.3, 0.5\} >$	$< S_5, \{0.3, 0.5\} >$	$< S_3, \{0.5\} >$	$< S_4, \{0.7\} >$

Table 3 The normalized hesitant fuzzy linguistic decision matrix $\tilde{B}_3$ given by the decision maker D_3

Alternatives	C_1	C_2	C_3	C_4
A_1	$< S_2, \{0.5, 0.6\} >$	$< S_4, \{0.6\} >$	$< S_6, \{0.7\} >$	$< S_3, \{0.6\} >$
A_2	$< S_2, \{0.5, 0.6\} >$	$< S_4, \{0.7\} >$	$< S_3, \{0.5\} >$	$< S_7, \{0.7\} >$
A_3	$< S_3, \{0.6\} >$	$< S_6, \{0.5, 0.6\} >$	$< S_3, \{0.4\} >$	$< S_7, \{0.5, 0.6\} >$
A_4	$< S_2, \{0.4, 0.5\} >$	$< S_3, \{0.6\} >$	$< S_6, \{0.8\} >$	$< S_5, \{0.5\} >$
A_5	$< S_2, \{0.4, 0.5\} >$	$< S_4, \{0.2, 0.4\} >$	$< S_6, \{0.4\} >$	$< S_6, \{0.6\} >$

(C_3) and after-disaster process capacity (C_4), whose weight vector is given as $w = (0.2, 0.3, 0.3, 0.2)^T$. Three decision makers, denoted by $D_k (k = 1, 2, 3)$, are invited to provide their evaluation information to candidate alternatives $A_i (i = 1, 2, 3, 4, 5)$ with respect to attributes C_j $(j = 1, 2, 3, 4)$. Due to the uncertainty of the evaluation process, the five emergency alternatives $A_i (i = 1, 2, 3, 4, 5)$ are evaluated by using the hesitant fuzzy linguistic information under the above four attributes with the linguistic term set: $S = \{s_0$: extremely poor; s_1: very poor; s_2: poor; s_3: slightly poor; s_4: fair; s_5: slightly good; s_6: good; s_7: very good; s_8: extremely good$\}$. Then, the individual hesitant fuzzy linguistic decision matrices $B_k = (b_{ij}^k)_{5\times 4} (k = 1, 2, 3)$ are constructed as shown in Tables 1, 2, 3, where $b_{ij}^k = \left\langle s_{\theta(b_{ij}^k)}, h(b_{ij}^k)\right\rangle$ is a HFLN. In what follows, we utilize the proposed method in this paper to select the most desirable emergency alternative(s), which involves the following steps:

Step 1. Construct the hesitant linguistic fuzzy matrix. Since all the attributes are the benefit type, there is no need to normalize the hesitant fuzzy linguistic decision matrix, i.e., the normalized hesitant fuzzy linguistic decision matrix $\tilde{B}^k$ is equal to $B^k (k = 1, 2, 3)$.

Step 2. By Eq. (30), we can compute the supports $\mathrm{Sup}(\tilde{b}_{ij}^k, \tilde{b}_{ij}^l) (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ between the decision maker D_k and D_l. Further, we can construct the support matrix among different decision makers, which are shown as follows.

$$\mathrm{Sup}(D_1, D_2) = \mathrm{Sup}(D_2, D_1) = \begin{bmatrix} 0.9750 & 0.8375 & 0.7750 & 0.8750 \\ 0.9625 & 0.8750 & 0.9375 & 0.8250 \\ 0.9625 & 0.7500 & 0.8500 & 0.7000 \\ 0.9375 & 0.7375 & 0.8875 & 0.9250 \\ 0.9250 & 0.8125 & 0.8750 & 0.9000 \end{bmatrix},$$

$$\mathrm{Sup}(D_1, D_3) = \mathrm{Sup}(D_3, D_1) = \begin{bmatrix} 0.9500 & 0.8875 & 0.7750 & 0.9125 \\ 0.9625 & 0.9000 & 0.9875 & 0.9125 \\ 0.9625 & 0.9250 & 0.9000 & 0.7750 \\ 0.9125 & 0.9500 & 0.7750 & 0.9875 \\ 0.9500 & 0.9250 & 0.9875 & 1.0000 \end{bmatrix},$$

$$\mathrm{Sup}(D_2, D_3) = \mathrm{Sup}(D_3, D_2) = \begin{bmatrix} 0.9750 & 0.9500 & 0.5500 & 0.8500 \\ 0.9250 & 0.9750 & 0.9250 & 0.7375 \\ 0.9250 & 0.6750 & 0.8500 & 0.8375 \\ 0.9750 & 0.7875 & 0.6625 & 0.9375 \\ 0.9500 & 0.7875 & 0.8875 & 0.9000 \end{bmatrix}.$$

Step 3. Calculate the weights π_{ij}^k of the decision makers associated with the evaluation value $\tilde{b}_{ij}^k (k = 1, 2, 3)$ by Eq. (31).

Further, we can construct the weight matrices of decision makers, which are shown as follows.

$$\pi(D_1)=\begin{bmatrix}0.3324 & 0.3263 & 0.3542 & 0.3369\\ 0.3362 & 0.3265 & 0.3362 & 0.3443\\ 0.3362 & 0.3474 & 0.3354 & 0.3246\\ 0.3295 & 0.3381 & 0.3480 & 0.3348\\ 0.3324 & 0.3400 & 0.3368 & 0.3372\end{bmatrix},$$

$$\pi(D_2)=\begin{bmatrix}0.3352 & 0.3338 & 0.3229 & 0.3293\\ 0.3319 & 0.3353 & 0.3290 & 0.3223\\ 0.3319 & 0.3149 & 0.3293 & 0.3328\\ 0.3367 & 0.3176 & 0.3333 & 0.3290\\ 0.3324 & 0.3230 & 0.3250 & 0.3256\end{bmatrix},$$

$$\pi(D_3)=\begin{bmatrix}0.3324 & 0.3398 & 0.3229 & 0.3338\\ 0.3319 & 0.3382 & 0.3348 & 0.3333\\ 0.3319 & 0.3377 & 0.3354 & 0.3426\\ 0.3338 & 0.3443 & 0.3186 & 0.3362\\ 0.3353 & 0.3370 & 0.3382 & 0.3372\end{bmatrix}.$$

Step 4. Aggregate all the individual hesitant fuzzy linguistic decision matrices $\tilde{B}^k=(\tilde{b}_{ij}^k)_{5\times4}$, $(k=1,2,3)$ into the collective one $\tilde{B}=(\tilde{b}_{ij})_{5\times4}$ by Eq. (33) (suppose $p=q=1$), which is shown in Table 4.

Step 5. By Eq. (33), we can calculate the supports $\mathrm{Sup}(b_{ij}, b_{iv})$ between the attributes C_j and C_v for the given alternative $A_i(i=1,2,3,4,5)$. Further, we can construct the support matrices among different attributes under the given alternatives, which are shown as follows.

$$\mathrm{Sup}(A_1)=\begin{bmatrix}- & 0.8631 & 0.8390 & 0.8277\\ 0.8631 & - & 0.9765 & 0.9645\\ 0.8390 & 0.9765 & - & 0.9586\\ 0.8277 & 0.9645 & 0.9586 & -\end{bmatrix},$$

$$\mathrm{Sup}(A_2)=\begin{bmatrix}- & 0.8631 & 0.8396 & 0.8277\\ 0.8631 & - & 0.9765 & 0.7821\\ 0.8396 & 0.9765 & - & 0.6718\\ 0.8277 & 0.7821 & 0.6718 & -\end{bmatrix},$$

$$\mathrm{Sup}(A_3)=\begin{bmatrix}- & 0.8338 & 0.9519 & 0.6899\\ 0.8338 & - & 0.8273 & 0.8011\\ 0.9519 & 0.8273 & - & 0.6834\\ 0.6899 & 0.8011 & 0.6834 & -\end{bmatrix},$$

$$\mathrm{Sup}(A_4)=\begin{bmatrix}- & 0.8391 & 0.7048 & 0.7896\\ 0.8391 & - & 0.8657 & 0.9477\\ 0.7048 & 0.8657 & - & 0.9180\\ 0.7896 & 0.9477 & 0.9180 & -\end{bmatrix},$$

$$\mathrm{Sup}(A_5)=\begin{bmatrix}- & 0.8951 & 0.8342 & 0.6845\\ 0.8951 & - & 0.8821 & 0.7324\\ 0.342 & 0.8821 & - & 0.8504\\ 0.6845 & 0.7324 & 0.8504 & -\end{bmatrix}.$$

Table 4 The collective hesitant fuzzy linguistic decision matrix $\tilde{B}$ by the HFLPGHM operator

Alternatives	C_1	C_2	C_3	C_4
A_1	$< s_2, \{0.57, 0.61, 0.60, 0.64\} >$	$< s_{3.68}, \{0.61\} >$	$< s_{3.81}, \{0.64\} >$	$< s_{4.70}, \{0.44, 0.48, 0.50, 0.54\} >$
A_2	$< s_{2.35}, \{0.37, 0.40, 0.41, 0.41\} >$	$< s_{4.67}\{0.58, 0.61, 0.64\} >$	$< s_{2.67}, \{0.68\} >$	$< s_7, \{0.58, 0.64\} >$
A_3	$< s_3, \{0.50\} >$	$< s_{4.48}, \{0.53, 0.60, 0.57, 0.63\} >$	$< s_{4.37}, \{0.34, 0.40, 0.37, 0.43\} >$	$< s_{7.02}, \{0.51, 0.53, 0.54, 0.57\} >$
A_4	$< s_{2.34}, \{0.40, 0.44, 0.47, 0.44, 0.47, 0.50\} >$	$< s_{3.34}, \{0.67\} >$	$< s_{4.69}, \{0.71\} >$	$< s_{5.67}, \{0.47\} >$
A_5	$< s_2, \{0.44, 0.50, 0.48, 0.53\} >$	$< s_{3.69}, \{0.34, 0.41, 0.40, 0.47\} >$	$< s_{4.73}, \{0.47\} >$	$< s_{5.37}, \{0.63\} >$

Table 5 The overall assessment value of each alternative determined by the HFLWPGHM operator

Alternatives	The overall assessment values
A_1	$< s_{3.65}$,{0.55, 0.57, 0.56, 0.57, 0.57, 0.58, 0.57, 0.59, 0.56, 0.58, 0.57, 0.58, 0.58, 0.59, 0.58, 0.60}$>$
A_2	$< s_{4.08}$,{0.55, 0.57, 0.56, 0.57, 0.57, 0.58, 0.57, 0.59, 0.58, 0.59, 0.59, 0.60, 0.57, 0.59, 0.58, 0.60, 0.59, 0.60, 0.59, 0.61, 0.60, 0.61, 0.61, 0.62}$>$
A_3	$< s_{4.65}$,{0.55, 0.56, 0.57, 0.58, 0.56, 0.57, 0.57, 0.58, 0.57, 0.58, 0.58, 0.59, 0.57, 0.58, 0.59, 0.60, 0.56, 0.57, 0.58, 0.58, 0.57, 0.58, 0.58, 0.59, 0.58, 0.58, 0.59, 0.60, 0.58, 0.59, 0.60, 0.60, 0.57, 0.58, 0.59, 0.59, 0.58, 0.59, 0.59, 0.60, 0.59, 0.59, 0.60, 0.61, 0.59, 0.60, 0.60, 0.61, 0.58, 0.59, 0.59, 0.60, 0.59, .59, 0.60, 0.61, 0.59, 0.60, 0.61, 0.61, 0.60, 0.61, 0.61, 0.62}$>$
A_4	$< s_{4.08}$, {0.57, 0.57, 0.57, 0.58, 0.58, 0.59} $>$
A_5	$< s_{4.06}$,{0.48, 0.49, 0.49, 0.51, 0.49, 0.50, 0.50, 0.52, 0.49, 0.51, 0.51, 0.52, 0.50, 0.52, 0.52, 0.53}$>$

Table 6 The collective hesitant fuzzy linguistic decision matrix $\tilde{B}$ by the HFLPGGHM operator

Alternatives	C_1	C_2	C_3	C_4
A_1	$< s_{2.00}$, {0.48, 0.56, 0.51, 0.60} $>$	$< s_{3.65}$, {0.60} $>$	$< s_{3.21}$, {0.63} $>$	$< s_{4.55}$, {0.42, 0.50, 0.46, 0.53} $>$
A_2	$< s_{2.31}$, {0.26, 0.36, 0.34, 0.45} $>$	$< s_{4.64}$, {0.56, 0.60, 0.63} $>$	$< s_{2.63}$, {0.66} $>$	$< s_{7.00}$, {0.59, 0.67} $>$
A_3	$< s_{3.00}$, {0.69} $>$	$< s_{4.18}$, {0.59, 0.62, 0.66, 0.70} $>$	$< s_{4.20}$, {0.33, 0.37, 0.39, 0.43} $>$	$< s_{6.89}$, {0.45, 0.48, 0.53, 0.57} $>$
A_4	$< s_{2.30}$, {0.39, 0.43, 0.43, 0.47, 0.47, 0.50} $>$	$< s_{3.14}$, {0.67} $>$	$< s_{4.54}$, {0.58} $>$	$< s_{5.64}$, {0.52} $>$
A_5	$< s_{2.00}$, {0.42, 0.45, 0.49, 0.53} $>$	$< s_{3.48}$, {0.34, 0.42, 0.41, 0.49} $>$	$< s_{4.59}$, {0.43} $>$	$< s_{5.30}$, {0.63} $>$

Step 6. By Eq. (36), we calculate the weight λ_{ij} associated with the collective evaluation value $\tilde{b}_{ij}(i = 1, 2, 3, 4, 5,\ j = 1, 2, 3, 4)$. Further, we can construct the weight matrix $\lambda = (\lambda_{ij})_{5\times 4}$ for all alternatives with respect to each attribute, which is shown as follows.

$$\lambda = \begin{bmatrix} 0.1893 & 0.3060 & 0.3036 & 0.2011 \\ 0.1939 & 0.3096 & 0.3126 & 0.1838 \\ 0.2040 & 0.3048 & 0.3049 & 0.1863 \\ 0.1882 & 0.3096 & 0.2957 & 0.2064 \\ 0.1974 & 0.3044 & 0.3093 & 0.1889 \end{bmatrix}.$$

Step 7. By Eq. (38), we can aggregate the hesitant fuzzy linguistic assessment values $\tilde{b}_{ij} = \left\langle s_{\theta(\tilde{b}_{ij})}, h(\tilde{b}_{ij})\right\rangle$ on all attributes $C_j(j = 1, 2, \ldots, n)$ of each alternative (suppose $p = q = 1$), and the aggregation results are shown in Table 5.

Step 8. By Eq. (4), we can calculate the score values $S(\tilde{b}_i)$ of all overall assessment values $\tilde{b}_i(i = 1, 2, 3, 4, 5)$:

$$S(\tilde{b}_1) = 2.0873, \quad S(\tilde{b}_2) = 2.3267, \quad S(\tilde{b}_3) = 2.1661,$$
$$S(\tilde{b}_4) = 2.4021, \quad S(\tilde{b}_5) = 1.8868.$$

Then, the ranking order of alternatives is given as follows:

$$A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5.$$

Therefore, A_4 is the best alternative.

In Step 4, if we use the HFLPGGHM operator in Eq. (34) to aggregate the individual hesitant fuzzy linguistic decision matrices $\tilde{B}^k = (\tilde{b}^k_{ij})_{m\times n}(k = 1, 2, 3)$ into the collective one $\tilde{B} = (\tilde{b}_{ij})_{5\times 4}$ by the HFLPGGHM operator (Suppose $p = q = 1$), we can obtain Table 6.

Similar to Steps 5–6, we can calculate the supports $\mathrm{Sup}(b_{ij}, b_{iv})(j, v = 1, 2, 3, 4,\ j \neq v)$ between the attributes C_j and C_v for the given alternative $A_i(i = 1, 2, 3, 4, 5)$, and further, we can construct the weight matrix $\lambda = (\lambda_{ij})_{5\times 4}$ for all alternatives with respect to each attribute. To save space, we do not list them here.

In Step 7, if we use the HFLWPGGHM operator in Eq. (39) to aggregate all the hesitant fuzzy linguistic assessment values $\tilde{b}_{ij} = \left\langle s_{\theta(\tilde{b}_{ij})}, h(\tilde{b}_{ij})\right\rangle$ of the alternative $A_i(i = 1, 2, 3, 4, 5)$ on all attributes $C_j(j = 1, 2, 3, 4)$, we can obtain the overall assessment values $\tilde{b}_i = \left\langle s_{\theta(\tilde{b}_i)}, h(\tilde{b}_i)\right\rangle (i = 1, 2, 3, 4, 5)$ of all alternatives (suppose $p = q = 1$), which is shown in Table 7.

By Eq. (40), we can calculate the scores of $S(\tilde{b}_i)$ of the overall values $\tilde{b}_i(i$=1,2,3,4,5):

$$S(\tilde{b}_1) = 1.9367, \quad S(\tilde{b}_2) = 2.1271, \quad S(\tilde{b}_3) = 2.1199,$$
$$S(\tilde{b}_4) = 2.2411, S(\tilde{b}_5) = 1.8230.$$

Table 7 The overall assessment value of each alternative determined by the HFLWPGGHM operator

Alternatives	The overall assessment values
A_1	$< s_{3.67}, \{0.54, 0.55, 0.56, 0.57, 0.55, 0.55, 0.56, 0.57, 0.55, 0.56, 0.57, 0.57, 0.55, 0.56, 0.57, 0.58\} >$
A_2	$< s_{3.80}, \{0.52, 0.54, 0.55, 0.57, 0.52, 0.55, 0.55, 0.58, 0.53, 0.56, 0.56, 0.59, 0.52, 0.54, 0.55, 0.57, 0.53, 0.55, 0.56, 0.58, 0.53, 0.56, 0.57, 0.59\} >$
A_3	$< s_{4.41}, \{0.50, 0.51, 0.51, 0.52, 0.51, 0.53, 0.52, 0.54, 0.51, 0.52, 0.52, 0.53, 0.52, 0.54, 0.53, 0.55, 0.50, 0.52, 0.51, 0.53, 0.52, 0.54, 0.53, 0.55, 0.51, 0.53, 0.52, 0.54, 0.53, 0.55, 0.54, 0.56, 0.50, 0.52, 0.51, 0.52, 0.52, 0.53, 0.53, 0.54, 0.51, 0.53, 0.52, 0.53, 0.53, 0.54, 0.53, 0.55, 0.51, 0.52, 0.51, 0.53, 0.52, 0.54, 0.53, 0.55, 0.51, 0.53, 0.52, 0.54, 0.53, 0.55, 0.54, 0.56\} >$
A_4	$< s_{3.86}, \{0.55, 0.56, 0.57, 0.56, 0.57, 0.58\} >$
A_5	$< s_{3.85}, \{0.44, 0.45, 0.45, 0.46, 0.46, 0.47, 0.46, 0.48, 0.46, 0.48, 0.47, 0.48, 0.48, 0.50, 0.49, 0.50\} >$

By the score function value $S(\tilde{b}_i)$ $(i = 1, 2, 3, 4, 5)$, we can get the exactly same ranking of alternatives: $A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$, which means that the most desirable alternative is also A_4. In specific decision-making process, decision makers can select different aggregation operators to aggregate hesitant fuzzy linguistic information according to the practical needs.

5.2 Comparison and discussion

In order to verify the effectiveness of the proposed method in this paper, we compare the proposed method with other existing method based on the hesitant fuzzy linguistic weighted Heronian mean (HFLWHM) operator proposed by Yu and Hou (2016). For the same multi-attribute group decision-making problem presented in Sect. 5.1, if we aggregate the individual hesitant fuzzy linguistic decision matrices $\tilde{B}^k = (\tilde{b}^k_{ij})_{5\times4}$, $(k = 1, 2, 3)$ into the collective hesitant fuzzy linguistic decision matrix $\tilde{B} = (\tilde{b}_{ij})_{5\times4}$ by the HFLWHM operator (Suppose $p = q = 1$) in Step 4, instead of the HFLPGHM operator proposed in this paper, then the collective hesitant fuzzy linguistic decision matrix is shown in Table 8. Further, we can obtain the overall evaluation value of each alternative by aggregating the evaluation information with respect to each attribute according to the HFLWHM operator (suppose $p = q = 1$). The overall evaluation values of alternatives are shown in Table 9.

By Eq. (3), we can calculate the score of each alternative, and the results are shown as follows:

$$S(\tilde{b}_1) = 2.0254,\ S(\tilde{b}_2) = 2.3860,\ S(\tilde{b}_3) = 2.2453,$$
$$S(\tilde{b}_4) = 2.3339,\ S(\tilde{b}_5) = 1.9111.$$

The five alternatives can be ranked as $A_2 \succ A_4 \succ A_3 \succ A_1 \succ A_5$, which means that the A_2 is the best alternative, while the best alternative is A_4 by the proposed method in this paper.

Similarly, for the same multi-attribute group decision-making problem presented in Sect. 5.1, if we utilize the hesitant fuzzy linguistic weighted geometric Heronian mean (HFLWGHM) operator proposed by Yu and Hou (2016) to aggregate the individual hesitant fuzzy linguistic decision matrices shown in Tables 1, 2, 3 and determine the overall evaluation value of each alternative (suppose $p = q = 1$), then we can obtain the ranking of the alternatives, which is given in Table 10. From Table 10, we can see that there are some differences between the operators proposed in this paper and that proposed by Yu and Hou (2016). The reason is that the former consider not only the interrelationships between input arguments, but also the relationships between the fused values by combing power average and Heronian mean operators, while the latter just consider the interrelationship of the individual argument based on the Heronian mean operator. In other words, the operators proposed in this paper consider more interactions among the input arguments in the course of information aggregation.

To further show the merits of the proposed operators in this paper, for the collective hesitant fuzzy linguistic decision matrix $\tilde{B}$ shown in Tables 4 and 6, if we use the hesitant fuzzy linguistic power weighted average (HFLPWA) and the hesitant fuzzy linguistic power weighted geometric (HFLPWG) operators proposed by Lin et al. (2014) to aggregate the evaluation information of all attributes, respectively, then the rankings of the alternatives are also given in Table 10. From Table 10, we can see that the ranking results obtained from the proposed operators in this paper are different from that obtained from that proposed operators by Lin et al. (2014). The ranking results obtained from the HFLWPGHM and HFLWPGGHM operators proposed in this paper are consistent, i.e., $A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$, while the ranking results obtained from the HFLPWA and HFLPWG operators proposed by Lin et al. (2014) are different from each other. Therefore, the proposed operators in this paper can provide robust ranking in the process of information fusion. In addition, the proposed operators in this paper include two parameters p and q, which can be used to reflect the decision maker's risk preferences, while the HFLPWA and HFLPWG operators given by Lin et al. (2014) do not include any param-

Table 8 The collective hesitant fuzzy linguistic decision matrix $\widehat{B}$ determined by the HFLWHM operator

Alternatives	C_1	C_2	C_3	C_4
A_1	$< s_{2.00}, \{0.19, 0.20, 0.19, 0.21\} >$	$< s_{3.67}, \{0.61\} >$	$< s_{3.81}, \{0.64\} >$	$< s_{4.71}, \{0.46, 0.49, 0.53, 0.55\} >$
A_2	$< s_{2.34}, \{0.09, 0.12, 0.11, 0.15\} >$	$< s_{4.67}, \{0.58, 0.61, 0.64\} >$	$< s_{2.68}, \{0.68\} >$	$< s_{7.00}, \{0.59, 0.62, 0.67\} >$
A_3	$< s_{3.00}, \{0.23\} >$	$< s_{4.42}, \{0.57, 0.62, 0.64, 0.68\} >$	$< s_{4.38}, \{0.46, 0.49, 0.52, 0.55\} >$	$< s_{7.01}, \{0.48, 0.52, 0.54, 0.57\} >$
A_4	$< s_{2.35}, \{0.15, 0.16, 0.15, 0.17, 0.16, 0.17\} >$	$< s_{3.39}, \{0.67\} >$	$< s_{4.71}, \{0.62\} >$	$< s_{5.67}, \{0.55\} >$
A_5	$< s_{2.00}, \{0.49, 0.52, 0.55, 0.57\} >$	$< s_{3.72}, \{0.39, 0.44, 0.45, 0.50\} >$	$< s_{4.71}, \{0.44\} >$	$< s_{5.35}, \{0.64\} >$

Table 9 The overall evaluation value of each alternative determined by the HFLWHM operator

Alternatives	The overall evaluation values
A_1	$< s_{3.57}$,{0.56, 0.56, 0.57, 0.58, 0.57, 0.58, 0.59, 0.60, 0.57, 0.58, 0.58, 0.59, 0.58, 0.59, 0.60, 0.61}$>$
A_2	$< s_{4.26}$,{0.55, 0.55, 0.57, 0.55, 0.56, 0.58, 0.56, 0.58, 0.58, 0.56, 0.57, 0.59, 0.57, 0.58, .59, 0.58, 0.59, 0.60, 0.57, 0.58, 0.59, 0.58, 0.58, 0.60, 0.58, 0.59, 0.61, 0.59, 0.59, 0.61, .59, 0.60, 0.62, 0.60, 0.61, 0.62}$>$
A_3	$< s_{4.75}$,{0.57, 0.58, 0.58, 0.59, 0.57, 0.58, 0.59, 0.59, 0.58, 0.59, 0.59, 0.60, 0.59, 0.60, 0.60, 0.61, 0.58, 0.59, 0.59, 0.60, 0.59, 0.60, 0.60, 0.61, 0.59, 0.60, 0.61, 0.61, 0.60, 0.61, 0.61, 0.62, 0.59, 0.59, 0.60, 0.61, 0.59, 0.60, 0.61, 0.61, 0.6, 0.61, 0.61, 0.62, 0.61, 0.61, 0.62, 0.63, 0.60, 0.61, 0.61, 0.62, 0.60, 0.61, 0.62, 0.62, 0.61, 0.62, 0.62, 0.63, 0.62, 0.62, 0.63, 0.64}$>$
A_4	$< s_{4.07}$,{0.58, 0.59, 0.59, 0.60, 0.59, 0.60}$>$
A_5	$< s_{3.99}$,{0.48, 0.50, 0.50, 0.51, 0.50, 0.51, 0.51, 0.52, 0.50, 0.52, 0.52, 0.53, 0.51, 0.52, 0.53, 0.54}$>$

Table 10 Comparisons with different operators

Operators	Parameters	Rankings
HFLWPGHM	$p = q = 1$	$A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$
HFLWPGGHM	$p = q = 1$	$A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$
HFLWHM	$p = q = 1$	$A_2 \succ A_4 \succ A_3 \succ A_1 \succ A_5$
HFLWGHM	$p = q = 1$	$A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$
HFLPWA	—	$A_3 \succ A_2 \succ A_4 \succ A_5 \succ A_1$
HFLPWG	—	$A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$

eters and cannot reflect the decision maker's risk preferences. According to the comparisons and analysis above, we can find that the operators proposed in this paper are better than the ones given by Lin et al. (2014) and Yu and Hou (2016).

In order to illustrate the influence of the parameters p and q on decision making of this example, for the collective hesitant fuzzy linguistic decision matrix shown in Table 4, we use different values of the parameters p and q in the HFLWPGHM operator to rank the alternatives. Table 11 lists the ranking order of the alternatives calculated by the HFLWPGHM operator as the parameters p and q change. Obviously, if the parameter p (or q) takes the value of 0, the HFLWPGHM operator ignores the interrelationships of the input arguments. With the change of the parameter p (or q), the score value of each alternative clearly shows variation tendency. For example, if the parameter p is fixed (Suppose $p = 1$), then the score value of each alternative becomes

Table 11 The ranking results by different values of p and q with the HFLWPGHM operator

Parameters	The score values of alternatives	Rankings
$p=1, q=0$	$S(\tilde{b}_1)=1.46$, $S(\tilde{b}_2)=1.67$, $S(\tilde{b}_3)=2.12$, $S(\tilde{b}_4)=2.56$, $S(\tilde{b}_5)=2.43$.	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
$p=1, q=1$	$S(\tilde{b}_1)=1.57$, $S(\tilde{b}_2)=1.72$, $S(\tilde{b}_3)=1.80$, $S(\tilde{b}_4)=2.56$, $S(\tilde{b}_5)=2.36$.	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
$p=1, q=2$	$S(\tilde{b}_1)=1.72$, $S(\tilde{b}_2)=1.84$, $S(\tilde{b}_3)=1.81$, $S(\tilde{b}_4)=2.68$, $S(\tilde{b}_5)=2.52$.	$A_4 \succ A_5 \succ A_2 \succ A_3 \succ A_1$
$p=1, q=3$	$S(\tilde{b}_1)=1.88$, $S(\tilde{b}_2)=1.95$, $S(\tilde{b}_3)=1.87$, $S(\tilde{b}_4)=2.83$, $S(\tilde{b}_5)=2.67$.	$A_4 \succ A_5 \succ A_2 \succ A_3 \succ A_1$
$p=1, q$=4	$S(\tilde{b}_1)=2.03$, $S(\tilde{b}_2)=2.06.$, $S(\tilde{b}_3)=1.94$, $S(\tilde{b}_4)=2.97$, $S(\tilde{b}_5)=2.81$.	$A_4 \succ A_5 \succ A_2 \succ A_1 \succ A_3$
$p=0, q=1$	$S(\tilde{b}_1)=1.62$, $S(\tilde{b}_2)=1.82$, $S(\tilde{b}_3)=1.63$, $S(\tilde{b}_4)=2.63$, $S(\tilde{b}_5)=2.38$.	$A_4 \succ A_5 \succ A_2 \succ A_3 \succ A_1$
$p=1, q=1$	$S(\tilde{b}_1)=1.57$, $S(\tilde{b}_2)=1.72$, $S(\tilde{b}_3)=1.80$, $S(\tilde{b}_4)=2.56$, $S(\tilde{b}_5)=2.36$.	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
$p=2, q=1$	$S(\tilde{b}_1)=1.68$, $S(\tilde{b}_2)=1.84$, $S(\tilde{b}_3)=1.96$, $S(\tilde{b}_4)=2.70$, $S(\tilde{b}_5)=2.54$.	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
$p=3, q=1$	$S(\tilde{b}_1)=1.82$, $S(\tilde{b}_2)=1.98$, $S(\tilde{b}_3)=2.09$, $S(\tilde{b}_4)=2.87$, $S(\tilde{b}_5)=2.70$.	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
$p=4, q=1$	$S(\tilde{b}_1)=1.96$, $S(\tilde{b}_2)=2.11$, $S(\tilde{b}_3)=2.21$, $S(\tilde{b}_4)=3.04$, $S(\tilde{b}_5)=2.84$.	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$

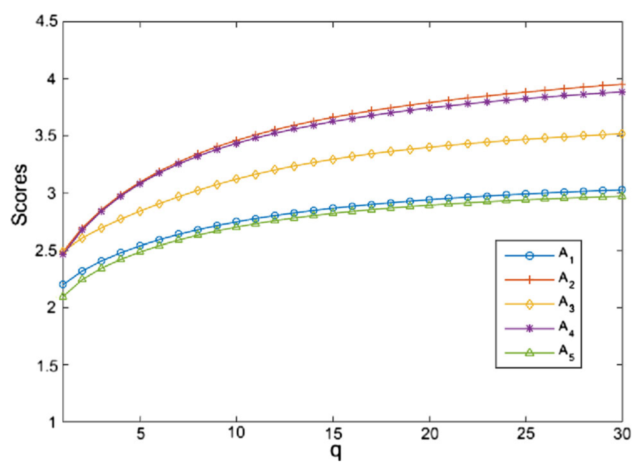

Fig. 2 Scores of alternatives by the HFLWPGHM operator ($p=1, q \in [0,30]$)

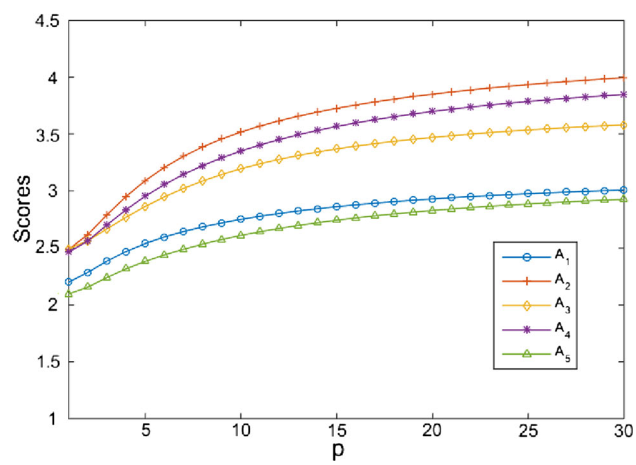

Fig. 3 Scores of alternatives by the HFLWPGHM operator ($p \in [0,30], q=1$)

bigger as the parameter q increases. This increasing trend is shown in Fig. 2. Similarly, if the parameter q is fixed (Suppose $q=1$), then the score value of each alternative also becomes bigger in as the parameter p increases. This increasing trend is shown in Fig. 3. Therefore, the parameter q (or p) can reflect the decision maker's risk preferences. For instance, the decision maker who is risk averter can choose the big value of the parameter q (or p), while decision maker who is risk lover can choose the small value of the parameter q (or p).

Similarly, to illustrate the influence of the parameters p, q on ranking results, for the collective hesitant fuzzy linguistic decision matrix shown in Table 6, we use different values of the parameters p, q in the HFLWPGGHM operator to rank the alternatives. Let the parameter p be fixed (Suppose $p=1$), and the variation tendency of the score values as the parameter $q$changes is shown in Fig. 4. From Fig. 4, we can see that the score value of each alternative obtained by the HFLWPGGHM operator becomes smaller as the parameter q increases. In addition, the discrimination between alternatives is not obvious. In the same way, let the parameter q be

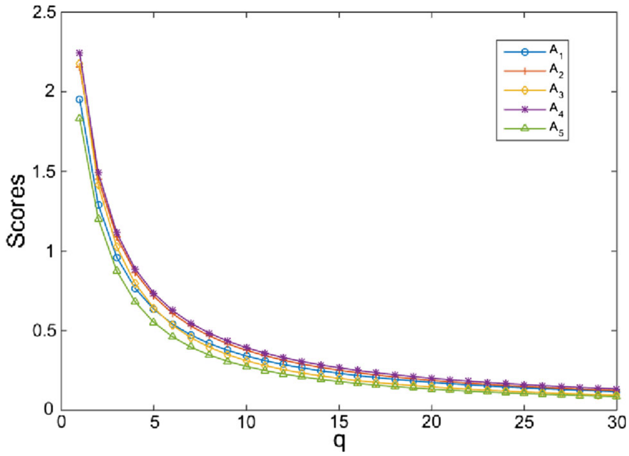

Fig. 4 Scores of alternatives by the HFLWPGGHM operator ($p = 1, q \in [0, 30]$)

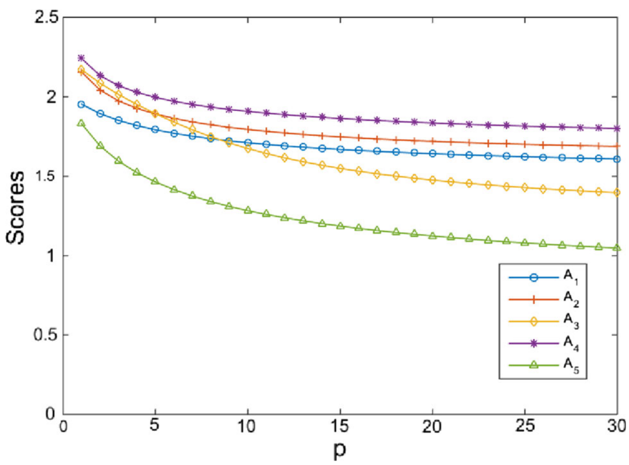

Fig. 5 Scores of alternatives by the HFLWPGGHM operator ($p \in [0, 30], q = 1$)

fixed (Suppose $q = 1$), and then, the variation tendency of the score values as the parameter p increases is shown in Fig. 5. From Fig. 5, we can see that the score value of each alternative obtained by the HFLWPGGHM operator also becomes smaller as the parameter p increases, but the discrimination between alternatives is obvious. Furthermore, we can find that the ranking results may be different for different values of the parameter $p(p < 10)$. In addition, from Figs. 4, 5, it is noted that as the value of the parameter p (or q) tends toward infinitude, the score value of each alternative tends to be a constant. Similar to the HFLWPGHM operator, this variation tendency of the parameter q (or p) can reflect the decision maker's risk attitude. In practical decision-making situations, decision makers can choose the appropriate value according to their risk preferences. For example, the decision maker who is risk averter can choose small value of the parameter q (or p). On the contrary, decision maker who is risk lover can choose the big value of the parameter q (or p).

6 Conclusions

In this paper, a novel approach is proposed to solve the multi-attribute group decision-making problem under hesitant fuzzy linguistic environment by combining the generalized Heronian mean operator and power average operator. Firstly, we proposed four novel aggregation operators, such as the hesitant fuzzy linguistic power generalized Heronian mean (HFLPGHM) operator, the hesitant fuzzy linguistic power generalized geometric Heronian mean (HFLPGGHM) operator, the hesitant fuzzy linguistic weighted power generalized Heronian mean (HFLWPGHM) operator and the hesitant fuzzy linguistic weighted power generalized geometric Heronian mean (HFLWPGGHM) operator. Secondly, some special cases of the proposed HFLPGHM and HFLPGGHM operators are investigated in detail. Thirdly, based on the proposed operators, a novel method is developed to deal with MAGDM problem under hesitant fuzzy linguistic environment. Finally, we illustrated the application of the developed method to select the most desirable emergency alternative(s) and compared the proposed operators with some existing ones. The comparison results demonstrate the effectiveness and practicality of the proposed approach. It is worth noting that the operators proposed in this paper consider much more information among the multi-input arguments by allowing the values being aggregated to support each other and can provide robust ranking in the process of information fusion. They can be used to other management domains in addition to emergency alternative selection. In the future research, we will focus on extending the aggregation operators and MAGDM method with dual hesitant fuzzy linguistic information as well as dual hesitant fuzzy uncertain linguistic information.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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