Soft Comput (2018) 22:6629–6643
https://doi.org/10.1007/s00500-017-2758-5

FOCUS

# A novel group decision-making model based on triangular neutrosophic numbers

**Mohamed Abdel-Basset**[1] · **Mai Mohamed**[1] · **Abdel-Nasser Hussien**[2] · **Arun Kumar Sangaiah**[3]

Published online: 5 August 2017


**Abstract** In group decision-making (GDM) model, experts often evaluate their opinion by using triangular fuzzy numbers. The preference relations with triangular fuzzy numbers are in consistent in nature, so we turned to neutrosophic in this paper. It is very important to take into account consistency of expert opinion and consensus degree in GDM. In order to distinguish the typical consistency, the concept of additive approximation consistency is proposed for triangular neutrosophic additive reciprocal matrices. The properties of triangular neutrosophic additive reciprocal matrices with additive approximation consistency are studied in detail. Second, by using $(n-1)$ restricted preference values, a triangular neutrosophic additive reciprocal preference relation with additive approximation consistency is constructed. The differences among expert's opinions are measured using consensus degree. For generating a collective triangular neutrosophic additive reciprocal matrix with additive approximation consistency, the neutrosophic triangular weighted aggregation operator is used. Finally, a novel algorithm for the group decision-making problem with triangular neutrosophic additive reciprocal preference relations is presented. A numerical example is carried out to illustrate the proposed definitions and algorithm.

**Keywords** Group decision making (GDM) · Triangular neutrosophic number · Additive approximation consistency · Neutrosophic triangular weighted aggregation operator (NTWAO)

Communicated by G. Yi.

✉ Arun Kumar Sangaiah
arunkumarsangaiah@gmail.com

Mohamed Abdel-Basset
analyst_mohamed@yahoo.com

Mai Mohamed
analyst_mohamed@zu.edu.eg

Abdel-Nasser Hussien
nasshra@gmail.com

1 Department of Operations Research, Faculty of Computers and Informatics, Zagazig University, Sharqiyah, Egypt

2 Department of Information System, Faculty of Computers and Informatics, Zagazig University, Sharqiyah, Egypt

3 School of Computing Science and Engineering, VIT University, Vellore, Tamil Nadu 632014, India

## 1 Introduction

Due to the increasing complexity of modern-life decisions problems, many organizations employ multiple experts to reach a decision, which is called as group decision making (GDM). It is difficult for an expert to be able to consider all aspects of a decision-making problem. All the experts may evaluate their judgments by using preference representation formats, such as fuzzy preference relations (Tanino 1984), multiplicative preference relations (Fan et al. 2006; Peneva and Popchev 2007), interval preference relation (Saaty and Vargas 1987) and linguistic frame work (Saaty 2008; Pedrycz and Song 2011) for modeling GDM problems. Moreover, to deal with the cases with incomplete information, different formats of incomplete preference relations have also been applied (Cabrerizo et al. 2010; Gong 2008; Herrera-Viedma et al. 2007; Xu and Chen 2008; Büyüközkan and Çifçi 2012). It is noted that a precise numerical value cannot reflect the ambiguous knowledge of the expert's preference level. To rationalize uncertainty associated with vagueness, fuzzy set theory has been established (Zadeh 1965; Wang and Chen 2007). It treats vague data in terms of set memberships. These set memberships include L-R fuzzy number, trapezoidal fuzzy numbers, triangular fuzzy numbers, interval numbers and others. The experts in a GDM process have been utilized some of the above fuzzy formats (Kaufmann and Gupta 1988). For further dealing with uncertainty and vagueness, the fuzzy set has been extended to present intuitionistic fuzzy set. The concept of intuitionistic fuzzy preference relation proposed in Atanassov (1986). One of the most important issues in GDM problems is the consistency of preference relations (Dubois 2011; Benítez et al. 2012) in order to avoid self-contradiction of decision makers. There exist many definitions of consistent preference relations such as, consistent multiplicative preference relations (Saaty 2008), additive preference relations with multiplicative and additive consistency (Tanino 1984; Herrera-Viedma et al. 2004), consistent interval multiplicative preference relations (Wang et al. 2005; Liu 2009; Liu et al. 2012), interval additive preference relations with multiplicative and additive consistency (Xu and Chen 2008; Liu et al. 2012; Meng et al. 2017, 2016), consistent triangular fuzzy multiplicative preference relations (Liu et al. 2014; Wang 2015) and triangular fuzzy additive preference relations with multiplicative consistency (Meng et al. 2017; Wang and Tong 2016). The consistency of intuitionistic fuzzy preference relation has been defined in Xu (2007). The consistency of intuitionistic fuzzy preference relation in the previous studies has been focused on the multiplicative consistency (Jiang et al. 2015). From analyzing the previous studies, it is obvious that:

1. Fuzzy preference relations have some drawbacks owing to the limitation of the fuzzy set.
2. Fuzzy set has only single valued function used to express evidence of acceptance and rejection at the same time in many practical situations.
3. The preference relations with triangular fuzzy numbers are inconsistent in nature (Liu et al. 2016).
4. And few approaches in intuitionistic fuzzy preference relation proposed to improve consistency.

We overcame all the above drawbacks by proposing a group decision-making model in neutrosophic environment. Because the problem domain should has a precise knowledge, otherwise the people have some uncertainty in assigning the preference evaluation values and this makes the decision-making process appear the characteristics of confirmation, refusal and indeterminacy. Smarandache (2005) suggested the concept of neutrosophic set, which is differentiated by truth-membership function, indeterminacy-membership function and falsity-membership function. So the neutrosophic set theory should be utilized to rationalize uncertainty associated with ambiguity in a manner analogous to human thought. It treats ambiguous data as possibility distributions in terms of set memberships. The experts should use neutrosophic set to determine their preference relations. In this paper, the approximate consistency of triangular neutrosophic additive reciprocal preference relation is applied to GDM problem and neutrosophic preference relations without consistency have been repaired. Consensus is the best way to group decision because it considers fears and contradictory ideas without animosity and terror but also it is much more intractable, owing to the conflicting opinions of experts and the difference in importance of those opinions in the decision-making process (Wu and Xu 2012; Samuel et al. 2017; Sangaiah et al. 2017; Sangaiah and Thangavelu 2013). So we need to consider consensus degree in the aggregation process. The neutrosophic triangular weighted aggregation operator (NTWAO) is given to overall triangular neutrosophic additive reciprocal preference relations based on the consensus degree. In the end, we give the structure of this paper. Section 2 introduces the basic preliminaries. In Sect. 3, a new definition of triangular neutrosophic additive reciprocal preference relations with additive approximation consistency is proposed and the properties are studied in detail. Section 4 shows the proposed algorithm for GDM problems with a new method of constructing triangular neutrosophic additive reciprocal preference relations by using only $(n-1)$ restricted preference values. In Sect. 5, a numerical example is offered to illustrate the proposed definitions and method. Finally, the main conclusions and future works are covered.

## 2 Preliminaries

In this part, the essential definitions involving neutrosophic set, single valued neutrosophic sets, triangular neutrosophic numbers, operations on triangular neutrosophic numbers and group decision-making problem are outlined.

**Definition 1** (Smarandache 2004) Let $X$ be a space of points (objects) and $x \in X$. A neutrosophic set $A$ in $X$ is defined by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $]^-0, 1^+[$. That is $T_A(x){:}X \rightarrow\ ]^-0, 1^+[$, $I_A(x) : X \rightarrow\ ]^-0, 1^+[$ and $F_A(x){:}X \rightarrow\ ]^-0, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3+$.

**Definition 2** (Smarandache 2004) Let $X$ be a universe of discourse. A single valued neutrosophic set $A$ over $X$ is an object having the form $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$, where $T_A(x) : X \rightarrow [0, 1]$, $I_A(x) : X \rightarrow [0, 1]$ and $F_A(x) : X \rightarrow [0, 1]$ with $0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3$ for all $x \in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity-membership degree of $x$ to $A$, respectively. For convenience, a SVN number is denoted by $A = $ (a, b, c), where $a, b, c \in [0, 1]$ and $a + b + c \leq 3$.

**Definition 3** (Liu and Wang 2014) Let $\alpha_{\tilde{a}}$ ,$\theta_{\tilde{a}}$, $\beta_{\tilde{a}} \epsilon [0, 1]$ and $a_1, a_2, a_3 \epsilon R$ such that $a_1 \leq a_2 \leq a_3$. Then a single valued triangular neutrosophic number, $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ is a special neutrosophic set on the real line set $R$, whose truth-membership, indeterminacy-membership and falsity-membership functions are given as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}} \left( \frac{x-a_1}{a_2-a_1} \right) & \text{if } a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}} & \text{if } x = a_2 \\ \alpha_{\tilde{a}} \left( \frac{a_3-x}{a_3-a_2} \right) & \text{if } a_2 < x \leq a_3 \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\theta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & \text{if } a_1 \leq x \leq a_2 \\ \theta_{\tilde{a}} & \text{if } x = a_2 \\ \frac{(x-a_2+\theta_{\tilde{a}}(a_3-x))}{(a_3-a_2)} & \text{if } a_2 < x \leq a_3 \\ 1 & \text{otherwise,} \end{cases} \tag{2}$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & \text{if } a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}} & \text{if } x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{(a_3-a_2)} & \text{if } a_2 < x \leq a_3 \\ 1 & \text{otherwise,} \end{cases} \tag{3}$$

where $\alpha_{\tilde{a}}, \theta_{\tilde{a}}$ and $\beta_{\tilde{a}}$ denote the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree, respectively. A single valued triangular neutrosophic number $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ may express an ill-defined quantity about $a$, which is approximately equal to $a$.

**Definition 4** (Liu and Wang 2014) Let $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \theta_{\tilde{b}}, \beta_{\tilde{b}} \rangle$ be two single valued triangular neutrosophic numbers and $\gamma \neq 0$ be any real number. Then,

1. Addition of two triangular neutrosophic numbers

$$\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle$$

2. Subtraction of two triangular neutrosophic numbers

$$\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle$$

3. Inverse of a triangular neutrosophic number

$$\tilde{a}^{-1} = \left\langle \left( \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \right\rangle, \text{ where } (\tilde{a} \neq 0)$$

4. Multiplication of triangular neutrosophic number by constant value

$$\gamma \tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma a_2, \gamma a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle & \text{if } (\gamma > 0) \\ \langle (\gamma a_3, \gamma a_2, \gamma a_1); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle & \text{if } (\gamma < 0) \end{cases}$$

5. Division of two triangular neutrosophic numbers

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \left\langle \left( \frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1} \right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \right\rangle & \text{if } (a_3 > 0, b_3 > 0) \\ \left\langle \left( \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1} \right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \right\rangle & \text{if } (a_3 < 0, b_3 > 0) \\ \left\langle \left( \frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3} \right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \right\rangle & \text{if } (a_3 < 0, b_3 < 0) \end{cases}$$

6. Multiplication of triangular neutrosophic numbers

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_3 > 0, b_3 > 0) \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_3 < 0, b_3 > 0) \\ \langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_3 < 0, b_3 < 0) \end{cases}$$

**Definition 5** (Chiclana et al. 2001) In a group decision-making problem with a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n$ and $(n \geq 2)$, the alternatives will be ranked from the best to the worst by making use of preference relations provided by a group of experts $E = \{e_1, e_2, \ldots, x_s$ $(s \geq 2)$. Each expert $e_k$ will compare every pair of alternatives to give a preference value in decision-making processes.

**Table 1** Abd-elbaset's $\frac{3}{2} - 3$ scale for preference ratio of alternatives

| Value of $\check{r}_{ij}$ | Explanation |
|---|---|
| $\check{r}_{ij} = \frac{3}{2}$ | Alternatives $i$ and $j$ are equally importance |
| $\frac{3}{2} < \check{r}_{ij} < 3$ | Alternative $i$ is preferred to $j$ |
| $0 < \check{r}_{ij} < \frac{3}{2}$ | Alternative $i$ is not preferred to $j$ |
| $\check{r}_{ij} = 3$ | Alternative $i$ is absolutely more important than alternative $j$ |

## 3 Additive approximation consistency of triangular neutrosophic additive reciprocal matrices

A single valued neutrosophic sets can be used in real-life applications, such as engineering and scientific applications. From Definition 2, a SVN number is denoted by $A = (a, b, c)$, where $0 \leq a+b+c \leq 3$. When the experts evaluate their judgments by using fuzzy numbers, the analysis in Liu et al. (2016) shows that the preference relations with fuzzy numbers are inconsistent in nature and for this reason, we focus here on additive approximation consistency of single valued triangular neutrosophic additive reciprocal matrices and its properties. Prior to give the definition of single valued triangular neutrosophic additive reciprocal preference relations, let us firstly assume that the scale system 0–3 is applied by all experts. The following triangular neutrosophic additive reciprocal preference relation is given in a GDM problem as:

$$\tilde{R} = (\check{r}_{ij})\, n \times n = \begin{bmatrix} (3/2, 3/2, 3/2) & (l_{12}, m_{12}, u_{12}) \cdots & (l_{1n}, m_{1n}, u_{1n}) \\ (l_{21}, m_{21}, u_{21}) & (3/2, 3/2, 3/2) \ldots & (l_{2n}, m_{2n}, u_{2n}) \\ \vdots & \vdots & \vdots \\ (l_{n1}, m_{n1}, u_{n1}) & (l_{n2}, m_{n2}, u_{n2}) \ldots & (3/2, 3/2, 3/2) \end{bmatrix}. \quad (4)$$

where $\check{r}_{ij}$ is interpreted as the neutrosophic number degree of the alternative $x_i$ over $x_j$ . $l_{ij}$ , $m_{ij}$ and $u_{ij}$ indicate the lower, median and upper bounds of the triangular neutrosophic number $\check{r}_{ij}$. $l_{ij}$ , $m_{ij}$ and $u_{ij}$ are nonnegative real numbers with $0 \leq l_{ij} \leq m_{ij} \leq u_{ij} \leq 3$ and have the additive reciprocity of $l_{ij} + u_{ji} = m_{ij} + m_{ji} = u_{ij} + l_{ji} = 3$, for all $i, j = 1, 2, \ldots, n$. The preference relation may be conveniently expressed as the matrix $\tilde{R} = (\check{r}_{ij})\, n \times n$, where $\check{r}_{ij} = T_{\check{r}}(x_i, x_j)$ , $I_{\check{r}}(x_i, x_j)$ and $F_{\check{r}}(x_i, x_j)$. It is interpreted as the preference ratio of the alternative $x_i$ over $x_j$ , for all $i, j = 1, 2, \ldots, n$, and it is illustrated in Table 1. For example, $\check{r}_{ij} = \frac{3}{2}$ indicates that there is no difference between $x_i$ and $x_j$, $\check{r}_{ij} = 3$ means that $x_i$ is absolutely preferred to $x_j$, and $\frac{3}{2} < \check{r}_{ij} < 3$ implies that $x_i$ is preferred to $x_j$.

### 3.1 Additive approximation consistency analysis

The additive consistency of triangular neutrosophic additive reciprocal preference relations $\tilde{R} = (\check{r}_{ij})\, n \times n$ can be expressed as

$$\check{r}_{ij} = \check{r}_{ik} + \check{r}_{kj} - \left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right), \quad (5)$$

where $i, j, k = 1, 2, \ldots, n$. The reason is based on the following analysis:

Making use of the operation laws of triangular neutrosophic numbers, Eq. (5) is also rewritten as

$$l_{ij} = l_{ik} + l_{kj} - \frac{3}{2}, \quad m_{ij} = m_{ik} + m_{kj} - \frac{3}{2},$$
$$u_{ij} = u_{ik} + u_{kj} - \frac{3}{2},$$

where $i, j, k = 1, 2, \ldots, n$. It is easily seen that $m_{ij} = m_{ik} + m_{kj} - \frac{3}{2}$ is correct. And applying $l_{ij} = l_{ik} + l_{kj} - \frac{3}{2}$ and $u_{ij} = u_{ik} + u_{kj} - \frac{3}{2}$, one can find that two following matrices possess additive consistency:

$$\begin{bmatrix} \frac{3}{2} & l_{12} \cdots & l_{1n} \\ l_{21} & \frac{3}{2} \ldots & l_{2n} \\ \vdots & \vdots \ldots & \vdots \\ l_{n1} & l_{n2} \ldots & \frac{3}{2} \end{bmatrix}, \begin{bmatrix} \frac{3}{2} & u_{12} \cdots & u_{1n} \\ u_{21} & \frac{3}{2} \ldots & u_{2n} \\ \vdots & \vdots \ldots & \vdots \\ u_{n1} & u_{n2} \ldots & \frac{3}{2} \end{bmatrix}. \quad (6)$$

It is convenient to construct three preference relations from a triangular neutrosophic additive preference relation $\tilde{R} = (\check{r}_{ij})\, n \times n = (l_{ij}, m_{ij}, u_{ij})\, n \times n$ as follows:

$$\check{r}^{l}_{ij} = \begin{cases} l_{ij}, & i < j \\ \frac{3}{2}, & i = j \\ u_{ij}, & i > j \end{cases} \quad (7)$$

$$\check{r}^{u}_{ij} = \begin{cases} u_{ij}, & i < j \\ \frac{3}{2}, & i = j \\ l_{ij}, & i > j \end{cases} \quad (8)$$

And $\check{r}^{m}_{ij} = m_{ij}$, for every $i, j = 1, 2, \ldots, n$. (9)

For a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ and $(n \geq 2)$ , there are $n!$ possible comparison matrices corresponding to the permutation of alternatives. Hence, we define the following function:

$$P : k \to P(k), \quad k = 1, 2, \ldots, n, \tag{10}$$

where the function $P$ denotes a permutation of $(1, 2, \ldots, n)$, $P(k_1) \neq P(k_2)$ when $k_1 \neq k_2$ , $(k_1, k_2 \in \{1, 2, \ldots, n\}$ .

From Eq. (10), the triangular neutrosophic additive reciprocal matrix with a permutation $P$ can be expressed as $\tilde{R}_P = \left(\check{r}_{p(i)p(j)}\right) n \times n$ where $\check{r}_{p(i)p(j)} = \left(l_{p(i)p(j)}, m_{p(i)p(j)}, u_{p(i)p(j)}\right)$. Similarly, three preference relations can be written as follows:

$$\check{r}^l_{p(i)p(j)} = \begin{cases} l_{p(i)p(j)}, & i < j \\ \frac{3}{2}, & i = j \\ u_{p(i)p(j)}, & i > j \end{cases}, \tag{11}$$

$$\check{r}^u_{p(i)p(j)} = \begin{cases} u_{p(i)p(j)}, & i < j \\ \frac{3}{2}, & i = j \\ l_{p(i)p(j)}, & i > j \end{cases}, \tag{12}$$

And $\check{r}^m_{p(i)p(j)} = m_{p(i)p(j)}$, for every $i, j = 1, 2, \ldots, n$. (13)

Then, we conclude a new definition of a triangular neutrosophic additive reciprocal preference relation:

**Definition 6** In order to check whether a triangular neutrosophic additive reciprocal preference relation $\tilde{R}$ is of additive approximation consistency or not, one should check the additive consistency of $\check{r}^l_{p(i)p(j)}$, $\check{r}^u_{p(i)p(j)}$ and $\check{r}^m_{p(i)p(j)}$. In other words, if $\tilde{R}$ is not of additive approximation consistency, there is at least one of $\check{r}^l_{p(i)p(j)}$, $\check{r}^u_{p(i)p(j)}$ and $\check{r}^m_{p(i)p(j)}$ without additive approximation consistency for any permutation of alternatives.

According to the characterization of additive consistency of additive reciprocal matrices in Herrera-Viedma et al. (2004), we can study the characterization of additive approximation consistency of triangular neutrosophic additive reciprocal preference relations:

**Proposition 1** *For an additive reciprocal preference relation $R = \left(r_{ij}\right) n \times n$, the following statements are equivalent:*

- $r_{ik} + r_{kj} + r_{ji} = \frac{3}{2}$, *for every $i, j, k$.*
- $r_{ik} + r_{kj} + r_{ji} = \frac{3}{2}$, *for every $i < j < k$.*

**Proposition 2** *For an additive reciprocal preference relation $R = \left(r_{ij}\right) n \times n$, the following statements are equivalent:*

- $r_{ik} + r_{kj} + r_{ji} = \frac{3}{2}$ , *for every $i, j, k$.*
- $r_{i(i+1)} + r_{(i+1)(i+2)} + \cdots + r_{(j-1)j} + r_{ji} = \frac{j-i+1}{2}$ , *for every $i < j$.*

**Proposition 3** *For an additive reciprocal preference relation $R = \left(r_{ij}\right) n \times n$ , the following statement is true:*

$$r_{ik} = 3 - r_{kj} = \left(3 - u_{ij}, 3 - m_{ij}, 3 - l_{ij}\right).$$

*Then, the characterization of additive approximation consistency of triangular neutrosophic additive reciprocal preference relations as follows:*

**Theorem 1** *A triangular neutrosophic additive reciprocal preference relations $\tilde{R}$ is of additive approximation consistency if and only if there is a permutation $P$ such that*

$$\begin{cases} l_{p(i)p(k)} + l_{p(k)p(j)} + u_{p(j)p(i)} = \frac{9}{2} \\ m_{p(i)p(k)} + m_{p(k)p(j)} + m_{p(j)p(i)} = \frac{9}{2} \\ u_{p(i)p(k)} + u_{p(k)p(j)} + l_{p(j)p(i)} = \frac{9}{2} \end{cases}. \tag{14}$$

*For every $i \leq k \leq j$.*

*Proof* It is seen that $\tilde{R}$ has additive approximation consistency, if and only if there is a permutation $P$ such that three additive reciprocal preference relations $\check{r}^l_{p(i)p(j)}$ , $\check{r}^u_{p(i)p(j)}$ and $\check{r}^m_{p(i)p(j)}$ are additively consistent. Otherwise, $\tilde{R}$ is said to be not of additive approximation consistency. Making use of Proposition 1, Eq. (14) is satisfied. Inversely, if Eq. (14) is satisfied, $\check{r}^l_{p(i)p(j)}$ , $\check{r}^u_{p(i)p(j)}$ and $\check{r}^m_{p(i)p(j)}$ are additively consistent for the permutation $P$. So we have proved the theorem. □

**Theorem 2** *For a triangular neutrosophic additive reciprocal preference relation $\tilde{R}$ with a permutation $P$, two following cases of statements are equivalent:*

$$\begin{cases} l_{p(i)p(k)} + l_{p(k)p(j)} + u_{p(j)p(i)} = \frac{9}{2}, \\ m_{p(i)p(k)} + m_{p(k)p(j)} + m_{p(j)p(i)} = \frac{9}{2}, \\ u_{p(i)p(k)} + u_{p(k)p(j)} + l_{p(j)p(i)} = \frac{9}{2}, \end{cases} \tag{I}$$

*For every $i \leq k \leq j$.*

$$\begin{aligned} & l_{p(i)p(i+1)} + l_{p(i+1)p(i+2)} + \ldots + l_{p(j-1)p(j)} \\ & \quad + u_{p(j)p(i)} = \tfrac{j-i+3}{2}, \\ & m_{p(i)p(i+1)} + m_{p(i+1)p(i+2)} + \cdots + m_{p(j-1)p(j)} \\ & \quad + m_{p(j)p(i)} = \tfrac{j-i+3}{2}, \\ & u_{p(i)p(i+1)} + u_{p(i+1)p(i+2)} + \cdots \\ & \quad + u_{p(j-1)p(i)} + l_{p(j)p(i)} = \tfrac{j-i+3}{2} \end{aligned} \tag{II}$$

*For $i < j$.*

*Proof* Since $\check{r}^l_{p(i)p(j)}$ , $\check{r}^u_{p(i)p(j)}$ and $\check{r}^m_{p(i)p(j)}$ are constructed from $\tilde{R}$ by using (11), (12) and (13), it is seen from Proposition 2 that two statements (I) and (II) are equivalent. □

## 4 A novel group decision-making model with triangular neutrosophic additive reciprocal matrices

Generally, every expert will make $\frac{n\times(n-1)}{2}$ judgments for a GDM problem with $n$ alternatives. When the number of alternatives is increasing, this makes the experts tired and leads to inconsistent judgments. Here, experts focus only on $(n-1)$ restricted judgments, which is similar to those given in Wang and Chen (2007, 2008). The specific process is composed of two main steps:

- Construction of triangular neutrosophic additive reciprocal preference relations with additive approximation consistency.
- Aggregation process by using NTWAO.

*Step 1* Suppose that there is a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ and a group of experts $\mathrm{E} = \{e_1, e_2, \ldots, e_s\}$ in a GDM problem. The expert focuses his/her attention not on $\frac{n\times(n-1)}{2}$ judgments, but on $(n-1)$ restricted judgments in decision making.

The expert $e_x$ , $x \in \{1, 2, \ldots, s\}$ compares $x_i$ over $x_j$ $(i \neq j)$ to give a triangular neutrosophic preference value $\check{r}_{ij}^x$ . Then eliminating $x_i$ from $X$, another preference value $\check{r}_{jk}^x$ is obtained by comparing $x_i$ over $x_k$ in the remaining $(n-2)$ alternatives. The expert $e_x$ repeats the above process, until the $(n-1)$ th preference value about two final alternatives is given. The expert $e_x$ gives his/her restricted $(n-1)$ preference values as $\check{r}_{12}^x, \check{r}_{23}^x, \ldots, \check{r}_{(n-1)n}^x$.

*Step 2* Construct a triangular neutrosophic additive reciprocal preference relation $\check{R}^x = (\check{r}_{ij}^x)n \times n$ with additive approximation consistency from the $(n-1)$ preference values by using Theorems 1 or 2. And to do this, we should estimate the missing preference values and check these estimation values.

*Step 3* Check estimation value $\check{r}_{ij}^x$ according to the standard of $0 \leq l_{ij}^x \leq m_{ij}^x \leq u_{ij}^x \leq 3$. Note that the estimation value $\check{r}_{ij}^x$ is obtained by using the additive consistency from the elements $\check{r}_{ik}^x$ and $\check{r}_{kj}^x$. It is further seen that $\check{r}_{ik}^x$ and $\check{r}_{kj}^x$ are given directly by the expert $e_x$ , or one of them is from the expert $e_x$ and another is indirectly obtained.

*Step 4* If the preference relation $\check{R}^x$ not be a triangular neutrosophic additive reciprocal matrix for $u_{ij}^x > 3$ or $l_{ij}^x < 0$, then go to the next step.

*Step 5* Do the following adjustment to obtain the acceptable preference relation $\check{R}^{x'} = (\check{r}_{ij}^{'x})n \times n$ with $u_{ij}^x < 3$ , by using the following explicit formula:

$$\check{r}_{ij}^{'x} = \frac{\check{r}_{ij}^x + c_x}{3 + 2c_x}, \tag{15}$$

where

$$c_x = \max\left\{u_{ij}^x - 3, 0 - l_{ij}^x\right\} \text{ For every } i, j = 1, 2, \ldots, n,\}$$

It is easy to see that $0 \leq l_{ij}^{'x} \leq m_{ij}^{'x} \leq u_{ij}^{'x} \leq 3$.

*Proof* Application of Theorems 1 or 2 yields the following:

$$\check{r}_{ij}^x = \check{r}_{ik}^x + \check{r}_{kj}^x - \left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right),$$

And because the scale of our system is 0–3, then the maximum range of $\check{r}_{ij}^x$ is $\frac{9}{2}$ and by applying Eq. (15), we ensure that, the obtained $\check{r}_{ij}^x$ does not exceed the specified range.

When $l_{ij}^x < 0$, apply Eq. (16), to ensure that, the obtained $\check{r}_{ij}^{'x}$ is not less than the specified range.

$$\check{r}_{ij}^{'x} = \frac{-\check{r}_{ij}^x + c_x}{3 + 2c_x}, \tag{16}$$

Let us consider the case of $l_{ij}^x > m_{ij}^x$ or $m_{ij}^x > u_{ij}^x$. We propose an adjustment method for making $l_{ij}^x > m_{ij}^x$ to $l_{ij}^x \leq m_{ij}^x$. Because the estimation value $\check{r}_{ij}^x$ is obtained by using the additive consistency from $\check{r}_{ik}^x$ and $\check{r}_{kj}^x$. It is further seen that $\check{r}_{ik}^x$ and $\check{r}_{kj}^x$ are given directly by the expert $e_x$ or one of them is from the expert $e_x$ and another is indirectly obtained. When $\check{r}_{ik}^x$ and $\check{r}_{kj}^x$ are given by the expert $e_x$, the adjustment process has the following possible cases:

The first case is, $k < i < j$ or $j < i < k$. By using additive approximation consistency given in Definition 6, we have the following:

$$l_{ij}^x = u_{ik}^x + l_{kj}^x - 0.5,$$
$$m_{ij}^x = u_{ik}^x + l_{kj}^x - 0.5,$$

Let,

$$u_{ik}^{'x} = u_{ij}^x - v_{ik}^{ux}, \quad l_{kj}^{'x} = l_{kj}^x - v_{kj}^{lx},$$
$$m_{kj}^{'x} = m_{kj}^x + v_{kj}^{mx}, \quad m_{ik}^{'x} = m_{ik}^x + v_{ik}^{mx},$$

where

$$u_{ik}^x + v_{kj}^{lx} + v_{ik}^{mx} + v_{kj}^{mx} \geq l_{ij}^x - m_{ij}^x = \Delta > 0 \text{ for}$$
$$v_{ik}^{ux} \geq 0, v_{kj}^{lx} \geq 0, v_{ik}^{mx} \geq 0 \text{ and } v_{kj}^{mx} \geq 0.$$

Then,

$$l_{ij}^{'x} = u_{ik}^x + l_{kj}^x - v_{ik}^{ux} - v_{kj}^{lx} - 0.5,$$
$$m_{ij}^{'x} = m_{ik}^x + m_{kj}^x + v_{ik}^{mx} + v_{kj}^{mx} - 0.5,$$

Then,

$$l_{ij}^{'x} \leq m_{ij}^{'x}.$$

The second case is, $i < k < j$ or $j < k < i$ . By using additive approximation consistency given in Definition 6, we have the following:

$$l_{ij}^{x} = u_{ik}^{x} + l_{kj}^{x} - 0.5,$$
$$m_{ij}^{x} = u_{ik}^{x} + l_{kj}^{x} - 0.5,$$

Then, we can get the following:

$$l_{ij}^{'x} = l_{ik}^{x} + u_{kj}^{x} - v_{ik}^{lx} - v_{kj}^{ux} - 0.5,$$
$$m_{ij}^{'x} = m_{ik}^{x} + m_{kj}^{x} + v_{ik}^{mx} + v_{kj}^{mx} - 0.5,$$

where $v_{ik}^{mx} + v_{kj}^{mx} + v_{ik}^{lx} + v_{kj}^{ux} \geq \Delta$ for $v_{kj}^{mx} \geq 0, v_{ik}^{mx} \geq 0, v_{ik}^{lx} \geq 0, v_{kj}^{ux} \geq 0$.

Then, $l_{ij}^{'x} \leq m_{ij}^{'x}$.

From the previous, we can conclude that:

In order to keep the original information as much as possible, we usually make $m_{ij}^{'x} - l_{ij}^{'x} = \Delta$.

When $m_{ij}^{x} > u_{ij}^{x}$, the method is similar to the previous and we keep the original information as much as possible by using the following equation:

$$u_{ij}^{x} - m_{ij}^{x} = \Delta, \tag{17}$$

*Step 6* After checking consistency of triangular neutrosophic additive reciprocal preference relation, each expert should determine the maximum truth-membership degree ($\alpha$), minimum indeterminacy-membership degree ($\theta$) and minimum falsity-membership degree ($\beta$) of each neutrosophic number as in Definition 3.

*Step 7* A collective triangular neutrosophic additive reciprocal preference relation should be derived by means of an aggregation procedure.

An aggregation operator of the single valued triangular neutrosophic number is calculated as follows:

Let $\tilde{a}_j = \left\langle \left(a_j, b_j, c_j\right); \alpha_{\tilde{a}_j}, \theta_{\tilde{a}_j}, \beta_{\tilde{a}_j} \right\rangle (j = 1, 2, \ldots n)$ be a collection of single valued triangular neutrosophic numbers. Then, neutrosophic triangular weighted aggregation operator (NTWAO) is defined as:

$$\text{NTWAO}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = (w_1\tilde{a}_1 + w_2\tilde{a}_2 + \cdots + w_n\tilde{a}_n), \tag{18}$$

$w = (w_1, w_2, \ldots w_n)^T$ is a weight vector of alternatives, $w_j \geq 0$, $j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} w_j = 1$. □

**Theorem 3** *Let* $\tilde{a}_j = \left\langle \left(a_j, b_j, c_j\right); \alpha_{\tilde{a}_j}, \theta_{\tilde{a}_j}, \beta_{\tilde{a}_j} \right\rangle (j = 1, 2, \ldots n)$ *be a collection of single valued triangular neutrosophic numbers,* $w = (w_1, w_2, \ldots w_n)^T$ *is a weight vector of* $\tilde{a}_j$ *with* $w_j \geq 0$, $j = 1, 2, \ldots, n$ *and* $\sum_{j=1}^{n} w_j = 1$. *Then their aggregated value by using NTWAO is also a neutrosophic triangular number:*

$$\textit{NTWAO}\,(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\left(\sum_{j=1}^{n} w_j\tilde{a}_j, \sum_{j=1}^{n} w_j\tilde{b}_j, \sum_{j=1}^{n} w_j\tilde{c}_j\right); \wedge_{j=1}^{n}\alpha_{\tilde{a}_j}, \vee_{j=1}^{n}\theta_{\tilde{a}_j}, \vee_{j=1}^{n}\beta_{\tilde{a}_j}\right).$$

*Proof* By using mathematical induction on $n$ as:

Let $\tilde{a}_1 = \langle(\tilde{a}_1, \tilde{b}_1, \tilde{c}_1); \alpha_{\tilde{a}1}, \theta_{\tilde{a}1}, \beta_{\tilde{a}1}\rangle$ and $\tilde{a}_2 = \langle(\tilde{a}_2, \tilde{b}_2, \tilde{c}_2); \alpha_{\tilde{a}2}, \theta_{\tilde{a}2}, \beta_{\tilde{a}2}\rangle$ be two single valued triangular neutrosophic numbers then,

For $n = 2$, we have the following:

$$w_1\tilde{a}_1 + w_2\tilde{a}_2 = \left(\left(\sum_{j=1}^{2} w_j\tilde{a}_j, \sum_{j=1}^{2} w_j\tilde{b}_j, \sum_{j=1}^{2} w_j\tilde{c}_j\right); \wedge_{j=1}^{2}\alpha_{\tilde{a}_j}, \vee_{j=1}^{2}\theta_{\tilde{a}_j}, \vee_{j=1}^{2}\beta_{\tilde{a}_j}\right).$$

For $n = k$, that is

$$w_1\tilde{a}_1 + w_2\tilde{a}_2 + \cdots + w_k\tilde{a}_k = \left(\left(\sum_{j=1}^{k} w_j\tilde{a}_j, \tilde{b}_j, \sum_{j=1}^{2} w_j\tilde{c}_j\right); \wedge_{j=1}^{k}\alpha_{\tilde{a}_j}, \vee_{j=1}^{k}\theta_{\tilde{a}_j}, \vee_{j=1}^{k}\beta_{\tilde{a}_j}\right).$$

Then, when $n = k + 1$, by using laws in Definition 4, we have the following:

$$\begin{aligned} & w_1\tilde{a}_1 + w_2\tilde{a}_2 + \cdots + w_{k+1}\tilde{a}_{k+1} \\ & = \left(\left(\sum_{j=1}^{k} w_j\tilde{a}_j, \sum_{j=1}^{k} w_j\tilde{b}_j, \sum_{j=1}^{k} w_j\tilde{c}_j\right); \wedge_{j=1}^{k}\alpha_{\tilde{a}_j}, \vee_{j=1}^{k}\theta_{\tilde{a}_j}, \vee_{j=1}^{k}\beta_{\tilde{a}_j}\right) \\ & \quad + \left(\left(w_{k+1}\tilde{a}_{k+1} + w_{k+1}\tilde{b}_{k+1} + w_{k+1}\tilde{c}_{k+1}\right); \alpha_{\tilde{a}_{k+1}}, \theta_{\tilde{a}_{k+1}}, \beta_{\tilde{a}_{k+1}}\right) \\ & \quad = \left(\left(\sum_{j=1}^{k+1} w_j\tilde{a}_j, \sum_{j=1}^{k+1} w_j\tilde{b}_j, \sum_{j=1}^{k+1} w_j\tilde{c}_j\right); \wedge_{j=1}^{k+1}\alpha_{\tilde{a}_j}, \vee_{j=1}^{k+1}\theta_{\tilde{a}_j}, \vee_{j=1}^{k+1}\beta_{\tilde{a}_j}\right). \end{aligned}$$

Then, we validated the proof. □

*Step 8* After obtaining a collective triangular neutrosophic additive reciprocal preference relation $\tilde{R}$, consensus degree of experts should be calculated as follows:

Let $\tilde{a}_j = \langle\left(a_j, b_j, c_j\right); \alpha_{\tilde{a}_j}, \theta_{\tilde{a}_j}, \beta_{\tilde{a}_j}\rangle (j = 1, 2, \ldots n)$ be a collection of single valued triangular neutrosophic numbers. Then, consensus degree (CD) is defined as:

$$\mathrm{CD}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = (w_1\tilde{a}_1^\varepsilon + w_2\tilde{a}_2^\varepsilon + \ldots + w_n\tilde{a}_n^\varepsilon)^{\varepsilon^{-1}}, \tag{19}$$

where $\varepsilon > 0$, it is the number of experts that are consensus on decision.

$w = (w_1, w_2, \ldots w_n)^T$ is a weight vector of neutrosophic preference relations, $w_j \geq 0,\ j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} w_j = 1$.

$$\mathrm{CD}\,(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\left(\sum_{j=1}^{n} w_j\tilde{a}_j^\varepsilon, \sum_{j=1}^{n} w_j\tilde{b}_j^\varepsilon, \sum_{j=1}^{n} w_j\tilde{c}_j^\varepsilon\right); \wedge_{j=1}^{n}\alpha_{\tilde{a}_j}, \vee_{j=1}^{n}\theta_{\tilde{a}_j}, \vee_{j=1}^{n}\beta_{\tilde{a}_j}\right).$$

*Proof* By using mathematical induction on n as:

Let $\tilde{a}_1 = \langle (a_1^\varepsilon, b_1^\varepsilon, c_1^\varepsilon); \alpha_{\tilde{a}1}^\varepsilon, \theta_{\tilde{a}1}^\varepsilon, \beta_{\tilde{a}1}^\varepsilon \rangle$ and $\tilde{a}_2 = \langle (a_2^\varepsilon, b_2^\varepsilon, c_2^\varepsilon); \alpha_{\tilde{a}2}^\varepsilon, \theta_{\tilde{a}2}^\varepsilon, \beta_{\tilde{a}2}^\varepsilon \rangle$ be two single valued triangular neutrosophic numbers.

For $n = 2$, we have the following:

$$w_1a_1^\varepsilon + w_2a_2^\varepsilon = \left(\left(\sum_{j=1}^{2} w_ja_j^\varepsilon, \sum_{j=1}^{2} w_jb_j^\varepsilon, \sum_{j=1}^{2} w_jc_j^\varepsilon\right); \wedge_{j=1}^{2}\alpha_{\tilde{a}_j}, \vee_{j=1}^{2}\theta_{\tilde{a}_j}, \vee_{j=1}^{2}\beta_{\tilde{a}_j}\right).$$

For $n = k$, that is

$$w_1a_1^\varepsilon + w_2a_2^\varepsilon + \cdots + w_ka_k^\varepsilon = \left(\left(\sum_{j=1}^{k} w_ja_j^\varepsilon, \sum_{j=1}^{2} w_jb_j^\varepsilon, \sum_{j=1}^{2} w_jc_j^\varepsilon\right); \wedge_{j=1}^{k}\alpha_{\tilde{a}_j}, \vee_{j=1}^{k}\theta_{\tilde{a}_j}, \vee_{j=1}^{k}\beta_{\tilde{a}_j}\right).$$

Then, when $n = k + 1$, by using laws in Definition 4, we have the following;

$$\begin{aligned} &w_1a_1^\varepsilon + w_2a_2^\varepsilon + \cdots + w_{k+1}a_{k+1}^\varepsilon \\ &= \left(\left(\sum_{j=1}^{k} w_ja_j^\varepsilon, \sum_{j=1}^{k} w_jb_j^\varepsilon, \sum_{j=1}^{k} w_jc_j^\varepsilon\right); \wedge_{j=1}^{k}\alpha_{\tilde{a}_j}, \vee_{j=1}^{k}\theta_{\tilde{a}_j}, \vee_{j=1}^{k}\beta_{\tilde{a}_j}\right) \\ &\quad + \left((w_{k+1}a_{k+1}^\varepsilon + w_{k+1}b_{k+1}^\varepsilon + w_{k+1}c_{k+1}^\varepsilon); \alpha_{\tilde{a}_{k+1}}, \theta_{\tilde{a}_{k+1}}, \beta_{\tilde{a}_{k+1}}\right) \\ &= \left(\left(\sum_{j=1}^{k+1} w_ja_j^\varepsilon, \sum_{j=1}^{k+1} w_jb_j^\varepsilon, \sum_{j=1}^{k+1} w_jc_j^\varepsilon\right); \wedge_{j=1}^{k+1}\alpha_{\tilde{a}_j}, \vee_{j=1}^{k+1}\theta_{\tilde{a}_j}, \vee_{j=1}^{k+1}\beta_{\tilde{a}_j}\right). \end{aligned}$$

Then, we validated the proof. □

To obtain crisp values of consensus degree, use the following equation:

Let $\tilde{a}_{ij} = \langle (a_1, b_1, c_1), \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ be a single valued triangular neutrosophic number, then,

$$S\left(\tilde{a}_{ij}\right) = \frac{1}{8}\left[a_1 + b_1 + c_1\right] \times \left(2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}}\right), \tag{20}$$

where $S\left(\tilde{a}_{ij}\right)$ is the score function of neutrosophic number.

*Step 9* Rank the alternatives.

The previous steps for solving group decision-making problems with triangular neutrosophic additive reciprocal matrices are shown in Fig. 1.

## 5 Illustrative example and comparison analysis

In this section, to illustrate the efficiency and applicability of the proposed algorithm, we consider a group decision-making problem about personal computer selection and make a comparison analysis of the proposed algorithm with other existing algorithms.

### 5.1 Illustrative example

Now we offer an example to illustrate the GDM method based on triangular neutrosophic additive reciprocal preference relation with additive approximation consistency. This example is that of selecting the best personal computer from four different alternatives:

1. The first type is Dell, black, fast and very expensive computer.
2. The second type is HP, black, very fast and expensive computer.
3. The third type is Toshiba, white, slow and very cheap computer.
4. The fourth type is Lenovo, red, fast and economic computer.

There exist three experts (financial expert ($e_1$), engineering expert ($e_2$), control expert ($e_3$)) for group decision-making problem. And by making an interview with each expert, their preference relations are as follows:

The financial expert ($e_1$) preference relation is:

$$\tilde{R}^1 = \begin{bmatrix} (3/2, 3/2, 3/2) & (2, 5/2, 3) & \mathrm{X} & \mathrm{X} \\ \mathrm{X} & (3/2, 3/2, 3/2) & (1/2, 1, 2) & \mathrm{X} \\ \mathrm{X} & \mathrm{X} & (3/2, 3/2, 3/2) & (3/2, 2, 3) \\ \mathrm{X} & \mathrm{X} & \mathrm{X} & (3/2, 3/2, 3/2) \end{bmatrix}.$$

The engineering expert ($e_2$) preference relation is:

$$\tilde{R}^2 = \begin{bmatrix} (3/2, 3/2, 3/2) & (1, 3/2, 2) & \mathrm{X} & \mathrm{X} \\ \mathrm{X} & (3/2, 3/2, 3/2) & (1/2, 1, 2) & \mathrm{X} \\ \mathrm{X} & \mathrm{X} & (3/2, 3/2, 3/2) & (3/2, 2, 5/2) \\ \mathrm{X} & \mathrm{X} & \mathrm{X} & (3/2, 3/2, 3/2) \end{bmatrix}.$$

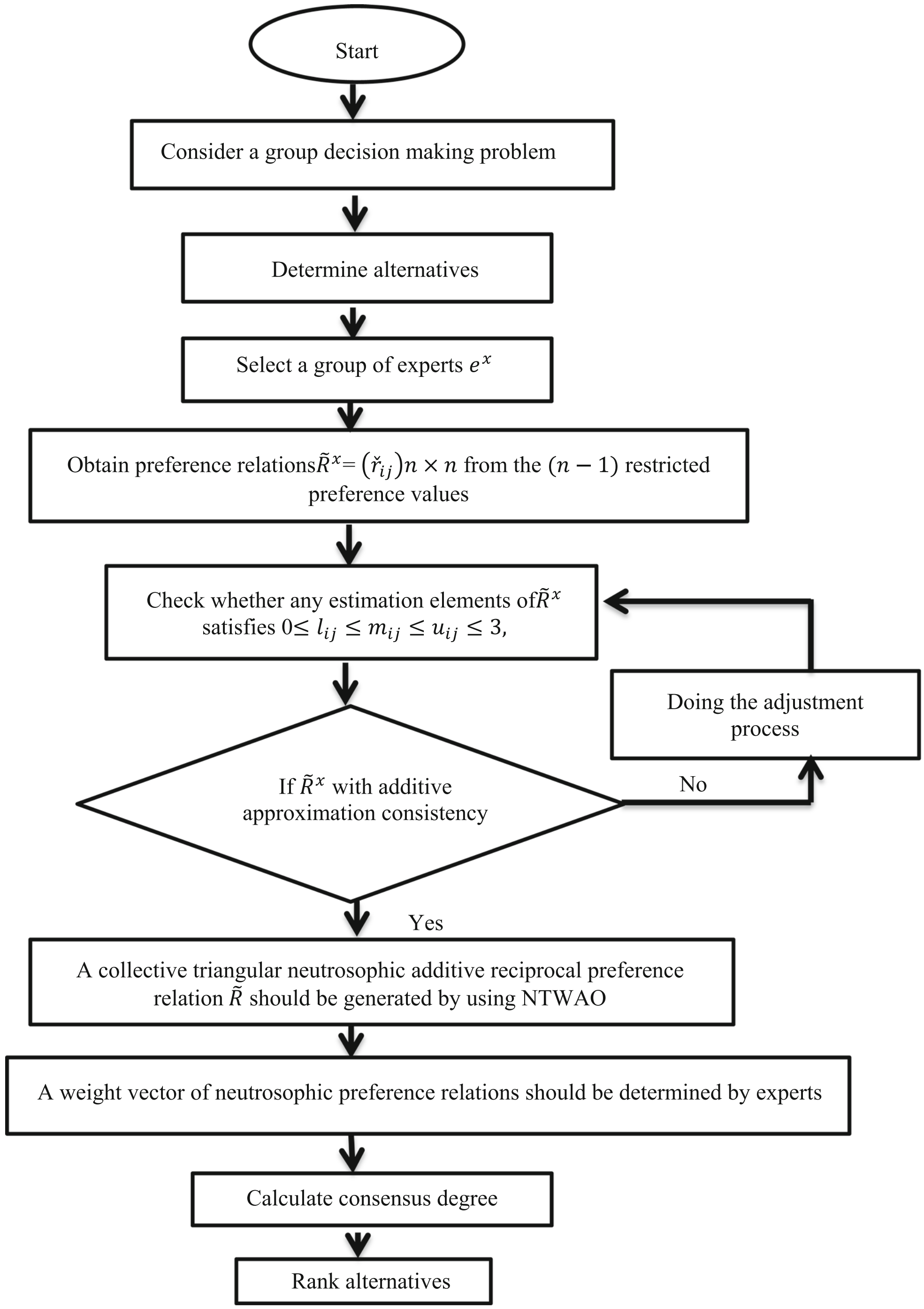


**Fig. 1** Schematic diagram for solving GDM problems with triangular neutrosophic additive reciprocal matrices

The control expert ($e_3$) preference relation is:

$$\tilde{R}^3 = \begin{bmatrix} (3/2, 3/2, 3/2) & X & (3/2, 2, 3) & X \\ X & (3/2, 3/2, 3/2) & X & X \\ X & X & (3/2, 3/2, 3/2) & (1, 3/2, 2) \\ X & (1/2, 4/5, 2) & X & (3/2, 3/2, 3/2) \end{bmatrix}.$$

where X indicates incomplete preference values.

And the hierarchical structure of this example is presented in Fig. 2.

It is obvious that the expert $e_1$ compares alternatives $x_1$ over $x_2$, $x_2$ over $x_3$ and $x_3$ over $x_4$ to give three preference values $\tilde{r}^1_{12}$, $\tilde{r}^1_{23}$ and $\tilde{r}^1_{34}$. The expert $e_2$ compares alternatives $x_1$ over $x_2$, $x_2$ over $x_3$ and $x_3$ over $x_4$ to give three preference values $\tilde{r}^1_{12}$, $\tilde{r}^1_{23}$ and $\tilde{r}^1_{34}$, and the expert $e_3$ compares alter-

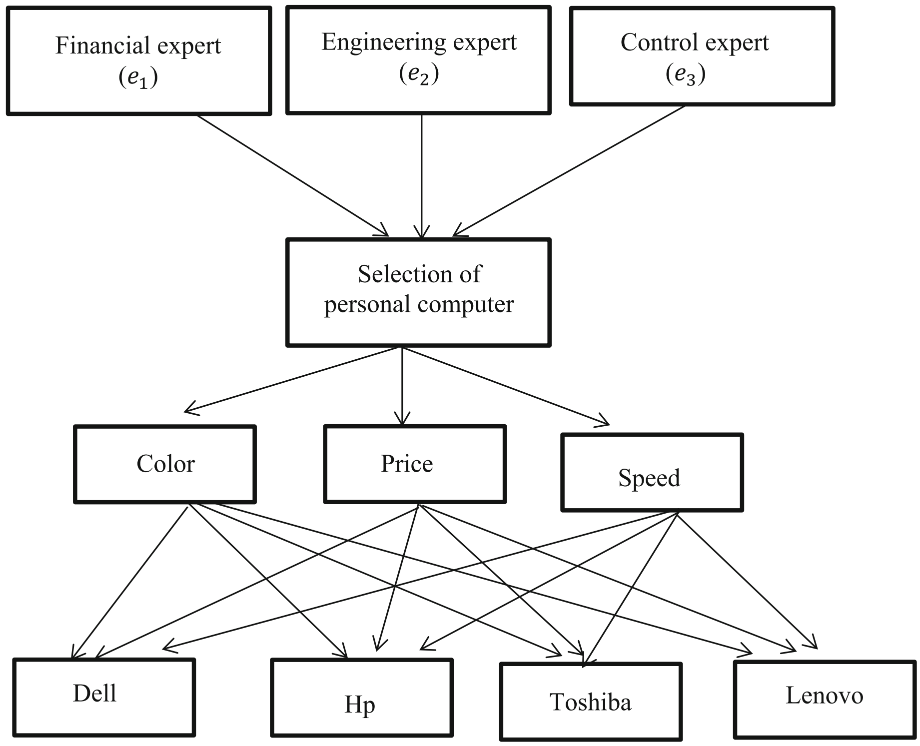


**Fig. 2** Hierarchical structure of the illustrative example

natives $x_1$ over $x_3$, $x_3$ over $x_4$ and $x_4$ over $x_2$ to give three preference values $\tilde{r}^1_{13}$, $\tilde{r}^1_{34}$ and $\tilde{r}^1_{42}$.

By applying Theorems 1 or 2, we can obtain the following:

$$\tilde{r}^1_{13} = \tilde{r}^1_{12} + \tilde{r}^1_{23} - (3/2, 3/2, 3/2) = (1, 2, 7/2),$$

$$\tilde{r}^1_{24} = \tilde{r}^1_{23} + \tilde{r}^1_{34} - (3/2, 3/2, 3/2) = (1/2, 3/2, 7/2),$$

$$\tilde{r}^1_{14} = \tilde{r}^1_{12} + \tilde{r}^1_{24} - (3/2, 3/2, 3/2) = (2, 3, 9/2).$$

Making use of the property of triangular neutrosophic additive reciprocal preference relation, one can obtain the following:

$$\tilde{r}^1_{21} = 3 - \tilde{r}^1_{12} = (0, 1/2, 1),$$

$$\tilde{r}^1_{31} = 3 - \tilde{r}^1_{13} = (1/2, 1, 2),$$

$$\tilde{r}^1_{32} = 3 - \tilde{r}^1_{23} = (1, 2, 5/2),$$

$$\tilde{r}^1_{41} = 3 - \tilde{r}^1_{14} = (-3/2, 0, 1),$$

$$\tilde{r}^1_{42} = 3 - \tilde{r}^1_{24} = (-1/2, 3/2, 5/2)$$

$$\tilde{r}^1_{43} = 3 - \tilde{r}^1_{34} = (0, 1, 3/2).$$

The additive preference relation of financial expert ($e_1$) is as follows:

$$\tilde{R}^1 = \begin{bmatrix} (3/2, 3/2, 3/2) & (2, 5/2, 3) & (1, 2, 7/2) & (2, 3, 9/2) \\ (0, 1/2, 1) & (3/2, 3/2, 3/2) & (1/2, 1, 2) & (1/2, 3/2, 7/2) \\ (1/2, 1, 2) & (1, 2, 5/2) & (3/2, 3/2, 3/2) & (3/2, 2, 3) \\ (-3/2, 0, 1) & (-1/2, 3/2, 5/2) & (0, 1, 3/2) & (3/2, 3/2, 3/2) \end{bmatrix}.$$

By applying Theorems 1 or 2, we can obtain the following:

$$\tilde{r}^2_{13} = \tilde{r}^2_{12} + \tilde{r}^2_{23} - (3/2, 3/2, 3/2) = (1/2, 3/2, 9/2),$$

$$\tilde{r}^2_{14} = \tilde{r}^2_{13} + \tilde{r}^2_{34} - (3/2, 3/2, 3/2) = (1/2, 3/2, 9/2),$$

$$\tilde{r}^2_{24} = \tilde{r}^2_{23} + \tilde{r}^2_{34} - (3/2, 3/2, 3/2) = (1/2, 3/2, 3).$$

Making use of the property of triangular neutrosophic additive reciprocal preference relation, one can obtain the following:

$$\tilde{r}^2_{21} = 3 - \tilde{r}^2_{12} = (1, 3/2, 2),$$

$$\tilde{r}^2_{31} = 3 - \tilde{r}^2_{13} = (-3/2, 3/2, 5/2),$$

$$\tilde{r}^2_{32} = 3 - \tilde{r}^2_{23} = (1, 2, 5/2),$$

$$\tilde{r}^2_{41} = 3 - \tilde{r}^2_{14} = (-5/2, 3/2, 5/2),$$

$$\tilde{r}^2_{42} = 3 - \tilde{r}^2_{24} = (-1/2, 3/2, 5/2),$$

$$\tilde{r}^2_{43} = 3 - \tilde{r}^2_{34} = (1/2, 1, 3/2).$$

The additive preference relation of engineering expert ($e_2$) is as follows:

$$\tilde{R}^2 = \begin{bmatrix} (3/2, 3/2, 3/2) & (1, 3/2, 2) & (1/2, 3/2, 9/2) & (1/2, 3/2, 11/2) \\ (1, 3/2, 2) & (3/2, 3/2, 3/2) & (1/2, 1, 2) & (1/2, 3/2, 3) \\ (-3/2, 3/2, 5/2) & (1, 2, 5/2) & (3/2, 3/2, 3/2) & (3/2, 2, 5/2) \\ (-5/2, 3/2, 5/2) & (-1/2, 3/2, 5/2) & (1/2, 1, 3/2) & (3/2, 3/2, 3/2) \end{bmatrix}.$$

By applying Theorems 1 or 2 and the property of triangular neutrosophic additive reciprocal preference relation, we can obtain the following:

$$\tilde{r}^3_{14} = \tilde{r}^3_{13} + \tilde{r}^3_{34} - (3/2, 3/2, 3/2) = (1, 2, 7/2),$$

$$\tilde{r}^3_{12} = \tilde{r}^3_{14} + \tilde{r}^3_{42} - (3/2, 3/2, 3/2) = (0, 13/10, 4),$$

$$\tilde{r}^3_{21} = 3 - \tilde{r}^3_{12} = (1, 17/10, 3),$$

$$\tilde{r}^3_{23} = \tilde{r}^3_{21} + \tilde{r}^3_{13} - (3/2, 3/2, 3/2) = (1, 11/5, 9/2),$$

$$\tilde{r}^3_{24} = \tilde{r}^3_{21} + \tilde{r}^3_{14} - (3/2, 3/2, 3/2) = (1/2, 11/5, 5),$$

$$\tilde{r}^3_{31} = 3 - \tilde{r}^3_{13} = (0, 1, 3/2),$$

$$\tilde{r}^3_{32} = 3 - \tilde{r}^3_{23} = (-3/2, 4/5, 2),$$

$$\tilde{r}^3_{41} = 3 - \tilde{r}^3_{14} = (-1/2, 1, 2),$$

$$\tilde{r}^3_{43} = 3 - \tilde{r}^3_{34} = (1, 3/2, 2).$$

The additive preference relation of control expert ($e_3$) is as follows:

$$\tilde{R}^3 = \begin{bmatrix} (3/2, 3/2, 3/2) & (0, 13/10, 4) & (3/2, 2, 3) & (2, 3, 9/2) \\ (0, 1/2, 1) & (3/2, 3/2, 3/2) & (1, 11/5, 9/2) & (1/2, 11/5, 5) \\ (0, 1, 3/2) & (-3/2, 4/5, 2) & (3/2, 3/2, 3/2) & (1, 3/2, 2) \\ (-1/2, 1, 2) & (1/2, 4/5, 2) & (1, 3/2, 2) & (3/2, 3/2, 3/2) \end{bmatrix}.$$

According to Definition 6, one can see that $\tilde{R}^1$ is not triangular neutrosophic additive reciprocal preference relations due to $u^1_{13} = 7/2 > 3$, $u^1_{14} = 9/2 = 4.5 > 3$, $u^1_{24} = 7/2 = 3.5 > 3$, $l^1_{41} = -3/2 = -1.5 < 0$, $l^1_{42} = -0.5 < 0$.

With regard to $\tilde{R}^1$, we use Eq. (15) with change parameters $c^1_{13}, c^1_{14} = 0.5$, $c^1_{24} = 0.5$ and we also use Eq. (16) because $l^1_{41} = -3/2 = -1.5 < 0$, $l^1_{42} = -0.5 < 0$.

After using Eqs. (15, 16), one can obtain the following:

$$\tilde{R}^1 = \begin{bmatrix} (3/2, 3/2, 3/2) & (2, 5/2, 3) & (1, 2, 1) & (2, 3, 1.25) \\ (0, 1/2, 1) & (3/2, 3/2, 3/2) & (1/2, 1, 2) & (1/2, 3/2, 1) \\ (1/2, 1, 2) & (1, 2, 5/2) & (3/2, 3/2, 3/2) & (3/2, 2, 3) \\ (1/2, 0, 1) & (1/4, 3/2, 5/2) & (0, 1, 3/2) & (3/2, 3/2, 3/2) \end{bmatrix}.$$

It is obvious that $u^1_{13} = 1 < m^1_{13} = 2$, $u^1_{14} = 1.25 < m^1_{14} = 3$,also $u^1_{24} = 1 < m^1_{24} = 1.5$. To deal with this special case, we should use Eq. (17).

Then,

$$\tilde{R}^1 = \begin{bmatrix} (3/2, 3/2, 3/2) & (2, 5/2, 3) & (1, 2, 2) & (2, 3, 3) \\ (0, 1/2, 1) & (3/2, 3/2, 3/2) & (1/2, 1, 2) & (1/2, 3/2, 3/2) \\ (1/2, 1, 2) & (1, 2, 5/2) & (3/2, 3/2, 3/2) & (3/2, 2, 3) \\ (1/2, 0, 1) & (1/4, 3/2, 5/2) & (0, 1, 3/2) & (3/2, 3/2, 3/2) \end{bmatrix}.$$

From $\tilde{R}^1$, one can see that, for each neutrosophic number $0 \le l_{ij} \le m_{ij} \le u_{ij} \le 3$, then $\tilde{R}^1$ is a triangular neutrosophic additive reciprocal preference relation.

According to Definition 6, one can see that $\tilde{R}^2$ is not triangular neutrosophic additive reciprocal preference relations due to $u^2_{13} = 9/2 = 4.5 > 3$, $u^2_{14} = 11/2 = 5.5 > 3$, $l^2_{31} = -3/2 = -1.5 < 0$, $l^2_{41} = -2.5 < 0$, and $l^2_{42} = -0.5 < 0$.

After using Eqs. (15, 16), one can obtain the following:

$$\tilde{R}^2 = \begin{bmatrix} (3/2, 3/2, 3/2) & (1, 3/2, 2) & (1/2, 3/2, 5/4) & (1/2, 3/2, 6/4) \\ (1, 3/2, 2) & (3/2, 3/2, 3/2) & (1/2, 1, 2) & (1/2, 3/2, 3) \\ (1/2, 3/2, 5/2) & (1, 2, 5/2) & (3/2, 3/2, 3/2) & (3/2, 2, 5/2) \\ (3/4, 3/2, 5/2) & (1/4, 3/2, 5/2) & (1/2, 1, 3/2) & (3/2, 3/2, 3/2) \end{bmatrix}.$$

It is obvious that $u^2_{13} = 1.25 < m^2_{13} = 1.5$. To deal with this special case, we should use Eq. (17).

Then,

$$\tilde{R}^2 = \begin{bmatrix} (3/2, 3/2, 3/2) & (1, 3/2, 2) & (1/2, 3/2, 3/2) & (1/2, 3/2, 6/4) \\ (1, 3/2, 2) & (3/2, 3/2, 3/2) & (1/2, 1, 2) & (1/2, 3/2, 3) \\ (1/2, 3/2, 5/2) & (1, 2, 5/2) & (3/2, 3/2, 3/2) & (3/2, 2, 5/2) \\ (3/4, 3/2, 5/2) & (1/4, 3/2, 5/2) & (1/2, 1, 3/2) & (3/2, 3/2, 3/2) \end{bmatrix}.$$

From $\tilde{R}^2$, one can see that, for each neutrosophic number $0 \le l_{ij} \le m_{ij} \le u_{ij} \le 3$, then $\tilde{R}^2$ is a triangular neutrosophic additive reciprocal preference relation.

According to Definition 6, one can see that $\tilde{R}^3$ is not triangular neutrosophic additive reciprocal preference relations due to $u^3_{12} = 4 > 3$, $u^3_{14} = 9/2 = 4.5 > 3$, $u^3_{23} = 9/2 = 4.5 > 3$, $u^3_{24} = 5 > 3$, $l^3_{32} = -3/2 = -1.5 < 0$, $l^3_{41} = -0.5 < 0$.

After using Eqs. (15, 16), one can obtain the following:

$$\tilde{R}^3 = \begin{bmatrix} (3/2, 3/2, 3/2) & (0, 13/10, 1.33) & (3/2, 2, 3) & (2, 3, 1.25) \\ (0, 1/2, 1) & (3/2, 3/2, 3/2) & (1, 11/5, 1.25) & (1/2, 11/5, 1.66) \\ (0, 1, 3/2) & (1/2, 4/5, 2) & (3/2, 3/2, 3/2) & (1, 3/2, 2) \\ (1/4, 1, 2) & (1/2, 4/5, 2) & (1, 3/2, 2) & (3/2, 3/2, 3/2) \end{bmatrix}.$$

It is obvious that $u^3_{14} = 1.25 < m^4_{14} = 3$, $u^3_{23} = 1.25 < m^3_{23} = 2.2$. To deal with this special case, we should use Eq. (17). Then,

$$\tilde{R}^3 = \begin{bmatrix} (3/2, 3/2, 3/2) & (0, 13/10, 133/100) & (3/2, 2, 3) & (2, 3, 3) \\ (1, 1/2, 1) & (3/2, 3/2, 3/2) & (1, 11/5, 11/5) & (1/2, 11/5, 83/50) \\ (0, 1, 3/2) & (1/2, 4/5, 2) & (3/2, 3/2, 3/2) & (1, 3/2, 2) \\ (1/4, 1, 2) & (1/2, 4/5, 2) & (1, 3/2, 2) & (3/2, 3/2, 3/2) \end{bmatrix}.$$

From $\tilde{R}^3$, one can see that, for each neutrosophic number $0 \le l_{ij} \le m_{ij} \le u_{ij} \le 3$, then $\tilde{R}^3$ is a triangular neutrosophic additive reciprocal preference relation.

After checking consistency of triangular neutrosophic additive reciprocal preference relations, each expert should determine the maximum truth-membership degree ($\alpha$), minimum indeterminacy-membership degree ($\theta$) and minimum falsity-membership degree ($\beta$) of single valued neutrosophic numbers as in Definition 3.

The consistent additive preference relation of financial expert ($e_1$) is as follows:

$$\tilde{R}^1 = \begin{bmatrix} \langle(3/2, 3/2, 3/2); 0.5, 0.3, 0.4\rangle & \langle(2, 5/2, 3); 0.7, 0.2, 0.5\rangle & \langle(1, 2, 2); 0.5, 0.2, 0.1\rangle & \langle(2, 3, 3); 0.5, 0.2, 0.1\rangle \\ \langle(0, 1/2, 1); 0.7, 0.2, 0.5\rangle & \langle(3/2, 3/2, 3/2); 0.8, 0.5, 0.3\rangle & \langle(1/2, 1, 2)0.5, 0.2, 0.1\rangle & \langle(1/2, 3/2, 3/2); 0.4, 0.5, 0.6\rangle \\ \langle(1/2, 1, 2); 0.6, 0.2, 0.3\rangle & \langle(1, 2, 5/2); 0.3, 0.1, 0.5\rangle & \langle(3/2, 3/2, 3/2); 0.6, 0.4, 0.2\rangle & \langle(3/2, 2, 3); 0.7, 0.2, 0.5\rangle \\ \langle(1/2, 0, 1); 0.6, 0.2, 0.3\rangle & \langle(1/4, 3/2, 5/2); 0.6, 0.2, 0.3\rangle & \langle(0, 1, 3/2); 0.9, 0.4, 0.6\rangle & \langle(3/2, 3/2, 3/2); 0.8, 0.5, 0.2\rangle \end{bmatrix}.$$

The consistent additive preference relation of engineering expert ($e_2$) is as follows:

$$\tilde{R}^2 = \begin{bmatrix} \langle(3/2, 3/2, 3/2); 0.8, 0.2, 0.6\rangle & \langle(1, 3/2, 2); 0.7, 0.2, 0.5\rangle & \langle(1/2, 3/2, 3/2); 0.4, 0.5, 0.6\rangle & \langle(1/2, 3/2, 6/4); 0.6, 0.2, 0.3\rangle \\ \langle(1, 3/2, 2); 0.5, 0.3, 0.4\rangle & \langle(3/2, 3/2, 3/2); 0.8, 0.5, 0.3\rangle & \langle(1/2, 1, 2)0.5, 0.2, 0.1\rangle & \langle(1/2, 3/2, 3); 0.5, 0.2, 0.1\rangle \\ \langle(1/2, 3/2, 5/2); 0.3, 0.1, 0.5\rangle & \langle(1, 2, 5/2); 0.3, 0.1, 0.5\rangle & \langle(3/2, 3/2, 3/2); 0.5, 0.3, 0.4\rangle & \langle(3/2, 2, 5/2); 0.7, 0.2, 0.5\rangle \\ \langle(3/4, 3/2, 5/2); 0.5, 0.2, 0.1\rangle & \langle(1, 2, 5/2); 0.3, 0.1, 0.5\rangle & \langle(1/2, 1, 3/2); 0.3, 0.1, 0.5\rangle & \langle(3/2, 3/2, 3/2); 0.6, 0.4, 0.2\rangle \end{bmatrix}.$$

The consistent additive preference relation of control expert ($e_3$) is as follows:

$$\tilde{R}^3 = \begin{bmatrix} \langle(3/2, 3/2, 3/2); 0.6, 0.4, 0.2\rangle & \langle(0, 13/10, 133/100); 0.7, 0.2, 0.5\rangle & \langle(3/2, 2, 3); 0.5, 0.2, 0.1\rangle & \langle(2, 3, 3); 0.5, 0.2, 0.1\rangle \\ \langle(0, 1/2, 1); 0.6, 0.2, 0.3\rangle & \langle(3/2, 3/2, 3/2); 0.5, 0.3, 0.4\rangle & \langle(1, 11/5, 11/5)0.3, 0.1, 0.5\rangle & \langle(1/2, 11/5, 83/50); 0.7, 0.2, 0.5\rangle \\ \langle(0, 1, 3/2); 0.5, 0.2, 0.1\rangle & \langle(1/2, 4/5, 2); 0.5, 0.2, 0.1\rangle & \langle(3/2, 3/2, 3/2); 0.8, 0.5, 0.3\rangle & \langle(1, 3/2, 2); 0.6, 0.2, 0.3\rangle \\ \langle(1/4, 1, 2); 0.6, 0.2, 0.3\rangle & \langle(1/2, 4/5, 2); 0.5, 0.2, 0.1\rangle & \langle(1, 3/2, 2); 0.5, 0.2, 0.1\rangle & \langle(3/2, 3/2, 3/2); 0.5, 0.3, 0.4\rangle \end{bmatrix}.$$

A collective triangular neutrosophic additive reciprocal preference relation $\tilde{R}$ should be derived by means of an aggregation procedure of each preference relation. The experts should calculate weight of each alternative, and we should use Eq. (19) to combine experts opinions in just one ray.

Suppose that, experts have been determined weights of alternatives as follows:

W $= (0.4, 0.1, 0.2, 0.3)^T$ then,

$$\tilde{R} = \begin{bmatrix} \langle(0.85, 0.85, 1.4); 0.5, 0.3, 0.5\rangle & \langle(1.225, 2, 2.6); 0.3, 0.5, 0.5\rangle & \langle(0.75, 1.5, 1.75); 0.5, 0.4, 0.6\rangle & \langle(1.6, 2.2, 2.4); 0.5, 0.50.6\rangle \\ \langle(1.025, 1.5, 2.2); 0.3, 0.3, 0.6\rangle & \langle(1.05, 1.75, 2.2); 0.3, 0.5, 0.5\rangle & \langle(0.7, 1.3, 1.55); 0.3, 0.5, 0.6\rangle & \langle(1, 1.6, 1.85); 0.5, 0.4, 0.5\rangle \\ \langle(0.675, 1.15, 1.6); 0.5, 0.4, 0.3\rangle & \langle(0.4, 1.07, 1.682); 0.5, 0.3, 0.5\rangle & \langle(1.3, 1.77, 2.32); 0.3, 0.5, 0.5\rangle & \langle(1.5, 2.17, 2.216); 0.5, 0.3, 0.\rangle \end{bmatrix}.$$

The next step is to calculate consensus degree of each alternative according to number of experts gathered in the same opinion ($\varepsilon$):

The weights vector of collective additive reciprocal preference relation are w $= (0.4, 0.3, 0.3),^T$ which is determined by experts. By using Eq. (19), we aggregate neutrosophic numbers in each column to a single valued neutrosophic number. The results are presented in Table 2. By using Eq. (20), we can calculate consensus degree as in Table 3.

As shown in Table 4, we can find that the best alternative is Lenovo with the largest consensus degree, followed by HP, Toshiba and Dell. And results are also illustrated in Fig. 3.

## 5.2 Comparison analysis, merits and contributions of the proposed algorithm

### *5.2.1 Comparison analysis*

The proposed algorithm to group decision making compared with other existing approaches in this subsection.

1. One can see that the preference relations with triangular fuzzy numbers are inconsistent in nature, motivated by the idea in Liu et al. (2016). So in this paper it is very important to define the consistency of triangular neutrosophic additive preference relation.
2. The group decision-making problems with intuitionistic fuzzy preference relations in Urena et al. (2015) check the consistency and calculate consensus degree between decision makers. However, they did not introduce any approach to enhance the consistency. But in our paper, we calculate consistency of preference relations and present a new method for calculating consensus degree. When some preference relations are inconsistent, we also proposed a novel method to improve consistency.
3. There are no approaches for group decision-making problems considering both the additive consistency and consensus degree of neutrosophic preference relations.
4. Thus the problem domain should be precise knowledge, otherwise the people would be handled some uncertainty in assigning the preference evaluation values. This makes the decision making process appear the various characteristics of confirmation, refusal and indeterminacy. So neutrosophic is very important and efficient in dealing with uncertainty and vagueness.
5. In Liao et al. (2015, 2016), the consistency of preference relations checks only after achieving consensus process, but in their work many fractional models existed and

**Table 2** Aggregated values of each alternative by using (NTWAO) according to number of experts gathered in the same opinion

| Experts | Dell | HP | Toshiba | Lenovo |
|---|---|---|---|---|
| $\varepsilon = 1$ | $\langle(0.85, 1.135, 1.7); 0.3, 0.4, 0.6\rangle$ | $\langle(0.925, 1.646, 2.18); 0.3, 0.5, 0.5\rangle$ | $\langle(1.1, 1.5, 1.9); 0.3, 0.5, 0.6\rangle$ | $\langle(1.4, 2, 2.17); 0.5, 0.5, 0.6\rangle$ |
| $\varepsilon = 2$ | $\langle(0.74, 1.36, 3); 0.3, 0.4, 0.6\rangle$ | $\langle(0.979, 2.86, 5); 0.3, 0.5, 0.5\rangle$ | $\langle(0.88, 2.35, 3.6); 0.3, 0.5, 0.6\rangle$ | $\langle(2, 4, 5); 0.5, 0.5, 0.6\rangle$ |
| $\varepsilon = 3$ | $\langle(0.66, 1.7, 5); 0.3, 0.4, 0.6\rangle$ | $\langle(1.1, 5, 12); 0.3, 0.5, 0.5\rangle$ | $\langle(0.9, 3.7, 7); 0.3, 0.5, 0.6\rangle$ | $\langle(3, 8, 11); 0.5, 0.5, 0.6\rangle$ |

**Table 3** Consensus degree according to number of experts gathered in the same opinion

| Experts | Consensus degree (CD) | | | |
|---|---|---|---|---|
| | Dell | HP | Toshiba | Lenovo |
| $\varepsilon = 1$ | 0.6 | 0.8 | 0.7 | 0.97 |
| $\varepsilon = 2$ | 0.8 | 1.4 | 1 | 2 |
| $\varepsilon = 3$ | 1.2 | 2.9 | 2 | 3.8 |

**Table 4** Ranking of alternatives

| Ranking |
|---|
| (1) Lenovo |
| (2) HP |
| (3) Toshiba |
| (4) Dell |

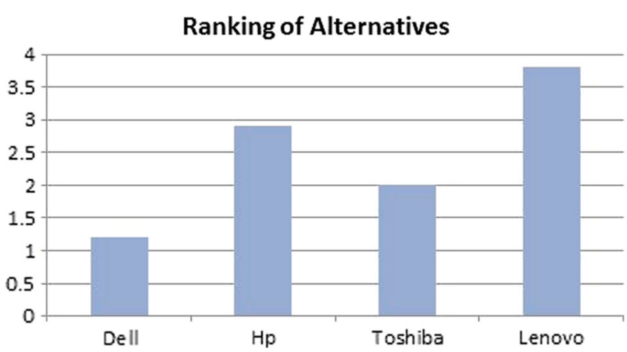


**Fig. 3** Priority of alternatives

needs to solve and so it is a time-consuming process . But in our wok we save time by introducing a new method for repairing preference relations and make it consistent if it is not.

### *5.2.2 Merits and contributions of the proposed algorithm*

1. Using neutrosophic to calculate consistency of additive preference relations for the first time.
2. Using only $(n-1)$ restricted triangular neutrosophic preference values instead of $\frac{n\times(n-1)}{2}$ judgments and it is a time saving process.
3. If additive preference relation is inconsistent, we can easily repair it and make it consistent.
4. A new method for calculating collective additive preference relation and calculating consensus degree according to number of experts gathered in the same opinion.
5. Optimal representation of the problem domain in our proposed computations by considering various features such as truthfulness, falseness, and indeterminacy.
6. We solved an illustrative example to show the efficiency and applicability of our algorithm.

## 6 Conclusion and future works

In order to simulate the uncertainty associated with vagueness in the real world environments, the experts give their judgments in terms of triangular neutrosophic additive reciprocal preference relations. In GDM process, the consistency of preference relations with neutrosophic numbers is important to reflect the rationality of decision makers. A new method has been proposed to construct a complete preference relation with additive approximation consistency from $(n-1)$ restricted triangular neutrosophic preference values. We have further defined the consensus degree among triangular neutrosophic additive reciprocal preference relations. A new algorithm for a group decision-making problem with triangular neutrosophic additive reciprocal preference relations has been presented to show the effectiveness of the new definitions. In the future, we will study the consistency of trapezoidal neutrosophic additive reciprocal preference relations and apply it in different practical problems.

**Acknowledgements** We all want to thank anonymous for the constructive suggestions that improved both the quality and clarity of the research.

**Compliance with ethical standards**

**Conflicts of interest** The authors declare that there is no conflict of interests regarding the publication of this article.

**Human and animal rights** This article does not contain any studies with human participants or animals performed by any of the authors.

**Informed consent** Informed consent was obtained from all individual participants included in the study.

## References


Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20(1):87–96

Benítez J, Delgado-Galván X, Izquierdo J, Pérez-García R (2012) Improving consistency in AHP decision-making processes. Appl Math Comput 219(2):2432–2441

Büyüközkan G, Çifçi G (2012) A new incomplete preference relations based approach to quality function deployment. Inf Sci 206:30–41

Cabrerizo FJ, Pérez IJ, Herrera-Viedma E (2010) Managing the consensus in group decision making in an unbalanced fuzzy linguistic context with incomplete information. Knowl-Based Syst 23(2):169–181

Chiclana F, Herrera F, Herrera-Viedma E (2001) Integrating multiplicative preference relations in a multipurpose decision-making model based on fuzzy preference relations. Fuzzy Sets Syst 122(2):277–291

Dubois D (2011) The role of fuzzy sets in decision sciences: old techniques and new directions. Fuzzy Sets Syst 184(1):3–28

Fan ZP, Ma J, Jiang YP, Sun YH, Ma L (2006) A goal programming approach to group decision making based on multiplicative preference relations and fuzzy preference relations. Eur J Oper Res 174(1):311–321

Gong ZW (2008) Least-square method to priority of the fuzzy preference relations with incomplete information. Int J Approx Reason 47(2):258–264

Herrera-Viedma E, Herrera F, Chiclana F, Luque M (2004) Some issues on consistency of fuzzy preference relations. Eur J Oper Res 154(1):98–109

Herrera-Viedma E, Chiclana F, Herrera F, Alonso S (2007) Group decision-making model with incomplete fuzzy preference relations based on additive consistency. IEEE Trans Syst Man Cybern B (Cybern) 37(1):176–189

Jiang Y, Xu Z, Gao M (2015) Methods for ranking intuitionistic multiplicative numbers by distance measures in decision making. Comput Ind Eng 88:100–109

Kaufmann A, Gupta MM (1988) Fuzzy mathematical models in engineering and management science. Elsevier Science Inc., Amsterdam

Liao H, Xu Z, Zeng XJ, Merigó JM (2015) Framework of group decision making with intuitionistic fuzzy preference information. IEEE Trans Fuzzy Syst 23(1):1211–1227

Liao H, Xu Z, Zeng XJ, Xu DL (2016) An enhanced consensus reaching process in group decision making with intuitionistic fuzzy preference relations. Inf Sci 329:274–286

Liu F (2009) Acceptable consistency analysis of interval reciprocal comparison matrices. Fuzzy Sets Syst 160(18):2686–2700

Liu P, Wang Y (2014) Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Neural Comput Appl 25(7–8):2001–2010

Liu F, Zhang WG, Wang ZX (2012) A goal programming model for incomplete interval multiplicative preference relations and its application in group decision-making. Eur J Oper Res 218(3):747–754

Liu F, Zhang WG, Fu JH (2012) A new method of obtaining the priority weights from an interval fuzzy preference relation. Inf Sci 185(1):32–42

Liu F, Zhang WG, Zhang LH (2014) Consistency analysis of triangular fuzzy reciprocal preference relations. Eur J Oper Res 235(3):718–726

Liu F, Pedrycz W, Zhang WG (2016) Limited rationality and its quantification through the interval number judgments with permutations. IEEE Trans Cybern 99:1–13

Meng F, Chen X, Zhang Y (2016) Consistency-based linear programming models for generating the priority vector from interval fuzzy preference relations. Appl Soft Comput 41:247–264

Meng F, Tan C, Chen X (2017) Multiplicative consistency analysis for interval fuzzy preference relations: A comparative study. Omega 68:17–38

Meng F, Lin J, Tan C, Zhang Q (2017) A new multiplicative consistency based method for decision making with triangular fuzzy reciprocal preference relations. Fuzzy Sets Syst 315:1–25

Pedrycz W, Song M (2011) Analytic hierarchy process (AHP) in group decision making and its optimization with an allocation of information granularity. IEEE Trans Fuzzy Syst 19(3):527–539

Peneva V, Popchev I (2007) Aggregation of fuzzy preference relations to multicriteria decision making. Fuzzy Optim Decis Mak 6(1):351–365

Saaty TL (2008) Decision making with the analytic hierarchy process. Int J Serv Sci 1(1):83–98

Saaty TL, Vargas LG (1987) Uncertainty and rank order in the analytic hierarchy process. Eur J Oper Res 32(1):107–117

Samuel OW, Asogbon GM, Sangaiah AK, Fang P, Li G (2017) An integrated decision support system based on ANN and Fuzzy_AHP for heart failure risk prediction. Expert Syst Appl 68:163–172

Sangaiah A, Thangavelu A (2013) An exploration of FMCDM approach for evaluating the outcome/success of GSD projects. Open Eng 3(3):419–435

Sangaiah AK, Gopal J, Basu A, Subramaniam PR (2017) An integrated fuzzy DEMATEL, TOPSIS, and ELECTRE approach for evaluating knowledge transfer effectiveness with reference to GSD project outcome. Neural Comput Appl 28(1):111–123

Smarandache F (2004) A geometric interpretation of the neutrosophic set—a generalization of the intuitionistic fuzzy set. arXiv preprint arXiv:math/0404520

Smarandache F (2005) A unifying field in logics: neutrosophic logic. neutrsophic logic. neutrosophy, neutrosophic set, neutrosophic probability. infinite study, neutrosophy, neutrosophic set, neutrosophic probability, American Research Press. ISBN 978-1-59973-080-6

Tanino T (1984) Fuzzy preference orderings in group decision making. Fuzzy Sets Syst 12(2):117–131

Urena R, Chiclana F, Fujita H, Herrera-Viedma E (2015) Confidence-consistency driven group decision making approach with incomplete reciprocal intuitionistic preference relations. Knowl-Based Syst 89:86–96

Wang ZJ (2015) Consistency analysis and priority derivation of triangular fuzzy preference relations based on modal value and geometric mean. Inf Sci 314:169–183

Wang TC, Chen YH (2007) Applying consistent fuzzy preference relations to partnership selection. Omega 35(1):384–388

Wang TC, Chen YH (2008) Applying fuzzy linguistic preference relations to the improvement of consistency of fuzzy AHP. Inf Sci 178(19):3755–3765

Wang ZJ, Tong X (2016) Consistency analysis and group decision making based on triangular fuzzy additive reciprocal preference relations. Inf Sci 361:29–47

Wang YM, Yang JB, Xu DL (2005) Interval weight generation approaches based on consistency test and interval comparison matrices. Appl Math Comput 167(1):252–273

Wu Z, Xu J (2012) A consistency and consensus based decision support model for group decision making with multiplicative preference relations. Decis Support Syst 52(3):757–767

Xu Z (2007) Intuitionistic fuzzy aggregation operators. IEEE Trans Fuzzy Syst 15(3):1179–1187

Xu Z, Chen J (2008) Group decision-making procedure based on incomplete reciprocal relations. Soft Comput A Fus Found Methodol Appl 12(3):515–521
Xu Z, Chen J (2008) Some models for deriving the priority weights from interval fuzzy preference relations. Eur J Oper Res 184(1):266–280
Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353