

# Reduced order modelling of linear time-invariant system using modified cuckoo search algorithm

Afzal Sikander<sup>1</sup>  · Padmanabh Thakur<sup>1</sup>

Published online: 3 April 2017  
© Springer-Verlag Berlin Heidelberg 2017

**Abstract** In this paper modified cuckoo search (MCS) algorithm is considered to develop reduced order model (ROM) of higher-order linear time-invariant systems. Firstly, the MCS algorithm has been employed to minimize the integral square error (ISE) between original and proposed ROM to obtain its unknown coefficients. Five systems of different order are considered to obtain their reduced order model. Finally, various performance indices, such as ISE, integral of absolute and integral of time multiplied by absolute error, have been estimated to reveal the efficacy of the proposed model. Also, time and frequency response characteristics of original higher-order model are compared with the proposed MCS-based and some of other existing techniques-based ROM available in the literature. Furthermore, the results are compared in terms of time response specifications such as rise time ( $t_r$ ) in second, settling time ( $t_s$ ) in second and maximum peak overshoot ( $M_p$ ) in percentage. It is revealed that the response of the proposed MCS-based ROM is much closer to the response of the original higher-order system.

**Keywords** Modified cuckoo search algorithm · Optimization · Order reduction · Integral square error

---

Communicated by V. Loia.

---

✉ Afzal Sikander  
afzal.sikander@hotmail.com  
Padmanabh Thakur  
tonu\_arth@rediffmail.com

<sup>1</sup> Department of Electrical Engineering, Graphic Era University, Dehradun, India

## 1 Introduction

In control system design, most of the systems are complex from analysis point of view. Such systems need to be replaced by a simpler system which would be the replica of the original system. The process of obtaining simpler system from high-order system is named as reduced order modelling. Therefore, this simpler/lower-order model is used to analyse behaviour of higher-order model by which simulation time reduces and design work becomes easier. In recent years, model order reduction has been explored extensively by many researches. Recently, [Ghosh and Senroy \(2013\)](#) suggested a technique for MOR which is based on balanced truncation technique and [Sikander and Prasad \(2015b\)](#) presented a simple technique for MOR in time domain using improved Hermite normal form. Also, [Desai and Prasad \(2013a\)](#) proposed a new technique for LTI systems which utilize Routh approximation (RA) ([Hutton and Friedland 1975](#)) and Big Bang–Big Crunch (BB-BC) optimization ([Erol and Eksin 2006](#)) for reduced order modelling. They have synthesized the reduced system by calculating the denominator polynomial using RA to preserve the stability, whereas the numerator polynomial is evaluated using BB-BC algorithm. Several techniques for reduced order modelling are reported in the literature ([Sambariya and Arvind 2016](#); [Biradar et al. 2016](#); [Sikander et al. 2016](#); [Sikander and Prasad 2017](#)).

The optimization approach is not new in the field of system engineering. Different authors have considered different approaches such as minimization of integral square of impulse response error ([Wilson 1970](#)), minimization of equation error ([Obinata and Inooka 1983](#)), minimizing weighted equation error ([Eitelberg 1981](#)) or minimization of  $L_1$  and  $L_\infty$  norm ([El-Attar and Vidyasagar 1978](#)).

Recently, nature-inspired optimization algorithms are being used for reduced order modelling. Most popular search

algorithm is genetic algorithm (GA) developed by Goldberg (1989) and particle swarm optimization (PSO) developed by Kennedy and Eberhart (1995). PSO is based on the common behaviour of some birds in a group. Harmony search (HS) algorithm suggested by Lee and Geem (2004) and Erol and Eksin (2006) proposed a new search method, namely Big Bang–Big Crunch (BB-BC) algorithm which is based on the theory of Universe. The application of this search algorithm is not limited to any specific field. Parmar et al. (2007a, d) used GA and PSO for reduced order modelling of liner SISO systems, respectively, whereas reduced order modelling for MIMO system using PSO is given by Abu-Al-Nadi et al. (2011). Different fitness functions are used by different authors such as ISE (Panda et al. 2009), IAE (Parmar et al. 2009) and norms (Salim and Bettayeb 2011).

Furthermore, a number of mixed methods are suggested by researchers in the literature for reduced order modelling. The concept of preserving the stability of the reduced system is being used in these mixed methods. Often, stability of the reduced model is achieved by reducing the denominator polynomial using stability preserving methods (Parmar et al. 2007c; Sikander and Prasad 2015c; Vishwakarma and Prasad 2008). Recently, stability equation method is being used to calculate stable denominator of the reduced model, and then PSO is applied to calculate numerator polynomial (Sikander and Prasad 2015a). Therefore, efficacy of the new techniques for reduced order modelling is in demand nowadays as it reduces the hardware complexity, cost of the system and computation time etc.

So this paper contributes a novel technique for order reduction in LTI systems which is based on modified cuckoo search algorithm (Walton et al. 2011). The proposed technique always provides stable reduced system for original stable system. The optimization approach is used to obtain reduced system by means of minimizing the ISE values as fitness function. The results obtained in this paper are compared with recently published work in terms of performance indices.

## 2 Description of the problem

Let the higher-order system is given as follows:

$$G_k(s) = \frac{N_{k-1}(s)}{D_k(s)} = \frac{\sum_{i=0}^{k-1} b_i s^i}{\sum_{i=0}^k a_i s^i} \quad (1)$$

where  $a_i$  and  $b_i$  are constants, whereas for unity steady-state output  $a_0 = b_0$ .

The objective is to obtain the reduced system of order ' $r$ ' ( $r < k$ ) such that it preserve all necessary characteristics of the higher-order system and represented as follows:

$$R_r(s) = \frac{N_{r-1}(s)}{D_r(s)} = \frac{\sum_{i=0}^{r-1} d_i s^i}{\sum_{i=0}^r c_i s^i} \quad (2)$$

where  $c_i$  and  $d_i$  are unknown constants.

## 3 Modified cuckoo search algorithm

Cuckoo search (CS) is an optimization technique/algorithm which is inspired by nature and recently developed by Yang and Deb (2008). This technique is based on the common behaviour of the cuckoo bird. All the cuckoo birds lay down their eggs into the other bird's nest for fertilization. It is possible that the other birds may recognize that it is not their eggs and then either they throw the eggs or form a new nest at new place which results in the evolution of cuckoo eggs (Yang and Deb 2010).

A set of host nest shows the cuckoo breeding analogy. Each nest carries an egg which is considered as a solution. A new nest is formed using Lévy flight (Brown et al. 2007; Viswanathan 2010), i.e. random walk. Success of resources random searches can be optimized using the Lévy flight movements (Humphries et al. 2012).

Yang and Deb (2008) gives the following three rules to combine the apply cuckoo species with the Lévy flight:

- For laying down the egg, the nest should be selected at random and dump by every cuckoo.
- The nest must be transferred to the next generation if good quality eggs are found in it.
- The probability of an alien egg which can be observed by the fixed number of host nest is  $p_a \in [0, 1]$ . In such cases the host nest either throws the alien egg or forms a new nest at any other place.

For easiness, the fraction of  $p_a$  of  $n$  nests, which are interchanged new nests, is considered the approximation of previous assumption.

The Lévy flight can be represented by the following relation for the generation of new solution  $Y^{(t+1)}$  of cuckoo  $i$  (Yang and Deb 2010)

$$Y_i^{(t+1)} = Y_i^{(t)} + a \otimes \text{Lévy}(\lambda) \quad (3)$$

where  $a$  ( $a > 0$ ) is a step size which is related to the level of the problem optimized by the technique. The random step size of the Lévy flights is calculated as follows:

$$\text{Lévy } \tilde{u} = t^{-\lambda}; \quad (1 < \lambda \leq 3) \quad (4)$$

Generally, the problems encountered in the CS algorithm have been modified in order to obtain effective solution. The worst nests probability  $p_a$  and the step size  $a$  used in the CS

algorithm were taken as constant which may lead to more number of iteration for optimal solution. The combination of  $p_a$  and  $a$  may provide the better solution comparatively (Walton et al. 2011). In this paper, the reduced system is achieved by modified cuckoo search algorithm assuming  $p_a$  and  $a$  as constraint for the optimization problem such that  $0.001 < Pa < 1$  and  $0.01 < a < 0.6$ .

### 4 Proposed methodology

The theory of optimization is being employed here to calculate the reduced numerator and denominator by minimizing the predefined fitness function. Although the reduced system may be any type/order, for simplification the following reduced system is considered in this paper.

$$R_2(s) = \frac{d_0 + d_1s}{c_0 + c_1s + c_2s^2} \tag{5}$$

where  $d_0, d_1, c_0, c_1$  and  $c_2$  are unknown coefficients of numerator and denominator, respectively, which are to be determined.

Therefore, the proposed algorithm is used to achieve the best values of the coefficients in Eq. 5, for reduce system, by minimizing the performance index described by the following equation

$$ISE(Z) = \int_0^\infty [y_1(t) - y_2(t)]^2 dt \tag{6}$$

Furthermore, to analyse the performance of the reduced system as compared to original system and the systems which are reduced by the other methods, the following performance indices are used here.

$$IAE = \int_0^\infty |y_1(t) - y_2(t)| dt \tag{7}$$

$$ITAE = \int_0^\infty t |y_1(t) - y_2(t)| dt \tag{8}$$

$$IRE = \int_0^\infty g^2(t) dt \tag{9}$$

where IAE, ITAE and IRE tend for integral of absolute error, integral of time multiplied absolute error and impulse response energy, respectively.  $y_1(t)$  and  $y_2(t)$  are the step responses of system under consideration and reduced order system obtained by proposed technique, respectively.  $g(t)$  is the impulse response of the system.

The steps are as follows to obtain reduced order system from any randomly given high-order system using new proposed technique.

*Step 1* Specify the fitness function as given in Eq. 6 and the number of chosen variables (say  $q$ ) as per Eq. 2 along with their range. Set the probability of the worst nests and step size also. Initializing a population of  $p$  host nests, then problem is summarized as

Minimize fitness function, subject to  $d_{iL} < d_i < d_{iU}$ ;  $c_{iL} < c_i < c_{iU}$ ;  $0.001 < Pa < 1$  and  $0.01 < a < 0.6$  where  $d_{iL}, c_{iL}$  and  $d_{iU}, c_{iU}$  are the lowest and highest values of the chosen variables, respectively, and  $i = 0, 1, \dots, q$ .

*Step 2* Obtain the value of  $Z_\alpha$  for a randomly selected cuckoo ( $\alpha$ ) and select a nest ( $\beta$ ) randomly among  $p$ .

*Step 3* if ( $Z_\alpha > Z_\beta$ ), then interchange  $\beta$  by the current obtained solution else go to *Step 4*.

*Step 4* Check whether the predefined stopping criterion is arrived or the maximum generation occurred or not; if yes, then the solution obtained in the current generation would be the best solution else go to *step 5*.

*Step 5* Abandon a fraction of worse nests with optimal value of probability  $p_a$  and step size  $a$ .

*Step 6* Using Eq. 3 the obtained solution must be updated by calculating  $Y_i^{(t+1)}$  and repeat this algorithm, until the predefined condition is arrived or the maximum generation occurred.

### 5 Numerical examples and results

*Example 1* A fourth-order system is chosen to reduce by the proposed technique which is previously considered by Desai and Prasad (2013a) and is given as follows:

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}$$

Using proposed technique, as discussed above, the achieved second-order reduced system is as follows:

$$R_2(s) = \frac{0.77s + 1.649}{s^2 + 2.548s + 1.649}$$

and the system reduced by RA and BB-BC (Desai and Prasad 2013a) is as follows:

$$R_2(s) = \frac{0.8058s + 0.7944}{s^2 + 1.65s + 0.7944}$$

whereas the same system was considered by Alsmadi et al. (2011) and reduced by GA-MOR method which is as follows:

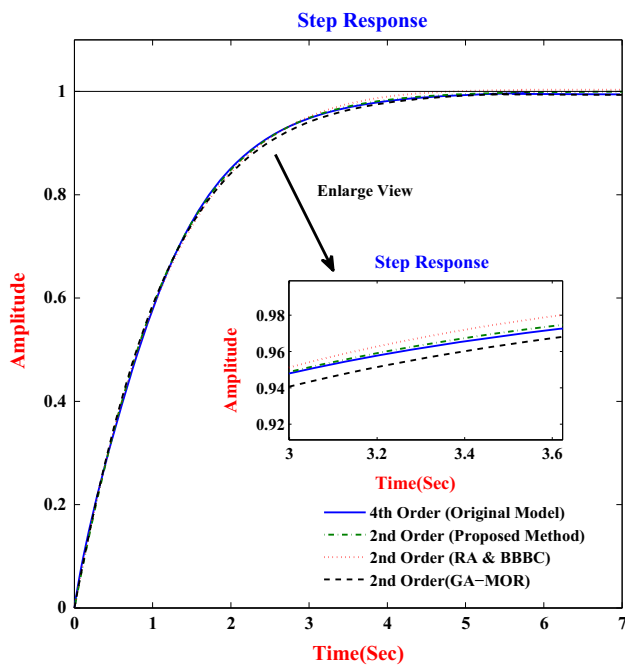


Fig. 1 Time response of original and reduced order model for Example 1

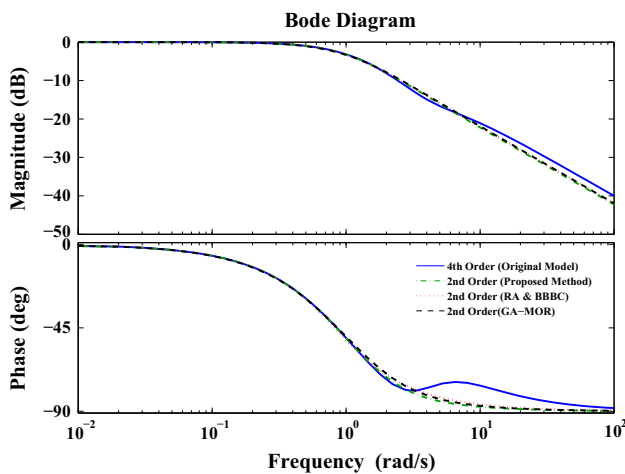


Fig. 2 Bode plots of original and reduced order model for Example 1

$$R_2(s) = \frac{0.4s + 1}{0.5s^2 + 1.5s + 1}$$

The step responses and bode plots of original system, reduced order model obtained by proposed method (MCSA), RA and BB-BC (Desai and Prasad 2013a) and GA-MOR method (Alsmadi et al. 2011) are depicted in Figs. 1 and 2, respectively. It is clearly observed that the reduced system obtained by proposed MCSA offers excellent close approximation to the system under consideration. Also, the comparative analysis of reduced system achieved by pro-

posed technique and those obtained by alternative technique in terms of ISE, IAE and ITAE values is depicted in Table 1 for Example 1. It is noticed that the proposed technique exhibits the ISE value, i.e.  $7.668 \times 10^{-5}$  which is much lesser than the values obtained by other methods and recently published work, i.e.  $2.18 \times 10^{-4}$  by Desai and Prasad (2013a) and  $2.3785 \times 10^{-4}$  by Alsmadi et al. (2011). Hence, it is clearly noticed that the proposed technique performs well as compared to other well-known methods. The bar chart of performance indices of different reduced systems is depicted in Fig. 3 from which the efficacy and powerfulness of the proposed technique in terms of ISE, IAE and ITAE values are observed. Furthermore, the performance of proposed algorithm for Example 1 is shown in Fig. 4.

Example 2 The second system which is to be reduced is an eighth-order system considered from Alsmadi et al. (2011), where Genetic algorithm approach with frequency selectivity is used to obtain reduced system. The system is given as follows:

$$G_8(s) = \frac{18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320}$$

By following the steps to reduce a system as discussed in Sect. 4, the reduced second-order model is given as:

$$R_2(s) = \frac{16.39s + 4.865}{s^2 + 6.627s + 4.865}$$

The same example has been taken into consideration by Alsmadi et al. (2011) and reduced model obtained by GA-MOR method is given as:

$$R_2(s) = \frac{16.9686s + 15.2295}{s^2 + 6.8996s + 15.2295}$$

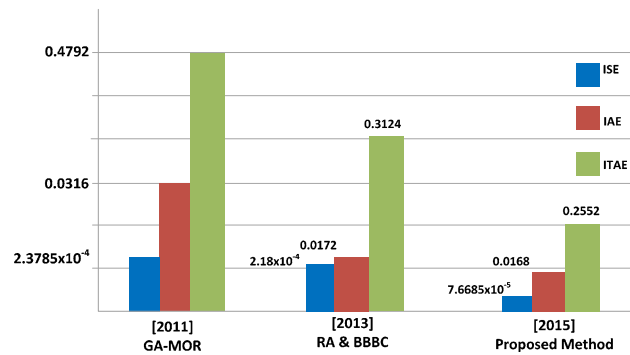
whereas the same example is also considered by Parmar et al. (2007c), where factor division algorithm and Eigen spectrum analysis is employed to obtain reduced system which is given as:

$$R_2(s) = \frac{24.11429s + 8}{s^2 + 9s + 8}$$

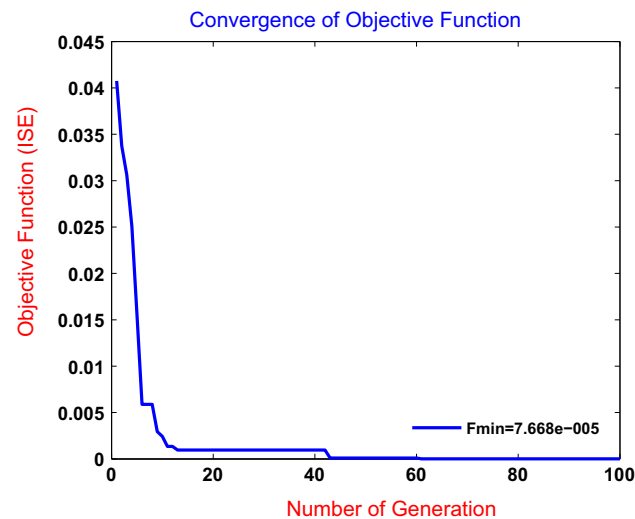
The time response and bode plots of original system and reduced system achieved by proposed technique (MCSA), GA-MOR method (Alsmadi et al. 2011) and factor division algorithm along with Eigen spectrum analysis (FDA and ESA) (Parmar et al. 2007c) are plotted in Figs. 5 and 6, respectively. It is observed that the response of reduced system achieved by proposed technique is much closer than

**Table 1** Performance analysis of proposed and existing reduction techniques for Example 1

Performance parameter	Original model	Proposed method	RA and BB-BC (Desai and Prasad 2013a)	GA-MOR (Alsmadi et al. 2011)	FDA and ESA (Parmar et al. 2007c)
ISE	–	$7.668 \times 10^{-5}$	$2.18 \times 10^{-4}$	$2.3785 \times 10^{-4}$	$2.64337 \times 10^{-4}$
IAE	–	0.0168	$1.52 \times 10^{-2}$	0.0316	0.0261
ITAE	–	0.2552	$1.12 \times 10^{-2}$	0.4794	0.3966
IRE	0.4518	0.4519	0.4585	0.4569	0.4516
$t_r$ (s)	2.2602	2.2568	2.2787	2.3413	2.2645
$t_s$ (s)	3.9307	3.8284	3.6199	4.0916	4.0177
$M_p$ (s)	0	0	0.2738	0	0

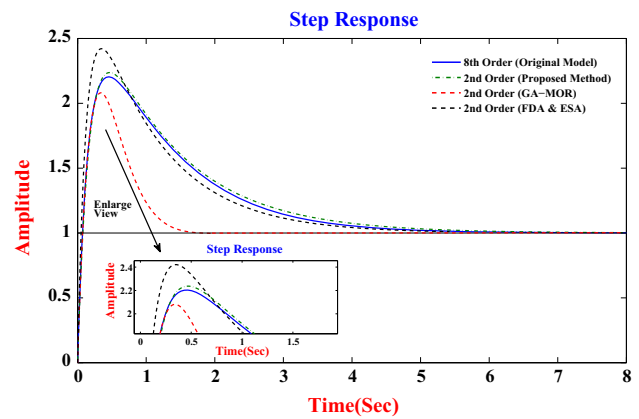


**Fig. 3** Bar chart of performance indices of reduced models for Example 1

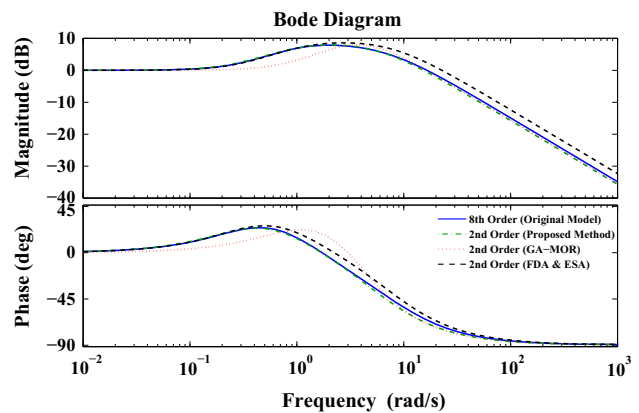


**Fig. 4** Performance of MCS algorithm for Example 1

the responses of reduced model obtained by alternative methods. In addition, the ISE, IAE and ITAE values of reduced models are also tabulated in Table 2 for example 2. Using the proposed method, the ISE value for Example 2 is found to be  $3.2978 \times 10^{-4}$ , whereas the lowest value of ISE, using the other well-known methods is only  $7.4277 \times 10^{-4}$ ,



**Fig. 5** Time response of original and reduced order systems for Example 2

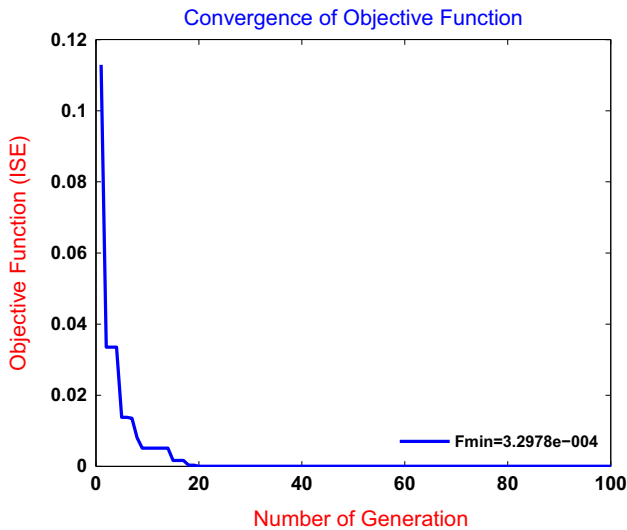


**Fig. 6** Bode plots of original and reduced order systems for Example 2

1.7924, and  $4.8090 \times 10^{-2}$  (Alsmadi et al. 2011; Parmar et al. 2007b,c), respectively. So, it is noticed that the ISE, IAE and ITAE value for proposed method is much lesser as compared to other methods. Also, the performance of proposed algorithm for Example 2 is shown in Fig. 7. Hence, the proposed method exhibits the better performance comparatively.

**Table 2** Performance analysis of proposed and existing reduction techniques for Example 2

Performance parameter	Original model	Proposed method	GA-MOR (Alsmadi et al. 2011)	FDA and ESA (Parmar et al. 2007c)	ESA and PA (Parmar et al. 2007b)
ISE	–	$3.2978 \times 10^{-4}$	$7.4277 \times 10^{-4}$	$4.8090 \times 10^{-2}$	1.7924
IAE	–	0.1378	1.2286	0.3007	0.3007
ITAE	–	2.6839	23.9325	5.8571	5.8571
IRE	23.0367	21.8760	24.2248	34.6928	34.6928
$t_r$ (s)	0.0569	0.0614	0.0583	0.0409	0.0409
$t_s$ (s)	4.8201	5.3070	1.5008	4.3942	4.3942
$M_p$ (%)	120.3504	123.6984	108.0844	142.1034	142.1439



**Fig. 7** Performance of MCS algorithm for Example 2

*Example 3* To show the powerfulness of the proposed technique, the third system considered in this paper is a tenth-order rational approximation of a thermal diffusion model (Parmar et al. 2007c) represented by the following transfer function

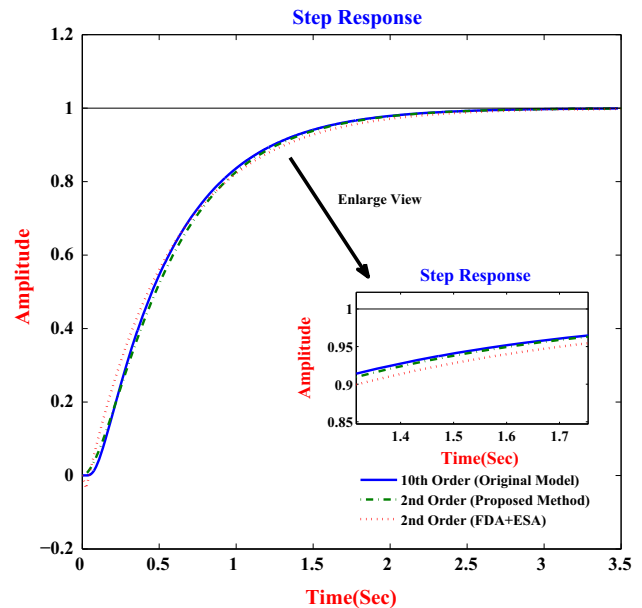
$$G_{10}(s) = \frac{540.70748 \times 10^{17}}{\prod_{i=1}^{10} (s + \lambda_i)}$$

where

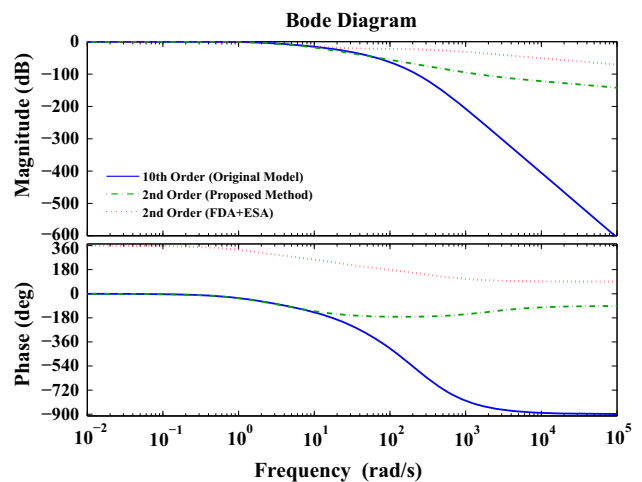
$$\begin{aligned} \lambda_1 &= 2.04, \quad \lambda_2 = 18.3, \quad \lambda_3 = 50.13, \quad \lambda_4 = 95.15, \\ \lambda_5 &= 148.85, \quad \lambda_6 = 205.16, \quad \lambda_7 = 252.21, \\ \lambda_8 &= 298.03, \quad \lambda_9 = 320.97, \quad \lambda_{10} = 404.16. \end{aligned}$$

By following the steps to reduce a system as discussed in Sect. 4, the reduced second-order model is given as:

$$R_2(s) = \frac{0.007842s + 16.06}{s^2 + 9.868s + 16.06}$$



**Fig. 8** Step response of original and reduced order model for Example 3



**Fig. 9** Frequency response of original and reduced order model for Example 3



**Table 3** Performance analysis of proposed and existing reduction techniques for Example 3

Performance parameter	Original model	Proposed method	FDA and ESA (Parmar et al. 2007c)	ESA and PA (Parmar et al. 2007b)	LSMR (Edgar 1975)
ISE	–	$1.5337 \times 10^{-4}$	$1.5 \times 10^{-3}$	1.3646	$4.3986 \times 10^{-4}$
IAE	–	0.0204	0.0456	0.0456	0.0581
ITAE	–	0.1613	0.3611	0.3610	0.4600
IRE	0.9031	0.9037	2.0875	2.0856	0.9045
$t_r$ (s)	1.0901	1.1304	1.2153	1.2153	1.0884
$t_s$ (s)	2.0314	2.0512	2.1743	2.1743	2.0124
$M_p$ (%)	0	0	0	0	0

with  $ISE = 1.5337 \times 10^{-4}$ , whereas the same system is reduced by FDA and ESA (Parmar et al. 2007c) and given as follows:

$$R_2(s) = \frac{-28.3902s + 647.6004}{s^2 + 359.999s + 647.6004}$$

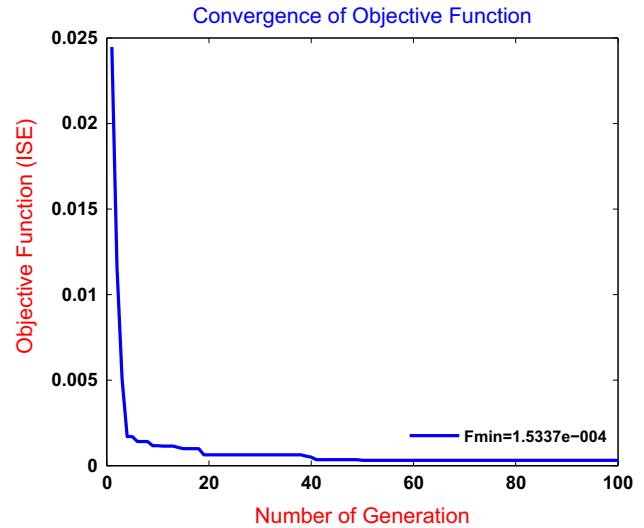
having  $ISE = 1.5 \times 10^{-3}$ ; here it is noticed that although the reduced system given by Parmar et al. (2007c) is stable but it is of non-minimum phase system.

Another method is proposed by Parmar et al. (2007b) in which the same system is reduced by using Eigen spectrum analysis and Pade approximation technique (ESA and PA). The ISE value for this system is given as 1.3646, and the transfer function of the reduced order model is given as:

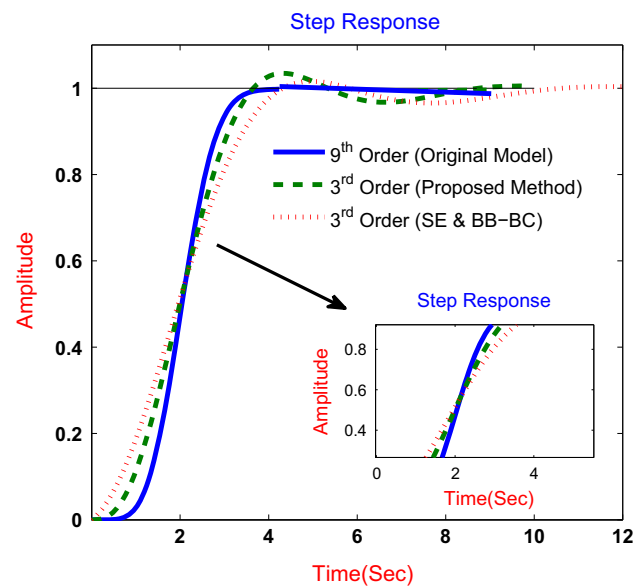
$$R_2(s) = \frac{-28.367s + 647.60193}{s^2 + 359.999s + 647.60193}$$

In this case the reduced order model is stable and almost same but it is also a non-minimum phase system, whereas the new proposed method always exhibits stable as well as minimum phase system. The responses of the reduced system achieved by proposed technique are much closer to the responses of the original system as depicted in Figs. 8 and 9. The efficacy of the proposed technique is clearly observed in Table 3 which presents the performance analysis of proposed and existing reduction techniques for Example 3. Furthermore, the performance of proposed algorithm is shown in Fig. 10 for this example. Hence, the new proposed method is better as compared to other well-known methods and recently published work.

*Example 4* Let us consider a ninth-order system (Desai and Prasad 2013b) having complex roots and described by the following transfer function.



**Fig. 10** Performance of MCS algorithm for Example 3



**Fig. 11** Step response of original and reduced order models for Example 4

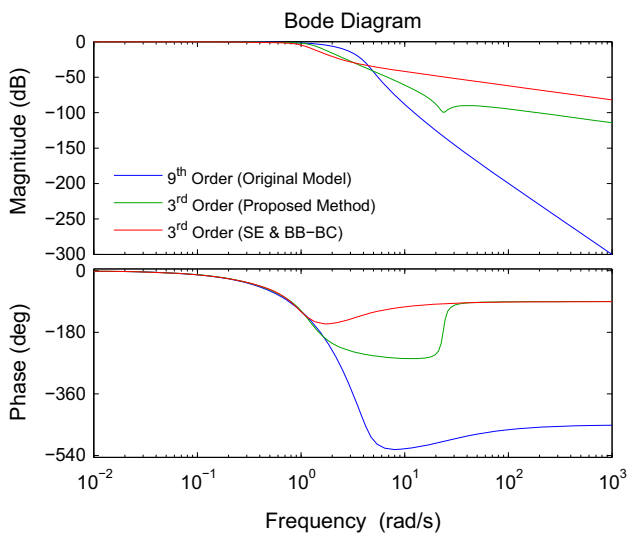


Fig. 12 Bode plot of original and reduced order models for Example 4

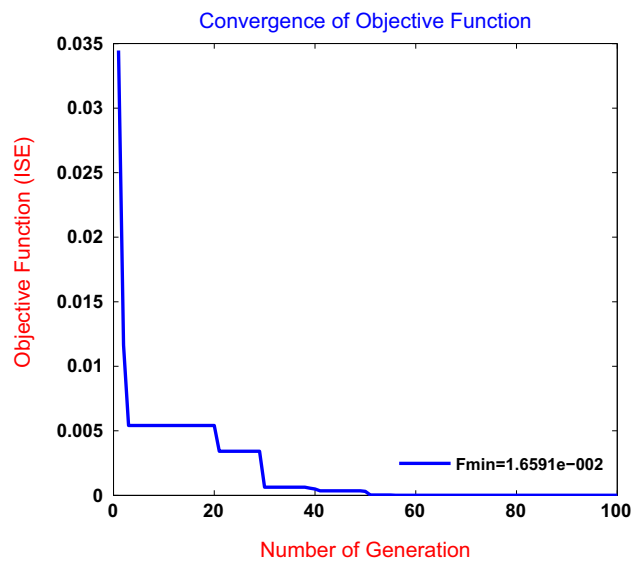


Fig. 13 Performance of MCS algorithm for Example 4

$$G_9(s) = \frac{s^4 + 35s^3 + 291s^2 + 1093s + 1700}{s^9 + 9s^8 + 66s^7 + 294s^6 + 1029s^5 + 2541s^4 + 4684s^3 + 5856s^2 + 4620s + 1700}$$

Reduced System obtained by proposed technique is given as

$$R_3(s) = \frac{0.001935s^2 + 0.005725s + 1.073}{s^3 + 1.681s^2 + 2.183s + 1.073}$$

and the reduced model obtained by stability equation method and Big Bang–Big Crunch (SE and BB-BC) algorithm (Desai and Prasad 2013b) is given as

$$R_3(s) = \frac{0.0789s^2 + 0.3142s + 0.493}{s^3 + 1.3s^2 + 1.34s + 0.493}$$

Figures 11 and 12 show the step response and Bode plot of original system, the reduced order model obtained by proposed method (MCS) and stability equation method along with Big Bang–Big Crunch (SE and BB-BC) algorithm

(Desai and Prasad 2013b), respectively. It is seen that the obtained reduced order model provides better close approximation to the original model as compared to SE and BB-BC method. Table 4 shows the comparison of ISE, IAE and ITAE values and time response specifications for different reduction methods for Example 4, and it is noticed that the proposed method is much better than other methods. The performance of proposed algorithm is shown in Fig. 13 also.

Example 5 Let us consider another example having repeated poles as

$$G_4(s) = \frac{1}{(s + 1)^4}$$

Table 4 Performance analysis of proposed and existing reduction techniques for Example 4

Performance parameter	Original model	Proposed method	SE and BB-BC (Desai and Prasad 2013b)	MPC and GA (Vishwakarma and Prasad 2009)	RMT (Mukherjee et al. 2005)
ISE	–	$1.6591 \times 10^{-2}$	$2.252 \times 10^{-2}$	$5.86 \times 10^{-2}$	$8.77 \times 10^{-2}$
IAE	–	0.2982	0.4917	0.2060	0.9359
ITAE	–	4.5968	7.5786	13.1748	14.4252
IRE	0.4705	0.3465	0.2686	0.6974	0.5085
$t_r$ (s)	1.5390	2.1457	2.7708	2.6033	2.9214
$t_s$ (s)	3.3554	7.6152	9.0779	5.1518	6.9056
$M_p$ (%)	0	1.7534	1.6015	0	0



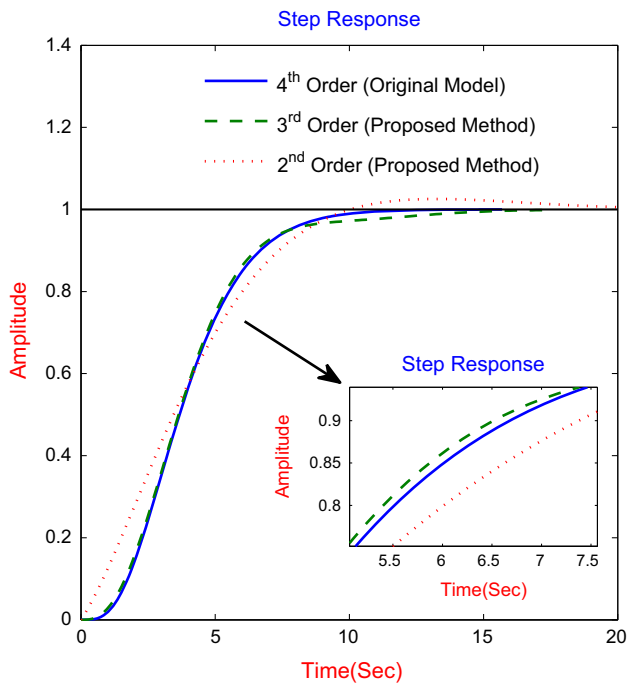


Fig. 14 Step response of original and reduced order models for Example 5

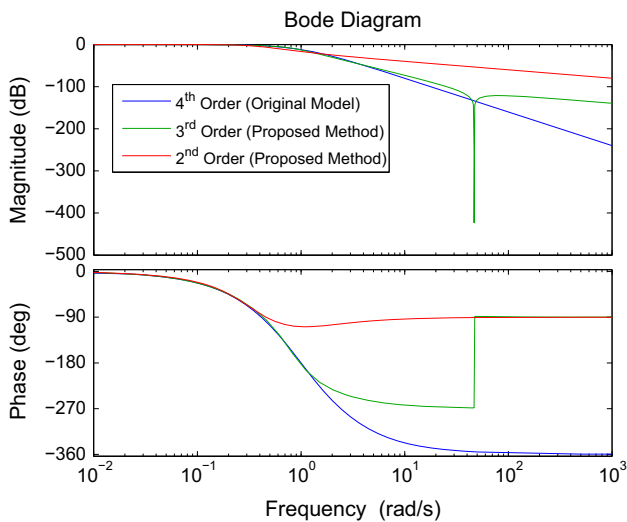


Fig. 15 Bode plot of original and reduced order models for Example 5

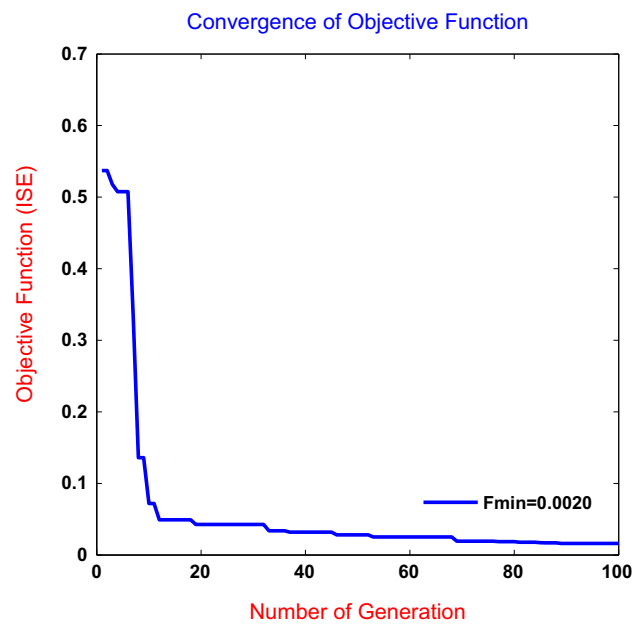


Fig. 16 Performance of MCS algorithm for Example 5

The reduced third- and second-order systems obtained by proposed method are as

$$R_3(s) = \frac{0.0001064s^2 + 0.2325}{s^3 + 1.238s^2 + 0.9371s + 0.2325}$$

$$R_2(s) = \frac{0.1s + 0.1158}{s^2 + 0.5202s + 0.1158}$$

Figures 14 and 15 show the step response and Bode plot of original and reduced order models (third and second order), respectively. The closeness of step responses of original and reduced order models reveals that the third-order model provides better approximation, to the original model having repeated poles, as compared to second-order model. Table 5 depicts the comparison of various performance measures and time response specifications of original and reduced order models for Example 5. The performance of proposed algorithm is shown in Fig. 16 also. Therefore, it is observed that the proposed third-order model exhibits better performance.

Table 5 Performance analysis of proposed reduction method for Example 5

Original/reduced model	ISE	IAE	ITAE	IRE	$t_r$ (s)	$t_s$ (s)	$M_p$ (%)
Original model (fourth order)	–	–	–	0.1562	4.9364	9.0842	0
Proposed method (third order)	0.0020	0.1689	6.5492	0.1551	4.8686	11.6140	0
Proposed method (second order)	0.0458	0.6967	0	0.1219	6.5553	15.6311	2.5588

## 6 Conclusion

In this paper, a novel technique of reduced order modelling of higher-order LTI systems, based on MCS algorithm, has been presented. In the proposed approach, the coefficients of reduced order model have been estimated by minimizing ISE between original and reduced order model. Five higher-order systems have been considered to achieve their reduced order model with proposed MCS and existing RA and BB-BC, GA, FDA and ESA, LSMR, ESA and PA-based algorithms. It is observed through time and frequency responses that the proposed MCS-based reduced order model gives more accurate and stable performance than the others. Further, the values of ISE, IAE, ITAE obtained with proposed model are found lower than the other existing reduced order models.

### Compliance with ethical standards

**Conflict of interest** All authors declare that they have no conflicts of interest.

**Ethical approval** This article does not contain any studies with human participants or animals performed by any of the authors.

## References

- Abu-Al-Nadi DI, Alsmadi OMK, Abo-Hammour ZS (2011) Reduced order modeling of linear MIMO systems using particle swarm optimization. In: 7th international conference on autonomic and autonomous systems, Venice, Italy, pp 62–66
- Alsmadi OMK, Abo-Hammour ZS, Al-Smadi AM, Abu-Al-Nadi DI (2011) Genetic algorithm approach with frequency selectivity for model order reduction of MIMO systems. *Math Comput Model Dyn Syst* 17(2):163–181. doi:10.1080/13873954.2010.540806
- Biradar S, Hote YV, Saxena S (2016) Reduced-order modelling of linear time invariant systems using big bang big crunch optimization and time moment matching method. *Appl Math Model* 40(15–16):7225–7244
- Brown CT, Liebovitch LS, Glendon R (2007) Lévy flights in Dobe Ju/hoansi foraging patterns. *Hum Ecol* 35(1):129–138
- Desai SR, Prasad R (2013a) A novel order diminution of LTI systems using big bang big crunch optimization and routh approximation. *Appl Math Model* 37(16–17):8016–8028. doi:10.1016/j.apm.2013.02.052
- Desai SR, Prasad R (2013b) A new approach to order reduction using stability equation and big bang big crunch optimization. *Syst Sci Control Eng Open Access J* 1:20–27
- Edgar TF (1975) Least squares model reduction using step response. *Int J Control* 22:261–270
- Eitelberg E (1981) Model reduction by minimizing the weighted equation error. *Int J Control* 34(6):1113–1123
- El-Attar RA, Vidyasagar M (1978) Order reduction by  $L_1$  and  $L_\infty$  norm minimization. *IEEE Trans Autom Control* 23(4):731–734
- Erol OK, Eksin I (2006) A new optimization method: big bang-big crunch. *Adv Eng Softw* 37:106–111. doi:10.1016/j.advengsoft.2005.04.005
- Ghosh S, Senroy N (2013) Balanced truncation approach to power system model order reduction. *Electr Power Compon Syst* 41:747–764. doi:10.1080/15325008.2013.769031
- Goldberg DE (1989) Genetic algorithms in search, optimization, and machine learning. Addison-Wesley, Boston 10.1007/s10589-009-9261-6
- Humphries NE, Weimerskirch H, Queiroz N, Southall EJ, Sims DW (2012) Foraging success of biological Lévy flights recorded in situ. *Proc Natl Acad Sci* 109(19):7169–7174
- Hutton MF, Friedland B (1975) Routh approximations for reducing order of linear, time-invariant systems. *IEEE Trans Autom Control* 20:329–337. doi:10.1109/TAC.1975.1100953
- Kennedy J, Eberhart R (1995) Particle swarm optimization. In: IEEE International Conference on Neural Networks, Perth, WA, vol 4, pp 1942–1948. doi:10.1109/ICNN.1995.488968
- Lee KS, Geem ZW (2004) A new structural optimization method based on the Harmony search algorithm. *J Comput Struct* 82:781–798
- Mukherjee S, Satakshi R, Mittal C (2005) Model order reduction using response-matching technique. *J Frankl Inst* 342:503–519
- Obinata G, Inooka H (1983) Authors reply to comments on model reduction by minimizing the equation error. *IEEE Trans Autom Control* 28:124–125
- Panda S, Yadav JS, Padidar NP, Ardil C (2009) Evolutionary techniques for model order reduction of large scale linear systems. *Int J Appl Sci Eng Technol* 5:22–28
- Parmar G, Mukherjee S, Prasad R (2007a) Reduced order modeling of linear dynamic systems using particle swarm optimized eigen spectrum analysis. *Int J Comput Math Sci* 1(31):45–52
- Parmar G, Mukherjee S, Prasad R (2007b) System reduction using eigen spectrum analysis and pade approximation technique. *Int J Comput Math* 84(12):1871–1880
- Parmar G, Mukherjee S, Prasad R (2007c) System reduction using factor division algorithm and eigen spectrum analysis. *Appl Math Model* 31(11):2542–2552. doi:10.1016/j.apm.2006.10.004
- Parmar G, Prasad R, Mukherjee S (2007d) Order reduction of linear dynamic systems using stability equation method and GA. *Int J Comput Inf Eng* 1(1):26–32
- Parmar G, Pandey MK, Kumar V (2009) System order reduction using GA for unit impulse input and a comparative study using ISE and IRE. In: International conference on advances in computing, communications and control, Mumbai, India, pp 23–24
- Salim R, Bettayeb M (2011)  $H_2$  and  $H_\infty$  optimal model reduction using genetic algorithms. *J Frankl Inst* 348:1177–1191. doi:10.1016/j.jfranklin.2009.10.016
- Sambariya DK, Arvind G (2016) High order diminution of LTI system using stability equation method. *Br J Math Comput Sci* 13(5):1–15. doi:10.9734/BJMCS/2016/23243
- Sikander A, Prasad R (2015a) Soft computing approach for model order reduction of linear time invariant systems. *Circuit Syst Signal Process*. doi:10.1007/s00034-015-0018-4
- Sikander A, Prasad R (2015b) Time domain order reduction method using improved Hermite Normal Form. In: National conference on emerging trends in electrical and electronics engineering, JMI, New Delhi, India, pp 224–229
- Sikander A, Prasad R (2015c) Linear time-invariant system reduction using a mixed methods approach. *Appl Math Model* 39(16):4848–4858
- Sikander A, Prasad R (2017) A new technique for reduced-order modelling of linear time-invariant system. *IETE J Res* 1–9. doi:10.1080/03772063.2016.1272436
- Sikander A, Uniyal I, Thakur P (2016) Hybrid method of reduced order modelling for LTI system using evolutionary algorithm. In: IEEE international conference on next generation computing technologies, Dehradun, India
- Viswanathan GM (2010) Fish in levy-flight foraging. *Nature* 465:1018–1019
- Vishwakarma CB, Prasad R (2008) System reduction using modified pole clustering and pade approximation. In: XXXII national systems conference, NSC 2008, pp 592–596

- Vishwakarma CB, Prasad R (2009) MIMO system reduction using modified pole clustering and genetic algorithm. *Model Simul Eng* 2009:1–5
- Walton S, Hassan O, Morgan K, Brown MR (2011) Modified cuckoo search: a new gradient free optimisation algorithm. *Chaos Solitons Fractals* 44:710–718. doi:[10.1016/j.chaos.2011.06.004](https://doi.org/10.1016/j.chaos.2011.06.004)
- Wilson DA (1970) Optimal solution of model reduction problem. *Proc Inst Electr Eng* 117(06):1161–1165
- Yang XS, Deb S (2008) *Nature-inspired metaheuristic algorithms*. Luniver Press, London
- Yang XS, Deb S (2010) Engineering optimisation by cuckoo search. *Int J Math Model Numer Optim* 1:330–343