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METHODOLOGIES AND APPLICATION

Design of digital IIR filter with low quantization error using hybrid optimization technique

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Abstract In this paper, a hybrid optimization technique based on particle swarm optimization (PSO) and artificial bee colony (ABC) algorithm is presented for the optimal design of infinite impulse response (IIR) filter with low quantization effect. In this method, different variants of PSO have been exhaustively tested, and the time varying coefficients-PSO (TVC-PSO) is used to formulate a new hybrid technique for better exploitation and exploration, which is further modified by sorting and replacement mechanism of Scout Bee from ABC algorithm. For designing IIR filter, an objective function is constructed that satisfies the absolute error including peak ripples in passband and stopband regions in frequency domain, while stability of designed filter is confirmed by exploiting the lattice form structure during iterative computation that also reduces computation complexity. Several attributes such as passband error (e_p), stopband error (e_s), and stopband attenuation (A_s) are used to measure the performance of proposed algorithm. The simulation results presented in this paper evidence that this technique can be effectively used for designing digital IIR filter with higher filter taps, and low quantization effect for fixed number of bits.

Keywords Hybrid technique · Evolutionary computation · IIR · Optimization · Quantization

1 Introduction

In recent decades, digital signal processing (DSP) has gained considerable attention due to its wide applications in numerous engineering fields of one-dimensional (1-D) and multidimensional signals. Typical applications include biomedical signal processing, adaptive filtering, harmonic estimation, satellite image processing, communication network, and power system (Kumar et al. 2012; Ahirwal et al. 2014a, b; Bhandari et al. 2015a). Digital filters are the frequency selective elements extensively used in various signal processing applications due to their reconfigurability and simplified design. In addition, digital filters have a sharper transient response, better stopband attenuation as compared to analog filters (Nongpiur et al. 2013). There are several applications such as system identification, adaptive filtering, biomedical signal processing (Hartmann et al. 2014) etc, where digital IIR filter is widely exploited. All such applications rely on efficient design of IIR filter, which can be designed either using a conventional method or by computer-based method. In early stage of research, conventional techniques based on analog filter design were exploited for the design of IIR filter due to their simplified design; however, they suffer from approximation error, quantization error due to truncation with a finite bit, poor efficiency, and the requirement of higher order of filter taps for prescribed behavior (Tang et al. 1998; Lang 2000). Therefore, a computer-based method using gradient-based optimization was presented in Kobayashi and Imai (1990), and subsequently, several algorithms were developed to explore the solution by either minimizing or maximizing the objective

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function formulated using different filter design objectives (Lang 2000; Kumar et al. 2010, 2013; Nongpiur et al. 2013). However, the gradient-based techniques are suitable for problem having unimodal surface. Thus, these techniques are not suitable for the design of IIR filter as its error surface is multimode.

A new technique using genetic algorithm (GA) was presented for the efficient design of IIR filter for system identification, and this was further improved in (Weidong and Fan; Etter et al. 1982). But, the limitation observed in all such techniques was dependence on initial population. If the diversity mechanism is not executed properly, GA converges into local minima/maxima, and high variation in the output parameters for each trial of execution is seen. Therefore, particle swarm optimization (PSO) inspired by fish schooling and bird flocking was used for designing IIR filters due to its fast convergence rate and better optimal solution as compared to GA (Krusienski and Jenkins 2003; Saha et al. 2011) and was further modified using different variants of PSO in (Bansal et al. 2011; Ahirwal et al. 2013). However, problem with PSO is still of getting trapped in local minima, and PSO is not immune to handle higher dimensional problems, which is order of filter in case of filter design.

Recently, artificial bee colony (ABC) algorithm inspired by intellectual scavenging conduct of honey bee swarm has emerged as a robust optimization for multimodal and unimodal error functions. Literature reviews reflects that this technique has been used for solving many complex engineering problems such as adaptive filtering (Ahirwal et al. 2014a), satellite image segmentation (Bhandari et al. 2015b), multirate system design (Kuldeep et al. 2015a, b). Karaboga (2009) have used ABC techniques for designing digital IIR filter using system identification, and this was further improved in Agrawal et al. (2015a). But, some hinders have been observed in ABC technique similar to PSO. As ABC technique adopts probabilistic mechanism for sorting best solution from the population and hence requires too many numbers of iterative trials and execution as compared to PSO. In conventional PSO, convergence is fast due to comparatively less function evaluation; however, it suffers to achieve global minima in large-scale optimization problems. To resolve this issue, an improved PSO based on scout mechanism of ABC algorithm was proposed, and used for designing filter bank (Rafi et al. 2013). Subsequently, several researchers have proposed improved PSO based on hybridization with other optimization techniques. Gong et al. (2010) and Zhang et al. (2014) have introduced a new hybrid PSO technique by governing mutation operation of DE on search space (particles) of improved bare-bones PSO (control parameter free) for power system optimization. The GA was also merged to develop another Hybrid PSO for determining design parameters for a higher-order sliding model controller (Cao et al. 2016). A new hybrid technique was also presented based on PSO and DE with binary search algorithm for designing optimal IIR filter (Sidhu et al. 2016). Thus, literature review on hybrid techniques for designing digital IIR filters evidences that several hybrid techniques discussed above, have been proposed. In these algorithms, complexity was quite high as both local and global search were conducted using two different optimization techniques. It is also evident that extensive work has been done toward design and development of IIR filter using various swarm-based techniques. However, there is no technique available in literature, which is applicable for higher filter taps, and has low quantization affect. Therefore, in this paper, a new hybrid technique is proposed with better exploration and exploitation abilities for designing improved IIR filter with simple stability constraint. This technique is also applicable for designing higher order filter with less quantization and truncation errors.

In above context, therefore, this paper presents an improved hybrid method based on ABC and PSO with time varying coefficients for solving nonlinear optimization constructed using the prescribed ripple in passband and stopband region. The presented technique gives a more stable design with less quantization effect.

2 Overview of swarm-based techniques

Swarm-based techniques are the subset of evolutionary computation and usually counted in artificial computation. The widely recognized algorithms are as follows.

2.1 Particle swarm optimization (PSO)

PSO algorithm was developed by inspiring from phenomena of communication behavior of birds, fish and insects, and successfully applied in optimal design of digital filter (Kennedy and Eberhart 1995; Sheng and Bing 2010; Shao et al. 2015; Sharma et al. 2016). In PSO technique, the optimal solution is obtained by following a random path governed by two variables: '*Pbest*' (local best component), which is the solution corresponding to current best solution of objective function and '*Gbest*' (global best component) which is the another solution corresponding to final best solution achieved in entire search. In first stage, initializing of population matrix of particles/swarm is performed. Each set of particle vector contains a possible solution of the given problem. In second stage, this swarm matrix is updated using (Kennedy and Eberhart 1995; Bansal et al. 2011):

$$v_n^{[i+1]} = \chi \left(w \cdot v_n^{[i]} + C_1 \cdot rand_1 \cdot (Pbest_n^{[i]} - pop_n^{[i]}) + C_2 \cdot rand_2 \cdot (Gbest^{[i]} - pop_n^{[i]}) \right) \quad (1)$$

In above Eq. (1), v_n represents the velocity matrix, whose dimensionality is same as of the population/swarm matrix, w

is the inertia weight that controls the search space by putting restriction on particles, χ is the constrained factor and ith is the current iteration cycle. C_1 and C_2 are the cognitive and social scaling parameters; $rand_1$ and $rand_2$ is the random number vector. The velocity associated with particles of swarm has to be in limit of certain range., now the population matrix is updated as (Kennedy and Eberhart 1995):

$$pop_n^{[i+1]} = pop_n^{[i]} + v_n^{[i+1]} \tag{2}$$

In the end, greedy based selection procedure is conducted for sorting and updating of *Pbest* and *Gbest*. PSO has been formulated in CAD program by following the pseudocode explained in Ahirwal et al. (2013).

2.2 Artificial bee colony (ABC)

Artificial bee colony (ABC) algorithm is a global, meta-heuristic search and optimization method inspired by intelligent foraging behavior of honey bees (Karaboga and Basturk 2007a; Karaboga 2010). This method consists three segments: employed, onlooker, and scout bees with food sources. In ABC, problem formulation for optimization is constructed by searching the best parameter vector from population entitled as '*Food*', which will minimize the objective function. In the first stage, search space is formed by initializing food matrix, and each row represents set of values (solution). Now, the employed bee phase starts with modification of search space, followed by evolution of fitness, and sorting of solution with best fitness value is performed. The second stage is onlooker bee phase, in which a solution is searched among the food particle left after employed bee phase based on certain parameter selection. During these phases, if a new solution has not been improved, then counter associated with each solution is incremented by one. In third phase (also known as scout bee phase), counter value is scanned, and if any of the counter value is found equal to permissible limit, then that corresponding solution (food particle) is initialized with a new value. The detailed analysis on ABC algorithm along with pseudocode can be found in many literature (Karaboga and Basturk 2007a; Karaboga 2010; Ahirwal et al. 2013).

3 Proposed hybrid technique

From literature review, it is evident that several attempts have been made to improve performance of conventional PSO for various applications. Rafi et al. (2013) have proposed the concept of hybridization of two optimization techniques for exploring the optimal solution. Originally, Rafi et al. (2013) have developed a hybrid technique for designing multirate filter banks based on PSO and ABC algorithms. In this technique, mechanism for updating population, and sorting of the best solution is governed by PSO, followed by replacement mechanism for unimproved solution of ABC algorithm. Here, after updating the velocity matrix, population matrixes are updated, and after execution of each steps, examination on velocity particles, and position matrix is carried out. If these values exceed the limit, they are brought back within the limits. After completion of evaluation of objective function using updated population matrix, quality of each solution is checked. This mechanism has been adopted from ABC algorithm in which concept of three bees such as employed bee, onlooker bee, and scout bee are used (Karaboga and Basturk 2007b; Karaboga 2010). During food search, if the employed bee could not succeed in an exploration of food source with improved quality in specified number of prescribed trails, known as the *limit* then it leaves the current food source and converts into a scout. Similarly, in hybrid PSO, if any vector whose solution is not improved, then its value is replaced by till known global best solution (*Gbest*) and the velocity vector corresponding to unimproved solution is replaced by the improved velocity recorded for *Gbest* In this way, swarm is updated in direction of an optimal solution that confirms the optimal point exploration, and resists the trapping in local minima. The proposed concept is a joint venture of two distinguishes robust meta-heuristic techniques, and thus, named as hybrid PSO. Later on, several researchers have developed improved hybrid techniques using different optimization techniques such GA and DE (Gong et al. 2010; Zhang et al. 2014).

Literature review on PSO technique evidences that several variant of conventional PSO such as constant weight inertia (CWI)-PSO, linearly decay inertia-PSO (LDI-PSO), dynamic inertia-PSO (DI-PSO) and time varying coefficients-PSO (TVC-PSO) have been proposed for different applications. A detailed discussion on these variants are given in Ahirwal et al. (2013) and the references therein. Therefore, in this work, a comparative study of performance of different variants of PSO, hybridizing with ABC algorithm is carried out. Based on performance, an improved hybrid technique is proposed using the time varying coefficients-PSO (TVC-PSO) and ABC algorithm for designing digital IIR filters. Following step are executed during the course of proposed hybrid PSO method, defined as:

Step 1: The initial parameters are defined such as: desired solution value (D), dimension of solution (N), search space size (M).
Step 2: The search space (particles) matrix is formulated by assigning a uniformly distributed random number in the range of lower limit (X_l) and upper limit (X_u), defined as:

$$pop_n^{[i=0]} = X_\text{l} + (X_\text{u} - X_\text{l}) \cdot rand_n\,[0, 1] \tag{3}$$

where n is the index of search space vector, ranging from 1 to M.

Step 3: Then, velocity matrix (v) associated with particles is formed within lower velocity limit (V_l) and upper velocity limit (V_u) as:

$$v_n^{[i=0]} = V_l + (V_u - V_l) \cdot rand_n\,[0, 1] \tag{4}$$

Step 4: Each solution of search space (*pop*) vector is used for evaluation of fitness function, and the solution with best fitness function values is considered as *Gbest*, while the initially formed search space is considered as *Pbest*.

Step 5: Now, algorithm enters into iterative computation stage ranging as $i = 1$ to i_{max} Firstly, inertia weight (w) or coefficients (C_1, C_2) are computed for suitable variant, then v is updated using Eq. (1), and at last, *pop*is updated using Eq. (2).

Step 6: Then, this updated *pop* is exploited for evaluation of fitness function. If newly generated solution vectors have gained better fitness value over previous vectors then, old solution in *Pbest* will be replaced with new one, and associated velocity is recorded correspondingly; else counter trail is increased by one.

Step 7: Now check whether current fitness value of any *Pbest* solution vector is better than current *Gbes* t value or not. If yes, then, that *Pbest* solution will replace the current *Gbest*, and corresponding velocity is recorded as global best velocity (*Best vel*), otherwise old *Gbest* is kept unchanged.

Step 8: Then, counter value corresponding to each individual solution vector of *pop* is checked. If it is equal to *limit*, then such solution is replaced by *Gbest* and v with *Best vel*.

Step 9: Now, check whether fitness values achieve 'Tol' (tolerable fitness value) or iteration cycles are completed or not, if yes, then *Gbest* holds the optimal solution, otherwise, go back to step 5 and follow the next steps.

A flowchart for the proposed method is depicted in Fig. 1. The proposed hybrid algorithm is very useful for non-convex and non-differentiable design problem of IIR filter with acceptable fidelity parameters using finest swarm size, while the complexity remains same as for conventional PSO.

4 Design of IIR filter using proposed improved hybrid technique

Digital IIR filter can be designed using transformation method, but it suffers from inefficiency and quantization effect. Therefore, a new state of art has been practiced in which modern meta-heuristic techniques have been employed. In the design procedure using these techniques, the coefficients of required filter is searched according to required performance. IIR filter is characterized by a linear constant difference equation, defined as (Proakis and Manolakis 2006; Saha et al. 2013):

$$\sum_{i=0}^{K} b_i y\,[n-i] = \sum_{j=0}^{L} a_j x\,[n-j] \tag{5}$$

where $y(n)$ is the output sequence when excited by the input sequence of $x(n)$, a_j and b_i are the coefficients that decide the nature of response. From Eq. (5), the output response can also be stated in frequency domain using z transform as (Proakis and Manolakis 2006):

$$\begin{aligned} Y(z)\left[1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_K z^{-K}\right] \\ = X(z)\left[a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_L z^{-L}\right], \\ \text{where } b_0 = 1 \end{aligned} \tag{6}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_L z^{-L}}{1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_K z^{-K}} \tag{7}$$

Hence, Eq. (7) characterizes the frequency response of IIR filter, and governing parameters are the coefficients of denominator and numerator polynomials.

4.1 Problem formulation

In this paper, an efficient design of optimal IIR filters is carried out using ABC technique, variants of PSO, and Hybrid methods, in which coefficients of IIR filter are successively explored until the error between outputs of proposed filter and desired filter is minimized. The error function, which is absolute error with ripple has been adopted as an objective function, computed in frequency domain and minimized, given as (Saha et al. 2014):

$$\begin{aligned} J = \Big[\textstyle\sum_{\omega\in\omega_p} abs\left(|H(\omega)| - D(\omega) - a_p\right) \\ + \textstyle\sum_{\omega\in\omega_s} abs\left(|H(\omega)| - D(\omega) - a_s\right)\Big] \end{aligned} \tag{8}$$

where $|H(\omega)|$ is the magnitude response of designed filter, a_p and a_s are the permissible ripples in passband and stopband, respectively. $D(\omega)$ is the desired frequency response, defined as:

$$D(\omega) = \begin{cases} 1, & \omega \in passband \\ 0, & \omega \in stopband, \end{cases} \tag{9}$$

4.2 Stability constraint

IIR filters are potential toward instability and thus require higher attention during their process of design. Stability of

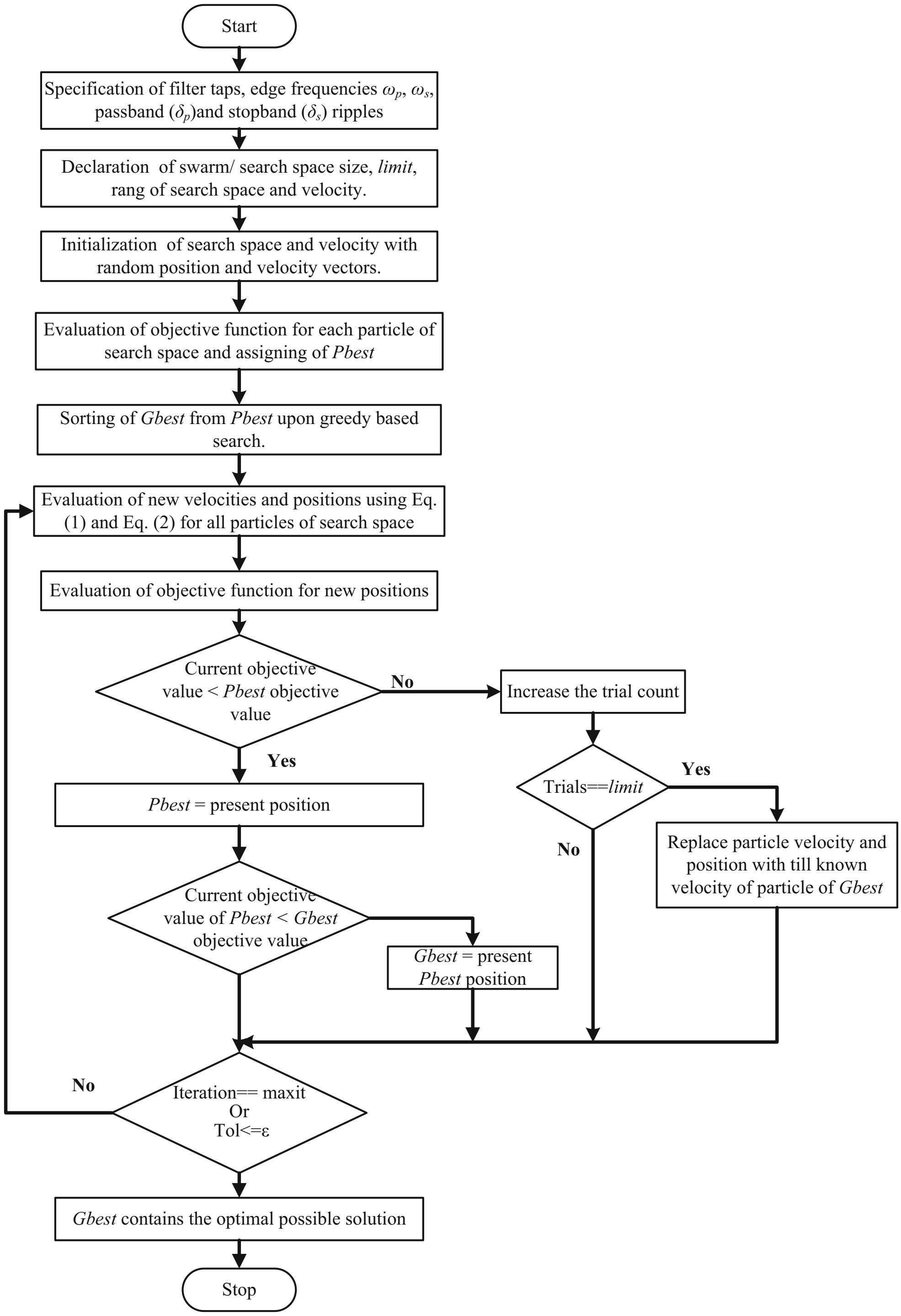


Fig. 1 A flowchart for the proposed Hybrid method (Rafi et al. 2013)

IIR filters are confirmed by ensuring the location of poles (roots of denominator polynomial), which should be lied in unity circle of z-plane. In early stage of research, researchers have utilized the concept of breaking higher-order transfer function into first and second order functions, in which denominator coefficients values are restricted in certain range for stability (Tang et al. 1998; Yu and Xinjie 2007). This method was suitable for lower-order filter design and was not computationally efficient. Moreover, the implemented filter in direct form was not able to achieve better optimal point. In this work, a different method is used in which instead of denominator polynomial in direct form, equivalent values in lattice form are substituted as shown in Eq. (10) (Sheng and Bing 2010):

$$pop_{n,m}^{[i]} = \left[a_{n,0}\ a_{n,1}\ a_{n,2}\ \cdots\ a_{n,L}\ :\ g_{n,L+1}\ g_{n,L+2}\ \cdots\ g_{n,L+K}\right]^{[i]} \quad (10)$$

where n corresponds to index of a solution vector from population matrix, m corresponds to a solution vector element. Each solution consists of D elements ($D = 2order + 1$). The element $a_{n,m}$ are the numerator coefficients, whereas $g_{n,m}$ are the lattice equivalents of denominator coefficients. The use of lattice coefficients is adopted because it makes easy to handle the stability as their values should be in limit of -1 to 1 for stable design. The values of lattice coefficients are converted back into direct form and substituted in Eq. (7). The conversation is computed by recursive computation of the following set of equations for each lattice coefficient (Agrawal et al. 2015b):

$$U_m(z) = U_{m-1} + g_m \cdot z^{-1} \cdot V_{m-1}, \quad m = 1, 2, \ldots, N-1. \text{ where } \mathrm{U}_0(z) = V_0 = 1 \quad (11)$$

$$V_m(z) = z^{-m} \cdot U(z^{-1}) \quad (12)$$

Now, the polynomial coefficients of Eq. (11) are the equivalent direct form coefficients:

$$\left[g_{i,L+1}\ g_{i,L+2}\ \cdots\ g_{i,L+K}\right] \to \left[1\ b_{i,1}\ \cdots\ b_{i,K}\right] \quad (13)$$

Upon substitution in Eq. (7), N point frequency sample response $|H(\omega)|$ is calculated and used for computation of objective function using Eq. (8).

4.3 Designing of IIR filter using proposed hybrid method

In this paper, a comprehensive experimental study has been performed for designing an optimal digital IIR filter using improved hybrid swarm-based techniques. For this purpose, search space is formulated, in which $m \times n$ matrix is initialized with some pseudo random vector, defined in Eq. (10), where m is the total possible solutions considered, and n ($n = L + K$) is the coefficient length of each solution, and a velocity matrix is also initialized with same dimension. The search space is modified by first updating the velocity matrix using Eq. (1), where scaling of w is performed in three distinguish ways, which result in constant weight inertia-PSO (CWI-PSO), linearly decay inertia-PSO (LDI-PSO), and dynamic inertia-PSO (DI-PSO), respectively. The detailed analysis of these variants can also be found in Ahirwal et al. (2013). In all these techniques, the value of χ is kept fixed at 1, while another variant of PSO, known as constrained factor inertia-PSO (CFI-PSO), is also exist, in which w is kept fixed to 1, and χ is initialized with 0.7213. Similarly, another variant has also been tested known as time varying coefficients-PSO (TVC-PSO), in which w and χ are fixed and C_1 and C_2 are made to swing accordingly (Sheng and Bing 2010):

Table 1 Pseudocode for checking and re-initializing the out of bound values

Pseudo code 1
⇒ *FOR m = 1:n*
⇒ *IF*($v_m \le -1$)
→$v_m = -1$**rand*
⇒ *ELSE IF*($v_m \ge 1$)
→v_m = *rand*
⇒ *END OF IF*
⇒ *END OF FOR*
Pseudo code 2
⇒ *FOR* $m = L+1{:}L+K$
→*IF*($g_{n,m} \le -1$)
$-g_{n,m} = -1$**rand*
→*ELSE IF*($g_{n,m} \ge 1$)
$-g_{n,m}$ = *rand*
→*END OF IF*
⇒ *END OF FOR*

$$\mathrm{C}_1 = C_{1_{\text{initial}}} - \left[\frac{\left(\left|C_{1_{\text{initial}}} - C_{1_{\text{final}}}\right|\right) \cdot i}{i_{\max}}\right] \quad (14)$$

$$C_2 = C_{2_{\text{initial}}} + \left[\frac{\left(\left|C_{2_{\text{initial}}} - C_{2_{\text{final}}}\right|\right) \cdot i}{i_{\max}}\right] \quad (15)$$

After velocity updating, its value is checked and if velocity associated with any element has been moved beyond the limit; then, it is enforced to stay back by initializing with a new value as shown in pseudocode-1 in Table 1. Experimentally the prescribed range of velocity matrix element is considered to be -1 to 1. After the velocity update, the position matrix (search space) is updated using Eq. (2), and similarly the elements corresponding to coefficients of lattice are checked. If the values of these coefficients moved beyond $|1|$, then they are reinitialized as shown in pseudocode-2 of Table 1. Now, the numerator and lattice coefficients from new positions are extracted, and used for evaluation of objective function using Eq. (8). The lattice coefficients are transformed in to direct form using Eqs. (11) and (12), after which the frequency response is computed by substituting the coefficients in Eq. (7), and at last the objective function is evaluated.

New solution (position) that has achieved better objective function value (J) with respect to previous local best solution (*Pbest*) is now accepted and copied over it. A similar analysis is carried out for the global solution, that is if the new any of '*Pbest*' solution has better objective function value than current '*Gbest*', then it replaces the previous global best solution (*Gbest*). Now in last stage the replacement mechanism is carried out in which scout bee concept of ABC technique is adopted. If any vector whose solution is not improved, then its value is replaced by till known global best solution that is *Gbest*. Also, the velocity vector corresponding to unimproved solution is also replaced by the velocity recorded for *Gbest*. In this way, the swarm is updated in the direction of an optimal solution. Thus, it confirms the optimal point exploration and resists the trapping in local minima.

5 Simulation results and discussion for efficient design of optimal IIR filter

In this section, the design of IIR filter using PSO, ABC and various developed hybrid PSO techniques are conducted. The comprehensive simulations have been carried out in order to analyze and select the suitable variant of PSO and then to develop the computationally improved hybrid version.

5.1 Design examples and parameter specifications

Various control parameters of different optimization techniques required in the proposed methodology are summarized in Table 2, which has been taken from the extensive analysis of literature that reflects their practice in various optimization problems such as filter designing and numerical optimization testing (Karaboga and Basturk 2007b; Sheng and Bing 2010). These values involve cognitive and social scaling parameters, inertia weight, limit, and modulation index. In Table 2, the value of C_1 and C_2 are taken from work proposed in Sheng and Bing (2010) and Bansal et al. (2011). Value of linearly varying w for LDI-PSO and CWI-PSO has been considered from Bansal et al. (2011). The suitable value for limit, velocity element (v), and swarm size (population size) are incorporated by experimental analysis. The inertia weight strategy for DI-PSO is modified as it observed from the analysis made in Agrawal et al. (2015b) that inertia weight should be in between 0.1 and 0.7, as it make the algorithm stable for IIR filter design. Previously, w in DI-PSO was governed as:

$$w = 0.5 + \frac{rand\,(\cdot)}{2} \tag{16}$$

whereas, in the new strategy is adopted, in which w is regenerated as:

$$w = ll + \frac{(ul - ll) \cdot rand\,(\cdot)}{2} \tag{17}$$

Table 2 Control parameters for the proposed technique and different algorithms

S. No.	Parameter name	CWI-PSO	CFI-PSO	LDI-PSO	MDI-PSO	TVC-PSO	ABC	Hybrid PSO-1	Hybrid PSO-2	Hybrid PSO-3
1	$[C_{1_{\text{initial}}}, C_{1_{\text{final}}}]$ (Sheng and Bing 2010; Bansal et al. 2011)	[2.05, 2.05]	[2.05, 2.05]	[2.05, 2.05]	2.05	2.50–0.50	–	2.05	2.05	2.50–0.50
2	$[C_{2_{\text{initial}}}, C_{2_{\text{final}}}]$ (Sheng and Bing 2010)	[2.05, 2.05]	[2.05, 2.05]	[2.05, 2.05]	2.05	0.50–2.50	–	2.05	2.05	0.50–2.50
3	w (Ahirwal et al. 2013)	–	–	0.75	As per Eq. (17)		–	As per Eq. (17)	1	0.707
4	$[w_{\max}, w_{\min}]$ (Bansal et al. 2011)	0.707	–	–	–	–	–	–	–	–
5	v (velocity element limit)	[−1, 1]	[−1, 1]	[−1, 1]	[−1, 1]	[−1, 1]	–	[−1, 1]	[−1, 1]	[−1, 1]
6	Maximum iteration $i_{\max}$	2000	2000	2000	2000	2000	2000	2000	2000	2000
7	Limit (Rafi et al. 2013)	–	–	–		–	100	100	100	100
8	χ	–	0.7213	–	–	–	–	–	–	–

Table 3 Design specification for the prototype IIR filter

S. No	Type of filter	Order of filter	Passband ω_p	Stopband ω_s	Passband permissible ripple	Stopband permissible ripple
Eaxmple-1	LPF	3	$[0,\ 0.4\pi]$	$[0.5\pi,\ \pi]$	0.01	0.001
Eaxmple-2		6	$[0,\ 0.4\pi]$	$[0.5\pi,\ \pi]$	0.01	0.001
Eaxmple-3		9	$[0,\ 0.4\pi]$	$[0.5\pi,\ \pi]$	0.01	0.001
Eaxmple-4		12	$[0,\ 0.4\pi]$	$[0.5\pi,\ \pi]$	0.01	0.001
Eaxmple-5 (Saha et al. 2013)		8	$[0,\ 0.45\pi]$	$[0.5\pi,\ \pi]$	0.001	0.0001
Eaxmple-6	HPF	3	$[0.6\pi,\ \pi]$	$[0,\ 0.5\pi]$	0.01	0.001
Eaxmple-7		6	$[0.6\pi,\ \pi]$	$[0,\ 0.5\pi]$	0.01	0.001
Eaxmple-8		9	$[0.6\pi,\ \pi]$	$[0,\ 0.5\pi]$	0.01	0.001
Eaxmple-9		12	$[0.6\pi,\ \pi]$	$[0,\ 0.5\pi]$	0.01	0.001
Eaxmple-10		8	$[0.5\pi,\ \pi]$	$[0,\ 0.45\pi]$	0.001	0.0001
Example-11 (Saha et al. 2013)		8	$[0.35\pi,\ \pi]$	$[0,\ 0.30\pi]$	0.001	0.0001

Table 4 mean, variance, standard deviation, best and worst values of J for each method for the 30 trials used to design the filter mentioned in Table 3 with the population size of 20

Population size 20									
Parameters	CWI-PSO	CFI-PSO	LDI-PSO	MDI-PSO	TVC-PSO	ABC	Hybrid-1	Hybrid-2	Hybrid-3
Example-1									
Mean	3.6632	2.8835	2.9024	2.9008	3.9968	3.2131	2.9142	2.8828	2.8829
Variance	0.3281	0.0000	0.0006	0.0003	1.2460	0.1558	3.3594	0.0000	0.0000
Std. dev.	0.5728	0.0026	0.0252	0.0165	1.1163	0.3947	1.8329	0.0032	0.0047
Best	3.2734	2.8776	2.8792	2.8785	2.8848	2.8791	1.2195	2.8795	2.8773
Worst	5.3389	2.8863	2.9766	2.9456	7.3534	4.2650	4.5257	2.8921	2.8930
Example-2									
Mean	3.6315	1.6420	1.8845	3.1329	3.1732	2.1642	2.5269	1.4354	1.3783
Variance	2.0103	0.4262	0.3948	1.9570	2.8109	0.4484	3.5219	0.2925	0.4666
Std. dev.	1.4179	0.6529	0.6283	1.3989	1.6766	0.6696	1.8767	0.5409	0.6831
Best	1.8459	0.5106	1.0264	1.7631	0.8219	1.1235	0.9569	0.5123	0.6765
Worst	7.6929	2.5951	3.6194	6.7847	6.6547	3.5721	10.5492	2.4053	2.7026
Example-3									
Mean	8.3635	2.2147	2.3712	4.7173	6.2245	3.6202	2.3557	1.8079	1.3397
Variance	13.9417	1.8803	3.6636	10.3552	15.4050	7.7945	1.8433	1.3528	0.3145
Std. dev.	3.7339	1.3712	1.9140	3.2180	3.9249	2.7919	1.3577	1.1631	0.5608
Best	3.6045	1.0671	0.4157	1.2151	0.6558	0.9026	0.3616	0.4640	0.4343
Worst	18.3944	6.6774	8.3548	12.0299	17.0637	11.4848	3.7114	3.6488	2.1677
Example-4									
Mean	13.3130	6.0239	3.8255	3.8255	4.2063	3.8255	3.8255	3.4156	2.5493
Variance	34.8286	16.6049	8.4398	8.4398	10.2615	8.4398	8.4398	5.8467	1.5612
Std. dev.	5.9016	4.0749	2.9051	2.9051	3.2034	2.9051	2.9051	2.4180	1.2495
Best	3.6695	1.0067	0.2391	0.2391	0.2586	0.2391	0.2391	0.9199	1.3438
Worst	25.4527	13.2549	8.1657	8.1657	11.2062	8.1657	8.1657	6.3375	4.6477
Example-5									
Mean	5.6509	3.8271	3.2048	4.1567	6.1314	3.7982	2.9142	2.6292	2.5021
Variance	3.7727	9.6262	3.4045	2.7336	5.1021	1.8019	3.3594	1.1423	4.1587
Std. dev.	1.9423	3.1026	1.8451	1.6534	2.2588	1.3424	1.8329	1.0688	2.0393

Table 4 continued

Population size 20									
Parameters	CWI-PSO	CFI-PSO	LDI-PSO	MDI-PSO	TVC-PSO	ABC	Hybrid-1	Hybrid-2	Hybrid-3
Best	2.3203	1.7235	1.3012	1.7284	2.7025	2.0797	1.2195	1.0222	0.8734
Worst	9.2596	13.4935	8.6996	6.8375	9.9032	5.9939	4.5257	4.2876	8.4707
Example-6									
Mean	3.7000	2.8744	2.8793	2.9104	2.9571	3.1628	3.2840	2.8716	2.8735
Variance	1.1381	0.0000	0.0002	0.0089	0.0485	0.2487	0.0000	0.0000	0.0000
Std. dev.	1.0668	0.0046	0.0146	0.0943	0.2202	0.4987	0.0000	0.0026	0.0051
Best	3.2523	2.8682	2.8691	2.8712	2.8692	2.8735	3.2839	2.8689	2.8701
Worst	7.5951	2.8829	2.9304	3.2611	3.6095	4.4999	3.2840	2.8785	2.8841
Example-7									
Mean	3.6418	2.1331	2.5029	2.9883	4.3243	2.5075	2.2318	1.3805	1.9658
Variance	1.9001	3.4540	1.5220	2.7801	8.6727	0.6581	3.1129	2.6203	0.3454
Std. dev.	1.3784	1.8585	1.2337	1.6674	2.9449	0.8113	1.7643	1.6187	0.5877
Best	2.0373	0.8181	1.0205	1.2201	1.2303	1.3635	0.9114	0.6802	1.1166
Worst	7.0622	8.7365	6.3983	7.4951	12.2923	4.1583	7.9235	2.6270	3.2688
Example-8									
Mean	6.7393	3.1555	2.0139	4.8904	7.1900	3.1026	2.5006	3.7199	1.5532
Variance	5.0451	6.1102	2.4207	20.9614	14.4708	1.9410	2.3766	22.9842	1.1337
Std. dev.	2.2461	2.4719	1.5559	4.5784	3.8041	1.3932	1.5416	4.7942	1.0647
Best	2.6090	0.3730	0.4106	0.6950	1.5728	1.4101	0.9053	0.5830	0.4862
Worst	10.6323	10.9342	5.8291	16.6708	12.8264	5.5780	4.2683	17.1780	4.3183
Example-9									
Mean	11.1487	6.9155	3.7556	5.1346	13.8242	6.3036	3.6331	5.3625	1.5532
Variance	35.4883	34.9540	8.0010	21.3037	26.8441	10.2254	8.1255	13.1216	1.1337
Std. dev.	5.9572	5.9122	2.8286	4.6156	5.1811	3.1977	2.8505	3.6224	1.0647
Best	3.1494	1.2128	0.9896	0.8037	4.5915	2.1010	0.9003	0.5362	0.4862
Worst	27.3040	19.9974	12.4287	16.4703	21.4831	12.3827	9.2004	10.7815	4.3183
Example-10									
Mean	7.3253	7.3253	4.0248	4.0295	2.8717	4.4070	4.0150	3.3729	2.8253
Variance	17.9151	17.9151	1.8680	5.9376	0.0000	2.9899	5.9499	3.0406	1.1815
Std. dev.	4.2326	4.2326	1.3668	2.4367	0.0034	1.7291	2.4392	1.7437	1.0870
Best	2.4437	2.4437	1.5918	0.7960	2.8681	1.1167	1.6204	1.2817	1.3894
Worst	17.4034	17.4034	6.3194	8.5725	2.8787	7.0223	12.8459	6.8739	4.7386

In above equation ll and ul are the suitable limits that are used to improve the convergence of algorithm during exploration, selected as 0.1 and 0.6, respectively (Kumar et al. 2012).

5.2 Comparison of Hybrid methods with swarm-based techniques.

In this section, the several examples are considered for the design of optimal IIR filter using PSO, ABC and various developed hybrid PSO techniques, which are listed in Table 3. The first experiment is carried out for analyzing the effect of search space matrix (swarm) on filter performance based on the value of J for the swarm-based techniques. In this experiment, all filters are designed using proposed methods for thirty independent trials, with different swarm size of 20, 30, 40 and 50. The necessary parameters are evaluated and summarized for respective swarm size in Tables 4, 5, 6 and 7. It is evident from Table 4 that, among CWI-PSO, LDI-PSO, CFI-PSO, MDI-PSO TVC-PSO and ABC algorithm, last three techniques of PSO family are able to achieve better value of J. Moreover, the standard deviation and mean value achieved by CFI-PSO is quite better than TVC-PSO and MDI-PSO, for the population size of 20 in lower-order filter examples. This leads to the development of three hybridized method using the concept of scout bee of ABC algorithm with these variant of PSO and entitled as Hybrid-1 (MDI-PSO

Table 5 mean, variance, standard deviation, best and worst values of J each method for the 30 trials used to design the filter mentioned in Table 3 with the population size of 30

Population size 30									
Parameters	CWI-PSO	CFI-PSO	LDI-PSO	MDI-PSO	TVC-PSO	ABC	Hybrid-1	Hybrid-2	Hybrid-3
Example-1									
Mean	3.0981	3.2730	2.8890	2.8939	2.8834	3.0566	3.2963	2.8804	2.8806
Variance	0.1165	0.0000	0.0002	0.0001	0.0000	0.1551	0.0000	0.0000	0.0000
Std. dev.	0.3413	0.0016	0.0123	0.0112	0.0034	0.3938	0.0000	0.0019	0.0026
Best	2.8847	3.2683	2.8769	2.8796	2.8802	2.8782	3.2963	2.8770	2.8774
Worst	4.0632	3.2751	2.9305	2.9169	2.8925	4.5275	3.2963	2.8836	2.8880
Example-2									
Mean	3.3708	2.0608	2.0163	2.8598	2.5051	1.9591	1.9268	1.5525	1.6553
Variance	3.8950	2.3252	0.3331	3.4001	1.8703	1.2288	1.0645	0.6028	0.2017
Std. dev.	1.9736	1.5249	0.5771	1.8439	1.3676	1.1085	1.0317	0.7764	0.4491
Best	0.8826	0.5214	1.2789	0.9315	0.9245	1.0080	0.6485	0.4939	0.9344
Worst	7.2865	6.9765	3.5431	7.3122	6.1492	4.1868	2.7161	2.8514	2.2458
Example-3									
Mean	4.7618	2.8500	2.4191	3.3970	4.5915	2.0153	2.1022	2.5024	0.9970
Variance	2.6138	9.4617	1.5861	5.9531	5.8167	0.5411	3.5102	5.6813	0.2122
Std. dev.	1.6167	3.0760	1.2594	2.4399	2.4118	0.7356	1.8735	2.3835	0.4607
Best	1.2280	0.3888	0.6366	1.1271	1.7274	0.7017	0.3754	0.4241	0.3391
Worst	7.1538	10.4422	5.6123	10.3421	10.8539	3.9373	5.4694	8.9930	1.8479
Example-4									
Mean	10.5223	3.2730	4.7365	8.3322	8.1248	5.5378	3.1985	3.5467	1.9401
Variance	34.3584	0.0000	8.5586	30.8900	10.0041	8.8422	4.9790	8.1247	1.6984
Std. dev.	5.8616	0.0016	2.9255	5.5579	3.1629	2.9736	2.2314	2.8504	1.3032
Best	4.1803	0.4845	0.6709	0.7962	3.0920	0.6722	1.5585	0.6156	0.9053
Worst	26.5140	3.2751	9.6909	20.4203	13.9477	9.2483	10.0483	9.5462	5.6062
Example-5									
Mean	5.9849	3.5904	3.2507	3.8419	4.4870	2.7688	3.6978	3.0105	1.9206
Variance	7.9014	18.7966	4.7658	5.4875	4.1067	0.6275	3.8498	3.4711	0.4949
Std. dev.	2.8109	4.3355	2.1831	2.3425	2.0265	0.7922	1.9621	1.8631	0.7035
Best	3.2047	1.2646	1.0496	1.6677	1.9925	1.5377	1.1158	0.7650	0.7229
Worst	14.5432	19.2665	10.5039	10.5913	9.7095	3.8050	7.9698	7.3630	2.8772
Example-6									
Mean	2.9351	3.2611	2.8738	2.8829	2.9571	2.8942	3.2840	2.8708	2.8702
Variance	0.0232	0.0000	0.0000	0.0001	0.0485	0.0015	0.0000	0.0000	0.0000
Std. dev.	0.1524	0.0015	0.0060	0.0121	0.2202	0.0386	0.0000	0.0005	0.0005
Best	2.8703	3.2582	2.8681	2.8682	2.8692	2.8680	3.2839	2.8702	2.8689
Worst	3.4997	3.2639	2.8914	2.9223	3.6095	3.0024	3.2840	2.8721	2.8708
Example-7									
Mean	3.0255	2.3320	1.6893	2.3812	1.6870	1.8255	2.1223	2.0111	1.6895
Variance	1.4577	3.0354	0.5402	1.3181	0.3062	0.1683	0.4350	0.5851	0.6695
Std. dev.	1.2073	1.7423	0.7350	1.1481	0.5533	0.4102	0.6595	0.7649	0.8182
Best	1.2968	0.6111	0.7998	0.7156	0.6445	1.1376	1.1065	0.7985	0.7392
Worst	6.7716	7.4095	3.2476	5.5866	2.4570	2.6002	3.0968	2.9436	3.6712
Example-8									
Mean	5.9937	2.6814	2.6868	3.6083	4.3163	3.0126	2.8283	1.7536	1.4179
Variance	9.7077	1.8409	13.8507	5.7925	11.4469	1.8997	15.2523	1.1574	0.6786

Table 5 continued

Population size 30									
Parameters	CWI-PSO	CFI-PSO	LDI-PSO	MDI-PSO	TVC-PSO	ABC	Hybrid-1	Hybrid-2	Hybrid-3
Std. dev.	3.1157	1.3568	3.7217	2.4068	3.3833	1.3783	3.9054	1.0758	0.8238
Best	0.9589	0.7289	0.7728	0.8349	0.8287	1.2869	0.9242	0.5018	0.5876
Worst	10.9898	5.0594	16.3366	9.6586	12.1608	5.4081	21.7762	4.2927	3.1323
Example-9									
Mean	9.4560	6.7035	3.2497	3.4103	7.6161	3.7402	4.7413	4.9074	3.9741
Variance	12.3711	52.8611	5.7110	3.5072	27.174	7.6041	16.5436	18.6410	5.9491
Std. dev.	3.5173	7.2706	2.3898	1.8727	5.2129	2.7576	4.0674	4.3175	2.4391
Best	5.3650	1.0944	0.3129	1.0319	1.7905	0.6975	1.0900	0.9550	0.5714
Worst	18.5403	26.3869	9.4322	7.3939	21.611	8.7326	11.3481	14.6229	9.2104
Example-10									
Mean	5.2151	2.8832	2.5805	3.5911	2.9571	2.7977	3.3534	3.4362	2.4659
Variance	3.7247	1.3422	1.4214	5.0962	0.0485	0.8003	5.3234	10.3287	1.2136
Std. dev.	1.9299	1.1585	1.1922	2.2575	0.2202	0.8946	2.3073	3.2138	1.1017
Best	2.5475	1.3950	0.6368	1.2837	2.8692	1.2579	1.0940	1.1427	1.2198
Worst	9.5995	6.1501	5.3918	8.6769	3.6095	4.7216	9.3086	11.1038	4.5934

Table 6 mean, variance, standard deviation, best and worst values of J for each method for the 30 trials used to design the filter mentioned in Table 3 with the population size of 40

Population size 40									
Parameters	CWI-PSO	CFI-PSO	LDI-PSO	MDI-PSO	TVC-PSO	ABC	Hybrid-1	Hybrid-2	Hybrid-3
Example-1									
Mean	2.9670	2.8811	2.8862	2.8881	3.2720	2.9625	3.2963	2.8819	3.2685
Variance	0.0386	0.0000	0.0000	0.0000	0.0000	0.0185	0.0000	0.0000	0.0000
Std. dev.	0.1966	0.0012	0.0048	0.0065	0.0024	0.1360	0.0000	0.0031	0.0016
Best	2.8778	2.8796	2.8790	2.8770	3.2644	2.8777	3.2963	2.8786	3.2653
Worst	3.6624	2.8835	2.8965	2.9020	3.2748	3.4182	3.2963	2.8882	3.2706
Example-2									
Mean	2.3795	2.2322	1.7910	1.8439	2.0566	1.6542	2.6219	1.5276	1.4659
Variance	1.2000	2.0888	0.1271	0.4910	0.7893	0.1744	5.2132	0.3786	0.5289
Std. dev.	1.0954	1.4453	0.3565	0.7007	0.8884	0.4176	2.2832	0.6153	0.7273
Best	0.9622	0.4957	1.3794	0.5717	0.6188	0.8535	0.8223	0.6716	0.4577
Worst	3.8453	6.1490	2.6700	2.9410	4.0694	2.5280	6.2845	2.4681	3.1500
Example-3									
Mean	5.0823	1.1452	2.4587	3.4710	2.7706	1.9745	2.8517	2.2212	1.9549
Variance	4.7464	0.2974	2.7799	7.6757	0.9942	1.2518	11.4507	2.0001	2.5881
Std. dev.	2.1786	0.5453	1.6673	2.7705	0.9971	1.1188	3.3839	1.4142	1.6088
Best	2.5485	0.4201	0.4379	0.4792	1.3885	0.6072	0.6526	0.8500	0.3888
Worst	11.1369	2.1782	5.6041	11.1230	5.3532	4.5960	3.7135	5.9462	6.4312
Example-4									
Mean	10.1212	2.2595	4.3421	6.7550	5.2280	3.5645	3.7587	3.5440	0.9975
Variance	21.5142	1.4884	11.0184	17.7442	7.1729	5.2003	13.7345	14.1789	0.3126
Std. dev.	4.6383	1.2200	3.3194	4.2124	2.6782	2.2804	3.7060	3.7655	0.5591
Best	3.9132	0.6616	0.9983	1.3479	1.6549	0.6442	0.7196	0.6259	0.2163
Worst	18.7743	5.3694	14.0458	18.7860	10.1141	7.9430	15.5797	11.8430	2.2269

Table 6 continued

Population size 40

Parameters	CWI-PSO	CFI-PSO	LDI-PSO	MDI-PSO	TVC-PSO	ABC	Hybrid-1	Hybrid-2	Hybrid-3
Example-5									
Mean	5.9578	2.5934	2.9559	5.1952	3.6673	2.8081	4.3442	2.8546	1.6738
Variance	9.9223	0.7036	2.4751	17.3171	1.2519	0.7361	4.3895	2.1280	0.5963
Std. dev.	3.1500	0.8388	1.5732	4.1614	1.1189	0.8580	2.0951	1.4588	0.7722
Best	1.7026	1.3908	0.6242	1.7629	2.2165	1.6043	5.3473	1.1723	0.6499
Worst	14.0124	4.2253	6.0513	18.2264	6.6400	4.5138	5.3473	6.4812	3.5823
Example-6									
Mean	2.9267	2.8721	2.8739	2.8796	3.2587	3.0451	3.2840	2.8713	3.2561
Variance	0.0070	0.0000	0.0000	0.0000	0.0000	0.1407	0.0000	0.0000	0.0000
Std. dev.	0.0839	0.0033	0.0045	0.0066	0.0031	0.3751	0.0000	0.0022	0.0023
Best	2.8703	2.8686	2.8677	2.8711	3.2514	2.8680	3.2840	2.8703	3.2507
Worst	3.1436	2.8791	2.8828	2.8971	3.2635	4.3694	3.2840	2.8780	3.2592
Example-7									
Mean	3.1042	1.4272	1.8679	2.1205	2.7499	1.6360	1.9400	1.9617	1.3697
Variance	1.2868	0.3612	0.3758	1.8773	16.0513	0.3853	0.4874	0.2195	0.0958
Std. dev.	1.1344	0.6010	0.6130	1.3701	4.0064	0.6207	0.6981	0.4685	0.3095
Best	1.8607	0.6843	0.9107	0.7240	0.5771	0.7240	0.6421	0.8913	0.6669
Worst	6.4785	2.4231	2.9797	6.4446	17.4258	2.9931	2.9418	2.6945	1.6807
Example-8									
Mean	4.4822	4.1151	2.1381	2.8446	3.7404	2.5028	1.4687	2.7357	1.5358
Variance	6.6777	31.9514	2.6339	4.1511	21.4462	2.5378	0.4984	8.4143	0.5413
Std. dev.	2.5841	5.6526	1.6229	2.0374	4.6310	1.5930	0.7060	2.9007	0.7357
Best	0.9355	0.3142	0.3873	0.4916	0.6944	0.3258	0.7379	0.3489	0.7706
Worst	10.8034	23.8783	6.1082	8.6126	17.3041	5.4414	2.6424	10.9052	3.1461
Example-9									
Mean	9.3474	3.5525	3.8889	5.9701	6.4683	4.7419	3.3965	4.0239	1.7382
Variance	18.5027	10.5908	4.4270	17.3540	7.1341	8.5769	6.9855	5.4782	1.7422
Std. dev.	4.3015	3.2544	2.1040	4.1658	2.6710	2.9286	2.6430	2.3406	1.3199
Best	4.4619	0.4803	1.5671	0.4452	2.2104	0.9405	0.4880	1.2484	0.5676
Worst	17.5570	9.7283	9.4273	12.9099	10.7099	10.8772	6.4121	8.3254	4.5836
Example-10									
Mean	5.2936	2.9372	3.9806	5.2843	3.2587	3.0013	3.5788	2.5542	2.3159
Variance	3.6868	2.5426	6.5332	16.7323	0.0000	0.6703	5.4082	0.8607	0.9094
Std. dev.	1.9201	1.5945	2.5560	4.0905	0.0031	0.8187	2.3255	0.9277	0.9536
Best	2.2789	0.8743	1.7786	1.1377	3.2514	0.9498	2.3278	1.2737	1.0667
Worst	9.2932	6.3128	12.1111	16.1955	3.2635	4.3677	2.3278	4.1715	3.8592

with ABC), Hybrid-2 (CFI-PSO with ABC) and Hybrid-3 (TVC-PSO with ABC), respectively. The hybridization has resulted in significant improvement in preformation of the proposed techniques tabularized in Table 4. Similar analysis has been performed for other swarm sizes, and performances are summarized in Tables 5, 6 and 7, respectively. It is evident from these Tables that the proposed Hybrid-3 performs better, when compared with above discussed techniques due to their time varying strategy of the control coefficients, which not only helps it for better exploration and exploitation, but also leads it to possess less computation time with superior ability of handling a higher order design problem efficiently than others. It has observed that the proposed Hybrid-3 method provides sustainable performance for all population sizes, which is also reflected from the evaluation of *mean*, obtained for various methods as depicted in Fig. 2.

In second experiment, the effects of population size on filter performance for different orders are studied specifically

Table 7 mean, variance, standard deviation, best and worst values of J for each method for the 30 trials used to design the filter mentioned in Table 3 with the population size of 50

Population size 50									
Parameters	CWI-PSO	CFI-PSO	LDI-PSO	MDI-PSO	TVC-PSO	ABC	Hybrid-1	Hybrid-2	Hybrid-3
Example-1									
Mean	2.9282	2.8809	2.8879	2.8908	2.8806	3.003	3.2963	2.8802	3.2683
Variance	0.0121	0.0000	0.0003	0.0001	0.0000	0.123	0.0000	8.2953	0.0000
Std. dev.	0.1099	0.0025	0.0163	0.0087	0.0023	0.351	0.0000	2.8802	0.0008
Best	2.8770	2.8774	2.8770	2.8850	2.8780	2.878	3.2963	2.8796	3.2671
Worst	3.2770	2.8874	2.9476	2.9146	2.8889	4.304	3.2963	2.8812	3.2697
Example-2									
Mean	3.4042	1.8491	1.7786	2.4942	1.4726	1.555	1.6142	1.6210	1.4020
Variance	2.7524	2.3042	0.7436	3.6071	0.3576	0.206	0.5232	2.8742	0.1384
Std. dev.	1.6590	1.5180	0.8624	1.8992	0.5980	0.454	0.7233	1.6953	0.3720
Best	1.1480	0.5681	0.6598	0.7102	0.6168	0.886	0.6188	0.8352	0.9438
Worst	7.0717	6.7217	4.2990	7.8292	2.4748	2.747	3.5874	2.3365	1.9733
Example-3									
Mean	3.9128	1.5867	2.3836	3.4601	1.5965	2.296	1.7533	2.4112	0.9524
Variance	2.7067	0.6803	1.7339	7.9661	0.5223	1.295	0.7120	8.4331	0.1780
Std. dev.	1.6452	0.8248	1.3168	2.8224	0.7227	1.138	0.8438	2.9040	0.4219
Best	1.8710	0.6105	0.7038	0.6377	0.5409	0.433	0.3792	0.3886	0.4943
Worst	6.9595	3.3739	5.0163	9.4157	3.1062	5.090	3.2083	5.5537	1.7830
Example-4									
Mean	10.9129	2.3085	3.5459	5.5100	4.2063	3.685	3.5956	4.1119	1.6990
Variance	48.0181	2.8985	6.9239	15.3170	10.2615	3.323	10.0448	33.9887	2.0406
Std. dev.	6.9295	1.7025	2.6313	3.9137	3.2034	1.823	3.1694	5.8300	1.4285
Best	3.5698	0.5398	0.7225	1.1233	0.2586	0.686	0.9478	0.5106	0.1275
Worst	32.0263	5.3062	10.3104	14.0958	11.2062	7.224	10.4747	13.4171	5.3439
Example-5									
Mean	5.6848	2.3149	3.4512	2.1653	3.2616	2.455	2.8038	3.2235	2.1804
Variance	15.4906	1.1384	3.9532	0.6626	2.3009	0.305	1.4373	17.5163	0.2185
Std. dev.	3.9358	1.0670	1.9883	0.8140	1.5169	0.552	1.1989	4.1853	0.4674
Best	1.6418	1.0218	1.1782	1.1525	1.7584	1.380	1.3463	0.8853	1.5755
Worst	17.6268	5.5975	7.8362	4.0317	7.9048	3.428	5.0625	10.7227	2.8752
Example-6									
Mean	2.9570	2.8716	2.8738	2.8776	2.8717	2.940	3.2840	2.8712	2.8735
Variance	0.0273	0.0000	0.0000	0.0000	0.0000	0.014	0.0000	0.0000	0.0000
Std. dev.	0.1653	0.0032	0.0050	0.0043	0.0034	0.118	0.0000	0.0024	0.0051
Best	2.8705	2.8679	2.8682	2.8723	2.8681	2.868	3.2840	2.8695	2.8701
Worst	3.4565	2.8784	2.8873	2.8858	2.8787	3.330	3.2840	2.8784	2.8841
Example-7									
Mean	2.3308	1.9320	2.0706	2.2202	1.9582	1.490	1.9472	1.2244	1.4561
Variance	0.9405	0.7536	0.7644	0.6636	4.6978	0.336	1.0949	0.2387	0.3122
Std. dev.	0.9698	0.8681	0.8743	0.8146	2.1674	0.580	1.0464	0.4886	0.5587
Best	0.7184	0.4773	0.9141	0.8758	0.3840	0.470	0.7483	0.4860	0.8057
Worst	4.3524	3.9058	3.6303	3.6947	9.5572	2.667	2.7737	1.8572	2.4960
Example-8									
Mean	5.1996	2.0809	2.2231	3.4917	3.1228	1.986	2.3429	1.9937	1.1003
Variance	6.1056	2.2142	2.3894	13.6616	7.6695	2.025	9.8523	2.3627	0.3300

Table 7 continued

Population size 50									
Parameters	CWI-PSO	CFI-PSO	LDI-PSO	MDI-PSO	TVC-PSO	ABC	Hybrid-1	Hybrid-2	Hybrid-3
Std. dev.	2.4710	1.4880	1.5458	3.6962	2.7694	1.423	3.1388	1.5371	0.5744
Best	1.7170	0.6752	0.5366	0.6864	0.3884	0.586	0.5387	0.3546	0.3171
Worst	11.0128	6.3002	6.0132	12.3704	10.6884	5.556	7.8174	4.4325	2.1012
Example-9									
Mean	10.6608	3.9063	4.1266	7.9172	5.3044	3.817	3.6368	2.2113	1.7785
Variance	8.1126	11.0866	6.3310	25.3977	21.5281	4.834	7.8957	1.1180	2.5197
Std. dev.	2.8483	3.3297	2.5161	5.0396	4.6398	2.199	2.8099	1.0574	1.5873
Best	5.4915	0.9912	0.7434	3.7434	1.0067	0.572	0.7774	0.5833	0.4567
Worst	14.5663	12.7645	9.4509	17.4400	16.3518	9.109	6.9354	4.0694	5.1375
Example-10									
Mean	6.1286	3.4577	3.5769	3.1804	2.8717	2.840	3.0012	2.3359	1.8970
Variance	10.0535	6.3476	6.8257	2.9566	0.0000	1.624	2.1605	1.0031	0.4368
Std. dev.	3.1707	2.5194	2.6126	1.7195	0.0034	1.274	1.4699	1.0015	0.6609
Best	2.2743	1.4542	1.0734	1.4218	2.8681	0.666	1.4936	1.0074	1.0996
Worst	15.8034	11.3490	11.7951	7.2154	2.8787	5.194	7.8424	3.7673	3.6048

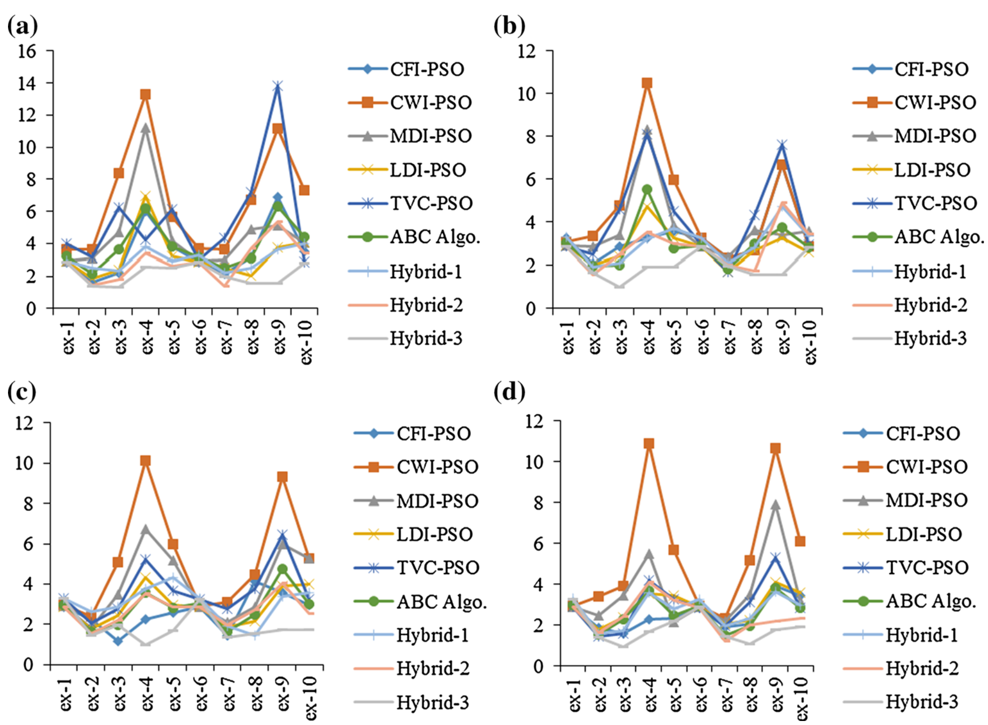


Fig. 2 **a** mean of J for quoted techniques for 10 used examples for the population size of 20. **b** mean of J for quoted techniques for 10 used examples for the population size of 30. **c** mean of J for quoted techniques for 10 used examples for the population size of 40. **d** mean of J for quoted techniques for 10 used examples for the population size of 50

for Hybrid-3 method. For this purpose, same filter specifications of example-5 and example-11 are utilized for different orders ranges from 2 to 15 with increment of 1, and in swarm sizes from 5 to 50 with linear increment of 5. It can be observed from Fig. 3 that the proposed technique works consistently efficient, for the entire range of filter orders and swarm sizes. On the basis of above discussed experiments, it is evident that the better performance in term of J can be achieved irrespective of filter order, which can be further utilized for designing the both LPF and HPF with less computation cost. In addition to this, the designed filter using proposed technique shows better fidelity parameter values

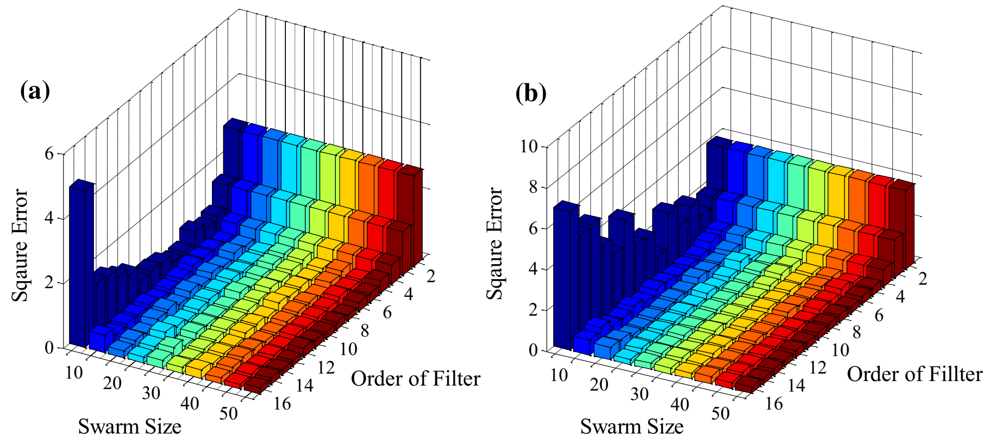


Fig. 3 **a** Population size effect on LPF filter for different order. **b** Population size effect on HPF filter for different order

Table 8 Comparative analysis of best fidelity parameters obtained for proposed algorithm with different techniques

S. No.	Technique	*J* (Eq. 8)		Passband error (e_p)		Stopband error (e_s)		Stopband edge fr. attenuation (dB)		CPU time elapsed in sec	
		LPF	HPF	LPF	HPF	LPF	HPF	LPF	HPF	LPF	HPF
1	Algorithm in (Saha et al. 2012b)	2.7808		0.4735	–	0.0628	–	25.2503	–	–	–
2	Algorithm in(Saha et al. 2011, 2012a, 2013)	2.7692	1.2453	0.5439	0.0303	0.0223	0.0148	34.5323	46.4287	–	–
3	PSO (Saha et al. 2011, 2012b, a, 2013)	2.1310	1.2453	0.2680	0.0303	0.0633	0.0148	25.1561	46.4287	–	–
4	CWI-PSO	3.9165	2.8108	0.1322	0.46216	1.06828	0.5029	14.2782	19.8948	97.0170	75.6916
	CFI-PSO	1.0218	1.1852	0.2988	0.1117	0.4696	0.4439	15.1558	15.1400	94.1934	94.6302
5	LDI-PSO	1.1365	1.400	0.2418	0.1798	0.2128	0.4580	24.7402	15.4859	74.5685	92.4150
6	MDI-PSO	1.3927	2.5807	0.2122	0.0355	0.3724	1.1033	18.1137	8.2184	72.7745	97.6410
	TVC-PSO	1.7584	0.9987	0.2395	0.1885	0.5043	0.3770	15.5230	16.2906	70.7153	70.3253
7	ABC	1.3804	1.0982	0.1358	0.1411	0.2722	0.1940	14.8225	17.2296	10263.73	69053.5290
8	Hybrid 1	1.1158	3.5446	0.1355	0.5140	0.2783	0.9121	21.1556	13.7253	65.4736	76.1285
9	Hybrid 2	0.7650	1.8818	0.2807	0.1943	0.3042	0.5240	17.8353	18.9843	51.1371	79.3889
10	Hybrid 3	0.7229	1.1942	0.0256	0.2115	0.2975	0.3628	17.0690	16.9325	47.2059	52.9779

than conventional ABC, PSO and its variant technique, which is summarized in Table 8.

5.3 Complexity of the algorithm

Computation complexity of the proposed method is measured in term of ‘O-notation’. It is evident from Table 9 that computation complexity of proposed hybrid method is $O(n^2)$, as only additional search and replacement of unimproved solution is executed. However, search mechanism of the proposed technique is improved as compared to other hybrid algorithm with less complexity. Computation time and total number of function evaluation (NFE) involved has been slightly increased compared to non-hybrid PSO.

5.4 Comparison with other existing methods

For the justification and significance of proposed technique in optimal design of IIR filter with other existing techniques, a numerical example has been taken and mentioned as example 6 and example 11 in Table 3 (Saha et al. 2013). The best performances corresponding to all developed methods are compared and tabularized in Table 8, which clearly indicates that the performance of proposed method is better than previously quoted versions of PSO and ABC algorithms. Moreover, the performance of the proposed technique is also evaluated by calculating the filter fidelity parameters such as: passband error, stopband error, and maximum stopband attenuation defined as (Rafi et al. 2013):

Table 9 Computational complexity measured in terms of O-notation

S. No.	Algorithm	Computational complexity in term of 'O-notation'
1	Hybrid PSO (Bare-bone PSO with directionally chaotic search)(Zhang et al. 2014)	$O(n^3)$
2	Hybrid PSO (TVC-PSO with DE) (Gong et al. 2010)	$O(n^3)$
3	Improved PSO (Rafi et al. 2013)	$O(n^2)$
4	CWI-PSO	$O(n^2)$
5	MDI-PSO	$O(n^2)$
6	TVC-PSO	$O(n^2)$
7	Proposed Hybrid-PSO(TVC-PSO with ABC Algo.)	$O\ n^2$

$$e_p = \sum_{\omega \in \omega_p} [D(\omega) - H(\omega)]^2 \tag{18}$$

$$e_s = \sum_{\omega \in \omega_s} [D(\omega) - H(\omega)]^2 \tag{19}$$

$$SE = e_p + e_s \tag{20}$$

$$A_{stop} = -20 \cdot \log_{10}(|H(\omega)|),\ at\ \omega = \omega_s \tag{21}$$

In previous techniques, the obtained solution for optimal IIR often suffers from quantization and truncation effect (Saha et al. 2011, 2012b, a, 2013), in which filter response is degraded during the truncating and quantizing the coefficients of numerator and denominator polynomials, as shown in Fig. 4a, b; therefore; computation cost would be a little bit higher for this system. However, the solution explored by proposed method has shown immunity to quantization effect, and sustained performance has recorded with lesser number of bits, as shown in Fig. 4c, d, respectively. These figures clearly indicate the better passband and stopband response with stability, where the poles lied inside the unity circle confirms the sustainable execution of stability mechanism, as depicted in Fig. 5. The power of exploring ability of proposed method is depicted in Fig. 6.

6 Conclusion

In this paper, an improved hybrid method is exploited for the design of optimal digital IIR filter based on minimization of nonlinear objective function constructed in frequency domain using prescribed passband and stopband ripples. The experiments based on statistical analysis evidence that the proposed hybrid method shows less deviation of fitness/ error function between best and worst values, when compared to

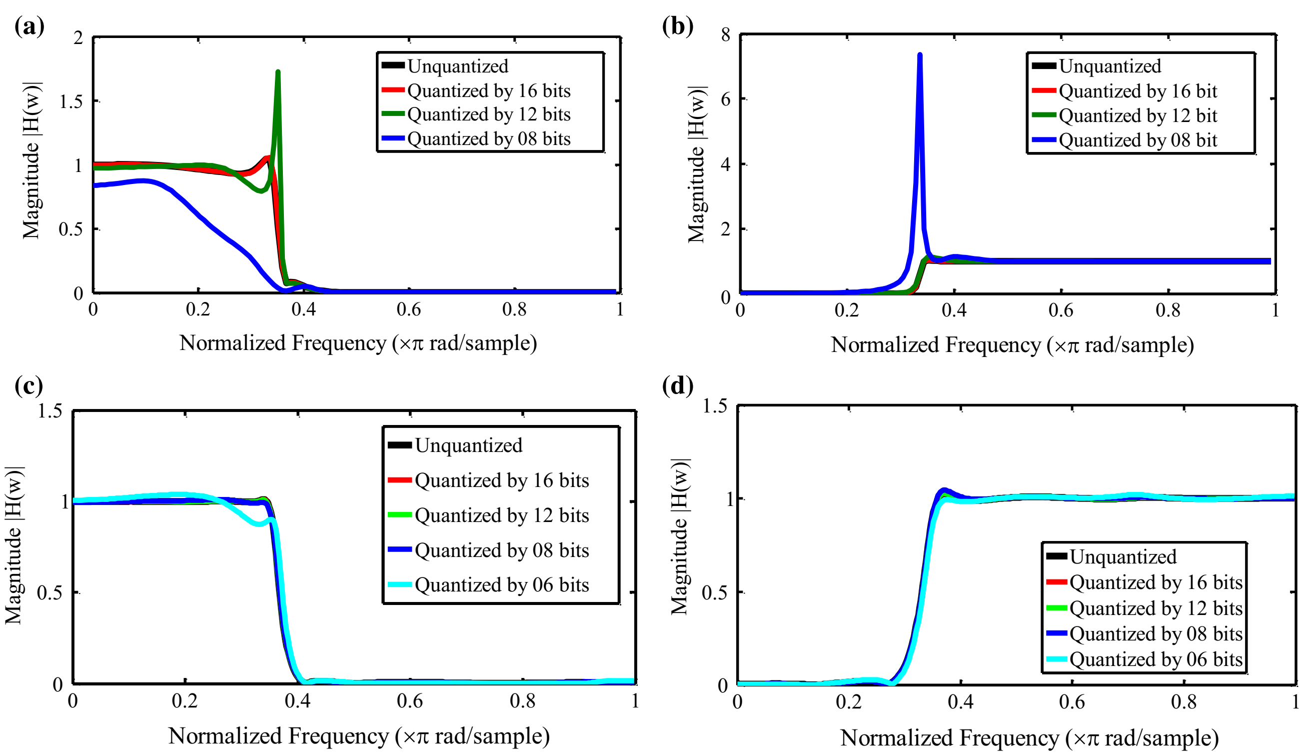


Fig. 4 Effect of truncation and quantization of filter tap coefficients obtained by **a**, **b** for algorithm proposed in (Saha et al. 2013) **c** Hybrid-3 for LPF, **d** Hybrid-3 for HPF

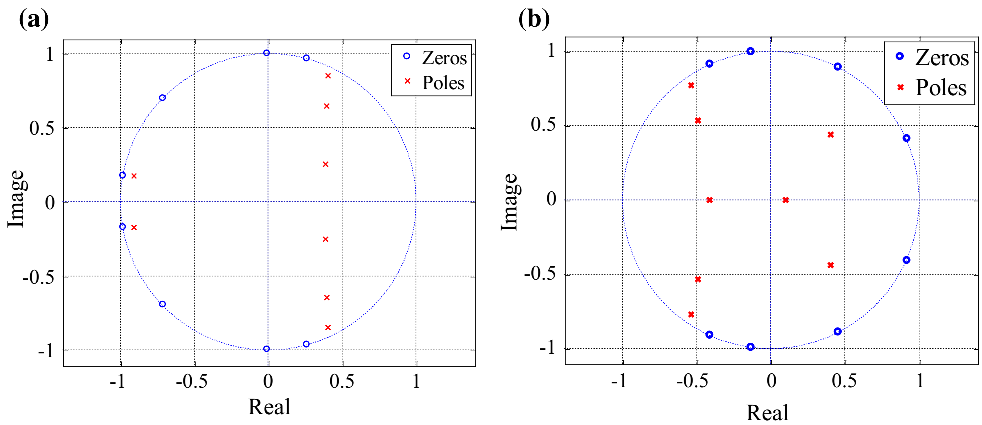
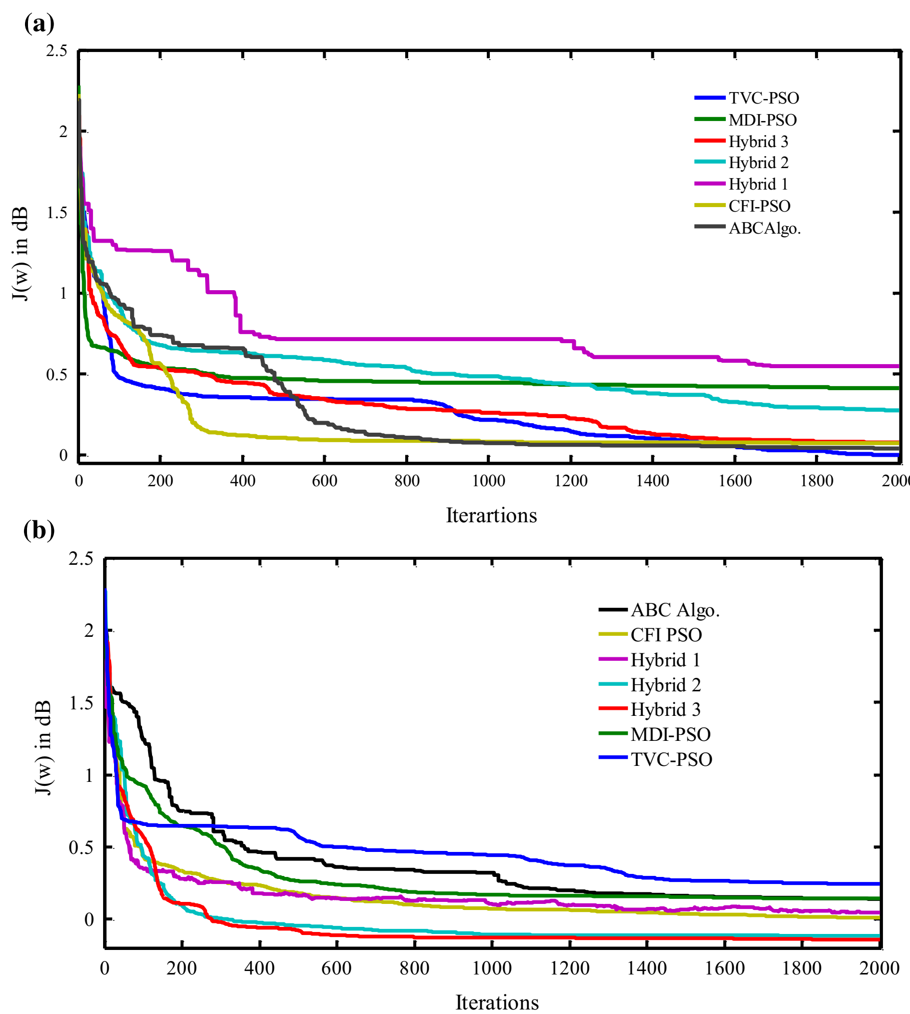


Fig. 5 **a** Poles and zero location for LPF designed using Hybrid-3. **b** Poles and zeros for HPF designed using Hybrid-3

Fig. 6 Convergence profile of various swarm-based methods. **a** HPF, **b** LPF



other optimization methods. The proposed algorithm also helps for non-convex and non-differentiable design problem of IIR filter with acceptable fidelity parameters using finest swarm size, while the complexity remains same as for conventional PSO. Several design examples have been included to demonstrate the effect of swarm sizes and efficiency of proposed method to handle large-scale optimization problem. The simulation results illustrate that the proposed technique is efficient in term of stability, and designed filter does not suffer from degradation due to quantization effect. Therefore, the designed filter can be realized by quantizing using six bits only, hence helps in fast realization of filters. The incorporation of sorting scheme enables the system to be self-intelligent for selecting the best solution out of the executed trials. The proposed technique can be extended for fractional delay IIR filter design and also for reconfigurable IIR filter design.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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