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METHODOLOGIES AND APPLICATION

# An intuitionistic fuzzy multi-criteria decision-making method based on non-hesitance score for interval-valued intuitionistic fuzzy sets

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**Abstract** Fuzzy numbers and intuitionistic fuzzy numbers are introduced in the literature to model problems involving incomplete and imprecise numerical quantities. Researchers from all over the world have been working in ranking of intuitionistic fuzzy numbers since 1985, but till date there is no common methodology that rank any two arbitrary intuitionistic fuzzy numbers. In order to improve the familiar ranking methods, a new non-hesitance score function for the theory of interval-valued intuitionistic fuzzy sets is introduced and the necessity for defining a new non-hesitance score function is explained using illustrative examples. In this paper, a new multi-criteria decision-making algorithm is established for decision problems involving interval-valued intuitionistic fuzzy numbers. Further the practicality of the proposed method is shown by solving an interval-valued intuitionistic fuzzy MCDM problem. Finally, an illustrative example is given to demonstrate the practicality and effectiveness of the proposed approach.

**Keywords** Intuitionistic fuzzy number · Interval-valued intuitionistic fuzzy number · Non-hesitance score · MCDM · IFMCDM

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## 1 Introduction

The data which are available for an alternative in an information system with respect to attributes may be imprecise or qualitative linguistic terms or incomplete in nature. To solve such problems with imprecision, ambiguity and uncertainty, the concept of fuzzy sets introduced by Zadeh (1965) is used. Theory of fuzzy sets has developed its own measures of qualitative information, which finds application in areas such as management, medicine and engineering, and any decision-making problem involving three steps, namely design of an information system (decision matrix) and collection of data from experts, finding of aggregated performance of each alternative with respect to all criteria, ranking of alternatives according to its aggregated performances. In interval-valued intuitionistic fuzzy decision problem, aggregated performance of an alternative is represented by interval-valued intuitionistic fuzzy numbers, and then, it is necessary for us to define a new score function which rank interval-valued intuitionistic fuzzy numbers.

Ranking of IFN assumes a vital part in problems with inadequate information. When compared with intuitionistic fuzzy number (IFN), interval-valued intuitionistic fuzzy number (IVIFN) is considered to be an important class of data in decision analysis. Unclear and uncertain information can be dealt better using IVIFN over IFN. By stretching out intuitionistic fuzzy numbers to IVIFNs, it is easy to handle unclear and uncertain information successfully, because the imprecise, membership and non-membership degrees are expressed only as ranges of values rather than exact value.

The main aim of this paper is to investigate the shortcomings of an accuracy function introduced by Sahin (2015) and Zhang and Xu (2015), and to define a new non-hesitance score function for better ranking of IVIFNs. The rest of this paper is organized as follows. After introduction, some necessary basic definitions are given in Sect. 2. In Sect. 3, a short review of ranking methods presented in Zhang and Xu (2015), Sahin (2015), Xu (2007c), Ye (2009) and Yang et al. (2015) is given and also a new non-hesitance score function is introduced. In Sect. 4, a new intuitionistic fuzzy multi-criteria decision-making method for solving MCDM problem is introduced by utilizing the proposed ranking approach and it is illustrated by solving numerical example. Finally, conclusions are given in Sect. 5.

## 2 Preliminaries

Some basic definitions are given in this section.

**Definition 2.1** (Atanassov 1986) Let $X$ be a nonempty set. An intuitionistic fuzzy set (IFS) $A$ in $X$ is defined by $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|x \in X\}$, where $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1], x \in X$ with the conditions $0 \le \mu_A(x)+\nu_A(x) \le 1, \forall x \in X$. The numbers $\mu_A(x), \nu_A(x) \in [0, 1]$ denote the degree of membership and non-membership of $x$ to lie in $A$, respectively. For each intuitionistic fuzzy subset $A$ in $X$, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called hesitancy degree of $x$ to lie in $A$.

**Definition 2.2** (Atanassov 1986) The complement $A^c$ of $A = \langle x, \mu_A(x), \nu_A(x) : x \in X\rangle$ is defined by $A = \langle x, \nu_A(x), \mu_A(x) : x \in X\rangle$.

**Definition 2.3** (Xu 2007c) Let $D[0, 1]$ be the set of all closed subintervals of the interval $[0, 1]$. An interval-valued intuitionistic fuzzy set on a set $X \neq \phi$ is an expression given by $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A : X \to D[0, 1], \nu_A : X \to D[0, 1]$ with the condition $0 < sup_x \mu_A(x) + sup_x \nu_A(x) \le 1$.

The intervals $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degree of belongingness and non-belongingness of the element $x$ to the set $A$. Thus for each $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ are closed intervals whose lower and upper end points are, respectively, denoted by $\mu_{A_L}(x)$, $\mu_{A_U}(x)$ and $\nu_{A_L}(x)$, $\mu_{A_U}(x)$. We denote $A = \left\{\langle x, [\mu_{A_L}(x), \mu_{A_U}(x)], [\nu_{A_L}(x), \nu_{A_U}(x)]\rangle : x \in X\right\}$ where $0 < \mu_{A_U}(x) + \nu_{A_U}(x) \le 1$

For each element $x \in X$, we can compute the unknown degree (hesitance degree) of belongingness $\pi_A(x)$ to $A$ as $\pi_A(x) = 1-\mu_A(x)-\nu_A(x) = [1-\mu_{A_U}(x)-\nu_{A_U}(x), 1-\mu_{A_L}(x) - \nu_{A_L}(x)]$. An intuitionistic fuzzy interval number (IFIN) is denoted by $A = ([a, b], [c, d])$ for convenience.

**Definition 2.4** (Xu 2007c) Let $A = ([a, b], [c, d])$ be an IVIFN. Then, the score function $S$ of $A$ and an accuracy function $H$ of $A$ are defined, respectively, as follows $S(A) = \frac{a+b-c-d}{2}$ and $H(A) = \frac{a+b+c+d}{2}$.

**Definition 2.5** (Xu 2007c) Let $A_j$ $(j = 1, \ldots, n) \in IVIFS(X)$. Then, the weighted arithmetic average operator is defined by,

$$F_w(A_1, A_2, \ldots, A_n) = \sum_{j=1}^{n} w_j A_j$$
$$= \left(\left[1 - \prod_{j=1}^{n}\left(1-\mu^L_{A_j}(x)\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1-\mu^U_{A_j}(x)\right)^{w_j}\right],\right.$$
$$\left.\times\left[\prod_{j=1}^{n}\left(\nu^L_{A_j}(x)\right)^{w_j}, \prod_{j=1}^{n}\left(\nu^U_{A_j}(x)\right)^{w_j}\right]\right)$$

where $w_j$ is the weight of $A_j$ $(j = 1, \ldots, n)$, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

**Definition 2.6** (Xu 2007c) Let $A_j$ $(j = 1, \ldots, n) \in IVIFS(X)$. Then, the weighted geometric average operator is defined by,

$$G_w(A_1, A_2, \ldots, A_n)$$
$$= \sum_{j=1}^{n} A_j^{w_j} = \left(\left[\prod_{j=1}^{n}\left(\mu^L_{A_j}(x)\right)^{w_j}, \prod_{j=1}^{n}\left(\mu^U_{A_j}(x)\right)^{w_j}\right],\right.$$
$$\left.\times\left[1 - \prod_{j=1}^{n}\left(1-\nu^L_{A_j}(x)\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1-\nu^U_{A_j}(x)\right)^{w_j}\right]\right)$$

where $w_j$ is the weight of $A_j$ $(j = 1, \ldots, n)$, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

**Definition 2.7** (Xu 2007c) Let $A_1$ and $A_2$ be two IVIFNs. Then, the ranking principle is defined as follows,

- If $S(A_1) > S(A_2)$, then $A_1 > A_2$
- If $S(A_1) < S(A_2)$, then $A_1 < A_2$
- If $S(A_1) = S(A_2)$, then
  - If $H(A_1) < H(A_2)$, then $A_1 < A_2$
  - If $H(A_1) > H(A_2)$, then $A_1 > A_2$
  - If $H(A_1) = H(A_2)$, then $A_1 \approx A_2$.

**Definition 2.8** (Ye 2009) Let $A = ([a, b], [c, d])$ be an IVIFN. Then, the novel accuracy score function $M$ of $A$ is defined as, $M(A) = a + b - 1 + \frac{c+d}{2}$.

**Definition 2.9** (Nayagam et al. 2011) Let $A = ([a, b], [c, d])$ be an IVIFN. Then, the new novel accuracy score function $L$ of $A$ is defined as, $L(A) = \frac{a-d(1-b)+b-c(1-a)}{2}$.

**Definition 2.10** (Sahin 2015) Let $A = ([a, b], [c, d])$ be an IVIFN. Then, the improved accuracy score function $K$ of $A$ is defined as, $K(A) = \frac{a+b(1-a-c)+b+a(1-b-d)}{2}$.

**Definition 2.11** (Zhang and Xu 2015) Let $A = ([a, b], [c, d])$ be an IVIFN. Then, the contrast degree between the membership and nonmembership degrees of IVIFN $A$ is defined as, $c(A) = \frac{1}{2}\left(\frac{(a-c)+(b-d)}{2} + 1\right)$.

**Definition 2.12** (Zhang and Xu 2015) Let $A = ([a, b], [c, d])$ be an IVIFN. Then, the new accuracy score function $\overline{f}$ of IVIFN $A$ is defined as, $\overline{f}(A) = \frac{1}{2}\left[\frac{(a-c)+(b-d)(1-a-c)}{2} + \frac{(b-d)+(a-c)(1-b-d)}{2} + 1\right]$.

## 3 A new non-hesitance Score function for interval-valued intuitionistic fuzzy numbers

In this section, a new non-hesitance score function is introduced to overcome the shortcomings of familiar methods presented in Zhang and Xu (2015), Nayagam et al. (2011), Sahin (2015), Xu (2007c), Ye (2009) and Yang et al. (2015).

Xu (2007c) has introduced the concept of score function and accuracy function to rank arbitrary IVIFNs. If $A = ([a_1, a_2], [a_3, a_4])$ and $B = ([b_1 = a_1 - \epsilon, b_2 = a_2 + \epsilon], [b_3 = a_3 - \epsilon, b_4 = a_4 + \epsilon])$ be any two IVIFNs. Then, by Definition 2.4, we get $S(A) = S(B) = \frac{a+b-c-d}{2}$, $H(A) = H(B) = \frac{a+b+c+d}{2} \Rightarrow A = B$. i.e., Xu's method does not rank the IVIFNs of above kind which is a very particular subclass of the class of IVIFNs.

*Example 3.1* Illogicality of Xu (2007c) approach: Let $A = ([0.1, 0.2], [0.3, 0.4])$ and $B = ([0, 0.3], [0.2, 0.5])$ be two IVIFNs. Then, by Definition 2.4 we get $S(A) = S(B) = -0.2$ and $H(A) = H(B) = 0.5 \Rightarrow A = B$ which is illogical because $A \neq B$.

Ye (2009) presented the ranking procedure for IVIFNs using a novel accuracy score function. But unfortunately his method also fails to cover even the class of symmetric IVIFNs. Nayagam et al.'s (2011) ranking method also has the same drawback as Xu's (2007c) in ranking symmetric IVIFNs. If $A = ([a_1, a_2], [a_3, a_4])$ and $B = ([b_1 = a_1 + \epsilon, b_2 = a_2 - \epsilon], [b_3 = a_3 - \epsilon, b_4 = a_4 + \epsilon])$ are any two IVIFNs. Then, by Definition 2.8, we get $M(A) = M(B) = a + b - 1 + \frac{c+d}{2} \Rightarrow A = B$ but $A \neq B$ which is anti-intuitive.

*Example 3.2* Illogicality of Ye (2009) accuracy score function in ranking IVIFNs: Let $A = ([0.13, 0.26], [0.17, 0.43])$ and $B = ([0.17, 0.22], [0.09, 0.51])$ be two IVIFNs. Then, by Definition 2.8 we get $M(A) = M(B) = -0.31 \Rightarrow A = B$ which is illogical.

To overcome the shortcomings of aforementioned ranking methods, Nayagam et al. (2011) have introduced the new novel accuracy score function to rank arbitrary IVIFNs. But due to the partial ordering of IVIFSs, his method also gives anti-intuitive results in some cases.

*Example 3.3* Illogicality of Nayagam et al.'s (2011) novel accuracy score function in ranking IVIFNs: Let $A = ([0.3, 0.3], [0.2, 0.2])$ and $B = ([0.3, 0.4], [0.2, 0.4])$ be two IVIFNs. Then, by Definition 2.9 we get $L(A) = L(B) = 0.16 \Rightarrow A = B$. But $A \neq B$ which is illogical.

Sahin (2015) has introduced the new improved accuracy score function to rank IVIFNs, and he claimed that his method is far better than the existing methods, but unfortunately his method also fails to rank IVIFNs in some places which is shown in the following Examples 3.4 and 3.5.

*Example 3.4* Illogicality of Sahin's (2015) ranking method: Let $A = ([0, 0], [c, d])$ and $B = ([0, 0], [e, f])$ be two IVIFNs. Then, by Definition 2.10 we get $K(A) = K(B) = 0 \Rightarrow A = B$. But $A \neq B$ which is illogical.

*Example 3.5* Illogicality of Sahin's (2015) ranking method: Let $A = ([0, 0], [0.1, 0.1])$ and $B = ([0, 0], [0.2, 0.3])$ be two IVIFNs. Then, by Definition 2.10 we get $K(A) = K(B) = 0 \Rightarrow A = B$. But $A \neq B$ which is anti-intuitive.

Zhang and Xu (2015) have pointed out the illogicality of Sahin's (2015) method and introduced the concept of contrast of an IVIFNs and proposed the new accuracy score function.

But their method also fails to give better results for the following subclass of IVIFNs.

If $A = ([a, b], [a, b])$ and $B = ([e, f], [e, f])$ are any two IVIFNs, then we get $c(A) = c(B) = \frac{1}{2}$ and $\overline{f}(A) = \overline{f}(B) = \frac{1}{2}$. For different values of $a, b, e, f \in [0, 0.5]$, we will get infinite number of IVIFNs which cannot be ranked by Zhang and Xu's (2015) ranking approach.

*Example 3.6* Let $A = ([0.3, 0.3], [0.3, 0.3])$ and $B = ([0.4, 0.4], [0.4, 0.4])$ be two IVIFNs. Then, by Definition 2.11 we get $c(A) = c(B) = \frac{1}{2}$ and $\overline{f}(A) = \overline{f}(B) = 0.5$. But $A \neq B$ which is anti-intuitive to the result of Zhang and Xu's (2015) method.

In order to overcome the shortcomings of all the above methods, a new non-hesitance score function is introduced to rank IVIFNs as follows:

**Definition 3.1** Let $A = ([a, b], [c, d])$ be an IVIFN. Then, the non-hesitance score $J$ of an IVIFN is defined as $J(A) = \frac{a+b+c-d+ab+cd}{3}$.

**Definition 3.2** Let $V = \{A = ([a_1, b_1], [c_1, d_1]), B = ([a_2, b_2], [c_2 = c_1 + \epsilon, d_2 = d_1 - \epsilon]) \in IVIFN$ with $a_1 < a_2, b_1 < b_2\}$ for different values of $a_1, a_2, b_1, b_2, c_1, d_1, \in [0, 1], 0 \leq \epsilon < \frac{(d_1 - c_1)}{2} \in [0, 1]$ we get an infinite subclass $V$ of the set of IVIFN. Our aim is to define a total order relation on the subclass $V$.

**Definition 3.3** Ranking Principle:

Let $A = ([a, b], [c, d]), B = ([e, f], [g, h])$ be any two IVIFN. If $J(A) < J(B)$, then $A < B$.

This new $<$ relation defines a total order on the subclass $V$ of IVIFNs which can be seen from the following theorem.

**Theorem 3.1** *If $A, B \in V$, then $J(A) < J(B)$.*

*Proof* Let $A, B \in V$. Therefore from Definition 3.2, without loss of generality we assume that, $A = ([a_1, b_1], [c_1, d_1])$, $B = ([a_2, b_2], [c_2 = c_1 + \epsilon, d_2 = d_1 - \epsilon]) \Rightarrow a_1 < a_2, b_1 < b_2, c_1 < c_2, d_1 > d_2$. Now we have to show that $J(B) - J(A) \geq 0$.

$$\begin{aligned}3(J(B) - J(A)) &= (a_2 + b_2 + c_2 - d_2 + a_2b_2 + c_2d_2) \\ &\quad - (a_1 + b_1 + c_1 - d_1 + a_1b_1 + c_1d_1) \\ &= (a_2 - a_1) + (b_2 - b_1) + (c_2 - c_1) \\ &\quad + (d_1 - d_2) + (a_2b_2 - a_1b_1) \\ &\quad + (c_2d_2 - c_1d_1).\end{aligned}$$

(Since $c_2d_2 = (c_1 + \epsilon)(d_1 - \epsilon) = c_1d_1 + \epsilon(d_1 - c_1) - \epsilon^2 > c_1d_1$) Since each term in the above sum is positive. Therefore, $3(J(B) - J(A)) \geq 0 \Rightarrow J(B) - J(A) \geq 0$. Hence, the proof. □

**Proposition 3.1** *Let $A$ be an IVIFN. Then*
*$J(A) = 0$ when $A = ([0, a], [0, a]), a \in [0, \frac{1}{2}]$.*
*$J(A) = 1$ when $A = ([1, 1], [0, 0])$.*

Significance of the proposed method is given in Table 1.

In the next section, a new algorithm for solving intuitionistic fuzzy multi-criteria decision-making problem based on the non-hesitance score function is established, and further, a numerical example is illustrated.

## 4 A new intuitionistic fuzzy multi-criteria decision-making method based on non-hesitance score function

Any decision-making problem involving following three steps:

1. design of an information system (decision matrix) and collection of data from experts; 2. finding of aggregated performance of each alternative with respect to all criteria; and 3. ranking of alternatives according to its aggregated performances. In interval-valued intuitionistic fuzzy decision problem, each entry in the decision matrix is represented by IVIFNs and the aggregated performance of an alternative is also represented by interval-valued intuitionistic fuzzy numbers.

We define a decision problem mathematically as follows:

Let $A = \{A_1, A_2, \ldots, A_m\}$ be the set of alternatives and let $C = \{C_1, C_2, \ldots, C_n\}$ be the set of criteria under which the performance of alternatives will be evaluated. Let $w_j$ be the weight vector given by the decision maker for each criteria $C_j$, where $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Let $R = (B_{ij})_{m\times n} = \big([a_{ij}, b_{ij}], [c_{ij}, d_{ij}]\big)$ be an interval- valued intuitionistic fuzzy decision matrix, where $a_{ij}, b_{ij}, c_{ij}, d_{ij} \in [0, 1]$ with $b_{ij} + d_{ij} \leq 1$, $i = 1, \ldots, m$ and $j = 1, 2, \ldots, n$. Each entry $B_{ij}$ in the decision matrix represents the performance degree of each alternative $A_i$ with respect to criteria $C_j$. That is $[a_{ij}, b_{ij}]$ represents the degree of alternative $A_i$ which satisfies criteria $C_j$, whereas $[c_{ij}, d_{ij}]$ represents degree of alternative $A_i$ which does not satisfy the criteria $C_j$.

In this decision problem, our aim is to choose the best alternative from the set of alternatives with respect to its aggregated performance under different criteria. In order to show the effectiveness of the proposed method, we establish an algorithm for solving interval-valued intuitionistic fuzzy multi-criteria decision- making (IVIFMCDM) problem as follows:

**Algorithm 4.1** *Step 1: Calculate the aggregated interval-valued intuitionistic fuzzy number (IVIFN) $B_i$ for each alternative $A_i (i = 1, 2, \ldots, m)$ using Definition 2.5 (2.6). i.e., $B_i = ([a_i, b_i], [c_i, d_i]) = F_{iw}(B_{i1}, B_{i2}, \ldots, B_{in})$ (or $B_i = ([a_i, b_i], [c_i, d_i]) = G_{iw}(B_{i1}, B_{i2}, \ldots, B_{in})$) according to each row in decision matrix. (This step is same as Sahin's algorithm.)*
*Step 2: Compute the non-hesitance score $J(B_i)$ for the aggregated IVIF value $B_i$ $(i = 1, 2, \ldots, m)$ using Definition 3.1.*
*Step 3: Rank the alternatives $A_1, A_2, \ldots, A_m$ by using their non-hesitance scores and choose the best alternative according to higher values of $J(B_i)$ using Definition 3.3.*

*Example 4.1* (Numerical example adopted from Herrera and Herrera-Viedma (2000) and illustrated in Sahin (2015):) Assume a panel with four possible alternatives to invest the money, which are a car company $A_1$, a food company $A_2$, a computer company $A_3$ and an arms company $A_4$. The investment company wants to decide a decision according to three criteria given by the risk analysis $C_1$, the growth analysis $C_2$ and the environmental impact analysis $C_3$.

Sahin has solved the above problem using his ranking method, but in 2015, Yang et al. have pointed out the inappropriacy of Sahin's (2015) accuracy function in ranking alternatives using Example 4.1 and also they have shown the efficiency of Sahin's (2015) accuracy function in ranking alternatives by solving the modified version of Example 4.1. In this paper, we show the inefficiency of Sahin's (2015) accuracy function using Example 4.2.

*Example 4.2* Let us reconsider the Example 4.1, suppose the four possible alternatives are to be re-evaluated according to the three criteria and the evaluated results are listed in the following matrix:

$$M = \begin{bmatrix} & C_1 & C_2 & C_3 \\ A_1 & ([0.25, 0.45], [0.25, 0.45]) & ([0.25, 0.45], [0.25, 0.45]) & ([0.25, 0.45], [0.25, 0.45]) \\ A_2 & ([0, 0], [0.1, 0.2]) & ([0, 0], [0, 0.3]) & ([0, 0], [0.15, 0.15]) \\ A_3 & ([0, 0], [0.1, 0.1]) & ([0, 0], [0.2, 0.2]) & ([0, 0], [0.1, 0.3]) \\ A_4 & ([0.1, 0.3], [0.1, 0.3]) & ([0.1, 0.3], [0.1, 0.3]) & ([0.1, 0.3], [0.1, 0.3]) \end{bmatrix}$$

**Decision Matrix**

Assume that the weights of $C_1$, $C_2$ and $C_3$ are 0.25, 0.50 and 0.25, respectively. Then, we apply Algorithm 4.1 to choose the optimal alternative.

Step 1: Aggregated performance of each alternative $A_i (i = 1, 2, 3, 4)$ with respect to criteria $C_1, C_2, C_3$ can be computed using Definition 2.5 as follows:

$$B_1 = ([0.25, 0.45], [0.25, 0.45]),$$
$$B_2 = ([0, 0], [0, 0.227951]),$$
$$B_3 = ([0, 0], [0.141421, 0.186121]),$$
$$B_4 = ([0.1, 0.3], [0.1, 0.3]).$$

Step 2: If we apply Definition 2.10 to each $B_i$ $(i = 1, 2, 3, 4)$, then we get $K(B_1) > K(B_4) > K(B_2) = K(B_3) \Rightarrow A_1 > A_4 > A_2 = A_3$. The improved accuracy score function introduced by Sahin (2015) fails to rank the four alternatives. Also if we apply Definition 2.11, Definition 2.12 we have $c(B_1) = c(B_4) > c(B_2) > c(B_3)$, $\overline{f}(B_1) = \overline{f}(B_4) > \overline{f}(B_2) > \overline{f}(B_3) \Rightarrow A_1 = A_4 > A_2 > A_3$. That is Zhang and Xu's (2015) method also fails to rank four alternative. By using Definition 3.1, we have $J(B_1) = 0.241667 > J(B_4) = 0.086667 > J(B_3) = -0.00613 > J(B_4) = -0.07598 \Rightarrow A_1 > A_3 > A_4 > A_2$ which favors human intuition. Therefore, our proposed non-hesitance score function $J$ ranks the alternatives correctly. Hence, the non-hesitance score function is better when it is compared with Sahin's and Fangwei et al.'s approach.

### 4.1 Future scope

Any intuitionistic fuzzy multi-criteria decision-making method involves three steps: 1. design of an information system (decision matrix) and collection of data from experts; 2. finding of aggregated performance of each alternative with respect to all criteria; and 3. ranking of alternatives according to its aggregated performances. In this paper, we have concentrated only on step 3. In interval-valued intuitionistic fuzzy decision problem, performance of alternatives is represented by interval-valued intuitionistic fuzzy numbers (IVIFNs), and then, it is necessary for us to define a complete ranking on the class of intuitionistic fuzzy numbers. This newly proposed ranking procedure will make any intuitionistic fuzzy decision-making algorithm more effective when it is compared with Nayagam et al. (2011), Sahin (2015) and

**Table 1** Significance of the proposed method

| Existing methods | Shortcomings of existing methods | Proposed method based on non-hesitance |
|---|---|---|
| Xu (2007c) | | |
| $A = ([0.1, 0.2], [0.3, 0.4])$ and $B = ([0, 0.3], [0.2, 0.5])$ | $S(A) = S(B) = -0.2$, $H(A) = H(B) = 0.5 \Rightarrow A \approx B$ | $J(A) = 0.113 > J(B) = 0.033 \Rightarrow A > B$ |
| Ye (2009) | | |
| $A = ([0.13, 0.26], [0.17, 0.43])$ and $B = ([0.17, 0.22], [0.09, 0.51])$ | $M(A) = M(B) = -0.31 \Rightarrow A = B$ | $J(A) = 0.079 > J(B) = 0.018 \Rightarrow A > B$ |
| Nayagam et al. (2011) | | |
| $A = ([0.3, 0.3], [0.2, 0.2])$ and $B = ([0.3, 0.4], [0.2, 0.4])$ | $L(A) = L(B) = 0.16 \Rightarrow A = B$ | $J(A) = 0.24 > J(B) = 0.23 \Rightarrow A > B$ |
| Sahin (2015) | | |
| $A = ([0, 0], [0.1, 0.1])$ and $B = ([0, 0], [0.2, 0.3])$ | $K(A) = K(B) = 0 \Rightarrow A = B$ | $J(A) = 0.003 > J(B) = -0.01 \Rightarrow A > B$ |
| Zhang and Xu (2015) | | |
| $A = ([0.3, 0.3], [0.3, 0.3])$ and $B = ([0.4, 0.4], [0.4, 0.4])$ | $c(A) = c(B) = \frac{1}{2}$ and $\overline{f}(A) = \overline{f}(B) = 0.5 \Rightarrow A = B$ | $J(A) = 0.26 < J(B) = 0.37 \Rightarrow A > B$ |

Zhang and Xu (2015) which is shown Table 1 and Example 4.2.

In this paper, we have introduced a new ranking procedure for ranking arbitrary IVIFNs using new non-hesitance score function. Xu (2013) has introduced some ranking procedures on the class of IVIFNs using different methodologies. Numerous ranking methods available in the literature fail in comparing arbitrary IVIFNs. That is, in some places their method rank two different IVIFNs as same due to the partial ordering of IVIFNs. The proposed ranking procedure also has the same drawback. Any two IVIFNs with equal non-hesitance score function need not imply the equality of the aforesaid IVIFNs. Though an ordering on the class of IVIFNs using non-hesitance score function is described in this paper, has the limitation that it is not a total order, paving way for future research. So our future aim is to improve the proposed ranking procedure to give a complete ranking on the class of IVIFNs.

Further, step 2 of decision algorithm involves different operators on the class of IVIFNs to get finer aggregated performance. Atanassov (2014) has introduced different operators on the class of IVIFNs and their generalizations. In future, our aim is to define a new operator (for obtaining the aggregated performance of alternatives) on the class of IVIFNs and compare it with the operators introduced in Atanassov (2014). Xu (2013) has introduced some ranking procedures on the class of IVIFNs using different methodologies, and further, our idea is to improve the proposed ranking procedure to give a total order on the class of IVIFNs for introducing a better intuitionistic fuzzy decision algorithm.

## 5 Conclusion

The class of interval-valued intuitionistic fuzzy numbers is an important subclass of the set of intuitionistic fuzzy numbers. IVIFNs are widely used in decision analysis and information systems. Diverse ranking methods are available for the class of IVIFNs, but none of them yields a total order on the class of IVIFNs. In order to improve the familiar ranking methods, a new non-hesitance score function for the theory of interval-valued intuitionistic fuzzy sets is introduced by which a total order will be attempted in near future and the necessity for defining a new non-hesitance score function is explained using illustrative examples. In this paper, a new multi-criteria decision-making method is established for decision problems involving interval-valued intuitionistic fuzzy numbers. Further, the practicality of the proposed method is shown by solving an interval-valued intuitionistic fuzzy MCDM problem.

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**Compliance with ethical standards**

**Conflict of interest** The authors declare that they have no conflict of interest.

**Ethical approval** This article does not contain any studies with human participants or animals performed by any of the authors.

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