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METHODOLOGIES AND APPLICATION

A multiple attribute interval type-2 fuzzy group decision making and its application to supplier selection with extended LINMAP method

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Abstract Supplier selection is a key issue in supply chain management, which directly impacts the manufacturer's performance. The problem can be viewed as a multiple attribute group decision making (MAGDM) that concerns many conflicting evaluation attributes, both being of qualitative and quantitative nature. Due to the increasing complexity and uncertainty of socio-economic environment, some evaluations of attributes are not adequately represented by numerical assessments and type-1 fuzzy sets. In this paper, we develop some linear programming models with the aid of multidimensional analysis of preference (LINMAP) method to solve interval type-2 fuzzy MAGDM problems, in which the information about attribute weights is incompletely known, and all pairwise comparison judgments over alternatives are represented by IT2FSs. First, we introduce a new distance measure based on the centroid interval between the IT2FSs. Then, we construct the linear programming model to determine the interval type-2 fuzzy positive ideal solution (IT2PIS) and corresponding attributes weight vector. Based on it, an extended LINMAP method to solve MAGDM problem under IT2FSs environment is developed. Finally, a supplier selection example is provided to demonstrate the usefulness of the proposed method.

Keywords Supplier selection · Linear programming techniques for multidimensional analysis of preference (LINMAP) method · Interval type-2 fuzzy sets (IT2FSs) · Multiple attribute group decision making (MAGDM)

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1 Introduction

With the advent of economic globalization and market competition intensifies, supply chain management (SCM) and supplier (vendor) management have gained a great deal of attention by the research community and industry. More and more companies have started to strengthen the cooperation with suppliers, especially to make compact relationship with strategic suppliers. Therefore, it is a very important concern to select the right supplier that suits the requirement of company's development and exhibits a sound development foreground. Supplier selection plays an important role in SCM, which has been viewed as a multiple attribute group decision making (MAGDM) problem. This process mainly involves the evaluation of different alternatives of suppliers based on various attributes, both of qualitative and quantitative character. A large number of decision-making methods for supplier selection, such as AHP (Akarte et al. 2002; Kahraman et al. 2003; Chan and Kumar 2007; Labib 2011), TOPSIS (Boran et al. 2009; Wang et al. 2009), VIKOR (Sanayei et al. 2010), QFD (Kahraman et al. 2006; Bevilacqua et al. 2006), goal programming (Amid et al. 2006) and other approaches (Samvedi et al. 2012; Pitchipoo et al. 2013; Ertay et al. 2011; Liu and Zhang 2011), have been published in this field during the last decades. A detailed review of the supplier selection approaches can be found in Ho et al. (2010) and Chai et al. (2013).

It is noted that all these previous decision methods of supplier selection cannot solve the MAGDM problems with incomplete decision preference information. In fact, there always exists such situation that the decision maker (DM) feels comfortable to express his/her decision in terms of preference relation instead offering a detailed numerical quantification. This problem can be solved using the LINMAP method. The LINMAP (Linear programming techniques for multidimensional analysis of preference) method developed by Srinivasan and Shocker (1973) is now one of the well-known classic decision-making methods in modern decision analysis theory, which can be used to evaluate the weights of attributes and the ideal alternative based on DMs' preference relations on pairwise comparisons of alternatives. Here, we construct a linear programming (LP) model to generate an optimal compromise alternative as the solution and calculate the distance to the ideal alternative. In the traditional LINMAP method, the decision information is usually known in advance and provided in a numeric form. However, due to the increasing complexity of socio-economic environment of the inherently subjective nature of human thinking, under many situations, numeric values are not adequate to model real practical decision-making problems. Recently, some researchers have extended the LINMAP method to a variety of different fuzzy environments. For example, Xia et al. (2006) extended the LINMAP method to type-1 fuzzy environment and developed an approach for solving the ensuing MAGDM problems. Li and Wan (2013) put forward a new LINMAP method to handle MAGDM with multiple types of attributes and considered a case with incomplete weight information. Bereketli et al. (2011) investigated a new fuzzy LINMAP method and applied it to WEEE treatment strategies' evaluation. Wan and Li (2013) presented a fuzzy LINMAP method to solve the heterogeneous MAGDM problems, in which the attributes with multiple formats of information, and further considered the comparisons of alternatives with hesitation degrees. Chen (2014) and Wang and Liu (2013) extended the LINMAP to interval-valued intuitionistic fuzzy sets (IVIFSs) environment and proposed the methodology to solve MAGDM problems. Zhang and Xu (2014) proposed the interval programming method for hesitant fuzzy MAGDM with incomplete preference over alternatives and showed its applicability to energy project selection problem.

Evidently, the LINAMP method has been extended to many different fuzzy environments to handle MAGDM problems. However, little attention has been paid to extending the LINMAP into high-type fuzzy environment. In practice, for supplier selection problems, most of the evaluation information is not known and many factors are affected by uncertainty. As a result, the traditional type-1 fuzzy sets (T1FSs) might be insufficient to model practical situations because of the increasing complexity of the supplier selection problem. In such cases, type-2 fuzzy sets (T2FSs) could be considered as one of the most useful techniques for handling vague and uncertainty. T2FSs was first proposed by Zadeh (1975), which can be regarded as an extension of T1FSs. They are characterized by two membership functions: primary membership function (PMF) and secondary membership function (SMF). T2FSs can deal with the fuzziness and uncertainty characteristics of fuzzy complex systems more effectively than the traditional T1FSs. Therefore, some theoretical results have been achieved in T2FSs (Karnik and Mendel 2001; Liang and Mendel 2000; Hagras 2004; Wu and Mendel 2007; John and Coupland 2007; Liu 2008; Zhou et al. 2008, 2011; Chiclana and Zhou 2013; Greenfield and Chiclana 2013a, b; Greenfield et al. 2009, 2012). For example, Greenfield and Chiclana (2013b) proposed a new efficient and accurate method for defuzzification of the generalized type-2 fuzzy sets. Greenfield et al. (2012) studied the sampling method defuzzification for T2FSs based on experimental evaluation. Chiclana and Zhou (2013) developed a new type-reduction method of T2FSs with the aid of type-1 OWA operator. Interval type-2 fuzzy sets (IT2FSs) are the most widely used in type-2 fuzzy sets, given that their computational complexity is much lower than the general type-2 fuzzy sets (GT2FSs). Therefore, they are easy to use in real-world application areas, especially in multiple attribute group decision-making domains. In the past few years, many methods have been developed to extend and enrich the aggregation approaches and the MAGDM methodologies under interval type-2 fuzzy environment. For example, Zhou et al. (2008) and Zhou et al. (2011) proposed a new type of type-2 OWA operator for aggregating uncertain information with uncertain weights induced by type-2 linguistic quantifiers, and further applied them to breast cancer treatments. Greenfield et al. (2009) developed the collapsing method of defuzzification for IT2FSs. Greenfield and Chiclana (2013a) provided accuracy and complexity evaluation of defuzzification for IT2FSs, and obtained the conclusions that the collapsing defuzzifier and the Nie–Tan method are the most accurate, and the Nie–Tan Method is the least computationally complex. Chen and Lee (2010) developed an interval type-2 fuzzy TOPSIS method to solve MAGDM problems. Wang et al. (2012) investigated some optimization models for determining the attribute weights and further developed a new approach to handle the situations where the attribute values are characterized by IT2FSs. Chen et al. (2013) developed an extended QUALIFLEX method for handling MAGDM based on IT2FSs and gave a case study for medical decision making. Chen et al. (2013) developed an ELECTRE-base outranking method for MAGDM using IT2FSs and applied it to supplier selection. Wu and Mendel (2007) proposed a linguistic weighted average aggregation operator to handle multiple attribute hierarchical group decision making using fuzzy preference relations under interval type-2 fuzzy environment. Qin and Liu (2014) investigated a family of type-2 fuzzy aggregation operators based on Frank triangular norm and developed a new approach to MAGDM problems within the IT2FSs context. In addition, some authors have also extended the classical decision techniques to interval type-2 fuzzy environment (Chen et al. 2013; Ngan 2013; Chen and Lee 2010; Zhou et al. 2008; Qin and Liu 2015) and applied these decision methods to some practical applications, such as supplier selection (Chen 2013), weapon evaluation (Wu and Mendel 2010), medicine decision making (Chen et al. 2013), and water resource evaluation (Wang and Chen 2014).

Considering the values of theoretical and applications arising within IT2FSs in decision making and the actual need for supplier selection, it becomes necessary to develop a new method based on LP technique to solve MAGDM within IT2FSs and give it application to supplier selection problem. Motivated by this idea, in this paper, we extend the traditional LINMAP method to interval type-2 fuzzy environment and develop an interval type-2 fuzzy MAGDM method based on mathematical programming technique for handling real-life supplier selection problems with preference relation and incomplete weight information, in which all the attribute values and pairwise preference relations given by DMs are represented by IT2FSs. The essential objective of this paper is to develop a new interval type-2 fuzzy LINMAP method to solve MAGDM problem that considers the preference relation of pair order which is represented by IT2FSs. The existing methods involve a variety of fuzzy environment having a common shortcoming that they are all ignore the uncertainty of pair order itself. In actual decision-making situation, it is difficult for the decision maker to answer which alternative is absolutely preferable to another alternative, so IT2FSs can effectively cope with the vagueness and imprecise of such situation. Moreover, little research has been focused on group decision making based on LINMAP method. Therefore, it is meaningful and necessary to integrate the LINMAP method and its preference information within the interval type-2 fuzzy information. It can not only enhance the model ability of high-order uncertainties, but also address MAGDM problems with imprecise and uncertain decision information.

The paper is structured as follows: In Sect. 2, we briefly introduced some basic concepts related to T2FSs, IT2FSs and IT2F preference relations. In Sect. 3, we extend the LINMAP to develop an MAGDM method under interval type-2 fuzzy environment. In Sect. 4, we provide a case study concerns that supplier selection example and make a comparison analysis to demonstrate the applicability and validity of the proposed methodology. Finally, some conclusions and further research are covered in Sect. 5.

2 Preliminaries

In this section, we review some basic concepts of interval type-2 fuzzy sets (IT2FSs), fuzzy preference relations, to be directly used in the next sections.

2.1 The concepts of type-2 fuzzy sets and interval type-2 fuzzy sets

Type-2 fuzzy sets were first introduced by Zadeh (1975), and then developed by Mendel and al. The type-2 fuzzy sets can be viewed as a general extension of type-1 fuzzy sets that are characterized by two membership functions: primary membership function (PMF) and secondary membership function (SMF), with the additional dimension of membership function. Therefore, it is more capable for handling imprecision and imperfect information in real-world application. T2FSs are suitable for dealing with the situations in which the decision makers have vagueness in providing their decision preferences.

Definition 1 (Mendel and John 2002) Let X be the universe of discourse, a type-2 fuzzy set A can be represented by type-2 membership function $\mu_A(x, u)$as follows:

$$A = \{((x, u), \mu_A(x, u))|\forall x \in X, \forall u \in J_x \subseteq [0, 1]\} \tag{1}$$

where J_x denotes an interval in [0, 1]. Moreover, the type-2 fuzzy set can also be expressed as the following form:

$$\begin{aligned} A &= \int_{x \in X} \int_{u \in J_x} \mu_A(x, u)/(x, u) \\ &= \int_{x \in X} \left(\int_{u \in J_x} \mu_A(x, u)/u \right) /x \end{aligned} \tag{2}$$

where $J_x \subseteq [0, 1]$ is the primary membership at x, and $\int_{u \in J_x} \mu_A(x, u)/u$ indicates the second membership at$x$. For discrete space, $\int$ is replaced by Σ.

Definition 2 (Mendel and John 2002) Let $\tilde{A}$ be a type-2 fuzzy sets in the universe of discourse Xrepresented by a type-2 membership function $\mu_A(x, u)$. If all $\mu_A(x, u) = 1$, then $\tilde{A}$is called an interval type-2 fuzzy sets (IT2FSs). An interval type-2 fuzzy set can be regarded as a special case of the type-2 fuzzy sets, which is defined as follows:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x, u) = \int_{x \in X} \left(\int_{u \in J_x} 1/u \right) /x \tag{3}$$

It is obvious that the IT2FSs $\tilde{A}$ defined in X is completely determined by the primary membership which is called the footprint of uncertainty (FOU); the FOU can be expressed as follows:

$$\mathrm{FOU}(\tilde{A}) = \underset{x\in X}{\cup} J_x = \underset{x\in X}{\cup} \{(x,u)|u \in J_x \subseteq [0,1]\} \tag{4}$$

Because the operations on IT2FSs are very complex according to the decomposition theorem, so in applications the IT2FSs are usually considered in a certain simplified form. Here, we follow the results of Chen (2013), which adopted trapezoid interval type-2 fuzzy sets (TrIT2FSs) for solving MAGDM problems.

Definition 3 (Chen 2013) Let $\tilde{A}^L$ and $\tilde{A}^U$ be two generalized trapezoidal fuzzy numbers (TrT1FNs), where the height of a generalized fuzzy number is between zero to one. Let $h_{\tilde{A}}^L$ and $h_{\tilde{A}}^U$ be the heights of $\tilde{A}^L$ and $\tilde{A}^U$, respectively. An IT2TrFN $\tilde{A}$ in the universe of discourse X is defined as:

$$\begin{aligned}\tilde{A} &= \left[\tilde{A}^L, \tilde{A}^U\right] \\ &= \left[(a_1^L, a_2^L, a_3^L, a_4^L; h_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; h_A^U)\right]\end{aligned} \tag{5}$$

where $a_1^L, a_2^L, a_3^L, a_4^L, h_{\tilde{A}}^L, b_1^U, b_2^U, b_3^U, b_4^U, h_{\tilde{A}}^U$ are all real numbers and which satisfy the inequality $a_1^L \le a_2^L \le a_3^L \le a_4^L, b_1^U \le b_2^U \le b_3^U \le b_4^U, 0 \le h_{\tilde{A}}^L \le h_{\tilde{A}}^U \le 1$. The upper membership function (UMF) $\tilde{A}^U(x)$ and lower membership function (LMF) $\tilde{A}^L(x)$ are defined as follows:

$$\tilde{A}^U(x) = \begin{cases} \frac{(x-a_1^U)h_{\tilde{A}}^U}{a_2^U - a_1^U} & a_1^U \le x \le a_2^U \\ h_{\tilde{A}}^U & a_2^U \le x \le a_3^U \\ \frac{(a_4^U - x)h_{\tilde{A}}^U}{a_4^U - a_3^U} & a_3^U \le x \le a_4^U \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

and

$$\tilde{A}^L(x) = \begin{cases} \frac{(x-a_1^L)h_{\tilde{A}}^L}{a_2^L - a_1^L} & a_1^L \le x \le a_2^L \\ h_{\tilde{A}}^L & a_2^L \le x \le a_3^L \\ \frac{(a_4^L - x)h_{\tilde{A}}^L}{a_4^L - a_3^L} & a_3^L \le x \le a_4^L \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Based on the definition of the LMF and UMF, Karnik and Mendel (2001) proposed an algorithm to calculate the centroid of the IT2FSs, which is shown as follows:

$$C_{\tilde{A}}^L = \min_{\xi\in[a,b]} \frac{\int_a^\xi x\tilde{A}^U(x)\mathrm{d}x + \int_\xi^b x\tilde{A}^L(x)\mathrm{d}x}{\int_a^\xi \tilde{A}^U(x)\mathrm{d}x + \int_\xi^b \tilde{A}^L(x)\mathrm{d}x} \tag{8}$$

$$C_{\tilde{A}}^R = \max_{\xi\in[a,b]} \frac{\int_a^\xi x\tilde{A}^L(x)\mathrm{d}x + \int_\xi^b x\tilde{A}^U(x)\mathrm{d}x}{\int_a^\xi \tilde{A}^L(x)\mathrm{d}x + \int_\xi^b \tilde{A}^U(x)\mathrm{d}x} \tag{9}$$

where $C_{\tilde{A}}^L$ and $C_{\tilde{A}}^R$ are the endpoints of the centroid interval corresponding to IT2FSs $\tilde{A}$. Then, the ranking value of IT2FSs $\tilde{A}$ is calculated as follows:

$$R(\tilde{A}) = \frac{C_{\tilde{A}}^L + C_{\tilde{A}}^R}{2} \tag{10}$$

Based on the signed distance and the extension principle, Chen (2013) defined the interval type-2 signed distance as follows:

$$\begin{aligned} d(\tilde{A},\tilde{B}) = \frac{1}{8}\Big|&(b_1^L - a_1^L + b_2^L - a_2^L + b_3^L - a_3^L + b_4^L - a_4^L) \\ &+ 4(a_1^U - b_1^U) + 2(a_2^U - b_2^U) + 2(a_3^U - b_3^U) \\ &+ 4(a_4^U - b_4^U) + 3(a_2^U + a_3^U - a_1^U - a_4^U)\frac{h_{\tilde{A}}^L}{h_{\tilde{A}}^U} \\ &- 3(b_2^U + b_3^U - b_1^U - b_4^U)\frac{h_{\tilde{B}}^L}{h_{\tilde{B}}^U}\Big| \end{aligned} \tag{11}$$

where $\tilde{A}$ and $\tilde{B}$ are two IT2FNs.

2.2 Interval type-2 fuzzy preference relations

Fuzzy preference relation (FPR) is one of the most important concepts in decision problems. In traditional multidimensional analysis of preference, we only consider situations where the pairwise comparison between alternatives produces a numeric degree in between 0 and 1 (Li et al. 2005). However, under many real situations, due to the increasing complexity and uncertainty of social economic environment, especially to cope with high imperfect and imprecise whereby two or more sources of vagueness appear simultaneously, the traditional LINMAP method shows some limitations, because the decision makers (DMs) are often not sure enough when realizing all pairwise comparisons over alternative. Therefore, using interval type-2 fuzzy numbers instead of numerical values is more reasonable and justifiable. In this study, the preferences of DM's can be provided as interval type-2 fuzzy numbers given through pairwise comparisons between the alternatives.

Definition 4 For the DM D_p and each pair of alternative A_l and A_k, if the DM D_p prefers the alternative A_l and A_k to the degree of $\tilde{C}_p(k,l)$, we can define Ω^p as:

$$\Omega^p = \left\{(k,l)|A_k \succ_{\tilde{C}_p(k,l)} A_l(k,l \in M)\right\} \tag{12}$$

where Ω^p is a set of ordered pairs (k,l) provided by the DM D_p, and the degree of truth is expressed as an interval type-2 fuzzy number denoted here by $\tilde{C}_p(k,l) = (\underline{C}_{kl}^p, \overline{C}_{kl}^p)$.

Table 1 Linguistic terms of interval type-2 fuzzy preferences (Wang et al. 2012)

Linguistic meanings	Interval type-2 fuzzy numbers
Scarcely preferable (SCP)	[(0,0.1,0.15,0.3;1), (0.05,0.1,0.1,0.2;0.95)]
Moderate preferable (MP)	[(0.15,0.3,0.35,0.5;1), (0.2,0.25,0.3,0.4;0.95)]
Almost preferable (AP)	[(0.3,0.5,0.55,0.7;1), (0.4,0.45,0.5,0.6;0.95)]
Preferable (P)	[(0.5,0.7,0.75,0.9;1), (0.6,0.65,0.7,0.8;0.95)]
Strong preferable (STP)	[(0.7,0.9,0.95,1;1), (0.8,0.85,0.9,0.95;0.95)]

In practical decision problem, interval type-2 fuzzy numbers come with a certain semantics. In other words, there exist several different preference relations between interval type-2 numbers and linguistic terms. All these relations are shown in Table 1. For instance, $\tilde{C}_p(k,l) = [(0.5, 0.7, 0.75, 0.9; 1), (0.6, 0.65, 0.7, 0.8; 0.95)]$ indicates that the decision maker D_p prefers the alternative A_k to A_l, while $\tilde{C}_p(k,l) = [(0, 0.1, 0.15, 0.3; 1), (0.05, 0.1, 0.1, 0.2; 0.95)]$ shows that the decision maker D_p slightly prefers the alternative A_k to A_l. To simplify the computational overhead, we use the centroid of the truth degree $R(\tilde{C}_p(k,l))$ instead of $\tilde{C}_p(k,l)$.

3 LINMAP method for MAGDM using IT2FSs

3.1 The description of MAGDM problem

Consider a multiple attribute group decision-making problem. Let $A = \{A_1, A_2, \ldots, A_m\}$ be the discrete set of alternatives, $D = \{D_1, D_2, \ldots, D_q\}$ be the discrete set of decision makers (DMs) and let $e = (e_1, e_2, \ldots, e_q)^T$ be the associated weights (weight vector) of DMs, where $e_p \geq 0$ $(p = 1, 2, \ldots, q)$ and $\sum_{p=1}^{q} e_p = 1$. Let $C = \{C_1, C_2, \ldots, C_n\}$ be the set of attributes, and let $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be a set of weights (weight vector) of them, such that $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$. Let $T = (t_{ij}^p)_{m\times n}$ be the interval type-2 fuzzy decision information matrix, where $t_{ij}^p = [(a_{1(ij)}^{L(p)}, a_{2(ij)}^{L(p)}, a_{3(ij)}^{L(p)}, a_{4(ij)}^{L(p)}; h_{(ij)}^{L(p)}), (a_{1(ij)}^{U(p)}, a_{2(ij)}^{U(p)}, a_{3(ij)}^{U(p)}, a_{4(ij)}^{U(p)}; h_{(ij)}^{U(p)})]$ is described by IT2FN, which is given by DM D_P for the alternative A_i with respect to attribute C_j. The decision matrix is shown as follows:

$$T^p = (t_{ij}^p)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} t_{11}^p & t_{12}^p & \cdots & t_{1n}^p \\ t_{21}^p & t_{22}^p & \cdots & t_{2n}^p \\ \vdots & \vdots & \ddots & \vdots \\ t_{m1}^p & t_{m2}^p & \cdots & t_{mn}^p \end{pmatrix} \end{array} \quad (13)$$

In general, attributes are divided into two types: benefit attributes (the bigger, the better), and cost attributes (the smaller, the better). To maintain the consistency of the attribute values, we should transform the decision matrix $T^p = (t_{ij}^p)_{m\times n}$ into the corresponding normalized matrix $R^p = (r_{ij}^p)_{m\times n}$ unless all the attributes are of the same type. Here, we use the following formula to normalize the decision matrix $R^p = (r_{ij}^p)_{m\times n}$, as being proposed by Hu et al. (2013).

$$r_{ij}^p = \begin{cases} t_{ij}^p & \text{for benefit attribute } C_j \\ \left(t_{ij}^p\right)^c & \text{for cost attribute } C_j \end{cases} \quad (14)$$

where $(t_{ij}^p)^c$ is the complement of t_{ij}^p such that

$$\begin{aligned} &\left(t_{ij}^p\right)^c \\ &= \Bigg(\Bigg(1 - \frac{a_{4(ij)}^{L(p)}}{\max\limits_{1\le k\le 4}\{a_{k(ij)}^{L(k)}\}}, 1 - \frac{a_{3(ij)}^{L(p)}}{\max\limits_{1\le k\le 4}\{a_{k(ij)}^{L(k)}\}}, \\ &\quad 1 - \frac{a_{2(ij)}^{L(p)}}{\max\limits_{1\le k\le 4}\{a_{k(ij)}^{L(k)}\}}, 1 - \frac{a_{1(ij)}^{L(p)}}{\max\limits_{1\le k\le 4}\{a_{k(ij)}^{L(k)}\}}; h_{(ij)}^{L(p)} \wedge 1\Bigg), \\ &\quad \Bigg(1 - \frac{a_{4(ij)}^{U(p)}}{\max\limits_{1\le k\le 4}\{a_{k(ij)}^{L(k)}\}}, 1 - \frac{a_{3(ij)}^{U(p)}}{\max\limits_{1\le k\le 4}\{a_{k(ij)}^{L(k)}\}}, \\ &\quad 1 - \frac{a_{2(ij)}^{U(p)}}{\max\limits_{1\le k\le 4}\{a_{k(ij)}^{L(k)}\}}, 1 - \frac{a_{1(ij)}^{U(p)}}{\max\limits_{1\le k\le 4}\{a_{k(ij)}^{L(k)}\}}; h_{(ij)}^{U(p)} \wedge 1\Bigg)\Bigg) \end{aligned} \quad (15)$$

3.2 The distance measure between IT2FSs

The distance is an important measure to quantify the difference between two IT2FSs. In this subsection, we define a new distance measure based on centroid interval, which reads as follows:

$$\begin{aligned} d(\tilde{A}, \tilde{B}) = \int_0^1 \Big| &\left[C_{\tilde{A}}^L + x(C_{\tilde{A}}^R - C_{\tilde{A}}^L)\right] \\ &- \left[C_{\tilde{B}}^L + x(C_{\tilde{B}}^R - C_{\tilde{B}}^L)\right]\Big| \mathrm{d}x \end{aligned} \quad (16)$$

where $C_{\tilde{A}}^L, C_{\tilde{A}}^R, C_{\tilde{B}}^L, C_{\tilde{B}}^R$ are the reference centroid points of $\tilde{A}$ and $\tilde{B}$.

In what follows, we prove that the provided distance measure satisfies the properties of the metric space.

Theorem 1 *Let $\tilde{A}, \tilde{B}, \tilde{C} \in \Omega$ be two IT2FNs. Then, the metric distancedin a set Ω is a real function $d : \Omega \times \Omega \to R$, which satisfies the following three axioms:*

(1) $0 \leq d(\tilde{A}, \tilde{B}) \leq 1$. *In particular,* $d(\tilde{A}, \tilde{B}) = 0 \Leftrightarrow \tilde{A} = \tilde{B}$ *(Positivity);*
(2) $d(\tilde{A}, \tilde{B}) = d(\tilde{B}, \tilde{A})$ *(Symmetry);*
(3) $d(\tilde{A}, \tilde{C}) \leq d(\tilde{A}, \tilde{B}) + d(\tilde{B}, \tilde{C})$ *(Triangle inequality).*

Proof (1) Since $0 \leq C_{\tilde{A}}^{L}, C_{\tilde{A}}^{R}, C_{\tilde{B}}^{L}, C_{\tilde{B}}^{R} \leq 1$, then

$$\begin{aligned} d(\tilde{A}, \tilde{B}) &= \int_0^1 \left| \left[C_{\tilde{A}}^{L} + x\left(C_{\tilde{A}}^{R} - C_{\tilde{A}}^{L}\right)\right] \right. \\ &\quad \left. - \left[C_{\tilde{B}}^{L} + x\left(C_{\tilde{B}}^{R} - C_{\tilde{B}}^{L}\right)\right] \right| \mathrm{d}x \\ &\leq \int_0^1 \left| C_{\tilde{A}}^{R} - C_{\tilde{B}}^{L} \right| \mathrm{d}x \leq \int_0^1 \mathrm{d}x = 1 \end{aligned} \tag{17}$$

ie.,

$$0 \leq d(\tilde{A}, \tilde{B}) \leq 1 \tag{18}$$

In particular, one has

$$\begin{aligned} d(\tilde{A}, \tilde{B}) = 0 &\Leftrightarrow C_{\tilde{A}}^{L} = C_{\tilde{B}}^{L}, C_{\tilde{A}}^{R} = C_{\tilde{B}}^{R} \\ &\Leftrightarrow \tilde{A}^{U}(x) = \tilde{B}^{U}(x) \text{ and } \tilde{A}^{L}(x) = \tilde{B}^{L}(x) \\ &\Leftrightarrow \tilde{A} = \tilde{B} \end{aligned} \tag{19}$$

(2) It follows from Eq. (16), then we have

$$\begin{aligned} d(\tilde{A}, \tilde{B}) &= \int_0^1 \left| \left[C_{\tilde{A}}^{L} + x\left(C_{\tilde{A}}^{R} - C_{\tilde{A}}^{L}\right)\right] \right. \\ &\quad \left. - \left[C_{\tilde{B}}^{L} + x\left(C_{\tilde{B}}^{R} - C_{\tilde{B}}^{L}\right)\right] \right| \mathrm{d}x \\ &= \int_0^1 \left| \left[C_{\tilde{B}}^{L} + x\left(C_{\tilde{B}}^{R} - C_{\tilde{B}}^{L}\right)\right] \right. \\ &\quad \left. - \left[C_{\tilde{A}}^{L} + x\left(C_{\tilde{A}}^{R} - C_{\tilde{A}}^{L}\right)\right] \right| \mathrm{d}x \\ &= d(\tilde{B}, \tilde{A}) \end{aligned} \tag{20}$$

(3) Since

$$\begin{aligned} &d(\tilde{A}, \tilde{C}) \\ &= \int_0^1 \left| \left[C_{\tilde{A}}^{L} + x\left(C_{\tilde{A}}^{R} - C_{\tilde{A}}^{L}\right)\right] - \left[C_{\tilde{C}}^{L} + x\left(C_{\tilde{C}}^{R} - C_{\tilde{C}}^{L}\right)\right] \right| \mathrm{d}x \\ &= \int_0^1 \left| \begin{array}{l} \left[C_{\tilde{A}}^{L} + x\left(C_{\tilde{A}}^{R} - C_{\tilde{A}}^{L}\right)\right] - \left[C_{\tilde{B}}^{L} + x\left(C_{\tilde{B}}^{R} - C_{\tilde{B}}^{L}\right)\right] \\ + \left[C_{\tilde{B}}^{L} + x\left(C_{\tilde{B}}^{R} - C_{\tilde{B}}^{L}\right)\right] - \left[C_{\tilde{C}}^{L} + x\left(C_{\tilde{C}}^{R} - C_{\tilde{C}}^{L}\right)\right] \end{array} \right| \mathrm{d}x \\ &\leq \int_0^1 \left| \left[C_{\tilde{A}}^{L} + x\left(C_{\tilde{A}}^{R} - C_{\tilde{A}}^{L}\right)\right] - \left[C_{\tilde{B}}^{L} + x\left(C_{\tilde{B}}^{R} - C_{\tilde{B}}^{L}\right)\right] \right| \mathrm{d}x \\ &\quad + \int_0^1 \left| \left[C_{\tilde{B}}^{L} + x\left(C_{\tilde{B}}^{R} - C_{\tilde{B}}^{L}\right)\right] - \left[C_{\tilde{C}}^{L} + x\left(C_{\tilde{C}}^{R} - C_{\tilde{C}}^{L}\right)\right] \right| \mathrm{d}x \\ &= d(\tilde{A}, \tilde{B}) + d(\tilde{B}, \tilde{C}) \end{aligned} \tag{21}$$

which completes the proof of Theorem 1. □

Example 1 Let $\tilde{A} = [(0.1, 0.3, 0.3, 0.5; 0.9), (0.3, 0.5, 0.5, 0.7; 1)]$ and $\tilde{B} = [(0.3, 0.5, 0.5, 0.7; 0.9), (0.5, 0.7, 0.7, 0.9; 1)]$ be two IT2FSs. Then, the distance between them is calculated as follows:

Based on the KM algorithm, we obtain the centroid intervals of $\tilde{A}$ and $\tilde{B}$ as follows:

$$C(\tilde{A}) = \left[C_{\tilde{A}}^{L}, C_{\tilde{A}}^{R}\right] = [0.3, 0.5],$$
$$C(\tilde{B}) = \left[C_{\tilde{B}}^{L}, C_{\tilde{B}}^{R}\right] = [0.5, 0.7]$$

In the virtue of Eq. (14), we obtain the distance between $\tilde{A}$ and $\tilde{B}$:

$$\begin{aligned} &d(\tilde{A}, \tilde{B}) \\ &= \int_0^1 \left| \left[C_{\tilde{A}}^{L} + x\left(C_{\tilde{A}}^{R} - C_{\tilde{A}}^{L}\right)\right] - \left[C_{\tilde{B}}^{L} + x\left(C_{\tilde{B}}^{R} - C_{\tilde{B}}^{L}\right)\right] \right| \mathrm{d}x \\ &= \int_0^1 |[0.3 + x(0.5 - 0.3)] - [0.5 + x(0.7 - 0.5)]| \mathrm{d}x = 0.2 \end{aligned}$$

To verify the validity of the proposed distance measure, we use the Chen's signed-based distance (Chen 2013) to verify this example. The results are calculated as follows:

Based on the signed-based distance, we obtain the distance of $\tilde{A}$ and $\tilde{B}$:

$$\begin{aligned} &d(\tilde{A}, \tilde{B}) \\ &= \frac{1}{8} |(0.3 - 0.1 + 0.5 - 0.3 + 0.5 - 0.3 + 0.7 - 0.5) \\ &\quad + 4(0.3 - 0.5) + 2(0.5 - 0.7) + 2(0.5 - 0.7) + 4(0.7 - 0.9) \\ &\quad + 3(0.5 + 0.5 - 0.3 - 0.7)\frac{1}{0.9} - 3(0.7 + 0.7 - 0.5 - 0.9)\frac{1}{0.9} \Bigg| \\ &= 0.2 \end{aligned}$$

It is easy to see that two methods produce the same results. This fact verifies that the proposed distance measure is valid and reasonable.

Remark 1 Compared with Chen's (2013) sign-based distance method, the proposed method uses a new distance based on the KM algorithm to use the centroid interval information, while Chen's method can only use the reference point information to derive the distance; much useful information is ignored in this distance method. Therefore, our proposed method can overcome the drawbacks of the previous method and in this manner avoid the information loss in the decision process.

3.3 The LINMAP method based on IT2FSs

3.3.1 Consistency and inconsistency measurements

Let $r^+ = (r_1^+, r_2^+, \ldots, r_n^+)$ denote an initial interval type-2 fuzzy positive ideal solution (IIT2FPIS), where $r_j^+ (j = 1, 2, \ldots, n)$ indicates the IT2FN with respect to attribute C_j. For convenience, we let $r_j^+ = [(1, 1, 1, 1; 1), (1, 1, 1, 1; 1)]$ $(j = 1, 2, \ldots, n)$ in actual computing process.

Consider a group decision-making (GDM) problem, for decision maker D_p, using Eq. (16), we can obtain the weighted distance between the alternative r_i^k and the IIT2FPIS r^+ as follows:

$$S_i^p = \sum_{j=1}^{n} \omega_j d\left(r_{ij}^p, r_j^+\right) \tag{22}$$

where

$$d\left(r_{ij}^p, r_j^+\right) = \int_0^1 \left\| \left[C^L_{r^p_{ij}} + x\left(C^R_{r^p_{ij}} - C^L_{r^p_{ij}}\right)\right] - \left[C^L_{r^+_j} + x\left(C^R_{r^+_j} - C^L_{r^+_j}\right)\right] \right\| \mathrm{d}x. \tag{23}$$

As mentioned above, the decision makers provide the incomplete preference relations between alternatives by a set of ordered pairs $\Omega^p = \left\{(k, l) | A_k \succ_{\tilde{C}_p(k,l)} A_l (k, l \in M)\right\}$ based on their experience and knowledge. Therefore, the weighted distance between each pair of alternatives $(k, l) \in \Omega^p$ and the IT2FPIS is calculated in the form:

$$S_l^p = \sum_{j=1}^{n} \omega_j d\left(r_{lj}^p, r_j^+\right) \tag{24}$$

and

$$S_k^p = \sum_{j=1}^{n} \omega_j d\left(r_{kj}^p, r_j^+\right) \tag{25}$$

The index $(S_l^p - S_k^p)^-$can be defined as:

$$\left(S_l^p - S_k^p\right)^- = \begin{cases} R(\tilde{C}_p(k,l))\left(S_k^p - S_l^p\right) & \text{if } S_l^p < S_k^p \\ 0 & \text{if } S_l^p \geq S_k^p \end{cases} \tag{26}$$

$$\left(S_l^p - S_k^p\right)^- = R\left(\tilde{C}_p(k,l)\right) \max\left\{S_k^p - S_l^p, 0\right\} \tag{27}$$

$$B^p = \sum_{(k,l)\in\Omega^p} \left(S_l^p - S_k^p\right)^- = \sum_{(k,l)\in\Omega^p} R\left(\tilde{C}_p(k,l)\right) \max\left\{S_l^p - S_k^p, 0\right\} \tag{28}$$

Here, we call B^p is the inconsistency index associated with decision maker D_p. Based on this index, we define a group inconsistency index B in the form:

$$B = \sum_{p=1}^{q} B^p = \sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} (S_l^p - S_k^p)^- = \sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} R(\tilde{C}_p(k,l)) \max\left\{S_l^p - S_k^p, 0\right\} \tag{29}$$

In a similar way, another measure index $(S_l^p - S_k^p)^+$ is introduced:

$$\left(S_l^p - S_k^p\right)^+ = \begin{cases} R\left(\tilde{C}_p(k,l)\right)\left(S_i^p - S_k^p\right) & \text{if } S_l^p \geq S_k^p \\ 0 & \text{if } S_l^p < S_k^p \end{cases} \tag{30}$$

$$\left(S_l^p - S_k^p\right)^+ = R\left(\tilde{C}_p(k,l)\right) \max\left\{S_l^p - S_k^p, 0\right\} \tag{31}$$

$$G^p = \sum_{(k,l)\in\Omega^p} \left(S_l^p - S_k^p\right)^+ = \sum_{(k,l)\in\Omega^p} R\left(\tilde{C}_p(k,l)\right) \max\left\{S_l^p - S_k^p, 0\right\} \tag{32}$$

Here, we call the symbol G^p is the inconsistency index associated with decision maker D_p. Based on this index, we can express a group inconsistency index G as follows:

$$G = \sum_{p=1}^{q} G^p = \sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} (S_l^p - S_k^p)^+ = \sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} R(\tilde{C}_p(k,l)) \max\left\{S_l^p - S_k^p, 0\right\} \tag{33}$$

$$\left(S_l^p - S_k^p\right)^+ - \left(S_l^p - S_k^p\right)^- = R\left(\tilde{C}_p(k,l)\right)\left(S_l^p - S_k^p\right) \tag{34}$$

It can be easily obtained from Eqs. (2) and (3) that

$$S_l^p - S_k^p = \sum_{j=1}^{n} \omega_j d\left(r_{lj}^p, r_j^+\right) - \sum_{j=1}^{n} \omega_j d\left(r_{kj}^p, r_j^+\right) = \sum_{j=1}^{n} \omega_j \left(d\left(r_{lj}^p, r_j^+\right) - d\left(r_{kj}^p, r_j^+\right)\right) \tag{35}$$

$$G - B = \sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} \left[\left(S_l^p - S_k^p\right)^+ - \left(S_l^p - S_k^p\right)^-\right] = \sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} R\left(\tilde{C}_p(k,l)\right)\left(S_l^p - S_k^p\right)$$

$$= \sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} \sum_{j=1}^{n} \omega_j R\left(\tilde{C}_p(k,l)\right) \times d\left(r_{lj}^p, r_j^+\right) - d\left(r_{kj}^p, r_j^+\right) \tag{36}$$

3.3.2 The linear programming models for MAGDM using IT2FSs

In general, the IIT2FPIS $r^+ = (r_1^+, r_2^+, \ldots, r_n^+)$ is provided a priori. To obtain the optimal weight vector ω, we can construct the mathematical programming model in the following format:

$$\begin{aligned} &\min\{B\} \\ &\text{s.t.} \begin{cases} G - B \geq h \\ \omega_j \geq \delta \ \ (j = 1, 2, \ldots, m), \sum_{j=1}^{n} \omega_j = 1 \end{cases} \end{aligned} \tag{37}$$

where h is a positive number given by decision makers a priori, and $\delta > 0$ is sufficiently small which ensures that the weights derived are not equal to zero.

Based on Eq. (37), this mathematical programming model can be rewritten as:

$$\begin{aligned} &\min\left\{\sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} R\left(\tilde{C}_p(k,l)\right) \max\left\{S_l^p - S_k^p, 0\right\}\right\} \\ &\text{s.t.} \begin{cases} \sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} \sum_{j=1}^{n} \omega_j R\left(\tilde{C}_p(k,l)\right) \\ \quad \times d\left(r_{lj}^p, r_j^+\right) - d\left(r_{kj}^p, r_j^+\right) \geq h \\ \omega_j \geq \delta \ \ (j = 1, 2, \ldots, m), \sum_{j=1}^{n} \omega_j = 1 \end{cases} \end{aligned} \tag{38}$$

To simplify this model, we let

$$z_{kl}^p = \max\left\{S_l^p - S_k^p, 0\right\} \tag{39}$$

Then, we can easily obtain the inequality $z_{kl}^p \geq (S_l^p - S_k^p) \Rightarrow (S_k^p - S_l^p) + z_{kl}^p \geq 0$.

According to Eqs. (24) and (25), it directly follows that

$$\sum_{j=1}^{n} \omega_j \left(d\left(r_{lj}^p, r_j^+\right) - d\left(r_{kj}^p, r_j^+\right)\right) + z_{kl}^p \geq 0 \tag{40}$$

Based on Eqs. (27), (28) and (40), we transform the above mathematical programming model into the following version:

$$\begin{aligned} &\min\left\{\sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} R\left(\tilde{C}_p(k,l)\right) z_{kl}^p\right\} \\ &\text{s.t.} \begin{cases} \sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} \sum_{j=1}^{n} \omega_j R\left(\tilde{C}_p(k,l)\right) \\ \quad \times d\left(r_{lj}^p, r_j^+\right) - d\left(r_{kj}^p, r_j^+\right) \geq h \\ \sum_{j=1}^{n} \omega_j \left(d\left(r_{lj}^p, r_j^+\right) - d\left(r_{kj}^p, r_j^+\right)\right) \\ \quad + z_{kl}^p \geq 0 \ \ (k,l) \in \Omega^p, (p = 1, 2, \ldots, q) \\ \omega_j \geq \delta \ \ (j = 1, 2, \ldots, m), \sum_{j=1}^{n} \omega_j = 1 \\ z_{kl}^p \geq 0 \ \ (k,l) \in \Omega^p, (p = 1, 2, \ldots, q) \end{cases} \end{aligned} \tag{41}$$

The above linear programming can be easily solved using a simplex method to obtain the optimal weight vector $\omega^* = (\omega_1^*, \omega_2^*, \ldots, \omega_n^*)^T$ and the IT2FPIS A^+. Then, we can calculate the distance $S_i^p (i \in A, p \in D)$ of each alternative A_i to the IT2FPIS A^+ using Eq. (16). Afterwards, we can use the Copland's social chance function to obtain the collective alternative ranking orders.

It is worth noting that the above model can be used in situations where the weights of attributes are completely unknown. However, in many practical situations, the weights information of attributes is not completely unknown, but partially known. In general, let Λ be the set of known attribute weight information, based on the related studies (Park and Kim 1997; Park 2004); the attribute weight information set Λ can be quantified as:

(1) A weak ranking: $\{\omega_i \geq \omega_j\} (i \neq j)$;
(2) A strict ranking: $\{\omega_i - \omega_j \geq \xi_i\} (\xi_i > 0)$;
(3) A ranking with multiples: $\{\omega_i \geq \xi_i \omega_j\} (i \neq j, 0 \leq \xi_i \leq 1)$;
(4) A ranking of differences: $\{\omega_i - \omega_j \geq \omega_k - \omega_l\} (i \neq j \neq k \neq l)$;
(5) An interval form: $\{\xi_i \leq \omega_i \leq \xi_i + \varepsilon_i\} (0 \leq \xi \leq \xi_i + \varepsilon \leq 1)$.

In such a case, the model (41) can be transformed into the following model:

$$\begin{aligned} &\min\left\{\sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} R\left(\tilde{C}_p(k,l)\right) z_{kl}^p\right\} \\ &\text{s.t.} \begin{cases} \sum_{p=1}^{q} \sum_{(k,l)\in\Omega^p} \sum_{j=1}^{n} \omega_j R\left(\tilde{C}_p(k,l)\right) \\ \quad \times d\left(r_{lj}^p, r_j^+\right) - d\left(r_{kj}^p, r_j^+\right) \geq h \\ \sum_{j=1}^{n} \omega_j d\left(r_{lj}^p, r_j^+\right) - d\left(r_{kj}^p, r_j^+\right) \\ \quad + z_{kl}^p \geq 0 \ \ (k,l) \in \Omega^p, (p = 1, 2, \ldots, q) \\ z_{kl}^p \geq 0 \ \ (k,l) \in \Omega^p, (p = 1, 2, \ldots, q) \\ \omega \in \Lambda \end{cases} \end{aligned} \tag{42}$$

It is noted that the model (42) can just cope with the situation where the importance levels of all the DMs are equal. Obviously, this assumption might not be acceptable all the times. Therefore, if we consider the situation that the weights of DMs are different, then the model (42) can be modified as:

$$\min\left\{\sum_{p=1}^{q}\sum_{(k,l)\in\Omega^p} R\left(\tilde{C}_p(k,l)\right) z_{kl}^{p}\right\}$$
$$\text{s.t.}\begin{cases}\sum_{p=1}^{q} qe_p\Big(\sum_{(k,l)\in\Omega^p}\sum_{j=1}^{n}\omega_j R\left(\tilde{C}_p(k,l)\right) \\ \quad\times\left(d\left(r_{lj}^{p},r_j^{+}\right)-d\left(r_{kj}^{p},r_j^{+}\right)\right)\Big)\geq h \\ \sum_{j=1}^{n}\omega_j\left(d\left(r_{lj}^{p},r_j^{+}\right)-d\left(r_{kj}^{p},r_j^{+}\right)\right) \\ \quad+z_{kl}^{p}\geq 0\ \ (k,l)\in\Omega^p,\ (p=1,2,\ldots,q) \\ z_{kl}^{p}\geq 0\ \ (k,l)\in\Omega^p,\ (p=1,2,\ldots,q) \\ \omega\in\Lambda\end{cases}\tag{43}$$

It can be easily obtained that when $e=(1/q,1/q,\ldots,1/q)^T$, then the model (43) is reduced to model (42).

3.3.3 Decision steps

In what follows, we develop an extended LINMAP method to solve interval type-2 MAGDM problem as shown in Figs. 1 and 2, which involves the following steps:

Step 1. Set up a group of DMs and identify all alternatives to be evaluation and evaluation attributes. Denote the sets of alternatives $A=\{A_1,A_2,\ldots,A_m\}$ and attributes $C=\{C_1,C_2,\ldots,C_n\}$, respectively.
Step 2. Provide the performance ratings $t_{ij}^{p}(i\in A,j\in C,p\in D)$ for alternative A_i with respect to C_j by decision maker D_p using IT2FNs, and then construct the interval type-2 fuzzy decision matrix $T^p=(t_{ij}^{p})_{m\times n}$, respectively.

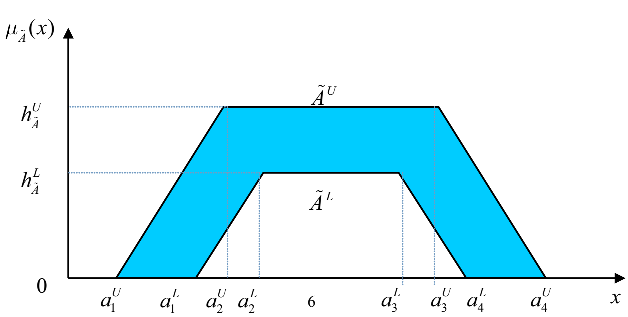

Fig. 1 An interval type-2 fuzzy sets $\tilde{A}$ and its geometric interpretation

Step 3. Normalized the decision matrices $T^p=(t_{ij}^{p})_{m\times n}$ using Eq. (14).
Step 4. Construct the individual decision-making preference relation between alternatives by $\Omega^p=\left\{(k,l)|A_k\succ_{\tilde{C}_p(k,l)}A_l(k,l\in M)\right\}$.
Step 5. Form the mathematical programming model based on Eq. (41), and the transform the model into the linear programming counterpart in Eq. (42).
Step 6. Solve the linear programming model using traditional simplex method, and then obtain the optimal-weighted vector ω and the IT2FPIS A^{+}.
Step 7. Calculate the distances $S_i^p(i=1,2\ldots,m;\ p=1,2,\ldots,q)$ between the alternative $A_i(i=1,2,\ldots,m)$ and the IT2FPIS A^{+}.
Step8. Rank the alternatives $A_i(i\in A)$ for each DM $D_p(p\in D)$ based on the increasing orders of the distances $S_i^p(i=1,2\ldots,m;\ p=1,2,\ldots,q)$, respectively.
Step 9. Rank the overall alternatives for the group using the Copeland's social choice function and determine the best alternative from the alternative set A.
Step 10. End.

4 Numerical example

In this section, we apply the extend interval type-2 fuzzy LINMAP method to a supplier selection problem. In this study, we assume that all the decision makers expect to form linguistic terms (see Table 2) to produce the linguistic value to express their decision preferences described by a trapezoid interval type-2 fuzzy set. Table 1 shows the linguistic terms "Very Low"(VL), "Low"(L), "Medium Low"(ML), "Medium"(M), "Medium High"(MH), "High"(H), "Very High"(VH) and their corresponding trapezoid interval type-2 fuzzy numbers (TrIT2FNs), respectively, which are shown in Fig. 3.

4.1 The supplier selection problem description

With the continuous development of economic globalization, the supply chain management has played an important role in marketing economic and become the most hot research topic in modern management science, which directly impact on the manufactures' performance. Green supplier selection is one of the most important problems in supply chain management. Consider a problem in a shipbuilding company, which aims to search for the best green supplier for purchasing the key components of its new ship equipments. After preliminary screening, five potential ship equipment suppliers (A_1, A_2, A_3, A_4, A_5) have been identified for further evaluation. Six attributes to be considered in the evaluation process are: C_1: *Green product innovation*; C_2: *Green*

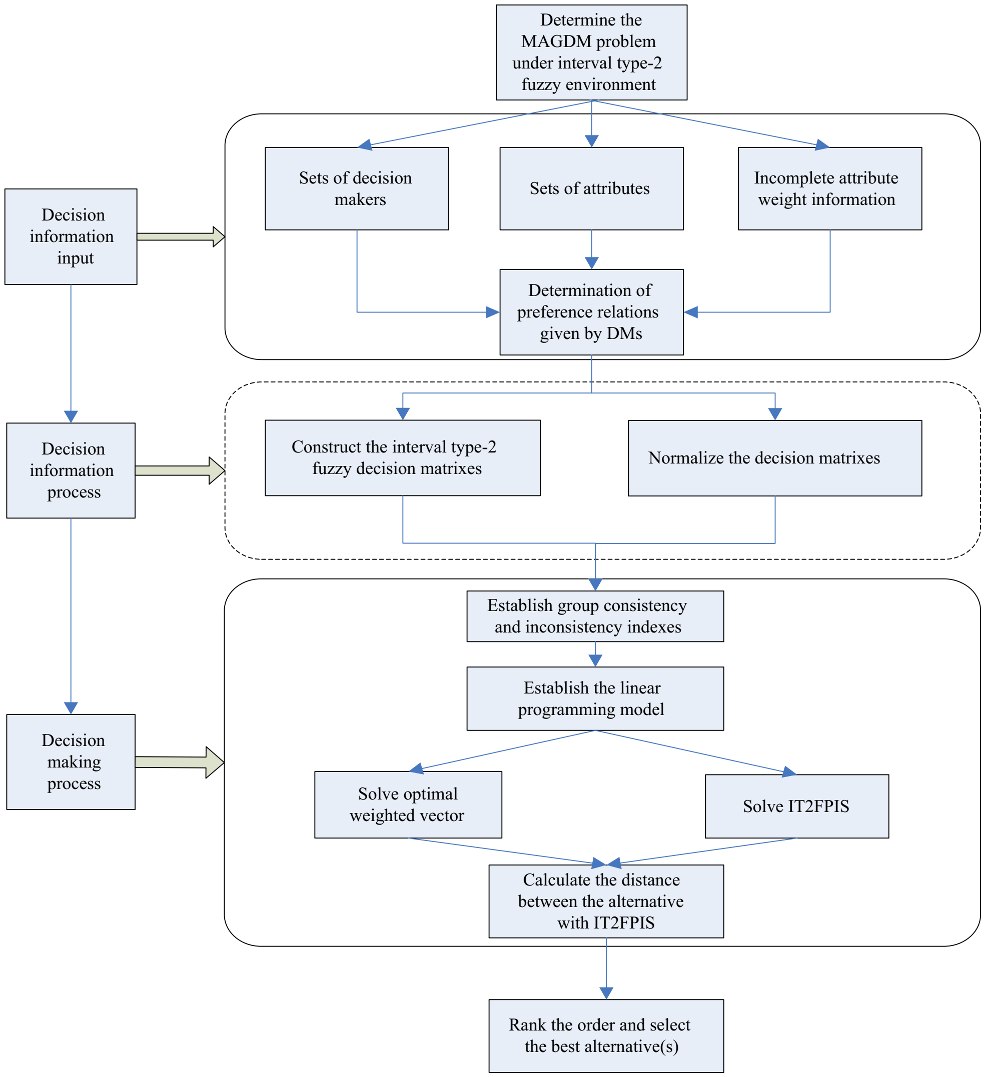

Fig. 2 Procedure of the extended LINMAP methodology

image; C_3: *Use of environmentally friendly technology*; C_4: *Green competencies*; C_5: *Environment management*; C_6: *Quality flexible management* (See Table 3). Three decision makers D_1, D_2, D_3 are invited to carry out the evaluation and $e = (0.25, 0.40, 0.35)^T$ be a set of weight vector of them. The decision matrices are listed in Tables 4, 5 and 6.

Table 2 Linguistic terms and their corresponding TrIT2FNs (Wang et al. 2012)

Linguistic terms	Symmetry trapezoid interval type-2 fuzzy number
Very Low (VL)	[(0, 0, 0, 0.1;1), (0, 0, 0, 0.05;0.9)]
Low (L)	[(0, 0.1, 0.1,0.3;1), (0.05, 0.1, 0.1,0.2;0.9)]
Medium Low (ML)	[(0.1, 0.3, 0.3,0.5;1), (0.2, 0.3, 0.3,0.4;0.9)]
Medium (M)	[(0.3, 0.5, 0.5,0.7;1), (0.4, 0.5, 0.5,0.6;0.9)]
Medium High (MH)	[(0.5, 0.7, 0.7,0.9;1), (0.6, 0.7, 0.7,0.8;0.9)]
High (H)	[(0.7, 0.9, 0.9,1;1), (0.8, 0.9, 0.9,0.95;0.9)]
Very High (VH)	[(0.9, 1, 1, 1;1), (0.95, 1, 1, 1;0.9)]

4.2 Illustration of the proposed method

Because all the attributes are of benefit type, so we do not require to normalize the decision matrices in advance. Assume that the three DMs have provided their comparison preference information between alternatives as follows:

$$\text{DM1}: \Omega^1 = \{\langle (A_2, A_1), SCP\rangle, \langle (A_1, A_3), P\rangle, \langle (A_2, A_3), MP\rangle\}$$

$$\text{DM2}: \Omega^2 = \{\langle (A_2, A_4), AP\rangle, \langle (A_3, A_5), STP\rangle, \langle (A_2, A_1), MP\rangle\}$$

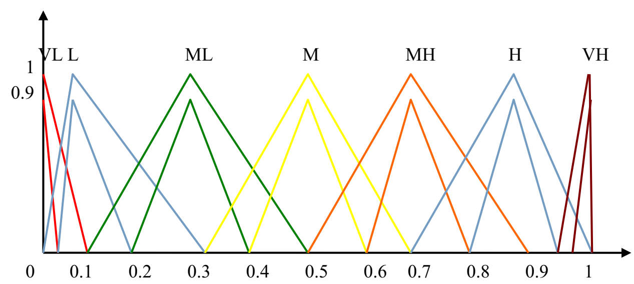

Fig. 3 The membership function for IT2FN linguistic term

Table 3 Attributes for evaluating supplier selection

Attribute	Description of attribute
Green product innovation	Green product innovation addresses environmental issues through product design and technique innovation
Green image	The ratio of green customers to total customers
Use of environmentally friendly technology	The application of the environmental science to conserve the natural environment and resources, and to curb the negative impacts of human involvement
Green competencies	Materials used in the supplied components that reduce the impact on natural resources ability to alter process and product for reducing the impact on natural resources
Environment management	Applying the management technique to seek to balance economic and supplier effective with the condition of environment
Quality management	Supply chain management activities and functions involved in determination of quality policy, quality planning and quality control

Table 4 The decision matrix $R^{(1)}$

Supplier	C_1	C_2	C_3	C_4	C_5	C_6
A_1	VH	ML	VH	VH	VH	M
A_2	H	M	L	MH	M	L
A_3	M	L	H	ML	VL	ML
A_4	VH	VH	ML	ML	VH	VL
A_5	ML	ML	L	MH	M	MH

Table 5 The decision matrix $R^{(2)}$

Supplier	C_1	C_2	C_3	C_4	C_5	C_6
A_1	VH	ML	H	MH	VH	L
A_2	H	M	H	M	ML	H
A_3	M	H	H	M	M	H
A_4	L	MH	ML	VH	H	M
A_5	L	VH	ML	ML	H	H

Table 6 The decision matrix $R^{(3)}$

Supplier	C_1	C_2	C_3	C_4	C_5	C_6
A_1	M	M	VH	VH	MH	VL
A_2	MH	VH	L	L	VH	MH
A_3	VH	VH	MH	H	VH	VH
A_4	M	ML	VH	H	M	VH
A_5	VL	VH	VL	VH	ML	ML

$$\begin{aligned}\text{DM3}: \Omega^3 = \{&\langle (A_1, A_4), SCP\rangle, \langle (A_2, A_3), P\rangle, \\ &\langle (A_2, A_4), MP\rangle, \langle (A_2, A_5), STP\rangle\}\end{aligned}$$

The attribute weights' information are partially known and are given as follows:

$$\begin{aligned}&\Lambda_1: \ 0.1 \le w_1 \le 0.3,\ 0.1 \le w_2 \le 0.4,\ w_3 \le w_4; \\ &\Lambda_2: \ 0.15 \le w_3 \le 0.20,\ 0.20 \le w_4 \le 0.30, \\ &\quad 0.10 \le w_6 \le 0.15; \\ &\Lambda_3: \ 0.10 \le w_5 \le 0.25, w_2 - w_1 \ge 0.10, \\ &\quad w_4 - w_2 \le w_3 - w_1.\end{aligned}$$

Therefore, we have

$$\begin{aligned}\Lambda &= \Lambda_1 \cup \Lambda_2 \cup \Lambda_3 \\ &= \Big\{0.1 \le w_1 \le 0.3,\ 0.1 \le w_2 \le 0.4,\ w_3 \le w_4, \\ &\quad 0.15 \le w_3 \le 0.20,\ 0.20 \le w_4 \le 0.30, \\ &\quad 0.10 \le w_6 \le 0.25,\ 0.10 \le w_5 \le 0.25, \\ &\quad w_2 - w_1 \ge 0.10, w_4 - w_2 \le w_3 - w_1, \\ &\quad \sum_{j=1}^{6} w_j = 1\Big\}\end{aligned}$$

Table 7 The distance between each alternative and IT2FPIS

	A_1	A_2	A_3	A_4	A_5	Ranking orders
D_1	0.324	0.285	0.296	0.307	0.314	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$
D_2	0.325	0.279	0.265	0.288	0.307	$A_3 \succ A_2 \succ A_4 \succ A_5 \succ A_1$
D_3	0.288	0.259	0.315	0.265	0.301	$A_2 \succ A_4 \succ A_1 \succ A_5 \succ A_3$

In the sequel, we construct the linear programming model based on Eq. (43), which is shown as follows:

$$\min\left\{\begin{array}{l}0.125z_{21}^1+0.714z_{13}^1+0.319z_{23}^1+0.512z_{24}^2+0.881z_{35}^2+0.319z_{21}^2+0.125z_{14}^3+\\ 0.714z_{23}^3+0.512z_{24}^3+0.881z_{25}^3\end{array}\right\}$$

$$\text{s.t.}\begin{cases}0.371\omega_1+0.247\omega_2-0.231\omega_3+0.169\omega_4-0.038\omega_5+0.173\omega_6\geq h\\ 0.085\omega_1-0.106\omega_2+0.465\omega_3+0.189\omega_4+0.270\omega_5-0.045\omega_6+z_{21}^1\geq 0\\ -0.271\omega_1-0.089\omega_2+0.085\omega_3-0.376\omega_4+0.554\omega_5+0.221\omega_6+z_{13}^1\geq 0\\ -0.185\omega_1+0.195\omega_2+0.381\omega_3-0.187\omega_4-0.284\omega_5+0.131\omega_6+z_{23}^1\geq 0\\ 0.065\omega_1+0.269\omega_2-0.291\omega_3+0.274\omega_4+0.386\omega_5-0.074\omega_6+z_{24}^2\geq 0\\ -0.106\omega_1+0.181\omega_2-0.295\omega_3+0.342\omega_4+0.191\omega_5+0.233\omega_6+z_{35}^2\geq 0\\ 0.113\omega_1-0.106\omega_2+0.214\omega_3+0.248\omega_4+0.386\omega_5-0.192\omega_6+z_{21}^2\geq 0\\ 0.353\omega_1-0.112\omega_2+0.324\omega_3-0.086\omega_4+0.024\omega_5-0.087\omega_6+z_{14}^3\geq 0\\ 0.136\omega_1+0.258\omega_2+0.293\omega_3+0.157\omega_4+0.193\omega_5-0.121\omega_6+z_{23}^3\geq 0\\ 0.097\omega_1-0.094\omega_2+0.487\omega_3+0.382\omega_4+0.235\omega_5+0.203\omega_6+z_{24}^3\geq 0\\ -0.337\omega_1+0.269\omega_2+0.098\omega_3+0.463\omega_4-0.091\omega_5+0.063\omega_6+z_{25}^3\geq 0\\ z_{21}^1,z_{13}^1,z_{23}^1,z_{24}^2,z_{35}^2,z_{21}^2,z_{14}^3,z_{23}^3,z_{24}^3,z_{25}^3\geq 0\\ 0.1\leq w_1\leq 0.3,\ 0.10\leq w_2\leq 0.40,\ w_3\leq w_4,\ 0.15\leq w_3\leq 0.20,\ 0.20\leq w_4\leq 0.30,\\ 0.10\leq w_6\leq 0.15,\ 0.10\leq w_5\leq 0.25,\ w_2-w_1\geq 0.1,\ w_4-w_2\leq w_3-w_1\\ w_1+w_2+w_3+w_4+w_5+w_6=1\end{cases}$$

Using the optimal software (Lingo), the optimal weight vector ω^* and the IT2FPIS A^+ can be obtained by the following results:

$$\omega^*=(0.114, 0.220, 0.162, 0.239, 0.105, 0.160)^T$$
$$A^+=[(0.872, 0.891, 0.894, 0.901; 1),$$
$$(0.934, 0.952, 0.965, 0.977; 1)]$$

Then, based on Eq. (16), we can calculate the distance $S_i^p (i = 1, 2, 3, 4, 5; p = 1, 2, 3)$ between the alternatives $A_i (i = 1, 2, \ldots, 5)$ to IT2FPIS A^+, with the results shown in Tables 7 and 8.

Therefore, the ranking order of five suppliers is generated as follows:

$$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$$

where the symbol "$\succ$" means "superior to". Obviously, the best supplier is A_2.

4.3 Comparisons and further discussion

To verify the validity of our proposed method, in this subsection, we make some comparisons with type-1 fuzzy LINMAP (T1-LINMAP) method proposed by Xia et al. (2006). and type-2 fuzzy LINMAP (T2-LINMAP) method proposed by Chen (2015), respectively. First, we used the type reduction method (Karnik and Mendel 2001) to transform the interval type-2 fuzzy linguistic terms information in this supplier selection example into type-1 fuzzy linguistic terms information, which is depicted in Table 9 and Fig 4.

4.3.1 Comparison with T1-LINMAP method

Using the T1-LINMAP method, we can construct the following linear programming model which corresponds toeach

Table 8 Copeland's social choice scores of all alternatives with respect to each DM

Supplier	Decision makers team			Copeland's social choice scores
	D_1	D_2	D_3	
A_1	5	5	3	4.30
A_2	1	2	1	1.40
A_3	2	1	5	2.65
A_4	3	3	2	2.67
A_5	4	4	4	4.00

decision maker $D_p (p = 1, 2, 3)$, respectively. Due to the space limitations, we consider decision maker D_1 as an example. The computational process proceeds in the following way:

Solving the model by simplex method, we obtain the optimal solution as:

$$
\min\left\{z_{21}^{1}+z_{13}^{1}+z_{23}^{1}\right\}
$$

$$
\text{s.t.}\left\{\begin{array}{l}
0.371\omega_1+0.247\omega_2-0.231\omega_3+0.169\omega_4-0.038\omega_5+0.025\omega_6+0.134v_{1l}-0.254v_{1m1}\\
-0.254v_{1m2}+0.542v_{1r}+0.434v_{2l}-0.137v_{2m1}-0.236v_{2m2}+0.435v_{2r}\\
+0.134v_{3l}-0.254v_{3m1}-0.254v_{3m2}+0.542v_{3r}+0.134v_{4l}-0.254v_{4m1}-0.254v_{4m2}\\
+0.542v_{4r}+0.134v_{5l}-0.254v_{5m1}-0.254v_{5m2}+0.542v_{5r}-z_{21}^{1}\le 0\\
0.371\omega_1+0.247\omega_2-0.231\omega_3+0.169\omega_4-0.038\omega_5-0.027\omega_6+0.134v_{1l}-0.254v_{1m1}\\
-0.254v_{1m2}+0.542v_{1r}+0.434v_{2l}-0.137v_{2m1}-0.236v_{2m2}+0.435v_{2r}\\
+0.134v_{3l}-0.254v_{3m1}-0.254v_{3m2}+0.542v_{3r}+0.134v_{4l}-0.254v_{4m1}-0.254v_{4m2}\\
+0.542v_{4r}+0.134v_{5l}-0.254v_{5m1}-0.254v_{5m2}+0.542v_{5r}-z_{13}^{1}\le 0\\
0.371\omega_1+0.247\omega_2-0.231\omega_3+0.169\omega_4-0.038\omega_5+0.124\omega_6+0.134v_{1l}-0.254v_{1m1}\\
-0.254v_{1m2}+0.542v_{1r}+0.434v_{2l}-0.137v_{2m1}-0.236v_{2m2}+0.435v_{2r}\\
+0.134v_{3l}-0.254v_{3m1}-0.254v_{3m2}+0.542v_{3r}+0.134v_{4l}-0.254v_{4m1}-0.254v_{4m2}\\
+0.542v_{4r}+0.134v_{5l}-0.254v_{5m1}-0.254v_{5m2}+0.542v_{5r}-z_{23}^{1}\le 0\\
0.371\omega_1+0.247\omega_2-0.231\omega_3+0.169\omega_4-0.038\omega_5-0.218\omega_6+0.134v_{1l}-0.254v_{1m1}\\
-0.254v_{1m2}+0.542v_{1r}+0.434v_{2l}-0.137v_{2m1}-0.236v_{2m2}+0.435v_{2r}\\
+0.134v_{3l}-0.254v_{3m1}-0.254v_{3m2}+0.542v_{3r}+0.134v_{4l}-0.254v_{4m1}-0.254v_{4m2}\\
+0.542v_{4r}+0.134v_{5l}-0.254v_{5m1}-0.254v_{5m2}+0.542v_{5r}-z_{21}^{1}\le 0\\
z_{21}^{1},z_{13}^{1},z_{23}^{1}\ge 0\\
v_{il},v_{im1},v_{im2},v_{ir}\ge 0\ (i=1,2\ldots,5)\\
0.10\le w_1\le 0.30,\ 0.10\le w_2\le 0.40,\ w_3\le w_4,\ 0.15\le w_3\le 0.20,\ 0.20\le w_4\le 0.30,\\
0.10\le w_6\le 0.15,\ 0.10\le w_5\le 0.25,\ w_2-w_1\ge 0.10,\ w_4-w_2\le w_3-w_1\\
w_1+w_2+w_3+w_4+w_5+w_6=1
\end{array}\right.
$$

Table 9 Linguistic terms and their corresponding trapezoid type-1 fuzzy number

Linguistic terms	Trapezoid type-1 fuzzy number
Very low (VL)	(0, 0, 0,0.75;0.95)
Low (L)	(0.025, 0.1, 0.1,0.25;0.95)
Medium low (ML)	(0.15, 0.3, 0.3,0.45;0.95)
Medium (M)	(0.35, 0.5, 0.5,0.65;0.95)
Medium high (MH)	(0.55, 0.7, 0.7,0.85;0.95)
High (H)	(0.75, 0.9, 0.9,0.975;0.95)
Very high (VH)	(0.925, 1, 1,1;0.95)

$$
\omega^{*}=(0.134, 0.216, 0.159, 0.257, 0.112, 0.122)^{T}
$$

$$
A^{+}=\begin{pmatrix}
(0.872, 0.891, 0.894, 0.901; 1), (0.934, 0.952, 0.965, 0.977; 1)\\
(0.872, 0.891, 0.894, 0.901; 1), (0.934, 0.952, 0.965, 0.977; 1)\\
(0.872, 0.891, 0.894, 0.901; 1), (0.934, 0.952, 0.965, 0.977; 1)
\end{pmatrix}
$$

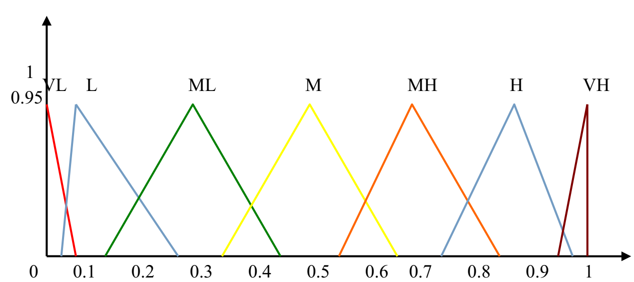

Fig. 4 The membership function for type-1 linguistic term

Table 10 The distance between each alternative and T1FPIS

	A_1	A_2	A_3	A_4	A_5	Ranking orders
D_1	0.314	0.285	0.296	0.307	0.325	$A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$
D_2	0.325	0.279	0.265	0.288	0.307	$A_3 \succ A_2 \succ A_4 \succ A_5 \succ A_1$
D_3	0.288	0.259	0.301	0.265	0.315	$A_2 \succ A_4 \succ A_1 \succ A_3 \succ A_5$

Then, using the Euclidean distance of each alternative $A_i (i = 1, 2, \ldots, 5)$ coming from the T1FPIS A^+, the results are:

$$d_1^1 = 0.336,\ d_2^1 = 0.236,\ d_3^1 = 0.217,$$
$$d_4^1 = 0.417,\ d_5^1 = 0.362$$

Similarly, we obtain the distance of each alternative $A_i (i = 1, 2, \ldots, 5)$ coming from the T1FPIS A^+ by decision makers D_2 and D_3, respectively.

$$d_1^2 = 0.316,\ d_2^2 = 0.272,\ d_3^2 = 0.341,$$
$$d_4^2 = 0.432,\ d_5^2 = 0.259$$
$$d_1^3 = 0.228,\ d_2^3 = 0.325,\ d_3^3 = 0.513,$$
$$d_4^3 = 0.424,\ d_5^3 = 0.347$$

Based on these distances, the ranking order is visualized in Table 10.

Then, using the Copeland's social choice function (Copeland 1951), the scores of the alternatives $A_i (i = 1, 2, \ldots, 5)$ can be obtained as depicted in Table 11.

From the Table 11, it is easy to produce the overall ranking order is:

$$A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$$

Therefore, the best supplier is A_2.

As it is seen from Table 11, the ranking order-based T1-LINMAP method is slightly different with our proposed method. The main reason is that when we transform the type-2 fuzzy sets into type-1 fuzzy sets, the type reduction method will leads to some information loss. In addition, the proposed method can both consider the absolute and relative preference relation using an interval type-2 fuzzy truth degree, while the T1-LINMAP method can only consider the absolute preference relation and ignore the relative preference truth degree; this is the main advantage of the T1-LINMAP method that often leads to some inconsistency phenomenon in actual decision problems.

Compared with the T1-LINMAP method, our proposed method has some advantages, which are shown as follows:

(1) The proposed method extends the LINMAP method to interval type-2 fuzzy environment, so it is more suitable to handle imprecision and imperfect information in real decision-making applications. Furthermore, if we let the UMF and LMF of the IT2FSs are equal, and set the preference relation $R(\tilde{C}_p(k,l)) = 1$, then the IT2FSs mathematical programming model conducted in this paper can reduce to the T1FSs mathematical programming model in Xia et al. (2006). Therefore, the T1-LINMAP method is a special case of our proposed method in this paper.

(2) The proposed method can solve MAGDM problems by constructing one linear programming model, while the T1-LINMAP method can only handle MADM problem; therefore, the proposed method is more general. In addition, our methods both consider the attribute's weight and decision maker's weight together, which means that it can better model the real decision problem.

(3) Our method is more reasonable, because it considers the preference relation with type-2 truth degree and incomplete attribute weights information, whereas the T1-LINMAP method and its extension did not consider the fuzzy truth degree; obviously, it is not in accordance with many actual decision situations.

Table 11 Copeland's social choice scores of all alternatives with respect to each DM based on TI-LINMAP method

Supplier	Decision makers team			Copeland's social choice scores
	D_1	D_2	D_3	
A_1	4	5	3	4.60
A_2	1	2	1	1.40
A_3	2	1	4	2.30
A_4	3	3	2	2.65
A_5	5	4	5	4.65

4.3.2 Comparison with T2-LINMAP method

In what follows, another comparative study was conducted to validate the results of the proposed method with Chen's (2015) interval type-2 fuzzy LINMAP method, which involves the following steps:

Step 1. Utilize the IT2WA operator to aggregate all individual decision matrixes into group decision matrix. Based on the definition of IT2FWA operator proposed by Hu et al. (2013), we can calculate the value of A_{ij}; the results are shown as follows:

$$A_{11} = [(0.690, 0.825, 0.825, 0.895; 0.95), (0.758, 0.825, 0.825, 0.860; 1)]$$
$$A_{12} = [(0.170, 0.370, 0.370, 0.570; 0.95), (0.270, 0.370, 0.370, 0.470; 1)]$$

$A_{13} = [(0.820, 0.960, 0.960, 1.000; 0.95), (0.890, 0.960, 0.960, 0.980; 1)]$

$A_{14} = [(0.740, 0.880, 0.880, 0.960; 0.95), (0.810, 0.880, 0.880, 0.960; 1)]$

$A_{15} = [(0.760, 0.895, 0.895, 0.965; 0.95), (0.828, 0.895, 0.895, 0.930; 1)]$

$A_{16} = [(0.025, 0.115, 0.115, 0.245; 0.95), (0.070, 0.115, 0.115, 0.198; 1)]$

$A_{21} = [(0.630, 0.830, 0.830, 0.965; 0.95), (0.730, 0.830, 0.830, 0.898; 1)]$

$A_{22} = [(0.510, 0.675, 0.675, 0.805; 0.95), (0.593, 0.675, 0.675, 0.740; 1)]$

$A_{23} = [(0.280, 0.420, 0.420, 0.580; 0.95), (0.350, 0.420, 0.420, 0.500; 1)]$

$A_{24} = [(0.245, 0.410, 0.410, 0.610; 0.95), (0.328, 0.410, 0.410, 0.510; 1)]$

$A_{25} = [(0.430, 0.595, 0.595, 0.725; 0.95), (0.513, 0.595, 0.595, 0.660; 1)]$

$A_{26} = [(0.455, 0.630, 0.630, 0.790; 0.95), (0.543, 0.630, 0.630, 0.710; 1)]$

$A_{31} = [(0.380, 0.545, 0.545, 0.675; 0.95), (0.463, 0.545, 0.545, 0.610; 1)]$

$A_{32} = [(0.595, 0.735, 0.735, 0.825; 0.95), (0.665, 0.735, 0.735, 0.780; 1)]$

$A_{33} = [(0.630, 0.830, 0.830, 0.965; 0.95), (0.730, 0.830, 0.830, 0.898; 1)]$

$A_{34} = [(0.390, 0.590, 0.590, 0.755; 0.95), (0.490, 0.590, 0.590, 0.673; 1)]$

$A_{35} = [(0.435, 0.550, 0.550, 0.630; 0.95), (0.493, 0.550, 0.550, 0.602; 1)]$

$A_{36} = [(0.620, 0.785, 0.785, 0.875; 0.95), (0.703, 0.785, 0.785, 0.830; 1)]$

$A_{41} = [(0.330, 0.465, 0.465, 0.615; 0.95), (0.398, 0.465, 0.465, 0.540; 1)]$

$A_{42} = [(0.460, 0.635, 0.635, 0.785; 0.95), (0.548, 0.635, 0.635, 0.710; 1)]$

$A_{43} = [(0.380, 0.545, 0.545, 0.675; 0.95), (0.463, 0.545, 0.545, 0.610; 1)]$

$A_{44} = [(0.630, 0.790, 0.790, 0.875; 0.95), (0.710, 0.790, 0.790, 0.832; 1)]$

$A_{45} = [(0.610, 0.785, 0.785, 0.895; 0.95), (0.698, 0.785, 0.785, 0.840; 1)]$

$A_{46} = [(0.435, 0.550, 0.550, 0.630; 0.95), (0.493, 0.550, 0.550, 0.602; 1)]$

$A_{51} = [(0.025, 0.115, 0.115, 0.245; 0.95), (0.070, 0.115, 0.115, 0.198; 1)]$

$A_{52} = [(0.700, 0.825, 0.825, 0.875; 0.95), (0.763, 0.825, 0.825, 0.850; 1)]$

$A_{53} = [(0.040, 0.145, 0.145, 0.275; 0.95), (0.093, 0.145, 0.145, 0.228; 1)]$

$A_{54} = [(0.480, 0.645, 0.645, 0.775; 0.95), (0.563, 0.645, 0.645, 0.710; 1)]$

$A_{55} = [(0.390, 0.590, 0.590, 0.750; 0.95), (0.490, 0.590, 0.590, 0.670; 1)]$

$A_{56} = [(0.440, 0.640, 0.640, 0.800; 0.95), (0.540, 0.640, 0.640, 0.720; 1)]$

Step 2. We can determine the evaluative ratings A_{+j} and A_{-j} of the approximate positive ideal and negative ideal solutions z_+ and z_- with respect to C_j, where the identify characteristics A_+ and A_- are calculated as follows:

$$
\begin{aligned}
A_+ = \Big\{ & \Big\langle C_1, \Big[A_{+1}^L, A_{+1}^U\Big]\Big\rangle, \Big\langle C_2, \Big[A_{+2}^L, A_{+2}^U\Big]\Big\rangle, \\
& \Big\langle C_3, \Big[A_{+3}^L, A_{+3}^U\Big]\Big\rangle, \Big\langle C_4, \Big[A_{+4}^L, A_{+4}^U\Big]\Big\rangle, \\
& \Big\langle C_5, \Big[A_{+5}^L, A_{+5}^U\Big]\Big\rangle, \Big\langle C_6, \Big[A_{+6}^L, A_{+6}^U\Big]\Big\rangle \Big\} \\
A_- = \Big\{ & \Big\langle C_1, \Big[A_{-1}^L, A_{-1}^U\Big]\Big\rangle, \Big\langle C_2, \Big[A_{-2}^L, A_{-2}^U\Big]\Big\rangle, \\
& \Big\langle C_3, \Big[A_{-3}^L, A_{-3}^U\Big]\Big\rangle, \Big\langle C_4, \Big[A_{-4}^L, A_{-4}^U\Big]\Big\rangle, \\
& \Big\langle C_5, \Big[A_{-5}^L, A_{-5}^U\Big]\Big\rangle, \Big\langle C_{,6}, \Big[A_{-6}^L, A_{-6}^U\Big]\Big\rangle \Big\}
\end{aligned}
$$

where

$$
\begin{aligned}
\Big[A_{+j}^L, A_{+j}^U\Big] = \Bigg[\Bigg(& \bigvee_{p=1}^{3} \left(\bigvee_{i=1}^{6} e_p a_{1i_j}^{Lp} \right), \bigvee_{p=1}^{3} \left(\bigvee_{i=1}^{6} e_p a_{1i_j}^{Lp} \right), \\
& \bigvee_{p=1}^{3} \left(\bigvee_{i=1}^{6} e_p a_{1i_j}^{Lp} \right), \bigvee_{p=1}^{3} \left(\bigvee_{i=1}^{6} e_p a_{1i_j}^{Lp} \right); \\
& \bigwedge_{p=1}^{3} \left(\bigwedge_{i=1}^{6} e_p h_{1i_j}^{p} \right) \Bigg), \Bigg(\bigvee_{p=1}^{3} \left(\bigvee_{i=1}^{6} e_p a_{1i_j}^{Up} \right), \\
& \bigvee_{p=1}^{3} \left(\bigvee_{i=1}^{6} e_p a_{1i_j}^{Up} \right), \bigvee_{p=1}^{3} \left(\bigvee_{i=1}^{6} e_p a_{1i_j}^{Up} \right), \\
& \bigvee_{p=1}^{3} \left(\bigvee_{i=1}^{6} e_p a_{1i_j}^{Up} \right); \bigwedge_{p=1}^{3} \left(\bigwedge_{i=1}^{6} e_p h_{1i_j}^{p} \right) \Bigg) \Bigg]
\end{aligned}
$$

The results are listed in Table 12.

Step 3. Utilize the interval type-2 Euclidean distance to calculate the results of $d_2(A_{ij}, A_{+j})$ and $d_2(A_{ij}, A_{-j})$ which are shown in the left two columns of Table 13. Due to the

Table 12 The evaluative ratings A_{+j} and A_{-j}

The evaluative rating A_{+j}	
A_{+1}	[(0.630, 0.830, 0.830, 0.965; 0.95), (0.730, 0.830, 0.830, 0.898; 1)]
A_{+2}	[(0.700, 0.825, 0.825, 0.875; 0.95), (0.763, 0.825, 0.825, 0.850; 1)]
A_{+3}	[(0.820, 0.960, 0.960, 1.000; 0.95), (0.890, 0.960, 0.960, 0.980; 1)]
A_{+4}	[(0.740, 0.880, 0.880, 0.960; 0.95), (0.810, 0.880, 0.880, 0.960; 1)]
A_{+5}	[(0.760, 0.895, 0.895, 0.965; 0.95), (0.828, 0.895, 0.895, 0.930; 1)]
A_{+6}	[(0.620, 0.785, 0.785, 0.875; 0.95), (0.703, 0.785, 0.785, 0.830; 1)]
The evaluative rating A_{-j}	
A_{-1}	[(0.025, 0.115, 0.115, 0.245; 0.95), (0.070, 0.115, 0.115, 0.198; 1)]
A_{-2}	[(0.170, 0.370, 0.370, 0.570; 0.95), (0.270, 0.370, 0.370, 0.470; 1)]
A_{-3}	[(0.040, 0.145, 0.145, 0.275; 0.95), (0.093, 0.145, 0.145, 0.228; 1)]
A_{-4}	[(0.245, 0.410, 0.410, 0.610; 0.95), (0.328, 0.410, 0.410, 0.510; 1)]
A_{-5}	[(0.390, 0.590, 0.590, 0.750; 0.95), (0.490, 0.590, 0.590, 0.670; 1)]
A_{-6}	[(0.035, 0.145, 0.145, 0.275; 0.95), (0.085, 0.145, 0.145, 0.235; 1)]

Table 13 Results of distances and closeness-based indices

A_{ij}	Use the city block distance ($\beta = 1$)			Use the Euclidean distance ($\beta = 2$)		
	$d_1(A_{ij}, A_{+j})$	$d_1(A_{ij}, A_{-j})$	CI_{ij}^1	$d_2(A_{ij}, A_{+j})$	$d_2(A_{ij}, A_{-j})$	CI_{ij}^2
A_{11}	0.027	0.688	0.962	0.031	0.689	0.957
A_{12}	0.441	0.000	0.000	0.442	0.000	0.000
A_{13}	0.000	0.789	1.000	0.000	0.790	1.000
A_{14}	0.000	0.457	1.000	0.000	0.459	1.000
A_{15}	0.000	0.300	1.000	0.000	0.303	1.000
A_{16}	0.646	0.027	0.040	0.647	0.028	0.042
A_{21}	0.000	0.693	1.000	0.000	0.695	1.000
A_{22}	0.143	0.299	0.676	0.148	0.300	0.669
A_{23}	0.518	0.212	0.290	0.519	0.232	0.309
A_{24}	0.457	0.000	0.000	0.459	0.000	0.000
A_{25}	0.294	0.015	0.049	0.296	0.019	0.060
A_{26}	0.144	0.476	0.768	0.146	0.477	0.766
A_{31}	0.279	0.414	0.597	0.281	0.286	0.504
A_{32}	0.085	0.356	0.807	0.086	0.359	0.807
A_{33}	0.123	0.665	0.844	0.131	0.667	0.836
A_{34}	0.290	0.167	0.365	0.293	0.168	0.364
A_{35}	0.337	0.049	0.127	0.338	0.059	0.147
A_{36}	0.000	0.619	1.000	0.000	0.626	1.000
A_{41}	0.350	0.343	0.495	0.351	0.344	0.494
A_{42}	0.181	0.260	0.589	0.184	0.262	0.587
A_{43}	0.403	0.386	0.489	0.404	0.387	0.489
A_{44}	0.098	0.359	0.786	0.099	0.362	0.785
A_{45}	0.110	0.190	0.633	0.112	0.192	0.632
A_{46}	0.226	0.394	0.635	0.227	0.396	0.636
A_{51}	0.693	0.000	0.000	0.694	0.000	0.000
A_{52}	0.000	0.441	1.000	0.000	0.446	1.000
A_{53}	0.789	0.000	0.000	0.790	0.000	0.000
A_{54}	0.235	0.222	0.486	0.236	0.224	0.487
A_{55}	0.300	0.000	0.000	0.303	0.000	0.000
A_{56}	0.139	0.481	0.776	0.142	0.482	0.772

space limited, we take $d_2(A_{11}, A_{+1})$ and $d_2(A_{11}, A_{-1})$as an example:

$$d_2(A_{11}, A_{+1}) = \left[\frac{1}{8}((0.690-0.630)^2+(0.825-0.830)^2 + (0.825-0.830)^2+(0.895-0.965)^2 + (0.758-0.730)^2+(0.825-0.830)^2 + (0.825-0.830)^2+(0.860-0.898)^2)\right]^{\frac{1}{2}} = 0.037$$

$$d_2(A_{11}, A_{-1}) = \left[\frac{1}{8}((0.690-0.025)^2+(0.825-0.115)^2 + (0.825-0.115)^2+(0.895-0.245)^2 + (0.758-0.070)^2+(0.825-0.115)^2 + (0.825-0.115)^2+(0.860-0.198)^2)\right]^{\frac{1}{2}} = 0.688$$

Step 4. Calculate the closeness-based index for each alternative based on interval type-2 Euclidean distance. The obtained results are listed in the last column of table. Take CI_{11}^2 as an example:

$$CI_{11}^2 = \frac{d_2(A_{11}, A_{-1})}{d_2(A_{11}, A_{+1})+d_2(A_{11}, A_{-1})} = \frac{0.688}{0.688+0.037} = 0.949$$

Step 5. As mentioned, the ordered pairs in Ω are (1, 3), (1,5), (2, 3), (2, 5), (3, 5). We set the parameter $\eta = 0.3$, and the incompletely known weight vector is denoted by $\omega = (\omega_1, \omega_2, \ldots, \omega_6)$, which satisfies $\omega_j \geq 0 (j = 1, 2, \ldots, 6)$ and $\sum_{j=1}^{6} \omega_j = 1$,

$$\Lambda = \{0.10 \leq w_1 \leq 0.30, 0.10 \leq w_2 \leq 0.40, w_3 \leq w_4, 0.15 \leq w_3 \leq 0.20, 0.20 \leq w_4 \leq 0.30, 0.10 \leq w_5 \leq 0.25, 0.10 \leq w_6 \leq 0.15, w_2 - w_1 \geq 0.15, w_4 - w_2 \leq w_3 - w_1\}$$

Therefore, for each ordered pair in set Ω, we can calculate $Z_{13} = 0 \vee \sum_{j=1}^{6} (C_{3j}^2 - C_{1j}^2)\omega_j$, $Z_{23} = 0 \vee \sum_{j=1}^{6} (C_{3j}^2 - C_{2j}^2)\omega_j$, $Z_{25} = 0 \vee \sum_{j=1}^{6} (C_{5j}^2 - C_{2j}^2)\omega_j$, $Z_{35} = 0 \vee \sum_{j=1}^{6} (C_{5j}^2 - C_{3j}^2)\omega_j$ and $Z_{46} = 0 \vee \sum_{j=1}^{6} (C_{6j}^2 - C_{4j}^2)\omega_j$, we use Chen's method to construct the following linear programming model:

$$\min\{Z_{13}+Z_{23}+Z_{25}+Z_{35}+Z_{45}\}$$
$$\text{s.t.}\begin{cases} -2.847\omega_1+1.742\omega_2-0.198\omega_3-0.175\omega_4-1.605\omega_5+1.111\omega_6 \geq 0.3 \\ -0.352\omega_1+0.806\omega_2-0.164\omega_3-0.636\omega_4-0.852\omega_5+0.959\omega_6+Z_{13} \geq 0 \\ -0.403\omega_1+0.125\omega_2+0.492\omega_3+0.364\omega_4+0.087\omega_5+0.234\omega_6+Z_{23} \geq 0 \\ -1.000\omega_1+0.281\omega_2+0.037\omega_3+0.487\omega_4-0.061\omega_5+0.007\omega_6+Z_{25} \geq 0 \\ -0.597\omega_1+0.156\omega_2-0.455\omega_3+0.123\omega_4-0.148\omega_5-0.227\omega_6+Z_{35} \geq 0 \\ -0.495\omega_1+0.374\omega_2-0.108\omega_3-0.513\omega_4-0.631\omega_5+0.138\omega_6+Z_{45} \geq 0 \\ Z_{13} \geq 0, Z_{23} \geq 0, Z_{25} \geq 0, Z_{35} \geq 0, Z_{45} \geq 0 \\ \omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6 = 1, \omega_j \geq 0 \ for\ all\ j \\ \omega \in \Lambda \end{cases}$$

Using the simplex method to solve the linear programming model, we obtain the optimal weight vector: $\omega = (0.153, 0.204, 0.231, 0.252, 0.117, 0.043)$.

Step 6. Calculate the corresponding results of the comprehensive closeness degree; the results are shown as follows:

$$\overline{WI_1^2} = 0.457, \overline{WI_2^2} = 0.725, \overline{WI_3^2} = 0.623, \overline{WI_4^2} = 0.511, \overline{WI_5^2} = 0.398$$

Therefore, the priority ranking order of the five suppliers consists of $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$; the result is the same with the proposed method.

In what follows, we use the city block distances $d_1(A_{ij}, A_{+j})$ and $d_1(A_{ij}, A_{-j})$ $(\beta = 1)$ to do detailed calculations. The results are shown in the left-hand portion of Table 13. The computed results of K_j are shown as follows:

$$K_1 = d_1(A_{+1}, A_{-1}) = 0.693 \quad K_2 = d_1(A_{+2}, A_{-2}) = 0.441$$
$$K_3 = d_1(A_{+3}, A_{-3}) = 0.789 \quad K_4 = d_1(A_{+4}, A_{-4}) = 0.457$$
$$K_5 = d_1(A_{+5}, A_{-5}) = 0.301 \quad K_6 = d_1(A_{+6}, A_{-6}) = 0.619$$

We use the Chen's method to construct the following linear programming model:

$$\min\{Z_{13}+Z_{23}+Z_{25}+Z_{35}+Z_{45}\}$$
$$\text{s.t.}\begin{cases}-2.855\omega_1+1.865_2-0.194\omega_3-0.038\omega_4-1.604\omega_5+1.117\omega_6\geq 0.3\\ -0.365\omega_1+0.807\omega_2-0.156\omega_3-0.635\omega_4-0.873\omega_5+0.960\omega_6+Z_{13}\geq 0\\ -0.402\omega_1+0.131\omega_2+0.499\omega_3+0.365\omega_4+0.078\omega_5+0.232\omega_6+Z_{23}\geq 0\\ -1.000\omega_1+0.324\omega_2+0.035\omega_3+0.486\omega_4-0.049\omega_5+0.008\omega_6+Z_{25}\geq 0\\ -0.594\omega_1+0.193\omega_2-0.463\omega_3+0.121\omega_4-0.127\omega_5-0.224\omega_6+Z_{35}\geq 0\\ -0.494\omega_1+0.410\omega_2-0.109\omega_3-0.299\omega_4-0.633\omega_5+0.141\omega_6+Z_{45}\geq 0\\ Z_{13}\geq 0,\ Z_{23}\geq 0,\ Z_{25}\geq 0,\ Z_{35}\geq 0,\ Z_{45}\geq 0\\ \omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6=1,\ \omega_j\geq 0 \text{ for all } j\\ \omega\in\Lambda\end{cases}$$

In the virtue of the simplex method considered to solve the linear programming model, we obtain the optimal weight vector to be: $\omega=(0.153, 0.204, 0.231, 0.252, 0.117, 0.043)$. Then, we calculate the corresponding results of the comprehensive closeness degree; the results are shown as follows:

$$\overline{WI_1^2}=0.399,\ \overline{WI_2^2}=0.732,$$
$$\overline{WI_3^2}=0.647,\ \overline{WI_4^2}=0.562,\ \overline{WI_5^2}=0.518$$

Therefore, the priority ranking order of the five suppliers comes as $A_2\succ A_3\succ A_4\succ A_5\succ A_1$, which is also equivalent to the ranking order obtained by solving the proposed method.

Compared with the Chen's T2-LINMAP method, the proposed method has some differences and advantages, which are shown as follows:

(1) The proposed method uses the interval type-2 fuzzy truth degree to capture the membership of each paired order preference relation, which means that the higher order uncertainty is taken into consideration. While Chen's method can only consider the absolutely paired order preference relation of the alternative. It fails to reflect the uncertainty of the actual decision process. Furthermore, if we set the initial preference relation $\tilde{C}_p(k,l)=1$, then the proposed linear model is equivalent to Chen's method.

(2) Compared with Chen's method, the proposed method uses a new distance measure to construct the linear programming model, this new distance based on KM algorithm to use the centroid interval information, while Chen's method can only use the reference point information to derive the distance; many useful information is ignored in this distance method. Therefore, our method can overcome the drawbacks of the previous method and avoid the information loss in the decision process.

(3) In Chen's method, the LMF and UMF are computed based on Zadeh's extension principle; the max–min$(\vee,\wedge)$operator is adopted to aggregate the decision information. Although the computation overhead is reduced, it generates a lot of information loss. In our method, we use the IT2F aggregation operator based on Frank norms, which is more general than Chen's method.

(4) The proposed method can solve MAGDM problems by constructing one linear programming model, while Chen's T2-LINMAP method can only handle MADM problem. Therefore, compared with the Chen's model, the proposed method integrates more useful information into the decision process. It is note worthy that in Chen's model, the first constrain can be regarded as a linear combination of the other constrains associated with identity vector, so it will lead to multicollinearity. Furthermore, based on linear programming theory, we know that when the large-scale problem or the paired order set exist inverted sequence problem are taken into consideration, this model may fail to obtain feasible solution. In the proposed method, we take both the attribute weight and the fuzzy truth degree into consideration; it can guarantee that the model always have optimal feasible solution because the associated vector is not equal to identity vector. Therefore, the proposed method is more suitable for solving MAGDM problem within the context of IT2FSs.

5 Conclusions and future works

With the economic globalization and highly competitive business environment, the supplier selection problem is one of the most important issues in the supper chain management, which directly impact on the manufactures' performance. From this perspective, developing and extending a new supplier selection decision-making method have primary significance. Though many fuzzy MADM methods have been used to supplier selection problem, all these methods cannot consider the decision maker's fuzzy preference relations with incomplete information, and also cannot solve the fuzzy group decision-making problems. So this work has focused on the group decision making under interval type-2 fuzzy environment for a supplier selection using the classical linear programming technique.

LINMAP method is a useful linear programming technique to solve the MAGDM problems, especially in the situation where the DMs' preference relations are taken into consideration. In this paper, we have extended the LINMAP method to handle MAGDM problems under interval type-2 fuzzy environment, in which all the preference relation information provided by DMs is represented by IT2FSs, and the weights of attributes are incompletely known. First, we have defined a new distance and the weighted distances based on the centroid interval KM algorithm between IT2FSs, and further construct a linear programming (LP) model according to the group decision consistency and inconsistency index with the incomplete preference relations. Then, we develop a LINMAP-based MAGDM method within IT2FSs; in this method, we can utilize the optimization technique to generate the optimal attribute vector and the positive ideal solution, and use the Copeland's social choice function to derive the ranking order of alternatives and obtain the best alternative(s) for the group. Finally, we use a real supplier selection example to illustrate the proposed method, and make a comparison analysis with the T1-LINMAP method; the result shows that the proposed method is more general and flexible than T1-LINMAP method and easy to be applied and spread.

In the future, we will continue our work to extend the proposed method to solve general type-2 fuzzy (GT2F) MAGDM problems and further consider some behavior and risk factors in actual decision-making problems, and give its applications to supplier selection under a variety of uncertainty environments. In addition, how to effectively solve the type-2 fuzzy linear programming model and make a sensitivity analysis are two critical issues to impact the validity of the proposed method, which will be considered in the future research.

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Compliance with ethical standards

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