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METHODOLOGIES AND APPLICATION

A group decision making approach for trapezoidal fuzzy preference relations with compatibility measure

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Abstract The purpose of this paper is to develop a new compatibility for the additive trapezoidal fuzzy preference relations and utilize it to determine the optimal weights of experts in the group decision making. First, a least deviation model to obtain the priority vector of the additive trapezoidal fuzzy preference relation is provided. Then compatibility index of two additive trapezoidal fuzzy preference relations is proposed and some desirable properties are investigated. The characteristic of the new compatibility is that it uses the deviation measure between an additive trapezoidal fuzzy preference relation and its characteristic preference relation based on consistency of the preference relation, which develops a theoretic basis for the application of additive trapezoidal fuzzy preference relations in group decision making. Then, in order to determine the weights of experts in the group decision making, we propose an optimal model based on the criterion of minimizing the compatibility index. Finally, an example shows the feasibility and effectiveness of the proposed method.

Keywords Group decision making · Additive trapezoidal fuzzy preference relation · Priority vector · Compatibility

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1 Introduction

Group decision making plays an important role in modern politics, economy, science and military and so on. It is a process during which experts express their preference and ranking or preferred decision alternatives. In the process of decision making, decision makers often need to compare alternatives with each other and construct the judgment matrices, which are also called preference relations, have been developed, including fuzzy preference relation (Chen et al. 2014; Yan and Ma 2015; Zhu and Xu 2014), multiplicative preference relation (Chiclana et al. 2001), interval preference relation (Chen et al. 2015; Wu and Chiclana 2014) linguistic preference relation (Alonso et al. 2009; Dong and Herrera-Viedma 2015; Dong et al. 2009), interval linguistic preference relation (García et al. 2012), and intuitionistic fuzzy preference relation (Zeng et al. 2013), etc.

There are two crucial problems worthy of investigating in group decision making, which are consistency and compatibility of preference relations. The consistency of preference relations is an important issue in group decision making. The lack of consistency can result in inconsistent conclusions. Saaty (1980) first introduced the definition of consistency of multiplicative preference relation by using the consistency ratio. Table 1 summarises the literatures dealing with consistency, consistency improving methods, consistency index decision making methods and consistency based GDM methods.

Based on consistency of preference relations, priority weights are used to rank the alternatives. There are a lot of techniques to derive priority weights in group decision making with different preference relations. They are shown in Table 2.

However, there is little investigation that derives priority weights of trapezoidal fuzzy preference relations.

Table 1 Summary for consistency of different preference relations (PRs)

Characteristics	PR and references
Definition of consistency	Multiplicative PR (Chiclana et al. 2009; Saaty 1980)
	Fuzzy PR (Herrera-Viedma et al. 2004; Xia et al. 2013; Xu et al. 2013a)
	Interval fuzzy PR (Wu and Chiclana 2014; Xu 2011)
	Linguistic PR (Alonso et al. 2009; Dong et al. 2008b)
	Triangular fuzzy PR (Liu et al. 2014)
	Trapezoidal fuzzy PR (Gong et al. 2013)
	Hesitant fuzzy PR (Wang and Xu 2015; Zhang and Wu 2014)
Consistency improving method	Multiplicative PR (Wu and Xu 2012; Xu and Wei 1999)
	Fuzzy PR (Liu et al. 2012b; Xia et al. 2013)
	Linguistic PR (Dong et al. 2015, 2008b)
Consistency index method	Fuzzy PR (Dong et al. 2008a; Meng and Chen 2015)
	Linguistic PR (Dong et al. 2008b)
Consistency based GDM	Multiplicative PR (Wu and Xu 2012; Xu et al. 2013b)
	Fuzzy PR (Meng and Chen 2015; Pérez et al. 2013; Xu et al. 2013b; Zhang et al. 2012)
	Interval PR (Wu and Chiclana 2014)
	Linguistic PR (Wu et al. 2015)
	Interval linguistic PR (García et al. 2012)

Table 2 Approaches to deriving priority weights of different PRs

PRs	Approaches
Multiplicative PRs	Eigenvalue method (Saaty 1980), Chi-square method (Wang et al. 2007)
Fuzzy PRs	Least-square method (Gong 2008), chi-square method (Wang et al. 2007)
Interval multiplicative PRs	Multi-objective optimization model (Conde and Pérez 2010), iterative algorithm (Lan et al. 2012), goal programming method (Wang and Elhag 2007), two-stage logarithmic goal programming method (Wang et al. 2005)
Interval fuzzy PRs	Straightforward approach (Genc et al. 2010), iterative algorithm (Liu et al. 2012a), logarithmic least square method (Wang and Chen 2014), goal programming method (Wang and Li 2012), linear programming method (Xu and Chen 2008)
Triangular fuzzy PRs	Logarithmic least square method (Wang 2015)

Compatibility is used to measure the consensus of rankings between the group and each individual. The lack of acceptable compatibility can bring the lack of decision making with preference relations because there is a significant difference among the preference relations proposed by experts in group decision making. Saaty (1994) was the first to propose compatibility of preference relations. For fuzzy decision making environment, some literatures about compatibility measures of difference preference relations are demonstrated in Table 3. From the Table 3, we can see that little attempt has been devoted to the issue on the compatibility of two additive trapezoidal fuzzy preference relations in the literatures.

Table 3 Compatibilities for different PRs under fuzzy environment

Types of PRs	References
Interval multiplicative PRs	Wang et al. (2013) and Zhou et al. (2014b)
Interval fuzzy PRs	Xu (2004)
Interval multiplicative linguistic PRs	Zhou et al. (2014a)
Interval additive linguistic PRs	Chen et al. (2011)

The aim of this paper is to develop a new compatibility for additive trapezoidal fuzzy preference relations and use it to determine the optimal weights for experts in group decision making. In order to do that, we define the compatibility degree and compatibility index of two additive trapezoidal fuzzy preference relations. Some properties of compatibility degree and compatibility index of two additive trapezoidal fuzzy preference relations are studied. We also construct a model to determine the weighting vector of weights in group decision making. Finally, some examples are given to illustrate the new approach.

The rest of this paper is organized as follows. In Sect. 2, we mainly introduce the definition and operational laws of trapezoidal fuzzy numbers. Two kinds of preference relations, additive and multiplicative trapezoidal fuzzy preference relations are defined. In Sect. 3, we investigate the relationship of additive and multiplicative trapezoidal fuzzy preference relations, and then an optimal model is presented to obtain the priority vector of additive trapezoidal fuzzy preference relation. In Sect. 4, the new compatibility degree and compatibility index of two additive trapezoidal fuzzy preference relations are developed and their properties are studied. We also propose an optimal model to determine the weights of experts in group decision making based on the compatibility of additive trapezoidal fuzzy preference relation. Section 5 provides an illustrative example to show the effectiveness of the proposed method and conclusions are made in Sect. 6.

2 Preliminaries

In this section, we introduce the trapezoidal fuzzy numbers and their operational laws, ranking of the trapezoidal fuzzy numbers, and additive and multiplicative trapezoidal fuzzy preference relations.

2.1 Trapezoidal fuzzy numbers and their operation laws

Definition 1 (Liou and Wang 1992) A fuzzy number $\tilde{A}$ on R is referred to be as a trapezoidal fuzzy number, if its membership function $f_{\tilde{A}} : R \to [0, 1]$ satisfies

$$f_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ \dfrac{x - a_4}{a_3 - a_4}, & a_3 \le x \le a_4, \\ 0, & \text{otherwise}, \end{cases}$$

where

$$f_{\tilde{A}}^L(x) = \frac{x - a_1}{a_2 - a_1}, \quad a_1 \le x \le a_2,$$

is called the left membership function, and

$$f_{\tilde{A}}^R(x) = \frac{x - a_4}{a_3 - a_4}, \quad a_3 \le x \le a_4,$$

is called the right membership function.

Obviously, $f_{\tilde{A}}^L(x)$ is a continuously increasing function, and $f_{\tilde{A}}^R(x)$ is a continuously decreasing function. The inverse functions $g_{\tilde{A}}^L(x)$ and $g_{\tilde{A}}^R(x)$ of $f_{\tilde{A}}^L(x)$ and $f_{\tilde{A}}^R(x)$, respectively, are shown as follows:

$$g_{\tilde{A}}^L(x) = a_1 + (a_2 - a_1)y, \quad y \in [0, 1],$$
$$g_{\tilde{A}}^R(x) = a_4 + (a_3 - a_4)y, \quad y \in [0, 1].$$

A trapezoidal fuzzy number can be denoted by using an ordered array (a_1, a_2, a_3, a_4). Specially, $\tilde{a} = (a, a, a, a)$ is a crisp number. In this paper, for convenience, we assume that (a_1, a_2, a_3, a_4) satisfies $0 < a_1 \le a_2 \le a_3 \le a_4$. Consider two trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$. The operational laws are as follows (Chen and Chen 2007):

(1) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$;
(2) $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$;
(3) $\tilde{A}_1 \otimes \tilde{A}_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2)$;
(4) $\lambda \otimes \tilde{A}_1 = (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1), \lambda > 0$;
(5) $\tilde{A}_1 \oslash \tilde{A}_2 = (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2)$, especially,

$$1 \oslash \tilde{A}_2 = (1/d_2, 1/c_2, 1/b_2, 1/a_2);$$

(6) $\log_\lambda \tilde{A}_1 = \begin{cases} (\log_\lambda a_1, \log_\lambda b_1, \log_\lambda c_1, \log_\lambda d_1), & \lambda > 1, \\ (\log_\lambda d_1, \log_\lambda c_1, \log_\lambda b_1, \log_\lambda a_1), & 0 < \lambda < 1. \end{cases}$

For convenience, we also denote $\tilde{A}_1 \oslash \tilde{A}_2$ as $\frac{\tilde{A}_1}{\tilde{A}_2}$.

2.2 Ranking of the trapezoidal fuzzy numbers

In order to rank the trapezoidal fuzzy numbers, Cheng (1999) designed an algorithm by using the intuition ranking method. Chu (2002) presented a centroid index to rank trapezoidal fuzzy numbers. Chen and Chen (2007) proposed a method considering the centroid points and the standard deviations of trapezoidal fuzzy number. Furthermore, Cheng (1998) proposed a distance method by using centroids points of trapezoidal fuzzy numbers:

$$R(\tilde{A}) = \sqrt{\bar{x}_{\tilde{A}}^2 + \bar{y}_{\tilde{A}}^2}, \tag{1}$$

where

$$\bar{x}_{\tilde{A}} = \frac{\int_{a_1}^{a_2} \left(x f_{\tilde{A}}^L\right) \mathrm{d}x + \int_{a_2}^{a_3} x \mathrm{d}x + \int_{a_3}^{a_4} \left(x f_{\tilde{A}}^R\right) \mathrm{d}x}{\int_{a_1}^{a_2} \left(f_{\tilde{A}}^L\right) \mathrm{d}x + \int_{a_2}^{a_3} \mathrm{d}x + \int_{a_3}^{a_4} \left(f_{\tilde{A}}^R\right) \mathrm{d}x}, \tag{2}$$

$$\bar{y}_{\tilde{A}} = \frac{\int_0^1 \left(y g_{\tilde{A}}^L\right) \mathrm{d}y + \int_0^1 \left(y g_{\tilde{A}}^R\right) \mathrm{d}y}{\int_0^1 \left(g_{\tilde{A}}^L\right) \mathrm{d}x + \int_0^1 \left(g_{\tilde{A}}^R\right) \mathrm{d}x}, \tag{3}$$

are the centroid points for trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, respectively. The larger the value $R(\tilde{A})$, the better the ranking of $\tilde{A}$.

In this paper, we use Cheng's distance method to rank trapezoidal fuzzy numbers because the distance index is more suitable than others to use in multiple criteria decision making with trapezoidal fuzzy numbers.

2.3 Trapezoidal fuzzy preference relation

Trapezoidal fuzzy preference relation is an extension of traditional preference relation. The multiplicative trapezoidal fuzzy preference relation and additive trapezoidal fuzzy preference relation can be defined as follows.

Definition 2 (Gong et al. 2013) Let $\tilde{R} = (\tilde{r}_{ij})_{n \times n}$ be a preference matrix, where $\tilde{r}_{ij} = (r_{ij1}, r_{ij2}, r_{ij3}, r_{ij4})$ is a trapezoidal fuzzy number, and $\frac{1}{9} \le r_{ij1} \le r_{ij2} \le r_{ij3} \le r_{ij4} \le 9, \forall i, j = 1, 2, \ldots, n$, then $\tilde{R}$ is called a multiplicative trapezoidal fuzzy preference relation, if

(i) $r_{ii1} = r_{ii2} = r_{ii3} = r_{ii4} = 1, \ i = 1, 2, \ldots, n$,
(ii) $r_{ij1} r_{ji4} = r_{ij2} r_{ji3} = r_{ij3} r_{ji2} = r_{ij4} r_{ji1} = 1, \ i, j = 1, 2, \ldots, n$.

Definition 3 (Gong et al. 2013) Let $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$ be a preference matrix, where $\tilde{a}_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ is a trapezoidal fuzzy number, and $0 \le a_{ij1} \le a_{ij2} \le a_{ij3} \le a_{ij4} \le 1, \forall i, j = 1, 2, \ldots, n$, then $\tilde{A}$ is called an additive trapezoidal fuzzy preference relation, if

(i) $a_{ii1} = a_{ii2} = a_{ii3} = a_{ii4} = 0.5,\ i = 1, 2, \ldots, n$,
(ii) $a_{ij1} + a_{ji4} = a_{ij2} + a_{ji3} = a_{ij3} + a_{ji2} = a_{ij4} + a_{ji1} = 1,\ i, j = 1, 2, \ldots, n$.

Consistency is to measure whether the preference relation can be used in decision making or not.

Definition 4 (Gong et al. 2013) A multiplicative trapezoidal fuzzy preference relation $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ is said to be completely consistent, if the following equation holds true:

$$\tilde{r}_{im} \otimes \tilde{r}_{mj} \otimes \tilde{r}_{ji} = \tilde{r}_{ij} \otimes \tilde{r}_{jm} \otimes \tilde{r}_{mi},\quad \forall i, j, m = 1, 2, \ldots, n, \quad i \ne j \ne m.$$

The consistent multiplicative trapezoidal fuzzy preference relation $\tilde{R}$ also can be given by Gong et al. (2013):

$$\tilde{r}_{ij} = \begin{cases} \dfrac{\tilde{w}_i}{\tilde{w}_j}, & i \ne j, \\ 1, & i = j, \end{cases} \tag{4}$$

where $\tilde{w}_i = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^{\mathrm{T}}$ is the fuzzy priority vector of $\tilde{R}$, $\tilde{w}_i = (w_{i1}, w_{i2}, w_{i3}, w_{i4})$ is the trapezoidal fuzzy number, $i = 1, 2, \ldots, n$.

Definition 5 (Gong et al. 2013) An additive trapezoidal fuzzy preference relation $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$ is said to be of completely additive consistency, if

$$\tilde{a}_{ij} \oplus \tilde{a}_{jm} = \tilde{a}_{im} \oplus \tilde{a}_{jj}, \quad \forall i, j, m = 1, 2, \ldots, n.$$

It can been from Definitions 4 and 5, consistency of trapezoidal fuzzy preference relations means transitivity, which is the basis of constructing the priority vectors of preference relations.

3 A least deviation model to obtain fuzzy priority vector of additive trapezoidal fuzzy preference relation

In order to obtain fuzzy priority vector of additive trapezoidal fuzzy preference relation, in this section, we develop a least deviation model by using the consistency of additive trapezoidal fuzzy preference relation.

Theorem 1 *Let $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ be a multiplicative trapezoidal fuzzy preference relation, where $\tilde{r}_{ij} = (r_{ij1}, r_{ij2}, r_{ij3}, r_{ij4})$. If*

$$\tilde{a}_{ij} = \widetilde{0.5} \oplus \log_{81} \tilde{r}_{ij}, \quad i, j = 1, 2, \ldots, n, \tag{5}$$

then $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$ is an additive trapezoidal fuzzy preference relation.

Proof By Eq. (5), we get

$$a_{ijk} = 0.5 + \log_{81} r_{ijk}, \quad a_{jik} = 0.5 + \log_{81} r_{jik}, \quad i, j = 1, 2, \ldots, n; \quad k = 1, 2, 3, 4,$$

then for all $i, j = 1, 2, \ldots, n$,

$$\begin{aligned} a_{ij1} + a_{ji4} &= 0.5 + \log_{81} r_{ij1} + 0.5 + \log_{81} r_{ji4} \\ &= 1 + \log_{81}(r_{ij1} r_{ji4}) = 1, \\ a_{ij2} + a_{ji3} &= 0.5 + \log_{81} r_{ij2} + 0.5 + \log_{81} r_{ji3} \\ &= 1 + \log_{81}(r_{ij2} r_{ji3}) = 1. \end{aligned}$$

Similarly, we can obtain

$$a_{ij3} + a_{ji2} = a_{ij4} + a_{ji1} = 1, \quad i, j = 1, 2, \ldots, n.$$

In addition, it is obvious that

$$a_{iik} = 0.5 + \log_{81} r_{iik} = 0.5, \quad i = 1, 2, \ldots, n; k = 1, 2, 3, 4.$$

Therefore, $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$ is the additive trapezoidal fuzzy preference relation.

Theorem 1 indicates that multiplicative trapezoidal fuzzy preference relation can be transformed into the additive trapezoidal fuzzy preference relation by using Eq. (5).

Theorem 2 *Let $\tilde{w}_i = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^{\mathrm{T}}$ be the fuzzy priority vector of additive trapezoidal fuzzy preference relation $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, $\tilde{w}_i = (w_{i1}, w_{i2}, w_{i3}, w_{i4}), i = 1, 2, \ldots, n$. If*

$$\tilde{a}_{ij} = \begin{cases} \widetilde{0.5} \oplus \log_{81} \frac{\tilde{w}_i}{\tilde{w}_j}, & i \ne j, \\ \widetilde{0.5}, & i = j, \end{cases} \tag{6}$$

then $\tilde{A}$ is completely consistent.

Proof By Eq. (6), we obtain

$$a_{ijk} = 0.5 + \log_{81} \frac{\tilde{w}_{ik}}{\tilde{w}_{j(5-k)}}, \quad i, j = 1, 2, \ldots, n; \quad k = 1, 2, 3, 4.$$

Then

$$a_{ijk}+a_{jmk}=0.5+\log_{81}\frac{w_{ik}}{w_{j(5-k)}}+0.5+\log_{81}\frac{w_{jk}}{w_{m(5-k)}}$$
$$=1+\log_{81}\frac{w_{ik}w_{jk}}{w_{j(5-k)}w_{m(5-k)}},$$

and

$$a_{imk}+a_{jjk}=0.5+\log_{81}\frac{w_{ik}}{w_{m(5-k)}}+0.5+\log_{81}\frac{w_{jk}}{w_{j(5-k)}}$$
$$=1+\log_{81}\frac{w_{ik}w_{jk}}{w_{j(5-k)}w_{m(5-k)}}.$$

Thus, we have

$$a_{ijk}+a_{jmk}=a_{imk}+a_{jjk},\quad i,j=1,2,\ldots,n;\quad k=1,2,3,4.$$

Therefore, based on Definition 5, $\tilde{A}$ is a completely consistent additive trapezoidal fuzzy preference relation, which complete the proof of Theorem 2.

As we can see from Theorem 2, if additive trapezoidal fuzzy preference relation $\tilde{A}$ is not completely consistent, then Eq. (6) dose not hold, i.e.,

$$a_{ijk}=0.5+\log_{81}\frac{w_{ik}}{w_{j(5-k)}},\quad i,j=1,2,\ldots,n;\quad k=1,2,3,4, \tag{7}$$

does not hold. Equation (7) is equivalent to Eq. (8):

$$a_{ijk}=0.5+\log_{81}w_{ik}-\log_{81}w_{j(5-k)},\quad i,j=1,2,\ldots,n;\quad k=1,2,3,4. \tag{8}$$

In order to obtain the fuzzy priority vector of $\tilde{A}$, we can construct a least deviation model as follows.

(M-1)

$$\min\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{4}\left(a_{ijk}-0.5-\log_{81}w_{ik}+\log_{81}w_{j(5-k)}\right)^2 \tag{9}$$

$$\text{s.t.}\begin{cases}0\le\sum_{i=1}^{n}w_{i1}\le 1,\\ \sum_{i=1}^{n}w_{i4}\ge 1,\\ 0\le w_{i1}\le w_{i2}\le w_{i3}\le w_{i4}\le 1,\\ i=1,2,\ldots,n.\end{cases}$$

Note that the first and second constraints in model (M-1) come from Sugihara et al. (2004). By solving model (M-1), we can get the fuzzy priority weights of $\tilde{A}$.

Example 1 Let $\tilde{A}=(\tilde{a}_{ij})_{4\times 4}$ be an additive trapezoidal fuzzy preference relation, where

$$\tilde{A}=\begin{bmatrix}(0,5,0.5,0.5,0.5) & (0.5,0.6,0.7,0.8) & (0.6,0.7,0.8,0.9) & (0.5,0.6,0.6,0.7)\\ (0.2,0.3,0.4,0.5) & (0,5,0.5,0.5,0.5) & (0.1,0.2,0.2,0.4) & (0.2,0.3,0.4,0.5)\\ (0.1,0.2,0.3,0.4) & (0.6,0.8,0.8,0.9) & (0,5,0.5,0.5,0.5) & (0.7,0.8,0.8,0.9)\\ (0.3,0.4,0.4,0.5) & (0.5,0.6,0.7,0.8) & (0.1,0.2,0.2,0.3) & (0,5,0.5,0.5,0.5)\end{bmatrix}.$$

By using model (M-1), we get the fuzzy priority vector $\tilde{w}=(\tilde{w}_1,\tilde{w}_2,\tilde{w}_3,\tilde{w}_4)$ of $\tilde{A}$, where

$$\tilde{w}_1=(0.3899,0.5274,0.6050,0.6050),$$
$$\tilde{w}_2=(0.1164,0.1575,0.1807,0.2017),$$
$$\tilde{w}_3=(0.3130,0.4725,0.4857,0.4857),$$
$$\tilde{w}_4=(0.1807,0.2444,0.2512,0.2512).$$

4 Compatibility measure of additive trapezoidal fuzzy preference relations

In this section, compatibility measure of additive trapezoidal fuzzy preference relations is introduced, and an optimal model is to developed to determine weights of experts in group decision making based on criterion of minimizing the deviation measure between the synthetic additive trapezoidal fuzzy preference relation and the synthetic characteristic fuzzy preference relation. Then a new approach of group decision making on the basis of compatibility measure of additive trapezoidal fuzzy preference relations is proposed.

4.1 Compatibility measure of additive trapezoidal fuzzy preference relations

In order to measure compatibility of additive trapezoidal fuzzy preference relations, we define the Hamming distance of trapezoidal fuzzy numbers as follows.

Definition 6 Let $\tilde{A}_1=(a_1,b_1,c_1,d_1)$ and $\tilde{A}_2=(a_2,b_2,c_2,d_2)$ be two trapezoidal fuzzy numbers, the Hamming distance between $\tilde{A}_1$ and $\tilde{A}_2$ is given by

$$d(\tilde{A}_1,\tilde{A}_2)=\frac{1}{4}(|a_1-a_2|)+(|b_1-b_2|)+(|c_1-c_2|)+(|d_1-d_2|). \tag{10}$$

Theorem 3 *Let $\tilde{A}_1,\tilde{A}_2,\tilde{A}_3$ be trapezoidal fuzzy numbers, then*

(1) $d(\tilde{A}_1, \tilde{A}_1) = 0$;
(2) $d(\tilde{A}_1, \tilde{A}_2) \geq 0$;
(3) $d(\tilde{A}_1, \tilde{A}_2) = d(\tilde{A}_2, \tilde{A}_1)$;
(4) *If* $d(\tilde{A}_1, \tilde{A}_2) = 0, d(\tilde{A}_1, \tilde{A}_3) = 0$, *then* $d(\tilde{A}_1, \tilde{A}_3) = 0$;
(5) $d(\tilde{A}_1, \tilde{A}_3) \leq d(\tilde{A}_1, \tilde{A}_2) + d(\tilde{A}_2, \tilde{A}_3)$.

Theorem 3 indicates that the Hamming distance measure of trapezoidal fuzzy numbers satisfies reflexivity, nonnegativity, commutativity, transitivity and triangle inequality.

Definition 7 Let $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, $\tilde{B} = (\tilde{b}_{ij})_{n\times n}$ be two additive trapezoidal fuzzy preference relations, then

$$C(\tilde{A}, \tilde{B}) = \sum_{i=1}^{n}\sum_{j=1}^{n} d(\tilde{a}_{ij}, \tilde{b}_{ij}),$$

is called the compatibility degree of $\tilde{A}$ and $\tilde{B}$.

It can be seen that the compatibility degree of additive trapezoidal fuzzy preference relations $\tilde{A}$ and $\tilde{B}$ is the sum of Hamming distance of all the corresponding elements from $\tilde{A}$ and $\tilde{B}$, which reflects the total difference between $\tilde{A}$ and $\tilde{B}$.

If $\tilde{a}_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ and $\tilde{b}_{ij} = (b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4})$, then the compatibility degree of $\tilde{A}$ and $\tilde{B}$ can be written as

$$C(\tilde{A}, \tilde{B}) = \frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{4} |a_{ijk} - b_{ijk}|. \tag{11}$$

Theorem 4 *Let* $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, $\tilde{B} = (\tilde{b}_{ij})_{n\times n}$ *and* $\tilde{F} = (\tilde{f}_{ij})_{n\times n}$ *be additive trapezoidal fuzzy preference relations, then*

(1) $C(\tilde{A}, \tilde{B}) \geq 0$;
(2) $C(\tilde{A}, \tilde{A}) = 0$;
(3) $C(\tilde{A}, \tilde{B}) = C(\tilde{B}, \tilde{A})$;
(4) *If* $C(\tilde{A}, \tilde{B}) = 0$ *and* $C(\tilde{B}, \tilde{F}) = 0$, *then* $C(\tilde{A}, \tilde{F}) = 0$;
(5) $C(\tilde{A}, \tilde{F}) \leq C(\tilde{A}, \tilde{B}) + C(\tilde{B}, \tilde{F})$.

Theorem 4 indicates that the compatibility degree of additive trapezoidal fuzzy preference relations is nonnegative, reflexive, commutative, transitive and satisfies triangle inequality.

Definition 8 Let $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, and $\tilde{B} = (\tilde{b}_{ij})_{n\times n}$ be additive trapezoidal fuzzy preference relations, If $C(\tilde{A}, \tilde{B}) = 0$, then $\tilde{A}$ and $\tilde{B}$ are perfectly compatible.

Theorem 5 *Let* $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$ *and* $\tilde{B} = (\tilde{b}_{ij})_{n\times n}$ *be additive trapezoidal fuzzy preference relations, then* $\tilde{A}$ *and* $\tilde{B}$ *are perfectly compatible if and only if* $\tilde{a}_{ij} = \tilde{b}_{ij}$ *for all* $i, j = 1, 2, \ldots, n$.

Definition 9 Let $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, $\tilde{B} = (\tilde{b}_{ij})_{n\times n}$ be additive trapezoidal fuzzy preference relations, then

$$\mathrm{CI}(\tilde{A}, \tilde{B}) = \frac{1}{n^2} C(\tilde{A}, \tilde{B}) \tag{12}$$

is called the compatibility index of $\tilde{A}$ and $\tilde{B}$.

As we can see from Definition 9, the compatibility index $\mathrm{CI}(\tilde{A}, \tilde{B})$ represents the average difference between $\tilde{A}$ and $\tilde{B}$. By Definition 9 and Theorems 4 and 5, we get the following conclusions.

Theorem 6 *Let* $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, $\tilde{B} = (\tilde{b}_{ij})_{n\times n}$ *and* $\tilde{F} = (\tilde{f}_{ij})_{n\times n}$ *be additive trapezoidal fuzzy preference relations, then*

(1) $\mathrm{CI}(\tilde{A}, \tilde{B}) \geq 0$;
(2) $\mathrm{CI}(\tilde{A}, \tilde{A}) = 0$;
(3) $\mathrm{CI}(\tilde{A}, \tilde{B}) = \mathrm{CI}(\tilde{B}, \tilde{A})$;
(4) $\mathrm{CI}(\tilde{A}, \tilde{F}) \leq \mathrm{CI}(\tilde{A}, \tilde{B}) + \mathrm{CI}(\tilde{B}, \tilde{F})$.
(5) $\mathrm{CI}(\tilde{A}, \tilde{B}) = 0$ *if and only if* $\tilde{A}$ *and* $\tilde{B}$ *are perfectly compatible.*

Definition 10 Let $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, $\tilde{B} = (\tilde{b}_{ij})_{n\times n}$ be additive trapezoidal fuzzy preference relations. If

$$\mathrm{CI}(\tilde{A}, \tilde{B}) \leq \alpha, \tag{13}$$

then $\tilde{A}$ and $\tilde{B}$ are of acceptable compatibility, where α is the threshold of acceptable compatibility.

As illustrated in Chen et al. (2011), we can take $\alpha = 0.1$ as the threshold of acceptable compatibility.

By Theorem 2, the consistency of additive trapezoidal fuzzy preference relation can be measured by its priority vector. Then we use characteristic preference relation to measure consistency of additive trapezoidal fuzzy preference relation by using the fuzzy priority vector.

Definition 11 Let $(\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^{\mathrm{T}}$ be the fuzzy priority vector of the additive trapezoidal fuzzy preference relation $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$, then $\tilde{W} = (\tilde{w}_{ij})_{n\times n}$ is called the characteristic preference relation of $\tilde{A}$, where

$$\tilde{w}_{ij} = (w_{ij1}, w_{ij2}, w_{ij3}, w_{ij4}),$$
$$w_{ijk} = \begin{cases} 0.5 + \log_{81} \frac{w_{ik}}{w_{j(5-k)}}, & i \neq j, \\ 0.5, & i = j, \end{cases} \tag{14}$$

$k = 1, 2, 3, 4$, and $\tilde{w}_i = (w_{i1}, w_{i2}, w_{i3}, w_{i4})$, $i = 1, 2, \ldots, n$, are fuzzy priority weights of $\tilde{A}$.

Theorem 7 *If* $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$ *is an additive trapezoidal fuzzy preference relation, then its characteristic preference* $\tilde{W} = (\tilde{w}_{ij})_{n\times n}$ *is a consistent additive trapezoidal fuzzy preference relation.*

Proof Let $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^{\mathrm{T}}$ be a fuzzy priority vector of $\tilde{A}$, and $\tilde{W} = (\tilde{w}_{ij})_{n\times n}$ be the characteristic preference relation of $\tilde{A}$, then by Eq. (14), for $i \neq j$,

$$w_{ijk} = 0.5 + \log_{81} \frac{w_{ik}}{w_{j(5-k)}}, \quad k = 1, 2, 3, 4.$$

Then

$$w_{ijk} + w_{ji(5-k)} = 0.5 + \log_{81} \frac{w_{ik}}{w_{j(5-k)}} + 0.5 + \log_{81} \frac{w_{j(5-k)}}{w_{ik}} = 1, \quad k = 1, 2, 3, 4. \tag{15}$$

By Definition 3, $\tilde{W} = (\tilde{w}_{ij})_{n\times n}$ is an additive trapezoidal fuzzy preference relation. Therefore, by Theorem 2, $\tilde{W} = (\tilde{w}_{ij})_{n\times n}$ is completely consistent.

Theorem 7 guarantees that the consistency of additive trapezoidal fuzzy preference relation can be measured by its characteristic preference relation, which is determined by the fuzzy priority vector of additive trapezoidal fuzzy preference relation.

Let $E = \{e_1, e_2, \ldots, e_m\}$ be a set of experts in group decision making. Assume that $\tilde{A}^{(k)} = (\tilde{a}_{ij}^{(k)})_{n\times n}$ is an additive trapezoidal fuzzy preference relation provided by expert e_k, and $\tilde{W}^{(k)} = (\tilde{w}_{ij}^{(k)})_{n\times n}$ is the characteristic preference relation of $\tilde{A}^{(k)}$. Assume that the weighting vector of experts is $L = (l_1, l_2, \ldots, l_m)^{\mathrm{T}}$, which satisfies $l_k \geq 0, k = 1, 2, \ldots, m$ and $\sum_{k=1}^{m} l_k = 1$.

Definition 12 Let $\tilde{A}^{(k)} = (\tilde{a}_{ij}^{(k)})_{n\times n}$ be an additive trapezoidal fuzzy preference relation provided by expert e_k in group decision making, $k = 1, 2, \ldots, m$, if

$$\bar{\tilde{a}}_{ij} = \bigoplus_{k=1}^{m} l_k \tilde{a}_{ij}^{(k)}, \quad i, j = 1, 2, \ldots, n, \tag{16}$$

then $\bar{\tilde{A}} = (\bar{\tilde{a}}_{ij})_{n\times n}$ is called the synthetic preference relation of $\tilde{A}^{(k)}$, where $L = (l_1, l_2, \ldots, l_m)^{\mathrm{T}}$, which satisfies $l_k \geq 0, k = 1, 2, \ldots, m$ and $\sum_{k=1}^{m} l_k = 1$.

Theorem 8 *Let $\bar{\tilde{A}} = (\bar{\tilde{a}}_{ij})_{n\times n}$ be the synthetic preference relation of additive trapezoidal fuzzy preference relation $\tilde{A}^{(k)} = (\tilde{a}_{ij}^{(k)})_{n\times n}$ determined by Eq. (15), $k = 1, 2, \ldots, m$. Then $\bar{\tilde{A}}$ is an additive trapezoidal fuzzy preference relation.*

Proof Since $\tilde{A}^{(k)} = (\tilde{a}_{ij}^{(k)})_{n\times n}$ is an additive trapezoidal fuzzy preference relation, by Definition 3, we get

$$a_{ij1}^{(k)} + a_{ji4}^{(k)} = a_{ij2}^{(k)} + a_{ji3}^{(k)} = a_{ij3}^{(k)} + a_{ji2}^{(k)} = a_{ij4}^{(k)} + a_{ji1}^{(k)} = 1, \quad i, j = 1, 2, \ldots, n.$$

Let $\bar{\tilde{a}}_{ij} = (\bar{a}_{ij1}, \bar{a}_{ij2}, \bar{a}_{ij3}, \bar{a}_{ij4})$. If $i = j$, then

$$\bar{a}_{iit} = \sum_{k=1}^{m} l_k a_{iit}^{(k)} = 0.5, \quad i = 1, 2, \ldots, n; \quad t = 1, 2, 3, 4.$$

If $i \neq j$, then

$$\bar{a}_{ij1} + \bar{a}_{ji4} = \sum_{k=1}^{m} l_k a_{ij1}^{(k)} + \sum_{k=1}^{m} l_k a_{ji4}^{(k)} = \sum_{k=1}^{m} l_k \left(a_{ij1}^{(k)} + a_{ji4}^{(k)}\right) = 1,$$

$$\bar{a}_{ij2} + \bar{a}_{ji3} = \sum_{k=1}^{m} l_k a_{ij2}^{(k)} + \sum_{k=1}^{m} l_k a_{ji3}^{(k)} = \sum_{k=1}^{m} l_k \left(a_{ij2}^{(k)} + a_{ji3}^{(k)}\right) = 1.$$

Similarly, we can obtain that

$$\bar{a}_{ij3} + \bar{a}_{ji2} = \bar{a}_{ij4} + \bar{a}_{ji1} = 1.$$

Thus, $\bar{\tilde{A}} = (\bar{\tilde{a}}_{ij})_{n\times n}$ is an additive trapezoidal fuzzy preference relation.

Theorem 9 *Let $\tilde{A}^{(k)} = (\tilde{a}_{ij}^{(k)})_{n\times n}$ be an additive trapezoidal fuzzy preference relation provided by expert e_k in group decision making, $k = 1, 2, \ldots, m$, and $\bar{\tilde{A}} = (\bar{\tilde{a}}_{ij})_{n\times n}$ be the synthetic preference relation of $\tilde{A}^{(k)}$, $L = (l_1, l_2, \ldots, l_m)^{\mathrm{T}}$ is the weighting vector of $\tilde{A}^{(k)}$, which satisfies $l_k \geq 0, k = 1, 2, \ldots, m$ and $\sum_{k=1}^{m} l_k = 1$. If $\tilde{A}^{(k)}$ all are consistent, then $\bar{\tilde{A}}$ is also consistent.*

Proof Based on Definition 5, we have

$$\tilde{a}_{ij}^{(k)} \oplus \tilde{a}_{jp}^{(k)} = \tilde{a}_{ip}^{(k)} \oplus \tilde{a}_{jj}^{(k)}, \quad i, j, \quad p = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, m.$$

Then by Eq. (15), it follows that

$$\bar{\tilde{a}}_{ij} \oplus \bar{\tilde{a}}_{jp} = \left(\bigoplus_{k=1}^{m} l_k \tilde{a}_{ij}^{(k)}\right) \bigoplus \left(\bigoplus_{k=1}^{m} l_k \tilde{a}_{jp}^{(k)}\right) = \bigoplus_{k=1}^{m} l_k \left(\tilde{a}_{ij}^{(k)} \bigoplus \tilde{a}_{jp}^{(k)}\right) = \bigoplus_{k=1}^{m} l_k \left(\tilde{a}_{ip}^{(k)} \bigoplus \tilde{a}_{jj}^{(k)}\right) = \left(\bigoplus_{k=1}^{m} l_k \tilde{a}_{ip}^{(k)}\right) \bigoplus \left(\bigoplus_{k=1}^{m} l_k \tilde{a}_{jj}^{(k)}\right) = \bar{\tilde{a}}_{ip} \oplus \bar{\tilde{a}}_{jj},$$

which means that $\bar{\tilde{A}} = (\bar{\tilde{a}}_{ij})_{n\times n}$ is consistent.

Theorem 9 guarantees that the synthetic preference relation satisfies consistency based on the consistency of all individual additive trapezoidal fuzzy preference relations.

Definition 13 Let $\tilde{W}^{(k)} = (\tilde{w}_{ij}^{(k)})_{n\times n}$ be the characteristic preference relation of additive trapezoidal fuzzy preference relation $\tilde{A}^{(k)} = (\tilde{a}_{ij}^{(k)})_{n\times n}, k = 1, 2, \ldots, m$. If for $i, j = 1, 2, \ldots, n$,

$$\bar{\tilde{w}}_{ij} = \bigoplus_{k=1}^{m} \left(l_k \tilde{w}_{ij}^{(k)}\right) = \left(\sum_{k=1}^{m} l_k w_{ij1}^{(k)}, \sum_{k=1}^{m} l_k w_{ij2}^{(k)}, \sum_{k=1}^{m} l_k w_{ij3}^{(k)}, \sum_{k=1}^{m} l_k w_{ij4}^{(k)}\right) \tag{17}$$

then $\bar{\tilde{W}} = (\bar{\tilde{w}}_{ij})_{n\times n}$ is called the synthetic characteristic preference relation of $\tilde{W}^{(k)}$, where $\bar{\tilde{w}}_{ij} = (\bar{w}_{ij1}, \bar{w}_{ij2}, \bar{w}_{ij3}, \bar{w}_{ij4})$, $L = (l_1, l_2, \ldots, l_m)^{\mathrm{T}}$ is the weighting vector of $\tilde{A}^{(k)}$, satisfying $l_k \geq 0, \sum_{k=1}^{m} l_k = 1$.

Theorem 10 *Let $\tilde{A}^{(k)} = (\tilde{a}_{ij}^{(k)})_{n\times n}$ be an additive trapezoidal fuzzy preference relation provided by expert e_k in group decision making, $k = 1, 2, \ldots, m$, and $\bar{\tilde{W}} = (\bar{\tilde{w}}_{ij})_{n\times n}$ be the synthetic characteristic preference relation of $\tilde{W}^{(k)}$. Then $\bar{\tilde{W}}$ is a consistent additive trapezoidal fuzzy preference relation.*

Proof The Theorem 10 can be proved by Theorems 7–9, immediately.

Theorem 10 indicates that the synthetic characteristic preference relation guarantees the continuity of consistency on the basis of that all the individual additive trapezoidal fuzzy preference relations are consistent.

Theorem 11 *Let $\tilde{A}^{(k)} = (\tilde{a}_{ij}^{(k)})_{n\times n}$ be an additive trapezoidal fuzzy preference relation provided by expert e_k in group decision making, $k = 1, 2, \ldots, m$, and $\tilde{W}^{(k)} = (\tilde{w}_{ij}^{(k)})_{n\times n}$ be the characteristic preference relation of $\tilde{A}^{(k)}$. Assume that $\bar{\tilde{A}} = (\bar{\tilde{a}}_{ij})_{n\times n}$ is the synthetic preference relation of $\tilde{A}^{(k)}$, and $\bar{\tilde{W}} = (\bar{\tilde{w}}_{ij})_{n\times n}$ is the synthetic characteristic preference relation of $\tilde{W}^{(k)}$, $L = (l_1, l_2, \ldots, l_m)^{\mathrm{T}}$ is the weighting vector of $\tilde{A}^{(k)}$, which satisfies $l_k \geq 0, k = 1, 2, \ldots, m$, and $\sum_{k=1}^{m} l_k = 1$. If $\tilde{A}^{(k)}$ and $\tilde{W}^{(k)}$ are of acceptable compatibility for $k = 1, 2, \ldots, m$, then $\bar{\tilde{A}}$ and $\bar{\tilde{W}}$ are of acceptable compatibility.*

Proof Assume that

$$\mathrm{CI}(\tilde{A}^{(k)}, \tilde{W}^{(k)}) \leq \alpha, \quad k = 1, 2, \ldots, m,$$

where α is the threshold of acceptable consistency. Then we get

$$\begin{aligned}
\mathrm{CI}(\bar{\tilde{A}}, \bar{\tilde{W}}) &= \frac{1}{4n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{4} \left|\bar{a}_{ijt} - \bar{w}_{ijt}\right| \\
&= \frac{1}{4n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{4} \left|\sum_{k=1}^{m} l_k a_{ijt}^{(k)} - \sum_{k=1}^{m} l_k w_{ijt}^{(k)}\right| \\
&= \frac{1}{4n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{4} \left|\sum_{k=1}^{m} l_k \left(a_{ijt}^{(k)} - w_{ijt}^{(k)}\right)\right| \\
&\leq \frac{1}{4n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{4} \sum_{k=1}^{m} l_k \left|a_{ijt}^{(k)} - w_{ijt}^{(k)}\right| \\
&= \sum_{k=1}^{m} l_k \left(\frac{1}{4n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{4} \left|a_{ijt}^{(k)} - w_{ijt}^{(k)}\right|\right) \\
&= \sum_{k=1}^{m} l_k \mathrm{CI}\left(\tilde{A}^{(k)}, \tilde{W}^{(k)}\right) \\
&\leq \sum_{k=1}^{m} l_k \alpha = \alpha \sum_{k=1}^{m} l_k = \alpha,
\end{aligned}$$

which means that $\bar{\tilde{A}}$ and $\bar{\tilde{W}}$ are of acceptable compatibility.

4.2 To determine the weighting vector of experts in group decision making

It can be seen that the less compatibility index of the synthetic preference relation and synthetic characteristic preference relation, the higher reliability of decision information provided by the experts. In order to determine the weights of experts, we can minimize the compatibility index of the synthetic preference relation and the synthetic characteristic preference relation.

For the convenience of computation, we use the deviation square instead of absolute deviation in compatibility index $\mathrm{CI}(\bar{\tilde{A}}, \bar{\tilde{W}})$. Based on the proof of Theorem 11, compatibility index of $\bar{\tilde{A}}$ and $\bar{\tilde{W}}$ can be rewritten as follows.

$$\begin{aligned}
&\mathrm{SCI}(\bar{\tilde{A}}, \bar{\tilde{W}}) \\
&= \frac{1}{4n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{4} (\bar{a}_{ijt} - \bar{w}_{ijt})^2 \\
&= \frac{1}{4n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{4} \left(\sum_{k=1}^{m} l_k a_{ijt}^{(k)} - \sum_{k=1}^{m} l_k w_{ijt}^{(k)}\right)^2 \\
&= \frac{1}{4n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{4} \left(\sum_{k=1}^{m} l_k \left(a_{ijt}^{(k)} - w_{ijt}^{(k)}\right)\right)^2 \\
&= \frac{1}{4n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{4} \left(\sum_{k_1=1}^{m} l_{k_1} \left(a_{ijt}^{(k_1)} - w_{ijt}^{(k_1)}\right)\right) \\
&\quad \times \left(\sum_{k_2=1}^{m} l_{k_2} \left(a_{ijt}^{(k_2)} - w_{ijt}^{(k_2)}\right)\right) \\
&= \frac{1}{4n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{4} \sum_{k_1=1}^{m} \sum_{k_2=1}^{m} l_{k_1} l_{k_2}
\end{aligned}$$

$$\times \left(a_{ijt}^{(k_1)} - w_{ijt}^{(k_1)}\right)\left(a_{ijt}^{(k_2)} - w_{ijt}^{(k_2)}\right)$$
$$= \sum_{k_1=1}^{m}\sum_{k_2=1}^{m} l_{k_1} l_{k_2} \left(\frac{1}{4n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{4}\left(a_{ijt}^{(k_1)} - w_{ijt}^{(k_1)}\right)\right.$$
$$\left.\times \left(a_{ijt}^{(k_2)} - w_{ijt}^{(k_2)}\right)\right). \quad (18)$$

Let $G = (g_{k_1k_2})_{n\times n}$, where

$$g_{k_1k_2} = \frac{1}{4n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{4}\left(a_{ijt}^{(k_1)} - w_{ijt}^{(k_1)}\right) \times \left(a_{ijt}^{(k_2)} - w_{ijt}^{(k_2)}\right), \quad k_1, \quad k_2 = 1, 2, \ldots, m. \quad (19)$$

And let $L = (l_1, l_2, \ldots, l_m)^{\mathrm{T}}$ be the experts' weighting vector. Then the compatibility index $\mathrm{SCI}(\tilde{\bar{A}}, \tilde{\bar{W}})$ can be regarded as the function of L.

Denoting $z(L) = \mathrm{SCI}(\tilde{\bar{A}}, \tilde{\bar{W}})$, then we obtain

$$z(L) = L^{\mathrm{T}} G L. \quad (20)$$

Therefore, the optimal model to determine experts' weights by minimizing the compatibility index of additive trapezoidal fuzzy preference relations is as follows.

(M-2)

$$\min \ \mathrm{SCI}(\tilde{\bar{A}}, \tilde{\bar{W}}) = L^{\mathrm{T}} G L$$
$$\text{s.t.} \begin{cases} \sum_{k=1}^{m} l_k = 1, \\ l_k \geq 0, \quad k = 1, 2, \ldots, m. \end{cases} \quad (21)$$

Let $R^{\mathrm{T}} = (1, 1, \ldots, 1)_{1\times m}$, then (M-2) can be written as

$$\min \ \mathrm{SCI}(\tilde{\bar{A}}, \tilde{\bar{W}}) = L^{\mathrm{T}} G L$$
$$\text{s.t.} \begin{cases} R^{\mathrm{T}} L = 1, \\ L \geq 0. \end{cases} \quad (22)$$

If we don't take the constraint $L \geq 0$ into account, then Eq. (21) can be expressed as follows.

(M-3)

$$\min \ \mathrm{SCI}(\tilde{\bar{A}}, \tilde{\bar{W}}) = L^{\mathrm{T}} G L$$
$$\text{s.t.} \ R^{\mathrm{T}} L = 1. \quad (23)$$

Obviously, if the global optimal solution to model (M-3) $L^* \geq 0$, then L^* is also the global optimal solution to model (M-2).

Theorem 12 *If $\tilde{\bar{A}}$ and $\tilde{\bar{W}}$ are not perfectly compatible, then optimal solution to model* (M-3) *is*

$$L^* = \frac{G^{-1}R}{R^{\mathrm{T}} G^{-1} R}. \quad (24)$$

Proof If $\tilde{\bar{A}}$ and $\tilde{\bar{W}}$ are not perfectly compatible, there exists $i_0, j_0 \epsilon \{1, 2, \ldots, n\}, t_0 \epsilon \{1, 2, 3, 4\}$, which satisfy

$$\left(\bar{a}_{i_0 j_0 t_0} - \bar{w}_{i_0 j_0 t_0}\right)^2 > 0.$$

Thus,

$$z(L) = L^{\mathrm{T}} G L = \frac{1}{4n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{4}(\bar{a}_{ijt} - \bar{w}_{ijt})^2 > 0.$$

By Eq. (19), we get $g_{k_1k_2} = g_{k_2k_1}, k_1, k_2 = 1, 2, \ldots, m$. Therefore, $G = (g_{k_1k_2})_{n\times n}$ is a nonsingular matrix. By constructing the Lagrange function corresponding to the model of Eq. (23):

$$J(L, \lambda) = L^{\mathrm{T}} G L + \lambda(R^{\mathrm{T}} L - 1), \quad (25)$$

where L is the Lagrange multiplier.

Taking the partial derivatives of Eq. (25) with respect to L and λ, and setting them to be equal to 0, we obtain that

$$\begin{cases} \dfrac{\partial J(L, \lambda)}{\partial L} = 0, \\ \dfrac{\partial J(L, \lambda)}{\partial \lambda} = 0. \end{cases} \quad (26)$$

By solving Eq. (26), we get

$$L^* = \frac{G^{-1}R}{R^{\mathrm{T}} G^{-1} R}.$$

With the fact that $\frac{\partial^2 J(L,\lambda)}{\partial L^2} = 2G$, which means that $z(L)$ is a strictly convex function, L^* is the unique optimal solution to model (M-3), which completes the proof of the theorem.

4.3 Group decision making with compatibility of additive trapezoidal fuzzy preference relations

Consider a group decision making problem. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of finite alternatives and $E = \{e_1, e_2, \ldots, e_n\}$ be a finite set of experts. e_k provides his/her own additive trapezoidal fuzzy preference relations $\tilde{A}^{(k)} = (\tilde{a}_{ij}^{(k)})_{n\times n}, k = 1, 2, \ldots, m$. The process of new approach can be summarized as follows.

Step 1 To determine the fuzzy priority vectors of $\tilde{A}^{(k)}$ by using model (M-1):

$$\tilde{w}^{(k)} = \left(\tilde{w}_1^{(k)}, \tilde{w}_2^{(k)}, \ldots, \tilde{w}_n^{(k)}\right)^{\mathrm{T}}, \quad k = 1, 2, \ldots, m.$$

Step 2 To calculate the characteristic preference relation $\tilde{W}^{(k)}$ on the basis of Eq. (14).

Step 3 To determine the weights of experts by using model (M-2):

$$L^* = \left(l_1^*, l_2^*, \ldots, l_m^*\right)^{\mathrm{T}}.$$

Step 4 To calculate the synthetic preference relation $\tilde{\bar{A}}$ based on Eq. (15).

Step 5 To determine the fuzzy priority vector of $\tilde{\bar{A}}$ by using model (M-1).

Step 6 To rank all the alternatives x_i by using centroids points and select the best one(s) in accordance with the centroids points.

Step 7 End.

5 Illustrative example

In this section, we develop an approach for investment selection. Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of four projects for investment, and let $E = \{e_1, e_2, e_3\}$ be a set of experts. Each expert compares four alternatives with respect to the main criterion of profit of investment and provides additive trapezoidal fuzzy preference relation $\tilde{A}^{(k)}$. They are listed as follows:

$$\tilde{A}^{(1)} = \begin{bmatrix} (0,5,0.5,0.5,0.5) & (0.4,0.5,0.6,0.6) & (0.5,0.6,0.7,0.7) & (0.3,0.4,0.4,0.6) \\ (0.4,0.4,0.5,0.6) & (0,5,0.5,0.5,0.5) & (0.1,0.2,0.3,0.4) & (0.3,0.4,0.5,0.7) \\ (0.3,0.3,0.4,0.5) & (0.6,0.7,0.8,0.9) & (0,5,0.5,0.5,0.5) & (0.3,0.4,0.4,0.6) \\ (0.4,0.6,0.6,0.7) & (0.3,0.5,0.6,0.7) & (0.4,0.6,0.6,0.7) & (0,5,0.5,0.5,0.5) \end{bmatrix},$$

$$\tilde{A}^{(2)} = \begin{bmatrix} (0,5,0.5,0.5,0.5) & (0.5,0.6,0.6,0.8) & (0.1,0.6,0.7,0.7) & (0.4,0.5,0.6,0.6) \\ (0.2,0.4,0.4,0.5) & (0,5,0.5,0.5,0.5) & (0.1,0.3,0.4,0.4) & (0.3,0.5,0.6,0.8) \\ (0.3,0.3,0.4,0.9) & (0.6,0.6,0.7,0.9) & (0,5,0.5,0.5,0.5) & (0.4,0.6,0.6,0.7) \\ (0.4,0.4,0.5,0.6) & (0.2,0.4,0.5,0.7) & (0.3,0.4,0.4,0.6) & (0,5,0.5,0.5,0.5) \end{bmatrix},$$

$$\tilde{A}^{(3)} = \begin{bmatrix} (0,5,0.5,0.5,0.5) & (0.4,0.5,0.6,0.7) & (0.4,0.5,0.6,0.7) & (0.3,0.4,0.5,0.6) \\ (0.3,0.4,0.5,0.6) & (0,5,0.5,0.5,0.5) & (0.1,0.2,0.3,0.4) & (0.3,0.5,0.6,0.7) \\ (0.3,0.4,0.5,0.6) & (0.6,0.7,0.8,0.9) & (0,5,0.5,0.5,0.5) & (0.3,0.5,0.6,0.6) \\ (0.4,0.5,0.6,0.7) & (0.3,0.4,0.5,0.7) & (0.4,0.4,0.5,0.7) & (0,5,0.5,0.5,0.5) \end{bmatrix},$$

With this information, we use the proposed approach to get the ranking of the alternatives, and the following steps are involved:

Step 1 To determine the fuzzy priority vectors of $\tilde{A}^{(k)}$ by using model (M-1):

$$\tilde{w}^{(k)} = \left(\tilde{w}_1^{(k)}, \tilde{w}_2^{(k)}, \ldots, \tilde{w}_n^{(k)}\right)^{\mathrm{T}}, \quad k = 1, 2, 3,$$

where

$$\tilde{w}_1^{(1)} = (0.2824577, 0.3423338, 0.3820866, 0.3820866);$$
$$\tilde{w}_2^{(1)} = (0.1820141, 0.1976465, 0.2462143, 0.3067168);$$
$$\tilde{w}_3^{(1)} = (0.2824577, 0.3067168, 0.3423338, 0.4264577);$$
$$\tilde{w}_4^{(1)} = (0.2530704, 0.4264577, 0.4264577, 0.4758771);$$
$$\tilde{w}_1^{(2)} = (0.2547505, 0.4276058, 0.4716760, 0.4716760);$$
$$\tilde{w}_2^{(2)} = (0.1641597, 0.2468783, 0.2766299, 0.2992121);$$
$$\tilde{w}_3^{(2)} = (0.3486867, 0.3486867, 0.3846232, 0.7205701);$$
$$\tilde{w}_4^{(2)} = (0.2282459, 0.2468783, 0.2766299, 0.3727378);$$
$$\tilde{w}_1^{(3)} = (0.2659209, 0.3222913, 0.3800290, 0.4241593);$$
$$\tilde{w}_2^{(3)} = (0.1713578, 0.2317993, 0.2733256, 0.3050651);$$
$$\tilde{w}_3^{(3)} = (0.2968004, 0.4014882, 0.4734139, 0.4734139);$$
$$\tilde{w}_4^{(3)} = (0.2659209, 0.2887596, 0.3404902, 0.4734140).$$

Step 2 To calculate the characteristic preference relation $\tilde{W}^{(k)}$ on the basis of Eq. (14), we get

$$\tilde{W}^{(1)} = \begin{bmatrix} (0,5000,0.5000,0.5000,0.5000) & (0.4812,0.5750,0.6500,0.6687) & (0.4062,0.5000,0.5500,0.5687) & (0.3812,0.4500,0.4750,0.5937) \\ (0.3313,0.3500,0.4250,0.5188) & (0,5000,0.5000,0.5000,0.5000) & (0.3063,0.3750,0.4500,0.5187) & (0.2813,0.3250,0.3750,0.5437) \\ (0.4313,0.4500,0.5000,0.5938) & (0.4813,0.5500,0.6250,0.6937) & (0,5000,0.5000,0.5000,0.5000) & (0.3813,0.4250,0.4500,0.6187) \\ (0.4063,0.5250,0.5500,0.6188) & (0.4563,0.6250,0.6750,0.7187) & (0.3813,0.5500,0.5750,0.6187) & (0,5000,0.5000,0.5000,0.5000) \end{bmatrix},$$

$$\tilde{W}^{(2)} = \begin{bmatrix} (0,5000,0.5000,0.5000,0.5000) & (0.4634,0.5991,0.6473,0.7402) \\ (0.2598,0.3527,0.4009,0.5366) & (0,5000,0.5000,0.5000,0.5000) \\ (0.4312,0.4312,0.4759,0.7366) & (0.5438,0.5527,0.6009,0.8366) \\ (0.3348,0.3527,0.4009,0.5866) & (0.4384,0.4741,0.5259,0.6866) \end{bmatrix}$$

$$\begin{matrix} (0.2634, 0.5241, 0.5688, 0.5688) & (0.4134, 0.5991, 0.6473, 0.6652) \\ (0.1634, 0.3991, 0.4473, 0.4652) & (0.3134, 0.4741, 0.5259, 0.5616) \\ (0, 5000, 0.5000, 0.5000, 0.5000) & (0.4848, 0.5527, 0.6009, 0.7616) \\ (0.2384, 0.3991, 0.4473, 0.5152) & (0, 5000, 0.5000, 0.5000, 0.5000) \end{matrix}\Bigg],$$

$$\tilde{W}^{(3)} = \left[\begin{matrix} (0, 5000, 0.5000, 0.5000, 0.5000) & (0.4688, 0.5375, 0.6125, 0.7063) \\ (0.2937, 0.3875, 0.4625, 0.5312) & (0, 5000, 0.5000, 0.5000, 0.5000) \\ (0.4187, 0.5125, 0.5875, 0.6312) & (0.4937, 0.5875, 0.6625, 0.7317) \\ (0.3938, 0.4375, 0.5125, 0.6312) & (0.4688, 0.5125, 0.5875, 0.7313) \end{matrix}\right.$$

$$\left.\begin{matrix} (0.3688, 0.4125, 0.4875, 0.5813) & (0.3688, 0.4875, 0.5625, 0.6062) \\ (0.2687, 0.3375, 0.4125, 0.5063) & (0.2687, 0.4125, 0.4875, 0.5312) \\ (0, 5000, 0.5000, 0.5000, 0.5000) & (0.3937, 0.5375, 0.6125, 0.6312) \\ (0.3688, 0.3875, 0.4625, 0.6063) & (0, 5000, 0.5000, 0.5000, 0.5000) \end{matrix}\right].$$

Step 3 To determine the weights of experts by using model (M-2), and we obtain

$$L^* = (0.1069, 0.6686, 0.2245).$$

Step 4 To calculate the synthetic preference relation $\bar{\tilde{A}}$ based on Eq. (15), and we get

$$\bar{\tilde{A}} = \left[\begin{matrix} (0, 5000, 0.5000, 0.5000, 0.5000) & (0.4669, 0.5669, 0.6000, 0.7562) \\ (0.2438, 0.4000, 0.4331, 0.5331) & (0, 5000, 0.5000, 0.5000, 0.5000) \\ (0.4337, 0.5230, 0.5562, 0.6562) & (0.6000, 0.6331, 0.7331, 0.9000) \\ (0.4000, 0.4438, 0.5331, 0.6331) & (0.3000, 0.4107, 0.5107, 0.7000) \end{matrix}\right.$$

$$\left.\begin{matrix} (0.3438, 0.4438, 0.4770, 0.5663) & (0.3669, 0.4669, 0.5562, 0.6000) \\ (0.1000, 0.2669, 0.3669, 0.4000) & (0.3000, 0.4893, 0.5893, 0.7000) \\ (0, 5000, 0.5000, 0.5000, 0.5000) & (0.3669, 0.5562, 0.5786, 0.6669) \\ (0.3331, 0.4214, 0.4438, 0.6331) & (0, 5000, 0.5000, 0.5000, 0.5000) \end{matrix}\right]$$

Step 5 To determine the fuzzy priority vector of $\bar{\tilde{A}}$ by using model (M-1), and we have

$$\tilde{w}_1 = (0.2712, 0.3391, 0.3641, 0.3807);$$
$$\tilde{w}_2 = (0.1509, 0.2382, 0.2770, 0.2770);$$
$$\tilde{w}_3 = (0.3465, 0.4388, 0.4687, 0.5297);$$
$$\tilde{w}_4 = (0.2314, 0.2702, 0.3086, 0.3994).$$

Step 6 Based on Eq. (1), the centroids points of $\tilde{w}_i (i = 1, 2, 3, 4)$ are as follows:

$$R(\tilde{w}_1) = 0.6077, R(\tilde{w}_2) = 0.5652, R(\tilde{w}_3) = 0.6709, R(\tilde{w}_4) = 0.5796.$$

Then we have

$$R(\tilde{w}_1) > R(\tilde{w}_2) > R(\tilde{w}_3) > R(\tilde{w}_4),$$

which means that $x_3 \succ x_1 \succ x_4 \succ x_2$. Therefore, the best alternative is x_3.

6 Conclusions

Consistency and compatibility measure play the important roles in GDM using additive trapezoidal fuzzy preference relations in the literature. In this paper, a new compatibility measure of additive trapezoidal fuzzy preference relations is highlighted and discussed. The main work presented in this paper is summarized as follows.

First, consistency of additive trapezoidal fuzzy preference relations is investigated, and a deviation optimal model to determine priority vector of additive trapezoidal fuzzy preference relations is developed.

Second, new compatibility measure for additive trapezoidal fuzzy preference relations is proposed in which the deviation measure between the additive trapezoidal fuzzy preference relation and its characteristic preference relation are taken into account. Some properties are also investigated to ensure the effectiveness of the new compatibility measure.

Third, an optimal model is developed to determine the weights of experts in group decision making with additive trapezoidal fuzzy preference relations, and the optimal solution to the model is investigated. However, the existence of solution to the model is not achieved, which is used to guarantees the efficiency of the proposed model.

Future research may be done on extending the compatibility measure to other types of preference relations in a similar way as Chiclana et al. (2013). Additional research should focus on development of management procedures of different preference relations and managing preference information in Web 2.0 contexts (Ureña et al. 2015).

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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