Soft Comput (2015) 19:2211–2224
DOI 10.1007/s00500-014-1403-9

METHODOLOGIES AND APPLICATION

A new method for multiple attribute group decision-making with intuitionistic trapezoid fuzzy linguistic information

Yanbing Ju · Shanghong Yang

Published online: 10 August 2014

Abstract With respect to multi-attribute group decision-making (MAGDM) problems in which the attribute values take form of intuitionistic trapezoid fuzzy linguistic numbers, some new aggregation operators are proposed, such as intuitionistic trapezoid fuzzy linguistic weighted geometric operator, intuitionistic trapezoid fuzzy linguistic ordered weighted geometric operator, intuitionistic trapezoid fuzzy linguistic hybrid weighted geometric operator, intuitionistic trapezoid fuzzy linguistic generalized weighted averaging operator, intuitionistic trapezoid fuzzy linguistic generalized ordered weighted averaging operator and intuitionistic trapezoid fuzzy linguistic generalized hybrid weighted averaging operator are proposed at first. Then, some desirable properties of these proposed operators are discussed, including monotonicity, idempotency, commutativity and boundedness. Furthermore, based on the proposed operators, some novel methods are developed to solve MAGDM problems with intuitionistic trapezoid fuzzy linguistic information under different cases. Finally, an illustrative example of emergency response capability evaluation is provided to illustrate the applicability and effectiveness of the proposed methods.

Keywords Multi-attribute group decision-making · Intuitionistic trapezoid fuzzy linguistic aggregation operators · Intuitionistic trapezoid fuzzy linguistic generalized aggregation operators

Communicated by V. Loia.

Y. Ju (✉) · S. Yang
School of Management and Economics, Beijing Institute of Technology, No. 5 Southern Street, Haidian District, Beijing 100081, China
e-mail: juyb@bit.edu.cn

S. Yang
e-mail: shanghong86@126.com

1 Introduction

Multi-attribute group decision-making (MAGDM) problems have been widely spread in real-life decision-making situations, in which decision-maker usually uses crisp numbers to express his/her preference on alternatives [(Fujimoto and Yamada (2006); Ju and Wang (2012); Oztekin et al. (2011); Oliveira and Sorensen (2005); Xu et al. (2012); Wang (2011a,b); Chen et al. (2011); Porcel and Herrera-Viedma (2010); Dursun and Karsak (2010)]. However, in most cases, due to time pressure, knowledge limitation and lack of data, the attributes involved in MAGDM problems usually cannot be expressed by crisp numbers, and some are more suitable to be denoted by fuzzy numbers, such as interval number, intuitionistic fuzzy number, linguistic variable, etc. The fuzzy set theory, initially introduced by Zadeh (1965), is one of the existing well-known methods for MAGDM problems. However, the fuzzy set only uses a membership degree function to describe the uncertainty, which is inadequate. To solve this issue, Atanassov (1986, 1999) proposed the concept of intuitionistic fuzzy set (IFS), an extension of fuzzy set, characterized by a membership function and a non-membership function. Obviously, the IFS can treat imperfect and imprecise information in a more flexible and effective manner and it has been widely applied since its appearance [Chen (2009); Li (2010); Xu (2007a,b); Zhang (2013a,b); Zhao et al. (2010)].

Since the linguistic variables (Zadeh 1975) were introduced, they have been widely applied to deal with vague information existing in MAGDM process. So far, several MAGDM approaches have been proposed for solving linguistic information, such as linguistic assessments consensus model (Herrera et al. 1996a), uncertain linguistic variables (Xu 2004), 2-tuple linguistic approach (Herrera and Martínez 2000), interval-valued 2-tuple linguistic approach (Lin et al. 2009) and trapezoid fuzzy 2-tuple linguistic approach

(Ju et al. 2013). Based on these linguistic approaches, research on the relative information aggregation operators has become a hot topic. Herrera et al. (1996b) proposed the linguistic ordered weighted averaging (LOWA) operators. Wei et al. (2009, 2011, 2013) proposed some generalized uncertain linguistic aggregating operators, generalized aggregating operators with 2-tuple linguistic information and uncertain linguistic Bonferroni mean operators. Xu (2006) presented some uncertain linguistic geometric aggregation operators. Zhang (2012, 2013a,b) presented some interval-valued 2-tuple linguistic aggregation operators. Ju et al. (2013) proposed trapezoid fuzzy 2-tuple linguistic aggregation operators. Furthermore, in recent years, a new method called trapezoid fuzzy linguistic variables has received a lot of attentions from researchers [Wan (2013); Zhang et al. (2013); Liu and Yu (2010); Wu and Cao (2013)] since it had been proposed by Xu (2005), which is the generalization of the uncertain linguistic variables in essence, but more suitable for processing vague information.

However, in the real world, decision-makers usually cannot completely express their opinions by a linguistic variable or an intuitionistic fuzzy number individually. Sometimes, they can express the information accurately by combining linguistic variables and intuitionistic fuzzy set. Therefore, on the basis of intuitionistic fuzzy set and linguistic variables, Wang and Li (2009) proposed the definition of intuitionistic linguistic set, which is a very useful tool to express a decision-maker's preferences when making decisions in uncertain or vague circumstances. Liu (2013a,b) proposed some intuitionistic linguistic generalized aggregation operators based on generalized ordered weighted averaging operators (Yager 2004). In addition, Liu and Jin (2012) defined the intuitionistic uncertain linguistic variables. Liu (2013a,b) further defined the interval intuitionistic uncertain linguistic variables and some aggregation operators by combining the interval-valued intuitionistic fuzzy number (Atanassov and Gargov 1989) and uncertain linguistic variable.

In order to process uncertain and inaccurate information more efficiently and precisely, it is necessary to make a further study on the extended form of the intuitionistic uncertain linguistic variables by combining trapezoid fuzzy linguistic variables and intuitionistic fuzzy set. For example, we can evaluate the response capabilities of emergency departments by the linguistic term set $S = \{s_1$ (extremely low); s_2 (very low); s_3 (low); s_4 (medium); s_5 (high); s_6 (very high); s_7 (extremely high)$\}$. Perhaps, we can use the trapezoid fuzzy linguistic variable $[s_\alpha, s_\beta, s_\theta, s_\tau]$ $(1\leq \alpha \leq \beta \leq \theta \leq \tau \leq 7)$ to describe the evaluation result, but this is not accurate, because it merely provides a linguistic range. Therefore, it is necessary to develop the concept of intuitionistic trapezoid fuzzy linguistic variable $[s_\alpha, s_\beta, s_\theta, s_\tau], (u, v)$ to describe the membership degree u and non-membership degree v to $[s_\alpha, s_\beta, s_\theta, s_\tau]$. This is the motivation of our study. It can be seen that the main advantage of an intuitionistic trapezoid fuzzy linguistic variable is that it comprises two parts: the trapezoid fuzzy linguistic variable can describe uncertain information more precise than linguistic variable and uncertain linguistic variable in qualitative, and the intuitionistic fuzzy number is adopted to demonstrate how much degree an attribute value belong to and not belong to the trapezoid fuzzy linguistic variable in quantitative, which makes the evaluation more accurate and objective.

In this paper, a novel concept called intuitionistic trapezoid fuzzy linguistic variable is presented and some new geometric operators for aggregating intuitionistic trapezoid fuzzy linguistic information are proposed, such as intuitionistic trapezoid fuzzy linguistic weighted geometric (ITrFLWG) operator, intuitionistic trapezoid fuzzy linguistic ordered weighted geometric (ITrFLOWG) operator and intuitionistic trapezoid fuzzy linguistic hybrid weighted geometric (ITrFLHWG) operator. Then some generalized aggregation operators are defined, i.e., intuitionistic trapezoid fuzzy linguistic generalized weighted geometric (ITrFLGWG) operator, intuitionistic trapezoid fuzzy linguistic generalized ordered weighted geometric (ITrFLGOWG) operator and intuitionistic trapezoid fuzzy linguistic generalized hybrid weighted geometric (ITrFLGHWG) operator. Furthermore, based on the proposed operators, some novel methods to MAGDM problems with intuitionistic trapezoid fuzzy linguistic information are developed under different situations. Finally, a numerical example of emergency response capability evaluation is given to illustrate the applications of the developed methods.

The rest of this paper is organized as follows. In Sect. 2, some basic definitions of trapezoid fuzzy linguistic variable, intuitionistic linguistic set and generalized ordered weighted averaging operator are reviewed, and intuitionistic trapezoid fuzzy linguistic variables are defined as well as operational and comparison laws. In Sect. 3, we propose some new operators for aggregating intuitionistic trapezoid fuzzy linguistic information and then study the desirable properties of these operators. In Sect. 4, some novel methods for MAGDM with intuitionistic trapezoid fuzzy linguistic information are developed under different situations. In Sect. 5, a numerical example of emergency response capability evaluation is given to illustrate the applications of the developed methods. The paper is concluded in Sect. 6.

2 Preliminaries

In this section, some definitions related to trapezoid fuzzy linguistic variable, intuitionistic linguistic set and generalized ordered weighted averaging operator are briefly reviewed. Based on which, the concept of intuitionistic trapezoid fuzzy linguistic number is defined as well as the operational and comparison laws.

2.1 Trapezoid fuzzy linguistic variables

A linguistic set is defined as a finite and completely ordered discrete term set $S = (s_1, s_2, \ldots, s_l)$, where l is the odd value. For example, when $l = 7$, the linguistic term set$S$can be defined as follows:

$S = \{s_1$ (extremely low); s_2 (very low); s_3 (low); s_4 (medium); s_5 (high); s_6 (very high); s_7 (extremely high)$\}$.

Definition 1 (Xu 2005) Let $\tilde{S} = \{s_\theta | s_1 \leq s_\theta \leq s_l, \theta \in [1, l]\}$ be the continuous form of the linguistic set S. $s_\alpha, s_\beta, s_\theta, s_\tau$ are four linguistic terms in $\tilde{S}$, and $s_1 \leq s_\alpha \leq s_\beta \leq s_\theta \leq s_\tau \leq s_l$ if $1 \leq \alpha \leq \beta \leq \theta \leq \tau \leq l$, then the trapezoid fuzzy linguistic variable (TFLV) is defined as $\tilde{s} = [s_\alpha, s_\beta, s_\theta, s_\tau]$.

In particular, if any two of α, β, θ, τ are equal, then $\tilde{s}$ reduces to a triangular fuzzy linguistic variable (Xu 2007a,b); if any three of α, β, θ, τ are equal, then $\tilde{s}$ reduces to an uncertain linguistic variable (Xu 2004).

2.2 Intuitionistic linguistic set

Based on intuitionistic fuzzy set and linguistic term set, Wang and Li (2009) presented their extension form, i.e., intuitionistic linguistic set, which is shown as follows.

Definition 2 (Wang and Li 2009) An intuitionistic linguistic set A in X can be defined as

$$A = \left\{ \langle x, [s_{\theta(x)}, (u(x), v(x))] \rangle | x \in X \right\}, \tag{1}$$

where $s_{\theta(x)} \in S$, $u(x) \in [0, 1]$, $v(x) \in [0, 1]$, and $u(x) + v(x) \leq 1, \forall x \in X$. $s_{\theta(x)}$ is a linguistic term, $u(x)$ represents the membership degree of an element x to the linguistic term $s_{\theta(x)}$, while $v(x)$ represents the non-membership degree of an element x to the linguistic term $s_{\theta(x)}$. Let $\pi(x) = 1 - u(x) - v(x)$, $\pi(x) \in [0, 1]$, $\forall x \in X$, then $\pi(x)$ is called a hesitancy degree of x to the linguistic term $s_{\theta(x)}$.

2.3 Intuitionistic trapezoid fuzzy linguistic numbers

Definition 3 (Ju and Yang 2013) An intuitionistic trapezoid fuzzy linguistic set $\tilde{A}$ in X can be defined as

$$\tilde{A} = \left\{ \langle x, [[s_{\alpha(x)}, s_{\beta(x)}, s_{\theta(x)}, s_{\tau(x)}], (u(x), v(x))] \rangle | x \in X \right\}, \tag{2}$$

where $s_{\alpha(x)}$, $s_{\beta(x)}$, $s_{\theta(x)}$, $s_{\tau(x)} \in \tilde{S}$, $u(x) \in [0, 1]$, $v(x) \in [0, 1]$, and $u(x) + v(x) \leq 1, \forall x \in X$. $[s_{\alpha(x)}, s_{\beta(x)}, s_{\theta(x)}, s_{\tau(x)}]$ is a trapezoid fuzzy linguistic variable, $u(x)$ represents the membership degree of an element x to the trapezoid fuzzy linguistic variable $[s_{\alpha(x)}, s_{\beta(x)}, s_{\theta(x)}, s_{\tau(x)}]$, while $v(x)$ represents the non-membership degree of an element x to the trapezoid fuzzy linguistic variable $[s_{\alpha(x)}, s_{\beta(x)}, s_{\theta(x)}, s_{\tau(x)}]$. Let $\pi(x) = 1 - u(x) - v(x)$, $\pi(x) \in [0, 1]$, $\forall x \in X$, then $\pi(x)$ is called a hesitancy degree of x to the trapezoid fuzzy linguistic variable $[s_{\alpha(x)}, s_{\beta(x)}, s_{\theta(x)}, s_{\tau(x)}]$.

In Eq. 2, $\langle [s_{\alpha(x)}, s_{\beta(x)}, s_{\theta(x)}, s_{\tau(x)}], (u(x), v(x)) \rangle$ is an intuitionistic trapezoid fuzzy linguistic number (ITrFLN). Obviously, $\tilde{A}$ is a set of intuitionistic trapezoid fuzzy linguistic numbers (ITrFLNs). For convenience, $\tilde{a} = \langle [s_{\alpha(\tilde{a})}, s_{\beta(\tilde{a})}, s_{\theta(\tilde{a})}, s_{\tau(\tilde{a})}], (u(\tilde{a}), v(\tilde{a})) \rangle$ is used to represent an ITrFLN.

Definition 4 (Ju and Yang 2013) Let $\tilde{a}_i = \langle [s_{\alpha(\tilde{a}_i)}, s_{\beta(\tilde{a}_i)}, s_{\theta(\tilde{a}_i)}, s_{\tau(\tilde{a}_i)}], (u(\tilde{a}_i), v(\tilde{a}_i)) \rangle$ and $\tilde{a}_j = \langle [s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j)) \rangle$ be two ITrFLNs and $\lambda \geq 0$, then the operational laws of ITrFLNs can be defined as follows:

$$\begin{aligned} \tilde{a}_i \oplus \tilde{a}_j = \Big\langle & [s_{\alpha(\tilde{a}_i)+\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_i)+\beta(\tilde{a}_j)}, \\ & s_{\theta(\tilde{a}_i)+\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_i)+\tau(\tilde{a}_j)}], \\ & (u(\tilde{a}_i) + u(\tilde{a}_j) - u(\tilde{a}_i)u(\tilde{a}_j), v(\tilde{a}_i)v(\tilde{a}_j)) \Big\rangle, \end{aligned} \tag{3}$$

$$\begin{aligned} \tilde{a}_i \otimes \tilde{a}_j = \Big\langle & [s_{\alpha(\tilde{a}_i)\times\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_i)\times\beta(\tilde{a}_j)}, \\ & s_{\theta(\tilde{a}_i)\times\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_i)\times\tau(\tilde{a}_j)}], \\ & (u(\tilde{a}_i)u(\tilde{a}_j), v(\tilde{a}_i) + v(\tilde{a}_j) - v(\tilde{a}_i)v(\tilde{a}_j)) \Big\rangle, \end{aligned} \tag{4}$$

$$\begin{aligned} \lambda\tilde{a}_i = \Big\langle & [s_{\lambda\times\alpha(\tilde{a}_i)}, s_{\lambda\times\beta(\tilde{a}_i)}, s_{\lambda\times\theta(\tilde{a}_i)}, s_{\lambda\times\tau(\tilde{a}_i)}], \\ & (1 - (1 - u(\tilde{a}_i))^\lambda, (v(\tilde{a}_i))^\lambda) \Big\rangle, \end{aligned} \tag{5}$$

$$\begin{aligned} \tilde{a}_i^\lambda = \Big\langle & [s_{(\alpha(\tilde{a}_i))^\lambda}, s_{(\beta(\tilde{a}_i))^\lambda}, s_{(\theta(\tilde{a}_i))^\lambda}, s_{(\tau(\tilde{a}_i))^\lambda}], \\ & ((u(\tilde{a}_i))^\lambda, 1 - (1 - v(\tilde{a}_i))^\lambda) \Big\rangle. \end{aligned} \tag{6}$$

Theorem 1 (Ju and Yang 2013) *Let* $\tilde{a}_i = \langle [s_{\alpha(\tilde{a}_i)}, s_{\beta(\tilde{a}_i)}, s_{\theta(\tilde{a}_i)}, s_{\tau(\tilde{a}_i)}], (u(\tilde{a}_i), v(\tilde{a}_i)) \rangle$ *and* $\tilde{a}_j = \langle [s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j)) \rangle$ *be two ITrFLNs and* $\lambda, \lambda_i, \lambda_j \geq 0$, *then*

(1) $\tilde{a}_i \oplus \tilde{a}_j = \tilde{a}_j \oplus \tilde{a}_i$,
(2) $\tilde{a}_i \otimes \tilde{a}_j = \tilde{a}_j \otimes \tilde{a}_i$,
(3) $\lambda(\tilde{a}_i \oplus \tilde{a}_j) = \lambda\tilde{a}_j \oplus \lambda\tilde{a}_i$,
(4) $\lambda_i\tilde{a}_i \oplus \lambda_j\tilde{a}_i = (\lambda_i + \lambda_j)\tilde{a}_i$,
(5) $\tilde{a}_i^{\lambda_i} \otimes \tilde{a}_i^{\lambda_j} = \tilde{a}_i^{\lambda_i+\lambda_j}$,
(6) $\tilde{a}_i^\lambda \otimes \tilde{a}_j^\lambda = (\tilde{a}_i \otimes \tilde{a}_j)^\lambda$.

Definition 5 (Ju and Yang 2013) Let $\tilde{a}_i = \langle [s_{\alpha(\tilde{a}_i)}, s_{\beta(\tilde{a}_i)}, s_{\theta(\tilde{a}_i)}, s_{\tau(\tilde{a}_i)}], (u(\tilde{a}_i), v(\tilde{a}_i)) \rangle$ be an ITrFLN, then the expected function $E(\tilde{a}_i)$ and the accuracy function $H(\tilde{a}_i)$ are defined as follows, respectively:

$$\begin{aligned} E(\tilde{a}_i) &= \frac{1 + u(\tilde{a}_i) - v(\tilde{a}_i)}{2} \times s_{(\alpha(\tilde{a}_i)+\beta(\tilde{a}_i)+\theta(\tilde{a}_i)+\tau(\tilde{a}_i))/4} \\ &= s_{(\alpha(\tilde{a}_i)+\beta(\tilde{a}_i)+\theta(\tilde{a}_i)+\tau(\tilde{a}_i))\times(1+u(\tilde{a}_i)-v(\tilde{a}_i))/8}, \end{aligned} \tag{7}$$

$$H(\tilde{a}_i) = (u(\tilde{a}_i) + v(\tilde{a}_i)) \times s_{(\alpha(\tilde{a}_i)+\beta(\tilde{a}_i)+\theta(\tilde{a}_i)+\tau(\tilde{a}_i))/4}$$

$$= s_{(\alpha(\tilde{a}_i)+\beta(\tilde{a}_i)+\theta(\tilde{a}_i)+\tau(\tilde{a}_i))\times(u(\tilde{a}_i)+v(\tilde{a}_i))/4}. \quad (8)$$

Theorem 2 *Let $\tilde{a}_i = \langle[s_{\alpha(\tilde{a}_i)}, s_{\beta(\tilde{a}_i)}, s_{\theta(\tilde{a}_i)}, s_{\tau(\tilde{a}_i)}], (u(\tilde{a}_i), v(\tilde{a}_i))\rangle$ and $\tilde{a}_j = \langle[s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j))\rangle$ be two ITrFLNs, then based on the expected function $E(\tilde{a}_i)$ and the accuracy function $H(\tilde{a}_i)$, the comparison laws of ITrFLNs are shown as follows:*

(1) If $E(\tilde{a}_i) > E(\tilde{a}_j)$, then $\tilde{a}_i > \tilde{a}_j$;
(2) If $E(\tilde{a}_i) = E(\tilde{a}_j)$, then

(a) If $H(\tilde{a}_i) > H(\tilde{a}_j)$, then $\tilde{a}_i > \tilde{a}_j$;
(b) If $H(\tilde{a}_i) = H(\tilde{a}_j)$, then $\tilde{a}_i = \tilde{a}_j$;
(c) If $H(\tilde{a}_i) < H(\tilde{a}_j)$, then $\tilde{a}_i < \tilde{a}_j$.

2.4 Generalized ordered weighted averaging (GOWA) operator

Definition 6 (Yager 2004) A GOWA operator of dimension n is a mapping GOWA: $R^n \to R$, such that,

$$\text{GOWA}(a_1, a_2, \ldots, a_n) = \left(\sum_{j=1}^{n} \omega_j b_j^{\lambda}\right)^{1/\lambda}, \quad (9)$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector which is correlative with GOWA, such that $\omega_j \in [0, 1]$, $j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} \omega_j = 1$, b_j is the jth largest element of real numbers $a_k (k = 1, 2, \ldots, n)$, and λ is a parameter such that $\lambda \in (-\infty, 0) \cup (0, +\infty)$.

3 Some aggregation operators based on intuitionistic trapezoid fuzzy linguistic numbers

Motivated by of the idea of intuitionistic uncertain linguistic weighted geometric average (IULWGA) operator, intuitionistic uncertain linguistic ordered weighted geometric (IULOWG) operator and intuitionistic uncertain linguistic hybrid geometric (IULHG) operator (Liu 2013a,b), we define three new aggregation operators for intuitionistic trapezoid fuzzy linguistic information, such as intuitionistic trapezoid fuzzy linguistic weighted geometric (ITrFLWG) operator, ITrFLOWG operator and intuitionistic trapezoid fuzzy linguistic hybrid geometric (ITrFLHG) operator as follows.

Definition 7 Let $\tilde{a}_j = \langle[s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j))\rangle$ $(j = 1, 2, \ldots, n)$ be a collection of the ITrFLNs. The intuitionistic trapezoid fuzzy linguistic weighted geometric (ITrFLWG) operator can be defined as follows, and ITrFLWG: $\Omega^n \to \Omega$:

$$\text{ITrFLWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \prod_{j=1}^{n} \tilde{a}_j^{\omega_j}, \quad (10)$$

where Ω is the set of all intuitionistic trapezoid fuzzy linguistic numbers, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\tilde{a}_j (j = 1, 2, \ldots, n)$, such that $\omega_j \in [0, 1]$, $j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} \omega_j = 1$.

Especially, if $\omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, then the ITrFLWG operator will be simplified to an intuitionistic trapezoid fuzzy linguistic geometric (ITrFLG) operator (Ju and Yang 2013).

Theorem 3 *Let $\tilde{a}_j = \langle[s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j))\rangle (j = 1, 2, \ldots, n)$ be a collection of the ITrFLNs. According to the operational laws of ITrFLNs, the ITrFLWG operator in Eq. 10 can be transformed into Eq. 11, which is still an ITrFLN.*

$$\begin{aligned} &ITrFLWG(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \\ &= \Bigg\langle [s_{\prod_{j=1}^{n} \alpha(\tilde{a}_j)^{\omega_j}}, s_{\prod_{j=1}^{n} \beta(\tilde{a}_j)^{\omega_j}}, s_{\prod_{j=1}^{n} \theta(\tilde{a}_j)^{\omega_j}}, \\ &\quad s_{\prod_{j=1}^{n} \tau(\tilde{a}_j)^{\omega_j}}], \left(\prod_{j=1}^{n} (u(\tilde{a}_j))^{\omega_j}, \right. \\ &\quad \left. 1 - \prod_{j=1}^{n} (1 - v(\tilde{a}_j))^{\omega_j}\right)\Bigg\rangle, \end{aligned} \quad (11)$$

Theorem 3 can be proven by mathematical induction. The steps in the proof are given as follows:

Proof (1) When $n = 1$, obviously, Eq. 11 is correct.
(2) When $n = 2$, since

$$\begin{aligned} \tilde{a}_1^{\omega_1} &= \langle[s_{(\alpha(\tilde{a}_1))^{\omega_1}}, s_{(\beta(\tilde{a}_1))^{\omega_1}}, s_{(\theta(\tilde{a}_1))^{\omega_1}}, s_{(\tau(\tilde{a}_1))^{\omega_1}}], \\ &\quad ((u(\tilde{a}_1))^{\omega_1}, 1 - (1 - v(\tilde{a}_1))^{\omega_1})\rangle, \\ \tilde{a}_2^{\omega_2} &= \langle[s_{(\alpha(\tilde{a}_2))^{\omega_2}}, s_{(\beta(\tilde{a}_2))^{\omega_2}}, s_{(\theta(\tilde{a}_2))^{\omega_2}}, s_{(\tau(\tilde{a}_2))^{\omega_2}}], \\ &\quad ((u(\tilde{a}_2))^{\omega_2}, 1 - (1 - v(\tilde{a}_2))^{\omega_2})\rangle, \end{aligned}$$

thus,

$$\begin{aligned} &\text{ITrFLWG}(\tilde{a}_1, \tilde{a}_2) = \tilde{a}_1^{\omega_1} \times \tilde{a}_2^{\omega_2} \\ &= (\langle[s_{(\alpha(\tilde{a}_1))^{\omega_1}}, s_{(\beta(\tilde{a}_1))^{\omega_1}}, s_{(\theta(\tilde{a}_1))^{\omega_1}}, s_{(\tau(\tilde{a}_1))^{\omega_1}}], \\ &\quad ((u(\tilde{a}_1))^{\omega_1}, 1 - (1 - v(\tilde{a}_1))^{\omega_1})\rangle) \\ &\quad \times (\langle[s_{(\alpha(\tilde{a}_2))^{\omega_2}}, s_{(\beta(\tilde{a}_2))^{\omega_2}}, s_{(\theta(\tilde{a}_2))^{\omega_2}}, s_{(\tau(\tilde{a}_2))^{\omega_2}}], \\ &\quad ((u(\tilde{a}_2))^{\omega_2}, 1 - (1 - v(\tilde{a}_2))^{\omega_2})\rangle) \\ &= \Bigg\langle [s_{\prod_{j=1}^{2} \alpha(\tilde{a}_j)^{\omega_j}}, s_{\prod_{j=1}^{2} \beta(\tilde{a}_j)^{\omega_j}}, s_{\prod_{j=1}^{2} \theta(\tilde{a}_j)^{\omega_j}}, \\ &\quad s_{\prod_{j=1}^{2} \tau(\tilde{a}_j)^{\omega_j}}], \left(\prod_{j=1}^{2} (u(\tilde{a}_j))^{\omega_j}, \right. \\ &\quad \left. 1 - \prod_{j=1}^{2} (1 - v(\tilde{a}_j))^{\omega_j}\right)\Bigg\rangle. \end{aligned}$$

(3) Suppose that when $n=k$, Eq. 11 is correct, i.e.,

$$\begin{aligned}&\text{ITrFLWG}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_k)\\&=\Bigg\langle[s_{\prod_{j=1}^{k}\alpha(\tilde{a}_j)^{\omega_j}},s_{\prod_{j=1}^{k}\beta(\tilde{a}_j)^{\omega_j}},s_{\prod_{j=1}^{k}\theta(\tilde{a}_j)^{\omega_j}},\\&\quad s_{\prod_{j=1}^{k}\tau(\tilde{a}_j)^{\omega_j}}],\left(\prod_{j=1}^{k}(u(\tilde{a}_j))^{\omega_j},\right.\\&\quad\left.1-\prod_{j=1}^{k}(1-v(\tilde{a}_j))^{\omega_j}\right)\Bigg\rangle,\end{aligned}$$

then, when $n=k+1$, we have

$$\begin{aligned}&\text{ITrFLWG}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_k,\tilde{a}_{k+1})\\&=\Big\langle[s_{\prod_{j=1}^{k}\alpha(\tilde{a}_j)^{\omega_j}},s_{\prod_{j=1}^{k}\beta(\tilde{a}_j)^{\omega_j}},s_{\prod_{j=1}^{k}\theta(\tilde{a}_j)^{\omega_j}},\\&\quad s_{\prod_{j=1}^{k}\tau(\tilde{a}_j)^{\omega_j}}],\left(\prod_{j=1}^{k}(u(\tilde{a}_j))^{\omega_j},\right.\\&\quad\left.1-\prod_{j=1}^{k}(1-v(\tilde{a}_j))^{\omega_j}\right)\Big\rangle\\&\quad\times\big\langle[s_{(\alpha(\tilde{a}_{k+1}))^{\omega_{k+1}}},s_{(\beta(\tilde{a}_{k+1}))^{\omega_{k+1}}},s_{(\theta(\tilde{a}_{k+1}))^{\omega_{k+1}}},\\&\quad s_{(\tau(\tilde{a}_{k+1}))^{\omega_{k+1}}}],((u(\tilde{a}_{k+1}))^{\omega_{k+1}},\\&\quad 1-(1-v(\tilde{a}_{k+1}))^{\omega_{k+1}})\big\rangle\\&=\Big\langle[s_{\prod_{j=1}^{k+1}\alpha(\tilde{a}_j)^{\omega_j}},s_{\prod_{j=1}^{k+1}\beta(\tilde{a}_j)^{\omega_j}},s_{\prod_{j=1}^{k+1}\theta(\tilde{a}_j)^{\omega_j}},\\&\quad s_{\prod_{j=1}^{k+1}\tau(\tilde{a}_j)^{\omega_j}}],\left(\prod_{j=1}^{k+1}(u(\tilde{a}_j))^{\omega_j},\right.\\&\quad\left.1-\prod_{j=1}^{k+1}(1-v(\tilde{a}_j))^{\omega_j}\right)\Big\rangle,\end{aligned}$$

therefore, when $n=k+1$, Eq. 11 is correct as well. Thus, Eq. 11 is correct for all n. □

Theorem 4 (Monotonicity) *Let $(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$ and $(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n)$ be two collections of ITrFLNs. For all $j=1,2,\ldots,n$, if $\tilde{a}'_j\le\tilde{a}_j$, then*

$$\mathit{ITrFLWG}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n)\le\mathit{ITrFLWG}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n).\tag{12}$$

Proof According to the expression of ITrFLWG in Eq. 11, we can know that

$$\begin{aligned}&\text{ITrFLWG}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n)\\&=\Bigg\langle[s_{\prod_{j=1}^{n}\alpha(\tilde{a}'_j)^{\omega_j}},s_{\prod_{j=1}^{n}\beta(\tilde{a}'_j)^{\omega_j}},\\&\quad s_{\prod_{j=1}^{n}\theta(\tilde{a}'_j)^{\omega_j}},s_{\prod_{j=1}^{n}\tau(\tilde{a}'_j)^{\omega_j}}],\\&\quad\left(\prod_{j=1}^{n}(u(\tilde{a}'_j))^{\omega_j},1-\prod_{j=1}^{n}(1-v(\tilde{a}'_j))^{\omega_j}\right)\Bigg\rangle,\\&\text{ITrFLWG}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\\&=\Bigg\langle[s_{\prod_{j=1}^{n}\alpha(\tilde{a}_j)^{\omega_j}},s_{\prod_{j=1}^{n}\beta(\tilde{a}_j)^{\omega_j}},\\&\quad s_{\prod_{j=1}^{n}\theta(\tilde{a}_j)^{\omega_j}},s_{\prod_{j=1}^{n}\tau(\tilde{a}_j)^{\omega_j}}],\\&\quad\left(\prod_{j=1}^{n}(u(\tilde{a}_j))^{\omega_j},1-\prod_{j=1}^{n}(1-v(\tilde{a}_j))^{\omega_j}\right)\Bigg\rangle,\end{aligned}$$

then, according to the expected function in Eq. 7, we can obtain

$$\begin{aligned}&E(\text{ITrFLWG}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n))\\&=s_{(\prod_{j=1}^{n}\alpha(\tilde{a}'_j)^{\omega_j}+\prod_{j=1}^{n}\beta(\tilde{a}'_j)^{\omega_j}+\prod_{j=1}^{n}\theta(\tilde{a}'_j)^{\omega_j}+\prod_{j=1}^{n}\tau(\tilde{a}'_j)^{\omega_j})\times(1+\prod\limits_{j=1}^{n}(u(\tilde{a}'_j))^{\omega_j}-(1-\prod\limits_{j=1}^{n}(1-v(\tilde{a}'_j))^{\omega_j}))/8},\\&E(\text{ITrFLWG}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n))\\&=s_{(\prod_{j=1}^{n}\alpha(\tilde{a}_j)^{\omega_j}+\prod_{j=1}^{n}\beta(\tilde{a}_j)^{\omega_j}+\prod_{j=1}^{n}\theta(\tilde{a}_j)^{\omega_j}+\prod_{j=1}^{n}\tau(\tilde{a}_j)^{\omega_j})\times(1+\prod\limits_{j=1}^{n}(u(\tilde{a}_j))^{\omega_j}-(1-\prod\limits_{j=1}^{n}(1-v(\tilde{a}_j))^{\omega_j}))/8}.\end{aligned}$$

Since $\tilde{a}'_j\le\tilde{a}_j$, according to Theorem 2, we have E(ITrFLWG$(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n))\le E$(ITrFLWG$(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$),

i.e., ITrFLWG$(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n)\le$ ITrFLWG$(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$. □

Theorem 5 (Idempotency) *Let $\tilde{a}_j=\tilde{a}$, for all $j=1,2,\ldots,n$, then*

$$\mathit{ITrFLWG}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\tilde{a}.\tag{13}$$

Proof Since $\tilde{a}_j = \tilde{a}$, thus

$$
\begin{aligned}
&\text{ITrFLWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\\
&= \Big\langle [s_{\prod_{j=1}^n \alpha(\tilde{a}_j)^{\omega_j}}, s_{\prod_{j=1}^n \beta(\tilde{a}_j)^{\omega_j}},\\
&\quad s_{\prod_{j=1}^n \theta(\tilde{a}_j)^{\omega_j}}, s_{\prod_{j=1}^n \tau(\tilde{a}_j)^{\omega_j}}],\\
&\quad \left(\prod_{j=1}^n (u(\tilde{a}_j))^{\omega_j}, 1 - \prod_{j=1}^n (1 - v(\tilde{a}_j))^{\omega_j}\right)\Big\rangle\\
&= \Big\langle [s_{\alpha(\tilde{a})^{\sum_{j=1}^n \omega_j}}, s_{\beta(\tilde{a})^{\sum_{j=1}^n \omega_j}}, s_{\theta(\tilde{a})^{\sum_{j=1}^n \omega_j}}, s_{\tau(\tilde{a})^{\sum_{j=1}^n \omega_j}}],\\
&\quad ((u(\tilde{a}))^{\sum_{j=1}^n \omega_j}, 1 - (1 - v(\tilde{a}))^{\sum_{j=1}^n \omega_j})\Big\rangle\\
&= \langle [s_{\alpha(\tilde{a})}, s_{\beta(\tilde{a})}, s_{\theta(\tilde{a})}, s_{\tau(\tilde{a})}], (u(\tilde{a}), v(\tilde{a}))\rangle\\
&= \tilde{a}.
\end{aligned}
$$

□

Theorem 6 (Boundedness) *Let* $\tilde{a}_{\min} = \min_{1 \le j \le n}\{\tilde{a}_j\}$ *and* $\tilde{a}_{\max} = \max_{1 \le j \le n}\{\tilde{a}_j\}$*, then*

$$\tilde{a}_{\min} \le \mathit{ITrFLWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \le \tilde{a}_{\max}. \tag{14}$$

Proof Since $\tilde{a}_{\min} \le \tilde{a}_j \le \tilde{a}_{\max}$, then

$$\prod_{j=1}^n \tilde{a}_{\min}^{\omega_j} \le \prod_{j=1}^n \tilde{a}_j^{\omega_j} \le \prod_{j=1}^n \tilde{a}_{\max}^{\omega_j},$$

thus, $\tilde{a}_{\min} \le \text{ITrFLWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \le \tilde{a}_{\max}$. □

Considering the weight of different ordered position of each intuitionistic trapezoid fuzzy linguistic argument, based on the ordered weighted geometric (OWG) (Xu and Da 2002) operator, we shall develop an intuitionistic trapezoid fuzzy linguistic ordered weighted geometric operator.

Definition 8 Let $\tilde{a}_j = \langle [s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j))\rangle$ $(j = 1, 2, \ldots, n)$ be a collection of the ITrFLNs. The intuitionistic trapezoid fuzzy linguistic ordered weighted geometric (ITrFLOWG) operator can be defined as follows, and ITrFLOWG: $\Omega^n \to \Omega$:

$$\text{ITrFLOWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \prod_{j=1}^n \tilde{a}_{\sigma(j)}^{w_j}, \tag{15}$$

where Ω is the set of all intuitionistic trapezoid fuzzy linguistic numbers, and $w = (w_1, w_2, \ldots, w_n)^T$ is an associated weight vector with ITrFLOWG, such that $w_j \in [0, 1]$, $j = 1,2,\ldots,n$ and $\sum_{j=1}^n w_j = 1$. $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$ such that $\tilde{a}_{\sigma(j-1)} \ge \tilde{a}_{\sigma(j)}$ for all $j = 2, 3, \ldots, n$. w_j is decided only by the jth position in the aggregation process. Therefore, w can also be called the position-weighted vector.

According to the method of determining position-weighted vector proposed by Wang and Xu (2008), w can be calculated by the combination number. The calculation formula is as follows:

$$w_{i+1} = \frac{c_{n-1}^i}{2^{n-1}}, \tag{16}$$

where the combination number $c_{n-1}^i (i = 0, 1, \ldots, n-1)$ can be denoted as $c_{n-1}^i = \frac{(n-1)!}{i!(n-i-1)!}$.

Theorem 7 *Let* $\tilde{a}_j = \langle [s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j))\rangle$ $(j = 1, 2, \ldots, n)$ *be a collection of the ITrFLNs. According to the operational laws of ITrFLNs, the ITrFLOWG operator in Eq. 15 can be transformed into Eq. 17, which is still an ITrFLN.*

$$
\begin{aligned}
&\mathit{ITrFLOWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\\
&= \Big\langle [s_{\prod_{j=1}^n \alpha(\tilde{a}_{\sigma(j)})^{w_j}}, s_{\prod_{j=1}^n \beta(\tilde{a}_{\sigma(j)})^{w_j}},\\
&\quad s_{\prod_{j=1}^n \theta(\tilde{a}_{\sigma(j)})^{w_j}}, s_{\prod_{j=1}^n \tau(\tilde{a}_{\sigma(j)})^{w_j}}],\\
&\quad \left(\prod_{j=1}^n (u(\tilde{a}_{\sigma(j)}))^{w_j}, 1 - \prod_{j=1}^n (1 - v(\tilde{a}_{\sigma(j)}))^{w_j}\right)\Big\rangle,
\end{aligned} \tag{17}
$$

Obviously, the ITrFLOWG operator has the properties of monotonicity, idempotency and boundedness, which can be proved similar to Theorems 4, 5, 6. Furthermore, the ITrFLOWG operator also has the property of commutativity. For example, the commutativity of ITrFLOWG is given as follows:

Theorem 8 (Commutativity) *If* $(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n)$ *is any permutation of* $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$*, then*

$$\mathit{ITrFLOWG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \mathit{ITrFLOWG}(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n). \tag{18}$$

Proof Since $(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n)$ is any permutation of $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$, then

$$
\begin{aligned}
\prod_{j=1}^n \alpha(\tilde{a}_{\sigma(j)})^{w_j} &= \prod_{j=1}^n \alpha(\tilde{a}'_{\sigma(j)})^{w_j},\\
\prod_{j=1}^n \beta(\tilde{a}_{\sigma(j)})^{w_j} &= \prod_{j=1}^n \beta(\tilde{a}'_{\sigma(j)})^{w_j}\\
\prod_{j=1}^n \theta(\tilde{a}_{\sigma(j)})^{w_j} &= \prod_{j=1}^n \theta(\tilde{a}'_{\sigma(j)})^{w_j},\\
\prod_{j=1}^n \tau(\tilde{a}_{\sigma(j)})^{w_j} &= \prod_{j=1}^n \tau(\tilde{a}'_{\sigma(j)})^{w_j}\\
\prod_{j=1}^n (u(\tilde{a}_{\sigma(j)}))^{w_j} &= \prod_{j=1}^n (u(\tilde{a}'_{\sigma(j)}))^{w_j},
\end{aligned}
$$

$$1-\prod_{j=1}^{n}(1-v(\tilde{a}_{\sigma(j)}))^{w_j}$$
$$=1-\prod_{j=1}^{n}(1-v(\tilde{a}'_{\sigma(j)}))^{w_j},$$

thus, $\text{ITrFLOWG}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\text{ITrFLOWG}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n)$. □

From Definitions 7 and 8, we can find that the ITrFLWG operator only weights the importance of intuitionistic trapezoid fuzzy linguistic argument itself, but ignore the importance of ordered position of each argument, while the ITrFLOWG operator only weights the ordered position of each intuitionistic trapezoid fuzzy linguistic argument, but ignore the importance of argument. To overcome this drawback and inspired by the hybrid average operator (Xu and Da 2003), it is very necessary to develop an intuitionistic trapezoid fuzzy linguistic hybrid weighted geometric operator.

Definition 9 Let $\tilde{a}_j=\langle[s_{\alpha(\tilde{a}_j)},s_{\beta(\tilde{a}_j)},s_{\theta(\tilde{a}_j)},s_{\tau(\tilde{a}_j)}],(u(\tilde{a}_j),v(\tilde{a}_j))\rangle\,(j=1,2,\ldots,n)$ be a collection of the ITrFLNs. The intuitionistic trapezoid fuzzy linguistic hybrid weighted geometric (ITrFLHWG) operator can be defined as follows, and ITrFLHWG: $\Omega^n\to\Omega$:

$$\text{ITrFLHWG}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\prod_{j=1}^{n}\tilde{b}_{\sigma(j)}^{w_j},\tag{19}$$

where Ω is the set of all intuitionistic trapezoid fuzzy linguistic numbers, $w=(w_1,w_2,\ldots,w_n)^T$ is an associated weight vector with ITrFLHWG, such that $w_j\in[0,1]$, $j=1,2,\ldots,n$ and $\sum_{j=1}^{n}w_j=1$; $\tilde{b}_j=\tilde{a}_j^{n\omega_j}\,(j=1,2,\ldots,n)$, n is the balancing coefficient and $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ is the weight vector of $\tilde{a}_j\,(j=1,2,\ldots,n)$, such that $\omega_j\in[0,1]$, $j=1,2,\ldots,n$ and $\sum_{j=1}^{n}\omega_j=1$; $(\tilde{b}_{\sigma(1)},\tilde{b}_{\sigma(2)},\ldots,\tilde{b}_{\sigma(n)})$ is a permutation of $(\tilde{b}_1,\tilde{b}_2,\ldots,\tilde{b}_n)$, such that $\tilde{b}_{\sigma(j-1)}\geq\tilde{b}_{\sigma(j)}$ for all $j=2,3,\ldots,n$.

Theorem 9 *Let $\tilde{a}_j=\left\langle[s_{\alpha(\tilde{a}_j)},s_{\beta(\tilde{a}_j)},s_{\theta(\tilde{a}_j)},s_{\tau(\tilde{a}_j)}],(u(\tilde{a}_j),v(\tilde{a}_j))\right\rangle\,(j=1,2,\ldots,n)$ be a collection of the ITrFLNs. According to the operations of ITrFLNs, the ITrFLHWG operator in Eq. 19 can be transformed into Eq. 20, which is still an ITrFLN.*

$$\begin{aligned}&\textit{ITrFLHWG}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\\&=\Big\langle[s_{\prod_{j=1}^{n}\alpha(\tilde{b}_{\sigma(j)})^{w_j}},\\&\quad s_{\prod_{j=1}^{n}\beta(\tilde{b}_{\sigma(j)})^{w_j}},s_{\prod_{j=1}^{n}\theta(\tilde{b}_{\sigma(j)})^{w_j}},s_{\prod_{j=1}^{n}\tau(\tilde{b}_{\sigma(j)})^{w_j}}],\\&\quad\left(\prod_{j=1}^{n}(u(\tilde{b}_{\sigma(j)}))^{w_j},1-\prod_{j=1}^{n}(1-v(\tilde{b}_{\sigma(j)}))^{w_j}\right)\Big\rangle.\end{aligned}\tag{20}$$

Similar to the ITrFLWG operator, the ITrFLHWG operator has the properties of monotonicity, idempotency and boundedness, which can be proved similar to Theorems 4, 5, 6. Furthermore, two special cases of the ITrFLHWG operator are shown as follows:

(1) When $w=(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n})^T$, the ITrFLHWG operator will be simplified to the ITrFLWG operator in Eq. 10.
(2) When $\omega=(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n})^T$, the ITrFLHWG operator will be simplified to the ITrFLOWG operator in Eq. 15.

By adding a parameter controlling the power to which the argument values are raised, we generalize the ITrFLWG operator, ITrFLOWG operator and ITrFLHWG operator, then based on the GOWA operator (Yager 2004), we define three new operators such as intuitionistic trapezoid fuzzy linguistic generalized weighted averaging (ITrFLGWA) operator, intuitionistic trapezoid fuzzy linguistic generalized ordered weighted averaging (ITrFLGOWA) operator and intuitionistic trapezoid fuzzy linguistic generalized hybrid weighted averaging (ITrFLGHWA) operator as follows:

Definition 10 Let $\tilde{a}_j=\langle[s_{\alpha(\tilde{a}_j)},s_{\beta(\tilde{a}_j)},s_{\theta(\tilde{a}_j)},s_{\tau(\tilde{a}_j)}],(u(\tilde{a}_j),v(\tilde{a}_j))\rangle(j=1,2,\ldots,n)$ be a collection of the ITrFLNs. The intuitionistic trapezoid fuzzy linguistic generalized weighted averaging (ITrFLGWA) operator can be defined as follows, and ITrFLGWA: $\Omega^n\to\Omega$:

$$\text{ITrFLGWA}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\left(\sum_{j=1}^{n}\omega_j\tilde{a}_j^{\lambda}\right)^{1/\lambda},\tag{21}$$

where Ω is the set of all intuitionistic trapezoid fuzzy linguistic numbers, and $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ is the weight vector of $\tilde{a}_j\,(j=1,2,\ldots,n)$, such that $\omega_j\in[0,1]$, j = 1,2,..., n and $\sum_{j=1}^{n}\omega_j=1$. λ is a parameter such that $\lambda\in(-\infty,0)\cup(0,+\infty)$.

Theorem 10 *Let $\tilde{a}_j=\langle[s_{\alpha(\tilde{a}_j)},s_{\beta(\tilde{a}_j)},s_{\theta(\tilde{a}_j)},s_{\tau(\tilde{a}_j)}],(u(\tilde{a}_j),v(\tilde{a}_j))\rangle\,(j=1,2,\ldots,n)$ be a collection of the ITrFLNs. According to the operations of ITrFLNs, the ITrFLGWA operator in Eq. 21 can be transformed into Eq. 22, which is still an ITrFLN.*

$$\begin{aligned}&\textit{ITrFLGWA}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\\&=\Big\langle[s_{(\sum_{j=1}^{n}\omega_j(\alpha(\tilde{a}_j))^{\lambda})^{1/\lambda}},s_{(\sum_{j=1}^{n}\omega_j(\beta(\tilde{a}_j))^{\lambda})^{1/\lambda}},\\&\quad s_{(\sum_{j=1}^{n}\omega_j(\theta(\tilde{a}_j))^{\lambda})^{1/\lambda}},s_{(\sum_{j=1}^{n}\omega_j(\tau(\tilde{a}_j))^{\lambda})^{1/\lambda}}],\\&\quad\left(\left(1-\prod_{j=1}^{n}(1-u(\tilde{a}_j)^{\lambda})^{\omega_j}\right)^{1/\lambda},\right.\\&\quad\left.1-\left(1-\prod_{j=1}^{n}(1-(1-v(\tilde{a}_j))^{\lambda})^{\omega_j}\right)^{1/\lambda}\right)\Big\rangle.\end{aligned}\tag{22}$$

Remark 1 When $\lambda \to 0$,

$$\begin{aligned}&\mathrm{ITrFLGWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\\&= \left\langle [s_{\prod_{j=1}^n \alpha(\tilde{a}_j)^{\omega_j}}, s_{\prod_{j=1}^n \beta(\tilde{a}_j)^{\omega_j}}, s_{\prod_{j=1}^n \theta(\tilde{a}_j)^{\omega_j}},\right.\\&\quad s_{\prod_{j=1}^n \tau(\tilde{a}_j)^{\omega_j}}], \left(\left(\prod_{j=1}^n u(\tilde{a}_j)\right)^{\omega_j},\right.\\&\quad \left.\left. 1-\prod_{j=1}^n (1-v(\tilde{a}_j))^{\omega_j}\right)\right\rangle. \qquad (23)\end{aligned}$$

The ITrFLGWA operator reduces to the intuitionistic trapezoid fuzzy linguistic weighted geometric (ITrFLWG) operator in Eq. 11.

Remark 2 When $\lambda = 1$,

$$\begin{aligned}&\mathrm{ITrFLGWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\\&= \left\langle [s_{\sum_{j=1}^n \omega_j \times \alpha(\tilde{a}_j)}, s_{\sum_{j=1}^n \omega_j \times \beta(\tilde{a}_j)},\right.\\&\quad s_{\sum_{j=1}^n \omega_j \times \theta(\tilde{a}_j)}, s_{\sum_{j=1}^n \omega_j \times \tau(\tilde{a}_j)}],\\&\quad \left.\left(1-\prod_{j=1}^n (1-u(\tilde{a}_j))^{\omega_j}, \left(\prod_{j=1}^n v(\tilde{a}_j)\right)^{\omega_j}\right)\right\rangle. \qquad (24)\end{aligned}$$

The ITrFLGWA operator reduces to the intuitionistic trapezoid fuzzy linguistic weighted average (ITrFLWA) operator (Ju and Yang 2013).

It is easy to see that the ITrFLGWA operator has such properties as monotonicity, idempotency and boundedness. They can be easily proven similar to Theorems 4, 5, 6, therefore the proofs are omitted.

Definition 11 Let $\tilde{a}_j = \langle [s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j))\rangle$ $(j = 1, 2, \ldots, n)$ be a collection of the ITrFLNs. The intuitionistic trapezoid fuzzy linguistic generalized ordered weighted averaging (ITrFLGOWA) operator can be defined as follows, and ITrFLGOWA: $\Omega^n \to \Omega$:

$$\mathrm{ITrFLGOWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\sum_{j=1}^n w_j \tilde{a}_{\sigma(j)}^{\lambda}\right)^{1/\lambda}, \qquad (25)$$

where Ω is the set of all intuitionistic trapezoid fuzzy linguistic numbers, and $w = (w_1, w_2, \ldots, w_n)^T$ is an associated weighted vector with ITrFLGOWA, such that $w_j \in [0, 1]$, $j =$ 1,2, …, n and $\sum_{j=1}^n w_j = 1$, which can be calculated by Eq. 16. $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$ such that $\tilde{a}_{\sigma(j-1)} \geq \tilde{a}_{\sigma(j)}$ for all $j = 2, 3, \ldots, n$. λ is a parameter such that $\lambda \in (-\infty, 0) \cup (0, +\infty)$.

Theorem 11 *Let $\tilde{a}_j = \langle [s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j))\rangle$ $(j = 1, 2, \ldots, n)$ be a collection of the ITrFLNs. According to the operations of ITrFLNs, the ITrFLGOWA operator in Eq. 25 can be transformed into Eq. 26, which is still an ITrFLN.*

$$\begin{aligned}&\mathrm{ITrFLGOWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\\&= \left\langle [s_{(\sum_{j=1}^n w_j(\alpha(\tilde{a}_{\sigma(j)}))^{\lambda})^{1/\lambda}}, s_{(\sum_{j=1}^n w_j(\beta(\tilde{a}_{\sigma(j)}))^{\lambda})^{1/\lambda}},\right.\\&\quad s_{(\sum_{j=1}^n w_j(\theta(\tilde{a}_{\sigma(j)}))^{\lambda})^{1/\lambda}}, s_{(\sum_{j=1}^n w_j(\tau(\tilde{a}_{\sigma(j)}))^{\lambda})^{1/\lambda}}],\\&\quad \left(1-\prod_{j=1}^n (1-u(\tilde{a}_{\sigma(j)})^{\lambda})^{w_j}\right)^{1/\lambda}, 1-\left(1-\prod_{j=1}^n\right.\\&\quad \left.\left.(1-(1-v(\tilde{a}_{\sigma(j)}))^{\lambda})^{w_j})^{1/\lambda}\right)\right\rangle. \qquad (26)\end{aligned}$$

It is obviously seen that the ITrFLGOWA operator has such properties as monotonicity, idempotency, commutativity and boundedness. They can be easily proven similar to Theorems 4, 5, 6 and 8, therefore the proofs are omitted.

Remark 3 When $\lambda \to 0$,

$$\begin{aligned}&\mathrm{ITrFLGOWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\\&= \left\langle [s_{\prod_{j=1}^n \alpha(\tilde{a}_{\sigma(j)})^{w_j}}, s_{\prod_{j=1}^n \beta(\tilde{a}_{\sigma(j)})^{w_j}},\right.\\&\quad s_{\prod_{j=1}^n \theta(\tilde{a}_{\sigma(j)})^{w_j}}, s_{\prod_{j=1}^n \tau(\tilde{a}_{\sigma(j)})^{w_j}}],\\&\quad \left.\left(\prod_{j=1}^n \left(u(\tilde{a}_{\sigma(j)})\right)^{w_j}, 1-\prod_{j=1}^n (1-v(\tilde{a}_{\sigma(j)}))^{w_j}\right)\right\rangle. \qquad (27)\end{aligned}$$

The ITrFLGOWA operator reduces to the intuitionistic trapezoid fuzzy linguistic ordered weighted geometric (ITrFLOWG) operator in Eq. 16.

Remark 4 When $\lambda = 1$,

$$\begin{aligned}&\mathrm{ITrFLGOWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\\&= \left\langle [s_{\sum_{j=1}^n w_j \times \alpha(\tilde{a}_{\sigma(j)})}, s_{\sum_{j=1}^n w_j \times \beta(\tilde{a}_{\sigma(j)})},\right.\\&\quad s_{\sum_{j=1}^n w_j \times \theta(\tilde{a}_{\sigma(j)})}, s_{\sum_{j=1}^n w_j \times \tau(\tilde{a}_{\sigma(j)})}],\\&\quad \left.\left(1-\prod_{j=1}^n \left(1-u(\tilde{a}_{\sigma(j)})\right)^{w_j}, \left(\prod_{j=1}^n v(\tilde{a}_{\sigma(j)})\right)^{w_j}\right)\right\rangle \qquad (28)\end{aligned}$$

The ITrFLGOWA operator reduces to the intuitionistic trapezoid fuzzy linguistic ordered weighted average (ITrFLOWA) operator (Ju and Yang 2013).

Definition 12 Let $\tilde{a}_j = \langle [s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j))\rangle (j = 1, 2, \ldots, n)$ be a collection of the ITrFLNs. The intuitionistic trapezoid fuzzy linguistic generalized hybrid weighted averaging (ITrFLGHWA) operator can be defined as follows, and ITrFLGHWA: $\Omega^n \to \Omega$:

$$\mathrm{ITrFLGHWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\sum_{j=1}^n w_j \tilde{b}_{\sigma(j)}^{\lambda}\right)^{1/\lambda}, \qquad (29)$$

where Ω is the set of all intuitionistic trapezoid fuzzy linguistic numbers; $w = (w_1, w_2, \ldots, w_n)^T$ is an associated weighted vector with ITrFLGHWA, such that $w_j \in [0, 1]$, $j = 1,2,\ldots,n$ and $\sum_{j=1}^{n} w_j = 1$, which can be calculated by Eq. 16; $\tilde{b}_j = \tilde{a}_j^{n\omega_j} (j = 1, 2, \ldots, n)$, n is the balancing coefficient and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\tilde{a}_j (j = 1, 2, \ldots, n)$, such that $\omega_j \in [0, 1]$, j=1, 2, …, n and $\sum_{j=1}^{n} \omega_j = 1$; $(\tilde{b}_{\sigma(1)}, \tilde{b}_{\sigma(2)}, \ldots, \tilde{b}_{\sigma(n)})$ is a permutation of $(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n)$, such that $\tilde{b}_{\sigma(j-1)} \geq \tilde{b}_{\sigma(j)}$ for all j = 2,3,…,n; λ is a parameter such that $\lambda \in (-\infty, 0) \cup (0, +\infty)$.

Theorem 12 *Let* $\tilde{a}_j = \langle [s_{\alpha(\tilde{a}_j)}, s_{\beta(\tilde{a}_j)}, s_{\theta(\tilde{a}_j)}, s_{\tau(\tilde{a}_j)}], (u(\tilde{a}_j), v(\tilde{a}_j))\rangle (j = 1, 2, \ldots, n)$ *be a collection of the ITrFLNs. According to the operations of ITrFLNs, the ITrFLGHWA operator in Eq. 29 can be transformed into Eq. 30, which is still an ITrFLN.*

$$\begin{aligned}
&ITrFLGHWA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\\
&= \Big\langle [s_{(\sum_{j=1}^{n} \omega_j (\alpha(\tilde{b}_{\sigma(j)}))^{\lambda})^{1/\lambda}}, s_{(\sum_{j=1}^{n} \omega_j (\beta(\tilde{b}_{\sigma(j)}))^{\lambda})^{1/\lambda}},\\
&\quad s_{(\sum_{j=1}^{n} \omega_j (\theta(\tilde{b}_{\sigma(j)}))^{\lambda})^{1/\lambda}}, s_{(\sum_{j=1}^{n} \omega_j (\tau(\tilde{b}_{\sigma(j)}))^{\lambda})^{1/\lambda}}],\\
&\quad \left(\left(1 - \prod_{j=1}^{n} \left(1 - u(\tilde{b}_{\sigma(j)})^{\lambda}\right)^{\omega_j} \right)^{1/\lambda}, \right.\\
&\quad \left. 1 - \left(1 - \prod_{j=1}^{n} (1 - (1 - v(\tilde{b}_{\sigma(j)}))^{\lambda})^{\omega_j} \right)^{1/\lambda} \right) \Big\rangle. \qquad (30)
\end{aligned}$$

Especially, when $\omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, *the ITrFLGHWA operator will be simplified to the ITrFLGOWA operator in Eq. 25.*

Remark 5 When $\lambda \to 0$,

$$\begin{aligned}
&\text{ITrFLGHWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\\
&= \Big\langle [s_{\prod_{j=1}^{n} \alpha(\tilde{b}_{\sigma(j)})^{w_j}}, s_{\prod_{j=1}^{n} \beta(\tilde{b}_{\sigma(j)})^{w_j}},\\
&\quad s_{\prod_{j=1}^{n} \theta(\tilde{b}_{\sigma(j)})^{w_j}}, s_{\prod_{j=1}^{n} \tau(\tilde{b}_{\sigma(j)})^{w_j}}],\\
&\quad \left(\prod_{j=1}^{n} \left(u(\tilde{b}_{\sigma(j)})\right)^{w_j}, 1 - \prod_{j=1}^{n} (1 - v(\tilde{b}_{\sigma(j)}))^{w_j} \right) \Big\rangle. \qquad (31)
\end{aligned}$$

The ITrFLGHWA operator reduces to the intuitionistic trapezoid fuzzy linguistic ordered weighted geometric (ITrFLHWG) operator in Eq. 20.

Remark 6 When $\lambda = 1$,

$$\begin{aligned}
&\text{ITrFLGHWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)\\
&= \Big\langle [s_{\sum_{j=1}^{n} w_j \times \alpha(\tilde{b}_{\sigma(j)})}, s_{\sum_{j=1}^{n} w_j \times \beta(\tilde{b}_{\sigma(j)})},\\
&\quad s_{\sum_{j=1}^{n} w_j \times \theta(\tilde{b}_{\sigma(j)})}, s_{\sum_{j=1}^{n} w_j \times \tau(\tilde{b}_{\sigma(j)})}],\\
&\quad \left(1 - \prod_{j=1}^{n} (1 - u(\tilde{b}_{\sigma(j)}))^{w_j}, \prod_{j=1}^{n} (v(\tilde{b}_{\sigma(j)}))^{w_j} \right) \Big\rangle. \qquad (32)
\end{aligned}$$

The ITrFLGHWA operator reduces to the intuitionistic trapezoid fuzzy linguistic ordered weighted average (ITrFLHWA) operator (Ju and Yang 2013).

4 Approaches to multi-attribute group decision-making with intuitionistic trapezoid fuzzy linguistic information

For multi-attribute group decision-making problems, in which the attribute values take the form of intuitionistic trapezoid fuzzy linguistic variables, two methods for MAGDM with intuitionistic trapezoid fuzzy linguistic information are developed under the known and unknown weight information of decision-makers, respectively.

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of attributes, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weight vector of the attributes, such that $\omega_j \in [0, 1]$, $j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} \omega_j = 1$. $DM = \{DM_1, DM_2, \ldots, DM_k\}$ is the set of decision-makers. $R_f = [\tilde{a}_{ij}^f]_{m \times n} (f = 1, 2, \ldots, k)$ is the intuitionistic trapezoid fuzzy linguistic decision matrix given by the decision-maker DM_f, where $\tilde{a}_{ij}^f = \Big\langle [s_{\alpha(\tilde{a}_{ij}^f)}, s_{\beta(\tilde{a}_{ij}^f)}, s_{\theta(\tilde{a}_{ij}^f)}, s_{\tau(\tilde{a}_{ij}^f)}], (u(\tilde{a}_{ij}^f), v(\tilde{a}_{ij}^f))\Big\rangle$ is intuitionistic trapezoid fuzzy linguistic number, denoting the assessment value of the alternative A_i with respect to the attribute C_j given by the decision-maker DM_f.

For different pre-conditions, if the weight vector of the decision-makers is known, we shall propose the MAGDM method based on the ITrFLWG operator; if the position-weighted information about the decision-makers is known, we shall propose the MAGDM method based on the ITrFLOWG operator; if both the weight vector and the position-weighted vector of the decision-makers are known, we shall propose the MAGDM method based on the ITrFLHWG operator. Due to the space limitation of the paper, we take the ITrFLWG operator and the ITrFLOWG operator for examples to give the steps. For the other proposed operators, the steps are similar and thus the approaches are omitted in this paper.

Method I

If the weight vector of the decision-makers is already known and defined as $w = (w_1, w_2, \ldots, w_k)^T$, we select the best alternative by the following steps:

Step 1: For each decision-maker $DM_f (f = 1, 2, \ldots, k)$, we utilize the weight vector of the attributes $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ and the ITrFLWG operator in Eq. 33 to calculate the overall assessment value $\tilde{a}_i^f$ of the alternative $A_i (i = 1, 2, \ldots, m)$ given by the decision-maker

DM_f $(f = 1, 2, \ldots, k)$ as follows:

$$\begin{aligned}\tilde{a}_i^f &= \text{ITrFLWG}(\tilde{a}_{i1}^f, \tilde{a}_{i2}^f, \ldots, \tilde{a}_{in}^f)\\ &= \prod_{j=1}^{n} (\tilde{a}_{ij}^f)^{\omega_j}\\ &= \Big\langle [s_{\prod_{j=1}^n \alpha(\tilde{a}_{ij}^f)^{\omega_j}}, s_{\prod_{j=1}^n \beta(\tilde{a}_{ij}^f)^{\omega_j}},\\ &\quad s_{\prod_{j=1}^n \theta(\tilde{a}_{ij}^f)^{\omega_j}}, s_{\prod_{j=1}^n \tau(\tilde{a}_{ij}^f)^{\omega_j}}],\\ &\quad \left(\prod_{j=1}^{n} (u(\tilde{a}_{ij}^f))^{\omega_j}, 1 - \prod_{j=1}^{n} (1 - v(\tilde{a}_{ij}^f))^{\omega_j}\right)\Big\rangle,\\ &\quad i = 1, 2, ..., m,\ f = 1, 2, ..., k \end{aligned} \tag{33}$$

where $\tilde{a}_i^f = \langle [s_{\alpha(\tilde{a}_i^f)}, s_{\beta(\tilde{a}_i^f)}, s_{\theta(\tilde{a}_i^f)}, s_{\tau(\tilde{a}_i^f)}], (u(\tilde{a}_i^f), v(\tilde{a}_i^f))\rangle$ is an intuitionistic trapezoid fuzzy linguistic number.

Step 2: For each alternative $A_i (i = 1, 2, \ldots, m)$, we utilize the weight vector of decision-makers $w = (w_1, w_2, \ldots, w_k)^T$ and the ITrFLWG operator in Eq. 34 to calculate the collective overall assessment value $\tilde{a}_i$ of the alternative $A_i (i = 1, 2, \ldots, m)$ determined by all decision-makers as follows:

$$\begin{aligned}\tilde{a}_i &= \text{ITrFLWG}(\tilde{a}_i^f, \tilde{a}_i^f, \ldots, \tilde{a}_i^f)\\ &= \prod_{f=1}^{k} (\tilde{a}_i^f)^{w_f}\\ &= \Big\langle [s_{\prod_{f=1}^k \alpha(\tilde{a}_i^f)^{w_f}}, s_{\prod_{f=1}^k \beta(\tilde{a}_i^f)^{w_f}},\\ &\quad s_{\prod_{f=1}^k \theta(\tilde{a}_i^f)^{w_f}}, s_{\prod_{f=1}^k \tau(\tilde{a}_i^f)^{w_f}}],\\ &\quad \left(\prod_{f=1}^{k} (u(\tilde{a}_i^f))^{w_f}, 1 - \prod_{f=1}^{k} (1 - v(\tilde{a}_i^f))^{w_f}\right)\Big\rangle,\\ &\quad i = 1, 2, ..., m \end{aligned} \tag{34}$$

where $\tilde{a}_i = \langle [s_{\alpha(\tilde{a}_i)}, s_{\beta(\tilde{a}_i)}, s_{\theta(\tilde{a}_i)}, s_{\tau(\tilde{a}_i)}], (u(\tilde{a}_i), v(\tilde{a}_i))\rangle$ is an intuitionistic trapezoid fuzzy linguistic number.

Step 3: Utilize the expected function in Eq. 35 to calculate the expected values $E(\tilde{a}_i)$ of the collective overall assessment values $\tilde{a}_i (i = 1, 2, \ldots, m)$.

$$\begin{aligned}E(\tilde{a}_i) &= \frac{1 + u(\tilde{a}_i) - v(\tilde{a}_i)}{2}\\ &\quad \times s_{(\alpha(\tilde{a}_i)+\beta(\tilde{a}_i)+\theta(\tilde{a}_i)+\tau(\tilde{a}_i))/4}\\ &= s_{(\alpha(\tilde{a}_i)+\beta(\tilde{a}_i)+\theta(\tilde{a}_i)+\tau(\tilde{a}_i))\times(1+u(\tilde{a}_i)-v(\tilde{a}_i))/8},\\ &\quad i = 1, 2, ..., m \end{aligned} \tag{35}$$

If there is no difference between two expected values $E(\tilde{a}_i)$ and $E(\tilde{a}_p)$ $(i, p = 1, 2, ..., m$ and $i \neq p)$, then we need to calculate the accuracy values $H(\tilde{a}_i)$ and $H(\tilde{a}_p)$ by Eq. 36.

$$\begin{aligned}H(\tilde{a}_i) &= (u(\tilde{a}_i) + v(\tilde{a}_i))\\ &\quad \times s_{(\alpha(\tilde{a}_i)+\beta(\tilde{a}_i)+\theta(\tilde{a}_i)+\tau(\tilde{a}_i))/4}\\ &= s_{(\alpha(\tilde{a}_i)+\beta(\tilde{a}_i)+\theta(\tilde{a}_i)+\tau(\tilde{a}_i))\times(u(\tilde{a}_i)+v(\tilde{a}_i))/4},\\ &\quad i = 1, 2, ..., m. \end{aligned} \tag{36}$$

Step 4: Rank all feasible alternatives according to Theorem 2 and select the most desirable alternative(s).
Step 5: End.

Method II

If the weight vector of the decision-makers is unknown, and information about the decision-makers are in secrecy. Therefore, we can select the best alternative by the following steps:

Step 1: See Step 1 in Method I.
Step 2: Calculate the position-weighted vector $w = (w_1, w_2, \ldots, w_k)^T$ of decision-makers DM_f $(f = 1, 2, \ldots, k)$ by Eq. 37.

$$w_{f+1} = \frac{c_{k-1}^f}{2^{k-1}}, \quad f = 0, 1, ..., k - 1 \tag{37}$$

Step 3. For each alternative $A_i (i = 1, 2, \ldots, m)$, we utilize the position-weighted vector of decision-makers $w = (w_1, w_2, \ldots, w_k)^T$ obtained in Step 2 and the ITrFLOWG operator in Eq. 11 to calculate the collective overall assessment value $\tilde{a}_i$ of the alternative $A_i (i = 1, 2, \ldots, m)$ determined by all decision-makers by Eq. 38.

$$\begin{aligned}\tilde{a}_i &= \text{ITrFLOWG}(\tilde{a}_i^f, \tilde{a}_i^f, \ldots, \tilde{a}_i^f)\\ &= \prod_{f=1}^{k} (\tilde{a}_i^{\sigma(f)})^{w_f}\\ &= \Big\langle [s_{\prod_{f=1}^k \alpha(\tilde{a}_i^{\sigma(f)})^{w_f}}, s_{\prod_{f=1}^k \beta(\tilde{a}_i^{\sigma(f)})^{w_f}},\\ &\quad s_{\prod_{f=1}^k \theta(\tilde{a}_i^{\sigma(f)})^{w_f}}, s_{\prod_{f=1}^k \tau(\tilde{a}_i^{\sigma(f)})^{w_f}}],\\ &\quad \left(\prod_{f=1}^{k} (u(\tilde{a}_i^{\sigma(f)}))^{w_f}, 1 - \prod_{f=1}^{k} (1 - v(\tilde{a}_i^{\sigma(f)}))^{w_f}\right)\Big\rangle,\\ &\quad i = 1, 2, ..., m \end{aligned} \tag{38}$$

where $(\tilde{a}_i^{\sigma(1)}, \tilde{a}_i^{\sigma(2)}, \ldots, \tilde{a}_i^{\sigma(k)})$ is a permutation of $(\tilde{a}_i^1, \tilde{a}_i^2, \ldots, \tilde{a}_i^k)$, such that $\tilde{a}_i^{\sigma(f-1)} \geq \tilde{a}_i^{\sigma(f)} (f = 2, 3, \ldots, k)$ and $\tilde{a}_i = \langle [s_{\alpha(\tilde{a}_i)}, s_{\beta(\tilde{a}_i)}, s_{\theta(\tilde{a}_i)}, s_{\tau(\tilde{a}_i)}], (u(\tilde{a}_i), v(\tilde{a}_i))\rangle$ is an intuitionistic trapezoid fuzzy linguistic number.

Step 4: See Step 3 in Method I.

Step 5: See Step 4 in Method I.
Step 6: End.

5 An illustrative example

A serious traffic accident happens in one city of China, emergency management center (EMC) of the government organizes relative departments to implement rescue activities. After this disaster disappearing, EMC want to make an evaluation on the emergency response capabilities of these departments, so as to provide guidance for the similar events in the future. There are five departments taking part in the rescue activities: A_1 is the healthy and medical department; A_2 is the transportation department, A_3 is the telecommunications department, A_4 is the power utility company, A_5 is the foods supply unit. The EMC must make the evaluation according to the following four attributes: (1) C_1 is the emergency forecasting capability; (2) C_2 is the emergency process capability; (3) C_3 is the emergency support capability; (4) C_4 is the after-disaster process capability. The weight vector of four attributes is $\omega = (0.3, 0.4, 0.2, 0.1)^T$. Four decision-makers $DM_f(f = 1, 2, 3, 4)$ are invited to evaluate the five possible departments $A_i(i = 1, 2, 3, 4, 5)$ with respect to the above four attributes by using the predefined linguistic term set $S = \{s_1$ (extremely low); s_2 (very low); s_3 (low); s_4 (medium); s_5 (high); s_6 (very high); s_7 (extremely high)$\}$, and four intuitionistic trapezoid fuzzy linguistic deci-

Table 1 Intuitionistic trapezoid fuzzy linguistic decision matrix R_1 given by DM_1

Alternatives	C_1	C_2	C_3	C_4
A_1	$\langle[s_2, s_3, s_4, s_6], (0.8, 0.1)\rangle$	$\langle[s_3, s_5, s_6, s_7], (0.7, 0.2)\rangle$	$\langle[s_1, s_3, s_5, s_6], (0.7, 0.2)\rangle$	$\langle[s_2, s_3, s_5, s_6], (0.5, 0.3)\rangle$
A_2	$\langle[s_3, s_4, s_5, s_6], (0.6, 0.3)\rangle$	$\langle[s_4, s_5, s_6, s_7], (0.5, 0.4)\rangle$	$\langle[s_2, s_3, s_4, s_6], (0.5, 0.2)\rangle$	$\langle[s_2, s_4, s_5, s_6], (0.7, 0.1)\rangle$
A_3	$\langle[s_1, s_2, s_6, s_7], (0.5, 0.4)\rangle$	$\langle[s_2, s_3, s_5, s_6], (0.7, 0.1)\rangle$	$\langle[s_3, s_4, s_6, s_7], (0.6, 0.2)\rangle$	$\langle[s_3, s_4, s_5, s_6], (0.7, 0.3)\rangle$
A_4	$\langle[s_2, s_4, s_5, s_6], (0.5, 0.4)\rangle$	$\langle[s_3, s_4, s_6, s_7], (0.7, 0.2)\rangle$	$\langle[s_3, s_4, s_5, s_6], (0.6, 0.3)\rangle$	$\langle[s_3, s_5, s_6, s_7], (0.7, 0.3)\rangle$
A_5	$\langle[s_1, s_2, s_5, s_6], (0.7, 0.1)\rangle$	$\langle[s_3, s_4, s_5, s_6], (0.6, 0.3)\rangle$	$\langle[s_2, s_3, s_5, s_6], (0.7, 0.3)\rangle$	$\langle[s_3, s_4, s_6, s_7], (0.6, 0.3)\rangle$

Table 2 Intuitionistic trapezoid fuzzy linguistic decision matrix R_2 given by DM_2

Alternatives	C_1	C_2	C_3	C_4
A_1	$\langle[s_4, s_5, s_6, s_7], (0.6, 0.3)\rangle$	$\langle[s_2, s_3, s_5, s_6], (0.7, 0.2)\rangle$	$\langle[s_1, s_2, s_5, s_6], (0.6, 0.2)\rangle$	$\langle[s_1, s_2, s_6, s_7], (0.7, 0.2)\rangle$
A_2	$\langle[s_3, s_4, s_6, s_7], (0.6, 0.2)\rangle$	$\langle[s_1, s_3, s_5, s_6], (0.6, 0.3)\rangle$	$\langle[s_2, s_4, s_5, s_6], (0.6, 0.4)\rangle$	$\langle[s_3, s_5, s_6, s_7], (0.8, 0.2)\rangle$
A_3	$\langle[s_2, s_3, s_5, s_6], (0.7, 0.1)\rangle$	$\langle[s_3, s_4, s_5, s_6], (0.6, 0.4)\rangle$	$\langle[s_3, s_5, s_6, s_7], (0.7, 0.3)\rangle$	$\langle[s_3, s_4, s_5, s_6], (0.6, 0.3)\rangle$
A_4	$\langle[s_3, s_4, s_5, s_6], (0.5, 0.5)\rangle$	$\langle[s_4, s_5, s_6, s_7], (0.4, 0.2)\rangle$	$\langle[s_1, s_2, s_6, s_7], (0.7, 0.2)\rangle$	$\langle[s_2, s_3, s_5, s_6], (0.6, 0.4)\rangle$
A_5	$\langle[s_2, s_3, s_4, s_6], (0.8, 0.1)\rangle$	$\langle[s_2, s_4, s_5, s_6], (0.5, 0.5)\rangle$	$\langle[s_3, s_4, s_6, s_7], (0.5, 0.3)\rangle$	$\langle[s_4, s_5, s_6, s_7], (0.5, 0.4)\rangle$

Table 3 Intuitionistic trapezoid fuzzy linguistic decision matrix R_3 given by DM_3

Alternatives	C_1	C_2	C_3	C_4
A_1	$\langle[s_3, s_4, s_5, s_6], (0.6, 0.4)\rangle$	$\langle[s_3, s_5, s_6, s_7], (0.7, 0.3)\rangle$	$\langle[s_3, s_4, s_5, s_6], (0.6, 0.3)\rangle$	$\langle[s_2, s_3, s_5, s_7], (0.7, 0.1)\rangle$
A_2	$\langle[s_4, s_5, s_6, s_7], (0.5, 0.2)\rangle$	$\langle[s_1, s_2, s_4, s_5], (0.7, 0.2)\rangle$	$\langle[s_2, s_3, s_5, s_6], (0.6, 0.2)\rangle$	$\langle[s_3, s_4, s_5, s_6], (0.5, 0.5)\rangle$
A_3	$\langle[s_2, s_4, s_5, s_6], (0.5, 0.5)\rangle$	$\langle[s_3, s_4, s_6, s_7], (0.5, 0.3)\rangle$	$\langle[s_4, s_5, s_6, s_7], (0.5, 0.4)\rangle$	$\langle[s_2, s_3, s_4, s_6], (0.8, 0.1)\rangle$
A_4	$\langle[s_2, s_3, s_5, s_6], (0.7, 0.2)\rangle$	$\langle[s_1, s_2, s_5, s_6], (0.6, 0.2)\rangle$	$\langle[s_1, s_2, s_5, s_7], (0.7, 0.1)\rangle$	$\langle[s_4, s_5, s_6, s_7], (0.6, 0.1)\rangle$
A_5	$\langle[s_1, s_3, s_5, s_7], (0.6, 0.3)\rangle$	$\langle[s_2, s_4, s_5, s_6], (0.6, 0.1)\rangle$	$\langle[s_3, s_5, s_6, s_7], (0.8, 0.2)\rangle$	$\langle[s_3, s_4, s_6, s_7], (0.6, 0.2)\rangle$

Table 4 Intuitionistic trapezoid fuzzy linguistic decision matrix R_4 given by DM_4

Alternatives	C_1	C_2	C_3	C_4
A_1	$\langle[s_2, s_4, s_5, s_6], (0.5, 0.4)\rangle$	$\langle[s_3, s_5, s_6, s_7], (0.8, 0.2)\rangle$	$\langle[s_1, s_3, s_5, s_6], (0.6, 0.3)\rangle$	$\langle[s_3, s_4, s_6, s_7], (0.6, 0.2)\rangle$
A_2	$\langle[s_1, s_2, s_6, s_7], (0.7, 0.2)\rangle$	$\langle[s_2, s_3, s_4, s_6], (0.6, 0.4)\rangle$	$\langle[s_4, s_5, s_6, s_7], (0.7, 0.2)\rangle$	$\langle[s_3, s_4, s_5, s_6], (0.5, 0.5)\rangle$
A_3	$\langle[s_3, s_4, s_6, s_7], (0.5, 0.3)\rangle$	$\langle[s_4, s_5, s_6, s_7], (0.5, 0.4)\rangle$	$\langle[s_2, s_4, s_5, s_6], (0.5, 0.5)\rangle$	$\langle[s_2, s_3, s_4, s_6], (0.8, 0.1)\rangle$
A_4	$\langle[s_1, s_2, s_5, s_6], (0.6, 0.2)\rangle$	$\langle[s_1, s_2, s_6, s_7], (0.7, 0.2)\rangle$	$\langle[s_2, s_3, s_5, s_6], (0.7, 0.2)\rangle$	$\langle[s_4, s_5, s_6, s_7], (0.6, 0.3)\rangle$
A_5	$\langle[s_3, s_5, s_6, s_7], (0.7, 0.3)\rangle$	$\langle[s_3, s_4, s_5, s_6], (0.6, 0.3)\rangle$	$\langle[s_3, s_4, s_5, s_7], (0.6, 0.2)\rangle$	$\langle[s_2, s_3, s_5, s_6], (0.7, 0.1)\rangle$

sion matrices $R_f = [\tilde{a}_{ij}^f]_{5\times 4}(f = 1, 2, 3, 4)$ are constructed as shown in Tables 1, 2, 3, 4, respectively.

Then, if the weight vector of decision-makers is known as $w = (0.25, 0.20, 0.30, 0.25)^T$, the proposed method is utilized to determine the most desirable alternative(s) under intuitionistic trapezoid fuzzy linguistic environment, which involves the following steps:

Method I

Step 1. Utilize the weight vector of the attributes $\omega = (0.3, 0.4, 0.2, 0.1)^T$ and the ITrFLWG operator in Eq. 33 to calculate the overall assessment value $\tilde{a}_i^f$ of the alternative $A_i(i = 1, 2, 3, 4, 5)$ given by the decision-maker DM_f $(f = 1, 2, 3, 4)$.

$$\tilde{a}_1^1 = \langle[s_{2.0477}, s_{3.6801}, s_{5.0300}, s_{6.3816}], (0.7045, 0.1822)\rangle,$$
$$\tilde{a}_2^1 = \langle[s_{2.9804}, s_{4.1289}, s_{5.1435}, s_{6.3816}], (0.5462, 0.3068)\rangle,$$
$$\tilde{a}_3^1 = \langle[s_{1.8346}, s_{2.8958}, s_{5.4772}, s_{6.4807}], (0.6136, 0.2410)\rangle,$$
$$\tilde{a}_4^1 = \langle[s_{2.6564}, s_{4.0903}, s_{5.4772}, s_{6.4807}], (0.6136, 0.2950)\rangle,$$
$$\tilde{a}_5^1 = \langle[s_{1.9896}, s_{3.0673}, s_{5.0920}, s_{6.0932}], (0.6481, 0.2452)\rangle;$$
$$\tilde{a}_1^2 = \langle[s_{2.0000}, s_{3.0963}, s_{503783}, s_{6.3816}], (0.6481, 0.2314)\rangle,$$
$$\tilde{a}_2^2 = \langle[s_{1.7826}, s_{3.6457}, s_{5.3783}, s_{6.3816}], (0.5837, 0.2844)\rangle,$$
$$\tilde{a}_3^2 = \langle[s_{2.6564}, s_{3.8367}, s_{5.1857}, s_{6.1879}], (0.6481, 0.2903)\rangle,$$
$$\tilde{a}_4^2 = \langle[s_{2.5946}, s_{3.6993}, s_{5.5780}, s_{6.5814}], (0.4981, 0.3249)\rangle,$$
$$\tilde{a}_5^2 = \langle[s_{2.3246}, s_{3.7521}, s_{4.9391}, s_{6.2840}], (0.5757, 0.3503)\rangle;$$
$$\tilde{a}_1^3 = \langle[s_{2.8808}, s_{4.2494}, s_{5.3783}, s_{6.4807}], (0.6481, 0.3146)\rangle,$$
$$\tilde{a}_2^3 = \langle[s_{1.9433}, s_{3.0601}, s_{4.8301}, s_{5.8420}], (0.5933, 0.2367)\rangle,$$
$$\tilde{a}_3^3 = \langle[s_{2.7019}, s_{4.0639}, s_{5.4549}, s_{6.5814}], (0.5241, 0.3708)\rangle,$$
$$\tilde{a}_4^3 = \langle[s_{1.4142}, s_{2.4754}, s_{5.0920}, s_{6.2840}], (0.6481, 0.1712)\rangle,$$
$$\tilde{a}_5^3 = \langle[s_{1.8346}, s_{3.8367}, s_{5.2811}, s_{6.5814}], (0.6355, 0.1943)\rangle;$$
$$\tilde{a}_1^4 = \langle[s_{2.1324}, s_{4.1289}, s_{5.4772}, s_{6.4807}], (0.6373, 0.2855)\rangle,$$
$$\tilde{a}_2^4 = \langle[s_{1.9433}, s_{3.0280}, s_{5.0095}, s_{6.4807}], (0.6364, 0.3197)\rangle,$$
$$\tilde{a}_3^4 = \langle[s_{2.9804}, s_{4.2494}, s_{5.5553}, s_{6.6837}], (0.5241, 0.3690)\rangle,$$
$$\tilde{a}_4^4 = \langle[s_{1.3195}, s_{2.3771}, s_{5.4772}, s_{6.4807}], (0.6581, 0.2106)\rangle,$$
$$\tilde{a}_5^4 = \langle[s_{2.8808}, s_{4.1557}, s_{5.2811}, s_{6.4807}], (0.6382, 0.2628)\rangle.$$

Step 2: Utilize the weight vector of decision-makers $w = (w_1, w_2, \ldots, w_k)^T$ and the ITrFLWG operator in Eq. 34 to calculate the collective overall assessment value $\tilde{a}_i$ of the alternative $A_i(i = 1, 2, 3, 4, 5)$.

$$\tilde{a}_1 = \langle[s_{2.2808}, s_{3.8203}, s_{5.3132}, s_{6.4359}], (0.6590,\ 0.2594)\rangle,$$
$$\tilde{a}_2 = \langle[s_{2.1256}, s_{3.4066}, s_{5.0592}, s_{6.2387}], (0.5962,\ 0.2852)\rangle,$$
$$\tilde{a}_3 = \langle[s_{2.5050}, s_{3.7325}, s_{5.4302}, s_{6.5008}], (0.5688,\ 0.3240)\rangle,$$
$$\tilde{a}_4 = \langle[s_{1.8371}, s_{3.0107}, s_{5.3783}, s_{6.4409}], (0.6088,\ 0.2453)\rangle$$
$$\tilde{a}_5 = \langle[s_{2.1974}, s_{3.6846}, s_{5.1636}, s_{6.3718}], (0.6268,\ 0.2574)\rangle.$$

Step 3: Utilize the expected function in Eq. 35 to calculate the expected values $E(\tilde{a}_i)$ of the collective overall assessment values $\tilde{a}_i(i = 1, 2, 3, 4, 5)$.

$$E(\tilde{a}_1) = s_{3.1228}, E(\tilde{a}_2) = s_{2.7579}, E(\tilde{a}_3) = s_{2.8269},$$
$$E(\tilde{a}_4) = s_{2.8408}, E(\tilde{a}_5) = s_{2.9814}.$$

Step 4: According to the descending order of expected values $E(\tilde{a}_i)(i = 1, 2, 3, 4, 5)$, all feasible alternatives $A_i(i = 1, 2, 3, 4, 5)$ are ranked as follows:

$$A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$$

Therefore, the best alternative is A_1.

If the weight vector of the decision-makers is unknown in advance, then the proposed method is utilized to determine the most desirable alternative(s) under intuitionistic trapezoid fuzzy linguistic environment, which involves the following steps:

Method II

Step 1: See Step 1 in Method I.
Step 2: Due to the weight vector of the decision-makers is unknown, Eq. 37 is utilized to calculate the position-weighted vector $w = (w_1, w_2, w_3, w_4)^T$ of decision-makers DM_f $(f = 1, 2, 3, 4)$.

$$w_1 = \frac{c_3^0}{2^3} = 0.125,\ w_2 = \frac{c_3^1}{2^3} = 0.375,$$
$$w_3 = \frac{c_3^2}{2^3} = 0.375,\ w_4 = \frac{c_3^3}{2^3} = 0.125.$$

Step 3. Utilize the position-weighted vector of decision-makers $w = (w_1, w_2, w_3, w_4)^T$ obtained in Step 2 and the ITrFLOWG operator in Eq. 38 to calculate the collective overall assessment value $\tilde{a}_i$ of the alternative $A_i(i = 1, 2, 3, 4)$.

$$\tilde{a}_1 = \langle[s_{2.3560}, s_{3.9687}, s_{5.3700}, s_{6.4558}], (0.6508, 0.2780)\rangle,$$
$$\tilde{a}_2 = \langle[s_{1.9847}, s_{3.3791}, s_{5.1383}, s_{6.3481}], (0.6119, 0.2948)\rangle,$$
$$\tilde{a}_3 = \langle[s_{2.4192}, s_{3.6133}, s_{5.4661}, s_{6.5309}], (0.5709, 0.3140)\rangle,$$
$$\tilde{a}_4 = \langle[s_{1.6084}, s_{2.7197}, s_{5.3416}, s_{6.4186}], (0.6264, 0.2227)\rangle,$$
$$\tilde{a}_5 = \langle[s_{2.0611}, s_{3.5533}, s_{5.1660}, s_{6.3448}], (0.6327,\ 0.2431)\rangle.$$

Step 4: Utilize the expected function in Eq. 35 to calculate the expected values $E(\tilde{a}_i)$ of the collective overall assessment values $\tilde{a}_i(i = 1, 2, 3, 4, 5)$.

$$E(\tilde{a}_1) = s_{3.1145}, E(\tilde{a}_2) = s_{2.7742}, E(\tilde{a}_3) = s_{2.8328},$$
$$E(\tilde{a}_4) = s_{2.8248}, E(\tilde{a}_5) = s_{2.9746}.$$

Step 5: According to the descending order of expected values $E(\tilde{a}_i)(i = 1, 2, 3, 4, 5)$, all feasible alternatives $A_i(i = 1, 2, 3, 4, 5)$ are ranked as follows:

$$A_1 \succ A_5 \succ A_3 \succ A_4 \succ A_2$$

Therefore, the best alternative is A_1 as well.

From the collective overall assessment values $\tilde{a}_i(i = 1, 2, 3, 4, 5)$ obtained by two proposed methods, we can make sure the degree that an alternative is good or bad by a trapezoid fuzzy linguistic variable directly, which is more important than the ranking sequence of the alternatives. For example, if all alternatives are bad, we should not select any one of them, rather than select the first of the ranking sequence. Moreover, we can know how much degree that an alternative belong to and not belong to a trapezoid fuzzy linguistic variable by the intuitionistic fuzzy number. This is the main advantage of our methods than other multi-attribute group decision-making methods.

6 Conclusions

In this paper, we have developed some new intuitionistic trapezoid fuzzy linguistic aggregation operators, such as ITrFLWG operator, ITrFLOWG operator, ITrFLHWG operator, intuitionistic trapezoid fuzzy linguistic generalized weighted averaging (ITrFLGWA) operator, intuitionistic trapezoid fuzzy linguistic generalized ordered weighted averaging (ITrFLGOWA) operator and intuitionistic trapezoid fuzzy linguistic generalized hybrid weighted averaging (ITrFLGHWA) operator. Then, we studied some desired properties of these operators, such as monotonicity, commutativity, idempotency and boundedness. Moreover, considering that the weight vector of the decision-makers may be known or unknown, we developed methods to deal with multi-attribute group decision-making problems under intuitionistic trapezoid fuzzy linguistic information based on the ITrFLWG operator and the ITrFLOWG operator, respectively. Then, the collective overall assessment value of each alternative is obtained and the best alternative is selected according to the expected function. Finally, an illustrative example was given to illustrate the developed methods. In the future, we shall continue working in the extension and applications of the developed operators. The main characteristic of our approach is that the final results can demonstrate the degree that an alternative is good or bad by a trapezoid fuzzy linguistic variable and the degree that an alternative belong to and not belong to the trapezoid fuzzy linguistic variable by the intuitionistic fuzzy number. Additionally, our approaches can be utilized to deal with the situations whether weight vector and position-weighted vector are known or not in a scientific and effective manner. In future researches, we will focus on developing the traditional decision-making methods under intuitionistic trapezoid fuzzy linguistic environment such as vlsekriterijumska optimizacija i kompromisno resenje in serbian (VIKOR), technique for order preference by similarity to ideal solution (TOPSIS), grey relational analysis (GRA), elimination et choice translating reality (ELECTRE), etc.

Acknowledgments This research is supported by Program for New Century Excellent Talents in University (NCET-13-0037), Natural Science Foundation of China (No. 70972007, 71271049) and Beijing Municipal Natural Science Foundation (No. 9102015, 9133020).

References

Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20:87–96
Atanassov KT (1999) Intuitionistic fuzzy sets: theory and applications. Physica-Verlag, New York
Atanassov KT, Gargov G (1989) Interval valued intuitionistic fuzzy sets. Fuzzy Set Syst 31(3):343–349
Chen RY (2009) A problem-solving approach to product design using decision tree induction based on intuitionistic fuzzy. Eur J Oper Res 196:266–272
Chen YH, Wang TC, Wu CY (2011) Multi-criteria decision making with fuzzy linguistic preference relations. Appl Math Model 35:1322–1330
Dursun M, Karsak EE (2010) A fuzzy MCDM approach for personnel selection. Expert Syst Appl 37:4324–4330
Fujimoto M, Yamada T (2006) An exact algorithm for the knapsack sharing problem with common items. Eur J Oper Res 171:693–707
Herrera F, Herrera-Viedma E, Verdegay JL (1996a) A model of consensus in group decision making under linguistic assessments. Fuzzy Sets Syst 78:73–87
Herrera F, Herrera-Viedma E, Verdegay JL (1996b) Direct approach processes in group decision making using linguistic OWA operators. Fuzzy Sets Syst 79:175–190
Herrera F, Martínez L (2000) A 2-Tuple fuzzy linguistic representation model for computing with words. IEEE Trans Fuzzy Syst 8:746–752
Ju YB, Wang AH (2012) Emergency alternative evaluation under group decision makers: a method of incorporating DS/AHP with extended TOPSIS. Expert Syst Appl 39:1315–1323
Ju YB, Liu XY, Yang SH (2013) Trapezoid fuzzy 2-tuple linguistic aggregation operators and their applications to multiple attribute decision making. J Intell Fuzzy Syst. doi:10.3233/IFS-131087
Ju YB, Yang SH (2013) Some aggregation operators with intuitionistic trapezoid fuzzy linguistic numbers and their application to multiple attribute group decision making. Technical Report, pp 1–9
Li DF (2010) Multiattribute decision making method based on generalized OWA operators with intuitionistic fuzzy sets. Expert Syst Appl 37:8673–8678

Lin J, Lan JB, Lin YH (2009) Multi-attribute group decision-making method based on the aggregation operators of interval 2-tuple linguistic information. J Jilin Norm Univ 1:5–9
Liu PD (2013a) Some generalized dependent aggregation operators with intuitionistic linguistic numbers and their application to group decision making. J Comput Syst Sci 79:131–143
Liu PD (2013b) Some geometric aggregation operators based on interval intuitionistic uncertain linguistic variables and their application to group decision making. Appl Math Model 37:2430–2444
Liu PD, Jin F (2012) Methods for aggregating intuitionistic uncertain linguistic variables and their application to group decision making. Inf Sci 205:58–71
Liu PD, Yu S (2010) The multiple-attribute decision making method based on the TFLHOWA operator. Comput Math Appl 60:2609–2615
Oztekin A, Kong ZJ, Delen D (2011) Development of a structural equation modeling-based decision tree methodology for the analysis of lung transplantations. Decis Support Syst 51:155–166
Oliveira ARL, Sorensen DC (2005) A new class of preconditioners for large-scale linear systems from interior point methods for linear programming. Linear Algebra Appl 394:1–24
Porcel C, Herrera-Viedma E (2010) Dealing with incomplete information in a fuzzy linguistic recommender system to disseminate information in university digital libraries. Knowl Based Syst 23:32–39
Wan SP (2013) Power average operators of trapezoidal intuitionistic fuzzy numbers and application to multi-attribute group decision making. Appl Math Model 37:4112–4126
Wang JQ, Li JJ (2009) The multi-criteria group decision making method based on multi-granularity intuitionistic two semantics. Sci Technol Inf 33:8–9
Wang Y, Xu ZS (2008) A new method of giving OWA weights. Math Pract Theory 38:51–61
Wang YJ (2011a) A fuzzy multi-criteria decision-making model based on lower and upper boundaries. Appl Math Model 35:3213–3224
Wang YJ (2011b) Fuzzy multi-criteria decision-making based on positive and negative extreme solutions. Appl Math Model 35:1994–2004
Wei GW (2009) Some generalized uncertain linguistic aggregating operators. In: 2009 IITA International Conference on Services Science, Management and Engineering, Zhangjiajie, pp 85–88
Wei GW (2011) Some generalized aggregating operators with linguistic information and their application to multiple attribute group decision making. Comput Ind Eng 61:32–38
Wei GW, Zhao XF, Lin R, Wang HJ (2013) Uncertain linguistic Bonferroni mean operators and their application to multiple attribute decision making. Appl Math Model 37:5277–5285
Wu J, Cao QW (2013) Same families of geometric aggregation operators with intuitionistic trapezoidal fuzzy numbers. Appl Math Model 37:318–327
Xu YJ, Merigó JM, Wang HM (2012) Linguistic power aggregation operators and their application to multiple attribute group decision making. Appl Math Model 36:5427–5444
Xu ZS (2004) Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment. Inf Sci 168:171–184
Xu ZS (2005) An approach based on similarity measure to multiple attribute decision making with trapezoid fuzzy linguistic variables. Fuzzy Syst Knowl Discov 3613:110–117
Xu ZS (2006) An approach based on the uncertain LOWG and induced uncertain LOWG operators to group decision making with uncertain multiplicative linguistic preference relations. Decis Support Syst 41:488–499
Xu ZS (2007a) Group decision making with triangular fuzzy linguistic variables. In: Intelligent Data Engineering and Automated Learning - IDEAL 2007, Lecture Notes in Computer Science, vol 4881. Springer, Heidelberg, pp 17–26
Xu ZS (2007b) Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making. Fuzzy Optim Decis Making 6:109–121
Xu ZS, Da QL (2002) The ordered weighted geometric averaging operators. Int J Intell Syst 17:709–716
Xu ZS, Da QL (2003) An overview of operators for aggregating information. Int J Intell Syst 18:953–969
Yager RR (2004) Generalized OWA aggregation operators. Fuzzy Optim Decis Making 3:93–107
Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–353
Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning. Inf Sci 8:199–249
Zhang HM (2012) The multiattribute group decision making method based on aggregation operators with interval-valued 2-tuple linguistic information. Math Comput Model 56:27–35
Zhang HM (2013) Some interval-valued 2-tuple linguistic aggregation operators and application in multiattribute group decision making. Appl Math Model 37:4269–4282
Zhang X, Jin F, Liu PD (2013) A grey relational projection method for multi-attribute decision making based on intuitionistic trapezoidal fuzzy number. Appl Math Model 37:3467–3477
Zhang ZM (2013) Generalized Atanassov's intuitionistic fuzzy power geometric operators and their application to multiple attribute group decision making. Inf Fusion 14:460–486
Zhao H, Xu ZS, Ni M, Liu S (2010) Generalized aggregation operators for intuitionistic fuzzy sets. Int J Intell Syst 25:1–30