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METHODOLOGIES AND APPLICATION

A simple design method for interval type-2 fuzzy pid controllers

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Abstract In this study, a design method for single Input interval type-2 fuzzy PID controller has been developed. The most important feature of the proposed type-2 fuzzy controller is its simple structure consisting of a single input variable. The presented simple structure gives an opportunity to the designer to form the type-2 fuzzy controller output in closed form formulation for the first time in literature. This formulation cannot be achieved with present type-2 fuzzy PID controller structures which have employed the Karnik-Mendel type reduction. The closed form solution is derived in terms of the tuning parameters which are chosen as the heights of lower membership functions of the antecedent interval type-2 fuzzy sets. Elaborations are done on the derived closed form output and a simple strategy is presented for a single input type-2 fuzzy PID controller design. The presented interval type-2 fuzzy controller structure still keeps the most preferred features of the PID controller such as simplicity and easy design. We will illustrate how the extra degrees of freedom provided by the antecedent interval type-2 fuzzy sets can be used to enhance the control performance on linear and nonlinear benchmark systems by simulations. Moreover, the type-2 fuzzy controller structure has been implemented on experimental pH neutralization. The simulation and experimental results will illustrate that the proposed type-2 fuzzy controller produces superior control performance and can handle nonlinear dynamics, parameter uncertainties, noise and disturbances better in comparison with the standard PID controllers. Hence, the results and analyses of this study will give the control engineers an opportunity to draw a bridge and connect the type-2 fuzzy logic and control theory.

Keywords Interval type-2 fuzzy PID controller design · Single input fuzzy inference Nonlinear control · Process independent control curve design

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1 Introduction

It is a known fact that conventional PID controllers are the most popular controllers used in industry due to their simple structure and low cost (Skogestad 2003; Zhuang and Atherton 1993). In literature, a wide variety of design methods have been reported on the tuning of PID controllers. Some of them are the Ziegler and Nichols (1942), Cohen and Coon (1953), internal model control (Morari and Zafiriou 1989), pole placement (Wang et al. 2009) design strategies. The use of PID controllers might be an efficient way in controlling linear systems, however when the process is nonlinear or the process model is uncertain, PID controllers may not achieve a satisfactory closed loop control performance.

Ordinary (type-1) fuzzy PID controllers are widely been used to control nonlinear systems and demonstrated significant performance improvements (Guzelkaya et al. 2003; Fuente et al. 2006). A wide literature survey on fuzzy PID controllers and applications of fuzzy PID control is presented in (Feng 2006). From the input-output relationship point of view, the structures of the type-1 fuzzy PID controllers are analogous to the conventional PID controllers (Galichet and Foulloy 1995; Qiao and Mizumoto 1996). It has been stated in (Feng 2006), that type-1 fuzzy PID controllers can be designed using one, two or three inputs. A one input fuzzy inference mapping for a systematic and optimal design of fuzzy PID controllers is proposed in (Hu et al. 1999). However in literature, fuzzy PID controllers with two inputs are more widely used and preferred. After the pioneer study by Qiao and Mizumoto (1996), several type-1 fuzzy PID control design strategies have been proposed (Woo et al. 2000; Guzelkaya et al. 2003; Oh et al. 2004; Duan et al. 2008; Karasakal et al. 2012).

Lately, almost all of the fuzzy logic control applications are focused on interval type-2 fuzzy logic controllers (IT2-FLC). The internal structure of the IT2-FLC is similar to the type-1 counterpart. However, the major difference is that at least one of the fuzzy sets (FSs) in the rulebase is an interval type-2 fuzzy set (IT2-FS) (Karnik et al. 1999). Thus, a type-reducer is needed to convert them into a type-1 fuzzy set before a defuzzification procedure can be performed (Liang and Mendel 2000). Generally, interval type-2 fuzzy logic systems achieve better control performance because of the additional degree of freedom provided by the Footprint of Uncertainty (FOU) in their membership functions (Kumbasar et al. 2012). Consequently, IT2-FLCs have attracted much research interests, especially in control applications, since they are much more powerful to handle uncertainties and nonlinearities (Hagras 2004). Thus, several applications such as liquid-level process control (Wu and Tan 2006); autonomous mobile robots (Martinez et al. 2009); plants control (Castillo et al. 2005); bioreactor control (Galluzzo et al. 2008); pH control (Kumbasar et al. 2011, 2012) have been handled and implemented successfully. Studies have been reported in the literature that the IT2-FLCs are generally more robust than type-1 fuzzy controllers (Wu and Tan 2010; Aliasghary et al. 2013). Wu (2012a) has shown that interval type-2 fuzzy controllers have smoother control surfaces than its type-1 counterpart. However, type-2 fuzzy logic controllers are difficult to analyse and design since their internal structure is typically more complex (Wu 2012a). Wu (2012b) also pointed out there are many decisions to be made in designing an IT2 FLC such as which type of and how many IT2-FS to use and how to construct the rulebase. In this context, several studies have been performed in order to design and investigate the input-output mapping of the IT2-FLCs. Aliasghary et al. 2013 proposed a systematical methodology to construct an interval type-2 fuzzy logic controller based on an existing PI control law. In (Biglarbegian et al. 2009), IT2-FLC PD/PI is designed based on the transient state response criteria. Moreover, optimization based design methods for interval type-2 fuzzy controllers have also been proposed (Oh et al. 2011; Castillo and Melin 2012; Maldonado et al. 2013).

Nevertheless, a systematic design for both of type-1 and type-2 fuzzy controllers is still a challenging problem due to the main difficulty in designing and determining the FSs and rulebase, i.e. the control surface. This issue is usually solved by extensively trial and error procedures. Besides, several tuning and design strategies have been proposed to overcome this difficulty for certain fuzzy control structures or processes (Bhatttacharya et al. 2004; Yesil et al. 2004; Wu and Tan 2006; Juang et al. 2008; Ahn and Truong 2009; Oh et al. 2009; Castillo and Melin 2012; Karasakal et al. 2012).

In this study, a simple design method has been developed for interval type-2 fuzzy PID controllers. The most important feature of the proposed type-2 fuzzy controller is its simple structure which consists of a single input variable, i.e. error signal. At first, we will derive the closed form formulation of the IT2-FLC output in terms of the design parameters which are chosen as the heights of lower membership functions of antecedent type-2 fuzzy sets, i.e. the extra degree of freedoms provided by the antecedent IT2-FSs. This gives the opportunity to design and analyze the type-2 fuzzy mapping in the error domain. In other words, the IT2-FLC design problem is transformed from a control surface generation to a control curve generation. After certain elaborations, a simple design method for tuning process independent type-2 fuzzy PID controllers is presented. It will be illustrated that the proposed IT2-FLC PID controller structure keeps the most preferred features of the PID such as simplicity and easy implementation while extra degree of freedom provided by the IT2-FSs is used to enhance the control performance. The effectiveness of the proposed interval type-2 fuzzy control structure is demonstrated on benchmark linear and nonlinear systems by simulations. Moreover, a real time application of this new method is accomplished on the G.U.N.T RT-552 pH value Control Trainer.

This paper is organized in 6 sections excluding the conclusion section. In Sect. 2, the general structure of the IT2-FLC PID controller is explained. In Sect. 3, the closed form of the proposed type-2 fuzzy controller output is derived and its properties are presented. In Sect. 4, the analyses and design method of the IT2-FLC are presented. In Sect. 5, simulation studies for linear and nonlinear systems are presented to show the beneficial sides of the proposed type-2 fuzzy control method. In Sect. 6, the experimental studies which are implemented on the pH neutralization process are presented. The results have been evaluated and discussed in the conclusion section.

2 The general structure of the single input IT2-FLC PID controller

In this section, the general structure of the proposed single-input interval type-2 fuzzy PID controller with one to one inference mechanism is presented. The proposed type-2 fuzzy PID control structure is illustrated in Fig. 1. Here, the IT2-FLC, which in fact acts like type-2 fuzzy time varying gain, is cascaded with a conventional PID controller.

The IT2-FLC structure uses a single input fuzzy mapping which makes analytical solutions possible. More detailed

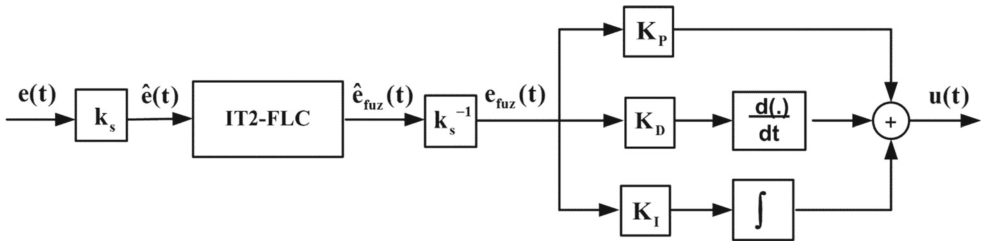

Fig. 1 Single input IT2-FLC PID control structure

information about the internal structure of the IT2-FLC is presented in Sect. 3. In this structure, a normalization method is performed on the error signal to guarantee that the input variable of the IT2-FLC is always within the range of [−1, +1] which is the universe of discourse of the input. The error signal is defined as $e = r - y$; where r is the reference input and y is the process output. The error signal is scaled and limited via the following transformation:

$$\hat{e}(t) = \begin{cases} +1 & k_s e(t) > +1 \\ k_s e & |k_s e(t)| \leq +1 \\ -1 & k_s e(t) < -1 \end{cases} \quad (1)$$

where $\hat{e}$ is the input variable of the IT2-FLC structure and k_s is a scaling factor defined as:

$$k_s = \frac{1}{r_0 - y_0} \quad (2)$$

where r_0 and y_0 are the initial reference and process output, respectively. Via this simple normalization it is guaranteed that $\hat{e} \in [-1, +1]$. Similarly, a rescaling is performed to the output $\hat{e}_{fuz}$ as follows:

$$e_{fuz}(t) = \hat{e}_{fuz}(t) k_s^{-1} \quad (3)$$

The second part of the proposed control structure is a conventional PID controller given as:

$$u(t) = K_p e_{fuz}(t) + K_I \int e_{fuz}(t) + K_D \frac{de_{fuz}(t)}{dt} \quad (4)$$

where K_P, K_I and K_D are the proportional gain, the integral gain and the derivative gain, respectively. For the tuning of the PID controller parameters various design strategies have been proposed (Skogestad 2003).

The output of the proposed type-2 fuzzy PID control, illustrated in Fig. 1, is in fact analogous to the conventional counterpart given in (4) and is formulated as:

$$u(t) = \hat{K}_p(t) e(t) + \hat{K}_I(t) \int e(t) + \hat{K}_D(t) \frac{de(t)}{dt} \quad (5)$$

However, here $\hat{K}_p(t)$, $\hat{K}_I(t)$ and $\hat{K}_D(t)$ are nonlinear time varying PID gains and are defined as:

$$\hat{K}_p(t) = K_p \cdot \alpha(t) \quad \hat{K}_I(t) = K_I \cdot \lambda(t) \quad \hat{K}_D = K_D(t) \cdot \beta(t) \quad (6)$$

where $\alpha(t)$, $\lambda(t)$ and $\beta(t)$ are nonlinear gains effecting the proportional, derivative and integral gains.

The presented control law in (6) is in fact analogous to a nonlinear PID controller. In control literature, several nonlinear PID controller structures and design methods have been proposed. The most known approaches are presented by Jiang and Gao (2001) and Su et al. (2005). In both design strategies, the nonlinear PID controller is designed based on a linear model. The design strategies are not straightforward and are computational complex since nonlinear functions such as sigmoid, exponential, etc. functions are employed in the control law to achieve a nonlinear control law. Moreover, in fuzzy control theory a similar single input type-1 fuzzy PID controller structure was presented by Hu et al. (1999) where the "max-min gravity" fuzzy reasoning method has been employed. In this structure, the design problem has been solved by employing Genetic algorithms to tune the controller parameters.

In this study, an alternative design method will be presented for the presented nonlinear control law in (6) where the FOU of the IT2-FLC will be used to design a simple and straightforward type-2 fuzzy PID control structure. Thus, as shown in Fig. 1, these three nonlinear gains will be affected from a type-2 fuzzy mapping. Here, only one nonlinear control gain will affect the proportional, derivative and integral control action (i.e. $\gamma(t) = \alpha(t) = \lambda(t) = \beta(t)$) which is defined as $\gamma(t) = e_{fuz}(t)/e(t)$. The output of the proposed IT2-FLC PID preserves the basic elements of the PID controller. It can be concluded that, IT2-FLC structure in fact acts like time varying interval type-2 fuzzy gain which effects the basic control actions of the PID. It should be noted that, the scaling factor (k_s) is not included in (6) since it does not change the nonlinear behavior of the controllers.

3 Analytical derivation of the IT2-FLC structure

In this section, the input output mapping of the proposed IT2-FLC structure is derived. In the presented type-2 fuzzy structure, the antecedent part of the fuzzy rule is defined with uniformly distributed symmetrical triangular interval type-2 fuzzy membership functions for simplicity. The linguistic values of the input "e" are denoted as $\tilde{A}_i$ where $i = \{-n,$

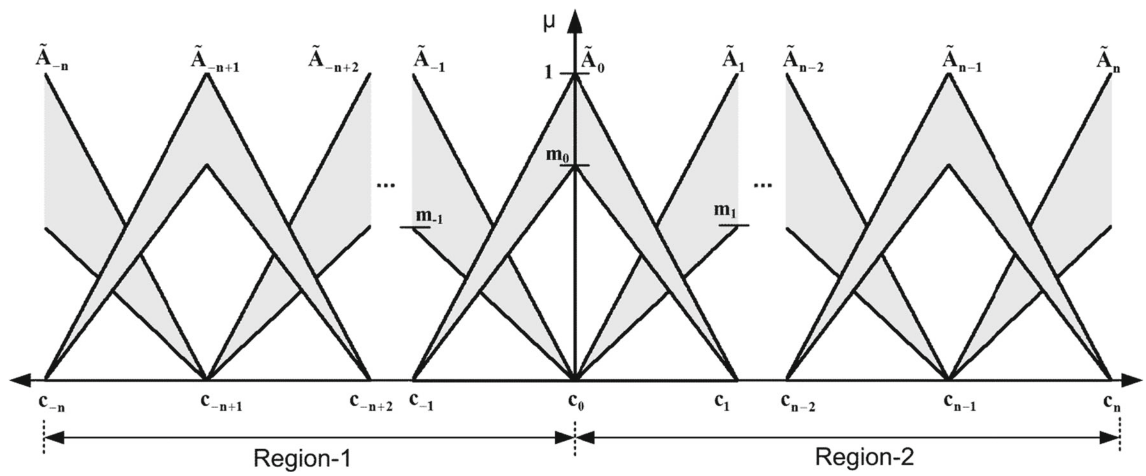

Fig. 2 Illustration of the interval type-2 triangular membership functions for "e".

$-n+1\ldots -1, 0, 1 \ldots n-1, n\}$. The defined type-2 fuzzy set ($\tilde{A}_i$) can be described in terms of an upper membership function ($\overline{\mu}\tilde{A}_i$) and a lower membership function ($\underline{\mu}\tilde{A}_i$). As shown in Fig. 2, m_i represents the height of the lower membership functions and c_i denotes the core of the $\tilde{A}_i$. Since the input IT2-FSs are symmetrical, the input domain (e) can be portioned in two main regions which are named as Region-1 ($e \in [c_{-n}, c_0]$) and Region-2 ($e \in [c_0, c_n]$). Besides, the defined interval type-2 fuzzy membership functions have the following properties:

(i) $\overline{\mu}\tilde{A}_i(e) + \overline{\mu}\tilde{A}_{i+1}(e) = 1, i = -n, \ldots, +n$
(ii) $\underline{\mu}\tilde{A}_i(e) = m_i \cdot \overline{\mu}\tilde{A}_i(e), i = -n, \ldots, n$
(iii) $m_{-i} = m_i, i = 1, \ldots, n$

The rule structure of the proposed IT2-FLC is defined as:

$$r_i : \text{IF } e \text{ is } \tilde{A}_i \text{THEN } e_{fuz} \text{ is } B_i, \quad i = 1, \ldots, N \tag{7}$$

where $N = 2n + 1$ is the total number of rules, while the consequent part is defined with crisp singleton values (B_i) which are uniformly distributed in the range of $[-1, 1]$.

Liang and Mendel (2000) demonstrated that the defuzzified output of an IT2-FLC can be calculated as:

$$e_{fuz} = \frac{e^l_{fuz} + e^r_{fuz}}{2} \tag{8}$$

where e^r_{fuz} and e^l_{fuz} are the end points of the type reduced set, and can be computed as follows:

$$e^l_{fuz} = \frac{\sum_{j=1}^{L} \overline{\mu}\tilde{A}_i(e) \cdot B_i + \sum_{j=L+1}^{N} \underline{\mu}\tilde{A}_i(e) \cdot B_i}{\sum_{j=1}^{L} \overline{\mu}\tilde{A}_i(e) + \sum_{j=L+1}^{N} \underline{\mu}\tilde{A}_i(e)} \tag{9}$$

$$e^r_{fuz} = \frac{\sum_{j=1}^{R} \underline{\mu}\tilde{A}_i(e) \cdot B_i + \sum_{j=R+1}^{N} \overline{\mu}\tilde{A}_i(e) \cdot B_i}{\sum_{j=1}^{R} \underline{\mu}\tilde{A}_i(e) + \sum_{j=R+1}^{N} \overline{\mu}\tilde{A}_i(e)} \tag{10}$$

Here, (L, R) is the solution set such that which minimize/maximize (9) and (10), respectively (Liang and Mendel 2000). As it has been illustrated in Fig. 2, the IT2-FLC uses fully overlapping triangular interval type-2 fuzzy sets in the sense of upper and lower fuzzy membership functions. Thus, it is always guaranteed that a crisp value of "e" belongs to two successive IT2-FSs, i.e. $\tilde{A}_i$ and $\tilde{A}_{i+1}$. As a consequence, the switching points (R, L) are always equal to "1" since for any crisp input only two rules ($N = 2$) are always activated. This gives the opportunity to derive a closed-form relation of the IT2-FLC. In this context, the type reduced can be derived as:

$$e^l_{fuz} = \frac{\overline{\mu}\tilde{A}_i(e) \cdot B_i + \underline{\mu}\tilde{A}_{i+1}(e) \cdot B_{i+1}}{\overline{\mu}\tilde{A}_i(e) + \underline{\mu}\tilde{A}_{i+1}(e)} \tag{11}$$

$$e^r_{fuz} = \frac{\underline{\mu}\tilde{A}_i(e) \cdot B_i + \overline{\mu}\tilde{A}_{i+1}(e) \cdot B_{i+1}}{\underline{\mu}\tilde{A}_i(e) + \overline{\mu}\tilde{A}_{i+1}(e)} \tag{12}$$

After replacing (11) and (12) in (8), a closed form mapping of the proposed type-2 fuzzy controller is obtained. The presented IT2-FLC output has the following properties.

(i) e_{fuz} is a continuous proportional function with respect to the input e.
(ii) e_{fuz} is symmetrical function with respect the input e, i.e. $e_{fuz}(e) = -e_{fuz}(-e)$.
(iii) If the input (e) is equal to zero, then $e_{fuz} = 0$. This is required to have zero steady state error.

4 The interval type-2 fuzzy controller design strategy

In this section, we will present the design strategy for the single input interval type-2 fuzzy controller. As it has been asserted in the Sect. 5, the type-2 fuzzy controller output can be explicitly represented in the error domain since it consists of a single input. This simplifies the IT2-FLC design method to a nonlinear control curve generation, instead of a control curve design. In this method, the heights (m_i) of lower membership functions of $\tilde{A}_i$ are considered as design parameters. Firstly, the effects of the design parameters on the IT2-FLC output are analyzed and then a process independent design method to generate control curves is proposed.

For the simplicity, a closed form of a "three rule" type-2 fuzzy inference is derived at first and then the effects of design parameters are investigated in detail. The parameters of IT2-FLC structure are set as $B_{-1} = -1$, $B_0 = 0$, $B_{+1} = +1$, $c_{-1} = -1$, $c_0 = 0$ and $c_1 = +1$. The antecedent and the consequent membership functions are illustrated in Fig. 3a.

The end points of the type reduced set can then be derived for the input Region-1 ($e \in [-1, 0]$) as follows:

$$e^l_{fuz} = \frac{\overline{\mu}\tilde{A}_{-1}(e)\cdot(-1) + \underline{\mu}\tilde{A}_0(e)\cdot 0}{\overline{\mu}\tilde{A}_{-1}(e) + \underline{\mu}\tilde{A}_0(e)}$$
$$e^l_{fuz} = \frac{-\overline{\mu}\tilde{A}_{-1}(e)}{\overline{\mu}\tilde{A}_{-1}(e) + \underline{\mu}\tilde{A}_0(e)} \tag{13}$$

$$e^r_{fuz} = \frac{\underline{\mu}\tilde{A}_{-1}(e)\cdot(-1) + \overline{\mu}\tilde{A}_0(e)\cdot(0)}{\underline{\mu}\tilde{A}_{-1}(e) + \overline{\mu}\tilde{A}_0(e)}$$
$$e^r_{fuz} = \frac{-\underline{\mu}\tilde{A}_{-1}(e)}{\underline{\mu}\tilde{A}_{-1}(e) + \overline{\mu}\tilde{A}_0(e)} \tag{14}$$

Similarly for the Region-2 ($e \in [0, +1]$), the type reduced set reduces to:

$$e^l_{fuz} = \frac{\underline{\mu}\tilde{A}_1(e)}{\overline{\mu}\tilde{A}_0(e) + \underline{\mu}\tilde{A}_1(e)} \tag{15}$$

Table 1 The derived expressions of e^l_{fuz} and e^r_{fuz}

	Region-1 $e \in [-1, 0]$	Region-2 $e \in [0, +1]$
e^l_{fuz}	$\frac{-\overline{\mu}\tilde{A}_{-1}(e)}{\overline{\mu}\tilde{A}_{-1}(e)+\underline{\mu}\tilde{A}_0(e)}$	$\frac{\underline{\mu}\tilde{A}_1(e)}{\overline{\mu}\tilde{A}_0(e)+\underline{\mu}\tilde{A}_1(e)}$
e^r_{fuz}	$\frac{-\underline{\mu}\tilde{A}_{-1}(e)}{\underline{\mu}\tilde{A}_{-1}(e)+\overline{\mu}\tilde{A}_0(e)}$	$\frac{\overline{\mu}\tilde{A}_1(e)}{\underline{\mu}\tilde{A}_0(e)+\overline{\mu}\tilde{A}_1(e)}$

$$e^r_{fuz} = \frac{\underline{\mu}\tilde{A}_1(e)}{\overline{\mu}\tilde{A}_0(e) + \underline{\mu}\tilde{A}_1(e)} \tag{16}$$

The derived analytical expressions of e^l_{fuz} and e^r_{fuz} for a "three rule" IT2-FLC are tabulated in Table 1 to investigate the effect of the design parameters (m_{-1}, m_0, m_1) on the output easily.

In this study, only Region-2 will be inspected in detail. However, the analyses that are going to be presented can be easily extended for Region-1 since the input and output membership functions are symmetrical and uniformly distributed. According to the derived expressions of e^l_{fuz} and e^r_{fuz} for Region-2, the following meta-rules can be derived to form a control action in order to obtain a satisfactory system performance.

(i) If the value of $\underline{\mu}\tilde{A}_1(e)$(i.e. m_1) decreases/increases then the value of e^l_{fuz} decreases/increases, respectively.

(ii) If the value of $\underline{\mu}\tilde{A}_0(e)$(i.e. m_0) decreases/increases then the value of e^r_{fuz} increases/decreases, respectively.
In the light of (i) & (ii) reminding that the defuzzified output of an IT2-FLC (e_{fuz}) is the average value of e^r_{fuz} and e^l_{fuz} values.

(iii) If the value of m_0 is decreased while m_1 is increased then the value of e_{fuz} is increased since the values of both e^r_{fuz} and e^l_{fuz} are increased. Thus, an aggressive control action is obtained.

(iv) If the value of m_0 is increased while m_1 is decreased then the value of e_{fuz} is decreased since the values of both e^r_{fuz} and e^l_{fuz} are decreased. Thus, a smooth control action is obtained.

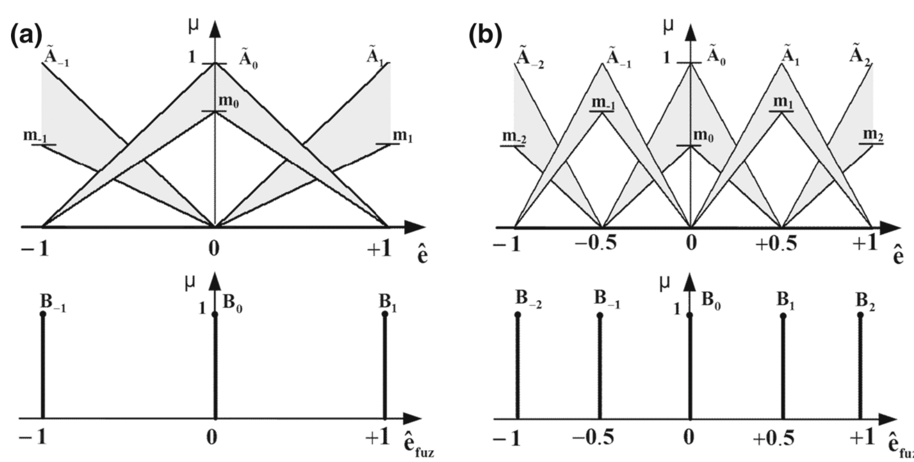

Fig. 3 Illustration of the antecedent and the consequent membership functions for **a** the "three rule" IT2-FLC and **b** the "five rule" IT2-FLC

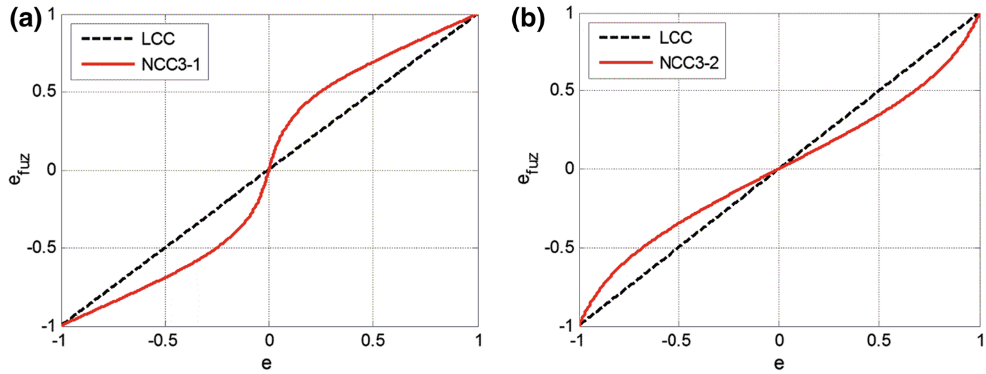

Fig. 4 Illustration of the nonlinear control curves **a** NCC3-1 and **b** NCC3-2

The presented meta-rules for a "three rule" type-2 fuzzy inference can easily be generalized for "N rule" inferences since for any crisp input value only two rules (N = 2) are always activated.

According to derived meta-rules, the following four nonlinear control curves are designed. The first three nonlinear control curves, Nonlinear Control Curve 3-1 (NCC3-1), Nonlinear Control Curve 3-1 (NCC3-2) and Nonlinear Control Curve 3-3 (NCC3-3), are generated from a "three rule" IT2-FLC as given in Fig. 3a. The last nonlinear control curve, Nonlinear Control Curve 5 (NCC5), is generated from a "five rule" IT2-FLC with the following parameters $B_{-2} = -1$, $B_{-1} = -0.5$, $B_0 = 0$, $B_1 = 0.5$, $B_2 = 1$, $c_{-2} = -1$, $c_{-1} = -0.5$, $c_0 = 0$, $c_1 = +0.5$ and $c_2 = +1$ as illustrated in Fig. 3b. All four nonlinear control curves are compared with a linear control curve (LCC) for an easier analysis. (The LCC can simple be constructed by choosing $m_1 = m_0 = m_{-1} = 1$, i.e, type-1 fuzzy membership functions)

1. Nonlinear control curve 3–1 (NCC3–1)

If m_1 is relatively bigger than m_0, i.e. $1 \geq m_1 = m_{-1} \geq m_0 \geq 0$, then an aggressive nonlinear control action is generated. For instance; if $m_1 = m_{-1}$ and m_0 are equal to 0.8 and 0.1, respectively then the control curve illustrated in Fig. 4a is obtained. This control curve has a high sensitivity when e is close to "0". This kind of a control curve can be preferred in order to have fast transient system response. However, the generated control curve is sensitive to the noises, especially around the set point value, i.e. $e = 0$.

2. Nonlinear control curve 3–2 (NCC3–2)

If m_1 is relatively smaller than m_0, i.e. $1 \geq m_0 \geq m_1 = m_{-1} \geq 0$, then a smooth control action is obtained. For instance; if $m_1 = m_{-1}$ and m_0 are equal to 0.2 and 0.9, respectively then the nonlinear control curve given in Fig. 4b is generated. The NCC3-2 has a low sensitivity when e is close to "0". Obviously, this nonlinear control curve can be preferred to have a robust closed loop control performance against parameter uncertainties and/or white noises.

3. Nonlinear control curve 3–3 (NCC3–3)

If m_1 is equal to m_0, i.e. $1 \geq m_1 = m_0 = m_{-1} \geq 0$, then an inverse S shaped nonlinear control action is generated. if m_1, m_{-1} and m_0 are both set to 0.2, the control curve illustrated in Fig. 5a is obtained. This kind of control action can be preferred if there exits expert/priori knowledge about the process gain, etc.

4. Nonlinear control curve 5 (NCC5)

In this design, a "five rule" IT2-FLC is used as illustrated in Fig. 3b. If both m_1 and m_2 are relatively bigger than m_0, i.e. $1 \geq m_2 \geq m_1 > m_0 \geq 0$, then an S shaped control curve is generated. For instance; if m_2, m_1 and m_0 are equal to 0.9, 0.9 and 0.1, respectively then the nonlinear control curve illustrated in Fig. 5b is obtained (Reminding that $m_2 = m_{-2}$ and $m_1 = m_{-1}$). The S shaped control curve is in fact a combination of the NCC3–1 and the NCC3–2. Similar to the NCC3–1, it has a high sensitivity when e is close to "1", while a low sensitivity when e is close to "0" similar to the NCC3–2.

The presented nonlinear control curves and properties of the interval type-2 fuzzy mapping are useful guidelines for designing the IT2-FLCs.

5 Simulation studies

In this section, the performances of the proposed interval type-2 fuzzy control structures are compared with conventional control schemes on four benchmark processes. These benchmark processes are integrating (System-I), non-minimum phase (System-II), high order (System-III), and nonlinear systems (System-IV), respectively. In the first two simulation studies (System-I and System II), we will

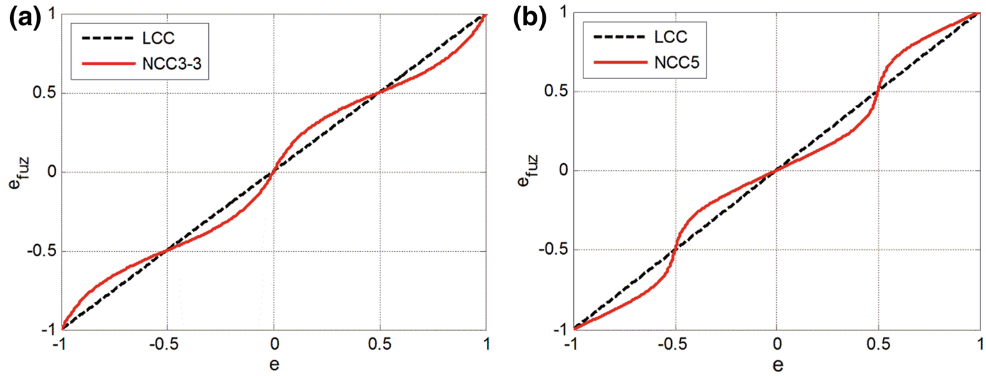

Fig. 5 Illustration of the nonlinear control curves **a** NCC3-3 and **b** NCC5

investigate if the system performance can be improved via the proposed type-2 fuzzy control structure if there does not exists any uncertainties. In the presented simulation results for System-III, since the controllers are designed based on the first order model approximation of the high order system, we will investigate how the proposed type-2 fuzzy controller handle modeling errors, i.e. parametric uncertainties. In the presented simulation studies for System-IV, we will investigate how the proposed type-2 fuzzy controller can handle nonlinear dynamics.

In the simulation studies, both the PID and IT2-FLC PID controllers are implemented as the discrete-time versions obtained with the bilinear transform with the sampling time $t_s = 0.1$s. The simulations were done on a personal computer with an Intel Pentium Dual Core T2370 1.73 GHz processor, 2.99 GB RAM, and software package MATLAB/Simulink 7.4.0. Note that the simulation solver option is chosen as ode5 (Dormand-prince) and the step size is fixed at a value of 0.1s throughout the simulation studies.

In order to make a fair comparison, three performance measures are considered. Two of these performance measures are selected from the classical transient system response criteria; namely, the settling time (T_s), and the overshoot (%OS) while the third performance measure is considered as Integral Absolute Error (IAE), which is defined as:

$$IAE = \int_0^{\infty} |r(t) - y(t)|\, dt \tag{17}$$

A unit step reference is applied to compare the performance of transient responses of the control systems,. Hereby, the scaling factor (k_s) of the IT2-FLC PID structure is set to "1" for all four simulation studies. Moreover, a step load disturbance is applied to observe the disturbance rejection performance of the control structures.

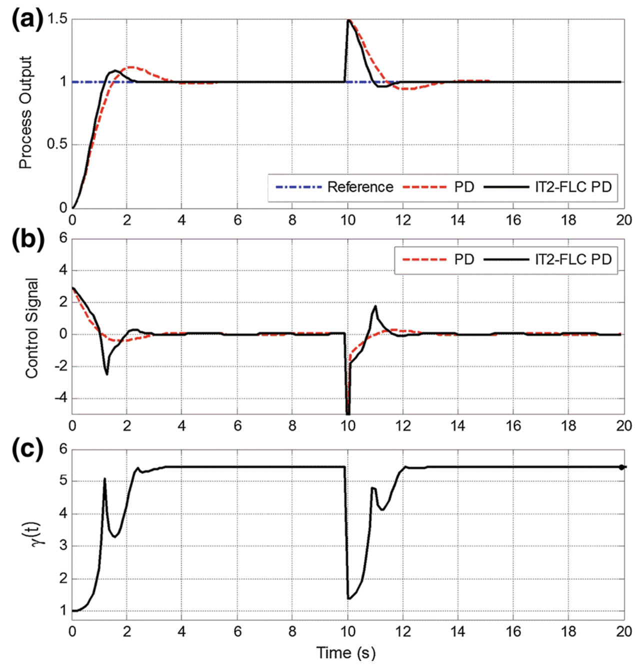

Fig. 6 Simulation results for System I: **a** the process outputs, **b** the control signals and **c** the variation of the interval type-2 fuzzy gain ($\gamma(t)$)

System I:

Consider the integrating process given by Mudi and Pal (1999) as

$$G_2(s) = \frac{1}{s(s+1)} \tag{18}$$

For this process, a PD controller is designed via the pole placement design procedure such that the overshoot will be 10 % and settling time will be 3 s. The controller parameters are set as $K_P = 2.9$, $K_D = 0.9$. Here, a "three rule" IT2-FLC is preferred with the design parameters as $m_{-1} = 0.9$, $m_0 = 0.1$ and $m_1 = 0.9$ to have an aggressive control action.

The unit step output responses and the load disturbance rejection performances of the controllers are illustrated in Fig. 6. The step output disturbance has been applied in

Table 2 Performance comparison of the controllers

	System I			System II			System III			System IV		
	T_s	OS	IAE	T_s	OS	IAE	T_s	OS	IAE	T_s	OS	IAE
Conventional controller	3.0 s	12 %	1.39	6.9 s	23 %	5.17	16 s	25 %	7.16	5.0 s	17 %	2.43
Type-2 fuzzy controller	1.5 s	8 %	1.04	6.7 s	14 %	4.86	11 s	6 %	6.36	3.5 s	8 %	1.63

10th second with magnitude of "0.5". The type-2 fuzzy proportional gain causes an aggressive control signal which improves the transient state and disturbance rejection performances. The variation of e_{fuz}/e is shown in Fig. 6c. As it can be clearly seen in Table 2, the proposed IT2-FLC PD structure ameliorates the overall performance and disturbance rejection performance of the process much better in every sense compared to the linear controller structure.

System II:

Consider the non-minimum phase process given by Astrom and Hagglund (2005),

$$G_3(s) = \frac{1-s}{(s+1)^3} \tag{19}$$

The PID parameters are calculated as $K_P = 1.2$, $K_D = 4$ and $K_I = 0.4$ via pole placement design method to obtain an oscillatory output response with a small rise time. Here, a "three rule" IT2-FLC is chosen with the design parameters as $m_{-1} = 0.4$, $m_0 = 0.9$ and $m_1 = 0.4$ to have a smooth control action. The unit step output responses and the load disturbance rejection performances of the PID and IT2-FLC PID are illustrated in Fig. 7 while the performance measures are tabulated in Table 2. The step output disturbance has been applied in 20th second with magnitude of "0.5".

As it can be seen from Fig. 7c, the type-2 fuzzy proportional gain causes a smoother control signal as the process output converges to the desired reference value which improves the system performance. IT2-FLC PID reduces the overshoot to 14 %; it also decreases the settling time to about 6.7 s and the total IAE value to 4.86 when it is compared to the PID.

System III:

The transfer function of the first handled process is given in (Yesil et al. 2008).

$$G_1(s) = \frac{1}{(s+1)^4} \tag{20}$$

An IMC based PI controller is designed based on the First Order Plus-Time Delay (FOPDT) model approximation and controller parameters are $K_P = 0.791$, $K_I = 0.37$ (Yesil et al. 2008). Here, a "three rule" IT2-FLC PI structure is preferred with the design parameters as $m_{-1} = 0.1$, $m_0 = 0.9$ and $m_1 = 0.1$ in order to have a smooth nonlinear control action. The unit step output and the load disturbance rejection performances of the PI and IT2-FLC PI are illustrated in Fig. 8. The step output disturbance has been applied in 50th second with magnitude of "0.5". The type-2 fuzzy proportional gain (i.e., the IT2-FLC output), which is given in Fig. 8c, causes a smoother control signal which improves the system performance. As it can be clearly seen in Table 2, the type-2 fuzzy PI controller structure reduced settling time and overshoot, significantly. Moreover, the proposed IT2-FLC PI structure compensated very effectively the output disturbance in a short period of time compared to the PI structure.

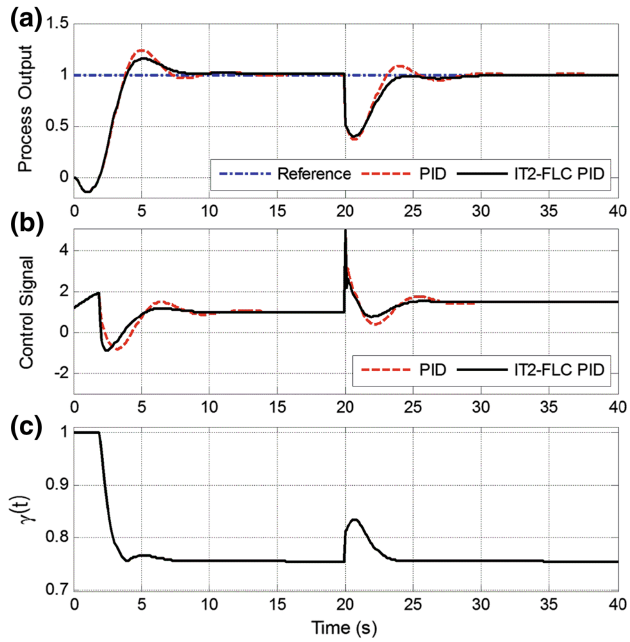

Fig. 7 Simulation results for System II: **a** the process outputs, **b** the control signals and **c** the variation of the interval type-2 fuzzy gain ($\gamma(t)$)

System IV:

Consider the nonlinear process given again by Mudi and Pal (1999) as

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 0.25y^2 = u(t) \tag{21}$$

Here, the controller parameters of the PID controller are optimized so as to minimize the IAE performance index via the BB–BC optimization algorithm (Erol and Eksin 2006).

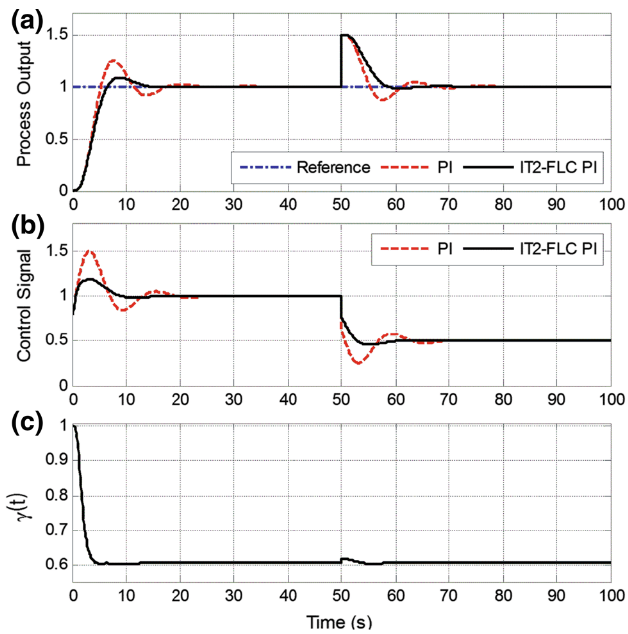

Fig. 8 Simulation results for System III: **a** the process outputs, **b** the control signals and **c** the variation of the interval type-2 fuzzy gain (γ(t))

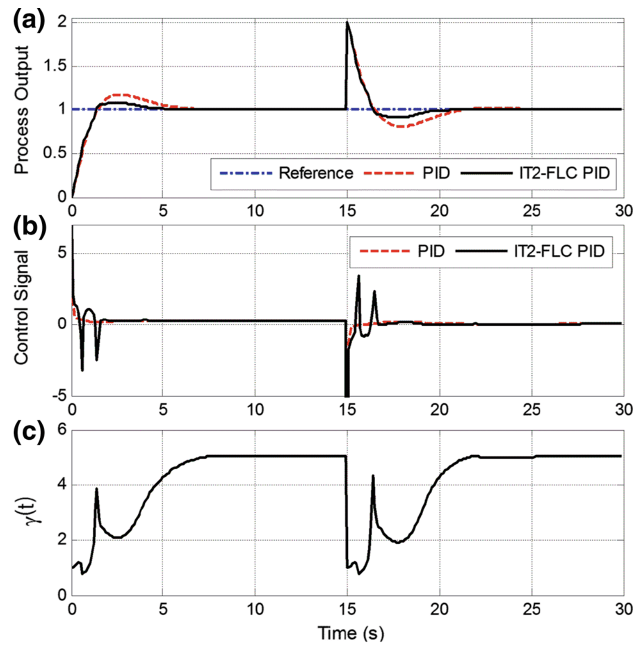

Fig. 9 Simulation results for System IV: **a** the process outputs, **b** the control signals and **c** the variation of the interval type-2 fuzzy gain (γ(t))

The optimum values of optimum parameters are found as $K_P = 2.5$, $K_I = 1.2$, $K_D = 1.8$. Here, a "five rule" IT2-FLC is chosen with the design parameters as $m_{-2} = 0.9$, $m_{-1} = 0.2$, $m_0 = 0.1$, $m_1 = 0.2$ and $m_2 = 0.9$ to have robust system performance against nonlinearities. The unit step output responses and the load disturbance rejection performances of the controllers are illustrated in Fig. 9 where the step output disturbance has been applied in 15th second with magnitude of "1".

As it is illustrated in Fig. 9, the IT2-FLC PID structure provides better transient state performances then the PID control structure. The type-2 fuzzy proportional gain action presented in Fig. 9c causes a relatively aggressive control signal which improves the system response as it converges to the reference value. The performance values of IT2-FLC PID structure are less than PID controller structure, as it can be clearly seen in Table 2. The IT2-FLC PID structure reduces the settling time to 3.5 s; it decreases the overshoot about 53 % and IAE value about 33 % and compensated very effectively the output disturbance in a short period of time when it is compared to the PID control structure.

6 Real-time control study

We evaluated the conventional and type-2 fuzzy PID control structures on an experimental neutralization process which inherit large amounts of nonlinearities and uncertainties caused by the internal dynamics and/or feedback sensors of the controlled system.

The experimental set-up handled in real-time control study is the G.U.N.T pH Value Control Trainer RT 552, which is illustrated in Fig.10 (G.U.N.T User Manual. http://www.gunt.de/networks/gunt/sites/s1/mmcontent/produktbilder/08055200/Datenblatt/08055200%202.pdf). This process has two input streams, one containing strong acid (HCl) and the other one strong base (NaOH). Since the pH process inherits severe nonlinearities and time varying behavior, it is a benchmark process to evaluate control performances (Kumbasar et al. 2011; Yesil et al. 2012). The G.U.N.T pH Value Control Trainer RT 552 experimental process can be described as the sodium hydroxide stream as input (F_b) and the pH as output while the acetic acid stream (F_a) is considered to be constant (Kumbasar et al. 2012). The parameters of the neutralization process are given in Table 3.

At first, in order to determine the parameters of the PID controller, the nonlinear pH neutralization process is represented by First Order Plus Dead Time (FOPDT) models that corresponds to

$$G(s) = \frac{Ke^{-Ls}}{\tau s + 1} \tag{22}$$

The model parameters are obtained based on the step response method (Astrom and Hagglund 2005) and are $K = 0.2$, $\tau = 200.75$ s, $L = 10$ s. Then, a conventional PID controller is designed based on the obtained FOPDT model. The PID parameters are tuned based on Internal Model Control (IMC) design procedure (Morari and Zafiriou 1989). In this structure, the PID inherits a low pass filter:

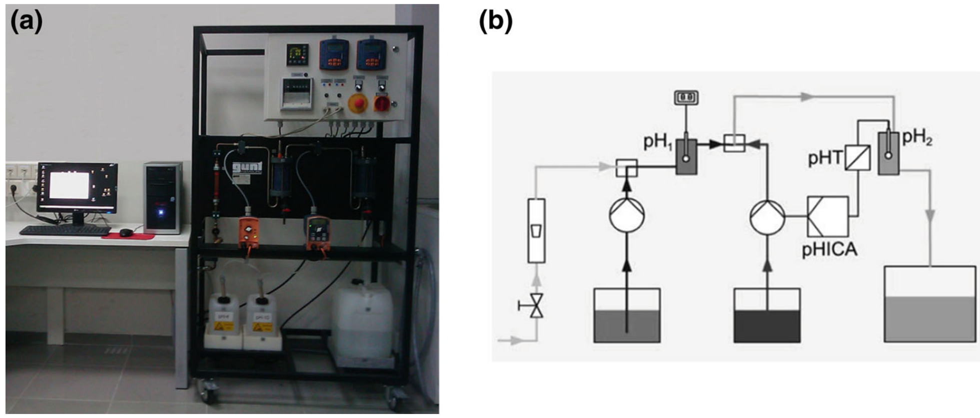

Fig. 10 **a** the general view of the G.U.N.T RT-552 pH process set **b** the schematic working diagram of the RT-552 pH process set

Table 3 Parameters of the pH neutralization process

Symbols	Description	Value
V	Volume of the CSTR	0.8 L
F_a	Flow rate of the influent stream	1 mL/s
F_b	Flow rate of the titrating stream	0–2.1 mL/s (0–100 %)
C_a	Concentration of the influent stream	6.3096 10^{-4}M
C_b	Concentration of the titrating stream	13 10^{-4}M

$$\mathrm{PID}(s) = \left(K_P + \frac{K_I}{s} + K_D s\right)\left(\frac{1}{1+\tau_f s}\right) \tag{23}$$

where controller parameters are defined as:

$$K_P = \frac{\tau + 0.5L}{K(\tau_c + L)}; \quad K_I = \frac{K_P}{\tau + 0.5L};$$
$$K_D = K_P\frac{0.5L\tau}{\tau + 0.5L}; \quad \tau_f = \frac{0.5L\tau_c}{\tau_c + L} \tag{24}$$

In this PID design procedure, the desired closed-loop time constantτ_c is chosen as 30 s so as to obtain moderate closed loop system response making comprise between IAE, overshoot and settling time. Then, the controller parameters are calculated as $K_P = 22.78$, $K_I = 0.111$, $K_D = 111.16$, $\tau_f = 3.75$ s.

In the real time control study, a "three rule" IT2-FLC is designed with the scaling factor $k_s = 0.29$. The design parameters are set as $m_{-1} = 0.2$, $m_0 = 0.9$, $m_1 = 0.2$ to have a robust closed loop control performance against parameter uncertainties, nonlinearities and measurement noises.

Similar to the simulation studies, both the PID and IT2-FLC PID controllers are implemented as the discrete-time versions obtained with the bilinear transform with the sampling time $T_s = 2$s (Kumbasar et al. 2012). The communication between the computer and experimental setup is established via MATLAB/OPC toolbox. It should be noted that the MATLAB/Simulink solver option is chosen as ode5

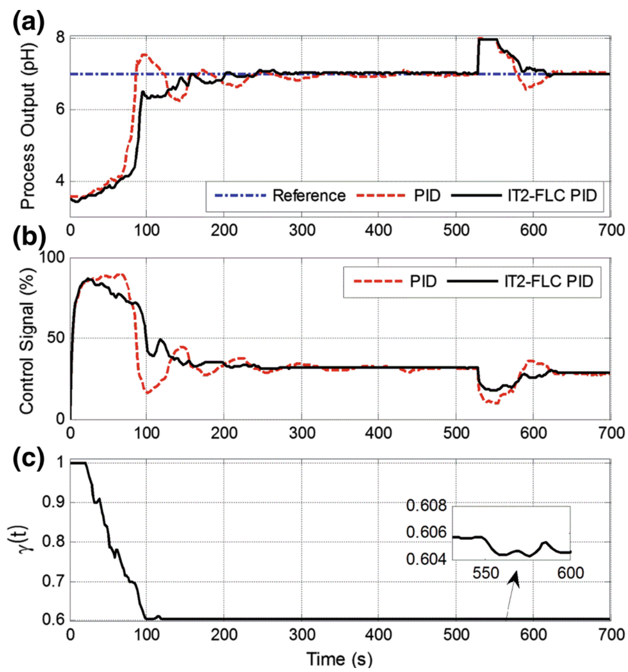

Fig. 11 Illustration of the **a** process outputs, **b** control signals and **c** variation of the interval type-2 fuzzy gain ($\gamma(t)$)

(Dormand-prince) and the step size is fixed at a value of 2 s throughout the real-time control studies.

In order to compare the control performance of PID and IT2-FLC PID, the reference value is chosen as 7 pH value since the pH process high sensitivity near the neutralization point (Fuente et al. 2006). Moreover, an additive step output disturbance is applied at this challenging operating point to observe the disturbance rejection performance of the control structures. The step output disturbance has been applied in 520th second with magnitude of "1".

The step input responses and the control signals of PID and IT2-FLC PID are illustrated in Fig. 11a, b, respectively. The variation of the interval type-2 fuzzy proportional gain

Table 4 Performance comparison of the implemented controllers

	T_s	OS	IAE
PID	158 s	9 %	187.83
IT2-FLC PID	126 s	0 %	202.07

is illustrated in Fig. 11c. It can be clearly seen that, the type-2 fuzzy proportional gain eliminates the oscillation in process output since a smooth nonlinear control curve is designed. The IT2-FLC output (i.e., $\gamma(t)$) reduces the feed-forward gain as the process output converges to desired reference value. The performance comparison of the controllers, according to the performance measures given in Sect. 3, is given in Table 4. The IT2-FLC PID structure reduces the overshoot to 0 and it decreases settling time by about 20 % when it is compared to the PID control structure. Moreover, the proposed type-2 fuzzy control structure compensated very effectively the output disturbance with the magnitude of "1". On the other hand, the total IAE performance value of the type-2 fuzzy PID control structure is relatively bigger than the conventional PID (around 7 %). However, it can be concluded that the IT2-FLC PID shows a better transient and disturbance rejection performance than its linear counterpart, particularly in handling nonlinearities and uncertainties.

7 Conclusions

In this study, a simple design method for single input interval type-2 fuzzy PID controllers has been proposed. The proposed interval type-2 fuzzy controller structure still keeps the most preferred features of the PID controller such as simplicity and easy design. The most important feature of the proposed IT2-FLC is its single input type-2 fuzzy internal structure which gives the opportunity to derive the output in a closed form in terms of the defined tuning parameters (m_i), i.e. the extra degrees of freedom provided by the antecedent interval type-2 fuzzy sets. Thus, the type-2 fuzzy controller output can be explicitly defined in the error domain based on the derived closed form. This simplifies the IT2-FLC design to a control curve generation, instead of a control curve generation. Hence, a process independent design method to form nonlinear control curves is presented and four curves are generated which are commonly used and preferred in nonlinear control theory.

The closed loop control performances of the proposed type-2 fuzzy control structure have been compared with conventional controllers on benchmark linear and nonlinear processes. The transient state and disturbance rejection performances of the implemented control structures are investigated and compared. It has been illustrated that the proposed interval type-2 fuzzy control structure ameliorates the transient state disturbance rejection performance of the process much better compared to the linear counterpart in both simulations and experiments.

Hence, the main contribution of the presented a type-2 fuzzy controller design method is to enhance the closed loop system performance via the extra degrees of freedom provided by the antecedent interval type-2 fuzzy sets. The presented analyses of the design method for the one input type-2 fuzzy inference will provide control engineers basic guidelines for tuning the single input type-2 fuzzy controller structures.

The proposed controller design method can be easily extended and generalized to IT2 FLCs with alternative interval type-2 fuzzy sets representing the input domain. However, it should be noted that the key principle of the proposed design methodology is "simplicity". For that reason, a simple rule base with triangular IT2-FSs is preferred. If a satisfactory closed loop control performance can be achieved via this simple structure, the use of alternative IT2-FSs; such as Gaussian, Sigmoidal, etc.; is not necessary and will obviously increase the computational cost.

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