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Towards a linear general type-2 fuzzy logic based approach for computing with words

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Abstract Within the last two decades, the paradigm of Computing With Words (CWW) has been gaining more attention. Mainly, CWW has an exciting vision which tries to tackle the problem of human intelligence by taking the human mind as a role model. The human intelligence has been investigated by various disciplines including psychology, philosophy, neuroscience, linguistics, computer science, and cognitive sciences. Notably, it is not a straightforward task to map the human's brain reasoning into computer processes. In this paper, we propose to facilitate such mapping by investigating a key element, which is to identify the step-by-step formation of perceptual judgments. Herein, we first introduce an approach that employs general type-2 fuzzy logic to dynamically model the human perceptions based on the human experience. This approach can be regarded as a step to enable the CWW vision. We have deployed the proposed approach in real-world settings and we will present two sets of real-world experiments which were conducted in the intelligent apartment (iSpace) in the University of Essex. The first set of experiments demonstrates the results of the proposed approach for the adaptive modeling of ambient luminance perception. In the second set of experiments, we show that our approach performs better in the rule base evaluation processing time and in output accuracy with comparison to an interval type-2 fuzzy logic system.

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1 Introduction

The mystery of human intelligence which encompasses complex reasoning, problem solving, decision-making and knowledge processing has been under the spotlight of scientists, philosophers and researchers for hundreds of years. With the advent of the information age, there has been a need to make the current systems, appliances, devices, and agents more intelligent, with the role model being the human mind. Supposedly, exploring the various aspects of human intelligence would lead to more natural communication in an everyday life of a human being with such systems, devices, appliances and agents. However, little is known about how the information is represented (Rangel et al. 2008), stored and processed in the human brain although this has been investigated from a range of disciplines including psychology, philosophy, neuroscience, linguistics, computer science, and cognitive sciences.

The paradigm of '*Computing with words*' (CWW), as coined by Zadeh (1996) in mid 1990s, can be regarded to have been established to focus on the problem of intelligence. The main inspiration of CWW has been identified to be the human ability to perform a wide variety of mental and physical tasks (Zadeh 2001), manipulate perceptions (Herrera et al. 2009; Mendel 2002), and comprehend natural language (Zadeh 1994) without the need for exact measurements and computation. As a matter of fact, all of these phenomena can be regarded as key points in a daily communication where human beings use their cognitive, perceptive and sensorial abilities in either conscious or unconscious manner. Correspondingly, there have been interpretations of computing with words as computing with *perceptions* (Mendel and Wu 2010). Within the literature, CWW is significantly associated with perceptions and mainly perceptual judgments (Mendel and Wu 2010), which can be considered as linguistic decisions (Herrera and Herrera-Viedma 2000) represented by 'words' in natural language. According to Mendel (Mendel 1999), 'words mean different things to different people' and therefore type-2 fuzzy logic should be employed in modeling words to cope with the linguistic uncertainty. Notably, type-1 fuzzy sets only offer limited scope for modeling uncertainty and hence they cannot handle high levels of uncertainty which are usually present in real world applications. For this reason, interval type-2 fuzzy logic has been extensively used in a wide range of applications including (Zarandi and Gamasaee 2012; Acampora et al. 2012) where interval type-2 is shown to perform better than its type-1 counterparts. In this paper, we will employ general type-2 fuzzy logic for representing words as we believe general type-2 fuzzy sets bear greater potential to model the linguistic uncertainty.

In order to shed some light on the operation of rather sophisticated human intelligence, we need to identify the step-by-step formation of perceptual judgments, which we consider as an important element to contribute to the CWW paradigm and to facilitate the mapping of the human's brain mechanism into computer processes. At this point, it is necessary to note that we will use the phrase 'perceptual judgment' interchangeably with 'perception', even though there is a considerable difference in the meanings that they convey. Yet, regarding the CWW paradigm, it is more appropriate to mention perceptual judgments as we will refer to them as the natural language representations of perceptions. Moreover, it is important to acknowledge that current CWW methodologies must be applied within the framework of existing computer architectures, all of which compute with numbers (Mendel 2002). In the same sense, we are proposing our conception for CWW using existing computer architectures.

The main objective of this particular work is to tackle the problem of intelligence while narrowing it down to the interpretation of human perceptual judgment within a specific application. To achieve that, we place our focus on the input–output relationship and employ backwards thinking for identifying and in turn modeling the human perception. Then, we apply the concept of granulation on the output linguistic propositions. Following this further, we propose to use a general type-2 fuzzy logic approach to address the problem of modeling perceptions, which are dynamically adapted depending on the human experience and represented using general type-2 fuzzy sets. Once the perception models are ready to use, we deploy them in a real-world environment with lay users. We also compare the proposed models with general type-2 fuzzy logic approach to previously employed models with interval type-2 fuzzy logic approach.

The paper is structured as follows: Sect. 2 will briefly identify the problem and the concepts that are related to the exploitation of the human ability to learn, to generalize and to judge. We will introduce the proposed architecture for CWW along with a framework for modeling human perceptual judgment in Sect. 3. Also, we will present a brief summary of the theory regarding the Linear General Type-2 (LGT2) fuzzy sets that will be employed for modeling human perceptions. Section 4 will present the experiments and results of modeling adaptive perceptual judgments and will also present the comparison of two fuzzy logic systems using different type-2 fuzzy logic approaches. The conclusions and future work are discussed in Sect. 5.

2 Decomposition of human intelligence and human judgment

Conceptually, the aim of this research is to mimic the exquisite human capability in performing a variety of daily tasks using perceptual judgments. In general, we believe that the investigation of human intelligence will provide the means to represent human compatible skills in computers, agents and most subservient subsystems (Roy 2000) such as cars, airplanes, together with man-made devices which are operated by humans. By practical terms, the way we foresee the machines to mimic human-like intelligence begins with the investigation of the human perceptual judgment. It is important to highlight that what we try to address (with the human perceptual judgment) is the moment when people make up their minds and form interpretations of the attended phenomena in a natural language. As put forward by Zadeh (1994), the use of linguistic values may be viewed as a form of data compression, also referred as granulation that aids mimicking the way in which humans interpret linguistic values. Furthermore, the fuzzy information granulation employed by humans provides the basis for CWW and can be viewed to be a mode of generalization (Zadeh 1997).

Hence, the course of human perceptual judgment is central to the process of making the information granular (or generalized), and vice versa. As pointed out by Pinker (1999), in order to bridge the computational theory of the mind and psychology, it is essential to discover the form of mental representations, and the processes that access them. In fact, the information encapsulated in an internal/mental representation is all that we can know about the world. However, little is known about how the mental representation of the objects, concepts, thoughts are constructed in the human brain and therefore, little is known about how humans generalize the sensory stimulus information. For this reason, unlike the traditional methods that formalize the cognition processes through the reception of the stimuli information to the determination of the environment state, we propose to take a reverse engineering approach. The viewpoint of backwards thinking helps to have a better understanding of the human perceptual judgment and is inspired from the interchange of the direction of inference in rule-based systems. By doing this, we believe that we can model the human perceptions in a much more realistic way.

Related to the problem of intelligence is the problem of perception, which is based on a hypothesis or a conclusion, used to interpret stimuli reaching us (Coren et al. 2004). It follows that, the nature of perception involves the imprecision and uncertainty present in both intra-user and inter-user information processing. The reason is that neither our sensors nor the hypothesis or conclusion we reach is exactly the same for everyone. Thus, the problem of perception contains a sort of experience granulation and people agree approximately rather than exactly. Furthermore, perception is not mere reception of a sensory stimulus, but a particular way of experiencing and organizing the stimulus (Geert 1983), by calling on stores of memory data and requiring classification, comparisons and myriad decisions (Coren et al. 2004). Therefore, it can be deduced that there is a need for memory in order to analyze the perceptions, in other words, our conscious experience. The last but not the least; perceptual judgment is a complicated process that has several subprocesses, including the final step of a discrete choice among available possibilities (Yang 2008). However, there is not enough evidence regarding how we form a mental representation of the outside environment or choose a possibility as a perceptual judgment. Hence, we propose a backwards thinking methodology starting from what we know as final actions or decisions to the potential perceptual judgments as a result of our granulation of conscious experience.

The need for memory (as mentioned above) is supported not only by psychology or psycholinguistics (Aitchison 2003; Field 2003), but also by neuroscience. As raised by Heekeren et al. (2008), one of the fundamental processes in the making of a perceptual decision is the contribution of memory. Our past sensory experiences, which are stored in memory and brought online in working memory, are combined with current sensory inputs to inform our perceptual decisions. Comparison of accumulated sensory evidence is a mechanism for perceptual decision-making (Heekeren et al. 2008). Moreover, human perceptual decision-making is influenced not only by the sensory information at hand, but also by factors such as attention, task difficulty, the prior probability of the occurrence of an event and the outcome of the decision (Heekeren et al. 2008). The neural architecture for perceptual decision-making can be viewed as a system that consists of four distinct but interacting processing modules (Heekeren et al. 2008). According to Heekeren et al. (2008), the first of these modules accumulates and compares sensory evidence; the second detects perceptual uncertainty or difficulty and signals when more attentional resources are required to process a task accurately; the third represents decision variables and includes motor and premotor structures; and the fourth is involved in performance monitoring, which detects when errors occur and when decision strategies need to be adjusted to maximize performance (Heekeren et al. 2008).

Mendel et al. (2010) stresses that CWW is not a replacement for traditional systems of computation, rather it is an addition. As was noted before, what CWW adds is the important capability to compute with information described in a natural language. Pursuing this further, we propose to include a new module standing for the memory in the ideal general framework of CWW where the human experience has a binding impact on the human perception. Clearly, experience is represented within the memory and has an effect on human capability of performing a wide variety of physical and mental tasks without exact measurements and computations. Besides, the memory has its own partitions such as long-term memory, short-term memory or working memory (Pinker 1999; Aitchison 2003; Field 2003). However, the analysis of the dimensions of memory is out of scope for this paper and initially, we hypothesize a simplified memory usage. In this paper, we will introduce the conception of a general-use memory, which we think is important to include in such systems.

Another angle and application area of CWW is decision-making, which bears substantial importance as it is a kind of perceptual judgment process that extends on time and resources. In other words, perceptual judgments as a consequence of our perceptions and way of reasoning can be viewed as the preliminary steps that will lead to making up our minds and giving a decision. According to Rangel et al. (2008), value-based decision making is pervasive in nature. In the same sense, human perceptual judgment as a subprocess of decision-making is supposed to be pervasive when dealing with values such as stimuli information. Also, we believe that the perceptual judgment is a matter of deciding between two opposite sides (e.g. dark or bright, true or false) of one category, in a fuzzy manner, that is having partial memberships rather than exact categorization. On the contrary, we believe that decision-making might involve more categories to choose from, not necessarily just two.

Admittedly, making decisions is one of the most basic tasks that human beings perform daily. As described by Gold and Shadlen (2007), a decision is a deliberative process that results in the commitment to a categorical proposition. Likewise, the linguistic propositions, which can be regarded as the heart of the fuzzy logic theory, are closely related to the judgments that humans make. In addition, the information granulation (Zadeh 1997, 2001; Herrera et al. 2009; in other terms generalization) prepares the grounds for such judgmental processes and hence, they can be communicated in an approximate manner. In this paper, we will take the perspective of human perceptual judgment as a subset of decision-making from the beginning, which is how the stimuli information is mapped into perceptions in the human mind, as we believe it is the initiative of the CWW paradigm.

In order to have a better understanding of how to model human perception, we need to consider the basics such as the environment, and the sensory apparatus. To put things into practice, we will pose an analogy between the human and the embedded agent in an intelligent environment regarding the information processing. When we are doing this, we have to make a couple of reasonable assumptions, which we will justify later. In the next section, we present the proposed CWW architecture with the additional abstract memory module that we think will bridge the gap of experience between the humans and the machines.

3 The proposed CWW architecture and linear general type-2 (LGT2) fuzzy sets

3.1 The proposed CWW architecture

In a broad sense, the proposed CWW framework, which will accomplish a human-like reasoning, needs to integrate memory for learning and adaptation purposes and experience folding. As illustrated in Fig. 1, the abstract memory module (Element 4 in Fig. 1) is foreseen to have a double-way information flow while forming an internal representation (arrow A in Fig. 1), reasoning within CWW Engine (arrow B in Fig. 1) and producing the final actions or words (arrow C in Fig. 1) that are specialized according to the system/application requirements. It is worthwhile to mention that these information flows have been drawn with the inspiration from the neural basis of human perceptual decision-making as described in (Heekeren et al. 2008). However, we will not go into the details of each particular module in this paper, since the center of scope is the step-by-step formation of human perceptual judgment and how we can mimic this in machine processes. Therefore, we will briefly bring the steps together which are illustrated in Fig. 1 and consider them in a nutshell for the reason that they support our line of research.

Initially, we need to take into account the sensory information (Element 1 in Fig. 1) from the environment, which is available through our various sensors such as the eyes, the ears, the skin, etc. Depending on the task that we are performing, we attend to one or more of our senses. In other words, we have a sort of attention filter (Element 2 in Fig. 1) that we use to focus on the bits of information that we think is necessary for that specific task. Upon reception of the attended sensory input, there is the input-judgment mapping (Element 3 in Fig. 1) using the internal representations within the human brain. However, we do not have sufficient scientific evidence regarding how these representations are created, stored or processed. Besides, we cannot know the outcome of this step, which can be referred as the cognitive result of such mapping, unless it is expressed in a natural language.

After this level, we cannot proceed with forward thinking, as there is a gap between the stimuli information (Element 1 in Fig. 1) and the perceptual judgment (Element 5 in Fig. 1), which is ideally represented in a natural language as words or phrases. On the other hand, using traditional reasoning or decision-making systems, we can have access to the inputs and outputs as well as the rules of the system especially when a human operator is involved for control purposes. The underlying architecture for such systems is based on a Fuzzy Logic System (FLS), which is illustrated in Fig. 2b. As an analogy, it can be noticed that the abstract CWW Engine conception consists of the

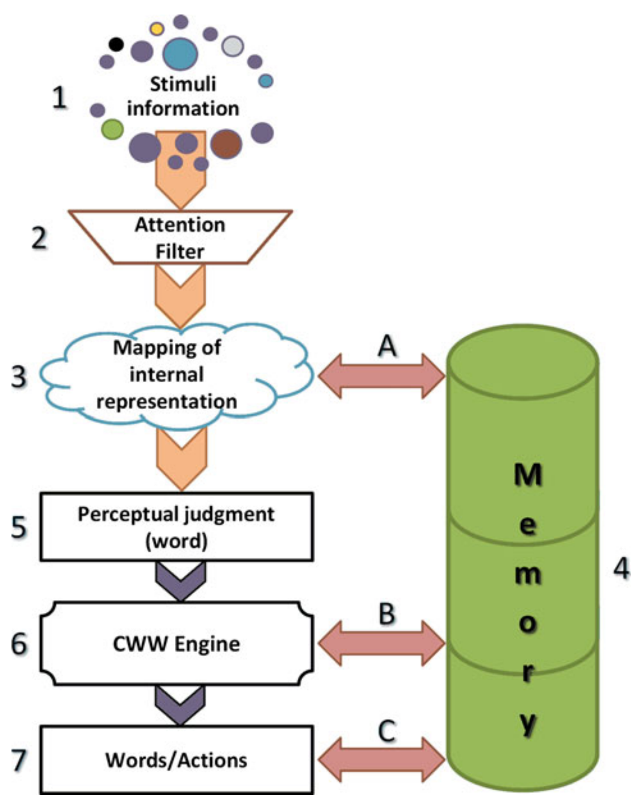

Fig. 1 Proposed CWW architecture

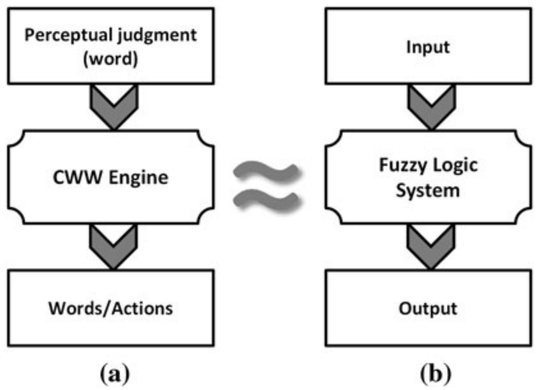

Fig. 2 Analogy between **a** CWW Engine and **b** FLS

traditional FLS from the point where the input is taken (perceptual judgment), and then processed (CWW Engine) before an output is given. We use the label 'CWW Engine' to identify the heart of the overall CWW conception, which consists of the modules such as the abstract counterparts of the ones in an FLS i.e. rule base, inference engine and defuzzifier. This hypothesis is also backed with the argument of Zadeh (2010) where he puts forward that CWW should be an addition to the traditional systems of computation. The analogy is visualized in Fig. 2.

Consequently, we consider a backwards thinking, that is, a reverse engineering viewpoint in order to model the human perceptual judgment, which is communicated in words. Pursuing this, we propose that what CWW conception needs to embody is similar to the traditional way of input–output mapping that uses a rule base for reasoning. Also, we will apply it on a real-world system (a reading application where we have access to the inputs and outputs as well as the rules of the system).

3.2 Dedicated CWW architecture and the backwards thinking approach

We have developed an application where the human perception of the ambient light level (luminance) is central to decide on the level of reading light levels in an intelligent environment. The important thing to note is that the application uses a rule based system, markedly a type-2 FLS similar to Bilgin et al. (2012a, b), to control the ceiling lights. The apparatus we use to sense the ambient light level, which is analogous to the human eye, is a collection of light sensors that can be referred to as the eyes of the intelligent embedded agent. The purpose is to model the human perception of the indoor ambient light levels, which is in turn used to decide on the ceiling light levels.

Figure 3 illustrates the process of backward thinking to ultimately have an insight on what words represent as perceptions (Element 3 in Fig. 3) in the form of light sensor values (Element 1 in Fig. 3) for this specific case. The usage of memory is represented in a module (Element 2 in Fig. 3) dedicated for accumulation and comparison of sensory evidence as supported by the literature (Pinker 1999; Aitchison 2003; Field 2003; Heekeren et al. 2008). The aftermath of this process in the human brain is a perceptual judgment (depicted in the rectangle H in Fig. 3) but there is not enough evidence for us to hypothesize it in machine processes. For this reason, we opt to use the traditional FLS (depicted in rectangle M in Fig. 3) to our advantage as follows. We have access to the real-world output of the user preference in a numeric format (Element 5 in Fig. 3). This numeric information is indeed the defuzzified collective output of the FLS. After defuzzification, we fuzzify the output numeric value to get the stimulated output linguistic labels using the same output fuzzy sets designed by the expert of the FLS. We make the assumption that, this is an adapted preference of the user, in other words, the user is content with the output light level and there is no more interaction with the system through the user interface. Hence, without looking at the collective fuzzy consequents coming from the rule base, we treat this fuzzified numeric output value as the word output of CWW Architecture.

From there, we can make a granulation using the output linguistic labels and input light sensor values attached to each and every output using the experience, which is accumulated in the memory (Element 2 in Fig. 3). First, we categorize all the input light sensor readings as per output linguistic label and we obtain a range of crisp input values for the same output linguistic label. What we do with these values can be referred to as a (reverse) granulation, as we will use the output label as a basis to interpret the human input light level perception. Table 1 illustrates the structure of the data as experience, which is kept in the memory module for accumulation and comparison. The 'Light Sensor Value' column shows the light sensor reading x_i, where $i = 1 \ldots Q$ and Q is the number of unique light sensor readings. It is important to note that x_i is an aggregation (specifically an average) of 3 individual light sensors located in various parts of the intelligent environment. The second column, named as 'Count of Light Sensor Value' has a dedicated sub-column for each and every output linguistic label $Label_j$, where $j = 1 \ldots M$ and M is the number of output linguistic labels. Last, k_{ji} is the corresponding number of occurrences of the light sensor reading x_i per output linguistic label $Labelj$.

Before proceeding to the details of the modeling process, it is important to refer to the theoretical background which can be found in Bilgin et al. (2012a, b) as we propose to model the interpretations of the human perceptual judgment using LGT2 fuzzy sets which will be presented in the following subsection.

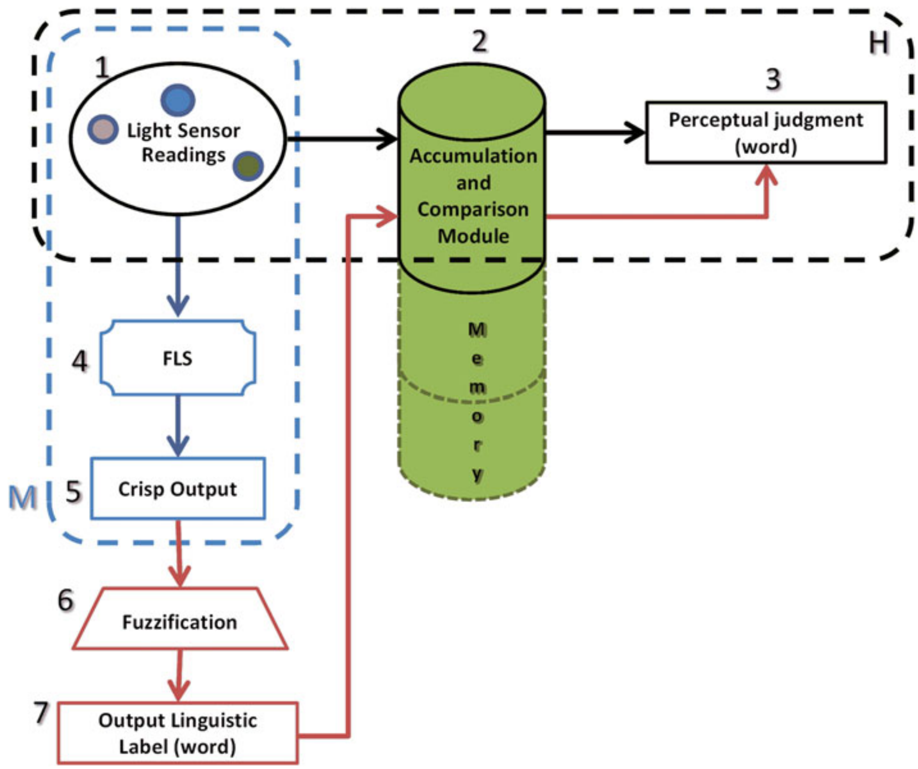

Fig. 3 Specialized CWW architecture focusing on the backwards thinking viewpoint to model the human perceptual judgment

Table 1 Structure of the data as experience in memory module for accumulation and comparison

Light sensor value	Count of light sensor value				
	$Label_1$	$Label_2$	.	.	$Label_j$
x_1					
x_2			k_{ji}		
.					
x_i					

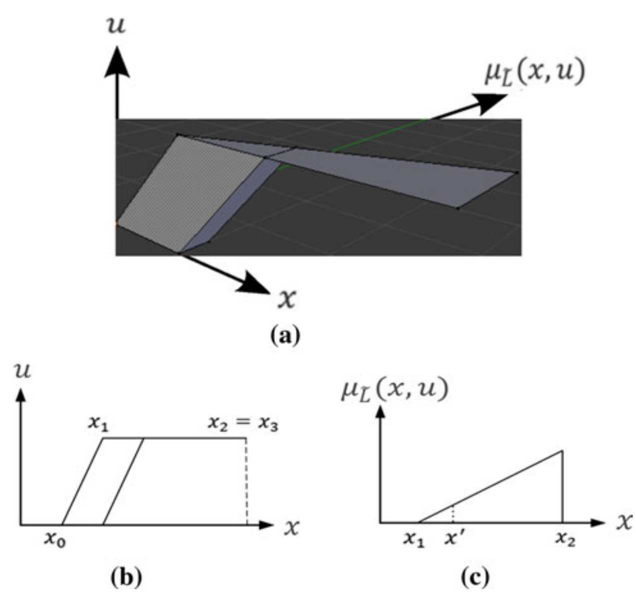

Fig. 4 Visualization of an LGT2 fuzzy set. **a** 3D view, **b** front view, **c** top view

3.3 Background on Linear General Type-2 (LGT2) fuzzy sets

In this subsection, we present theoretical background for LGT2 fuzzy sets, which we have introduced recently in Bilgin et al. (2012a, b). We will illustrate the prototype of the LGT2 fuzzy set which represents a right shoulder trapezoidal fuzzy membership function, namely $\tilde{L}$. Some common perspectives to visualize the LGT2 are illustrated in Fig. 4. The front view as seen in Fig. 4b is similar to the front view of an interval type-2 fuzzy set. However, the difference is in the third dimension, which can be visualized in Fig. 4a, c. In fact, for an interval type-2 set, the secondary degree always equals to 1 whereas for a general type-2 set it ranges in the interval [0, 1].

As shown in Fig. 4c, the top view of the prototype projected on x–$\mu_{\tilde{L}}(x,u)$ plane is a right triangle. In particular, it will be a right edged-right triangle for a right shoulder membership function, whereas it will be a left edged-right triangle for a left shoulder membership function. In order to find the secondary degree of any x value, we have to distinguish a singleton and a non-singleton case.

3.3.1 Singleton case

For a linear general type-2 fuzzy set $\tilde{L}$, the fuzzification of a singleton input $x = x'$ yields a vertical slice. Depending on the support of the upper membership function and the support of the lower membership function, we distinguish between the circumstances where $\bar{\mu}_{\tilde{L}}(x') = \underline{\mu}_{\tilde{L}}(x')$ and $\bar{\mu}_{\tilde{L}}(x') \neq \underline{\mu}_{\tilde{L}}(x')$.

For the first condition ($\bar{\mu}_{\tilde{L}}(x') = \underline{\mu}_{\tilde{L}}(x')$), in order to calculate the secondary degree $\mu_{\tilde{L}}(x', u)$, we make use of the concept of similarity of triangles. Equation (1) expresses the formulation:

$$\mu_{\tilde{L}}(x', u) = \frac{x' - x_1}{x_2 - x_1} \tag{1}$$

where $[x_0, x_1, x_2, x_3]$ represent the parameters of the trapezoidal type-1 upper membership function as depicted in Fig. 4b. Note that, in this case, the secondary degree is the amplitude on the u-$\mu_{\tilde{L}}(x, u)$ plane which can be interpreted as the length of a vertical line as shown in Fig. 5a.

The second condition $\bar{\mu}_{\tilde{L}}(x') \neq \underline{\mu}_{\tilde{L}}(x')$ requires that the vertical slice is a 2-D shape of either a triangle or a trapezoid as illustrated in Fig. 5b, c. In this case, we calculate the center of gravity, also known as the centroid, of the vertical slice. Hence, the secondary degree is found by the centroid calculation.

$$\mu_{\tilde{L}}(x', u) = f_{x'}^{cg}(\tilde{L}) = \frac{\sum_{k=1}^{M} u_k * \mu_{\tilde{L}}(x', u_k)}{\sum_{k=1}^{M} u_k} \tag{2}$$

where $u_k \in [\underline{\mu}_{\tilde{L}}(x'),\ \bar{\mu}_{\tilde{L}}(x')]$, M is the number of discretization points.

3.3.2 Non-Singleton case

As stated in Bilgin et al. (2012a, b), modeling a word involves handling of the uncertainty within perceptions which is why non-singleton input is comparatively more realistic for modeling a word. Herein, we present the theoretical aspects of using the LGT2 approach. For a linear general type-2 fuzzy set $\tilde{L}$, the fuzzification of a type-1 non-singleton input $x = x'_{ns}$ yields a tilted slice. Regardless of the shape of the type-1 fuzzy input membership function X, by using sup-star composition with minimum inference on X and each of the type-1 upper $\overline{FOU(\tilde{L})}$ and lower $\underline{FOU(\tilde{L})}$ membership functions of the LGT2 fuzzy set, we find two distinct points P_u and P_l, respectively, on x–u plane as follows:

$$\bar{x}_{k,max} = \sup_x \int_{x_k \in X} \left[\mu_X(x_k) \star \mu_{\overline{FOU(\tilde{L})}}(x_k)\right] \Big/ x \tag{3}$$

$$P_u(x_u, u_u) = \left(\bar{x}_{k,max}, \mu_{\overline{FOU(\tilde{L})}}(\bar{x}_{k,max})\right) \tag{4}$$

where k is the discretization index and $k = 1\ldots N$. If we let the value of x_k at which the supremum occurs be $\bar{x}_{k,max}$, then $x_u = \bar{x}_{k,max}$ and u_u follows from the right hand side of the parentheses in Eq. (4). Likewise, P_l can be written as:

$$\bar{x}_{k,max} = \sup_x \int_{x_k \in X} \left[\mu_X(x_k) \star \mu_{\underline{FOU(\tilde{L})}}(x_k)\right] \Big/ x \tag{5}$$

$$P_l(x_l, u_l) = \left(\underline{x}_{k,max}, \mu_{\underline{FOU(\tilde{L})}}(\underline{x}_{k,max})\right) \tag{6}$$

Similar to the singleton case, we distinguish between the circumstances where $P_u = P_l$ and $P_u \neq P_l$. For the first condition, in order to calculate the secondary degree $\mu_{\tilde{L}}(x', u')$ where $x' = x_u = x_l$ and $u' = u_u = u_l$ (Singleton case) we make use of the concept of similarity of triangles and the formulation is the same as in Eq. (1). However, for the second condition $P_u \neq P_l$, we need to modify the centroid calculation to find the secondary degree $\mu_{\tilde{L}}(x'_{ns}, u)$, as this case requires traversing on both x axis and u axis, whereas we used to traverse only on u. axis in the singleton fuzzification case.

In order to find the corresponding $\mu_{\tilde{L}}(x_p, u_p)$ for a point P, which is along the line C defined by the two points P_u and P_1, we make use of the distance between two points and the slope m where $m = (u_l - u_u)/(x_l - x_u)$. Hence, the centroid of a tilted slice, which is in turn the secondary degree of the non-singleton input x'_{ns}, is calculated as follows:

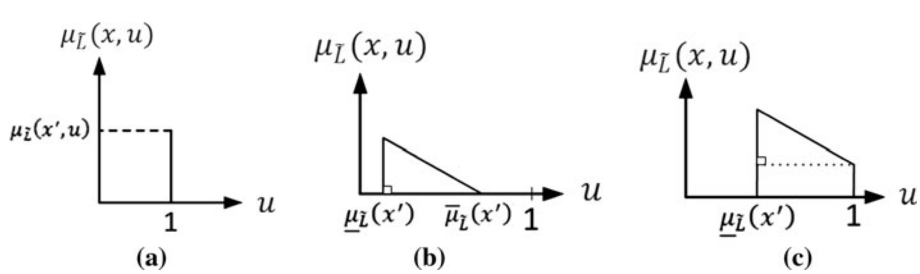

Fig. 5 Section of the vertical slice showing **a** the secondary degree $\mu_{\tilde{L}}(x', u)$ for the condition $\bar{\mu}_{\tilde{L}}(x') = \underline{\mu}_{\tilde{L}}(x')$, **b** the 2-D shape of $\mu_{\tilde{L}}(x', u)$ for the condition $\bar{\mu}_{\tilde{L}}(x') \neq \underline{\mu}_{\tilde{L}}(x')$—triangle when $\bar{\mu}_{\tilde{L}}(x') < 1$, **c** the 2-D shape of $\mu_{\tilde{L}}(x', u)$ for the condition $\bar{\mu}_{\tilde{L}}(x') \neq \underline{\mu}_{\tilde{L}}(x')$—trapezoid when $\bar{\mu}_{\tilde{L}}(x') = 1$

$$\mu_{\tilde{L}}(x'_{ns}, u) = f^{cg}_{x'_{ns}}(\tilde{L}) = \frac{\sum_{k=1}^{M} u_k * \mu_{\tilde{L}}(x_p, u_p)}{\sum_{k=1}^{M} u_k} \quad (7)$$

where k is the discretization index over the u axis, and $u_p = u_k$. Note that we need to calculate the value for x_p using the line equation as follows:

$$x_p = x_l + (u_p - u_l) * m \quad (8)$$

3.4 The modeling process

In the modeling process, we will exploit the efficiency of LGT2 fuzzy sets and show that they provide a better granulation for the user perceptual judgment which can be referred to be a decision between two opposite sides. LGT2 fuzzy sets provide the means to model such opposition with their simple and compact design, adding more functionality to the system point of view and offering a human-interpretable perspective of modeling perception. In the light of the scientific evidence, we will encourage the use of the two opposite sides for one category. For instance, for the illumination level, we will granulate the two opposite sides as 'dark' and 'bright' for the interpretation of the human light stimuli. Moreover, the mapping of the output linguistic labels (which can be more than just two in number) to these two perceptual judgments will easily be handled by LGT2 fuzzy sets using their characteristic feature for embedding the qualifiers in the third dimension.

In order to interpret the human ambient luminance perception, we make an assumption on the input–output relationship, which we extract from the data where we found out that the input and output relationship yields an inversely proportional figure. Then, we granulate the range of x_i to be the interpretation of its corresponding $Label_j$ in an inverted manner. For example, we hypothesize that when the level of output ceiling lights was *high*, the user's illumination stimuli information must have been *low* (and this triggered the human to switch the light to a high level). In fact, we argue that the user's perceptual judgment for the ambient light level must have been the opposite of the controlled ceiling light level. It is worthwhile to note that this hypothesis is based on the input–output relationship and is application dependent. The formulation for the interpretation of the input stimuli which is being sensed is shown in Eq. (9).

$$StimuliGranule_{M-j+1} = Label_j \quad (9)$$

where $j = 1\ldots M$ and M is the number of output linguistic labels (M is also the number of input stimuli information granules). We will refer to *StimuliGranule* as the representation of what has been sensed in response to an output linguistic label. For example, for a reading application, sensed illumination level can be a stimuli granule and if there are five output linguistic labels ($M = 5$, i.e. very low, low, medium, high, very high), then the *very high* output light level (corresponding to $j = 5$) must have been stimulated by a *very low* illumination stimuli granule (corresponding to $M - j + 1 = 1$).

As a next step, the range of the light input stimuli that we have accumulated in the memory as an experience is analyzed according to the number of occurrences for each light sensor reading. According to Laming (2004), sensory experience in general is characterized by self-adjustment to the *prevailing* level of stimulation. Using this information, we hypothesize that the prevailing level of stimulation can be extracted from the experience by finding the most frequent light sensor values associated with each output label. Then, we propose to adopt these most prevailing values to be the *base transition points* that will mark the support of the fuzzy sets (as shown in Fig. 6) and in turn will facilitate the placement of the shoulder membership functions in an adaptive manner. Consequently, the most crucial step in our proposed framework is to obtain the most frequent values as the base transition points for each of the stimuli granules which we will later generalize into perceptions or perceptual judgments. This step can be formalized as choosing x_i having $max(k_{ji})$ for each $StimuliGranule_{M-j+1}$.

$$P\_Granule(j) = x_{i(max)}X\text{ , where } i_{max} \text{ is found from } max(k_{ji}) \quad (10)$$

After having extracted $x_{i(max)}$ points, we position them on the horizontal x-axis as shown in Fig. 4, where 'minimum' is $min(x_i)$ and 'maximum' is $max(x_i)$ present in the experience over the course of the life-cycle. Moreover, $U(s)$ indicates the start and $U(e)$ indicates the end of the universe of discourse for x. By nature, the base transition points (denoted as $P\_Granule(\#)$) are ordered in line with their meaning. For example, the lower values

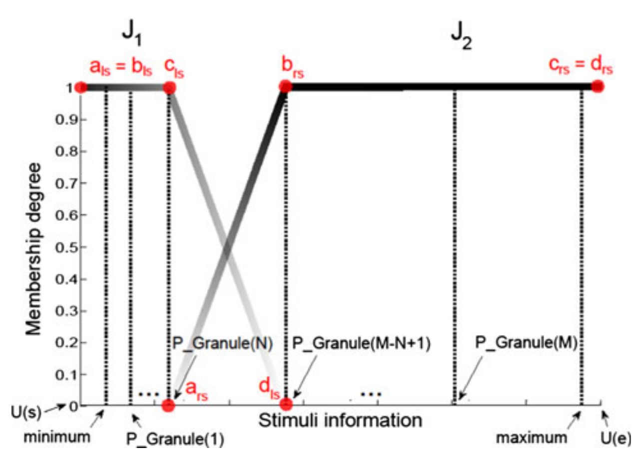

Fig. 6 The structural mapping of the base transition points $P\_Granule(\#)$ for each granule to judgment J_c where $c = 1, 2$ and the positioning of the LGT2 fuzzy sets

indicate the *darker* perception and the higher ones indicate the *brighter* perception. Another crucial step in the mapping process is to find the stimuli granules that belong to one of the two opposite sides with a primary membership degree of '1'. According to the theory behind the LGT2 sets, we use two shoulder type primary membership functions and position them as seen in Fig. 6 where the identification parameters of the membership functions are marked with dots and are labeled as $[a_{ls}, b_{ls}, c_{ls}, d_{ls}]$ for the left shoulder and $[a_{rs}, b_{rs}, c_{rs}, d_{rs}]$ for the right shoulder. The mapping of the parameters of the left shoulder membership function to the stimuli granules is formulated as follows:

$$a_{ls} = b_{ls} = U(s) \tag{11}$$

$$c_{ls} = P\_Granule(N) \tag{12}$$

$$d_{ls} = P\_Granule(M - N + 1) \tag{13}$$

where $N = \frac{M-1}{2}$ for odd M, and $N = \frac{M}{2}$ otherwise. Moreover, the mapping of the parameters of the right shoulder membership function to the stimuli granules is as follows:

$$a_{rs} = P\_Granule(N) \tag{14}$$

$$b_{rs} = P\_Granule(M - N + 1) \tag{15}$$

$$c_{rs} = d_{rs} = U(e) \tag{16}$$

Also, the following equations are important in positioning of the both of the membership functions:

$$c_{ls} = a_{rs} \tag{17}$$

$$d_{ls} = b_{rs} \tag{18}$$

In the same way as the primary membership functions, we continue with the structural mapping of the third dimension showing the secondary membership functions for the LGT2 fuzzy sets. In theory (Bilgin et al. 2012a, b), the secondary memberships will be linear functions in the third dimension. Figure 7 demonstrates the right triangle shaped secondary membership functions for both of the opposing sides belonging to one category, e.g. dark and bright for the ambient light level. In Fig. 7, each triangle represents the quantification of the third dimension for the corresponding perceptual judgment J_c where $c = 1, 2$. The labels for the input linguistic modifiers (e.g. 'very', 'fairly', etc.) can be adopted from the output linguistic propositions and be manifold with regards to the application requirements. Also, it is possible to have no modifier (*no_modifier*) which is also embedded in the model as seen in Fig. 7.

According to the number of output linguistic labels, M, where $M > 2$, the number of granules to be quantified in the third dimension of LGT2 will be $\left(\frac{M-1}{2}\right) + 1$ (at most) for odd M (there is a 'medium' label e.g. [very low, low,

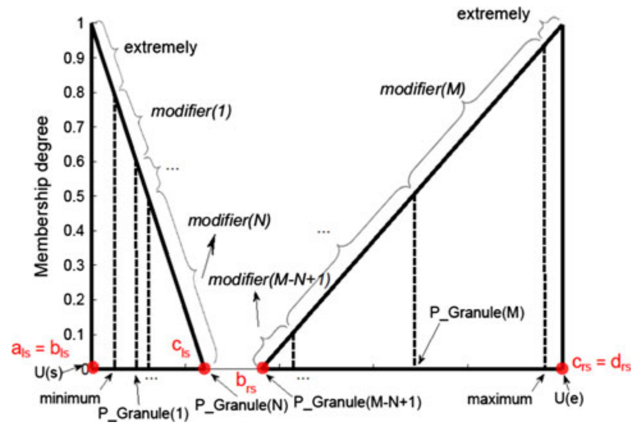

Fig. 7 The structural mapping of the transition points for the quantification of the third dimension

medium, high, very high]), where '+1' corresponds to an additional label for the granulation of the 'extreme' cases. For example, if an input sensor value that is smaller than the minimum value is stimulated, then the modifier 'extreme' can very well represent the perception of this stimuli information. Similar applies for the values larger than the maximum value. On the other hand, for even M (there is no label for 'medium' e.g. [very low, low, high, very high]), the maximum number of granules to be quantified in the third dimension of LGT2 will be $\left(\frac{M}{2}\right) + 1$. It is important to note that this is an assumption depending on the sequencing pattern of the output linguistic labels.

Pursuing this direction, we have two more steps till we draw the model that will incorporate both inter-user and intra-user uncertainties. First, we obtain LGT2 fuzzy sets for each individual user which have no uncertainty on the primary membership [i.e. zero Footprint Of Uncertainty (FOU)], and second, we aggregate various LGT2 fuzzy sets (for the individual users) which form a primary FOU as shaded in Fig. 8 to represent the inter-user and intra-user uncertainties.

Figure 8 demonstrates the aggregation of the left shoulder membership functions (MFs) for individual LGT2 fuzzy sets.

As can be seen, Eq. (11) has been applied in Fig. 8. Furthermore, the parameters of the aggregated FOU for the left shoulder MFs are calculated as follows:

$$c_{umf} = max\left(P\_Granule(N)_l\right) \tag{19}$$

$$d_{umf} = max\left(P\_Granule(M - N + 1)_l\right) \tag{20}$$

$$c_{lmf} = min\left(P\_Granule(N)_l\right) \tag{21}$$

$$d_{lmf} = min\left(P\_Granule(M - N + 1)_l\right) \tag{22}$$

where $l = 1 \ldots L$ and L is the number of LGT2 sets to be aggregated.

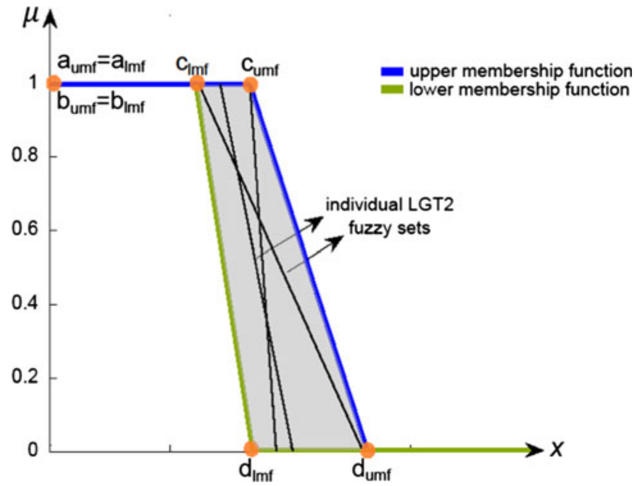

Fig. 8 Aggregation of primary MFs of individual LGT2 fuzzy sets with zero FOU to produce an adaptive LGT2 fuzzy set with shaded FOU

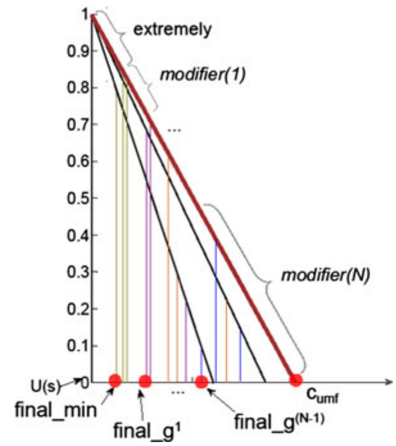

Fig. 9 Aggregation of secondary MFs of individual LGT2 fuzzy sets to produce an adaptive secondary MF with marked base transition points

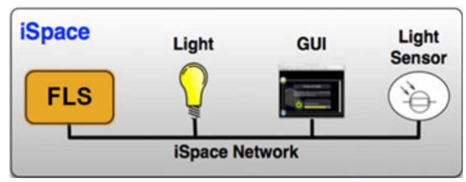

Fig. 10 The common architecture of the applications developed for this study

Similarly, we apply Eq. (16) and the rest of the parameters of the aggregated FOU for the right shoulder MFs are calculated as follows:

$$a_{umf} = min\left(P\_Granule(N)_l\right) \tag{23}$$

$$b_{umf} = min\left(P\_Granule(M-N+1)_l\right) \tag{24}$$

$$a_{lmf} = max\left(P\_Granule(N)_l\right) \tag{25}$$

$$b_{lmf} = max\left(P\_Granule(M-N+1)_l\right) \tag{26}$$

The next step is to aggregate the secondary membership functions as shown in Fig. 9. Using the same idea as before, we will calculate the base transition points of the aggregated left shoulder MF according to Equations (27) and (28), where $w = 1\ldots(N-1)$. The purpose of the aggregated base transition points (marked with dots in Fig. 9) is to facilitate the quantification of the third dimension.

$$final\_min = min(minimum_l) \tag{27}$$

$$final\_g^w = min\left(P\_Granule(w)_l\right) \tag{28}$$

For example, as shown in Fig. 9, the *final_min* will be a transition point between the modifier 'extremely' (hence the linguistic label e.g. extremely dark) and the *modifier*(1) (hence the linguistic label e.g. very dark). Furthermore, $final\_g^{(N-1)}$ will be a transition point between the *modifier(N* − 1) (hence the linguistic label e.g. very dark) and the *modifier(N)* (hence the linguistic label e.g. dark *(no_modifier)*). Similar to Eqs. (27)–(28), the calculation of the parameters for the base transition points of the aggregated right shoulder MF can be formulated as follows where $t = M-N+2\ldots M$.

$$final\_max = max(maximum_l) \tag{29}$$

$$final\_g^t = max\left(P\_Granule(t)_l\right) \tag{30}$$

4 Experiments and results

We have performed two sets of real-world experiments using modified versions of our Fuzzy Task Agent presented in (Bilgin et al. 2012a, b) in the intelligent apartment, iSpace (shown in Fig. 11), located at the University of Essex, Colchester, UK. The iSpace is a purpose-built and fully-furnished two-bedroom apartment which includes a spacious open plan kitchen and living area, bathroom, master bedroom and a study. It has distributed sensors and actuators that are connected in a homogenous manner over the iSpace network by the use of UPnP middleware. Fig. 10 illustrates the common architecture of the developed applications where we show the communication of the light sensors, the graphical user interfaces (GUI) and the ceiling lights over on the iSpace network.

The two sets of experiments are described in four subsections, each of them being a step that prepares a basis for the following one. First, we will start with single user experiments where we collect the user experience per participant and thereby construct the individual LGT2

Fig. 11 Users A, S and J participating in experiments in the intelligent apartment iSpace, University of Essex, UK

fuzzy sets. We will then detail how to create a collective model using these data that we have collected. The first two subsections described below will constitute the first set of experiments. In the second set of experiments, we will present a comparison of the rule base evaluation processing times between the two fuzzy logic systems where we have conducted the single user experiment using interval type-2 (IT2) and LGT2 fuzzy sets. The rule base evaluation consists of calculating the reading light output based on the number of rules, and the processing times that will be reported are based on the complete rule bases for the two FLSs. Finally, we will report on the accuracy of the outputs of both of the systems together with the satisfaction of the participants.

4.1 Real-world experiments with lay users

The first set of experiments was carried on with 3 participants. We have collected data on a daily basis from each of the participants to construct their luminance perception model. The aim is to customize the system for each user and let the embedded agent adapt to the user's preferences for the reading light level at a specific time of the day. We have chosen to run the experiments at noontime which corresponds to the time interval from 10.00 a.m. to 2.00 p.m. (BST). Also, we have chosen this time interval due to the fact that we have encountered most of the light sensor value changes and fluctuations during this period.

We have logged the light sensor readings and the corresponding output linguistic labels for users A, S and J. Table 2 illustrates some portion of the data as experienced in the accumulation and comparison module (Element 2 in Fig. 3). Due to space constraints, we will only show the light sensor values for the output linguistic label 'High' and the number of occurrences for each of them. As highlighted in Table 2, the most frequent input light sensor value for the output linguistic label 'High' is ~92.6667, which will be interpreted to be the prevailing level of stimulation (Laming 2004) for the stimuli granule '*Low*', which is found using Eq. (9).

In the same fashion, we extract all the other transition points from the user experience which have been collected over 3 days. Figure 12a shows the LGT2 fuzzy sets for the user A's perception of the ambient light level where the blue

Table 2 Experiment data for user A's experience in memory

Light Sensor Value	*Count of Light Sensor Value for each Output Linguistic Label*				
	Very High	High	Medium	Low	Very Low
…	…		…	…	…
90.3334		1			
90.6667		1			
92.6667		2			
93		1			
93.3334		1			
96.6667		1			
97		1			
99.3334		1			
99.6667		1			

MF (left shoulder) represents the perception of *dark* and the red one (right shoulder) represents the perception of *bright*. On the horizontal *x*-axis, light sensor lux values are shown and the four points respectively marked with a lozenge (◇), a circle (○), an asterisk (*) and a star (★) correspond to the transition points where the linguistic qualifiers change in the third dimension as shown in Fig. 12b and c. As explained previously, the third dimension is modeled to indicate the qualifier 'extremely' for the values smaller than the minimum stimulated, meaning that the light sensor values smaller than the 'minimum' are perceived as *extremely dark*. The same applies to the values larger than 'maximum' to be interpreted as *extremely bright*.

For the second and third participants, users S and J, the LGT2 model of ambient luminance perception (when the user is reading) is shown in Figs. 13 and 14.

4.2 General type-2 approach to model inter-user uncertainty

As noted before, the LGT2 fuzzy sets for individual user's models have a primary Footprint of Uncertainty (FOU) as zero. However, when we put all the three different individual user's models together, we have a collective model of LGT2 fuzzy sets as shown in Fig. 15 where the adaptation of the FOU in the primary MFs is highlighted.

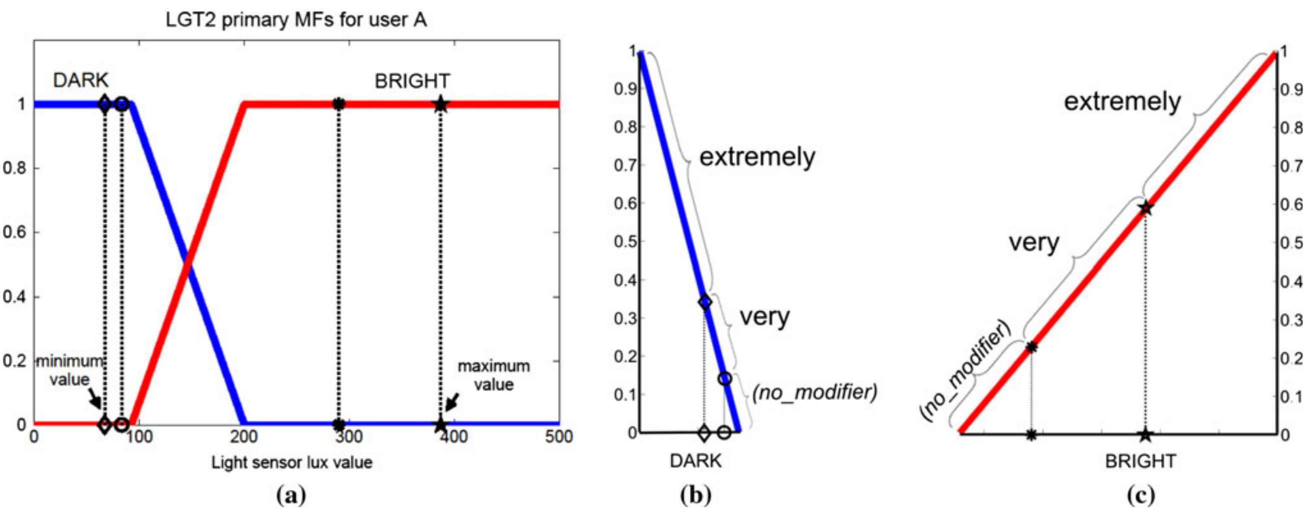

Fig. 12 LGT2 model of ambient luminance perception for user A when reading, **a** primary MFs; secondary MF for **b** ‘dark’, **c** ‘bright’

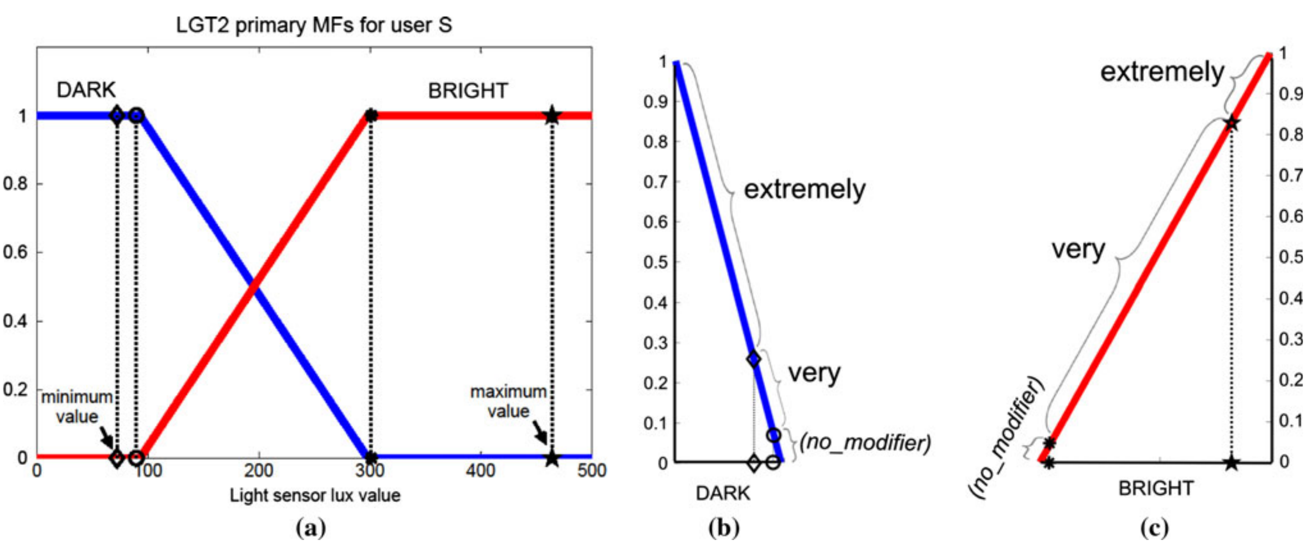

Fig. 13 LGT2 model of ambient luminance perception for user S when reading **a** primary MFs; secondary MF for **b** ‘dark’, **c** ‘bright’

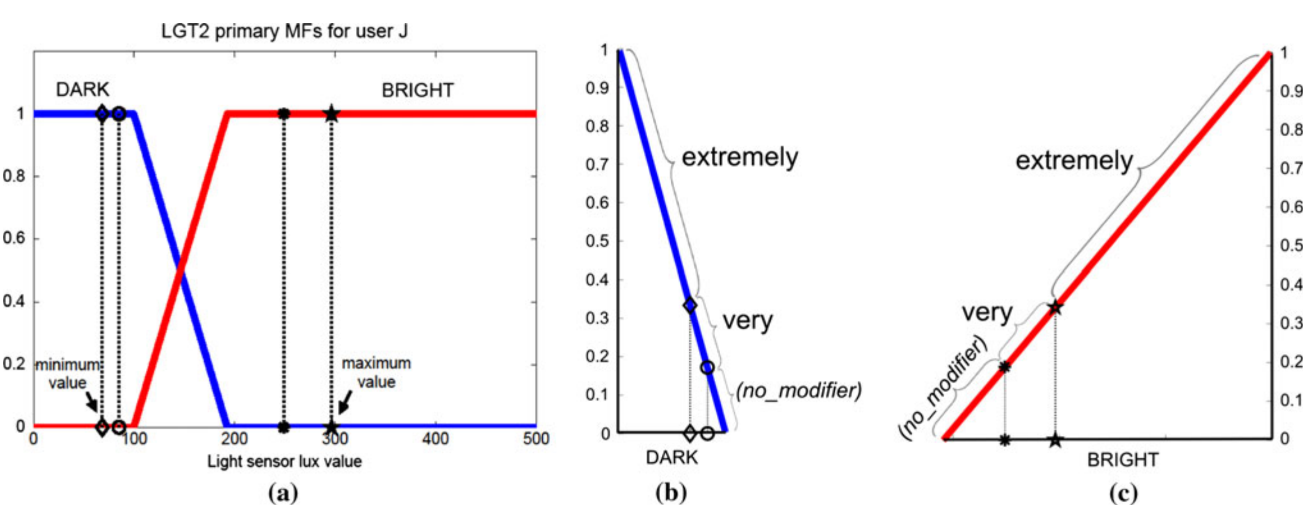

Fig. 14 LGT2 model of ambient luminance perception for user J when reading **a** primary MFs; secondary MF for **b** ‘dark’, **c** ‘bright’

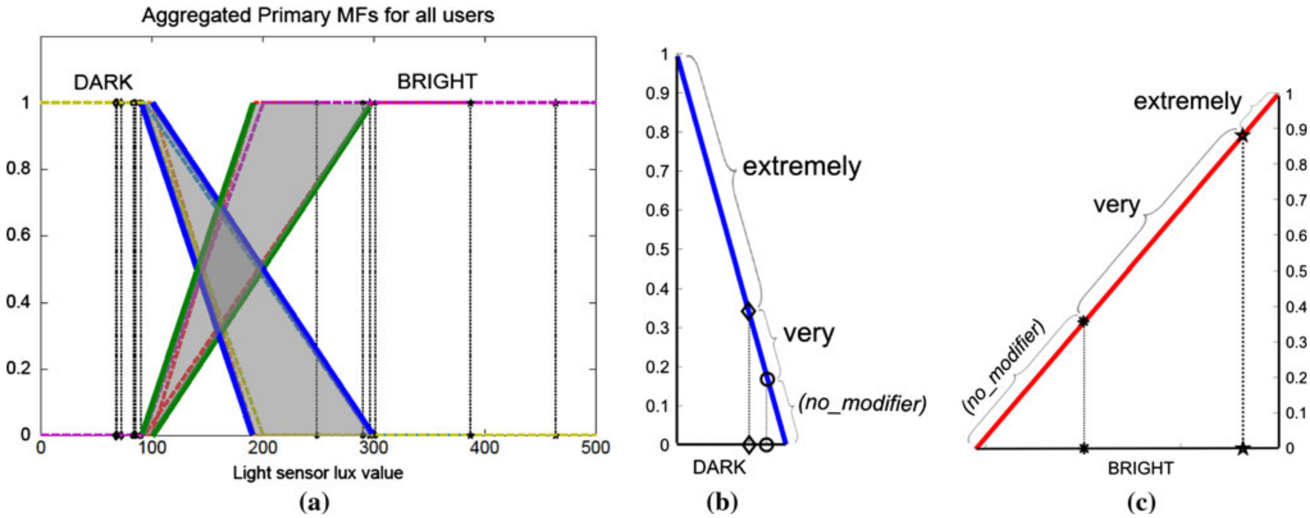

Fig. 15 Adaptive LGT2 model of ambient light level luminance for users A, S and J when reading **a** aggregated primary MFs with the FOUs shaded; aggregated secondary MFs for **b** 'dark', **c** 'bright'

Using Eqs. (10)–(30), we have obtained an adaptive model for the ambient light level perception for various users when they are reading at noontime, which can be promoted to be of real-world usage for libraries, or other public places of similar purpose. As such, the adaptive modeling of human perceptions can be considered as the preliminary steps that will enable to bring CWW paradigm into life. We believe these models will evolve and replace the expert designed fuzzy sets to allow for a better representation of human intelligence.

4.3 zSlices implementation of the LGT2 fuzzy sets

After having created the collective model of perceptions, we proceed to the second set of experiments where we have put the theory into practice using the zSlices implementation (Wagner and Hagras 2010). We have used two inputs, hence two linguistic variables which are the ambient luminance perception that has been modeled in the previous sub-section; and the time of day that has been designed using expert knowledge. Figures 16, 17, 18 demonstrate the linguistic labels that have been used for the two linguistic variables as well as the zSlices implementation of them.

The decision of the number of slices has been based on the Greater Common Divisor (GCD) calculation between the secondary degrees of the ambient luminance perception model in Fig. 15. In other words, we have calculated the GCD of the secondary degrees considering both of the linguistic labels for the light, which are *dark* and *bright*. By doing so, we were able to discretize the third dimension, which is an interval between [0,1], into equal valued slices that will allow for a fair establishment of zSlices for both sets, as the number of slices used in the linguistic variables within the whole system should be equal. Furthermore, we have applied simplification on this number, which was calculated to be 50 in the beginning, so that we could have a real-time deployment on embedded devices. Therefore, we kept the number of slices to be 5 throughout the LGT2 FLS. However, it should be noted that this simplification might result in disregarding a linguistic modifier due to merging of the slices. In fact, we have encountered a similar case where we had to disregard the modifier 'extremely' for the linguistic label 'bright' for the ambient light level linguistic variable. All of the linguistic labels that have been used in the system are illustrated in 2D view in Fig. 16, and in 3D view with different colors in Figs. 17 and 18.

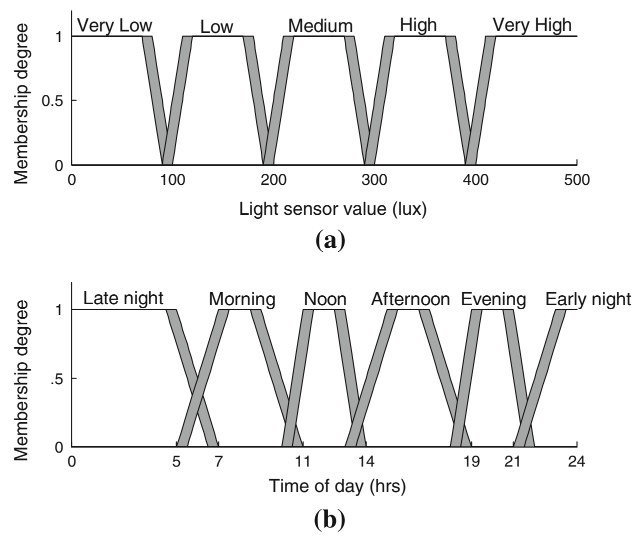

Fig. 16 2D view of Interval Type-2 fuzzy sets for **a** ambient luminance, **b** time of day

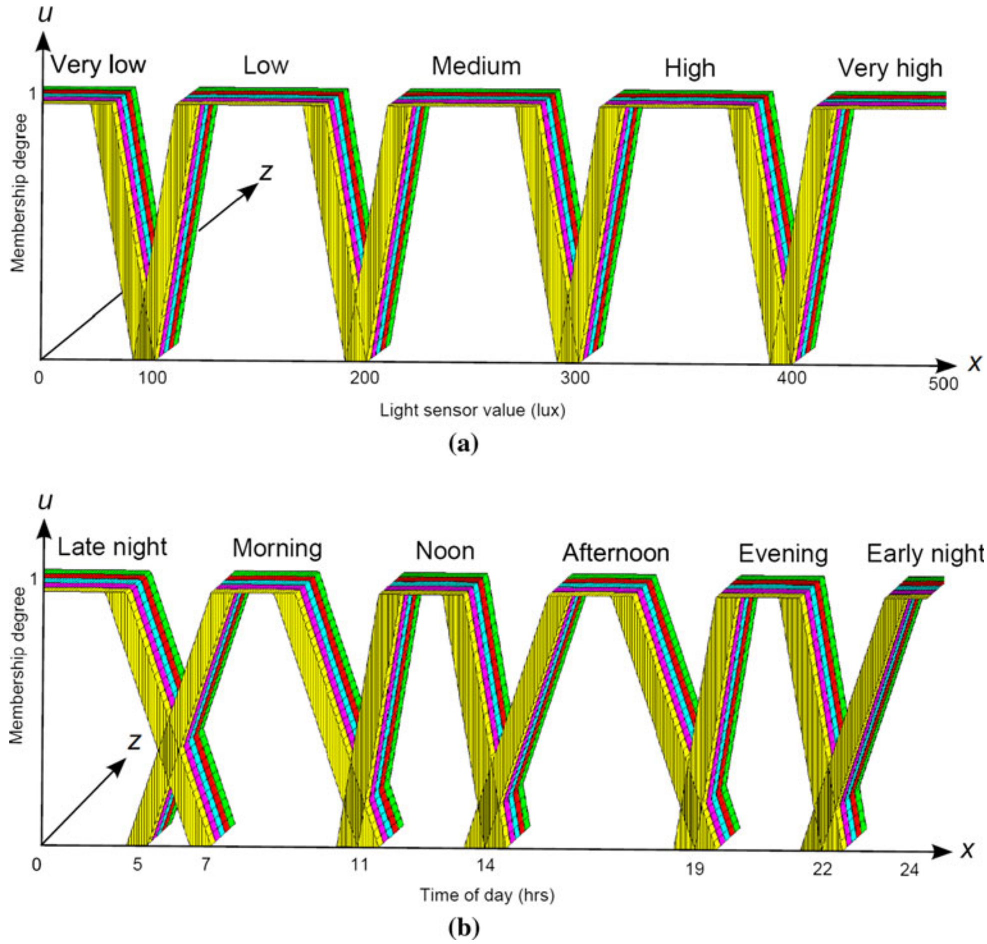

Fig. 17 3D view of Interval Type-2 fuzzy sets for **a** ambient luminance, **b** time of day

As visualized in the Figs. 16, 17 and 18, we have used three different modifiers which are ‘extremely’, ‘very’ and ‘no modifier’. For the ambient luminance perception model, these modifiers have been drawn from the expert-designed linguistic labels that are used in the initial FLS. For the time linguistic variable, the labels have been designed using expert knowledge where the time of day is considered as ‘early’ or ‘late’ for 24-h period. The modifiers employed for the time linguistic variable are consistent with the ambient light level linguistic variable; hence we have also used ‘extremely’, ‘very’ and ‘no modifier’ for the time input. Altogether, the linguistic labels that are employed for the ambient light level perception are ‘extremely dark’, ‘very dark’, ‘dark’, ‘bright’ and ‘very bright’, and for the time input are ‘extremely early’, ‘very early’, ‘early’, ‘late’, ‘very late’ and ‘extremely late’.

4.4 Comparison of the interval type-2 and LGT2 fuzzy rule base evaluation processing times

One of the main motivations for employing fuzzy logic based intelligent embedded agents in an intelligent environment is that fuzzy systems are easy to understand and the rule base is human interpretable. The construction of a complete rule base is marked with the number of linguistic variables and their number of linguistic labels. In order to account for all possibilities, we have constructed complete rule bases for both systems where we have employed IT2 and LGT2 fuzzy sets. For an IT2 FLS, the linguistic labels for the ambient light level perception are ‘very low’, ‘low’, ‘medium’, ‘high’ and ‘very high’; and for the time input are ‘early night’, ‘morning’, ‘noon’, ‘afternoon’, ‘evening’ and ‘late night’. Accordingly, a complete IT2 rule base consists of 30 rules. Table 3 lists all the rules for a single user (User A).

On the other hand, for an LGT2 system, the linguistic labels for the ambient light level perception are ‘dark’ and ‘bright’ that makes up the two fuzzy sets. The modifiers are designed to be placed in the third dimension and one can employ as many as needed. They do not appear in the rule base, as they are encoded within the zSlices. In other words, they are quantified in the third dimension and do not constitute another linguistic variable. Also, for the time input, the two linguistic labels are ‘early’ and ‘late’. Accordingly, the complete rule base for an LGT2 system contains 4 rules as seen in Table 4.

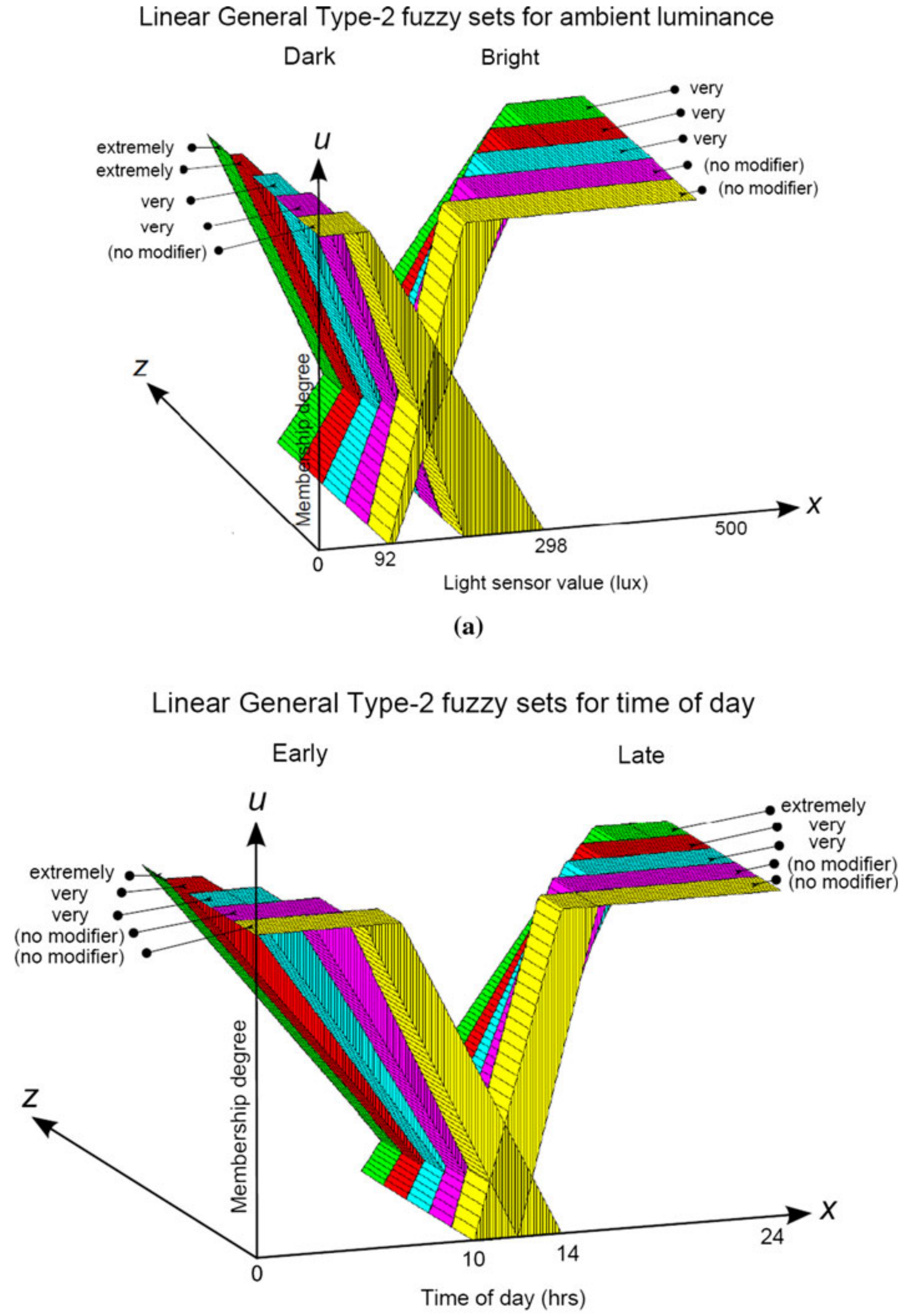

Fig. 18 Linear General Type-2 fuzzy sets for **a** ambient luminance where 'dark': [0, 298] and 'bright': [92. 7, 500], **b** time of day where 'early': [0, 14] and 'late': [10, 24]

We have calculated the processing time of the two FLSs considering only the rule base evaluation time in milliseconds based on the adaptation phase data for user A. The results show that the LGT2 FLS drastically outperforms the IT2 FLS. For example, the evaluation of 30 rules for an IT2 FLS takes up to minimum 168 ms within (approximately) 72-min-runtime. Also, the maximum processing time within the specified runtime has been recorded to be 250 ms for an IT2 FLS. In contrast, the LGT2 system has been recorded to take up to maximum of 5 ms for the rule base evaluation. Consequently, the LGT2 FLS has performed considerably better than the IT2 FLS regarding the rule base evaluation processing time.

Equally important, the limited computational and memory capabilities of embedded agents should be considered in real-world applications since both FLSs use embedded controllers in an ambient intelligent environment. The computational overhead inherent to Karnik–Mendel type-

Table 3 Complete interval type-2 rule base for a single user (user A)

If Time of Day is AFTERNOON and Ambient Light Level is HIGH then Ceiling Lights is Medium
If Time of Day is EARLY NIGHT and Ambient Light Level is HIGH then Ceiling Lights is Medium
If Time of Day is EVENING and Ambient Light Level is HIGH then Ceiling Lights is Medium
If Time of Day is LATE NIGHT and Ambient Light Level is HIGH then Ceiling Lights is Medium
If Time of Day is MORNING and Ambient Light Level is HIGH then Ceiling Lights is Low
If Time of Day is NOON and Ambient Light Level is HIGH then Ceiling Lights is Very Low
If Time of Day is AFTERNOON and Ambient Light Level is LOW then Ceiling Lights is High
If Time of Day is EARLY NIGHT and Ambient Light Level is LOW then Ceiling Lights is High
If Time of Day is EVENING and Ambient Light Level is LOW then Ceiling Lights is High
If Time of Day is LATE NIGHT and Ambient Light Level is LOW then Ceiling Lights is Very High
If Time of Day is MORNING and Ambient Light Level is LOW then Ceiling Lights is Medium
If Time of Day is NOON and Ambient Light Level is LOW then Ceiling Lights is Medium
If Time of Day is AFTERNOON and Ambient Light Level is MEDIUM then Ceiling Lights is Medium
If Time of Day is EARLY NIGHT and Ambient Light Level is MEDIUM then Ceiling Lights is High
If Time of Day is EVENING and Ambient Light Level is MEDIUM then Ceiling Lights is High
If Time of Day is LATE NIGHT and Ambient Light Level is MEDIUM then Ceiling Lights is High
If Time of Day is MORNING and Ambient Light Level is MEDIUM then Ceiling Lights is Medium
If Time of Day is NOON and Ambient Light Level is MEDIUM then Ceiling Lights is Low
If Time of Day is AFTERNOON and Ambient Light Level is VERY HIGH then Ceiling Lights is Low
If Time of Day is EARLY NIGHT and Ambient Light Level is VERY HIGH then Ceiling Lights is Medium
If Time of Day is EVENING and Ambient Light Level is VERY HIGH then Ceiling Lights is Low
If Time of Day is LATE NIGHT and Ambient Light Level is VERY HIGH then Ceiling Lights is Medium
If Time of Day is MORNING and Ambient Light Level is VERY HIGH then Ceiling Lights is Very Low
If Time of Day is NOON and Ambient Light Level is VERY HIGH then Ceiling Lights is Very Low
If Time of Day is AFTERNOON and Ambient Light Level is VERY LOW then Ceiling Lights is High
If Time of Day is EARLY NIGHT and Ambient Light Level is VERY LOW then Ceiling Lights is High
If Time of Day is EVENING and Ambient Light Level is VERY LOW then Ceiling Lights is High
If Time of Day is LATE NIGHT and Ambient Light Level is VERY LOW then Ceiling Lights is Very High
If Time of Day is MORNING and Ambient Light Level is VERY LOW then Ceiling Lights is Medium
If Time of Day is NOON and Ambient Light Level is VERY LOW then Ceiling Lights is High

Table 4 Complete LGT2 rule base for a single user (user A)

If Time of Day is Early and Ambient Light Level is Bright then Ceiling Lights is Low
If Time of Day is Late and Ambient Light Level is Bright then Ceiling Lights is Medium
If Time of Day is Early and Ambient Light Level is Dark then Ceiling Lights is Medium
If Time of Day is Late and Ambient Light Level is Dark then Ceiling Lights is High

reduction method is directly proportional to the number of rules (Lynch et al. 2005); hence, decreasing the number of rules allows for a reduced overhead for real-time systems. In our experiments, the results show that we have achieved a more responsive system by using LGT2 fuzzy sets despite the fact that general type-2 fuzzy systems are computationally more complex. The chart in Fig. 19 illustrates the processing times for both IT2 and LGT2 FLSs based on the data collected from user A. It can also be noted that even though both systems were run on equal period of runtime, the LGT2 FLS was able to evaluate the rule base more frequently, and hence be more responsive. This has caused the samples that are recorded by the software to be much more in number as seen on the horizontal axis of Fig. 19a. It can also be interpreted that the LGT2 FLS has performed more rule base evaluations due to the reduced overhead.

The spread of the data markers has been zoomed in in Fig. 19b. As shown, the IT2 FLS requires a wider window for the processing time with an average of 201.16 ms, and a standard deviation of 19.61 ms. On the other hand, the LGT2 FLS provides a significantly smaller window to operate on with an average of 0.92 ms and a standard deviation of 0.59 ms. The statistical values are calculated based on the entire runtime (approximately 72 min) of both systems. One of the most important things to realize herein is that the operational time on embedded devices is essential to estimate in advance so that they can be utilized in an efficient manner. Therefore, using an LGT2 FLS, as in this case, would reduce the overhead on the real-time processing of the embedded controller and would allow for an effective use of the hardware.

Similarly, the processing times of both systems based on the data collected from user H and T are presented in Fig. 20a and b, respectively. It is important to note that the charts in Fig. 20 consist of mixed data from the two phases of learning and

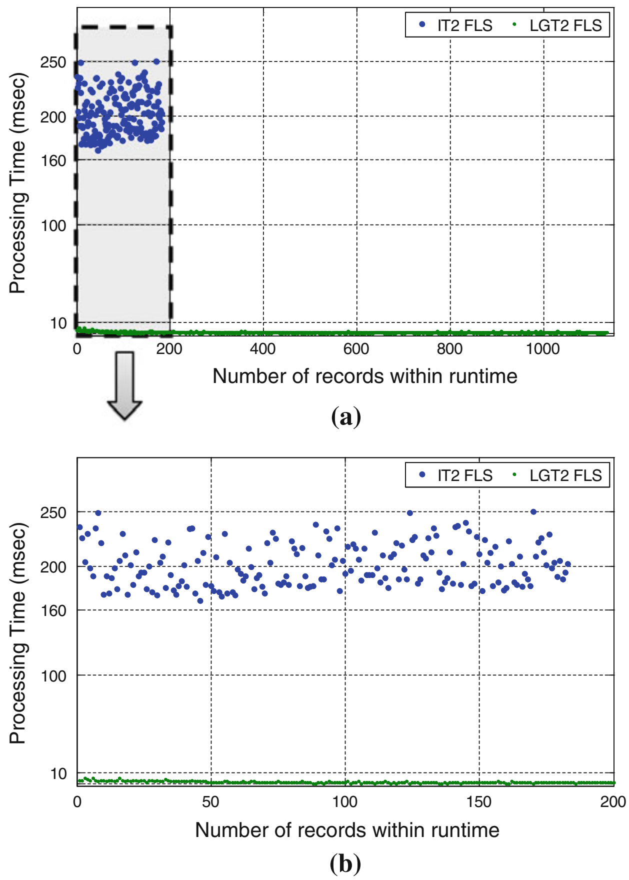

Fig. 19 **a** Comparison of the rule base evaluation processing time between IT2 and LGT2 fuzzy system for a 72-min run in adaptation phase for user A. **b** Zoomed in spread of IT2 FLS processing times within runtime

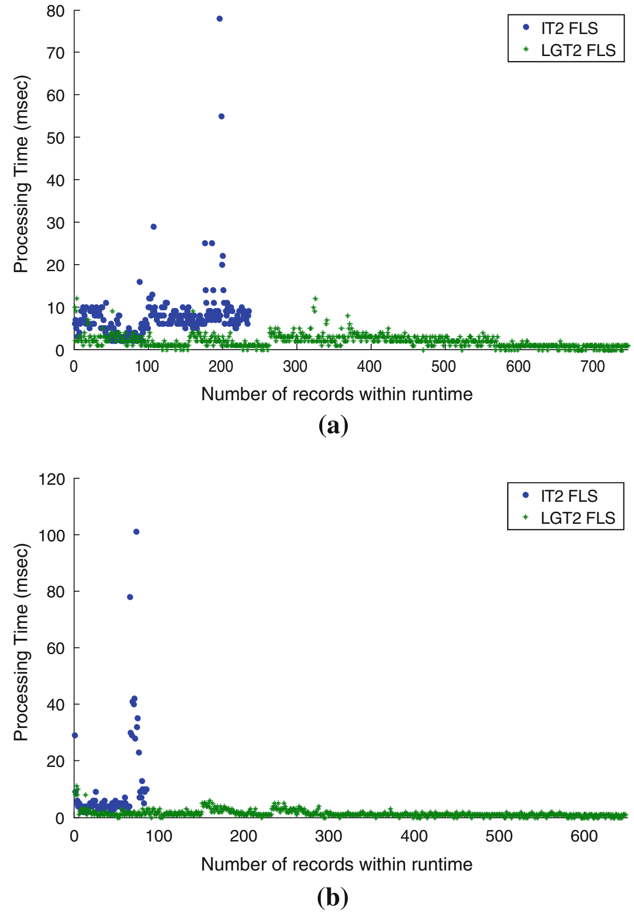

Fig. 20 Rule base evaluation processing times for learning and early adaptation phases of IT2 FLS and LGT2 FLS based on data collected from **a** user H, **b** user T

adaptation and also, the run times are not necessarily equal. This is why; the processing times for both systems are reported to be closer in value. However, it should be noted that both systems start with an empty rule base and as the users interact with the system, the rules are created and added to the rule base according to the user preferences. Hence, it is expected to have similar processing times as and when the number of rules is closer or equal for both IT2 FLS and LGT2 FLS. It can also be deduced from the charts in Fig. 20 that the FLSs might not produce as many outputs depending on the changes in the environment such as the light sensor values. As seen, the systems undergo the majority of the learning phase at the early stages. Afterwards, the adaptation phase starts to dominate the operation of the FLS. Still, the trends convey that the processing times for an LGT2 FLS outperform those of an IT2 FLS.

4.5 Comparison of the ITS FLS and LGT2 FLS on output accuracy and user satisfaction

After reducing the number of rules and the computational overhead, we need to consider the accuracy of both fuzzy systems together with the user satisfaction. For this purpose, we have deployed both FLSs in the intelligent apartment iSpace, University of Essex, UK.

For this experiment, we have applied two major phases as for learning and adaptation. First of all, the systems need to learn the user preferences. In other terms, FLSs need to learn the rule base consisting of the user preferences through interacting with the participants who use a graphical user interface to communicate their choices (Fig. 10). We have run the IT2 and LGT2 fuzzy systems separately to allow the users to create a complete rule base consisting of 30 rules and 4 rules, successively for each FLS. The second phase was employed for adaptation purposes. Both FLSs were run and the users were questioned on which system they preferred considering their comfort. By doing so, we were able to investigate the user satisfaction during the adaptation phase. In addition, we have performed comparison on the recorded output values in order to find out the performance accuracy of both FLSs.

We have conducted this experiment with three participants (Fig. 21). The rule bases collected from all

Fig. 21 Participants (users H and T) reading and assessing the accuracy of both fuzzy systems in the intelligent apartment iSpace, University of Essex, UK

Table 5 Complete IT2 rule base for users H and T

Time of day	Ambient light level	Ceiling lights	
		User H	User T
AFTERNOON	HIGH	Very Low	Very Low
EARLY NIGHT	HIGH	Medium	Very Low
EVENING	HIGH	Low	Very Low
LATE NIGHT	HIGH	Low	Very Low
MORNING	HIGH	Very Low	Very Low
NOON	HIGH	Very Low	Very Low
AFTERNOON	LOW	Low	Very Low
EARLY NIGHT	LOW	Very Low	Very Low
EVENING	LOW	Low	Very Low
LATE NIGHT	LOW	Very Low	Very Low
MORNING	LOW	Very Low	Very Low
NOON	LOW	Very Low	Very Low
AFTERNOON	MEDIUM	Very Low	Very Low
EARLY NIGHT	MEDIUM	Very Low	Very Low
EVENING	MEDIUM	Very Low	Very Low
LATE NIGHT	MEDIUM	Very Low	Very Low
MORNING	MEDIUM	Very Low	Very Low
NOON	MEDIUM	Very Low	Very Low
AFTERNOON	VERY HIGH	Low	Very Low
EARLY NIGHT	VERY HIGH	Very Low	Very Low
EVENING	VERY HIGH	Very Low	Very Low
LATE NIGHT	VERY HIGH	Medium	Very Low
MORNING	VERY HIGH	Very Low	Very Low
NOON	VERY HIGH	Very Low	Very Low
AFTERNOON	VERY LOW	Medium	Medium
EARLY NIGHT	VERY LOW	Very Low	Medium
EVENING	VERY LOW	Medium	Medium
LATE NIGHT	VERY LOW	Low	Medium
MORNING	VERY LOW	High	Low
NOON	VERY LOW	High	Medium

Table 6 Complete LGT2 rule base for users H and T

Time of day	Ambient light level	Ceiling lights	
		User H	User T
Early	Bright	Very Low	Very Low
Late	Bright	Low	Very Low
Early	Dark	High	Medium
Late	Dark	Medium	Medium

participants for both FLSs are displayed in Tables 3, 4, 5, 6. All of the participants have found the LGT2 FLS to be more responsive and accurate in a lighting scenario.

Furthermore, the accuracy of the both FLSs is compared and demonstrated in graphs (Figs. 22, 23) where we have taken the unique output values generated by both FLSs in a sorted manner for users H and T, respectively. As can be seen, LGT2 FLS is able to deliver a wider and more importantly a smoother range of ceiling light values. In other words, the changes in the output values of LGT2 FLS are much gentler and hence the LGT2 FLS provides more comfortable reading experience for the user for this specific application. In general, it can also be concluded that

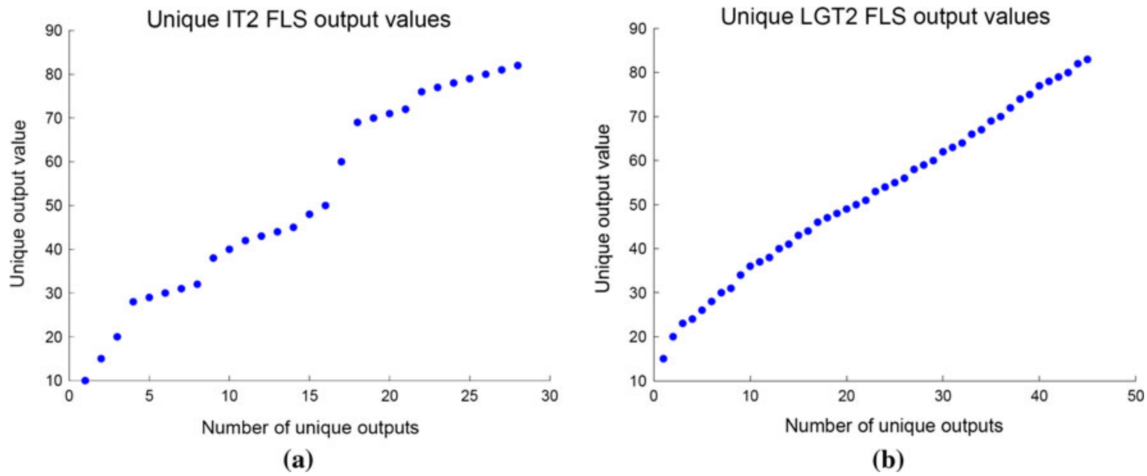

Fig. 22 Unique output values for ceiling lights based on user H learning and adaptation data for **a** IT2 FLS, **b** LGT2 FLS

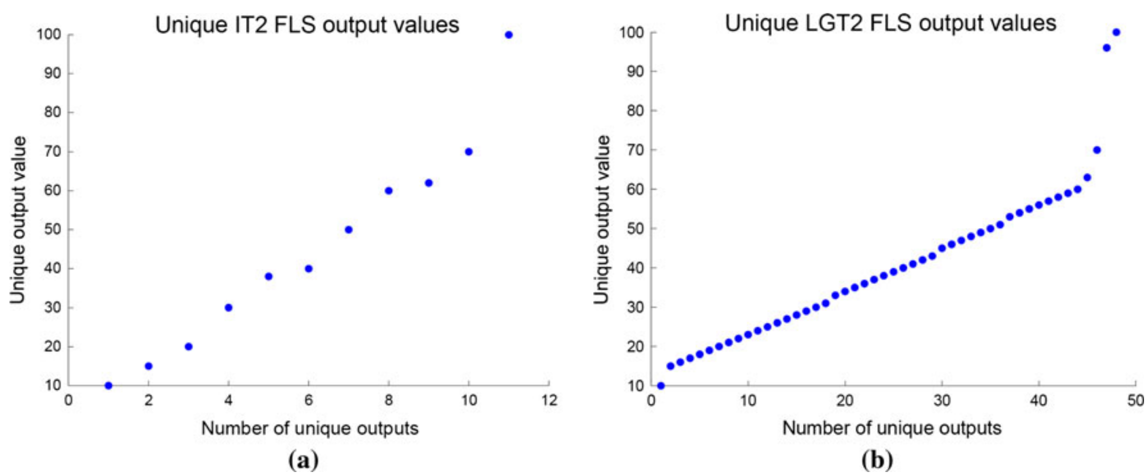

Fig. 23 Unique output values for ceiling lights based on user T learning and adaptation data for **a** IT2 FLS, **b** LGT2 FLS

interval type-2 FLSs are likely to show more intermittent behavior regarding the output compared to a general type-2 FLS.

5 Conclusions and future work

It is a very challenging task to bridge the gap between the computer processes and the human's brain reasoning. Today, one may consider the proximity of the human intelligence to today's machine intelligence to be still far from reach. However, a recently established research area in the field of fuzzy logic, which is the paradigm of computing with words (CWW), encourages the researchers to consider the human mind as a role model. In this study, we have detailed the problem of human intelligence from a perspective of CWW paradigm and proposed a novel way to realistically interpret the human perceptions in machine processes. We have taken the human experience into account in order to achieve a more natural representation of human perception, which is one of the key elements to mimic the human mind. Also, we have detailed how to model the human perceptual judgment using LGT2 fuzzy sets and the backwards thinking approach that has been inspired from exhaustive inter-disciplinary literature review mainly on psycholinguistics and neuroscience. Hence, we have demonstrated the benefits of LGT2 fuzzy sets in a real-world application. We have shown that we are able to not only quantify the third dimension in words, therefore serve the purpose of CWW, but also measure the quantification of the third dimension to indicate how profound the human perceptual judgments are.

Furthermore, we have compared the performance of two FLSs employing IT2 and LGT2 fuzzy sets. By using LGT2 FLS, we were able to significantly reduce the number of rules. Looking at the rule base evaluation processing times, it can be deduced that an LGT2 FLS can be more responsive and produce a much faster response despite the fact that general type-2 fuzzy systems are computationally more complex. Also, by looking at the generated outputs from both systems for different users, it can be concluded that LGT2 FLSs are likely to show more smooth behavior regarding the output compared to an interval type-2 FLS. And finally, the users have assessed both FLSs without being informed about the system specifications and agreed that LGT2 FLS provides a much more comfortable reading experience.

As part of future research, we will continue exploring the different aspects of human reasoning and will extend on the work involving the other modules of the proposed CWW framework which have not been referred to in this paper.

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