Soft Comput (2013) 17:849–865
DOI 10.1007/s00500-012-0949-7

METHODOLOGIES AND APPLICATION

Analysis and synthesis of laser forming process using neural networks and neuro-fuzzy inference system

Kuntal Maji · D. K. Pratihar · A. K. Nath

Published online: 4 November 2012

Abstract To apply laser forming process in reality, it is required to know the relationships between the deformed shape and scanning paths along with heating conditions. The deformation due to laser scanning depends on various factors, namely laser power, scan speed, spot diameter, scan position, number of scans, and many others. This article presents soft computing-based methods to predict deformations for a set of heating conditions, and also to determine the heating lines and heat conditions, in order to get a desired shape (i.e., inverse analysis). A novel attempt has been made in this paper to carry out analysis and synthesis (inverse analysis) of laser forming process using both genetic-neural network (GA-NN) and genetic adaptive neuro-fuzzy inference system (GA-ANFIS). During the analysis, laser power, scan speed, spot diameter, scan position and number of scans are taken as inputs and bending angle is considered as the output. A batch mode of training has been used for both the approaches with the help of some experimental data. The performances of the developed approaches have been tested on some real experimental data. Both the approaches are found to be effective to predict the bending angles and carry out the process synthesis successfully. GA-NN approach is found to perform better than the GA-ANFIS approach in predicting the bending angles, and both the approaches are able to provide comparable predictions in inverse analysis.

Communicated by Y. Jin.

K. Maji · D. K. Pratihar (✉) · A. K. Nath
Department of Mechanical Engineering, Indian Institute of Technology, Kharagpur, Kharagpur 721 302, India
e-mail: dkpra@mech.iitkgp.ernet.in

K. Maji
e-mail: kuntalmajiiitkgp@gmail.com

A. K. Nath
e-mail: aknath@mech.iitkgp.ernet.in

Keywords Laser forming · Analysis · Synthesis · Neural networks · Neuro-fuzzy inference system · Genetic algorithm

1 Introduction

Laser forming is a flexible thermal forming process used to make complex curved shapes from flat sheet metal by controlled laser irradiations. It is a complex thermo-elasto-plastic process that depends on various process parameters, such as laser power, scan speed, spot diameter, scan positions, number of scans, thermo-physical properties of the workpiece material and its dimensions. It has got many advantages over conventional mechanical forming like absence of hard tooling, more flexibility, better precision and accuracy without any spring-back. Depending on the specific combinations of component geometries and laser process conditions, mainly three mechanisms, namely temperature gradient mechanism (TGM), buckling mechanism (BM) and upsetting mechanism (UM) may work. In TGM, due to laser scan, a stiff temperature gradient is set along thickness of the sheet, and when the thermal stress exceeds the temperature-dependent yield stress, the material deforms plastically. Therefore, after the heating and cooling cycle, the sheet finally bends towards the laser beam. When thermal diffusivity is high and moment of inertia becomes low due to less sheet thickness, BM works and bending direction depends upon the initial stress and loading condition of the sheet. In UM, nearly uniform temperature and hence, nearly equal plastic deformation occurs at the top and bottom layers, but due to the increased sheet thickness offering more moment of inertia, buckling is prevented and uniform compression (in plane strain) is resulted with a slight bending towards the laser beam. The process is applicable to rapid prototyping in shipbuilding, automobile, aerospace industries and precision alignment and adjustments in microelectronics industries. However, the process has not become so popular and is not applied to large scale industry because of the lack of automation or due to the difficulty of determining the process parameters and scanning patterns to produce any desired shape within a reasonable time. Analytical models become difficult and cumbersome, and numerical models are more time consuming to predict the deformed shape due to multiple heating lines and multiple scans, and to solve the inverse problem. This paper presents neural network and fuzzy logic-based methods used for developing the models of bending angle and conducting inverse analysis of the laser forming process.

The remaining part of this paper has been organized as follows: Sect. 2 deals with the literature review. Section 3 describes the experimental set-up, method of experimentation and data collection. The methods of analysis and synthesis used in the present study are explained in Sect. 4. Results are stated and discussed in Sect. 5. Conclusions are drawn and the scopes for future work are indicated in Sect. 6.

2 Literature review

Several attempts were made by various researchers for modeling and analysis of laser forming process using different techniques, namely analytical, numerical and empirical ones (Shen and Vollertsen 2009). Some of them are discussed here. A number of analytical models were developed by various investigators (Vollertsen 1994; Kyrsanidi et al. 2000; Cheng and Lin 2001; Shen et al. 2006a) based on the theory of heat transfer and elasto-plastic mechanics to predict bending angle in laser forming process. However, these models become difficult and cumbersome to determine the deformations for multiple scans, as the absorptivity of the workpiece surface, thermal and material properties change with temperature and time, during the laser forming process. Numerical models give better insight into the process through transient temperature, and stress–strain distribution. Several numerical models are also available in the literature (Vollertsen et al. 1993; Hu et al. 2001; Wu and Ji 2002; Zhang and Michaleris 2004; Zhang et al. 2007; Griffiths et al. 2010; Hu et al. 2012) to predict bending angles in laser forming process using finite element method, but those models needed temperature dependent material properties and took long computation time particularly for the multiple laser irradiations. Moreover, determining the heating lines and heat conditions to achieve a desired (deformed) shape by laser forming, i.e., solving the inverse problem or carrying out process synthesis is less likely by either analytical or numerical method. Empirical models developed based on experimental data and heuristic approaches are helpful in such situations to solve both the problems, i.e., predicting the bending angle (i.e., analysis) and carrying out synthesis of laser forming process without considering the thermo-mechanical complexity of the process.

Recently, various non-conventional nonlinear regression methods based on different soft computing techniques like neural networks, fuzzy logic, genetic algorithms and others have been used by many researchers (Nasrabadi and Hashemi 2008; Khashei and Bijari 2010; Akbilgic and Bozdogan 2011; Martino et al. 2010, 2011) for the analysis of data and to achieve good prediction accuracy in different applications. The methods based on soft computing and hybrid algorithms (Whitley et al. 1990; Keller et al. 1992; Jang 1993; Herrera et al. 1995; Pratihar et al. 1999) can also be used for the analysis of complex manufacturing process like laser forming. Some of the studies on modeling of laser forming process using soft computing techniques are discussed here. Cheng and Lin (2000) used three supervised neural networks and regression analysis to develop models of bending angle in laser forming process. The process parameters, namely laser power, scan speed, spot diameter and workpiece geometries including thickness were taken as input parameters and bending angle was considered as output to develop the models. The performances of the developed models were verified with experimental data and the radial basis function neural network was found to perform better than the other models in predicting bending angle. A back-propagation neural network was utilized by Dragos et al. (2000) to predict the bending angle in laser bending of sheet metal. The laser power, beam diameter, scanning speed, thickness of the material and number of scans were considered as the input parameters of the process and bending angle was the output. The bending angles determined by the neural network method were found to be in good correlation with the experimental values. The proposed method was found to be helpful for carrying out online simulation and automatic control of the laser bending process. Casalino and Ludovico (2002) also used a back-propagation neural network to predict bending angle and to select process parameters in laser bending under both the TGM and BM. The developed model was verified with experimental data and was found to be satisfactory. Chen et al. (2002) proposed an adaptive fuzzy neural network to predict bending angle in laser forming process. Energy density (defined as laser power divided by the product of scan velocity and spot diameter), width, thickness of sheet, and scanning path curvature were taken as four inputs and bending angle was considered as the output of the network to establish the model. Good correlation was found between the results obtained from the model and experiments. Liu and Yao (2002) proposed a response surface methodology based optimization method for the synthesis of laser forming process. The proposed method was made robust by considering the error propagation technique as an additional response and optimized via desirability function approach. A fuzzy control system was developed by Kuo and Wu (2002) for controlling the laser bending process. The developed control system was found to be effective in controlling the laser bending process and it increased the manufacturing efficiency. The proposed method was found to be feasible and effective for producing some shapes. Cheng and Yao (2004) presented a process synthesis methodology for laser forming of a class of shapes based on genetic algorithm (GA). The effects of GA control parameters and the types of fitness function on the synthesis process were discussed. The synthesis process was experimentally validated through several cases under diverse conditions. A model for bending angle in laser forming was established by Shen et al. (2006b) using adaptive network fuzzy inference system (ANFIS). The laser forming process parameters considered in the model were laser power, beam diameter, scanning velocity and thickness of the plate. The performance of the ANFIS model was optimized as a function of both the type and number of membership functions. The prediction of the ANFIS model was found to be satisfactory by comparing with the experimental data. Guarino et al. (2007) presented a neural network-based method for modeling the laser-assisted forming of thin aluminium alloy sheet. Good correlation was found between the trends of experimental and calculated values of the variables through the sensitivity analysis on the developed neural network model. A back-propagation neural network model was developed by Wang et al. (2008) for modeling and optimization of the laser bending of aluminum alloy sheet. The developed model was able to predict the bending angle and process parameters within a reasonable accuracy. A computational procedure based on the minimization of a vectorial fitness function was proposed by Carlone et al. (2008) for the inverse analysis of laser forming process. The vectorial fitness function was calculated by comparing the target surface with the reference deformed surface. The reference deformed surface was determined using FEM and the required scan strategy was obtained by minimizing the vectorial fitness function. Nguyen et al. (2009) developed an artificial neural network (ANN)-based model for the predictions of angular distortion and transverse shrinkage during the plate forming by induction heating. The developed model could predict the deformation satisfactorily and with less time compared to finite element analysis. Du and Wang (2010) presented an improved back-propagation neural network model based on double chain quantum genetic algorithm for predicting the bending angle more accurately and optimization in laser bending of sheet metal. Neural network-based model was also developed by Gisario et al. (2011) for the correction and adjustment of bending angle in mechanically bent samples using laser-assisted bending. Good correlation was found between the experimental and model predicted results in both interpolative and extrapolative conditions.

Several techniques were presented to model laser forming process to predict bending angle and final shape of the sheet metal using analytical, numerical and soft computing-based approaches. However, not enough studies had been reported on the synthesis or inverse analysis of laser forming process. The forward and reverse mappings of electric discharge machining process were carried out by Maji and Pratihar (2010) using GA-tuned ANFIS considering both linear and nonlinear membership functions and the latter was found to give better results in terms of accuracy in predictions. In the present paper, GA-tuned neural network and neuro-fuzzy inference system have been used for predicting the bending angle and carrying out inverse analysis of laser forming process.

3 Experimental work

This section describes the experimental set-up used and procedure adopted to carry out the experiments. The method of data collection has also been explained.

3.1 Experimental set-up and method of experiments

Laser bending experiments are conducted on a Ytterbium doped fiber laser (refer to Fig. 1) of 2 kW maximum power

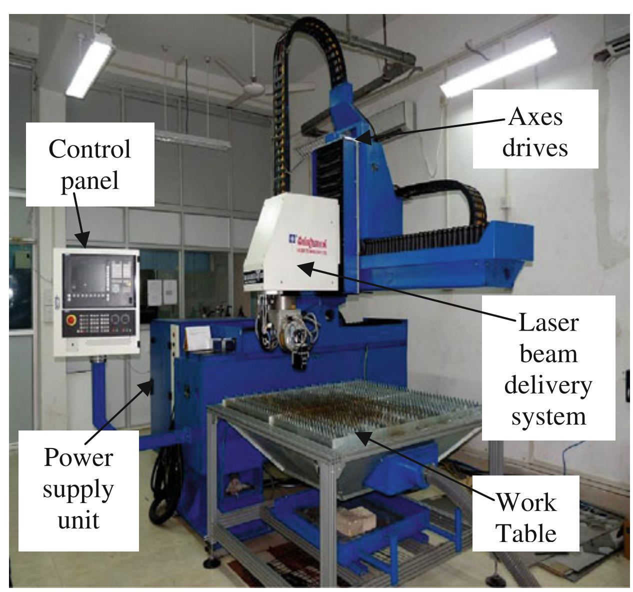


Fig. 1 2 kW Fiber laser system

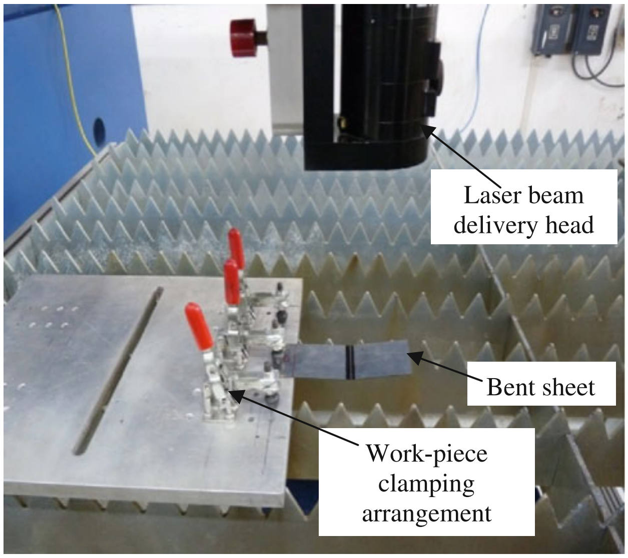


Fig. 2 Experimental set-up

and 1.06 µm wavelength. AISI304 stainless steel has been taken as the workpiece material and samples of size $120 \times 40 \times 0.5$ mm^3 are prepared. Black ink is used on the area of the work-piece to be irradiated by laser to increase the absorptivity. One end of the workpiece is clamped on a metal plate and laser scans are performed at different distances from the free end, as shown in Fig. 2.

Five input variables, namely laser power, scan speed, spot diameter, scan position and number of scans are taken as the inputs and bending angle is considered as the output. Scan position (denoted by r) is the non-dimensional distance from the free edge of the work-piece and defined as the ratio of effective free length (EFL) to total free length (TFL). TFL and EFL are the distances from the free edge to the scan position and clamped edge, respectively. After laser scanning, deflections of the bent samples are measured using a coordinate measuring gauge and the bending angles are calculated using triangulation method.

3.2 Experimental design and data collection

A series of experiments have been designed according to face-centered central composite design (FCCD) of experiments (Montgomery 2001). The five-factor CCD consists of 43 runs ($2^5 + 2 \times 5 + 1$) and each factor having three levels, i.e., low, medium and high. The 43 runs are distributed as 32 factorial points, 10 axial points and one center point. The factorial points represent the runs, where the upper and lower limits of the five input variables are chosen. The axial points denote the runs, where all but one of the factors is set at their respective mid-values. The centre point indicates the run, where mean values of all the input variables are taken. The ranges of the input variables have been considered in such a way to activate the temperature gradient mechanism (TGM) of laser forming process, and decided through some initial experiments, as given in Table 1. For each combination of the input variables, experiments are conducted for three times and thus, a total of 129 (i.e., 43×3) experiments are carried out. The experimental data have been collected according to the design of experiments and the corresponding sets of input–output values are given in “Appendix A”.

4 Analysis and synthesis of laser forming process

Analysis and synthesis of laser forming process have been carried out using neural network and neuro-fuzzy inference system. A batch mode of training has been adopted for the neural network and neuro-fuzzy system with the help of a binary-coded genetic algorithm (GA) (Pratihar 2008). In the batch mode of training, if the number of training data becomes less than that of the parameters of the network, it becomes mathematically undetermined. Thus, the batch mode requires a large number of training data but only 129 experimental data are available, which are inadequate for the training of the neural network. Non-linear regression analysis has been carried out (with the help of Minitab 14 software) based on the above experimental data using the least square method to obtain an equation for the output, i.e., bending angle (A) in terms of the input variables, as given below.

$$\begin{aligned} A = {} & -90.8617 + 0.232447p + 0.468033v + 8.93851d \\ & + 3.26021r + 0.0534127n - 6.70880 \times 10^{-4}p^2 \\ & - 9.15703 \times 10^{-4}v^2 - 40.3819d^2 - 2.52214r^2 \\ & + 0.00756133n^2 + 4.09091 \times 10^{-5}pv + 0.1772pd \\ & - 0.00723333pr + 0.00558167pn - 0.0105051vd \\ & - 0.00742424vr - 0.00281061vn + 0.813333dr \\ & - 0.372667dn + 0.1615rn \end{aligned} \tag{1}$$

The numerical values associated with different terms in Eq. (1) are the regression coefficients obtained using the

Table 1 Input variables and their ranges

SL. no.	Input variables	Symbol	Minimum value	Mid-value	Maximum value
1	Laser power (W)	p	225	250	275
2	Scan speed (mm/s)	v	250.0	266.5	283
3	Spot diameter (mm)	d	0.500	0.625	0.750
3	Scan position	r	0.25	0.50	0.75
3	Number of scans	n	5	10	15

least square method for error (in predictions) minimization. A total of 1,000 training data have been used, which include 129 experimental data (shown in "Appendix A") and 871 data generated using the regression Eq. (1). The additional 871 training data are generated from the regression Eq. (1) by randomly varying the input parameters (p, v, d, r, n) within their respective ranges and calculating the corresponding outputs (A). The training data generated in this way are distributed randomly within the ranges of the input variables.

A detailed report of the above regression analysis is available elsewhere (Maji et al. 2012). The test data to be used for the verification of the developed models are also collected separately through experiments. The experimental test data have been collected by considering random combinations of different values of the input variables lying within their respective ranges and these data have not been used for training of the developed neural network and neuro-fuzzy system-based models. The regression model (refer to Eq. 1) has been tested for 15 test cases (refer to "Appendix B") and found to give an average absolute deviation in predictions of 7.82 %. Both neural network and neuro-fuzzy models are developed to predict the bending angle in laser forming process. The performances of the neural network and neuro-fuzzy-based approaches have been compared between them and with that of the statistical model to predict the bending angles. Moreover, comparisons are made of the performances of these two approaches in case of inverse analysis or process synthesis.

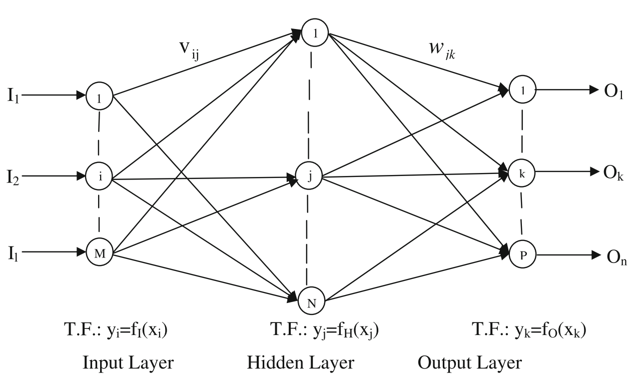


Fig. 3 Architecture of neural network used for modeling of laser forming process

4.1 Modeling and analysis of laser forming process to predict bending angle

Both neural network and neuro-fuzzy system-based models have been developed to predict bending angle in laser forming process as discussed below.

4.1.1 Genetic algorithm-tuned neural network (GA-NN)

Neural network (NN) can be considered as a nonlinear statistical data modeling tool (Pratihar 2008). It has been used here to model the laser forming process based on some experimental data for the estimation of bending angles. In the present work, a neural network consisting of three layers of neurons, i.e., input layer, hidden layer and output layer has been considered, as shown in Fig. 3. The number of neurons of the input and output layers has been kept equal to that of input and output variables, respectively. The transfer functions used in the input, hidden and output layers are denoted by $f_{Ii}(x_i)$, $f_{Hj}(x_j)$ and $f_{Ok}(x_k)$, and the corresponding coefficients are represented by c_{Ii}, c_{Hj} and c_{Ok}, respectively. Inputs and outputs of the different layers, i.e., input, hidden and output layers are denoted by I_{Ii}, H_{Ij}, O_{Ik} and I_{Oi}, H_{Oj}, O_{Ok} for the ith, jth and kth neurons, respectively. Now, the forward calculations through different layers are given below.

$$I_{Oi} = f_{Ii}(I_{Ii}), \quad \text{where, } i = 1, 2, \ldots M$$

$$H_{Ij} = \sum_{i=1}^{M} v_{ij} \times I_{oi}, \quad \text{where, } j = 1, 2, \ldots N$$

$$H_{Oj} = f_{Hj}(H_{Ij} + b),$$
where, $j = 1, 2, \ldots N$ and b is the bias value.

$$\mathrm{O}_{Ik} = \sum_{j=1}^{N} w_{jk} \times H_{oj}, \quad \text{where, } k = 1, 2, \ldots P,$$

$$O_{ok} = f_{Ok}(O_{Ik} + b),$$
where $k = 1, 2, \ldots P$ and b is the bias value.

The NN has been trained in a batch mode with the help of a binary-coded GA. The GA-string represents the synaptic weights (i.e., v_{ij} and w_{jk}) of the NN, coefficients of the transfer functions (c_{Ii}, c_{Hj} and c_{ok}) used and bias values (b), as shown in Fig. 4.

Each parameter is represented using ten bits in the GA-string. These parameters are decoded; real values are calculated and supplied to the neural network to make it ready before passing the training data. The mean absolute error

Fig. 4 GA-string used for representing the parameters of GA-NN Approach

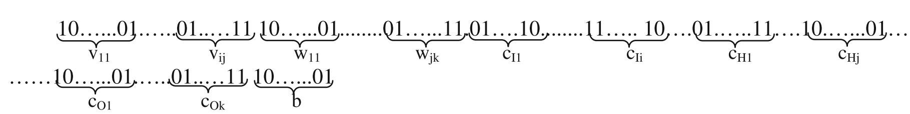
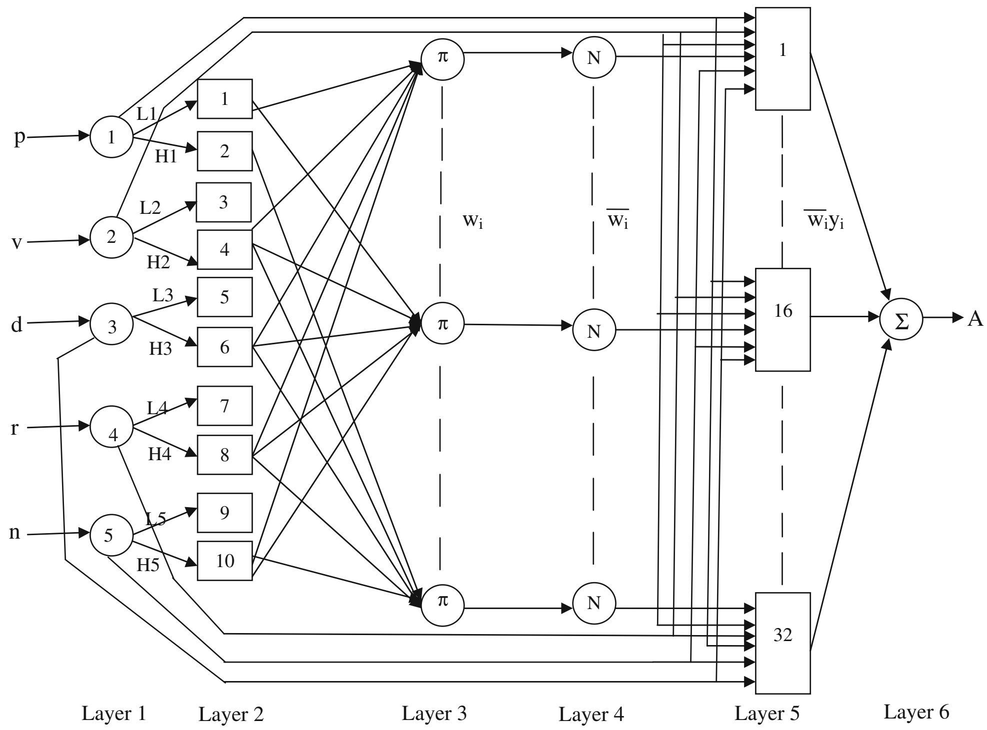


Fig. 5 ANFIS architecture used for modeling of laser forming process



has been used as the fitness of the GA-string. Thus, the fitness of a GA-string is calculated like the following:

$$f = \frac{1}{L}\frac{1}{P}\sum_{n=1}^{L}\sum_{k=1}^{P}|T_{Okn} - O_{Okn}|, \tag{2}$$

where L indicates the number of training scenarios and P represents the number of outputs. The population of GA-strings is then modified using the operators like tournament selection, single point crossover and bit-wise mutation. To determine the optimum structure of the network, the fitness values are calculated for different numbers of hidden neurons, and the number of hidden neurons giving the minimum fitness value is selected. A detailed parametric study is performed for the selection of suitable GA-parameters during the training (using a batch mode) and optimization of the network. Thus, the GA, through its search, determines an optimal network, which will be used for making the predictions.

4.1.2 Genetic algorithm-tuned adaptive neuro-fuzzy inference system (GA-ANFIS)

Adaptive neuro-fuzzy inference (ANFIS) (Jang 1993) system has been used to model the laser forming process. The working principle and architecture of the system expressing the non-linear relationships between the inputs and outputs of laser forming process are explained here. Five inputs and one output have been considered to develop the model, as shown in Fig. 5. A batch mode of training using one thousand data has been adopted for the neuro-fuzzy system as mentioned earlier. Each of the five inputs has been represented using two linguistic terms and there is a maximum of $2^5 = 32$ possible combinations (rules) of them. According to first-order Takagi and Sugeno's model of FLC, the output of each rule can be expressed as follows:

$$y_i = a_{1i} + a_{2i}p + a_{3i}v + a_{4i}d + a_{5i}r + a_{6i}n, \tag{3}$$

where $i = 1, 2, 3, \ldots, 32$; $a_{1i}, a_{2i}, a_{3i}, a_{4i}, a_{5i},$ and a_{6i} are the coefficients.

The functions of the different layers are described as follows: Layer 1 is the input layer of the network, where five nodes represent the five inputs, and it passes the inputs to the next layer. The outputs of these nodes are the same with the corresponding inputs. In layer 2, membership values (μs) for a set of inputs are calculated corresponding to their appropriate linguistic terms, i.e., low, high, etc. The number of nodes in layer 3 is kept equal to the number of rules, i.e., $2^5 = 32$, and these are generally denoted by the symbol π. This layer calculates the firing strengths (w) of the nodes as the product of the corresponding membership values (μ) for different combinations of input variables. The number of nodes of layer 4 is kept the same with that of the previous layer and these nodes are indicated by the symbol N. The normalized firing strength of each node of this layer is calculated as the ratio of firing strength of that node to the sum of strengths of all fired rules. Layer 6 consists of one node, as there is one output. The output is calculated as the sum of the products of normalized firing strengths and outputs of the corresponding fired rules. The performance of an ANFIS depends on membership function distributions of the input variables and coefficients of the rules (refer to Eq. 3). The membership function distributions of the linguistic terms used for five inputs have been considered to be bell-shaped and asymmetric in nature as given below.

$$\mu_j = \frac{1}{1 + \left|\frac{(I - c_j)}{a_j}\right|^{2b_j}}, \tag{4}$$

where I is the input, and a_j, b_j and c_j are the parameters of the membership function of jth linguistic term corresponding to an input. The initial membership function distributions for the input:laser power (p) is shown in Fig. 6. A binary-coded GA is utilized for tuning of the ANFIS, in which the string carries information of the parameters of the membership functions for inputs and coefficients of the rules as shown in Fig. 7. The fitness of the GA-string has been calculated according to the Eq. (2). The GA through its extensive search determines the optimal ANFIS, which will be used for making the predictions related to laser forming process.

A batch mode of training has been used in both the above approaches. In this mode of training, the parameters of the network are updated after all the training examples are passed or in other way, it can be said that the cost function is defined by the average error of the network for the whole training data set. From the statistical learning theory, a batch mode of training can be viewed as a form of statistical inference and, therefore, it is well suited for solving nonlinear regression problems. The batch mode of training may inject adaptability to the network, as it is implemented using the average effect of the training data. It is important to mention that once the training of the network using the batch mode (off-line) is over, it can be used for making on-line predictions.

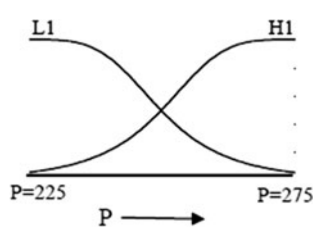


Fig. 6 Membership function distributions for laser power, p (w)

4.2 Process synthesis or inverse analysis

Process synthesis or inverse analysis aims to determine the set of input process parameters, corresponding to a set of desired outputs. It may be difficult to carry out the said inverse analysis using the obtained regression equation, as the associated transformation matrix may not be invertible. This problem of process synthesis of laser forming could be handled effectively using the soft computing tools, namely neural networks, fuzzy logic and genetic algorithm, etc., as they have the ability to establish complex relationships between a set of inputs and outputs based on some experimental data. In order to get a desired curved surface from a flat sheet metal using laser forming process, multiple scans are required at different positions to obtain different amounts of deformation. Now, to achieve different amounts of deformation or bending angles (A) and different scan positions (r), the required process parameters (p, v, d, n) are to be determined. Therefore, the inverse problem has been formulated and solved by considering (A and r) as inputs and (p, v, d and n) as outputs. The following section describes inverse analysis of the laser forming process using neural network and neuro-fuzzy inference system.

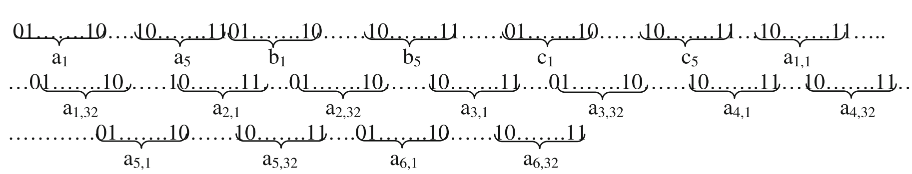
4.2.1 Genetic algorithm-tuned neural network (GA-NN)

GA-tuned NN model has been developed for conducting inverse analysis of the laser forming process. A neural network with two inputs (A and r), one hidden layer and four outputs (p, v, d and n) has been used to carry out the inverse analysis. To form a 2D surface, it is required to generate different bending angles (A) at different positions (r), which can be calculated from the desired geometry of the surface to be formed. Therefore, the bending angle (A) and scan position (r) have been considered as inputs for inverse analysis. Three transfer functions, namely linear, Gaussian and log-sigmoid are used in the input, hidden and output layers, respectively. A similar procedure as discussed above has been followed to determine the optimal network for inverse predictions in laser forming process.

4.2.2 Genetic algorithm-tuned adaptive neuro-fuzzy inference system (GA-ANFIS)

ANFIS model has also been developed to carryout inverse analysis of the laser forming process. Bending angle

Fig. 7 GA-string uesd for representing the parameters of GA-ANFIS Approach



(A) and scan position (r) are taken as inputs, and laser power (p), scan speed (v), spot diameter (d) and number of scans (n) are considered as the outputs for the inverse analysis. Each input has been represented using three linguistic terms, i.e., LW: Low; M: Medium; H: High, and they are assumed to have bell-shaped membership functions. The number of rules for these two inputs becomes equal to $3 \times 3 = 9$. The outputs of the network have been calculated following the same procedure, as explained above.

In both the above approaches used for conducting the forward and inverse analyses, the fitness of the GA has been calculated as the mean absolute error in predictions in order to obtain the optimum parameters of the networks. It is important to mention here that a GA, in principle, solves a maximization problem. However, in the present study, the optimization problem has been formulated as a minimization problem, where the aim is to minimize the fitness, i.e., mean absolute error in predicting the outputs. To solve a minimization problem using a GA, either the minimization problem has to be converted into a corresponding maximization problem or a slight modification is to be made (i.e., a greater than sign is to be replaced by a less than sign) in the tournament selection scheme used in it. In the present paper, the second option has been utilized, where the solution having the minimum fitness value has been selected for the mating pool.

5 Results and discussion

The accuracies of different approaches, i.e., GA-NN, GA-ANFIS and regression analysis have been compared in predicting bending angles of a laser forming process. In case of inverse analysis, the performances of the GA-NN and GA-ANFIS have been compared to predict the process parameters.

5.1 Results of modeling of bending angle

The aim of developing the models of bending angle in laser forming process is to determine the deformed shape of the sheet metal for a set of heating lines. Results of the developed models are stated and discussed below.

5.1.1 Results of GA-NN

To decide the optimum structure of the network (decided by the number of hidden neurons) to be used to model the laser forming process, a parametric study is conducted. The fitness values are determined for different number of hidden neurons of the network after keeping the GA-parameters initially fixed as follows: population size $N = 100$, maximum number of generations $G = 100$, probability of crossover $p_c = 0.90$ and probability of mutation $p_m = 0.001$. The minimum fitness value (i.e., 0.401513) is obtained for six number of hidden neurons (refer to Fig. 8) and, therefore, the structure of the network is determined as $5-6-1$, as displayed in Fig. 9. Now, to obtain the optimal network to be used for making the predictions, a systematic GA-parametric study has been conducted, as the performance of a GA depends on its parameters, namely crossover probability, mutation probability, population size and maximum number of generations. The results of GA-parametric study are shown in Fig. 10. The optimal values of GA-parameters are found to be as follows: crossover probability $p_c = 0.70$, mutation probability $p_m = 0.002$, population size $N = 130$, maximum number of generations $G = 50{,}000$. The minimum value of fitness is obtained as $f = 0.232031$ at the generation number of $G = 44{,}401$. The performance of the optimized network has been tested on 15 experimental test cases and the average absolute percent deviation in predictions is found to be equal to 7.98.

To decide the optimal architecture of the network, the fitness values are calculated for different numbers of hidden neurons varying from 2 to 15 using a set of fixed GA-parameters (i.e., $p_c = 0.9$, $p_m = 0.001$, $N = 100$ and

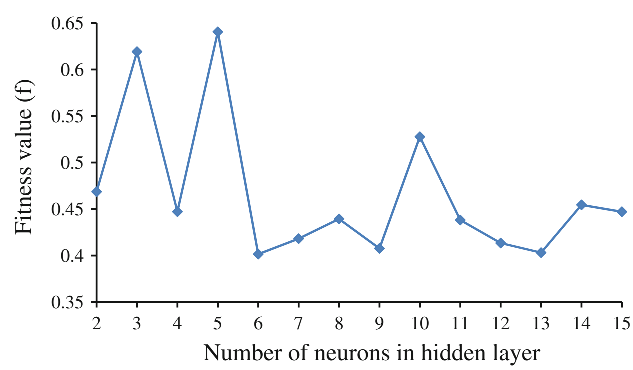


Fig. 8 Optimum fitness values for different number of hidden neurons of the network

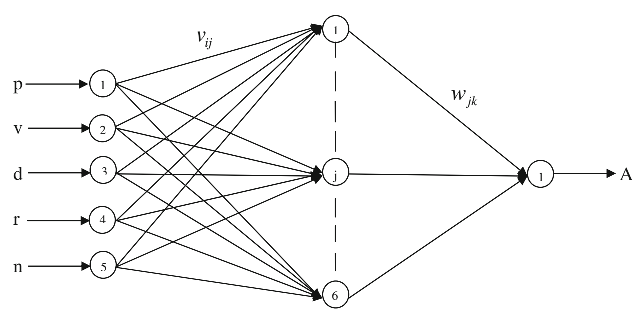
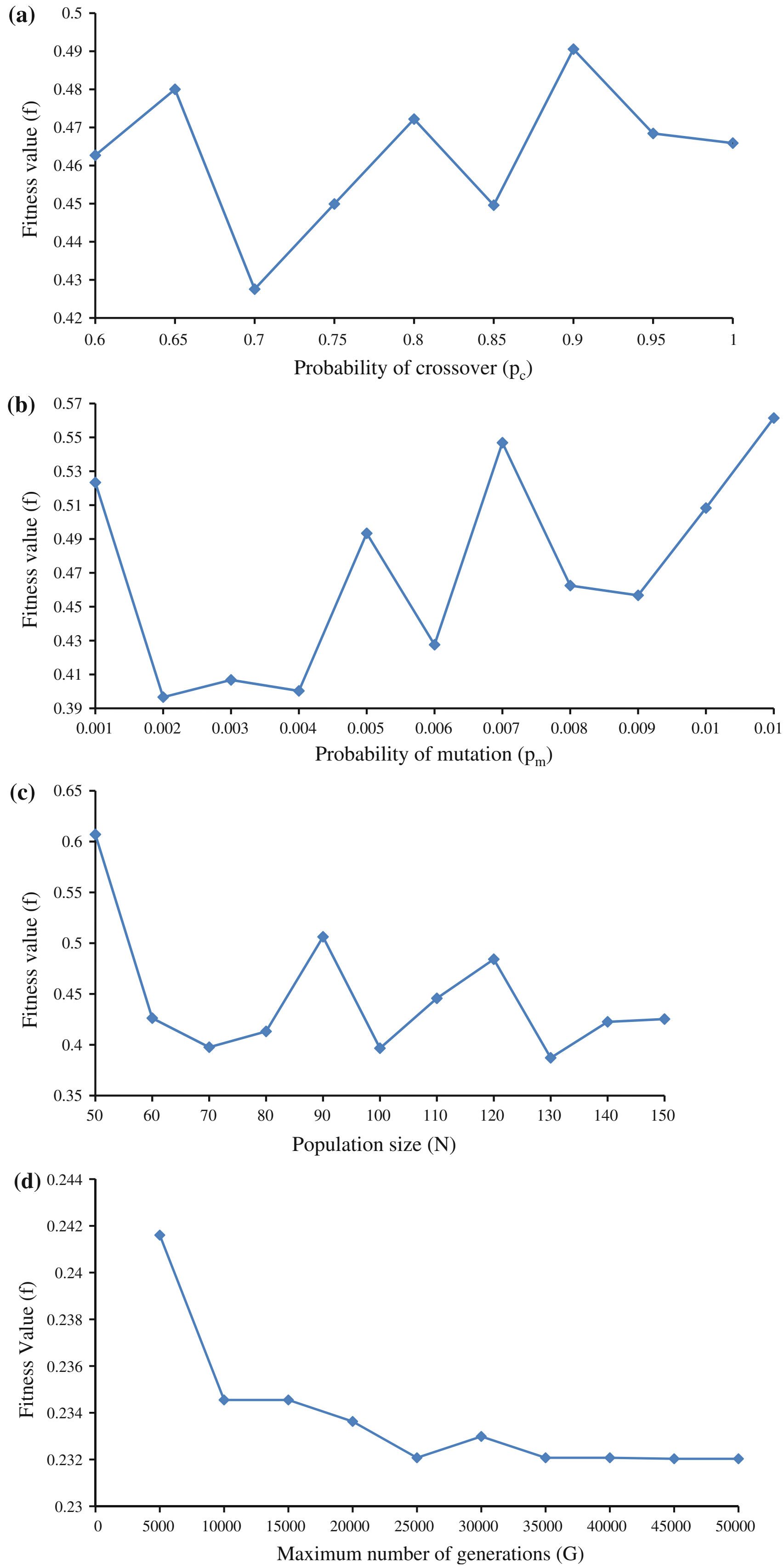


Fig. 9 GANN architecture used for modeling of laser forming process

Fig. 10 Results of GA-parametric study for the GANN model: **a** fitness versus p_c; **b** fitness versus p_m; **c** fitness versus N; **d** fitness versus G



$G = 100$). Thus, the network providing the minimum fitness value has been identified. It is important to mention here that NN-architecture determined in this way might not be the true optimal but could be a near-optimal one. In order to carry out the detailed GA-parametric study for each number of hidden neurons (varying from 2 to 15), a huge number of experiments are to be carried out. For simplicity, the above study has been conducted for a fixed set of GA-parameters (as mentioned above) initially by varying the number of hidden neurons in the range of (2, 15). However, once the number of hidden neurons of the network is decided, the GA-parameters have been determined through a detailed and systematic study.

5.1.2 Results of GA-ANFIS

A binary-coded GA is employed to determine the optimal network used for the prediction of bending angle in the laser forming process. The optimal set of GA-parameters is obtained through a systematic parametric study. The following GA-parameters are found to yield the best results: crossover probability $p_c = 0.80$, mutation probability $p_m = 0.001$, population size $N = 100$, maximum number of generations $G = 50{,}000$. The minimum value of fitness (i.e., $f = 0.009965$) is found to obtain at the 35952nd generation. The performance of the optimized network has been tested on 15 experimental test cases and the value of average absolute percent deviation in predictions is seen to be equal to 9.55.

The performances of two soft computing-based approaches (i.e., GA-NN and GA-ANFIS) and statistical regression analysis have been compared for 15 test cases in terms of average absolute percent deviation in predictions of bending angle. The values of average absolute percent deviations in prediction of bending angle (A) are obtained as 7.82, 7.98 and 9.55 by the regression analysis, GA-NN and GA-ANFIS approaches, respectively, as shown in Fig. 11 and Table 2. From these results, it can

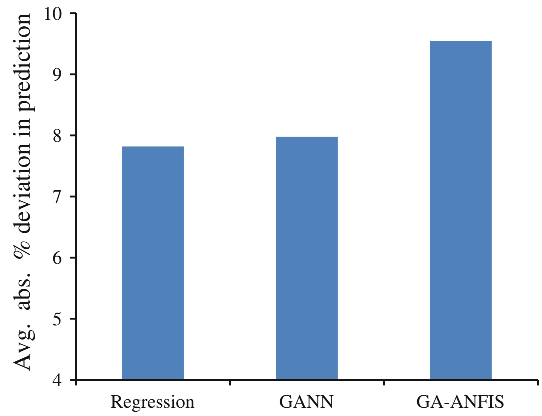


Fig. 11 Comparisons of prediction accuracy of different models to predict bending angles

Table 2 Comparison of prediction accuracy of different models for bending angle

Test no.	Experimental bending angle	Prediction accuracy of different models								
		Regression model			GANN model			GA-ANFIS model		
		Predicted value	Abs. deviat on	Abs. % deviation	Predicted value	Abs. deviation	Abs. % deviation	Predicted value	Abs. deviation	Abs. % deviation
1	5.93	5.77	0.15	2.62	5.77	0.16	2.72	6.12	0.19	3.24
2	5.82	6.43	0.61	10.54	6.44	0.62	10.68	6.48	0.66	11.41
3	6.37	6.83	0.46	7.29	6.68	0.31	4.94	7.06	0.69	10.77
4	11.85	12.69	0.84	7.07	12.44	0.59	4.97	13.04	1.19	10.07
5	5.72	6.48	0.76	13.31	6.36	0.64	11.20	6.49	0.77	13.50
6	5.91	5.10	0.81	13.73	5.05	0.86	14.60	5.41	0.49	8.37
7	8.18	8.45	0.27	3.35	8.41	0.23	2.83	8.86	0.68	8.31
8	8.93	9.64	0.71	7.92	9.72	0.79	8.85	9.87	0.94	10.56
9	6.95	6.87	0.08	1.10	7.01	0.06	0.89	7.02	0.07	1.04
10	8.29	8.47	0.18	2.12	8.43	0.14	1.66	8.75	0.46	5.61
11	8.70	9.14	0.44	5.01	9.28	0.58	6.72	9.48	0.78	8.98
12	8.41	9.65	1.24	14.75	9.68	1.27	15.08	9.78	1.37	16.32
13	8.61	9.32	0.71	8.24	9.62	1.0	11.71	9.48	0.87	10.10
14	6.08	6.96	0.88	14.54	6.91	0.83	13.61	7.01	0.93	15.28
15	10.48	11.08	0.60	5.69	11.45	0.97	9.29	11.50	1.02	9.71
Average of absolute % deviation				7.82			7.98			9.55
Standard deviation				4.50			4.65			3.93

be seen that the GA-NN approach has performed slightly better than the GA-ANFIS approach. The GA-NN approach gives prediction accuracy close to that of the regression model. Both the regression analysis and GA-NN models are found to perform better than the GA-ANFIS model in terms of accuracy in predictions. However, the variation in error for different test cases is seen to be less in the GA-ANFIS approach compared to that of the other approaches (refer to standard deviation values of different approaches given in Table 2). This could be due to inherent adaptive nature of the GA-ANFIS approach obtained by adjusting the membership function distributions of the variables. In case of GA-ANFIS approach, the prediction accuracy can be further improved using more number of linguistic terms (say three per input instead of two) for the inputs but at the cost of more computation. Moreover, it is felt that there is a chance of further improvement of the performances of the developed GA-NN and GA-ANFIS approaches.

The performances of the developed soft computing-based models (both GA-NN and GA-ANFIS) for predicting bending angles have been further verified using twofold cross-validation approach. The entire training data set of 1,000 has been divided into two sub-sets of equal size at random. One sub-set is used for the training of NN and the other one is utilized for validation, and the training and testing data are interchanged also. The advantages of this method are that both the training and test sets are large, and each data point is used for both the training and validation. The similar method has been used for both the GA-NN and GA-ANFIS models.

For the GA-NN model, the optimized architectures of the network are determined as 5-9-1 and 5-6-1 for two combinations of the data sub-sets. The optimal fitness values for these two combinations of the data sets are found to be equal to 0.178318 and 0.218035, after running the GA for a maximum of 50,000 generations with the optimized sets of GA-parameters: ($p_c = 0.8$, $p_m = 0.002$, $N = 70$) and ($p_c = 0.7$, $p_m = 0.002$, $N = 100$), respectively. The values of average absolute percent error in predictions of bending angles for these two combinations are seen to be equal to 2.98 and 3.37, respectively. For the GA-ANFIS approach, the optimized fitness values are determined as 0.010107 and 0.009317 for the above two combinations of data sets by running the GA with the optimal parameters: ($p_c = 0.7$, $p_m = 0.001$, $N = 90$) and ($p_c = 0.6$, $p_m = 0.001$, $N = 80$), respectively. For both these cases, the GA is run for a maximum of 50,000 generations. The values of average absolute percent error in predictions of bending angles are obtained as 2.43 and 2.59, respectively, for the two combinations of the data sets. Therefore, the performances of both the GA-NN and GA-ANFIS approaches are found to be reasonably good.

5.2 Results of inverse analysis

Process synthesis or inverse analysis of the laser forming process is carried out to determine the process parameters, such as laser power (p), scan speed (v), spot diameter (d) and number of scans (n) to get a desired bending angle (A) at a particular scan position (r). Both the GA-NN and GA-ANFIS models have been used for the said purpose.

5.2.1 Results of GA-NN

Bending angle (A) and scan position (r) are considered as the inputs, and laser power (p), scan speed (v), spot diameter (d) and number of scans (n) are taken as the outputs for inverse analysis of the laser forming process. To decide the architecture of the GA-tuned neural network (GA-NN), the number of hidden neurons is determined through a thorough parametric study. The fitness values of the network are calculated for different number of neurons (varying from 2 to 15) at the hidden layer by considering the following constant GA-parameters: $N = 100$, $G = 100$, $p_c = 0.90$ and $p_m = 0.001$, and the minimum fitness value ($f = 0.226306$) has been obtained for 11 numbers of hidden neurons. Therefore, the optimum architecture of the NN for conducting the inverse analysis is found to be as 2-11-4, as shown in Fig. 12.

To determine the optimized network to be used for inverse predictions, a parametric study is conducted to find the optimal set of GA-parameters. The following GA-parameters are seen to yield the best results: $p_c = 1.0$, $p_m = 0.001$, $N = 60$, $G = 50{,}000$. The minimum value of fitness (i.e., $f = 0.217208$) is obtained at the generation number of $G = 44{,}052$. For the test data, this approach yields the average absolute percent deviation in predictions as 5.34, 3.09, 12.36 and 12.42 for predicting p, v, d and n, respectively.

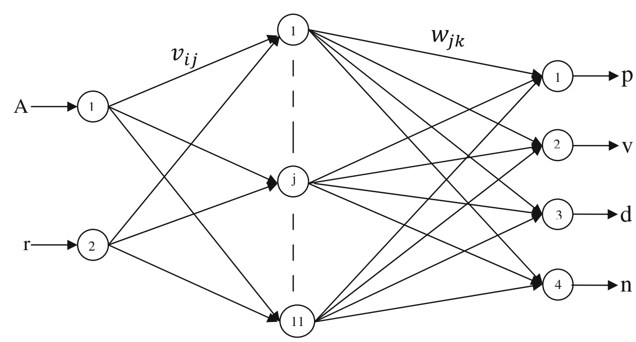


Fig. 12 NN architecture used for inverse analysis of laser forming process

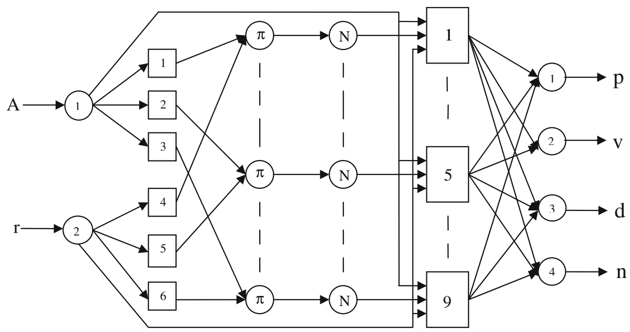


Fig. 13 ANFIS architecture used for inverse analysis of laser forming process

5.2.2 Results of GA-ANFIS

Inverse analysis has been conducted to predict the process parameters, such as laser power (p), scan speed (v), spot diameter (d) and number of scans (n) for the given bending angle (A) and scan position (r) using a GA-tuned ANFIS model. The ANFIS architecture used for the inverse analysis is shown in Fig. 13. It is to be noted that bell-shaped membership function distributions have been assumed for the inputs of ANFIS model.

The GA-parametric study has been carried out to optimize this neuro-fuzzy system. The optimum GA-parameters are found to be as follows: $p_c = 1.00, p_m = 0.002, N = 100$ and $G = 50{,}000$. The minimum fitness value (i.e., $f = 0.061454$) has been obtained at 27990th generation. The performance of the developed ANFIS model has been tested in terms of percent deviation in predictions of p, v, d and n for the 15 test cases. The values of average absolute percent deviation in predictions of p, v, d and n have been turned out to be equal to 5.44, 2.93, 12.29 and 16.31, respectively.

The performances of the two developed approaches, namely GA-NN and GA-ANFIS are found to be satisfactory for conducting inverse analysis of laser forming process, as shown in Tables 3, 4, 5 and 6. The two approaches have been compared in terms of average absolute percent deviations in predictions of the process parameters, as displayed in Fig. 14. For predicting the number of scans, the GA-NN approach is seen to perform better than the GA-ANFIS, whereas their performances are found to be comparable to predict the other process parameters. However, there is a chance of further improvement of their performances, which are mainly dependent on their knowledge base. A lot of research is going on to design and develop suitable knowledge base of an expert system, so that it can be adaptive and at the same time, predict accurately. It is also important to mention that the performances of these approaches are data-dependent. It is to be noted that the inverse model cannot be developed using the obtained regression equation, as the transformation matrix representing the input–output relationships becomes non-square and hence, singular.

Twofold cross-validation method is also applied for the verification of the developed models used in inverse

Table 3 Comparison of prediction accuracy of different models for laser power

Test no.	Actual value of laser power (W)	Prediction accuracy of different models					
		GANN model			GA-ANFIS model		
		Predicted value	Abs. deviation	Abs. % deviation	Predicted value	Abs. deviation	Abs. % deviation
1	230	237	7	3.04	243	13	5.65
2	270	239	31	11.48	241	29	10.74
3	240	241	1	0.42	241	1	0.42
4	260	260	0	0	257	3	1.15
5	240	239	1	0.42	242	2	0.83
6	230	239	9	3.91	239	9	3.91
7	270	248	22	8.15	245	25	9.26
8	240	250	10	4.17	249	9	3.75
9	270	241	29	10.74	246	24	8.89
10	260	248	12	4.62	246	14	5.38
11	230	249	19	8.26	245	15	6.52
12	240	249	9	3.75	245	5	2.08
13	230	248	18	7.83	249	19	8.26
14	260	240	20	7.69	240	20	7.69
15	270	255	15	5.56	251	19	7.04
Average of absolute % deviation				5.34			5.44
Standard deviation				3.50			3.19

Table 4 Comparison of prediction accuracy of different models for scan speed

Test no.	Actual value of scan speed (mm/s)	Prediction accuracy of different models					
		GANN model			GA-ANFIS model		
		Predicted value	Abs. deviation	Abs. % deviation	Predicted value	Abs. deviation	Abs. % deviation
1	258.33	272.82	14.49	5.61	277.54	19.21	7.44
2	275.00	269.5	5.5	2	266.98	8.02	2.92
3	275.00	270.59	4.41	1.6	267.33	7.67	2.79
4	258.33	262.57	4.24	1.64	260.89	2.56	0.99
5	258.33	269.46	11.13	4.31	264.22	5.89	2.28
6	275.00	271.4	3.6	1.31	268.43	6.57	2.39
7	258.33	266.62	8.29	3.21	265.61	7.28	2.82
8	275.00	266.42	8.58	3.12	268.05	6.95	2.53
9	275.00	270.63	4.37	1.59	274.54	0.46	0.17
10	258.33	267.37	9.04	3.5	262.55	4.22	1.63
11	258.33	265.97	7.64	2.96	265.2	6.87	2.66
12	275.00	266.35	8.65	3.15	265.57	9.43	3.43
13	258.33	267.09	8.76	3.39	269.04	10.71	4.15
14	258.33	271.1	12.77	4.94	268.03	9.7	3.75
15	275.00	263.96	11.04	4.01	263.98	11.02	4.01
Average of absolute % deviation				3.09			2.93
Standard deviation				1.25			1.59

Table 5 Comparison of prediction accuracy of different models for spot diameter

Test no.	Actual value of spot diameter (mm)	Prediction accuracy of different models					
		GANN model			GA-ANFIS model		
		Predicted value	Abs. deviation	Abs. % deviation	Predicted value	Abs. deviation	Abs. % deviation
1	0.550	0.644	0.094	17.09	0.653	0.103	18.73
2	0.700	0.647	0.053	7.57	0.655	0.045	6.43
3	0.550	0.638	0.088	16	0.642	0.092	16.73
4	0.700	0.623	0.077	11	0.605	0.095	13.57
5	0.700	0.655	0.045	6.43	0.665	0.035	5
6	0.550	0.641	0.091	16.55	0.647	0.097	17.64
7	0.550	0.635	0.085	15.45	0.631	0.081	14.73
8	0.700	0.625	0.075	10.71	0.619	0.081	11.57
9	0.700	0.637	0.063	9	0.642	0.058	8.29
10	0.550	0.627	0.077	14	0.623	0.073	13.27
11	0.550	0.640	0.090	16.36	0.63	0.080	14.55
12	0.700	0.634	0.066	9.43	0.629	0.071	10.14
13	0.550	0.627	0.077	14	0.623	0.073	13.27
14	0.700	0.640	0.060	8.57	0.645	0.055	7.86
15	0.550	0.623	0.073	13.27	0.619	0.069	12.55
Average of absolute % deviation				12.36			12.29
Standard deviation				3.48			3.94

analysis of laser forming process. In case of GA-NN model, the optimized architecture is determined as 2-9-4 for both the combinations of data sets. During the training using two combinations of data sets, the minimum fitness values are obtained as 0.219391 and 0.213003 corresponding to the optimized GA-parameters as ($p_c = 0.8$, $p_m = 0.004$, $N = 100$) and ($p_c = 0.9$, $p_m = 0.001$, $N = 120$), respectively. The GA is run for a maximum of

Table 6 Comparison of prediction accuracy of different models for number of scans

Test no.	Actual value of Number of scans	Prediction accuracy of different models					
		GANN model			GA-ANFIS model		
		Predicted value	Abs. deviation	Abs. % deviation	Predicted value	Abs. deviation	Abs. % deviation
1	6	6	0	0	7	1	16.67
2	6	7	1	16.67	7	1	16.67
3	8	7	1	12.5	7	1	12.5
4	14	14	0	0	14	0	0
5	8	7	1	12.5	7	1	12.5
6	6	6	0	0	7	1	16.67
7	8	10	2	25	10	2	25
8	14	10	4	28.57	10	4	28.57
9	6	7	1	16.67	8	2	33.33
10	8	9	1	12.5	9	1	12.5
11	12	11	1	8.33	11	1	8.33
12	14	10	4	28.57	10	4	28.57
13	12	9	3	25	10	2	16.67
14	6	6	0	0	7	1	16.67
15	12	12	0	0	12	0	0
Average of absolute % deviation				12.42			16.31
Standard deviation				10.53			9.33

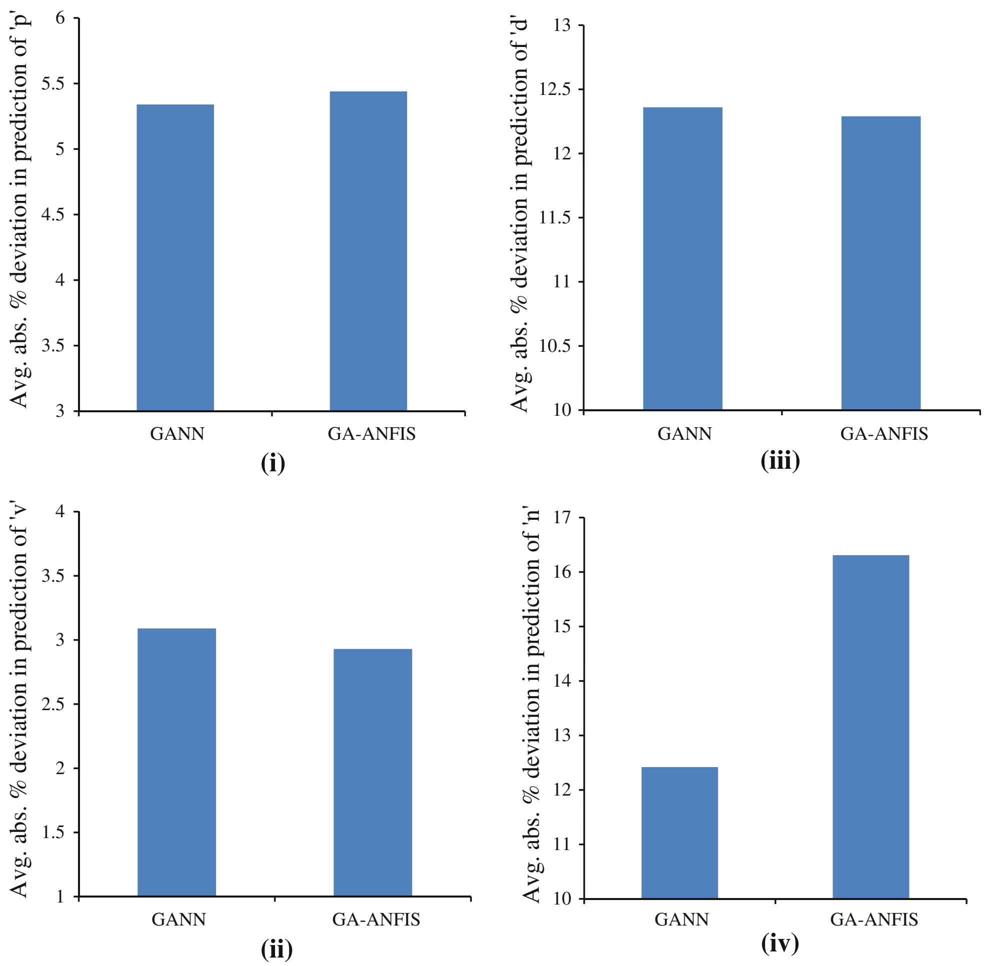


Fig. 14 Comparisons of the performances of GA-NN and GA-ANFIS models in predicting the process parameters (inverse analysis): **i** laser power (p); **ii** scan speed (v); **iii** spot diameter (d); **iv** Number of scans (n)

50,000 generations. The values of average absolute percent error in predicting the process parameters: p, v, d and n are found to be equal to 4.47, 3.21, 10.88 and 12.27, and 4.89, 3.26, 10.90 and 12.99, respectively, for the two combination of data sets. For GA-ANFIS model, the minimum fitness values are obtained as 0.061496 and 0.058811 during the training carried out using the optimized GA-parameters: ($p_c = 0.8$, $p_m = 0.001$, $N = 160$) and ($p_c = 1.0, p_m = 0.001, N = 150$), respectively, for the two combinations of data sets. The GA is allowed to run for a maximum of 50,000 generations. The values of average absolute percent error in predicting the process parameters: p, v, d and n have been obtained as 4.57, 3.20, 10.96 and 13.71, and 4.88, 3.45, 11.12 and 13.10, respectively, for the two combinations of data sets.

6 Conclusions and scope for future work

It is a novel attempt to carry out both the analysis and synthesis of laser forming process using neural network and neuro-fuzzy inference system. A batch mode of training is adopted. In case of predicting the bending angles, the performances of GA-NN and GA-ANFIS models are compared between them and with that of the conventional regression analysis for 15 test cases obtained through real experiments. Both the GA-NN and GA-ANFIS approaches are able to predict the bending angles in the laser forming process, and their performances are found to be comparable with that of the regression model. Moreover, the GA-NN approach has performed a slightly better than the GA-ANFIS approach. It has happened due to the reason that the GA faces a more difficult task for determining optimal knowledge base of ANFIS model compared to that of the NN model. In case of inverse analysis, both the models are able to predict the process parameters within reasonable accuracy limit. It is important to mention that it may be difficult to predict the process parameters using conventional regression equation, as the transformation matrix may become singular. The performances of the developed approaches have also been tested through twofold cross-validation method and found to be satisfactory. In the present paper, this method of process synthesis has been used for 2D shapes only. However, this work can be extended further to produce more complex 2D and 3D shapes by considering both the mechanisms (namely TGM and UM) of laser forming process. In the present study, the performances of GA-NN and GA-ANFIS have been compared for both the analysis and synthesis problems of laser forming process. In future, their performances will be compared with those of other soft computing-based regression methods available in the literature.

Appendix A: Experimental data collected according to CCD to develop the model of bending angle

SL. no.	Input parameters					Output: bending angle (°)		
	p (W)	v (mm/s)	d (mm)	r	n	A_1	A_2	A_3
1	225	250.0	0.500	0.25	5	4.81	5.34	5.07
2	275	250.0	0.500	0.25	5	6.71	6.61	6.82
3	225	283.0	0.500	0.25	5	4.06	4.52	3.76
4	275	283.0	0.500	0.25	5	4.56	4.20	5.00
5	225	250.0	0.750	0.25	5	3.97	3.74	3.55
6	275	250.0	0.750	0.25	5	5.69	5.85	6.04
7	225	283.0	0.750	0.25	5	2.14	1.95	2.18
8	275	283.0	0.750	0.25	5	6.45	6.29	6.68
9	225	250.0	0.500	0.75	5	4.12	4.21	4.19
10	275	250.0	0.500	0.75	5	4.41	4.73	4.56
11	225	283.0	0.500	0.75	5	2.22	2.27	2.17
12	275	283.0	0.500	0.75	5	5.05	4.95	5.00
13	225	250.0	0.750	0.75	5	3.56	3.63	3.73
14	275	250.0	0.750	0.75	5	6.12	6.23	5.94
15	225	283.0	0.750	0.75	5	1.74	1.58	1.66
16	275	283.0	0.750	0.75	5	3.89	4.08	4.20
17	225	250.0	0.500	0.25	15	11.30	11.50	11.06
18	275	250.0	0.500	0.25	15	15.21	14.47	14.83
19	225	283.0	0.500	0.25	15	8.58	8.65	8.79
20	275	283.0	0.500	0.25	15	13.00	12.40	12.80
21	225	250.0	0.750	0.25	15	8.07	8.28	7.90
22	275	250.0	0.750	0.25	15	15.47	15.28	15.40
23	225	283.0	0.750	0.25	15	7.16	7.21	7.26
24	275	283.0	0.750	0.25	15	12.24	12.47	11.93
25	225	250.0	0.500	0.75	15	11.70	11.06	11.25
26	275	250.0	0.500	0.75	15	14.20	14.34	14.47
27	225	283.0	0.500	0.75	15	9.48	9.54	9.36
28	275	283.0	0.500	0.75	15	12.53	12.46	12.5
29	225	250.0	0.750	0.75	15	7.98	8.09	8.28
30	275	250.0	0.750	0.75	15	15.20	15.28	15.10
31	225	283.0	0.750	0.75	15	6.46	6.30	6.38
32	275	283.0	0.750	0.75	15	12.25	12.31	12.37
33	225	266.5	0.625	0.50	10	6.33	6.45	6.21
34	275	266.5	0.625	0.50	10	11.13	10.84	10.95
35	250	250.0	0.625	0.50	10	8.87	8.95	9.04
36	250	283.0	0.625	0.50	10	8.67	8.84	8.56
37	250	266.5	0.500	0.50	10	9.30	8.78	9.02
38	250	266.5	0.750	0.50	10	7.44	7.75	8.35
39	250	266.5	0.625	0.25	10	8.88	9.00	8.60
40	250	266.5	0625	0.75	10	9.25	8.78	8.97
41	250	266.5	0.625	0.50	5	4.60	4.85	5.08
42	250	266.5	0.625	0.50	15	13.67	13.80	13.56
43	250	266.5	0.625	0.50	10	9.24	9.13	9.19

Appendix B: Data collected for testing the models of bending angle

SL. no.	Input parameters					Output: bending angle
	p (W)	*v* (mm/s)	*d* (mm)	*r*	*n*	*A* (°)
1	230	258.33	0.550	0.30	6	5.93
2	270	275.00	0.700	0.60	6	5.82
3	240	275.00	0.550	0.40	8	6.37
4	260	258.33	0.700	0.70	14	11.85
5	240	258.33	0.700	0.70	8	5.72
6	230	275.00	0.550	0.40	6	5.91
7	270	258.33	0.550	0.60	8	8.18
8	240	275.00	0.700	0.30	14	8.93
9	270	275.00	0.700	0.30	6	6.95
10	260	258.33	0.550	0.40	8	8.29
11	230	258.33	0.550	0.70	12	8.70
12	240	275.00	0.700	0.60	14	8.41
13	230	258.33	0.550	0.30	12	8.61
14	260	258.33	0.700	0.40	6	6.08
15	270	275.00	0.550	0.60	12	10.48

References

Akbilgic O, Bozdogan H (2011) Predictive subset selection using regression trees and RBF neural networks hybridized with the genetic algorithm. Eur J Pure Appl Math 4(4):467–485

Carlone P, Palazzo GS, Pasquino R (2008) Inverse analysis of the laser forming process by computational modeling and methods. Comput Math Appl 55:2018–2032

Casalino G, Ludovico AD (2002) Parameter selection by an artificial neural network for a laser bending process. IMechE Part B J Eng Manuf 216:1517–1520

Chen DJ, Xiang YB, Wu SC, Li MQ (2002) Application of fuzzy neural network to laser bending process of sheet metal. Mater Sci Technol 18:677–680

Cheng PJ, Lin SC (2000) Using neural networks to predict bending angle of sheet metal formed by laser. Int J Mach Tools Manuf 40:1185–1197

Cheng PJ, Lin SC (2001) An analytical model to estimate angle formed by laser. J Mater Process Technol 108:314–319

Cheng JG, Yao YL (2004) Process synthesis of laser forming by genetic algorithm. Int J Mach Tools Manuf 44:1619–1628

Dragos V, Dan V, Kovacevic R (2000) Prediction of the laser sheet bending using neural network. In: IEEE international symposium on circuits and systems, pp 686–689

Du Y, Wang X (2010) Improved BP network to predict bending angle in the laser bending process for sheet metal. In: International conference on intelligent system design and engineering application, Cairo, Egypt, pp 839–843

Gisario A, Barletta M, Conti C, Guarino S (2011) Springback control in sheet metal bending by laser-assisted bending: experimental analysis, empirical and neural network modeling. Opt Lasers Eng 49:1372–1383

Griffiths J, Edwardson SP, Dearden G, Watkins KG (2010) Finite element modeling of laser forming at macro and micro scales. Phys Procedia 5:371–380

Guarino S, Ucciardello N, Tagliaferri V (2007) An application of neural network solutions to modeling of diode laser assisted forming process of AA6082 thin sheets. Key Eng Mater 344:325–332

Herrera F, Lozano M, Verdegay JL (1995) Tuning fuzzy logic controllers by genetic algorithms. Int J Approx Reason 12:293–315

Hu Z, Labudovic M, Wang H, Kovacevic R (2001) Computer simulation and experimental investigation of sheet metal bending using laser beam scanning. Int J Mach Tools Manuf 41:589–607

Hu J, Dang D, Shen H, Zhang Z (2012) A finite element model using multi-layered shell element in laser forming. Opt Laser Technol 44:1148–1155

Jang JSR (1993) ANFIS: adaptive-network-based fuzzy inference system. IEEE Trans Syst Man Cybern 23(3):665–685

Keller JM, Yager RR, Tahani H (1992) Neural nework implementation of fuzzy logic. Fuzzy Sets Syst 45(1):1–12

Khashei M, Bijari M (2010) An artificial neural network (p, d, q) model for time series forecasting. Expert Syst Appl 37:479–489

Kuo HC, Wu LJ (2002) Automation of heat bending in shipbuilding. Comput Ind 48:127–142

Kyrsanidi AK, Kermanidis TB, Pantelakis SG (2000) An analytical model for the prediction of distortions caused by the laser forming process. J Mater Process Technol 104:94–102

Liu C, Yao YL (2002) Optimal and robust design of the laser forming process. J Manuf Processes 4:52–66

Maji K, Pratihar DK (2010) Forward and reverse mappings of electrical discharge machining process using adaptive network-based fuzzy inference system. Expert Syst Appl 37:8566–8574

Maji K, Pratihar DK, Nath AK (2012) Experimental investigations, modeling and optimization of multi-scan laser forming of AISI 304 stainless steel sheet. Int J Adv Manuf Technol (under review)

Martino FD, Loia V, Sessa S (2010) Fuzzy transforms method and attribute dependency in data analysis. Inf Sci 180:493–505

Martino FD, Loia V, Sessa S (2011) Fuzzy transforms method in prediction data analysis. Fuzzy Sets Syst 180:146–163

Montgomery DC (2001) Design and analysis of experiments. Wiley, New York

Nasrabadi E, Hashemi SM (2008) Robust fuzzy regression analysis using neural networks. Int J Uncertain Fuzziness Knowl Based Syst 16(4):579–598

Nguyen TT, Yang YS, Bae KY, Choi SN (2009) Prediction of deformations of steel plate by artificial neural network in forming process with induction heating. J Mech Sci Technol 23:1211–1221

Pratihar DK (2008) Soft computing. Narosa Publishing House, New Delhi

Pratihar DK, Deb K, Ghosh A (1999) A genetic-fuzzy approach for mobile robot navigation among moving obstacles. Int J Approx Reason 20:145–172

Shen H, Vollertsen F (2009) Modeling of laser forming—a review. Comput Mater Sci 46:834–840

Shen H, Shi Y, Yao Z, Hu J (2006a) An analytical model for estimating deformation in laser forming. Comput Mater Sci 37:593–598

Shen H, Shi YJ, Yao ZQ, Hu J (2006b) Fuzzy logic model for bending angle in laser forming. Mater Sci Technol 22:981–986

Vollertsen F (1994) An analytical model for laser bending. Lasers Eng 2:261–276

Vollertsen F, Geiger M, Li WM (1993) FDM-and FEM simulation of laser forming: a comparative study. In: Proceedings of the fourth

international conference on technology of plasticity, pp 1793–1798
Wang X, Xu W, Chen H, Wang J (2008) Parameter prediction in laser bending of aluminum alloy sheet. Front Mech Eng China 3(3):293–298
Whitley D, Starkweather T, Bogart C (1990) Genetic algorithms and neural networks: optimizing connections and connectivity. Parallel Comput 14:347–361
Wu S, Ji Z (2002) FEM simulation of the deformation field during the laser forming of sheet metal. J Mater Process Technol 121: 269–272
Zhang L, Michaleris P (2004) Investigation of Lagrangian and Eulerian finite element methods for modeling the laser forming process. Finite Element Anal Des 40:383–405
Zhang P, Guo B, Shan DB, Ji Z (2007) FE simulation of laser curve bending of sheet metals. J Mater Process Technol 184:157–162