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Uncertain aggregate production planning

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Abstract Based on uncertainty theory, multiproduct aggregate production planning model is presented, where the market demand, production cost, subcontracting cost, etc., are all characterized as uncertain variables. The objective is to maximize the belief degree of obtaining the profit more than the predetermined profit over the whole planning horizon. When these uncertain variables are linear, the objective function and constraints can be converted into crisp equivalents, the model is a nonlinear programming, then can be solved by traditional methods. An example is given to illustrate the model and the converting method.

Keywords Aggregate production planning · Uncertain variable · Uncertain distribution

1 Introduction

The goal of making aggregate production planning (APP) is to determine the optimal product quantity, inventory level, etc., to meet the demand for all products over a finite planning horizon for obtaining the maximum profit or minimum cost. Since Holt et al. (1955) proposed the HMMS rule, a lot of researchers have developed various types of models and approaches to solve APP decision making problems. Zhang et al. (2012) built a mixed integer linear programming (MILP) model to characterize mathematically the problem of APP with capacity expansion in a manufacturing system including multiple activity centers, and developed a hybrid heuristic combining beam search with capacity shifting, which was capable of producing a high quality solution within reasonable computational time. Ramezanian et al. (2012) developed an MILP model for general two-phase aggregate production planning systems, and designed a genetic algorithm for solving this problem. Bergstrom and Smith (1970) generalized the HMMS approach to a multiproduct formulation, which was further extended by Hausman and Mcclain (1971) to a stochastic programming model to deal with the randomness of product demand. Bitran and Yanassee (1984) considered the problems of determining production plans over a number of time periods under stochastic demands. Fung et al. (2003) developed a fuzzy multiproduct aggregate production planning model whose solutions were introduced to cater to different scenarios under various decision making preferences by using parametric programming, best balance and interactive techniques. Wang and Fang (2001) presented a fuzzy linear programming method for solving APP problems with multiple objectives where the product price, unit cost to subcontract, work force level, production capacity and market demand were fuzzy in nature. Then an interactive solution procedure was developed to provide a compromise solution. Wang and Liang (2005) provided an interactive possibilistic linear programming approach for solving APP problems with fuzzy demand, interrelated operating costs, and capacity. Based on ranking methods of fuzzy numbers and tabu search, Baykasoglu and Gocken (2010) proposed a direct solution method to solve fuzzy multi-objective aggregate production planning problem. The parameters of the problem were defined as triangular fuzzy numbers.

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However, in the real APP decision making problems, randomness and fuzziness usually coexist. Fuzzy random variable is a strong tool to deal with the above problems (Kwakernaak 1978, 1979; Liu 2001a, b). Ning et al. (2006) established a multiproduct aggregate production planning (APP) decision making model in fuzzy random environments. The objective was to maximize the chance of obtaining the profit more than the predetermined profit over the whole planning horizon. In the model, the market demand, production cost, maximum capital level, etc., were all characterized as fuzzy random variables. A hybrid optimization algorithm combining fuzzy random simulation, genetic algorithm (GA), neural network (NN) and simultaneous perturbation stochastic approximation (SPSA) algorithm was proposed to solve the model.

When historical data are not available to estimate a probability distribution, we have to invite some domain experts to evaluate their belief degree that each event will occur. Since human beings usually overweight unlikely events, the belief degree may have much larger variance than the real frequency. Perhaps some people think that the belief degree is subjective probability. However, Liu (2012) showed that it is inappropriate because probability theory may lead to counterintuitive results in this case. In order to deal with this phenomena, uncertainty theory was founded by Liu (2007) and refined by Liu (2010a). Nowadays uncertainty theory has become a branch of mathematics for modeling human uncertainty, and have been developed and applied widely to operational research, risk analysis, reliability, comprehensive evaluation, portfolio selection, transportation planning, etc. (Liu 2009a, b, 2010b, 2011, 2012; Yan 2009; Yang et al. 2009, 2012; Liu and Ha 2010; Rong 2011; Liu and Chen 2012; Li et al. 2012a, b). Liu (2011) proposed an uncertain comprehensive evaluation (UCE) method, where all weight values of indices in evaluated system were characterized as uncertain variables to constitute a vector, and all the corresponding remarks to evaluated indices were also characterized as uncertain variables to constitute a matrix. Liu (2012) presented an analytic method to solve a class of uncertain differential equations. Liu and Chen (2012) introduced an uncertain currency model, derived a currency option pricing formula for uncertain currency market, and discussed some mathematical properties. Liu and Ha (2010) proved that the expected value of monotone function of uncertain variable was just a Lebesgue–Stieltjes integral of the function with respect to its uncertainty distribution, and gave some useful expressions of expected value of function of uncertain variables. Rong (2011) provided two new models of economic order quantity (EOQ), where the holding cost, shortage cost and ordering cost per unit were assumed to be uncertain variables. The models could be converted into deterministic equivalents and solved by 99-method. Yan (2009) provided two new models for portfolio selection, where the securities were assumed to be uncertain variables. The original problems could be converted into their crisp equivalents when the returns were chosen as some special uncertain variables such as rectangular uncertain variable, triangular uncertain variable, trapezoidal uncertain variable and normal uncertain variable.

Motivated by all the literature mentioned above, this paper will present an uncertain APP model based on uncertainty theory, where the market demand, production cost, subcontracting cost, etc., are all characterized as uncertain variables. The objective function and constraints can be converted into crisp equivalents when they are linear uncertain variables. Then the model can be solved by traditional methods. At the end of this paper, an example is given to illustrate the model and the converting method.

2 Uncertain variable

Definition 1 Liu (2007) Let Γ be a nonempty set, τ a σ-algebra over Γ, and M an uncertain measure, M meets the three axioms: (1) (normality axiom) $M\{\Gamma\} = 1$; (2) (duality axiom) $M\{\Lambda\} + M\{\Lambda^c\} = 1$ for any event Λ. (3) (subadditivity axiom) For every countable sequence of events $\{\Lambda_i\}, M\{\bigcup_{i=1}^{\infty} \Lambda_i\} \leq \sum_{i=1}^{\infty} M\{\Lambda_i\}$. Then the triplet (Γ, τ, M) is called an uncertainty space.

Definition 2 Liu (2007) An uncertain variable is a measurable function from an uncertainty space (Γ, τ, M) to the set of real numbers, i.e., for any Borel set B of real numbers, the set $\{\xi \in B\} = \{r \in \Gamma | \xi(r) \in B\}$ is an event.

Definition 3 Liu (2007) The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = M\{\xi \leq x\} \tag{1}$$

for any real number x.

Definition 4 Liu (2007) An uncertain variable ξ is called linear if it has a linear uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a, \\ (x-a)/(b-a), & \text{if } a \leq x \leq b \\ 1, & \text{if } x \geq b \end{cases} \tag{2}$$

denoted by $L(a, b)$ where a and b are real numbers with $a < b$.

For other special uncertain distributions, see Liu (2007).

Definition 5 Liu (2007) An uncertain variable ξ is called zigzag if it has a zigzag uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \le a, \\ (x-a)/2(b-a), & \text{if } a \le x \le b \\ (x+c-2b)/2(c-b), & \text{if } b \le x \le c \\ 1, & \text{if } x \ge c \end{cases} \tag{3}$$

denoted by $Z(a, b, c)$ where a, b, c are real numbers with $a < b < c$.

Definition 6 Liu (2007) An uncertain variable ξ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad x \in R \tag{4}$$

denoted by $N(e, \sigma)$ where e and σ are real numbers with $\sigma > 0$.

Theorem 1 Liu (2007) *Assume that ξ_1 and ξ_2 are independent linear uncertain variables $L(a_1, b_1)$ and $L(a_2, b_2)$, respectively. Then the sum $\xi_1 + \xi_2$ is also a linear uncertain variable $L(a_1 + a_2, b_1 + b_2)$, i.e.,*

$$L(a_1, b_1) + L(a_2, b_2) = L(a_1 + a_2, b_1 + b_2). \tag{5}$$

The product of a linear uncertain variable $L(a, b)$ and a scalar number $k > 0$ is also a linear uncertain variable $L(ka, kb)$, i.e.,

$$kL(a, b) = L(ka, kb) \tag{6}$$

Theorem 2 Liu (2007) *The product of a linear uncertain variable $L(a, b)$ and a scalar number $k < 0$ is also a linear uncertain variable $L(kb, ka)$, i.e.,*

$$kL(a, b) = L(kb, ka) \tag{7}$$

3 Formulation for uncertain APP model

Assume that a company produces N types of products to meet the market demands over a planning horizon T in uncertain environments. For convenience, the notations used in this paper are described in Table 1, where the notations D_{nt}, g_{nt}, j_{nt}, z_{nt}, d_{nt}, e_{nt}, h_t, l_t, i_{nt}, m_{nt}, v_{nt}, r_{nt}, $W_{t\max}$, $M_{t\max}$, $V_{t\max}$ and $C_{t\max}$ are characterized as uncertain variables, Q_{nt}, O_{nt}, S_{nt}, I_{nt}, B_{nt}, H_t and L_t are decision variables, $n = 1, 2, \ldots, N,\ t = 1, 2, \ldots, T$.

In an APP decision making problem, the profit function can be defined as follows,

$$\begin{aligned} f = & \sum_{n=1}^{N}\sum_{t=1}^{T} r_{nt}(I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt}) \\ & - \sum_{n=1}^{N}\sum_{t=1}^{T}(g_{nt}Q_{nt} + j_{nt}O_{nt} + z_{nt}S_{nt} + d_{nt}I_{nt} + e_{nt}B_{nt}) \\ & - \sum_{t=1}^{T}(h_t H_t + l_t L_t), \end{aligned} \tag{8}$$

where r_{nt}, g_{nt}, j_{nt}, z_{nt}, d_{nt}, e_{nt}, h_t, and l_t are uncertain variables, the term $\sum_{n=1}^{N}\sum_{t=1}^{T} r_{nt}(I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt})$ is the total revenue, and the term $\sum_{n=1}^{N}\sum_{t=1}^{T}(g_{nt}Q_{nt} + j_{nt}O_{nt} + z_{nt}S_{nt} + d_{nt}I_{nt} + e_{nt}B_{nt})$ is the total production cost, and $\sum_{t=1}^{T}(h_t H_t + l_t L_t)$ is the cost of changing labor level including the costs to hire and lay off workers. It is obvious that the profit function f is an uncertain variable.

In the real APP decision making problems with uncertain coefficients, the demand D_{nt} cannot be predicted precisely. Therefore, the decision can only be made to meet the market demand within a permitted fluctuation scope at a predetermined confidence level. If the decision maker hopes that the belief degree of satisfying the market demand within a permitted fluctuation scope is at least λ, then the constraints on product-inventory are as follows,

$$\begin{aligned} M\{ & | I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} \\ & + B_{nt} - D_{nt} | \le p\} \ge \lambda, \end{aligned} \tag{9}$$

where p is the permitted fluctuation scope, λ is the predetermined confidence level, $0 < \lambda \le 1, n = 1, 2, \ldots, N$, and $t = 1, 2, \ldots, T$.

If the decision maker hopes that the belief degree of balancing the labor level in two successive periods within a permitted fluctuation scope is at least β, the constraints on labor level can be described as follows,

$$\begin{aligned} M\Bigg\{ & \Bigg| \sum_{n=1}^{N} i_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - L_t \\ & - \sum_{n=1}^{N} i_{nt}(Q_{nt} + O_{nt}) \Bigg| \le q \Bigg\} \ge \beta, \end{aligned} \tag{10}$$

where q is the permitted fluctuation scope, β is the predetermined confidence level, $0 < \beta \le 1$, and $t = 1, 2, \ldots, T$.

If the decision maker expects that the belief degree that the hours of labor used by all products in period t do not exceed the maximum labor level available in the period is at least ς, the constraints on labor usage are as follows,

$$M\left\{ \sum_{n=1}^{N} i_{nt}(Q_{nt} + O_{nt}) \le W_{t\max} \right\} \ge \varsigma, \tag{11}$$

where ς is the predetermined confidence level, $0 < \varsigma \le 1$, and $t = 1, 2, \ldots, T$.

If the decision maker wishes that the belief degree that the hours of machine usage by all products in period t does not exceed the maximum machine capability available in the period is at least δ, the constraints on machine usage are as follows,

$$M\left\{ \sum_{n=1}^{N} m_{nt}(Q_{nt} + O_{nt}) \le M_{t\max} \right\} \ge \delta, \tag{12}$$

Table 1 Notation

Notation	Meaning
N	Types of products
T	Planning horizon
f	Profit function over T
D_{nt}	Demand for the nth product in period t (units)
g_{nt}	Production cost in regular time per unit of the nth product in period t ($/unit)
Q_{nt}	Production in regular time per unit of the nth product in period t (units)
j_{nt}	Production cost in overtime per unit of the nth product in period t ($/unit)
O_{nt}	Production in overtime per unit of the nth product in period t (units)
z_{nt}	Subcontracting cost per unit of the nth product in period t ($/unit)
S_{nt}	Subcontracting quantity of the nth product in period t (units)
d_{nt}	Inventory carrying cost per unit of the nth product in period t ($/unit)
I_{nt}	Inventory level of the nth product in period t (units)
e_{nt}	Backorder cost per unit of the nth product in period t ($/unit)
B_{nt}	Backorder level of the nth product in period t (units)
h_t	Cost to hire one worker in period t ($/man-hour)
H_t	Workers hired in period t (man-hour)
l_t	Cost to lay off one worker in period t ($/man-hour)
L_t	Workers laid off in period t (man-hour)
i_{nt}	Hours of labor per unit of the nth product in period t (man-hour/unit)
m_{nt}	Hours of machine usage per unit of the nth product in period t (machine-hour/unit)
v_{nt}	Warehouse spaces per unit of the nth product in period t (ft^2/unit)
r_{nt}	Sales revenue per unit of the nth product in period t ($/unit)
$W_{t\max}$	Maximum labor level available in period t (man-hour)
$M_{t\max}$	Maximum machine capacity available in period t (machine-hour)
$V_{t\max}$	Maximum warehouse space available in period t (ft^2)
$C_{t\max}$	Maximum capital level available in period t($)

where δ is the predetermined confidence level, $0 < \delta \le 1$, and $t = 1, 2, \ldots, T$.

If the decision maker expects that the belief degree that the warehouse space used by all products in period t does not exceed the maximum warehouse space available in the period is at least σ, the constraints on warehouse space are as follows,

$$M\left\{\sum_{n=1}^{N} v_{nt} I_{nt} \le V_{t\max}\right\} \ge \sigma, \tag{13}$$

where σ is the predetermined confidence level, $0 < \sigma \le 1$, and $t = 1, 2, \ldots, T$.

If the decision maker hopes that the belief degree that all the costs in period t do not exceed the maximum capital level available in the period is at least τ, the constraints on capital are as follows,

$$M\left\{\sum_{n=1}^{N}(g_{nt}Q_{nt} + j_{nt}O_{nt} + z_{nt}S_{nt} + d_{nt}I_{nt} + e_{nt}B_{nt}) + h_t H_t + l_t L_t \le C_{t\max}\right\} \ge \tau, \tag{14}$$

where τ is the predetermined confidence level, $0 < \tau \le 1$, and $t = 1, 2, \ldots, T$.

The non-negativity constraints on decision variables are as follows,

$$Q_{nt}, O_{nt}, S_{nt}, I_{nt}, B_{nt}, H_t, L_t \ge 0, \tag{15}$$

where $n = 1, 2, \ldots, N$, and $t = 1, 2, \ldots, T$.

In many APP decision problems, a decision-maker is usually concerned about the profit rather than the cost. Moreover, the decision maker usually predetermines a number of total profit over the whole planning horizon, and wants to maximize the chance that the real profit exceeds the predetermined value. In such cases, the following uncertain APP model can be constructed,

$$\begin{cases} \max M\{f \ge f_0\} \\ \text{subject to:} \\ (9) - (15) \end{cases} \tag{16}$$

where f is given by (8), f_0 is the predetermined profit by the decision-maker.

Table 2 Uncertain variables

Uncertain variable	Distribution
D_{nt}	$L(a_{D_{nt}}, b_{D_{nt}})$
g_{nt}	$L(a_{g_{nt}}, b_{g_{nt}})$
j_{nt}	$L(a_{j_{nt}}, b_{j_{nt}})$
z_{nt}	$L(a_{z_{nt}}, b_{z_{nt}})$
d_{nt}	$L(a_{d_{nt}}, b_{d_{nt}})$
e_{nt}	$L(a_{e_{nt}}, b_{e_{nt}})$
h_t	$L(a_{h_t}, b_{h_t})$
l_t	$L(a_{l_t}, b_{l_t})$
i_{nt}	$L(a_{i_{nt}}, b_{i_{nt}})$
m_{nt}	$L(a_{m_{nt}}, b_{m_{nt}})$
v_{nt}	$L(a_{v_{nt}}, b_{v_{nt}})$
r_{nt}	$L(a_{r_{nt}}, b_{r_{nt}})$
$W_{t\max}$	$L(a_{W_{t\max}}, b_{W_{t\max}})$
$M_{t\max}$	$L(a_{M_{t\max}}, b_{M_{t\max}})$
$V_{t\max}$	$L(a_{V_{t\max}}, b_{V_{t\max}})$

4 Solving method

Suppose that all the uncertain variables in Model (16) can be characterized as linear ones, as shown in Table 2, the model can be converted into crisp equivalent, and the steps can be described as follows.

Step 1: conversion of objective function

From Eq. (8) and Theorems 1 and 2, it is obtained that $f - f_0$ is the uncertain variable $L(A, B)$ where

$$\begin{aligned} A = &\sum_{n=1}^{N}\sum_{t=1}^{T} a_{r_{nt}}(I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} \\ &- I_{nt} + B_{nt}) - \sum_{n=1}^{N}\sum_{t=1}^{T}(b_{g_{nt}}Q_{nt} + b_{j_{nt}}O_{nt} + b_{z_{nt}}S_{nt} \\ &+ b_{d_{nt}}I_{nt} + b_{e_{nt}}B_{nt}) - \sum_{t=1}^{T}(b_{h_t}H_t + b_{l_t}L_t) - f_0, \\ B = &\sum_{n=1}^{N}\sum_{t=1}^{T} b_{r_{nt}}(I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} \\ &- I_{nt} + B_{nt}) - \sum_{n=1}^{N}\sum_{t=1}^{T}(a_{g_{nt}}Q_{nt} + a_{j_{nt}}O_{nt} + a_{z_{nt}}S_{nt} \\ &+ a_{d_{nt}}I_{nt} + a_{e_{nt}}B_{nt}) - \sum_{t=1}^{T}(a_{h_t}H_t + a_{l_t}L_t) - f_0. \end{aligned}$$

Then we have

$$\begin{aligned} M\{f \ge f_0\} &= 1 - M\{f - f_0 < 0\} \\ &= 1 - \frac{0 - A}{B - A} = \frac{B}{B - A}. \end{aligned} \tag{17}$$

Step 2: conversion of product-inventory constraints

From Eq. (9) and Theorems 1 and 2, it is obtained that

$$\begin{aligned} &M\{-p \le I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} - D_{nt} \le p\} \\ &= M\{I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} \\ &\quad + B_{nt} - D_{nt} - p \le 0\} \\ &\quad - M\{I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} \\ &\quad + B_{nt} - D_{nt} + p \le 0\} \end{aligned}$$

Then $I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} - D_{nt} - p$ is uncertain variable $L(I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} - b_{D_{nt}} - p,\ I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} - a_{D_{nt}} - p)$, and $I_{nt\text{-}1} - B_{nt\text{-}1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} - D_{nt} + p$ is uncertain variable $L(I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} - b_{D_{nt}} + p, I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} - a_{D_{nt}} + p)$.

Then the product-inventory constraints (9) are converted into

$$\frac{2p}{b_{D_{nt}} - a_{D_{nt}}} \ge \lambda. \tag{18}$$

Step 3: conversion of labor level constraints

From Eq. (10) and Theorems 1 and 2, it is obtained that

$$\begin{aligned} &M\left\{-q \le \sum_{n=1}^{N} i_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - L_t \right. \\ &\qquad \left. - \sum_{n=1}^{N} i_{nt}(Q_{nt} + O_{nt}) \le q\right\} \\ &\quad = M\left\{\sum_{n=1}^{N} i_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - L_t \right. \\ &\qquad \left. - \sum_{n=1}^{N} i_{nt}(Q_{nt} + O_{nt}) - q \le 0\right\} \\ &\qquad - M\left\{\sum_{n=1}^{N} i_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - L_t \right. \\ &\qquad \left. - \sum_{n=1}^{N} i_{nt}(Q_{nt} + O_{nt}) + q \le 0\right\}. \end{aligned}$$

While the term $\sum_{n=1}^{N} i_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - L_t - \sum_{n=1}^{N} i_{nt}(Q_{nt} + O_{nt}) - q$ is the uncertain variable $L(\sum_{n=1}^{N} a_{i_{nt-1}}(Q_{nt-1} + O_{nt-1}) + H_t - L_t - \sum_{n=1}^{N} b_{i_{nt}}(Q_{nt} + O_{nt}) - q, \sum_{n=1}^{N} b_{i_{nt-1}}(Q_{nt-1} + O_{nt-1}) + H_t - L_t - \sum_{n=1}^{N} a_{i_{nt}}(Q_{nt} + O_{nt}) - q)$, and the term $\sum_{n=1}^{N} i_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - L_t - \sum_{n=1}^{N} i_{nt}(Q_{nt} + O_{nt}) + q$ is the uncertain variable $L(\sum_{n=1}^{N} a_{i_{nt-1}}(Q_{nt-1} + O_{nt-1}) + H_t - L_t - \sum_{n=1}^{N} b_{i_{nt}}(Q_{nt} + O_{nt}) + q, \sum_{n=1}^{N} b_{i_{nt-1}}(Q_{nt-1} + O_{nt-1}) + H_t - L_t - \sum_{n=1}^{N} a_{i_{nt}}(Q_{nt} + O_{nt}) + q)$. So we have

$$\frac{0-(\sum_{n=1}^{N}a_{i_{nt-1}}(Q_{nt-1}+O_{nt-1})+H_t-L_t-\sum_{n=1}^{N}b_{i_{nt}}(Q_{nt}+O_{nt})-q)}{\sum_{n=1}^{N}(Q_{nt-1}+O_{nt-1})(b_{i_{nt-1}}-a_{i_{nt-1}})+\sum_{n=1}^{N}(Q_{nt}+O_{nt})(b_{i_{nt}}-a_{i_{nt}})}$$
$$-\frac{0-(\sum_{n=1}^{N}a_{i_{nt-1}}(Q_{nt-1}+O_{nt-1})+H_t-L_t-\sum_{n=1}^{N}b_{i_{nt}}(Q_{nt}+O_{nt})+q)}{\sum_{n=1}^{N}(Q_{nt-1}+O_{nt-1})(b_{i_{nt-1}}-a_{i_{nt-1}})+\sum_{n=1}^{N}(Q_{nt}+O_{nt})(b_{i_{nt}}-a_{i_{nt}})}$$
$$=\frac{2q}{\sum_{n=1}^{N}(Q_{nt-1}+O_{nt-1})(b_{i_{nt-1}}-a_{i_{nt-1}})+\sum_{n=1}^{N}(Q_{nt}+O_{nt})(b_{i_{nt}}-a_{i_{nt}})}$$

Then the labor level constraints Eq. (10) are converted into

$$\frac{2q}{C+D}\geq\beta \tag{19}$$

where

$$C=\sum_{n=1}^{N}(Q_{nt-1}+O_{nt-1})(b_{i_{nt-1}}-a_{i_{nt-1}}),$$
$$D=\sum_{n=1}^{N}(Q_{nt}+O_{nt})(b_{i_{nt}}-a_{i_{nt}}).$$

Step 4: conversion of labor usage constraints

From Eq. (11) and Theorems 1 and 2, it is obtained that $\sum_{n=1}^{N} i_{nt}(Q_{nt}+O_{nt})-W_{t\max}$ is the uncertain variable $L(\sum_{n=1}^{N}a_{i_{nt}}(Q_{nt}+O_{nt})-b_{W\ t\max},\sum_{n=1}^{N}b_{i_{nt}}(Q_{nt}+O_{nt})-a_{W_{t\max}})$. Then the labor usage constraints are converted into

$$\frac{-\sum_{n=1}^{N}a_{i_{nt}}(Q_{nt}+O_{nt})+b_{W_{t\max}}}{\sum_{n=1}^{N}(b_{i_{nt}}-a_{i_{nt}})(Q_{nt}+O_{nt})+b_{W_{t\max}}-a_{W_{t\max}}}\geq\varsigma \tag{20}$$

Step 5: conversion of machine usage constraints

From Eq. (12) and Theorems1 and 2, it is obtained that $\sum_{n=1}^{N} m_{nt}\ (Q_{nt}+O_{nt})-M_{t\max}$ is the uncertain variable $L(\sum_{n=1}^{N}a_{m_{nt}}(Q_{nt}+O_{nt})-b_{M_{t\max}},\sum_{n=1}^{N}b_{m_{nt}}(Q_{nt}+O_{nt})-a_{M_{t\max}})$.

Then the machine usage constraints are converted into

$$\frac{-\sum_{n=1}^{N}a_{m_{nt}}(Q_{nt}+O_{nt})+b_{M_{t\max}}}{\sum_{n=1}^{N}(b_{m_{nt}}-a_{m_{nt}})(Q_{nt}+O_{nt})+b_{M_{t\max}}-a_{M_{t\max}}}\geq\sigma \tag{21}$$

Step 6: conversion of warehouse space constraints

From Eq. (13) and Theorems 1 and 2, it is obtained that $\sum_{n=1}^{N}v_{nt}I_{nt}-V_{t\max}$ is the uncertain variable $L(\sum_{n=1}^{N}a_{v_{nt}}I_{nt}-b_{V_{t\max}},\sum_{n=1}^{N}b_{v_{nt}}I_{nt}-a_{V_{t\max}})$.

Then the warehouse space constraints are converted into

$$\frac{-\sum_{n=1}^{N}a_{v_{nt}}I_{nt}+b_{V_{t\max}}}{\sum_{n=1}^{N}(b_{v_{nt}}-a_{v_{nt}})I_{nt}+b_{V_{t\max}}-a_{V_{t\max}}}\geq\delta \tag{22}$$

Step 7: conversion of capital constraints

From Eq. (14) and Theorems 1 and 2, it is obtained that $\sum_{n=1}^{N}(g_{nt}Q_{nt}+j_{nt}O_{nt}+z_{nt}S_{nt}+d_{nt}I_{nt}+e_{nt}B_{nt})+h_tH_t+l_tL_t-C_{t\max}$ is the uncertain variable $L(E, F)$, where $E=\sum_{n=1}^{N}(a_{g_{nt}}Q_{nt}+a_{j_{nt}}O_{nt}+a_{z_{nt}}S_{nt}+a_{d_{nt}}I_{nt}+a_{e_{nt}}B_{nt})+a_{h_t}H_t+a_{l_t}L_t-b_{C_{t\max}}$, and $F=\sum_{n=1}^{N}(b_{g_{nt}}Q_{nt}+b_{j_{nt}}O_{nt}+b_{z_{nt}}S_{nt}+b_{d_{nt}}I_{nt}+b_{e_{nt}}B_{nt})+b_{h_t}H_t+b_{l_t}L_t-a_{C_{t\max}}$.

Then the capital constraints are converted into

$$\frac{-E}{F-E}\geq\tau \tag{23}$$

Therefore, the crisp equivalent of APP Model (16) is made as follows,

$$\begin{cases}\max\quad(17)\\ \text{subject to}:\\ (18)-(23)\end{cases} \tag{24}$$

It is obvious that model (24) is a nonlinear programming. The model can be solved by many traditional methods.

5 An example

A food company produces two types of products to meet the market demand during two periods (denoted by Period 1 and Period 2, respectively) in uncertain environments. The basic data are shown in Table 3. It can be seen that there are 52 uncertain variables in this problem. In addition, the parameters in model (16) are given as follows, $I_{10}=0, I_{20}=0, B_{10}=0, B_{20}=0, i_{10}=0,\ i_{20}=0, \lambda=0.6,\ \beta=0.7, \varsigma=0.7,\ \delta=0.9,\ \sigma=0.7, \tau=0.8, p=100,\ q=100, f_0=9{,}000$.

The objective function can be converted into the following form.

Table 3 Basic data

Item	Period 1	Period 2
D_{1t}	$L(80, 150)$	$L(65, 100)$
D_{2t}	$L(65, 80)$	$L(70, 95)$
g_{1t}	$L(3, 8)$	$L(4, 10)$
g_{2t}	$L(4, 7)$	$L(4, 8)$
j_{1t}	$L(4, 8)$	$L(3, 8)$
j_{2t}	$L(3, 9)$	$L(3, 10)$
z_{1t}	$L(3, 9)$	$L(3, 10)$
z_{2t}	$L(2, 8)$	$L(3, 8)$
d_{1t}	$L(0.3, 0.8)$	$L(0.4, 0.8)$
d_{2t}	$L(0.3, 0.6)$	$L(0.3, 0.6)$
e_{1t}	$L(0.3, 0.7)$	$L(0.4, 0.7)$
e_{2t}	$L(0.4, 0.8)$	$L(0.3, 0.6)$
h_t	$L(3, 8)$	$L(4, 8)$
l_t	$L(3, 8)$	$L(3, 8)$
i_{1t}	$L(3, 6)$	$L(3, 6)$
i_{2t}	$L(4, 8)$	$L(4, 9)$
m_{1t}	$L(3, 8)$	$L(4, 8)$
m_{2t}	$L(4, 6)$	$L(3, 7)$
v_{1t}	$L(35, 70)$	$L(40, 70)$
v_{2t}	$L(30, 80)$	$L(30, 55)$
r_{1t}	$L(40, 70)$	$L(35, 70)$
r_{2t}	$L(45, 60)$	$L(45, 65)$
$W_{t\max}$	$L(30, 80)$	$L(20, 90)$
$M_{t\max}$	$L(35, 70)$	$L(40, 70)$
$V_{t\max}$	$L(150, 300)$	$L(0, 300)$
$C_{t\max}$	$L(300, 800)$	$L(200, 1{,}000)$

$$\begin{aligned}&(67Q_{11}+66O_{11}+67S_{11}-0.3I_{11}-0.3B_{11}\\&+66Q_{12}+67O_{12}+67S_{12}-70.4I_{12}+69.6B_{12}\\&+56Q_{21}+57O_{21}+58S_{21}-60.3I_{21}+59.6B_{21}\\&+61Q_{22}+62O_{22}+62S_{22}-65.3I_{22}+64.7B_{22}\\&-3H_1-3L_1-4H_2-3L_2-9000)/(35Q_{11}+34O_{11}\\&+36S_{11}+5.5I_{11}-4.6B_{11}+41Q_{12}+40O_{12}\\&+42S_{12}-34.6I_{12}+35.3B_{12}+18Q_{21}+21O_{21}\\&+21S_{21}-14.7I_{21}+15.4B_{21}+24Q_{22}+27O_{22}\\&+25S_{22}-19.7I_{22}+20.3B_{22}+5H_1+5L_1\\&+4H_2+5L_2).\end{aligned} \tag{25}$$

The the constraints can be converted into the following form.

$$\begin{aligned}&67Q_{11}+66O_{11}+67s_{11}-0.3I_{11}-0.3B_{11}+66Q_{12}\\&+67O_{12}+67S_{12}-70.4I_{12}+69.6B_{12}+56Q_{21}\\&+58S_{21}-60.3I_{21}+59.6B_{21}+61Q_{22}+62O_{22}\\&+62S_{22}+57O_{21}-65.3I_{22}+64.7B_{22}-3H_1-3L_1\\&-4H_2-3L_2-9{,}000>0.\end{aligned} \tag{26}$$

$$\begin{aligned}&32Q_{11}+32O_{11}+31s_{11}-5.8I_{11}+4.3B_{11}+25Q_{12}\\&+27O_{12}+25S_{12}-35.8I_{12}+34.3B_{12}+38Q_{21}\\&+36O_{21}+37S_{21}-45.6I_{21}+44.2B_{21}+37Q_{22}\\&+35O_{22}+37S_{22}-45.6I_{22}+44.4B_{22}-8H_1\\&-8L_1-8H_2-8L_2-9000<0.\end{aligned} \tag{27}$$

$$200/(3(Q_{11}+O_{11})+4(Q_{21}+O_{21}))\geq 0.7. \tag{28}$$

$$\begin{aligned}&200/(3(Q_{11}+O_{11})+4(Q_{21}+O_{21})+3(Q_{12}+O_{12})\\&+5(Q_{22}+O_{22}))\geq 0.7.\end{aligned} \tag{29}$$

$$\begin{aligned}&(-3(Q_{11}+O_{11})-4(Q_{21}+O_{21})+80)/(3(Q_{11}\\&+O_{11})+4(Q_{21}+O_{21})+50)\geq 0.7.\end{aligned} \tag{30}$$

$$\begin{aligned}&(-3(Q_{12}+O_{12})-4(Q_{22}+O_{22})+90)/(3(Q_{12}\\&+O_{12})+5(Q_{22}+O_{22})+70)\geq 0.7.\end{aligned} \tag{31}$$

$$\begin{aligned}&(-3(Q_{11}+O_{11})-4(Q_{21}+O_{21})+70)/(5(Q_{11}\\&+O_{11})+2(Q_{21}+O_{21})+35)\geq 0.9.\end{aligned} \tag{32}$$

$$\begin{aligned}&(-4(Q_{12}+O_{12})-3(Q_{22}+O_{22})+70)/(4(Q_{12}\\&+O_{12})+4(Q_{22}+O_{22})+30)\geq 0.9.\end{aligned} \tag{33}$$

$$(-35I_{11}-30I_{21}+300)/(35I_{11}+50I_{21}+150)\geq 0.7. \tag{34}$$

$$(-40I_{12}-30I_{22}+300)/(30I_{12}+25I_{22}+300)\geq 0.7. \tag{35}$$

$$\begin{aligned}&(-3Q_{11}-4O_{11}-3S_{11}-0.3I_{11}-0.3B_{11}-4Q_{21}\\&-3O_{21}-2S_{21}-0.3I_{21}-0.4B_{21}-3H_1-3L_1\\&+800)/(5Q_{11}+4O_{11}+6S_{11}+0.5I_{11}+0.4B_{11}\\&+5H_1+5L_1+3Q_{21}+6O_{21}+6S_{21}+0.3I_{21}\\&+0.4B_{21}+500)\geq 0.8.\end{aligned} \tag{36}$$

$$\begin{aligned}&(-4Q_{12}-3O_{12}-3S_{12}-0.4I_{12}-0.4B_{12}-4Q_{22}\\&-3O_{22}-3S_{22}-0.3I_{22}-0.3B_{22}-4H_2-3L_2\\&+1000)/(6Q_{12}+5O_{12}+7S_{12}+0.4I_{12}+0.3B_{12}\\&+4H_2+5L_2+4Q_{22}+7O_{22}+5S_{22}+0.3I_{22}\\&+0.3B_{22}+800)\geq 0.8.\end{aligned} \tag{37}$$

$$\begin{aligned}&Q_{11}\geq 0;Q_{12}\geq 0;Q_{21}\geq 0;Q_{22}\geq 0;\\&O_{11}\geq 0;O_{12}\geq 0;O_{21}\geq 0;O_{22}\geq 0;\\&S_{11}\geq 0;S_{12}\geq 0;S_{21}\geq 0;S_{22}\geq 0;\\&B_{11}\geq 0;B_{12}\geq 0;B_{21}\geq 0;B_{22}\geq 0;\\&I_{11}\geq 0;I_{12}\geq 0;I_{21}\geq 0;I_{22}\geq 0;\\&H_1\geq 0;H_2\geq 0;L_1\geq 0;L_2\geq 0.\end{aligned} \tag{38}$$

Up to now, the model (16) can be converted into the crisp one with the objective (25) and constraints (26)–(38). It is a nonlinear programming. We use software Lingo to solve the model. The optimal objective value is 1, and the optimal solution (production plan) is shown in Table 4.

Table 4 Optimal production plan

Variables	Period 1	Period 2	Variables	Period 1	Period 2
Q_{1t}	0.9792	0.9782	Q_{2t}	1.0043	1.0043
O_{1t}	0.9800	0.9809	O_{2t}	1.5338	1.3112
S_{1t}	0.9965	0.9861	S_{2t}	1.0146	1.0112
I_{1t}	1.0143	0.9539	I_{2t}	0.7500	0.4497
B_{1t}	1.0077	251.2463	B_{2t}	1.0257	1.0217
H_t	0.9887	0.9887	L_t	0.9896	1.8593

6 Conclusion and future research

This paper presents an uncertain APP model based on uncertainty theory. The objective function and constraints can be converted into crisp equivalents when they are linear uncertain variables. Then the model can be solved by traditional methods. Similarly, the objective function and constraints can also be converted into crisp equivalents when they are other uncertain variables, such as zigzag uncertain variable, normal uncertain variable, etc. Very importantly, if the uncertain distributions of the market demand, production cost, subcontracting cost, etc. do not belong among one same type, it may be impossible that the model is converted into crisp equivalent. In the situation, uncertain simulation can be used to estimate the values of objective function and constraint functions, then an intelligent algorithm (such as genetic algorithm) can be employed to solve the model.

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