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A new order relation on fuzzy soft sets and its application

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Abstract In this paper, a new order relation on fuzzy soft sets, called soft information order, is introduced and its application to decision-making is investigated. It is shown that the collection of all fuzzy soft sets (over a given universe set), equipped with this new order, forms a complete Heyting algebra. The representation theorem of fuzzy soft sets with respect to the soft information order is also obtained. We initiate the concepts of soft set satisfaction problems and their solutions. An algorithm is presented to solve such decision-making problems.

Keywords Fuzzy soft sets · Soft information order · Complete Heyting algebras · Soft set satisfaction problems

1 Introduction

Human activities and natural phenomena are full of uncertainty, including subjective uncertainty and objective uncertainty. In order to describe these uncertainties, mathematicians, economists and engineers in different research fields have proposed various mathematical theories. Probability theory (Kallenberg 1997), fuzzy set theory (Zadeh 1965), vague set theory (Gau and Buehrer 1993), interval mathematics theory (Gorzalzany 1987), rough set theory (Pawlak 1982), etc., are all very efficient tools for dealing with different types of uncertainties. Free from their inherent deficiency and difficulties for dealing with uncertain problems existing in the above theories (Molodtsov 1999; Maji et al. 2003), Molodtsov (1999) initiated a novel concept of soft sets as a new mathematical tool for solving uncertainties. A soft set is a tuple which associates with a set of parameters and a mapping from the parameter set into the power set of a universe set. Therefore, in fact, a soft set is a parameterized family of subsets of a universe set. Related studies demonstrate that soft set theory has its convenient and simplicity in application. Now it has been applied in several directions such as stability and regularization, game theory, soft analysis, theory of universal algebra, relation analysis (Maji et al. 2003; Aktaş and Çağman 2007; Jun 2008; Jun and Park 2008; Feng et al. 2008, 2010a; Gong et al. 2010; Babitha and Sunil 2010), especially decision-making (Chen et al. 2005; Roy and Maji 2007; Kong et al. 2009; Maji et al. 2002; Sezgin and Atagun 2011; Feng et al. 2010b, 2011, 2012).

The issue of ordering structures on a system is researched by working mathematicians and theoretical computer researchers continuously. In the present paper, we introduce a new order relation on fuzzy soft sets, which is called soft information order. Then we focus on the properties of this soft information ordering structure on fuzzy soft sets. These conclusions allow us to re-understand some operations defined between soft sets from the perspective of ordering relation, such as extended intersection and restricted union.

For the processing of information under uncertainty, combination and extraction are two basic operations. To

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give these operations in soft theory is necessary and natural. Therefore, according to the order relation defined here, we put forward the concept of soft set satisfaction problems in the application of this study. An algorithm for obtaining the solution of soft set satisfaction problems is given, which can be used for some decision-making problems.

The rest of this paper is organized as follows. Section 2 briefly reviews some basic notions on fuzzy soft sets firstly. The concept of soft information order is proposed. The properties of fuzzy soft sets over a universe set with respect to this new order relation are shown in Sect. 3. In the last part of this paper, we discuss soft set satisfaction problems and give a method of obtaining solution of them.

2 Preliminaries

In this section, first, we present some basic definitions and results of soft set theory. Let U be an initial universe set and let E be a set of parameters which usually are initial attributes, characteristics, or properties of objects in U. In fact, it is very important for decision-making that how to take an adequate and necessary parametrization for these related objects in solving some real-life problems.

Definition 1 (Roy and Maji 2007) Let U be an initial universe set and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A).

Definition 2 (Roy and Maji 2007) Let $\mathcal{F}(U)$ denote the set of all fuzzy sets of U. Let $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to \mathcal{F}(U)$.

Clearly, fuzzy soft sets extend classical soft sets by substituting fuzzy subsets for just crisp subsets of the universe. Note also that a fuzzy set could be viewed as a fuzzy soft set whose parameter set is a singleton. Hence fuzzy soft sets extend both soft sets and fuzzy sets. The following example further shows that fuzzy soft sets can also be regarded as a natural generalization of intuitionistic/grey/vagues sets.

Example 1 An intuitionistic fuzzy set (Atanassov 1986) A in a universe U is an object of the form

$$A = \{(u, \mu_A(u), \nu_A(u)) \mid u \in U\},$$

where, for all $u \in U$, $\mu_A(u) \in [0, 1]$ and $\nu_A(u) \in [0, 1]$ are called the membership degree and the non-membership degree, respectively, of u, and furthermore satisfy $\mu_A(u) + \nu_A(u) \leq 1$.

Now, let $A = \{(u, \mu_A(u), \nu_A(u)) \mid u \in U\}$ be an intuitionistic fuzzy set in U. Putting $E = \{\mu, \nu\}$ as the parameter set, we can define a fuzzy soft set $S_A = (F, E)$ over U, where $F(\mu) = \mu_A$ and $F(\nu) = \nu_A$.

In this formal point of view, we can say that every intuitionistic fuzzy set can be seen as a fuzzy soft set. Actually, from the parameterized view of soft set theory, one can easily observe that intuitionistic fuzzy sets are fuzzy soft sets with only two parameters. That is, intuitionistic fuzzy sets only consider two distinct (bipolar) aspects—the membership and non-membership degrees for each object in the universe of discourse. However, it is worth noting that much more differing parameters can be used in the setting of fuzzy soft sets. This will give a more general scheme for modelling uncertainty.

Note also that Deschrijver and Kerre (2007) have shown that intuitionistic fuzzy set, grey sets and vagues sets are mathematically equivalent. So it follows immediately that fuzzy soft sets can also be regarded as a common generalization of all these models for modelling imprecision.

We introduce two order relations between two fuzzy soft sets, including the fuzzy soft subset relation $\tilde{\subseteq}$ and a new order relation $\sqsubseteq$ called the soft information order.

Definition 3 (Feng et al. 2010a) For two fuzzy soft sets (F, A) and (G, B) over a common universe U, we write $(F, A) \tilde{\subseteq} (G, B)$, and we say (F, A) is a fuzzy soft subset of (G,B), if

(i) $A \subseteq B$, and
(ii) $\forall \varepsilon \in A$, $F(\varepsilon)$ is a fuzzy subset of $G(\varepsilon)$.

Definition 4 For two fuzzy soft sets (F, A) and (G, B) over a common universe U, we write $(F, A) \sqsubseteq (G, B)$, and call it soft information order if

(i) $A \subseteq B$, and
(ii) $\forall \varepsilon \in A$, $G(\varepsilon)$ is a fuzzy subset of $F(\varepsilon)$.

Naturally, if (F, A) and (G, B) are two classic soft sets, $(F, A) \sqsubseteq (G, B)$ means

(i) $A \subseteq B$, and
(ii) $\forall \varepsilon \in A$, $G(\varepsilon) \subseteq F(\varepsilon)$.

Example 2 Consider three fuzzy soft sets (F, A), (G, B) and (H, C) over a same universe set $U = \{h_1, h_2, h_3, h_4, h_5\}$. Here U represents the set of houses. Let $A = \{e_1, e_2\}$, $B = \{e_2, e_3\}$ and $C = \{e_1, e_2, e_3\}$ represent the sets of some attributes that Mr. X, Mr. Y and Mr. Z have considered in choosing houses respectively, and

$$\begin{aligned}
F(e_1) &= \{1/h_1, 0.5/h_2, 0.5/h_3, 1/h_4, 0.7/h_5\}\\
F(e_1) &= \{1/h_1, 0.5/h_2, 0.5/h_3, 1/h_4, 0.7/h_5\}\\
F(e_2) &= \{0.6/h_1, 0.6/h_2, 0.9/h_3, 0.8/h_4, 1/h_5\}\\
G(e_2) &= \{0.5/h_1, 0.4/h_2, 0.7/h_3, 0.8/h_4, 0.8/h_5\}\\
G(e_3) &= \{0.4/h_1, 0.5/h_2, 0.4/h_3, 0.6/h_4, 0.8/h_5\}\\
H(e_1) &= \{0.8/h_1, 0.5/h_2, 0.4/h_3, 1/h_4, 0.6/h_5\}\\
H(e_2) &= \{0.5/h_1, 0.6/h_2, 0.8/h_3, 0.8/h_4, 0.8/h_5\}\\
H(e_3) &= \{0.4/h_1, 0.6/h_2, 0.5/h_3, 0.8/h_4, 1/h_5\}
\end{aligned}.$$

Clearly, $(F,A)\sqsubseteq(H,C)$ and $(G,B)\tilde{\subseteq}(H,C)$.

Everyone, in the above example of choosing houses, can be seen as an independent judge. From the fact indicated by the formula $(F,A)\sqsubseteq(H,C)$ shown here, we can intuitively understand the meaning of the soft information order. First, the order relation $A\subseteq C$ presents that Mr. Z has taken into account more factors than Mr. X when they choose houses. Meanwhile, the scoring standards of the "judge" Z is more strict than X's. That is, for a same attractiveness, the scores of these houses that Z gives are all lower than theirs corresponding scores that X presents. Thus, according to the two decisive factors parameter sets and soft mappings of fuzzy soft sets, (H,C) is stronger than (F,A).

Now, further we demonstrate the relationship between this two order relations defined in Definitions 3 and 4.

Proposition 1 Let (F,A) and (G,B) be two fuzzy soft sets over a universe set U. Then $(F,A)\sqsubseteq(G,B)$ if, and only if $(F^c,A)\tilde{\subseteq}(G^c,B)$, where $F^c(e)(x)=1-F(e)(x)$, $\forall e\in A$ and $x\in U$.

Proof By Definitions 3 and 4, we can obtain the conclusion straightforwardly. □

Definition 5 (Maji et al. 2003) A fuzzy soft set (F,A) over U is said to be a null soft set, if for all $e\in A$, $F(e)=\emptyset$.

In Maji et al. (2003), a null soft set (F,A) is denoted by $(\emptyset,A)$. In general, $\emptyset$ means a function with empty domain. Thus, in order to avoid conflict with other symbols in this paper, we use symbol $(F_\emptyset,A)$ to denote a null soft set with a parameter set A.

Definition 6 (Maji et al. 2003) A fuzzy soft set (F,A) over U is said to be an absolute soft set denoted by $\tilde{A}$, if for all $e\in A, F(e)=U$.

Definition 7 (Ali et al. 2009) The extended intersection of two fuzzy soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C=A\cup B$, and $\forall e\in C$,

$$H(e)=\begin{cases} F(e), & \text{if } e\in A-B;\\ G(e), & \text{if } e\in B-A;\\ F(e)\cap G(e), & \text{if } e\in A\cap B.\end{cases}$$

We write $(F,A)\sqcap_\varepsilon(G,B)=(H,C)$.

It should be noted that, in this paper, if the sets A and B are fuzzy sets, then $A\cap B$ and $A\cup B$ denote the operations fuzzy intersection and fuzzy union of fuzzy sets, respectively. That is, the membership functions are defined as

$$(A\cap B)(x)=\min\{A(x),B(x)\},(A\cup B)(x)=\max\{A(x),B(x)\},$$

for all x.

Definition 8 (Ali et al. 2009) Let (F,A) and (G,B) be two fuzzy soft sets over a same universe U such that $A\cap B\neq\emptyset$. The restricted union of (F,A) and (G,B) is denoted by $(F,A)\cup_{\mathcal{R}}(G,B)$, and is defined as $(F,A)\cup_{\mathcal{R}}(G,B)=(H,C)$ where $C=A\cap B$ and for all $e\in C$, $H(e)=F(e)\cup G(e)$.

For the convenience of discussion of the following, we will generalize the definition of union of fuzzy soft sets from restricted union to rational restricted union.

Definition 9 Let (F,A) and (G,B) be two fuzzy soft sets over a same universe U. $(F,A)\sqcup_{\mathcal{R}}(G,B)$, called rational restricted union of (F,A) and (G,B), is defined as follows:

$$(F,A)\sqcup_{\mathcal{R}}(G,B)=\begin{cases}(F,A)\cup_{\mathcal{R}}(G,B), & \text{if } A\cap B\neq\emptyset;\\ (\emptyset,\emptyset), & \text{if } A\cap B=\emptyset.\end{cases}$$

where $\emptyset:\emptyset\to\mathcal{F}(U)$ is a function with empty domain.

3 Properties of ordering structure on fuzzy soft sets

In this section, we will study the soft information ordering structure on fuzzy soft sets which are over a same universe set. We can recognize some operations on fuzzy soft sets from the point of view of ordering relation.

First, we introduce some basic notions in lattice theory. Suppose that $(L,\leq)$ is a partially ordered set and $A\subseteq L$. We write $\bigvee_L A$ and $\bigwedge_L A$ for the least upper bound and the greatest lower bound of A in L, respectively, if they exist. For simplicity, they will always be denoted by $\vee A$ and $\wedge A$, respectively.

Let L be a partially ordered set. For all $a,b\in L$, if $a\vee b$ and $a\wedge b$ exist, then L is called a lattice. If $\vee A$ exists for every subset $A\subseteq L$, we call L a complete lattice.

Definition 10 (Gierz et al. 2003) A complete Heyting algebra (cHa) is a complete lattice which satisfies the following infinite distributive law:

$$x\wedge[\vee Y]=\vee\{x\wedge y: y\in Y\}.$$

Proposition 2 Let (F,A) and (G,B) be two fuzzy soft sets over a same universe U. Then, with respect to the soft information order,

$$(F,A) \vee (G,B) = (F,A) \sqcap_{\varepsilon} (G,B)$$

and

$$(F,A) \wedge (G,B) = (F,A) \sqcup_{\mathcal{R}} (G,B).$$

Proof By Definitions 7 and 8, we can obtain these conclusions straightforwardly. It should be noted that $(\emptyset,\emptyset)$ is the only lower bound of (F,A) and (G,B) if $A \cap B = \emptyset$. Hence $(F,A) \wedge (G,B) = (\emptyset,\emptyset)$ is clear for $A \cap B = \emptyset$. □

Proposition 3 Let $\{(F_i,A_i) : i \in I\}$ be fuzzy soft sets over a same universe U. Then

$$\bigvee_{i\in I}(F_i,A_i) = \left(H, \bigcup_{i\in I} A_i\right),$$

where $H : \bigcup_{i\in I} A_i \to \mathcal{F}(U)$ is defined as follows: $\forall e \in \bigcup_{i\in I} A_i$, let $I^{(e)} = \{i \in I : e \in A_i\}$, $H(e)(x) = \bigwedge_{i\in I^{(e)}} F_i(e)(x)$.

Proof Clearly $(H, \bigcup_{i\in I} A_i)$ is an upper bound of $\{(F_i,A_i) : i \in I\}$. Suppose that (G,B) is another upper bound of $\{(F_i,A_i) : i \in I\}$. Thus $\bigcup_{i\in I} A_i \subseteq B$. $\forall e \in \bigcup_{i\in I} A_i, i \in I^{(e)}$ and $x \in U$, we have $G(e)(x) \le F_i(e)(x)$. Then

$$G(e)(x) \le \bigwedge_{i\in I^{(e)}} F_i(e)(x) = H(e)(x),$$

that is, $G(e) \subseteq H(e)$. This proves that $(H, \bigcup_{i\in I} A_i) \sqsubseteq (G,B)$. Therefore, we obtain that

$$\bigvee_{i\in I}(F_i,A_i) = \left(H, \bigcup_{i\in I} A_i\right).$$

□

Let U be an initial universe set and E be a set of parameters. $\mathcal{F}_{U,E}$ (or simply $\mathcal{F}$ when this does not lead to confusions) denotes the set of all fuzzy soft sets over the set U, which have a parameter subset of E, i.e.,

$$\mathcal{F} = \{(F,A) : (F,A) \text{ is a fuzzy soft set over } U, \text{ where } A \subseteq E\}$$

The null soft set $(F_\emptyset, E)$ is the greatest element on the set $\mathcal{F}$. As the example of choosing houses shown in the above, the null soft set $(F_\emptyset, E)$ is a evaluation that each house in this region does not have any attractive attribute enumerated in the parameter set E. By Proposition 2, we know that $\mathcal{F}_{U,E}$ is a lattice with respect to the soft information order. Furthermore, the following conclusion can be drawn from Proposition 3.

Theorem 1 $\mathcal{F}_{U,E}$ is a complete lattice with respect to the soft information order. The top element is $(F_\emptyset, E)$, and the bottom element is $(\emptyset,\emptyset)$.

Theorem 2 $(\mathcal{F}_{U,E}, \sqsubseteq)$ is a complete Heyting algebra.

Proof Let $(F,A), (G_i,B_i)(i \in I)$ be fuzzy soft sets over the universe set U. By Theorem 1, we need to show the following equation holds:

$$(F,A) \wedge \left[\bigvee_{i\in I}(G_i,B_i)\right] = \bigvee_{i\in I}[(F,A) \wedge (G_i,B_i)].$$

First, the two fuzzy soft sets in both sides of the equation have a same parameter set $A \cap (\bigcup_{i\in I} B_i)$. We write

$$(F,A) \wedge (G_i,B_i) = (H_i, A \cap B_i),$$

$$\bigvee_{i\in I}(G_i,B_i) = \left(G, \bigcup_{i\in I} B_i\right),$$

$$(F,A) \wedge \left(G, \bigcup_{i\in I} B_i\right) = \left(J, A \cap \left(\bigcup_{i\in I} B_i\right)\right),$$

and

$$\bigvee_{i\in I}(H_i, A \cap B_i) = \left(H, A \cap \left(\bigcup_{i\in I} B_i\right)\right).$$

$\forall e \in A \cap (\bigcup_{i\in I} B_i)$, let $I^{(e)} = \{i \in I : e \in B_i\}$. By Proposition 3, we have $G(e)(x) = \bigwedge_{i\in I^{(e)}} G_i(e)(x)$. It follows that

$$J(e)(x) = F(e)(x) \vee \left[\bigwedge_{i\in I^{(e)}} G_i(e)(x)\right].$$

Meanwhile, $\forall e \in A \cap (\bigcup_{i\in I} B_i)$, $\forall i \in I^{(e)}$, we have

$$H_i(e)(x) = F(e)(x) \vee G_i(e)(x).$$

Then, by Proposition 3 again,

$$H(e)(x) = \bigwedge_{i\in I^{(e)}} [F(e)(x) \vee G_i(e)(x)]$$

$$= F(e)(x) \vee \left[\bigwedge_{i\in I^{(e)}} G_i(e)(x)\right] = J(e)(x).$$

According to the proof in the above, now we can obtain that

$$\left(H, A \cap \left(\bigcup_{i\in I} B_i\right)\right) = \left(J, A \cap \left(\bigcup_{i\in I} B_i\right)\right).$$

□

Complete Heyting algebras have been studied in many research areas of mathematics including order theory, domain theory and category theory. Some important properties (Gierz et al. 2003; Johnstone 1982) can help us to understand the characteristic of the soft information ordering structure over fuzzy soft sets in depth.

At last, a representation theorem of fuzzy soft sets, including a level characterization and a decomposition of fuzzy soft sets, is presented on this soft information ordering structure.

Definition 11 (Feng et al. 2010b) Let (F, A) be a fuzzy soft set over a universe set U, and let $\lambda \in [0, 1]$. Define $(F, A)_\lambda$ to be a soft set (H, A) such that

$$H(e) = \{x \in U : F(e)(x) \geq \lambda\}.$$

$\lambda(F, A)_\lambda$ is defined to be a fuzzy soft set (G, A) such that for all $e \in A$ and $x \in U$,

$$G(e)(x) = \begin{cases} \lambda, & \text{if } F(e)(x) \geq \lambda; \\ 0, & \text{otherwise.} \end{cases}$$

Example 3 Consider a fuzzy soft set (F, A) over a universe set $U = \{h_1, h_2, h_3, h_4, h_5\}$. Suppose that the parameters set $A = \{e_1, e_2\}$ and

$$F(e_1) = \{0.9/h_1, 0.7/h_2, 0.2/h_3, 0.4/h_4, 1/h_5\},$$
$$F(e_2) = \{0/h_1, 0.2/h_2, 0.8/h_3, 0.5/h_4, 0/h_5\}.$$

By the definition above, for example $(F, A)_{0.6}$ is a soft set (G, A), where

$$(G, A) = (G(e_1) = \{h_1, h_2, h_5\}, G(e_2) = \{h_3\}).$$

Similarly, we have $0.8(F, A)_{0.8}$ is a fuzzy soft set (H, A), and

$$(H, A) = (H(e_1) = \{0.8/h_1, 0.8/h_5\}, H(e_2) = \{0.8/h_3\}).$$

Proposition 4 Let (G_i, A) be a fuzzy soft set over U for all $i \in I$. Then

$$\bigwedge_{i \in I} (G_i, A) = (G, A),$$

where $G(e) = \bigcup_{i \in I} G_i(e), \forall e \in A$.

Proof Clearly, for all $i \in I$, we have $(G, A) \sqsubseteq (G_i, A)$. Assume that (H, A) is a lower bound of the set $\{(G_i, A) : i \in I\}$. Then, for all $e \in A$, we have $G_i(e) \subseteq H(e)$. Thus $\bigcup_{i \in I} G_i(e) \subseteq H(e)$, i.e., $G(e) \subseteq H(e)$. The proof above shows that $(H, A) \sqsubseteq (G, A)$. Now we obtain that $\bigwedge_{i \in I} (G_i, A) = (G, A)$.

Theorem 3 Let (F, A) be a fuzzy soft set over a same universe set U. Then

$$(F, A) = \bigwedge_{\lambda \in [0,1]} \lambda(F, A)_\lambda$$

and

$$(F, A) = \bigvee_{e \in A} (H_e, \{e\}),$$

where $H_e : \{e\} \to \mathcal{F}_U$ is defined by $H_e(e) = F(e)$.

Proof We write $\lambda(F, A)_\lambda = (F^{(\lambda)}, A)$ and $\bigwedge_{\lambda \in [0,1]} (F^{(\lambda)}, A) = (G, A)$. $\forall e \in A$ and $x \in U$, by Proposition 4, we have

$$G(e)(x) = \left[\bigcup_{\lambda \in [0,1]} F^{(\lambda)}(e) \right](x) = \bigvee_{\lambda \in [0,1]} F^{(\lambda)}(e)(x)$$
$$= \bigvee_{F(e)(x) \geq \lambda} \lambda = F(e)(x).$$

Thus $(F, A) = (G, A) = \bigwedge_{\lambda \in [0,1]} \lambda(F, A)_\lambda$.

For another part of this conclusion, according to Proposition 3, we can write

$$\bigvee_{e \in A} (H_e, \{e\}) = (Q, A).$$

For any a $d \in A$, there is only one fuzzy soft set $(H_d, \{d\}) \in \{(H_e, \{e\}) : e \in A\}$ such that its parameter set contains the parameter d. Therefore, by Proposition 3 the soft mapping Q is defined as $Q(d)(x) = H_d(d)(x)$ for all x. Thus $Q(d) = H_d(d) = F(d)$. It proves that $Q = F$. Then we have $\bigvee_{e \in A} (H_e, \{e\}) = (F, A)$. □

By the definition of supremum, the latter conclusion of Theorem 3 is also easy to prove directly.

4 A related application with the soft information order

For real-life applications, inference under some constraint conditions plays an important role in most cases. As in the example of choosing houses, everyone in a family will put forward some attributes which are considered to be important for him, and give the scores for all these optional houses. While, the family as a whole has to choose some most important parameters according to actuality such as income, the number of family members, working location and etc. Based on these facts and related data, we need to make a decision which house is their suitable choice. In fact, there exist two basic ways, that is, combination of information and extraction of information on a local domain, for dealing with information in solving these similar problems. To address this important issue, a two-tuple composed of a fuzzy soft set and a parameters set called a soft set satisfaction problem is presented in this section. The research here has received a revelation of the theory of constraints (Montanari 1974; Lauriere 1978; Bistarelli 2004).

Definition 12 Let U be an initial universe set and $\Theta = \{(F_i, A_i) : i = 1, 2, \ldots, n\}$ be a family of fuzzy soft sets over U. If K is a set of parameters such that $K \subseteq \bigcup_{i \in I} A_i$, then we call (Θ, K) a soft set satisfaction problem over U.

Definition 13 Let (F, A) be a fuzzy soft set over U and $B \subseteq A$. The projection of the fuzzy soft set (F, A) over the set B, written $(F, A)^{\downarrow B}$, is defined to be a new fuzzy soft set (G, B) such that $G(e) = F(e)$ for all $e \in B$.

Each soft set is a expression of a piece of information. Therefore, a projection of a fuzzy soft set defined here is the extraction of information on a local domain. We take the supremum of two soft sets (with respect to the soft information order) as the combination of them. Then naturally we define the solution of a soft set satisfaction problem to be the projection of the combination of soft sets on the parameters set of the problem discussed here.

Definition 14 Let (Θ, K) be a soft set satisfaction problem over a universe set U, where $\Theta = \{(F_i, A_i) : i = 1, 2, \ldots, n\}$. We call (H, K) an abstract solution of the problem (Θ, K), if

$$(H, K) = \left(\bigvee_{i=1}^{n}(F_i, A_i)\right)^{\downarrow K}.$$

And the set $\bigcap_{e \in K} H(e)$ is called the solution of the problem (Θ, K).

By Proposition 3 and Definition 13, we can get the following straightforward conclusion.

Corollary 1 Let (Θ, K) be a soft set satisfaction problem over a universe set U, where $\Theta = \{(F_i, A_i) : i = 1, 2, \ldots, n\}$. If (H, K) is an abstract solution of this problem, then $\forall e \in K$,

$$H(e) = \bigcap_{i \in I^{(e)}} F_i(e),$$

where $I^{(e)} = \{i \in I : e \in A_i\}$.

Example 4 Let $U = \{h_1, h_2, \ldots, h_8\}$ be a universe set which represents the set of houses. Consider three soft sets (F, A), (G, B) and (H, C) over U. Here $A = \{e_1, e_2\}$, $B = \{e_2, e_3\}$ and $C = \{e_1, e_3, e_4\}$ represent the attributes which have been considered in choosing houses by three different members in a family, respectively, and

$$\begin{aligned}
&(F, A) = (F(e_1) = \{h_1, h_8\}, F(e_2) = \{h_1, h_6, h_7\}),\\
&(G, B) = (G(e_2) = \{h_1, h_7\}, G(e_3) = \{h_1, h_4, h_5\}),\\
&(H, C) = (H(e_1) = \{h_1, h_4, h_8\},\\
&H(e_3) = \{h_1, h_4\}, H(e_4) = \{h_2, h_3, h_7\}).
\end{aligned}$$

Suppose that the family has chosen A as the parameters set finally. For the soft set satisfaction problem $P = (\{(F, A), (G, B), (H, C)\}, A)$, we obtain that its abstract solution is

$$(J, A) = (J(e_1) = \{h_1, h_8\}, J(e_2) = \{h_1, h_7\}),$$

and the solution of the problem P is $\{h_1\}$.

We give an algorithm for obtaining the abstract solution and the solution of a problem: Let $U = \{o_1, o_2, \ldots, o_m\}$ be an initial universe set and E be a set of parameters. Here $P = (\{(F_i, A_i) : i = 1, 2, \ldots, n\}, K)$ is a soft set satisfaction problem over U.

1. Input these fuzzy soft sets $(F_i, A_i)(i = 1, 2, \ldots n)$.
2. Input the parameters set K. Give an order for the elements of K in accordance with inputting sequence. We denote it by $K = \{e_1, e_2, \ldots, e_r\}$.
3. For each $e_i \in K$, take a tabular $X^{(i)}$: Get the set

   $$S^{(i)} = \{F_j^{(i)} : j = 1, 2, \ldots, l\}$$

   consisting of mappings in soft sets $\{(F_i, A_i) : i = 1, 2, \ldots n\}$, where $F_j^{(i)} \in S^{(i)}$ if and only if $e_i \in A_j^{(i)}$. Place these corresponding elements in the tabular $X^{(i)}$ with

   $$X^{(i)}(p, q) = F_p^{(i)}(o_q)(p = 1, 2 \ldots, l, q = 1, 2 \ldots, m),$$

   here $X^{(i)}(p, q)$ refers to the element in row p, column q of the tabular $X^{(i)}$.
4. Compute $\min_p X^{(i)}(p, q)$ for each a fixed q, and denote it by $l_{i,q}(q = 1, 2, \ldots, m)$.
5. Get a new tabular $L_{r \times m}$. The elements of this tabular are $l_{i,q}(i = 1, 2, \ldots, r; q = 1, 2, \ldots, m)$. The fuzzy soft set corresponded with the tabular $L_{r \times m}$ is the abstract solution of the problem P.
6. Compute the value of $\min\{l_{i,j} : i = 1, 2, \ldots, m\}$, and we write v_j. Then the solution of P is $\{v_1/o_1, v_2/o_2, \ldots, v_m/o_m\}$.

Example 5 Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be a universe set which represent houses. Here we use e_1, e_2, e_3, e_4 to present some attractive attributes of houses. Consider three fuzzy soft sets (F, A), (G, B) and (H, C) over U, where the sets of parameters considered by different persons are as follows: $A = \{e_1, e_2, e_3\}$, $B = \{e_2, e_4\}$, $C = \{e_1, e_3, e_4\}$, and

(1) $(F, A) = (F(e_1) = \{0.6/h_1, 0.2/h_2, 0/h_3, 0.9/h_4, 0.5/h_5, 0.1/h_6\}$, $F(e_2) = \{0.4/h_1, 0.5/h_2, 0.9/h_3, 0.5/h_4, 0.5/h_5, 0.8/h_6\}$, $F(e_3) = \{0.5/h_1, 0.5/h_2, 0.4/h_3, 0.2/h_4, 0.6/h_5, 0.5/h_6\})$;

(2) $(G, B) = (G(e_2) = \{0.5/h_1, 0.5/h_2, 0.8/h_3, 0.6/h_4, 0.6/h_5, 0.9/h_6\}$, $G(e_4) = \{0.6/h_1, 0.8/h_2, 1/h_3, 0.5/h_4, 0.6/h_5, 0.4/h_6\})$;

(3) $(H, C) = (H(e_1) = \{0.7/h_1, 0.3/h_2, 0.1/h_3, 0.8/h_4, 0.5/h_5, 0.2/h_6\}$, $H(e_3) = \{0.7/h_1, 0.4/h_2, 0.4/h_3, 0.3/h_4, 0.5/h_5, 0.6/h_6\}$, $H(e_4) = \{0.4/h_1, 0.9/h_2, 0.8/h_3, 0.5/h_4, 0.7/h_5, 0.3/h_6\})$.

Table 1 Tabular representation of fuzzy soft sets on e_1

$S^{(1)}$	h_1	h_2	h_3	h_4	h_5	h_6
F	0.6	0.2	0	0.9	0.5	0.1
H	0.7	0.3	0.1	0.8	0.5	0.2
min	0.6	0.2	0	0.8	0.5	0.1

Table 2 Tabular representation of fuzzy soft sets on e_2

$S^{(2)}$	h_1	h_2	h_3	h_4	h_5	h_6
F	0.4	0.5	0.9	0.5	0.5	0.8
G	0.5	0.5	0.8	0.6	0.6	0.9
min	0.4	0.5	0.8	0.5	0.5	0.8

Table 3 Tabular L representation of fuzzy soft set (M, D)

	h_1	h_2	h_3	h_4	h_5	h_6
e_1	0.6	0.2	0	0.8	0.5	0.1
e_2	0.4	0.5	0.8	0.5	0.5	0.8

Let the parameters set $D = \{e_1, e_2\}$. Then $P = (\{(F,A),(G,B),(H,C)\}, D)$ is a soft set satisfaction problem. We give the solution of P according to the algorithm shown above.

The three tables constructed in the process of computing solution are shown as follows, where Tables 1 and 2 have been created for the parameters e_1 and e_2, respectively. For example, for the parameter e_1, we get a set $S^{(1)} = \{F, H\}$ according to the third rule of this algorithm firstly. We input the corresponding values $F(h_i)$ and $H(h_i)$ for h_i sequentially in Table 1. Afterwards we compute the minimum between $F(h_i)$ and $H(h_i)$ for each h_i. Then Table 1 is obtained. We can also get the second tabular for the parameter e_2. Assume that (M, D) is the abstract solution of the problem P. Table 3 represents the fuzzy soft set (M, D), i.e.,

$$\begin{aligned}(M,D) &= (M(e_1)\\ &= \{0.6/h_1, 0.2/h_2, 0/h_3, 0.8/h_4, 0.5/h_5, 0.1/h_6\},\\ M(e_2) &= \{0.4/h_1, 0.5/h_2, 0.8/h_3, 0.5/h_4, 0.5/h_5, 0.8/h_6\}).\end{aligned}$$

Therefore, the solution of the problem P is $\{0.4/h_1, 0.2/h_2, 0/h_3, 0.5/h_4, 0.5/h_5, 0.1/h_6\}$.

According to the solution shown above, we can conclude that the houses h_4 and h_5 are adaptive choices.

5 Conclusion

In this paper, we proposed the concept of soft information order between fuzzy soft sets. We found that, with respect to this new ordering, the supremum and the infimum of two fuzzy soft sets correspond, respectively, to the operations extensive intersection and restricted union. A new meaning of these operations from the perspective of ordering relation is shown. Furthermore, we study the character of ordering structure on fuzzy soft sets. We have shown that the set of all fuzzy soft sets over a universe set is a complete Heyting algebra. The representations of fuzzy soft sets with respect to the information order are given. For practical applications, we have presented the concept of soft set satisfaction problems and given an algorithm for obtaining solution of them.

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