Soft Comput (2012) 16:1691–1695
DOI 10.1007/s00500-012-0853-1

ORIGINAL PAPER

On mixed fuzzy topological spaces and countability

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Published online: 11 May 2012

Abstract In this paper, we introduce a new type of mixed fuzzy topological space. We define countability on mixed fuzzy topological spaces. We investigate its different quasi type properties.

Keywords Bitopological spaces · Fuzzy sets · Fuzzy topology · First countable space · Second countable space

1 Introduction

The different notions of topological space has been generalized and expanded in many ways in the last half century. The introduction of the notions of bitopological space and mixed topological space are two major developments in topological spaces. Bitopological spaces has recently been studied by Ganguly and Singha (1984), Tripathy and Sarma (2011b, 2012) and many others. For a detailed account on Bitopological spaces, one may refer to Hussain (1966). Mixed topology is a technique of mixing two topologies on a set to get a third topology. The works on mixed topology is due to Buck (1952), Cooper (1971), Wiweger (1961), Das and Baishya (1995) and many others.

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Mixed topology lies in the theory of strict topology of the spaces of continuous functions on locally compact spaces. The study of classical mixed topology is very old. Buck (1952) introduced and investigated the concept of strict topology. This has been applied in the study of spectral synthesis, spaces of bounded holomorphic functions and multipliers of Banach algebras. Conway (1967) studied strict topology and compactness in the space of measures. Cooper (1971) applied strict topology in investigating some results in function spaces. Orlicz (1955) has applied mixed topology in summability theory. It has been applied for the study in non-locally compact spaces, interpolation theorems for analytic functions and Schauder decomposition by Subramanian (1972) and Gupta et al. (1983). There are many other applications of mixed topology.

Zadeh introduced the concept of fuzzy sets in the year 1965. Since then the notion of fuzziness has been applied for the study in all the branches of science and technology. It has been applied for studying different classes of sequences of fuzzy numbers by Tripathy and Baruah (2010), Tripathy and Borgohain (2008), Tripathy and Dutta (2010), Tripathy and Sarma (2011a) and many workers on sequence spaces in the recent years. The notion of fuzziness has been applied in topology and different notions of fuzzy topological spaces have been introduced and investigated by many researches in topological spaces. Different properties of fuzzy topological spaces have been investigated by Arya and Singal (2001a, b), Chang (1968), Das and Baishya (1995), Ganster et al. (2005), Ganguly and Singha (1984), Ghanim et al (1984), Katsaras and Liu (1977), Petricevic (1998), Srivastava et al. (1981), Warren (1978), Wong (1974a, b) and many others. Recently mixed fuzzy topological spaces are being investigated from different aspect by Das and Baishya (1995).

2 Preliminaries

Let X be a non-empty set and I, the unit interval [0, 1]. A fuzzy set A in X is characterized by a function $\mu_A : X \to I$ where μ_A is called the membership function of A and $\mu_A(x)$ representing the membership grade of x in A. The empty fuzzy set is defined by $\mu_\Phi(x) = 0$ for all $x \in X$. In addition, X can be regarded as a fuzzy set in itself defined by $\mu_X(x) = 1$ for all $x \in X$. Further, an ordinary subset A of X can also be regarded as a fuzzy set in X if its membership function is taken as usual characteristic function of A that is $\mu_A(x) = 1$ for all $x \in A$ and $\mu_A(x) = 0$ for all $x \in X - A$. Two fuzzy set A and B are said to be equal if $\mu_A = \mu_B$. A fuzzy set A is said to be contained in a fuzzy set B, written as $A \subseteq B$, if $\mu_A \le \mu_B$. Complement of a fuzzy set A in X is a fuzzy set A' in X defined by $\mu_A = 1 - \mu_{A^c}$. We write $A^c =$ co$A$. Union and intersection of a collection $\{A_i : i \in N\}$ of fuzzy sets in X, are written as $\cup_{i\in N} A_i$ and $\cap_{i\in N} A_i$, respectively. The membership functions are defined as follows:

$$\mu_{\bigcup_{i\in N} A_i}(x) = \sup\{\mu_{A_i}(x) : i \in N\} \quad \text{for all } x \in X,$$

and $$\mu_{\bigcap_{i\in N} A_i}(x) = \inf\{\mu_{A_i}(x) : i \in N\} \quad \text{for all } x \in X.$$

A *fuzzy topology* τ on X is a collection of fuzzy sets in X such that $\Phi, X \in \tau$; If $A_i \in \tau$, $i \in N$ then $\cup_{i\in N} A_i \in \tau$ and if $A, B \in \tau$ then $A \cap B \in \tau$. The pair (X, τ) is called a fuzzy topological space (fts). Members of τ are called open fuzzy sets and the complement of an open fuzzy set is called a closed fuzzy set. If (X, τ) is a fts then, the closure and interior of a fuzzy set A in X, denoted by *cl A* and *int A*, respectively, are defined as $cl\ A = \cap\{B$: B is a closed fuzzy set in X and $A \subseteq B\}$ and $int\ A = \cup\{V : V$ is an open fuzzy set in X and $V \subseteq A\}$. Clearly, *cl A* (respectively, *int A*) is the smallest (respectively, largest) closed (respectively, open) fuzzy set in X containing (respectively, contained in) A. If there are more than one topologies on X, then the closure and interior of A with respect to a fuzzy topology τ on X will be denoted by τ-*cl A* and τ-*int A*.

Definition 2.1 A collection $\mathscr{B}$ of open fuzzy sets in a fts X is said to be an open base for X if every open fuzzy set in X is a union of members of $\mathscr{B}$.

Definition 2.2 If A is a fuzzy set in X and B is a fuzzy set in Y, then $A \times B$ is a fuzzy set in $X \times Y$ defined by $\mu_{AxB}(x, y) = \min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$ and for all $y \in Y$.

Definition 2.3 Let f be a function from X into Y. Then for each fuzzy set B in Y, the inverse image of B under f, written as $f^{-1}[B]$, is a fuzzy set in X defined by $\mu_{f^{-1}[B]}(x) = \mu_B(f(x))$ for all $x \in X$.

Definition 2.4 A fuzzy set A in a fuzzy topological space (X, τ) is called a neighbourhood of a point $x \in X$ if and only if there exists $B \in \tau$ such that $B \subseteq A$ and $A(x) = B(x) > 0$.

Definition 2.5 A fuzzy point x_α is said to be quasi-coincident with A, denoted by $x_\alpha\ q\ A$, if and only if $\alpha + A(x) > 1$ or $\alpha > (A(x))^c$.

Definition 2.6 A fuzzy set A is said to be quasi-coincident with B, denoted by AqB, if and only if there exists a $x \in X$ such that $A(x) + B(x) > 1$.

It is clear from the above definition, if A and B are quasi-coincident at x both $A(x)$ and $B(x)$ are not zero at x and hence A and B intersect at x.

Definition 2.7 A fuzzy set A in a fts (X, τ) is called a quasi-neighbourhood of x_λ if and only if $A_1 \in \tau$ such that $\overline{A_1} \subseteq A$ and x_λ qA_1. The family of all Q-neighbourhood of x_λ is called the system of Q-neighbourhood of x_λ. Intersection of two quasi-neighbourhood of x_λ is a quasi-neighbourhood.

3 Main results

Das and Baishya (1995) have defined the mixed fuzzy topology with respect to a fuzzy point x_λ. Recently, Ali (2009) has defined the mixed fuzzy topology on taking the neighbourhood of a fuzzy point. In this paper, we have considered a fuzzy set. Thus, our definition of mixed fuzzy topology generalizes the existing type of construction of mixed fuzzy topology.

We introduce a new type of mixed fuzzy topological space in this section.

Theorem 3.1 *Let* (X, τ_1) *and* (X, τ_2) *be two fuzzy topological spaces. Consider the collection of fuzzy sets* $\tau_1(\tau_2) = \{A \in I^X :$ *For any fuzzy set B in X with AqB, there exists* τ_2*-open set* A_1 *such that* A_1qB *and* τ_1*-closure* $\overline{A_1} \subseteq A\}$. *Then this family of fuzzy sets will form a topology on X and this topology we call a mixed fuzzy topology on X.*

Proof First, we verify that this family will form a topology on X.

(T_1) $\overline{0} \in \tau_1(\tau_2)$ since $\overline{0}$ is not quasi-coincident with any fuzzy set A in X, and therefore, there does not arise any questions of violation of the condition of being member of $\tau_1(\tau_2)$. $\overline{1} \in \tau_1(\tau_2)$ since for any fuzzy set A, $Aq\overline{1}$ and there exists τ_2-open set A_1 with $A_1q\overline{1}$ and τ_1-closure $\overline{A_1} \subseteq \overline{1}$.
(T_2) To show that intersection of any two members of $\tau_1(\tau_2)$ is again a member of $\tau_1(\tau_2)$.
Let $A_1, A_2 \in \tau_1(\tau_2)$. We show that $A_1 \cap A_2 \in \tau_1(\tau_2)$.
Since $A_1 \in \tau_1(\tau_2)$, so for any fuzzy set A in X with AqA_1 there exists τ_2-open set B_1 such that B_1qA and τ_1-closure $\overline{B_1} \subseteq A_1$.
Also, $A_2 \in \tau_1(\tau_2)$ so for any fuzzy set A in X with A_qA_2

there exists τ_2-open set B_2 such that B_2qA and τ_1-closure $\overline{B_2} \subseteq A_2$.

Now, B_1, B_2 are τ_2-open set implies $B_1 \cap B_2 \in \tau_1(\tau_2)$.

We have $\overline{B_1 \cap B_2} \subseteq \overline{B_1} \cap \overline{B_2} \subseteq A_1 \cap A_2$, where $\overline{B_1 \cap B_2}$ is the τ_1-closure of $B_1 \cap B_2$.

Thus, we get a τ_2-open set $B_1 \cap B_2$ such that $(B_1 \cap B_2)qA$ and hence $\overline{B_1 \cap B_2} \subseteq A_1 \cap A_2$.

Therefore, we have $A_1 \cap A_2 \in \tau_1(\tau_2)$.

(T_3) Let $\{A_\lambda : \lambda \in \wedge\}$ be a family of open sets. We need to show that $\underset{\lambda \in \wedge}{\vee} A_\lambda \in \tau_1(\tau_2)$.

Since $A_\lambda \in \tau_1(\tau_2)$ $\forall \lambda \in \wedge$, thus for any fuzzy set A such that AqA_λ, for all $\lambda \in \wedge$.

$\Rightarrow$ There exists $x \in X$ such that $A(x) + A_\lambda(x) > 1$.

$$\Rightarrow A(x) + \underset{\lambda \in \wedge}{\text{Max}}\{A_\lambda(x)\} > 1.$$

$$\Rightarrow A(x) + \underset{\lambda \in \wedge}{\vee} A_\lambda(x) > 1.$$

$$\Rightarrow A\, q(\underset{\lambda \in \wedge}{\vee} A_\lambda).$$

Since $A_\lambda \in \tau_1(\tau_2)$ and AqA_λ, for any fuzzy set A, so there exists τ_2-open fuzzy open set B_λ such that $B_\lambda qA$ and τ_1-closure $cl(B_\lambda) \leq A_\lambda$.

But we know that arbitrary union of member of τ_2 is also a member of τ_2 and so $\vee_{\lambda \in \wedge} B_\lambda \in \tau_2$ and $(\vee_{\lambda \in \wedge} B_\lambda)qA$. On considering the τ_1-closure, we have

$$\vee(cl(B_\lambda) \leq cl(\underset{\lambda \in \wedge}{\vee} B_\lambda) \leq \underset{\lambda \in \wedge}{\vee} A_\lambda \Rightarrow \underset{\lambda \in \wedge}{\vee} A_\lambda \in \tau_1(\tau_2).$$

Therefore, this collection $\tau_1(\tau_2)$ of fuzzy subsets of X is a fuzzy topology on X. We call $(X, \tau_1(\tau_2))$ a mixed fuzzy topological space.

Remark 3.1 In order to have a clear idea about the mixed fuzzy topology $\tau_1(\tau_2)$ defined in Theorem 3.1, we consider the following example.

Example 3.1 Consider the set of real numbers R and let us consider the collection of fuzzy sets $\{\overline{0}, \overline{1}, B_{x,n}\}$ in R, where $B_{x,n}$ is defined by

$$B_{x,n}(t) = \begin{cases} nt - nx + 1, & x - \frac{1}{n} \leq t \leq x; \\ nt - nx - 1, & x \leq t \leq x + \frac{1}{n}; \\ 0, & \text{otherwise.} \end{cases}$$

Then the collection $\tau_1 = \{\overline{0}, \overline{1}, B_{x,n}\}$ will form a fuzzy topology on R.

Consider the fuzzy set in R defined by

$$A_{x,n}(t) = \begin{cases} 1, & x - \frac{1}{n} \leq t \leq x + \frac{1}{n} \\ 0, & \text{otherwise} \end{cases}$$

Then the collection $\tau_2 = \{\overline{0}, \overline{1}, A_{x,n}\}$ will form a fuzzy topology on R.

Now, consider the collection $\tau_1(\tau_2) = \{A \in I^R$: For any $B \in I^R$ with AqB there exists τ_2-open set $A_{x,n}$ such that $A_{x,n}qB$ and τ_1-closure $\overline{A_{x,n}} \subseteq A\}$.

First, we show that the collection of fuzzy sets $\tau_1(\tau_2)$ will form a mixed fuzzy topology on R.

$\overline{0} \in \tau_1(\tau_2)$. Let $B \in \boldsymbol{I}^R$, $\overline{0}$ is not quasi-coincident with B. since we cannot find a τ_2-open set $A_{x_0,n}$ such that $A_{x_0,n}qB$ and τ_1-closure $\overline{A_{x_0,n}} \subseteq \overline{0}$.

Also, $\overline{1} \in I^R$. Let $B \neq \overline{0} \in I^R$, we have $Bq\overline{1}$.

Let the fuzzy set B be defined by

$$B(t) = \begin{cases} 0.5, & x - 0.5 \leq t \leq x + 0.5, \\ 0, & \text{otherwise.} \end{cases}$$

Obviously, $Bq\,\overline{1}$, $A_{x_0,\frac{1}{3}}qB$ (Since $A_{x_0,\frac{1}{3}}(t) + B(t) > 1$) and τ_1-closure $\overline{A_{x_0,\frac{1}{3}}} \subseteq \overline{1}$.

Thus, for any $B \in I^R$ with $\overline{1}qB$, there exists τ_2-open set $A_{x_0,\frac{1}{3}}$ such that $A_{x_0,\frac{1}{3}}qB$ and the τ_1-closure $\overline{A_{x_0,\frac{1}{3}}} \subseteq \overline{1}$. Hence $\overline{1} \in \tau_1(\tau_2)$.

Next, let A_1, $A_2 \in \tau_1(\tau_2)$, We have to show that $A_1 \wedge A_2 \in \tau_1(\tau_2)$.

Let B be any fuzzy set in R such that $B\ q\ (A_1 \wedge A_2)$.

$$\Rightarrow B(t) + (A_1 \wedge A_2)(t) > 1, \text{ for some } t \in R.$$
$$\Rightarrow B(t) + \text{Min}\{A_1(t), A_2(t)\} > 1.$$
$$\Rightarrow B(t) + A_1(t) > 1 \text{ and } B(t) + A_2(t) > 1.$$
$$\Rightarrow BqA_1 \text{ and } BqA_2.$$

Since, A_1, $A_2 \in \tau_1(\tau_2)$, so there exists τ_2-open sets $A_{x_1,n}$, $A_{x_2,n}$ such that $A_{x_1,n}qB$ and $A_{x_2,n}qB$ and the τ_1-closure $\overline{A_{x_1,n}} \subseteq A_1$ and $\overline{A_{x_2,n}} \subseteq A_2$.

Now, $A_{x_1,n}qB$ and $A_{x_2,n}qB$.

$$\Rightarrow A_{x_1,n}(t) + B(t) > 1 \text{ and}$$
$$A_{x_2,n}(t) + B(t) > 1, \quad \text{for some } t \in R. \tag{1}$$

and τ_2 is a fuzzy topology, therefore, $A_{x_1,n} \wedge A_{x_2,n} \in \tau_2$.

$$\text{We have } (A_{x_1,n} \wedge A_{x_2,n})(t) + B(t) = \text{Min}\{A_{x_1,n}(t),\ A_{x_2,n}(t)\} + B(t) > 1.$$

$$\Rightarrow (A_{x_1,n} \wedge A_{x_2,n})qB.$$

Thus, for any fuzzy set B in R with $Bq(A_1 \wedge A_2)$, there exists τ_2-open set $A_{x_1,n} \wedge A_{x_2,n}$such that $(A_{x_2,n} \wedge A_{x_2,n})qB$ and the τ_1-closure $\overline{A_{x_1,n} \wedge A_{x_2,n}} = \overline{A_{x_1,n}} \wedge \overline{A_{x_2,n}} \subseteq A_1 \wedge A_2$.

Therefore, $A_1 \wedge A_2 \in \tau_1(\tau_2)$.

Finally, let us consider a collection $\{A_i : i \in \Delta\}$ of fuzzy sets in $\tau_1(\tau_2)$.

We have to show that $\vee_{i \in \Delta} A_i \in \tau_1(\tau_2)$.

Let B be any fuzzy set in R such that $Bq(\vee_{i \in \Delta} A_i)$.

$$\begin{aligned} &\Rightarrow B(t) + \text{Max}\{A_i(t) :\ i \in \Delta\} > 1, \text{ for some } t \in R \\ &\Rightarrow BqA_i \text{ for some } i \in \Delta. \end{aligned} \tag{2}$$

Since $A_i \in \tau_1(\tau_2)$ and BqA_i, for some i, so there exists τ_2-open set $A_{x_i,n}$ such that $Bq\ A_{x_i,n}$ and τ_1-closure $\overline{A_{x_i,n}} \subseteq A_i$.

Let $Bq\ A_{x_i,n}$ for some $i \in \Delta$

$$\Rightarrow B(t) + A_{x_i,n}(t) > 1$$
$$\Rightarrow B(t) + \underset{i\in\Delta}{\text{Max}}\{A_{x_i,n}(t)\} > 1$$
$$\Rightarrow B(t) + \underset{i\in\Delta}{\vee} A_{x_i,n}(t) > 1$$
$$\Rightarrow B\text{q} \underset{i\in\Delta}{\vee} A_{x_i,n}$$

Since $A_{x_i,n}$ are open in $\tau_2 \Rightarrow \vee_{i\in\Delta} A_{x_i,n}$ is open in τ_2 and τ_1-closure $\overline{\vee A_{x_i,n}} \subseteq \vee A_\text{i}$.

Therefore, if $Bq \vee A_\text{i}$ then there exist τ_2-open set $\vee_{i\in\Delta} A_{x_i,n}$ such that $Bq\ A_{x_i,n}$ and τ_1-closure $\overline{\vee A_{x_i,n}} \subseteq \vee A_\text{i}$.

Hence $\vee_{i\in\Delta}\ A_\text{i} \in \tau_1(\tau_2)$ and so $\tau_1(\tau_2)$ is a mixed fuzzy topology on R.

4 Countability on mixed fuzzy topological spaces

First, we list some known definitions, those will be used for establishing the results of this section.

Definition 4.1 Let U_{CQ} be a family of Q-neighbourhood of a fuzzy point x_λ in X. A subfamily B_{CQ} of U_{CQ} is said to be a Q-neighbourhood base of U_{CQ} if for any $A \in U_{CQ}$ there exists $B \in B_{CQ}$ such that $B < A$.

Definition 4.2 A fuzzy topological space (X,δ) is said to be Q-first countable space if and only if every fuzzy point in X has countable Q-neighbourhood base.

Definition 4.3 Let U_C be a family of neighbourhoods of a fuzzy point x_λ in X. A subfamily B_C of U_C is said to be a neighbourhood base of U_C, if for any $A \in U_C$, there exists $B \in U_C$ such that $B < A$.

Definition 4.4 A fuzzy topological space (X,δ) is said to be first countable space if and only if every fuzzy point in X has a countable neighbourhood base.

The following definition is an alternative to the Definition 4.4.

Definition 4.5 Let (X,δ) be a fuzzy topological space. Then X is said to be a first countable space, if for each fuzzy point $x_\lambda (0 \le \lambda \le 1)$ there exists a countable class of fuzzy open sets B_{x_α} such that $\lambda < U(x)$, for all $U \in B_{x_\alpha}$ and if $\lambda < V(x)$ for some fuzzy open set V then there exists $W \in B_{x_\alpha}$ such that $W \subseteq V$.

Definition 4.6 A fuzzy topological space (X,τ) is said to be C_{II} if there exists a countable base for τ.

We now introduce the following definitions:

Definition 4.7 Let $(X, \tau_1(\tau_2))$ be a mixed fuzzy topological space. Let U_λ be a family of neighbourhood of a fuzzy point x_λ in X. A subfamily B_λ of U_λ is said to be a neighbourhood base of U_λ if for any $A \in U_\lambda$ there exists $B \in B_\lambda$ such that $B < A$.

Definition 4.8 Let $(X, \tau_1(\tau_2))$ be a mixed fuzzy topological space. Then X is said to be first countable space if every fuzzy point in X has a countable neighbourhood base.

Definition 4.9 Let $(X, \tau_1(\tau_2))$ be a mixed fuzzy topological space. Then X is said to be Q-first countable space (i.e. Q–C_I space) if every fuzzy point in X has a Q-neighbourhood base.

Theorem 4.1 *Let $(X, \tau_1(\tau_2))$ be a C_I-space. Then it is a Q–C_I space.*

Proof Let e be any fuzzy point in X. Consider a sequence $\{\mu_n\}_{n\in N}$ in $(1 - \lambda,1]$ converging to $1 - \lambda$ and let $x_{\mu_n} = e_n$.

Since X is a C_I-space for each $n \in N$, there exists a countable open neighbourhood base $\{B_\text{n}\}$ of e_n. We have for each member B of $\{B_n\}$, $B(x) \ge \mu_n > 1 - \lambda$.

$$\Rightarrow \lambda + \text{B}(x) > 1.$$
$$\Rightarrow x_\lambda qB.$$

Hence B is a Q-neighbourhood of e. Thus, the collection $\{B_\text{n}\}$ is a family of open Q-neighbourhoods of "e" and hence this family will be a countable family of Q-neighbourhoods of "e".

Let A be an arbitrary Q-neighbourhood of "e". Hence $A(x) > 1 - \lambda$. Since $\mu_n > 1 - \lambda$, so there exists $m \in N$ such that $A(\text{x}) \ge \mu_m 1 - \lambda \Rightarrow e_m \in A$ and A is an open neighbourhood of "e_m".

Thus, there exists a member $B \in \{B_n\}$ such that $B < A$ and $B(x) > \mu_m > 1 - \lambda$ and so B is a Q-neighbourhood base of "e".

Hence $(X, \tau_1(\tau_2))$ is a Q–C_I space.

Proposition 4.2 *If $(X, \tau_1(\tau_2))$ is C_{II}, then it is also Q–C_I.*

Proof Let $(X, \tau_1(\tau_2))$ be C_II, so there exists a countable base for $\tau_1(\tau_2)$.

Let $\mathscr{B}$ be a countable base for $\tau_1(\tau_2)$. We have to show that $(X, \tau_1(\tau_2))$ is Q–C_I. It is sufficient to establish that every fuzzy point in X has a countable Q-neighbourhood base.

Let e be a fuzzy point in $(X, \tau_1(\tau_2))$.

Let A be any subset of X satisfying Aqe. Hence there exists $B \in \mathscr{B}$ such that eqB and $B < A$. Therefore, B is a Q-neighbourhood of the fuzzy point "e".

Let $\boldsymbol{U}$ be the family of all those members $B \in \mathscr{B}$. Clearly, this collection is a countable collection of Q-nbd.

of e. That is, the fuzzy point e has a countable Q-neighbourhood base and hence $(X, \tau_1(\tau_2))$ is a Q–C_I space.

Proposition 4.3 *Let τ_1 and τ_2 be two fuzzy topologies for X and if the mixed fuzzy topology $\tau_1(\tau_2)$ is Q-first countable, then τ_2 is also Q-first countable.*

Proof Let x_λ be an arbitrary fuzzy point in X.

Since $\tau_1(\tau_2)$ is Q–C_I space, therefore there exists a Q-neighbourhood base for every fuzzy point x_λ. Let $A \in B_Q$, where B_Q is the countable collection of $\tau_1(\tau_2)$ Q-neighbourhood base at x_λ. Then A is $\tau_1(\tau_2)$ Q-nbd. of x_λ.

$\Rightarrow$ There exists $B \in \tau_1(\tau_2)$ such that $B \subseteq A$ and $x_\lambda qB$.

We know that $\tau_1(\tau_2) \subseteq \tau_2$.

Therefore, $B \in \tau_1(\tau_2) \Rightarrow B \in \tau_2$ and $B \subseteq A$, $x_\lambda qB$.

i.e. A is also a τ_2–Q–nbd. of x_λ.

Thus, every member $A \in B_Q$ is τ_2–Q-neighbourhood of x_λ.

$\Rightarrow B_Q$ is also τ_2-Q-neighbourhood base at x_λ.

Hence, τ_2 is also Q–C_I space.

Acknowledgments The authors thank the referees and the Editor for their comments those improved the presentation of the paper.

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