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ORIGINAL PAPER

An intuitionistic fuzzy group decision making method using entropy and association coefficient

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Abstract Group decision making is a process in which experts rank and choose the most desirable alternatives based on some accepted criteria. The aim of this paper was to introduce a method to solve group decision making problems with Atanassov's intuitionistic fuzzy sets. First, the weight of each criterion is calculated using intuitionistic fuzzy entropy. Then, the total criteria weight vector is calculated by aggregating the calculated weights. Using the obtained weight vector, the alternatives are ranked based on the association coefficient of the performance of alternatives related to each criterion and the positive ideal intuitionistic fuzzy set value and the negative ideal intuitionistic fuzzy set value. Finally, to show the application of the proposed method, it is implemented in software vendor selection.

Keywords Group decision making · Atanassov's intuitionistic fuzzy set · Entropy weight · Association coefficient

1 Introduction

Fuzzy logic and fuzzy sets theory (FSs) proposed by Zadeh (1965) has represented a means for handling vagueness and impreciseness in the real-life situations. Atanassov (1986, 1989, 1994) introduced a generalization of Zadeh's fuzzy set called Atanassov's intuitionistic fuzzy set (IFS). Each IFS is characterized by a membership function and non-membership function. Atanassov's IFS are useful to deal with uncertainty and vagueness. Today, Atanassov's IFS has become one of the most applicable subjects in many different scientific fields, including medical diagnosis (Khatibi and Montazer 2009; De et al. 2001), clustering (Xu et al. 2008; Chaira 2010), pattern recognition (Boran 2009; Hung and Yang 2008; Vlachos and Sergiadis 2007; Zhang and Fu 2006; Hung and Yang 2004b; Liang and Shi 2003; Dengfeng and Chuntian 2002; Tizhoosh 2008), and IFS topology (Mursaleen et al. 2010; Yılmaz 2010; Mursaleen and Mohiuddine 2009; Samanta and Mondal 2002). Gau and Buehrer (1993) introduced the vague set, but Bustince and Burillo (1996) showed that it is an equivalence of Atanassov's IFS. Many relations and operators related to Atanassov's IFS have been studied by researchers, such as distance measure (Grzegorzewski 2004; Szmidt and Kacprzyk 2000; Wang and Xin 2005), similarity measure (Li 2004; Li et al. 2005, 2007; Hung and Yang 2004a), and IFS entropy (Zhang et al. 2009; Burillo and Bustince 1996; Ye 2010a, b). Xu and Yager (2006a) developed some geometric aggregation based on Atanassov's IFS, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator, and the intuitionistic fuzzy hybrid geometric (IFHG) operator. Xu (2007), moreover, developed the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator and the intuitionistic fuzzy hybrid aggregation (IFHA) operator.

There has been much investigation on group decision making (GDM) with Atanassov's IFS by researchers. Atanassov et al. (2005) used IFS to solve a multi-criteria, multi-person, and multi-measurement GDM problem. Li et al. (2009) developed a fractional programming model based on TOPSIS for solving multi-attribute group decision-making problems using Atanassov's IFS. There is some

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research which is done on aggregation operators of decision-making process. As an illustration, Wei (2010b) proposed two new aggregation operators: induced intuitionistic fuzzy ordered weighted geometric (I-IFOWG) operator and induced interval-valued intuitionistic fuzzy ordered weighted geometric (I-IVIFOWG) operator, and developed them to solve the MAGDM problems, in which both the attribute weights and the expert weights take the form of real numbers. Liu and Wang (2007) presented a new method for solving multi-criteria decision-making problem in an intuitionistic fuzzy environment to measure the degrees to which alternatives satisfy and do not satisfy the decision-makers requirement. Boran et al. (2009) combined TOPSIS method with Atanassov's IFS to select appropriate supplier in a GDM procedure. Xu and Yager (2008) presented two new aggregation operators: dynamic intuitionistic fuzzy weighted averaging (DIFWA) operator and uncertain dynamic intuitionistic fuzzy weighted averaging (UDIFWA) operator and introduced some methods, including the basic unit-interval monotonic (BUM) function-based method, normal distribution-based method, exponential distribution-based method, and average age method, to determine the weight vectors associated with these operators. They also investigated the dynamic multi-attribute decision-making problems with Atanassov's intuitionistic fuzzy information. Wei (2010a) introduced an optimization model based on the basic ideal of traditional gray relational analysis (GRA) method, by which the attribute weights can be determined. They investigated the multiple-attribute decision-making problems with intuitionistic fuzzy information. In this model, the information about attribute weights is incompletely known, and the attribute values take the form of Atanassov's intuitionistic fuzzy numbers. Xu (2010) also developed a method based on distance measure for GDM with interval-valued intuitionistic fuzzy matrices. Wu and Zhang (2011) present the concept of the intuitionistic fuzzy weighted entropy, which is a natural extension of the entropy for Atanassov's IFSs. They calculated the criteria weights according to the minimum entropy and use it to solve the multi-criteria decision-making. They also based on Atanassov's IFS score function and accuracy function ranked the alternatives. Ye (2010a, b) proposed a method for multi-criteria decision-making based on entropy weight. He utilized IFS entropy measure to compute the criterion weights and ranked the alternative with respect to weighted correlation coefficients.

Considering the growing ambiguity and complexity of today's decision-making process, the existence of the suitable decision-making method which handles imprecise and complicated situations will be more and more essential. MCDM is one of the widely used decision-making methodologies, which is applied in various areas. Due to the majority of decision makers (DMs) in the real world, they usually tend to give their preferences for each alternative based on a number of predetermined criteria in an uncertain situation. Thus, they are not completely confident about their preferences, and consequently, their attitudes are blended with some amount of uncertainty (hesitation) degree. This situation can be completely dealt with in the best way utilizing the Atanassov's IFS concept. Moreover, Atanassov's IFS allows DMs to assign the membership and non-membership degree to each alternative, and it also enables them to overcome the existing uncertainty. In the decision-making process, sometimes, the information about criteria weight is completely unknown or incompletely known because of time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain. In addition, there has been carried out a little investigation on GDM with completely unknown criteria weights. The entropy of a fuzzy set is a measure of the fuzziness of a fuzzy set. Although this is called entropy due to the concept's intrinsic similarity to Shannon's entropy, the two functions measure fundamentally different types of uncertainty (Szmidt and Kacprzyk 2001). In this paper, an entropy-based method to determine the criterion weights for each decision-making matrix is utilized and a model to aggregate the calculated weights is proposed to calculate the final entropy weights. In statistics and engineering, association (correlation) is often used. By association analysis, the joint relationship of two variables can be examined with an interdependence measure of two variables (Chaudhuri and Bhattacharya 2001). In real-life situations, we face some situations where instead of measured values of two alternatives, the rank of two alternatives is based on different qualitative criteria. In this case, association measure is utilized to correlate two alternatives. Such situations arise when we have qualitative criteria rather than quantitative ones. Therefore, using association to rank the alternatives can be a suitable choice.

Using the entropy and the association coefficient, the weight of criteria for decision-making process is calculated considering the relation among the criteria and incomplete information related to each criterion. In this paper, we combined the entropy as it is a good method for calculating the weights of each criterion according to its information and the association coefficient since it is a good idea for considering the relation among criteria. Thus, a strong method will be created for signifying the weights of criteria in a decision-making process for encountering the incomplete information and complicated relation among the criteria.

Here, a GDM model with respect to the weighted association coefficient degree combined intuitionistic fuzzy environment is developed to choose the best alternative. We present an evaluating process according to the weighted association coefficient degree to rank the alternatives by comparing them with PIIFS and NIIFS.

As presented above, the contributions of this paper are (1) the research underlines the gap in the GDM for involving uncertainty and precise information; moreover, it takes into account the DM's hesitancy. (2) A new method has been developed to find the completely unknown criterion weights using the IFS entropy measure. First, we find the entropy weights for each decision matrix; afterwards, the obtained weights are aggregated to reach the final criterion weights. (3) To compare the alternatives, an evaluation formula is proposed using the weighted intuitionistic fuzzy association coefficient between an alternative and the positive ideal alternative and negative ideal alternative. The most desirable alternatives can be selected or ranked according to the weighted intuitionistic fuzzy association coefficient.

This study attempts to present a high accuracy level method that will be more compatible with the real world and also to be able to give optimal results in comparison with other proposed methods.

The rest of this paper is organized as follows: in Sect. 2, some basic concepts, operators, relations, and their properties related to Atanassov's IFS are explored. Section 3 deals with the concept of IFS entropy. Section 4 develops a distance-based GDM method using entropy under intuitionistic fuzzy information, and Sect. 5 applies the method on a case study. Finally, the conclusion is given in Sect. 6.

2 Atanassov's intuitionistic fuzzy sets

Let us start with a short definition of simple fuzzy set (FS) introduced by Zadeh (1965). Fuzzy sets are an extension shape of crisp sets. Fuzzy sets are represented as a means for handling vagueness and impreciseness in real-life situations. If $X = \{x_1, x_2, \ldots, x_n\}$ is a universe of discourse, then Zadeh's fuzzy set is showed as follows:

$$F = \{\langle x, \mu_{\mathrm{F}}(X)\rangle | x \in X\} \tag{1}$$

where $\mu_{\mathrm{F}}(X)$ is a fundamental component of each FS and called membership degree and also the non-membership degree equals $1 - \mu_{\mathrm{F}}(X)$ where

$$\mu_{\mathrm{F}} : X \to [0, 1]; \quad \mu_{\mathrm{F}} \in [0, 1] \tag{2}$$

In the real world, our attitudes or preferences usually come with uncertainty (hesitation) degree and this situation could not be defined by simple FS. To deal with this situation, Atanassov 1989, 1994 and Atanassov et al. 2005 introduced a generalization of Zadeh's fuzzy set called Atanassov's IFS. Let $X = \{x_1, x_2, \ldots, x_n\}$ denote a universe of discourse; then an Atanassov's IFS A is defined as follows:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{3}$$

such that $\mu_A(x)$ and $\nu_A(x)$, respectively, are membership and non-membership degree where

$$\mu_A(x) : X \to [0, 1] \tag{4}$$

$$\nu_A(x) : X \to [0, 1] \tag{5}$$

With this condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1; \quad \forall x \in X \tag{6}$$

But besides $\mu_A(x)$ and $\nu_A(x)$ for each Atanassov's IFS Adefined on $X = \{x_1, x_2, \ldots, x_n\}$, Atanassov presented another function called hesitancy (uncertainty) degree where

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{7}$$

Evidently, each fuzzy set F can be showed as the following Atanassov's IFS:

$$F = \{\langle x, \mu_{\mathrm{F}}(x), 1 - \mu_{\mathrm{F}}(x)\rangle | x \in X\} \tag{8}$$

It is obvious that for each fuzzy set F in X, there is

$$\pi_{\mathrm{F}}(x) = 1 - \mu_{\mathrm{F}}(x) - [1 - \mu_{\mathrm{F}}(x)] = 0 \tag{9}$$

Definition 1 Let A and B be two Atanassov's IFSs that are defined on $X = \{x_1, x_2, \ldots, x_n\}$; then basic operations on Atanassov's IFSs are defined as follows (Atanassov 1986):

$$A \cup B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))) | x \in X\} \tag{10}$$

$$A \cap B = \{(x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))) | x \in X\} \tag{11}$$

$$A + B = \{(x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)) | x \in X\} \tag{12}$$

$$A.B = \{(x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)) | x \in X\} \tag{13}$$

$$\gamma A = \{(x, 1 - (1 - \mu_A(x))^{\gamma}, (\nu_A(x))^{\gamma}) | x \in X\}; \quad \gamma \ge 0 \tag{14}$$

$$A^{\gamma} = \{(x, (\nu_A(x))^{\gamma}, 1 - (1 - \nu_A(x))^{\gamma}) | x \in X\}; \quad \gamma \ge 0 \tag{15}$$

In addition, for the two Atanassov's IFS A and B, there exist the following properties:

$$A \subset B \quad \text{If} \quad \forall x \in X \quad \mu_A(x) \le \mu_B(x) \quad \text{and} \quad \nu_A(x) \ge \nu_B(x) \tag{16}$$

$$A = B \quad \text{If} \quad \forall x \in X \quad \mu_A(x) = \mu_B(x) \quad \text{and} \quad \nu_A(x) = \nu_B(x) \tag{17}$$

Definition 2 If $e = (\mu_e, \nu_e, \pi_e)$, then e is called an Atanassov's intuitionistic fuzzy number (IFN), where

$$\mu_e \in [0, 1], \quad \nu_e \in [0, 1], \quad \mu_e + \nu_e \le 1, \quad \pi_e = 1 - \mu_e - \nu_e \tag{18}$$

for each IFN $e = (\mu_e, \nu_e, \pi_e)$ if μ_e gets larger and ν_e gets smaller, then the IFN e gets greater. We can say that $e^+ =$

$(1,0,0)$ and $e^{-}=(0,1,0)$ are the largest and the smallest IFNs, respectively.

Definition 3 Let $e_{ij}=\left(\mu_{ij},v_{ij}\right)$ and $e_{km}=\left(\mu_{km},v_{km}\right)$ be two IFNs, then basic mathematic operations on them are defined as follows (Xu 2007):

$$e_{ij}+e_{km}=\left(\mu_{ij}+\mu_{km}-\mu_{ij}.\mu_{km},v_{ij}.v_{km}\right) \tag{19}$$

$$e_{ij}.\,e_{km}=\left(\mu_{ij}.\mu_{km},v_{ij}+v_{km}-v_{ij}.v_{km}\right) \tag{20}$$

$$\alpha e_{ij}=\left(1-\left(1-\mu_{ij}\right)^{\alpha},v_{ij}^{\alpha}\right);\quad \alpha\geq 0 \tag{21}$$

$$\left(e_{ij}\right)^{\alpha}=\left(\mu_{ij}^{\alpha},1-\left(1-v_{ij}\right)^{\alpha}\right);\quad \alpha\geq 0 \tag{22}$$

Definition 4 Let $e_i=(\mu_i,v_i)(i=1,2,\ldots,n)$ be a collection of IFNs and $w=(w_1,w_2,\ldots,w_n)^{\mathrm{T}}$ be the weight vector of $e_i(i=1,2,\ldots,n)$; then the IFWG operator is defined as follows (XU and Yager 2006b):

$$\mathrm{IFWG}_w(e_1,e_2,e_3,\ldots,e_n)=\prod_{i=1}^{n}e_i^{w_i}=\left(\prod_{i=1}^{n}\mu_i^{w_i},1-\prod_{i=1}^{n}(1-v_i)^{w_i}\right) \tag{23}$$

where $w_i\geq\succ 0,\sum_{i=1}^{n}w_i=1,\ (i=1,2,\ldots,n)$.

Definition 5 Let c be a mapping $c:(\phi(X))^2\rightarrow[0,1]$, the called $c(A,B)$ is the association coefficient of the two IFSs A and B which satisfies the following properties:

1. $0\leq c(A,B)\leq 1$;
2. $c(A,B)=1$ if and only if $A=B$;
3. $c(A,B)=c(B,A)$.

Many various methods and measures have been proposed to calculate the association coefficient of IFSs. Xu et al. (2008) proposed the association coefficient of A and B as follows:

$$c(A,B)=\frac{\sum_{i=1}^{n}\left(\mu_A(x_i).\mu_B(x_i)+v_A(x_i).v_B(x_i)+\pi_A(x_i).\pi_B(x_i)\right)}{\max\left(\sum_{i=1}^{n}\left(\mu_A^2(x_i)+v_A^2(x_i)+\pi_A^2(x_i)\right),\sum_{i=1}^{n}\left(\mu_B^2(x_i)+v_B^2(x_i)+\pi_B^2(x_i)\right)\right)} \tag{24}$$

And in such case where the weights of elements are taken into account, the above formula is transformed as follows:

$$c(A,B)=\frac{\sum_{i=1}^{n}\omega_i\left(\mu_A(x_i).\mu_B(x_i)+v_A(x_i).v_B(x_i)+\pi_A(x_i).\pi_B(x_i)\right)}{\max\left(\sum_{i=1}^{n}\omega_i\left(\mu_A^2(x_i)+v_A^2(x_i)+\pi_A^2(x_i)\right),\sum_{i=1}^{n}\omega_i\left(\mu_B^2(x_i)+v_B^2(x_i)+\pi_B^2(x_i)\right)\right)} \tag{25}$$

where $\omega=(\omega_1,\omega_2,\ldots,\omega_n)$ is the weight vector of $x_i(i=1,2,\ldots,n)$, and $\sum_{i=1}^{n}\omega_i=1,\ \omega_i\geq 0$.

3 Entropy on Atanassov's IFS

Atanassov's IFS is a useful idea for encountering the uncertainty during the decision-making process. To explain more about this idea, suppose $\tilde{I}=(\mu,v,\pi)$ be an IFN, where μ denotes the membership degree, v denotes the non-membership, and π is the hesitancy degree which denotes the uncertainty degree of a decision maker' attitude or the environment uncertainty.

In the fuzzy decision-making process, each decision maker gives the membership degree μ. Suppose $\mu=0.2$ then $v=1-\mu=0.8$. If the DM does not confide in his preference, we could take his uncertainty degree as a decision-making input. In a classic fuzzy situation, if the giving attitude be wrong, the process of providing DM's preferences should be repeated or at least a sensitivity analysis will be required as μ is changed.

In a real-world situation, the DM is usually not confident about his/her or her attitude; or the environment condition is such that the DM's attitude is not enough for creating confidence in the decision-making process. Therefore, the survey should be repeated as before with the change of preference.

To tackle this plight in this paper, we present two solutions based on IFS as follows:

1. Taking π from experts (DMs): here after taking μ from DM, request him to provide the uncertainty degree to giving μ. In this case, $v\neq 1-\mu$ thus, $v=1-(\mu+\pi)$ should be replaced.
2. Defining π by evaluation team: If an uncertain environment causes to change the experts' attitude, it is better to calculate the hesitation degree by evaluation team. Therefore, the non-membership degree is calculated by $v=1-(\mu+\pi)$, and it should be replaced instead of $v=1-\mu$.

Based on the description above, the proposed method extracts the non-membership degree μ from DM's idea in a real-world decision-making process. If the DM is not confident about his/her or her attitude, we should signify the hesitancy degree by π.

Owing to this fact that the proposed method can handle the hesitation of DM or the uncertain environment, the results will not be changed with change of DM's attitude in the defined uncertainty interval.

For example, assume $\mu = 0.2 \to v = 0.8$. If the DM doubts about his preference or we guess from his/her or her ability and experiences $\pi = 0.5$, we have $\tilde{I} = (0.2, 0.3, 0.5)$. Thus, the IFS number considers any change in μ and v from 0 to 50% cumulatively. Hence, our calculation is not necessary to be repeated with any 0–50% change in μ or v.

If we consider the classic fuzzy number for each change of μ from 0 to 50%, the decision-making process should be repeated. It means that the IFSs achieve the robust results.

Entropy measure of fuzzy sets is an important topic in fuzzy set theory. Entropy of fuzzy sets describes the fuzziness degree of a fuzzy set (Zeng and Li 2006). Many methods have been proposed by researchers for calculating fuzzy set and IFS entropy. Burillo and Bustince (1996) studied, in a general way, the concept of entropy and distance for Atanassov's IFS, and gave an axiom definition of fuzzy entropy to measure the degree of intuitionism of an Atanassov's IFS. Szmidt and Kacprzyk (2001) proposed a non-probabilistic geometric entropy measure for Atanassov's IFS. Hung and Yang (2006) proposed their axiom definition of entropy of IVIFS and Atanassov's IFS by exploiting the concept of probability. Ye (2010a, b) introduced two measures for calculating intuitionistic fuzzy entropy as follows:

Definition 6 Let A be an Atanassov's IFS in a universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. Then the two entropy measures of Atanassov's IFS A are defined as follows (Fig. 1):

$$E_1(A) = \left\{ \sin \frac{\pi \times [1 + \mu_A(x) - v_A(x)]}{4} + \sin \frac{\pi \times [1 - \mu_A(x) + v_A(x)]}{4} - 1 \right\} \times \frac{1}{\sqrt{2} - 1} \tag{26}$$

$$E_1(A) = \left\{ \cos \frac{\pi \times [1 + \mu_A(x) - v_A(x)]}{4} + \cos \frac{\pi \times [1 - \mu_A(x) + v_A(x)]}{4} - 1 \right\} \times \frac{1}{\sqrt{2} - 1} \tag{27}$$

For $E_1(A)$ and $E_1(A)$ there are the following properties:

(P1) $E_1(A) = E_2(A) = 0$ (minimum), if A is a crisp set;
(P2) $E_1(A) = E_2(A) = 1$ (maximum), if $\mu_A(x) = v_A(x)$ for any $x \in X$

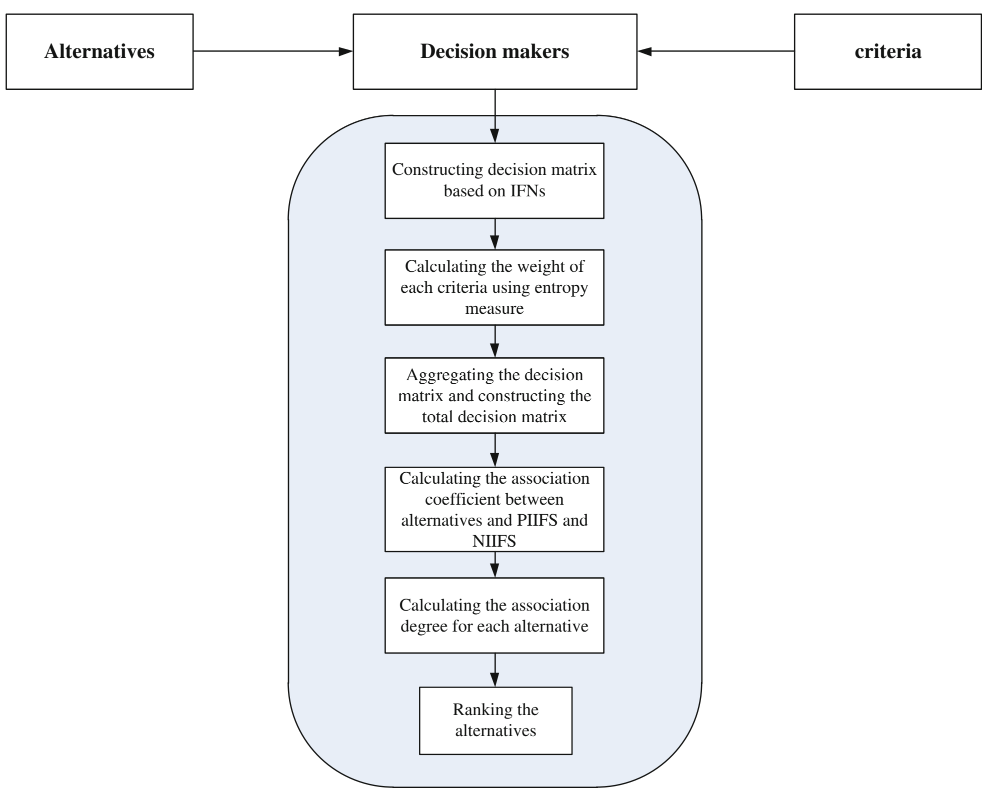

Fig. 1 Proposed GDM model based on IFNs and entropy

(P3) $E_1(A) \leq E_1(A)$ and $E_2(A) \leq E_2(A)$ if A less fuzzy than B;

(P4) $E_1(A) = E_1(A^c)$ and $E_2(A) = E_2(A^c)$.

4 A proposed method for Group decision making model based on IFNs

Group decision making process is included of a number of experts as decision makers that give their preferences to choose an alternative. Decision makers are not always certain about their given preferences and have some degree of uncertainty. Therefore, IFN is a suitable option to deal with these situations. Now $e_{jk}^{(i)} = \left(\mu_{jk}^{(i)}, v_{jk}^{(i)}, \pi_{jk}^{(i)}\right)$ denoted a given preference by ith decision maker, $i = (1,2,\ldots,n)$ based on kth criteria $k = (1,2,\ldots,p)$, for jth alternative $j = (1,2,\ldots,m)$, where $\mu_{jk}^{(i)} + v_{jk}^{(i)} \leq 1$ and $\left(\mu_{jk}^{(i)}, v_{jk}^{(i)}\right) \subseteq [0,1]$ the GDM based on IFN is presented in Table 1.

We develop a new method for GDM by using entropy weights-based distance of IFNs. Entropy measure of Atanassov's IFS is used for calculating the weight of criteria based on the decision maker's preferences. The positive ideal intuitionistic fuzzy set (PIIFS) and the negative ideal intuitionistic fuzzy set (NIIFS) are used to compare the alternatives. The proposed method consists of the following steps:

Step 1 Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of candidate alternatives, $D = \{D_1, D_2, \ldots, D_n\}$ be a set of decision makers (whose weight vector is $\varepsilon_i = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$, $\sum_{i=1}^{n} \varepsilon_i = 1$), and $C = \{C_1, C_2, \ldots, C_p\}$ be a set of criteria (with respect to the weight vector $w_k = (w_1, w_2, \ldots, w_p)$ where $\sum_{k=1}^{p} w_k = 1$). The decision makers $D_i (i = 1,2,\ldots,n)$ give their preferences $e_{jk}^{(i)} = \left(\mu_{jk}^{(i)}, v_{jk}^{(i)}, \pi_{jk}^{(i)}\right)$ based on IFNs for each alternative A_j and discreet criterion C_k. The decision matrix $X^{(i)} = \left(e_{jk}^{(i)}\right)_{m\times p}$ is constructed for ith decision maker D_i (Table 2).

Step 2 Calculate the weight of each criterion for each decision matrix. The weight $w_k^{(i)}$ of the criterion C_k $(k = 1,2,\ldots,p)$ and decision maker D_i $(i = 1,2,\ldots,n)$ is calculated according to the following entropy weights model Ye (2010a, b):

$$\delta_k^{(i)} = \frac{1 - E_k^{(i)}}{p - \sum_{k=1}^{p} E_k^{(i)}} \tag{28}$$

where $\delta_k^{(i)} \in [0,1]$, $\sum_{k=1}^{p} \delta_k^{(i)} = 1$ and based on Eq. (26) $E_k^{(i)}$ is calculated by:

$$E_{jk}^{(i)} = \frac{1}{m}\sum_{j=1}^{m}\left\{\left\{\sin\frac{\pi\times\left[1+\mu_{jk}^{i}(x)-v_{jk}^{i}(x)\right]}{4} + \sin\frac{\pi\times\left[1-\mu_{jk}^{i}(x)+v_{jk}^{i}(x)\right]}{4} - 1\right\}\times\frac{1}{\sqrt{2}-1}\right\} \tag{29}$$

or

$$E_{jk}^{(i)} = \frac{1}{m}\sum_{j=1}^{m}\left\{\left\{\cos\frac{\pi\times\left[1+\mu_{jk}^{i}(x)-v_{k}^{i}(x)\right]}{4} + \cos\frac{\pi\times\left[1-\mu_{jk}^{i}(x)+v_{jk}^{i}(x)\right]}{4} - 1\right\}\times\frac{1}{\sqrt{2}-1}\right\} \tag{30}$$

and $0 \leq E_{jk}^{(i)} \leq 1$; $(i = 1,2,\ldots,n)(j = 1,2,\ldots,m)$, $(k = 1,2,\ldots,p)$.

Step 3 Calculate the aggregated weight vector for discrete criteria to reach collective weight for each criterion $\bar{\omega}_k = (\bar{\omega}_1, \bar{\omega}_2, \ldots, \bar{\omega}_p)$, using the following operator:

$$\bar{\omega}_k = \sum_{i=1}^{n} \varepsilon_i \delta_k^{(i)}, \quad (k = 1,2,\ldots,p) \tag{31}$$

Step 4 Utilize $\text{IFWG}_{\varepsilon}$ to aggregate all given preferences for each alternative based on each criterion and then construct the total collective decision matrix $Q = (q_{jk})_{m\times p}$ as Table 3:

Table 1 Atanassov's intuitionistic fuzzy number in GDM model

	C_1	C_2	…	C_k	…	C_p
A_1	$\{e_{11}^1,\ldots,e_{11}^n\}$	$\{e_{12}^1,\ldots,e_{12}^n\}$	…	$\{e_{1k}^1,\ldots,e_{1k}^n\}$	…	$\{e_{1p}^1,\ldots,e_{1p}^n\}$
A_2	$\{e_{21}^1,\ldots,e_{21}^n\}$	$\{e_{22}^1,\ldots,e_{22}^n\}$	…	$\{e_{2k}^1,\ldots,e_{2k}^n\}$	…	$\{e_{2p}^1,\ldots,e_{2p}^n\}$
⋮	⋮	⋮	…	⋮	…	⋮
A_j	$\{e_{j1}^1,\ldots,e_{j1}^n\}$	$\{e_{j2}^1,\ldots,e_{j2}^n\}$	…	$\{e_{jk}^1,\ldots,e_{jk}^i\}$	…	$\{e_{jp}^1,\ldots,e_{jp}^n\}$
⋮	⋮	⋮	…	⋮	…	⋮
A_m	$\{e_{m1}^1,\ldots,e_{m1}^n\}$	$\{e_{m2}^1,\ldots,e_{m2}^n\}$	…	$\{e_{mk}^1,\ldots,e_{mk}^n\}$	…	$\{e_{mp}^1,\ldots,e_{mp}^n\}$

Table 2 Intuitionistic fuzzy preferences matrix (decision matrix) $X^{(i)}$ for D_i

	C_1	C_2	…	C_p
A_1	$e^i_{11} = (\mu^i_{11}, \nu^i_{11})$	$e^i_{12} = (\mu^i_{12}, \nu^i_{12})$	…	$e^i_{1p} = \left(\mu^i_{1p}, \nu^i_{1p}\right)$
A_2	$e^i_{21} = (\mu^i_{21}, \nu^i_{21})$	$e^i_{22} = (\mu^i_{22}, \nu^i_{22})$	…	$e^i_{2p} = \left(\mu^i_{2p}, \nu^i_{2p}\right)$
⋮	⋮	⋮	…	⋮
A_m	$e^i_{m1} = (\mu^i_{m1}, \nu^i_{m1})$	$e^i_{m2} = (\mu^i_{m2}, \nu^i_{m2})$	…	$e^i_{mp} = \left(\mu^i_{mp}, \nu^i_{mp}\right)$

Table 3 Total collective matrix Q

	C_1	C_2	…	C_p
A_1	$q_{11} = (\mu_{11}, \nu_{11})$	$q_{12} = (\mu_{12}, \nu_{12})$	…	$q_{1p} = (\mu_{1p}, \nu_{1p})$
A_2	$q_{21} = (\mu_{21}, \nu_{21})$	$q_{22} = (\mu_{22}, \nu_{22})$	…	$q_{2p} = (\mu_{2p}, \nu_{2p})$
⋮	⋮	⋮	…	⋮
A_m	$q_{m1} = (\mu_{m1}, \nu_{m1})$	$q_{m2} = (\mu_{m2}, \nu_{m2})$	…	$q_{mp} = (\mu_{mp}, \nu_{mp})$

$$\mathrm{IFWG}_{\varepsilon}(e_1, e_2, e_3, \ldots, e_n) = \prod_{i=1}^{n} e_i^{\varepsilon_i} = \left(\prod_{i=1}^{n} \mu_i^{\varepsilon_i}, 1 - \prod_{i=1}^{n} (1 - \nu_i)^{\varepsilon_i}\right) \tag{32}$$

where

$$\varepsilon_i \succ 0, \quad \sum_{i=1}^{n} \varepsilon_i = 1 (i = 1, 2, \ldots, n).$$

where

$$q_{jk} = \left(\mu_{jk}, \nu_{jk}\right) = \mathrm{IFWG}_{w_k}\{\{\ldots, e_1, \ldots\}, \ldots \{\ldots, e_n\}\} \quad k = 1, 2, \ldots, p, \quad j = 1, 2, \ldots, m.$$

For ease of calculation, let $F_j^k = \left(q_j^1, q_j^2, \ldots, q_j^k\right)$ be a set of collective preferences for alternative A_j.

Step 5 Define the PIIFS: $l^+ = (l_1^+, l_2^+, \ldots, l_p^+)$ where $l_k^+ = (1, 0, 0)$ and the NIIFS: $l^- = (l_1^-, l_2^-, \ldots, l_p^-)$, where $l_k^- = (0, 1, 0) (k = 1, 2, \ldots, p)$, and also the total preference set $F_j (j = 1, 2, \ldots, m)$.

Step 6 Calculate the association amount between each total preference set F_j and the PIIFS l^+ and NIIFS $l^-$by using the Eq. (25) as follows:

$$c(F_j, l^+) = \frac{\sum_{k=1}^{m} \omega_k \left(\mu_{jk}.1 + \nu_{jk}.0 + \pi_{jk}.0\right)}{\max\left(\sum_{k=1}^{m} \omega_k \left(\mu_{jk}^2 + \nu_{jk}^2 + \pi_{jk}^2\right), \sum_{i=1}^{n} \omega_k (1 + 0 + 0)\right)} = \frac{\sum_{k=1}^{m} \omega_k.\mu_{jk}}{\max\left(\sum_{k=1}^{m} \omega_k \left(\mu_{jk}^2 + \nu_{jk}^2 + \pi_{jk}^2\right), \sum_{i=1}^{n} \omega_k\right)} \tag{33}$$

$$c(F_j, l^-) = \frac{\sum_{k=1}^{m} \omega_k \left(\mu_{jk}.0 + \nu_{jk}.1 + \pi_{jk}.0\right)}{\max\left(\sum_{k=1}^{m} \omega_k \left(\mu_{jk}^2 + \nu_{jk}^2 + \pi_{jk}^2\right), \sum_{k=1}^{m} \omega_k (0 + 1 + 0)\right)} = \frac{\sum_{k=1}^{m} \omega_k.\nu_{jk}}{\max\left(\sum_{k=1}^{m} \omega_k \left(\mu_{jk}^2 + \nu_{jk}^2 + \pi_{jk}^2\right), \sum_{i=1}^{n} \omega_k\right)} \tag{34}$$

where $\omega_k (k = 1, 2, \ldots, p)$ is the calculated weight from Step 5 for each criterion.

Step 7 Calculate the association degree C_j for each alternative as follows:

$$C_j = \frac{c(F_j, l^-)}{c(F_j, l^+) + c(F_j, l^-)}; \quad j = 1, 2, \ldots, m \tag{35}$$

Since

$$c(F_j, l^+) + c(F_j, l^-) = \frac{\sum_{k=1}^{m} \omega_k.\mu_{jk}}{\max\left(\sum_{k=1}^{m} \omega_k \left(\mu_{jk}^2 + \nu_{jk}^2 + \pi_{jk}^2\right), 1\right)} + \frac{\sum_{i=1}^{n} \omega_k.\nu_{jk}}{\max\left(\sum_{k=1}^{m} \omega_k \left(\mu_{jk}^2 + \nu_{jk}^2 + \pi_{jk}^2\right), 1\right)} = \frac{\sum_{k=1}^{m} \omega_k \left(\mu_{jk} + \nu_{jk}\right)}{\max\left(\sum_{k=1}^{m} \omega_k \left(\mu_{jk}^2 + \nu_{jk}^2 + \pi_{jk}^2\right), 1\right)} \tag{36}$$

Then, Eq. (35) can be written as

$$C_j = \frac{\sum_{k=1}^{m} \omega_k.\nu_{jk}}{\sum_{k=1}^{m} \omega_k \left(\mu_{jk} + \nu_{jk}\right)}; \quad (j = 1, 2, \ldots, m) \tag{37}$$

Step 8 Rank the alternatives A_j based on the calculated amount of association degree C_j, where the greater value is the better alternative.

Step 9 End.

5 Software vendor selection

In this section, we will consider a decision-making problem to rank and choose software producing company as an alternative which is used to produce mobile phone software for a mobile phone factory. To increase customer attraction and competitive advantage, this factory has to improve its products. To this end, the production managers of this factory decide to add a software package to their mobile phones. This company has a board in the production unit including three active members who decide the choice of software producing company. Therefore, they decide to buy this software package from software producing company. A large number of companies have candidates, and after prequalification, five producers have been shortlisted. In addition, the mobile phone factory prefers its required software to be produced based on the software quality model (ISO/IEC9126-1)

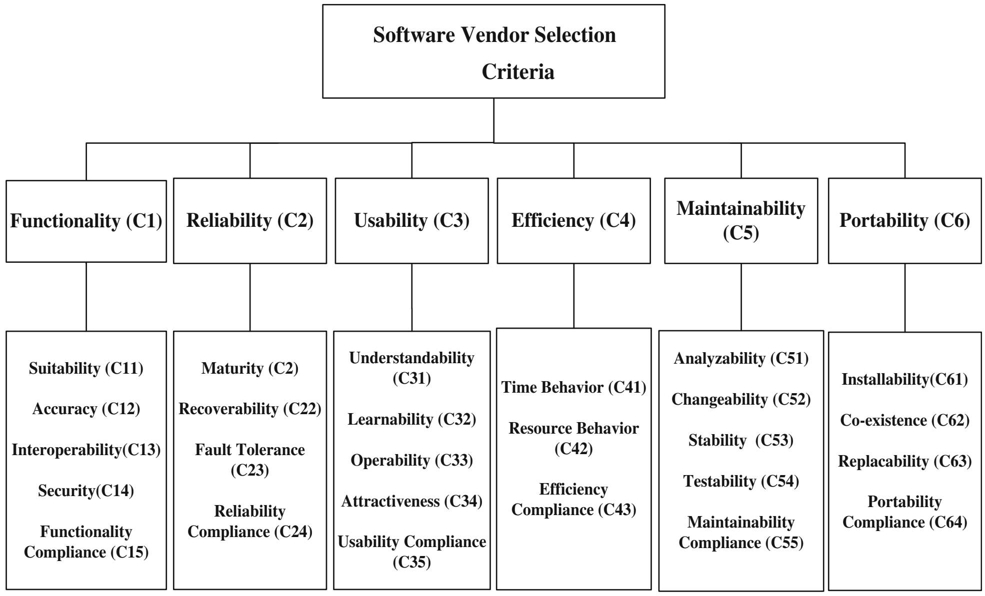

Fig. 2 Software quality model

(Fig. 2). Based on ISO/IEC9126-1:2001 standard, the six main criteria for software are defined as follows:

1. Functionality (C_1): the capability of the software product to provide functions which meet the stated or implied requirements when the software is in use under specified conditions.
2. Reliability (C_2): the capability of the software product to maintain a specified level of performance when used under specified conditions.
3. Usability (C_3): the capability of the software product to be understood, learned, and used, and to be attractive to the user under specified conditions.
4. Efficiency (C_4): the capability of the software product to provide appropriate performance, relative to the amount of resources used, under stated conditions.
5. Maintainability (C_5): the capability of the software product to be modified. Modifications may include corrections, improvements, or adaptation of the software to changes in environment, in requirements, and in functional specifications.
6. Portability (C_6): the capability of the software product to be transferred from one environment to another.

Based on the abovementioned information, there are three product managers as decision makers, $D = \{D_1, D_2, D_3\}$, and because of the difference among decision maker's experience, education, etc., each decision maker has different weights for decision-making process. Therefore, $\varepsilon = (0.3, 0.4, 0.3)$. And there are five software producer companies as alternatives $A = \{A_1, A_2, \ldots, A_5\}$. Now we use our proposed method step by step to rank and choose the alternatives as follows:

Step 1 After defining the alternatives and criteria, each decision maker gives his preferences for each alternative based on the discrete criteria under intuitionistic fuzzy information. Then we construct the three decision matrices $X^{(i)}$ (Tables 4, 5, 6) as follows:

For example $e_{11}^{1} = (0.7, 0.2)$ the element of the first row and column (Table 4), 0.7 denotes the degree that D_1 prefers alternative A_1 based on criterion C_1, and 0.2 denotes the degree that D_1 does not prefer alternative A_1 based on criterion C_1.

Step 2 Based on Tables 4, 5 and 6, we first calculate the entropy amount for each criterion using Eq. (29), and then

Table 4 Intuitionistic fuzzy preferences matrix $X^{(1)}$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.7, 0.2)	(0.4, 0.4)	(0.6, 0.3)	(0.5, 0.3)	(0.3, 0.5)	(0.8, 0.2)
A_2	(0.7, 0.1)	(0.4, 0.3)	(0.2, 0.5)	(0.5, 0.4)	(0.5, 0.3)	(0.7, 0.2)
A_3	(0.8, 0.1)	(0.5, 0.4)	(0.3, 0.1)	(0.4, 0.2)	(0.5, 0.2)	(0.4, 0.3)
A_4	(0.3, 0.2)	(0.8, 0.1)	(0.2, 0.4)	(0.4, 0.2)	(0.9, 0.1)	(0.5, 0.3)
A_5	(0.7, 0.2)	(0.7, 0.2)	(0.3, 0.6)	(0.6, 0.3)	(0.3, 0.4)	(0.5, 0.2)

Table 5 Intuitionistic fuzzy preferences matrix $X^{(2)}$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.4, 0.5)	(0.2, 0.3)	(0.6, 0.2)	(0.7, 0.2)	(0.7, 0.1)	(0.8, 0.1)
A_2	(0.6, 0.3)	(0.4, 0.2)	(0.3, 0.1)	(0.5, 0.4)	(0.6, 0.3)	(0.7, 0.1)
A_3	(0.5, 0.4)	(0.7, 0.1)	(0.4, 0.3)	(0.7, 0.2)	(0.5, 0.3)	(0.5, 0.1)
A_4	(0.5, 0.5)	(0.5, 0.4)	(0.6, 0.1)	(0.4, 0.2)	(0.4, 0.5)	(0.3, 0.4)
A_5	(0.3, 0.6)	(0.7, 0.1)	(0.6, 0.1)	(0.5, 0.3)	(0.8, 0.2)	(0.4, 0.5)

Table 6 Intuitionistic fuzzy preferences matrix $X^{(3)}$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.6, 0.1)	(0.8, 0.1)	(0.1, 0.3)	(0.6, 0.3)	(0.1, 0.8)	(0.6, 0.2)
A_2	(0.3, 0.1)	(0.2, 0.5)	(0.5, 0.4)	(0.1, 0.2)	(0.2, 0.7)	(0.4, 0.2)
A_3	(0.6, 0.1)	(0.2, 0.7)	(0.3, 0.6)	(0.3, 0.3)	(0.6, 0.2)	(0.4, 0.2)
A_4	(0.1, 0.7)	(0.2, 0.6)	(0.8, 0.2)	(0.3, 0.5)	(0.4, 0.3)	(0.2, 0.2)
A_5	(0.3, 0.2)	(0.3, 0.2)	(0.5, 0.4)	(0.6, 0.3)	(0.7, 0.1)	(0.6, 0.3)

calculate the weight of each criterion by (28) for each table as follows:

For $X^{(1)}$ we have

$$E_1^{(1)} = 0.7189, \quad E_2^{(1)} = 0.8432, \quad E_3^{(1)} = 0.9266,$$
$$E_4^{(1)} = 0.9538, \quad E_5^{(1)} = 0.8318, \quad E_6^{(1)} = 0.8442$$

$$\delta_1^{(1)} = 0.3189, \quad \delta_2^{(1)} = 0.1779, \quad \delta_3^{(1)} = 0.0833,$$
$$\delta_4^{(1)} = 0.0524, \quad \delta_5^{(1)} = 0.1908, \quad \delta_6^{(1)} = 0.1767$$

$$\delta^{(1)} = (0.3189, 0.1779, 0.0833, 0.0524, 0.1908, 0.1767)^{\mathrm{T}};$$

For $X^{(2)}$ we have

$$E_1^{(2)} = 0.9581, \quad E_2^{(2)} = 0.8386, \quad E_3^{(2)} = 0.8521,$$
$$E_4^{(2)} = 0.8771, \quad E_5^{(2)} = 0.7658, \quad E_6^{(2)} = 0.7873$$

$$\delta_1^{(2)} = 0.0455, \quad \delta_2^{(2)} = 0.1752, \quad \delta_3^{(2)} = 0.1606,$$
$$\delta_4^{(2)} = 0.1324, \quad \delta_5^{(2)} = 0.2543, \quad \delta_6^{(2)} = 0.2309$$

$$\delta^{(2)} = (0.0455, 0.1752, 0.1606, 0.1324, 0.2543, 0.2309)^{\mathrm{T}};$$

For $X^{(3)}$we have

$$E_1^{(3)} = 0.8111, \quad E_2^{(3)} = 0.7930, \quad E_3^{(3)} = 0.9685,$$
$$E_4^{(3)} = 0.9518, \quad E_5^{(3)} = 0.7375, \quad E_6^{(3)} = 0.9309$$

$$\delta_1^{(3)} = 0.2340, \quad \delta_2^{(3)} = 0.2564, \quad \delta_3^{(3)} = 0.0390,$$
$$\delta_4^{(3)} = 0.0597, \quad \delta_5^{(3)} = 0.3252, \quad \delta_6^{(3)} = 0.0856$$

$$\delta^{(3)} = (0.2340, 0.2564, 0.0390, 0.0597, 0.3252, 0.0856)^{\mathrm{T}},$$

where $\sum_{k=1}^{6} \delta_k^{(1)} = \sum_{k=1}^{6} \delta_k^{(2)} = \sum_{k=1}^{6} \delta_k^{(3)} = 1$, $k = (1, 2, \ldots, 6)$.

Step 3 Utilizing Eq. (31) we aggregate all the entropy weight vectors $\delta^{(i)} i = (1, 2, 3)$ into a collective one:

$$\bar{\omega} = 0.3\delta^{(1)} + 0.4\delta^{(2)} + 0.3\delta^{(3)}$$
$$= (0.1841, 0.2004, 0.1009, 0.0871, 0.2565, 0.1711)^{\mathrm{T}}$$

Step 4 Aggregate all the intuitionistic fuzzy decision matrices $X^{(i)}$ into a total collective decision matrix $Q = (q_{jk})_{5\times 6}$ (see Table 7):

Step 5 Define the PIIFSl^+, NIIFSl^-, and the total preference set $F_j (j = 1, 2, 3, 4, 5)$

$$l^+ = ((1, 0, 0), (1, 0, 0), (1, 0, 0), (1, 0, 0), (1, 0, 0), (1, 0, 0))^{\mathrm{T}}$$

$$l^- = ((0, 1, 0), (0, 1, 0), (0, 1, 0), (0, 1, 0), (0, 1, 0), (0, 1, 0))^{\mathrm{T}}$$

$$F_1 = ((0.53, 0.31, 0.16), (0.37, 0.28, 0.35), (0.35, 0.26, 0.39)$$
$$(0.57, 0.29, 0.14), (0.30, 0.52, 0.18), (0.73, 0.16, 0.11))^{\mathrm{T}}$$

Table 7 Total collective matrix Q

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.53, 0.31)	(0.37, 0.28)	(0.35, 0.26)	(0.57, 0.29)	(0.30, 0.52)	(0.73, 0.16)
A_2	(0.51, 0.19)	(0.32, 0.32)	(0.31, 0.33)	(0.25, 0.36)	(0.41, 0.46)	(0.59, 0.16)
A_3	(0.61, 0.23)	(0.43, 0.43)	(0.34, 0.36)	(0.41, 0.26)	(0.53, 0.24)	(0.44, 0.19)
A_4	(0.26, 0.51)	(0.44, 0.40)	(0.47, 0.23)	(0.33, 0.35)	(0.51, 0.34)	(0.31, 0.31)
A_5	(0.39, 0.39)	(0.56, 0.16)	(0.46, 0.38)	(0.53, 0.32)	(0.57, 0.24)	(0.48, 0.36)

$$F_2=((0.51,0.19,0.20),(0.32,0.32,0.36),(0.31,0.33,0.36),(0.25,0.36,0.39),(0.41,0.46,0.13),(0.59,0.16,0.25))^{\mathrm{T}}$$

$$F_3=((0.61,0.23,0.11),(0.43,0.43,0.16),(0.34,0.36,0.30),(0.41,0.26,0.33),(0.53,0.24,0.23),(0.44,0.19,0.36))^{\mathrm{T}}$$

$$F_4=((0.26,0.51,0.23),(0.44,0.40,0.16),(0.47,0.23,0.20),(0.33,0.35,0.32),(0.51,0.34,0.15),(0.31,0.31,0.38))^{\mathrm{T}}$$

$$F_5=((0.39,0.39,0.22),(0.56,0.16,0.28),(0.46,0.38,0.16),(0.53,0.32,0.15),(0.57,0.24,0.19),(0.48,0.36,0.16))^{\mathrm{T}}$$

$$\bar{\varpi}=(0.1841,0.2004,0.1009,0.0871,0.2565,0.1711)^{\mathrm{T}}$$

Step 6 Then based on aggregated weight vector $\bar{\varpi}$, we calculate the association degree of each alternative, using Eq. (35):

$$c_1=0.4163,\quad c_2=0.4267,\quad c_3=0.3710$$
$$c_4=0.4829,\quad c_5=0.3684$$

Step 7 Based on the calculated amount of association degree, rank the alternatives $A_j(j=1,2,3,4,5)$ which the greatest one is the best choice:

$$A_4\succ A_2\succ A_1\succ A_3\succ A_5$$

Therefore, A_4 is the best company to produce the required software for mobile phone factory.

Now, we make an experimental comparison among the Attanasov's intuitionistic fuzzy algorithm, fuzzy data, and crisp data to show the efficiency of our proposed algorithm. In what follows, the proposed method based on fuzzy sets is represented, and in the next section, the proposed algorithm is applied with crisp data to show the efficiency of the proposed decision-making method with Atanassov's IFS.

5.1 Proposed algorithm based on fuzzy sets

In this section, we apply the proposed decision-making algorithm to handle the case study with fuzzy data.

Step 1 In this step, experts give their preferences (μ) for each alternative regarding discrete criteria based on fuzzy set approach. The results are shown in Tables 8, 9 and 10, which are obtained from our data-gathering step illustrated in Tables 4, 5 and 6.

The element at first raw and first column ($\mu_{11}=0.7$) shows that the first expert believes about 70% for satisfying the C_1 by alternative A_1. In this procedure, the hesitancy degree is not important since we think our experts are sure about their fuzzy decisions.

Table 8 Fuzzy decision matrix $X^{(1)}$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.7	0.4	0.6	0.5	0.3	0.8
A_2	0.7	0.4	0.2	0.5	0.5	0.7
A_3	0.8	0.5	0.3	0.4	0.5	0.4
A_4	0.3	0.8	0.2	0.4	0.9	0.5
A_5	0.7	0.7	0.3	0.6	0.3	0.5

Table 9 Fuzzy decision matrix $X^{(2)}$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.4	0.2	0.6	0.7	0.7	0.8
A_2	0.6	0.4	0.3	0.5	0.6	0.7
A_3	0.5	0.7	0.4	0.7	0.5	0.5
A_4	0.5	0.5	0.6	0.4	0.4	0.3
A_5	0.3	0.7	0.6	0.5	0.8	0.4

Table 10 Fuzzy decision matrix $X^{(3)}$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.6	0.8	0.1	0.6	0.1	0.6
A_2	0.3	0.2	0.5	0.1	0.2	0.4
A_3	0.6	0.2	0.3	0.3	0.6	0.4
A_4	0.1	0.2	0.8	0.3	0.4	0.2
A_5	0.3	0.3	0.5	0.6	0.7	0.6

Table 11 Total collective decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.55	0.40	0.33	0.60	0.28	0.73
A_2	0.50	0.32	0.31	0.29	0.39	0.58
A_3	0.62	0.4	0.33	0.44	0.53	0.43
A_4	0.25	0.43	0.46	0.36	0.52	0.31
A_5	0.40	0.53	0.45	0.56	0.55	0.49

Step 2 Aggregate the fuzzy decision matrix to reach the collective decision matrix (see Table 11).

Step 3 Utilize the fuzzy entropy measures to calculate the entropy and the weights of each criterion for each decision matrix. The results are shown in Table 12, where $\sum_{k=1}^{6}\delta_k^{(1)}=\sum_{k=1}^{6}\delta_k^{(2)}=\sum_{k=1}^{6}\delta_k^{(3)}=1$, $k=(1,2,\ldots,6)$.

Step 4 Utilizing Eq. (31), we aggregate all the entropy weight vectors $\delta^{(i)}i=(1,2,3)$ into a collective one:

$$\begin{aligned}\bar{\varpi}&=0.3\delta^{(1)}+0.4\delta^{(2)}+0.3\delta^{(3)}\\&=(0.3054,0.1749,0.1453,0.4393,0.1605,0.1194)^{\mathrm{T}}\end{aligned}$$

Table 12 Amount of entropy and weight of each criterion for each decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
$X^{(1)}$						
E_1	0.8143	0.9020	0.8271	0.9769	0.8418	0.8957
δ_1	0.2502	0.1320	0.2330	0.0311	0.2132	0.1405
$X^{(2)}$						
E_2	0.5894	0.8707	0.9582	0.9143	0.9020	0.8707
δ_2	0.4589	0.1445	0.0467	0.0958	0.1095	0.1445
$X^{(3)}$						
E_3	0.8545	0.7587	0.8234	0.8545	0.8360	0.9396
δ_3	0.1559	0.2585	0.1892	0.1559	0.1757	0.0647

Table 13 The score of each alternative

Alternative	Score
A_1	0.6632
A_2	0.8318
A_3	0.7061
A_4	0.8478
A_5	0.6718

Step 5 Calculate the score of alternatives using the fuzzy association measures and the aggregated criterion weights. In Table 13, the score of each alternative is calculated.

Step 6 Rank the alternatives in which the greatest one is the best one.

$A_4 \succ A_2 \succ A_3 \succ A_5 \succ A_1$

According to provided preference, the decision makers have hesitancy degree π, means the membership degree could change between interval $[\mu, \mu + \pi]$. Therefore, to show the effect of hesitancy degree on the decision-making results, we run the algorithm with regard to this fact that the expert opinion is changed from (μ) to $(\mu + \pi)$ in Tables 8, 9 and 10.

The decision preferences and the decision matrices for each decision maker are presented in Tables 14, 15 and 16.

Based on Tables 14, 15 and 16, the entropy amount for each criterion and the weight of each criterion for each decision-making are shown in Table 17, where $\sum_{k=1}^{6} \delta_k^{(1)} = \sum_{k=1}^{6} \delta_k^{(2)} = \sum_{k=1}^{6} \delta_k^{(3)} = 1$, $k = (1, 2, \ldots, 6)$. The collective entropy weight vectors is

$$\bar{\omega} = 0.3\delta^{(1)} + 0.4\delta^{(2)} + 0.3\delta^{(3)} \\ = (0.1841, 0.1710, 0.1649, 0.1195, 0.1637, 0.1970)^{\mathrm{T}}.$$

The total collective decision matrix $Q = (q_{jk})_{5\times 6}$ is shown in Table 18. The association degree of each alternative is showed in Table 19. The final ranking is

$A_4 \succ A_1 \succ A_5 \succ A_2 \succ A_3$.

Table 14 Fuzzy decision matrix $X^{(1)}$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.8	0.6	0.7	0.7	0.5	0.8
A_2	0.9	0.7	0.5	0.6	0.7	0.8
A_3	0.9	0.6	0.9	0.8	0.8	0.7
A_4	0.8	0.9	0.6	0.8	0.9	0.7
A_5	0.8	0.8	0.4	0.7	0.6	0.8

Table 15 Fuzzy decision matrix $X^{(2)}$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.5	0.7	0.8	0.7	0.9	0.9
A_2	0.7	0.8	0.9	0.6	0.7	0.9
A_3	0.6	0.9	0.7	0.8	0.7	0.9
A_4	0.5	0.6	0.9	0.8	0.5	0.6
A_5	0.4	0.9	0.9	0.7	0.8	0.5

Table 16 Fuzzy decision matrix $X^{(3)}$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.9	0.9	0.7	0.7	0.2	0.8
A_2	0.9	0.5	0.6	0.8	0.3	0.8
A_3	0.9	0.3	0.4	0.7	0.8	0.8
A_4	0.3	0.4	0.8	0.5	0.7	0.8
A_5	0.8	0.8	0.6	0.7	0.9	0.7

Table 17 Amount of entropy and weights of each criterion

	C_1	C_2	C_3	C_4	C_5	C_6
E_1	0.5159	0.7450	0.8194	0.7759	0.7534	0.7099
δ_1	0.2881	0.1517	0.1075	0.1334	0.1467	0.1726
E_2	0.9498	0.6230	0.5010	0.7759	0.7283	0.6004
δ_2	0.0276	0.2070	0.2739	0.1230	0.1492	0.2196
E_3	0.5010	0.7534	0.8670	0.8253	0.6539	0.6689
δ_3	0.2888	0.7534	0.8670	0.8253	0.6539	0.6689

Table 18 The total collective matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.71	0.72	0.73	0.70	0.44	0.83
A_2	0.83	0.65	0.65	0.66	0.52	0.83
A_3	0.79	0.54	0.63	0.76	0.77	0.80
A_4	0.49	0.60	0.76	0.68	0.68	0.69
A_5	0.64	0.83	0.60	0.70	0.76	0.65

Table 19 The association degree of each alternatives

Alternative	Score
A_1	0.3068
A_2	0.3016
A_3	0.2841
A_4	0.3536
A_5	0.3054

Obviously, the result of our proposed method is different if we use fuzzy sets as the only input to our decision-making method. The experts have some hesitancy degree as π, in which they are not sure about the proposed μ as the performance of each alternative related to each criterion. Thus, if we use different values of μ in the range of $[\mu, \mu + \pi]$, different results for final ranking of alternatives will be achieved. This is not effective in a decision-making method as the experts who cannot choose the right μ as their opinion according to their hesitancy degree with the range of $[\mu, \mu + \pi]$ for each alternative related to each criterion, the result of full ranking will be changed by changing the expert opinion in the range of $[\mu, \mu + \pi]$. In this situation, we need to a decision-making method which supports the hesitancy degree of the experts.

5.2 Proposed algorithm based on crisp data

In this section, we apply the proposed algorithm to handle the case study with crisp (decisive) data.

Step 1 Construct the decision matrix for each decision maker based on crisp data. To provide the decisive data, we convert fuzzy data from Tables 8, 9 and 10 as follows:

If $\mu_F < 0.5 \Rightarrow \mu_D = 0$ and, if $\mu_F \geq 0.5 \Rightarrow \mu_D = 1$.

The results are represented in Tables 20, 21 and 22.

The value 1 in the first row and first column in Table 22 denotes which decision maker completely confident that alternative A_1 satisfies criteria C_1. The value ‘0’ in the first row and second column means the decision makers are completely confident that the alternative A_1 does not satisfy the criterion C_2.

Step 2 Given that the decision preferences are based on the crisp data, we cannot utilize the entropy weights; thus, we ask the decision maker to rate the criteria from 1 to 5. Then we calculate the relative importance computed by score/sum of the scores in Table 23.

Step 3 Aggregate all the decision matrices $X^{(i)}$ into a total collective decision matrix using weighted average operator (Table 24).

Step 4 Calculate the score of each alternative based on the criterion weights. See Table 25.

Table 20 Preferences matrix $X^{(1)}$ based on crisp data

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	1	0	1	1	0	1
A_2	1	0	0	1	1	1
A_3	1	1	0	0	1	0
A_4	0	1	0	0	1	1
A_5	1	1	0	1	0	1

Table 21 Preferences matrix $X^{(2)}$ based on crisp data

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0	0	1	1	1	1
A_2	1	0	0	1	0	1
A_3	1	1	0	1	1	1
A_4	1	1	1	0	0	0
A_5	0	1	1	1	1	0

Table 22 Preferences matrix $X^{(3)}$ based on crisp data

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	1	1	0	1	0	1
A_2	0	0	1	0	0	0
A_3	1	0	0	0	1	0
A_4	0	0	1	0	0	0
A_5	0	0	1	1	1	1

Table 23 The criterion weights

Criterion	Score	Relative importance
Functionality (C_1)	3	0.126
Reliability (C_2)	4	0.166
Usability (C_3)	5	0.208
Product price (C_4)	4	0.166
Maintainability (C_5)	3	0.126
Portability (C_6)	5	0.208

Table 24 Collective decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.66	0.33	0.66	1	0.33	1
A_2	0.66	0	0.33	0.66	0.33	0.66
A_3	1	0.66	0	0.33	1	0.33
A_4	0.33	0.66	0.66	0	0.33	0.33
A_5	0.33	0.66	0.66	1	0.66	0.66

Step 5 Rank the alternative based on the higher score.

$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$

Now, we rank the alternatives based on crisp data by converting the data of Tables 14, 15 and 16 as follows, to signify the effectiveness of the proposed Atanassov's IFS-based decision-making method:

$$\text{If}(\mu_F + \pi) \geq 0.5 \Rightarrow \mu_D = 0 \quad \text{and}, \quad \text{if}(\mu_F + \pi) \geq 0.5 \Rightarrow \mu_D = 1.$$

The results are represented in Tables 26, 27 and 28. The aggregated decision matrix, total collective decision matrix, using weighted average operator based on the calculated criterion weights is calculated as illustrated in Table 23 (Table 29). The score of each alternative is shown based on the criterion weights in Table 30. The ranking result of the alternatives based on the higher score is

$A_3 > A_1 > A_4 > A_2 = A_5.$

The results of proposed algorithm based on three different type data (IFS, simple fuzzy and decisive data) are presented in Fig. 3).

Table 25 The score of each alternative

Alternative	Score
A_1	0.6908
A_2	0.4402
A_3	0.3716
A_4	0.3986
A_5	0.6749

Table 26 Preferences matrix $X^{(1)}$ based on crisp data

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	1	1	1	1	1	1
A_2	1	1	1	1	1	1
A_3	1	1	1	1	1	1
A_4	1	1	1	1	1	1
A_5	1	1	1	1	1	1

Table 27 Preferences matrix $X^{(1)}$ based on crisp data

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	1	1	1	1	0	1
A_2	1	1	1	1	1	1
A_3	1	1	1	1	1	1
A_4	1	1	1	1	1	1
A_5	0	1	1	1	1	1

Table 28 Preferences matrix $X^{(1)}$ based on crisp data

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	1	1	1	1	0	1
A_2	1	1	0	1	0	1
A_3	1	0	1	1	1	1
A_4	0	0	1	1	1	1
A_5	1	1	0	1	1	1

Table 29 Total decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	1	1	1	1	0.33	1
A_2	1	1	0.66	1	0.66	1
A_3	1	0.66	1	1	1	1
A_4	0.66	0.66	1	1	1	1
A_5	0.66	1	0.66	1	1	1

Table 30 The score of each alternative

Alternative	Score
A_1	0.9156
A_2	0.8864
A_3	0.9436
A_4	0.9007
A_5	0.8864

6 Conclusion

In real world, decision makers' attitudes are blended with some amount of uncertainty (hesitation) degree due to the lack of enough knowledge and information about alternatives. This situation can be completely dealt with in the best way using the intuitionistic fuzzy concept. In this paper, we proposed a method to solve GDM problems where the weight of criteria is completely unknown and the given preferences is based on Atanassov's IFS. For dealing with unknown information of criteria, we use entropy measure to find the weight of each criterion based on each decision matrix. To reach the total criterion weight vector, we aggregate all calculated criterion entropy weights. We use the association coefficient measure to compare alternatives with the PIIFS and NIIFS and calculate the association degree for each alternative to rank and choose the best one(s). To show the effectiveness of our method, we used it to solve a GDM problem in an illustrative with six criteria, five suppliers, and three decision makers. Thus, it is more

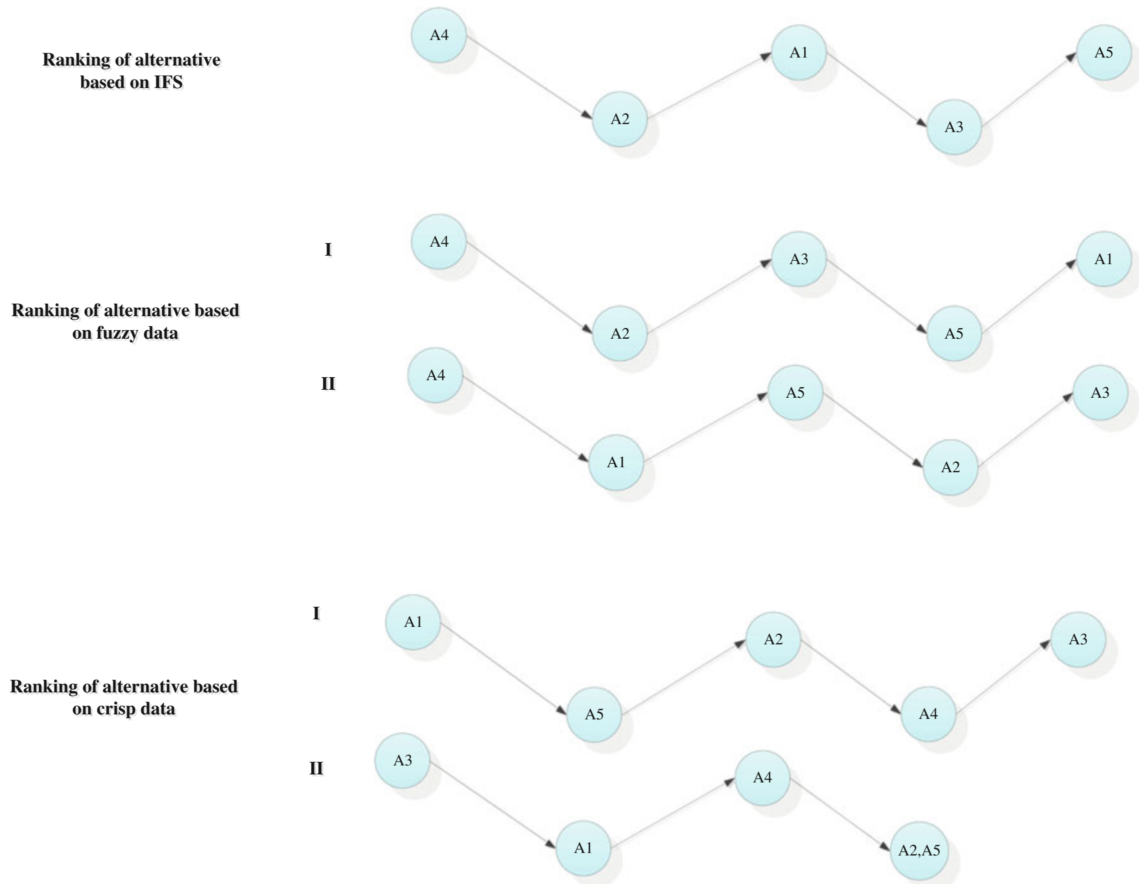

Fig. 3 The result of proposed method based on IFS, simple fuzzy, and decisive data

convenient to use it in a complicated and practical case with large amount of data.

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