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ORIGINAL PAPER

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The reliability of general vague fault-tree analysis on weapon systems fault diagnosis

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Abstract An algorithm of vague fault-tree analysis is proposed in this paper to calculate fault interval of system components from integrating expert's knowledge and experience in terms of providing the possibility of failure of bottom events. We also modify Tanaka et al's definition and extend the new usage on vague fault-tree analysis in terms of finding most important basic system component for managerial decision-making. In numerical verification, the fault of automatic gun is presented as a numerical example. For advanced experiment, a fault tree for the reactor protective system is adopted as simulation example and we compare the results with other methods. This paper also develops vague fault tree decision support systems (VFTDSS) to generate fault-tree, fault-tree nodes, then directly compute the vague fault-tree interval, traditional reliability, and vague reliability interval.

Keywords Vague fault tree analysis · Vague sets · Reliability analysis · Military application · Vague fault tree decision support systems (VFTDSS)

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1 Introduction

For fast technology innovation, new product development is getting much complicated not only on its system functions, but also on its system components. Therefore, system reliability analysis is an important issue both on the academic research and practice. Weapon systems are one of the most complicated system products in real world. Weapon systems include many different system components in order to integrate sophisticated functions under system command and control. Weapon system reliability problem is critical and important, because not only it is expensive product but also it might change history of a war or combat due to its ability at a specific time and in space.

The concept of fault-tree analysis (FTA) was developed by Bell telephone laboratories as a technique, which to perform a safety evaluation of the Minuteman launch control system in 1961. The Boeing Company modified the concept for computer utilization later. FTA is now widely used in many fields, such as in the nuclear reactor, chemical and aviation industries [11, 14, 16, 18, 26]. A fault tree is a model that graphically and logically represents the various combinations of possible events, both faulty and normal, occurring in a system that lead to the top undesired event.

The reliability of an item is the probability that the item will perform a specified function [11]. Traditionally, the reliability of a system behavior is fully characterized in the context of probability measures, and the outcome of the top event is certain and precise as long as the assignment of basic events are descent from reliable information. However, in real system, the information is inaccuracy and supposed to linguistic representation, the estimation of precise values of probability becomes very difficult in many cases. In order to handle the insufficient information, the fuzzy approach is used to evaluate the failure rate status. A great deal of literature works [2, 21, 22, 24] has been carried out in fuzzy reliable analysis.

Singer [25] has argued that the conventional fault tree does not concern the tolerances of the probability values of hazards. The causes of inaccurate relative frequencies are general non-stationary and non-ergodicity of natural phenomena especially in man-made systems. Thus, Singer proposed a method using fuzzy numbers to represent the relative frequencies of the basic events. He has shown the use of n-array possibilistic AND, OR, and NEG operators to construct possible fault tree. The concept of fuzzy probability was proposed by Walley [30, 31], and then there were some approaches about fuzzy fault-tree analysis have been introduced [7, 10, 13, 14, 16, 25–27].

In some circumstances, the experts can develop the fault-tree for one kind of product. However, they can't express the fuzziness under confirmable confidence level when a new type of product is developed. It usually occur in military weapon, because the new type of weapon system is usually based on previous product, and a lot of factors would influence weapon systems operation; but these factors usually have some uncertainty and linguistic ambiguity, such as:

- Most of weapon systems are too expensive or dangerous to measure experimentally. Instead, expert opinion is used to provide the fault information, but estimates usually are uncertain and the representations are supposed to be using linguistic.
- The normal or abnormal condition of system is incomplete defined, because weapon systems can operate their functions under some limited conditions, but can not rely on 100% system reliability.
- Weapon systems are constructed from different mechanical, electronic, and special materials. We cannot rule out any possibility on system failures including power systems, nature reasons, manual mistakes, and human factors.

Therefore, we suggest using vague set to evaluate weapon system reliability problems. The vague set can solve this kind of problem when the experts just can assign the range of failure events under un-confirmable confidence level.

The concept of vague set was proposed by Gau and Buehrer [8]. In 1995, Chen [3] presented the measures of similarity between vague sets. Recently, Chen [4] proposed fuzzy system reliability analysis based on vague set theory, where the reliabilities of the components of a system are represented by vague sets defined in the universe of discourse [0,1]. Chen's method has the advantages of modeling and analyzing fuzzy system reliability in a more flexible and more intelligent manner. However, Chen's method can just apply to some special case of general vague set. For solving this problem, a more general vague fault-tree analysis model is proposed in this paper.

It is difficult to solve vague and incomplete problems by traditional probability reliability. Therefore, this paper collects expert's knowledge and experience on the problem domain, and builds the possibility of failure of bottom event so as to consider a source of obtaining system reliability interval. To solve vague fault tree analysis, this paper modifies Tanaka et al's definition [27] on fuzzy fault-tree analysis and integrates vague set arithmetic operations to implement fault-tree analysis on weapon system fault diagnosis. In order to further illustrate the method of this paper and compare with other methods on fault-tree analysis, a fault-tree for the reactor protective system [26] is adopted as simulation example.

This paper is organized in six sections. Section 2 discusses definition of vague set and its operations. Section 3 proposes a new approach for vague fault-tree analysis, and represents an algorithm of vague fault-tree analysis. Automatic gun system is used to illustrate the algorithm of vague fault-tree analysis in Sect. 4. In Sect. 5, a simulation example is adapted and the results will be compared with other methods. The final section makes conclusions.

2 Vague set and its operations

In this section, we introduce the definitions and properties of vague set, and four arithmetic operations of triangle vague set.

2.1 Vague set

In 1965, Zadeh [32] proposed fuzzy sets to describe fuzzy phenomenon under a specific attribute. In a vague set V [8], for assigning a membership grade to every phenomenon, this membership grade is an interval of [0, 1]. This interval present a accept evidence of $x \in X$ and a decline evidence in the same time. In membership grade $\mu_V(x)$, vague set V uses a true membership function t_V and a false membership function f_v to represent lower bound (t_V) and upper bound $(1 - f_v)$. The interval $[t_V(x), 1 - f_V(x)]$can extend the fuzzy set of membership function. The membership grade $\mu_V(x)$ is not clear, but it locate in the sub-interval $[t_V(x), 1 - f_V(x)]$ (i.e.$t_V(x) \leq \mu_V(x) \leq 1 - f_V(x)$) and $0 \leq t_V(x) + f_V(x) \leq 1$. For example, if $[t_V(x_i), 1 - f_V(x_i)] = [0.7, 0.8]$, then $t_V(x_i) = 0.7$, $1 - f_V(x_i) = 0.8$, $f_V(x_i) = 0.2$. The result can explain that x_i belong to vague set V and accept evidence is 0.7; decline evidence is 0.2. If x_i is the vote result from ten people, it implies that seven people is agree, two people is reject, and one person is abandon. Figure 1 shows a vague set of real number$R$ [3]:

The uncertainty of x can described as the differential value of $(1 - f_V(x)) - t_V(x)$. If the differential value is little, it represent we are more certainly about x. If the differential

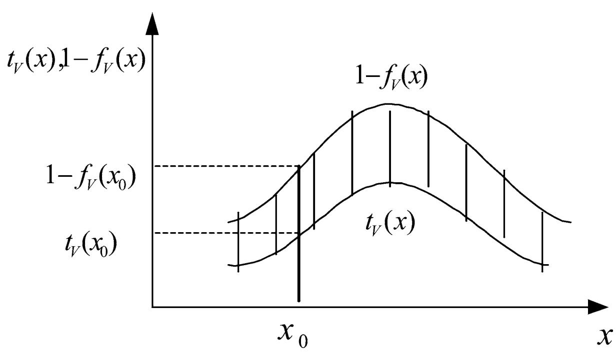

Fig. 1 Vague set explanation of real number R

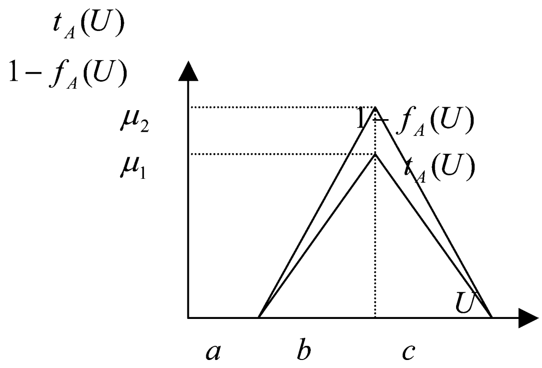

Fig. 2 A triangle vague set

value is great, it represent we are more uncertainly about x. If $1 - f_V(x) = t_V(x)$, then x is crisp and vague set is regress to fuzzy set.

2.2 Arithmetic operations of triangle vague sets

A simple triangle vague set is represented as: $\langle[(a, b, c); \mu_1], [(a, b, c); \mu_2]\rangle$, or more concise way as: $\langle[(a, b, c); \mu_1; \mu_2]\rangle$, as shown in Fig. 2.

From definition of triangle vague set, we propose four arithmetic operations for triangle vague sets in the following:

Let A and B are two vague set, as shown in Fig. 3 [13].

If two vague sets $t_A \neq t_B$, and $1 - f_A \neq 1 - f_B$, then the arithmetic operations are defined as:

$$A = \langle[(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2]\rangle \tag{1}$$

$$B = \langle[(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4]\rangle \tag{2}$$

$$\begin{aligned} A(+)B = &\langle[(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2]\rangle \\ &(+)\langle[(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4]\rangle \\ = &\langle[(a'_1 + a'_2, b_1 + b_2, c'_1 + c'_2); \min(\mu_1, \mu_3)], \\ &[(a_1 + a_2, b_1 + b_2, c_1 + c_2); \min(\mu_2, \mu_4)]\rangle \end{aligned} \tag{3}$$

$$\begin{aligned} A(-)B = &\langle[(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2]\rangle \\ &(-)\langle[(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4]\rangle \\ = &\langle[(a'_1 - c'_2, b_1 - b_2, c'_1 - a'_2); \min(\mu_1, \mu_3)], \\ &[(a_1 - c_2, b_1 - b_2, c_1 - a_2); \min(\mu_2, \mu_4)]\rangle \end{aligned} \tag{4}$$

$$\begin{aligned} A(\times)B = &\langle[(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2]\rangle \\ &(\times)\langle[(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4]\rangle \\ = &\langle[(a'_1 a'_2, b_1 b_2, c'_1 c'_2); \min(\mu_1, \mu_3)], \\ &[(a_1 a_2, b_1 b_2, c_1 c_2); \min(\mu_2, \mu_4)]\rangle \end{aligned} \tag{5}$$

$$\begin{aligned} A(/)B = &\langle[(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2]\rangle \\ &(/)\langle[(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4]\rangle \\ = &\langle[(a'_1/c'_2, b_1/b_2, c'_1/a'_2) \min(\mu_1, \mu_3)], \\ &[(a_1/c_2, b_1/b_2, c_1/a_2); \min(\mu_2, \mu_4)]\rangle\,, \end{aligned} \tag{6}$$

when $a_1 = a'_1$, $c_1 = c'_1$ and $a_2 = a'_2$, $c_2 = c'_2$, the vague sets of Fig. 3 become as Fig. 4, and its four arithmetic operations will be more easy.

3 Proposed vague fault-tree analysis

Chen [4] proposed a fuzzy system reliability analysis based on vague set theory. This method had the advantages of modeling and analyzing fuzzy system reliability in a more flexible and more intelligent manner. However, Chen's method can just apply to some special case of general vague set. For solving this problem, a more general vague fault-tree analysis model is proposed in this section.

In terms of implementing four arithmetic operations of triangle vague set in fault-tree analysis, this paper modifies Tanaka et al.'s fuzzy [27] fault-tree analysis definition and re-defines influence degrees of every bottom event as the following:

[Definition] q_{T_i} represents that q_T is not include the ith bottom event of failure interval (delete the ith bottom event). V denotes the difference between q_T and q_{Ti}. The larger of V represents the ith bottom event has greater influence on q_T, then:

$$\begin{aligned} V(q_T, q_{T_i}) \equiv\ &(a'_T - a'_{T_i}) + (a_T - a_{T_i}) + (b_T - b_{T_i}) \\ &+ (c'_T - c'_{T_i}) + (c_T - c_{T_i})\,, \end{aligned} \tag{7}$$

where $q_T = (a'_T, a_T, b_T, c_T, c'_T)$ and $q_{Ti} = (a'_{T_i}, a_{T_i}, b_{T_i}, c_{T_i}, c'_{T_i})$.

According to Sect. 2.2, this paper proposes five steps in order to implement vague fault-tree analysis in weapon system fault diagnosis. The concept and procedure of algorithm are shown in Fig. 5. These steps are also the basis of model for constructing vague fault-tree decision support system (VFTDSS).

Step 1 *To construct fault-tree diagram* To construct fault-tree diagram by fault-tree logical symbols and tracing back whole process from top to bottom events (as Fig. 6).

Step 2 *To obtain possible failure interval of bottom event* To obtain possible failure interval of bottom events according to expert's knowledge and experience.

Step 3 *To calculate possible failure interval of systems using vague set arithmetic operations* From fault-tree diagram and possible failure interval of bottom events, this step can calculate possible failure interval of systems using vague set arithmetic operations to obtain the failure interval of top event.

Step 4 *To calculate the reliability interval of top event* The reliability interval of top event is equal to one minus the failure interval of top event.

Step 5 *To find the most influential bottom event of system reliability* By the definition in this section, we delete the ith bottom event in the fault-tree diagram, and calculate $V(q_T, q_{T_i})$, $\forall i$ to find the most influential power (i.e. $\max_i V(q_T, q_{T_i})$) for the whole system.

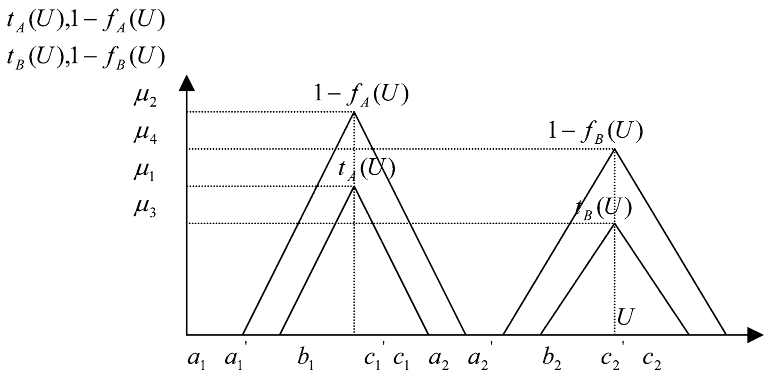

Fig. 3 Triangle vague set A and B

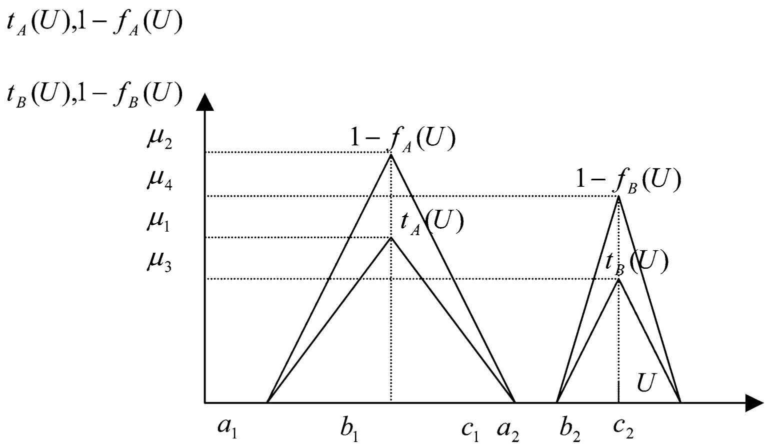

Fig. 4 Triangle vague set A and B (equal "end point" case)

4 Numerical verification

In this section, we present a case study of automatic gun in order to implement arithmetic operation. First of all, to construct fault tree including the top event ("automatic gun cannot fire"), the second event of automatic gun cannot fire ("firing assembly failure", "manual mistakes", and "feeder block"), and the bottom event ("manual mistakes", "body failure", "extractor failure", "spring failure", "feed frame failure", "out of machine oil", "pin firing too short", "bad weather", "machine oil filter dirty", "air filter dirty", "drive distortion", "copper dirt jam", and "oil dirt jam"). Fault tree integrates the top event, the second event, and the bottom event with "OR" and "AND" gate (Fig. 6). After fault tree is constructed, we generated the possibility failure interval of bottom failure in Table 1 from expert's knowledge and experience.

For connecting the fault tree diagram of "automatic gun cannot fire", this research uses logical node to describe "AND" gate with the sign of ∩, and "OR" gate with the sign of ∪. It can represent their relationship of parallel and series as (see Fig. 7):

$$\begin{aligned} T &= R \cup A \cup S \\ &= (B \cup W \cup X) \cup A \cup (C \cup D \cup E) \\ &= (B \cup (F \cup Y) \cup (G \cup Z)) \cup A \cup (C \cup D \cup E) \\ &= (B \cup (F \cup (H \cap I \cap J)) \\ &\quad \cup (G \cup (K \cap L \cap M))) \cup A \cup (C \cup D \cup E) \end{aligned} \tag{8}$$

where ∩ means relation of parallel and ∪ means series.

Let q_i represent the failure possibility of bottom event i, then the possibility of failure R can describe as:

$$q_R = 1 - (1 - q_B)(1 - q_W)(1 - q_X) \tag{9}$$

The failure possibility of S is:

$$q_S = 1 - (1 - q_C)(1 - q_D)(1 - q_E) \tag{10}$$

The failure possibility of W is:

$$q_W = 1 - (1 - q_F)(1 - q_Y) \tag{11}$$

The failure possibility of X is:

$$q_X = 1 - (1 - q_G)(1 - q_Z) \tag{12}$$

The failure possibility of Y is:

$$q_Y = q_H q_I q_J \tag{13}$$

The failure possibility of Z is:

$$q_Z = q_K q_L q_M \tag{14}$$

Then, the top event possibility of "automatic gun cannot fire" can be described as:

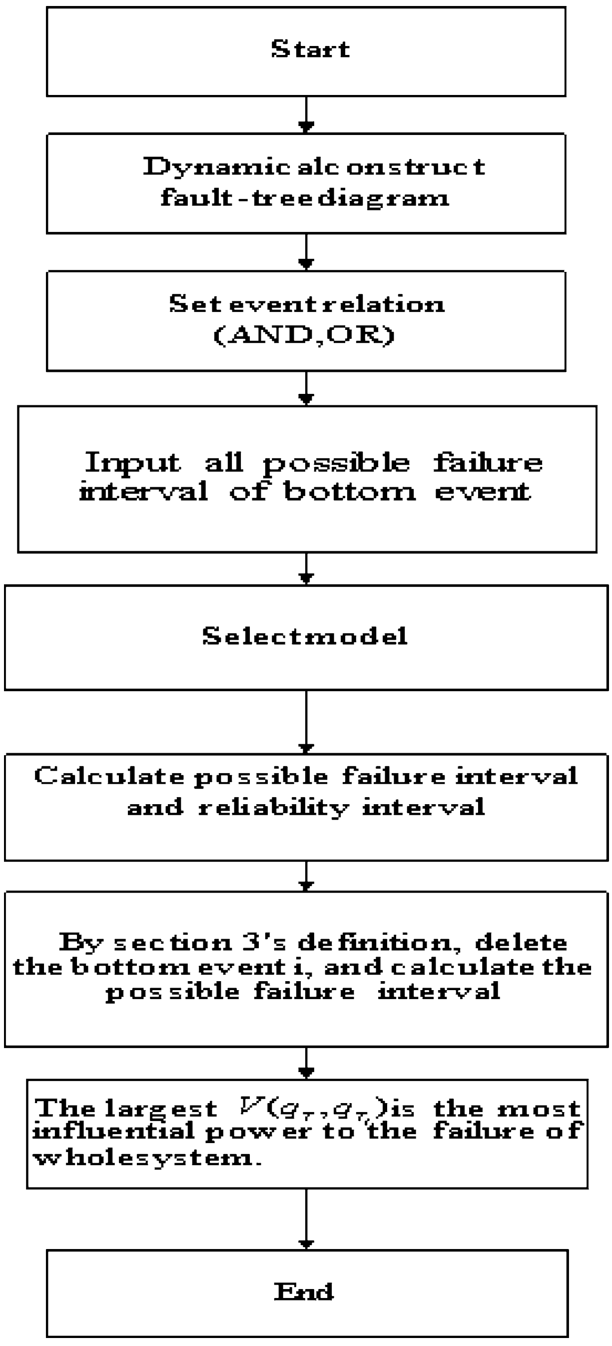

Fig. 5 The concept and procedure in the algorithm

$$
\begin{aligned}
q_T &= \{1-(1-q_R)(1-q_A)(1-q_S)\}\} \\
&= \{1-(1-q_B)(1-q_W)(1-q_X) \\
&\quad \times(1-q_A)(1-q_C)(1-q_D)(1-q_E)\} \\
&= \{1-(1-q_B)(1-q_F)(1-q_Y)(1-q_G) \\
&\quad \times(1-q_Z)(1-q_A)(1-q_C)(1-q_D)(1-q_E)\} \\
&= \{1-(1-q_B)(1-q_F)(1-q_Hq_Iq_J)(1-q_G) \\
&\quad \times(1-q_Kq_Lq_M)(1-q_A)(1-q_C)(1-q_D)(1-q_E)\}
\end{aligned}
\tag{15}
$$

4.1 Traditional reliability

Traditionally, probability method is the method for dealing with the heterogeneous problems, and probability can only show the randomness of success or failure events. This method is constrained to its usage on the condition of great amount of data sample and all of event outcomes are under certainty. However, a lot of uncertainty factors cause fuzziness in the procedure of weapon system evaluation, for example: statistics uncertainty, model uncertainty, and data uncertainty. These uncertainty factors will limit the understanding of system component failure due to the reason of incomplete data. Also, the traditional reliability method is lack of ability to make statistical estimate. Therefore, traditional reliability method is hard to calculate failure possibility of system and its component in a precise way because of the incomplete data. We calculate failure possibility of automatic gun based on data of Table 1 as the following:

$$
\begin{aligned}
q_T &= \{1-(1-q_B)(1-q_F)(1-q_Hq_Iq_J) \\
&\quad \times(1-q_G)(1-q_Kq_Lq_M)(1-q_A) \\
&\quad \times(1-q_C)(1-q_D)(1-q_E)\} \\
&= \{1-(1-0.005)(1-0.007) \\
&\quad \times(1-0.006\times0.004\times0.003) \\
&\quad \times(1-0.008)(1-0.005\times0.005\times0.009) \\
&\quad \times(1-0.0001)(1-0.006)(1-0.007)(1-0.005)\} \\
&= \{1-(0.995)(0.993)(1)(0.992)(1)(0.9999) \\
&\quad \times(0.994)(0.993)(0.995)\} \\
&= 1-0.96250 \\
&= 0.03750
\end{aligned}
$$

After the above calculation, we find that the failure probability of “automatic gun cannot fire” is 0.03750 and the reliability of “automatic gun can fire” is 0.96250.

4.2 Proposed method

According to arithmetic operations of triangle vague set (3) to (8), the failure range of “automatic gun cannot fire” can be described as:

$$
\begin{aligned}
q_T &= \{1-(1-q_B)(1-q_F)(1-q_Hq_Iq_J)(1-q_G) \\
&\quad \times(1-q_Kq_Lq_M)(1-q_A)(1-q_C)(1-q_D)(1-q_E)\} \\
&= \{1(-)\quad \langle[(0.992, 0.995, 0.998); 0.8], \\
&\quad [(0.99, 0.995, 1.0); 0.9]\rangle \\
&\quad (\times)\langle[(0.992, 0.993, 0.994); 0.8], \\
&\quad [(0.991, 0.993, 0.997); 0.9]\rangle \\
&\quad (\times)\langle[(1, 1, 1); 0.6], [(1, 1, 1); 0.7]\rangle \\
&\quad (\times)\langle[(0.991, 0.992, 0.993); 0.7], \\
&\quad [(0.99, 0.992, 0.994); 0.8]\rangle \\
&\quad (\times)\langle[(1, 1, 1); 0.6], [(1, 1, 1); 0.8]\rangle \\
&\quad (\times)\langle[(0.99974, 0.9999, 0.99995); 0.8], \\
&\quad [(0.9997, 0.9999, 0.99999); 0.9]\rangle
\end{aligned}
$$

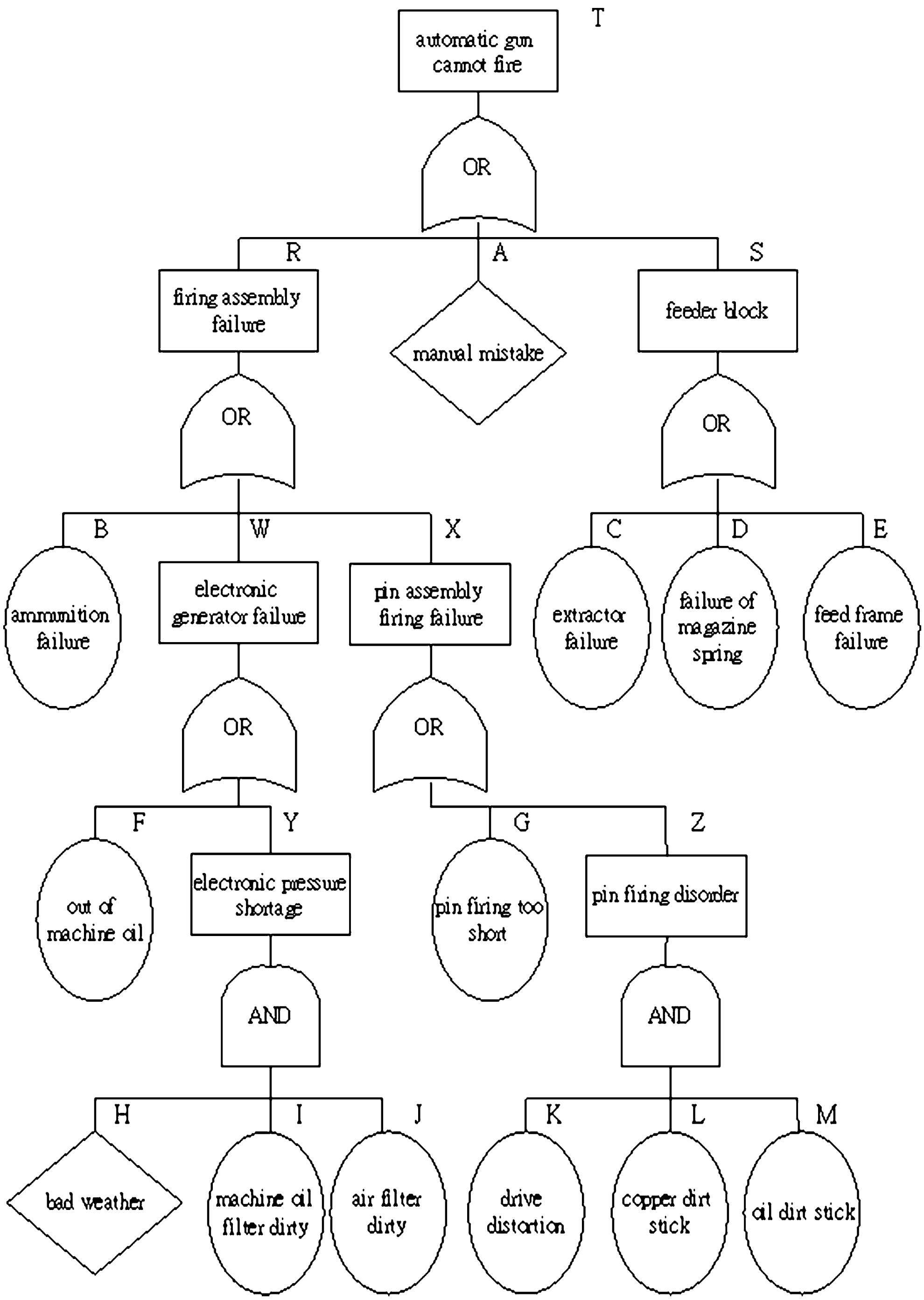

Fig. 6 Fault-tree diagram of automatic gun failure

$$
\begin{aligned}
&(\times)\langle[(0.992, 0.994, 0.995); 0.8],\\
&[(0.99, 0.994, 0.997); 0.8]\rangle\\
&(\times)\langle[(0.991, 0.993, 0.993); 1.0],\\
&[(0.991, 0.993, 0.993); 1.0]\rangle\\
&(\times)\langle[(0.994, 0.995, 0.996); 0.8],\\
&[(0.993, 0.995, 0.997); 0.9]\rangle\}\\
=&\{1(-)\quad\langle[(0.95270, 0.96250, 0.96934); 0.6],\\
&[(0.94596, 0.96250, 0.97818); 0.7]\rangle\}\\
=&\langle[(0.03066, 0.03750, 0.04730); 0.6],\\
&[(0.02182, 0.03750, 0.05404); 0.7]\rangle
\end{aligned}
$$

After the above calculation, we find that the failure interval of "automatic gun cannot fire" can be described as the following and in Fig. 8:

$$
\begin{aligned}
&\langle[(0.03066, 0.03750, 0.04730); 0.6],\\
&\quad[(0.02182, 0.03750, 0.05404); 0.7]\rangle
\end{aligned}
$$

Table 1 The possible range of bottom event failure

Failure possibility	a_i	a_i'	b_i	c_i'	c_i	$\mu_{1-f_A(U)}$	$\mu_{t_A(U)}$
q_A	0.00001	0.00005	0.0001	0.00026	0.0003	0.9	0.8
q_B	0.0	0.002	0.005	0.008	0.01	0.9	0.8
q_C	0.003	0.005	0.006	0.008	0.01	0.8	0.8
q_D	0.007	0.007	0.007	0.009	0.009	1.0	1.0
q_E	0.003	0.004	0.005	0.006	0.007	0.9	0.8
q_F	0.003	0.006	0.007	0.008	0.009	0.9	0.8
q_G	0.006	0.007	0.008	0.009	0.01	0.8	0.7
q_H	0.003	0.005	0.006	0.007	0.009	0.7	0.6
q_I	0.002	0.003	0.004	0.007	0.008	0.9	0.8
q_J	0.001	0.002	0.003	0.004	0.005	0.9	0.8
q_K	0.004	0.004	0.005	0.006	0.006	1.0	1.0
q_L	0.003	0.004	0.005	0.006	0.009	0.8	0.6
q_M	0.008	0.0085	0.009	0.0095	0.01	0.8	0.7

Then, the reliability interval of "automatic gun can fire" is as the following and can be described as Fig. 9.

$$\langle[(0.95270, 0.96250, 0.96934); 0.6],\\ [(0.94596, 0.96250, 0.97818); 0.7]\rangle\}$$

According to equation (7), we calculate q_{T_α} as the followings:

$$q_{T_A} = \langle[(0.03061, 0.03741, 0.04705); 0.6],\\ [(0.02181, 0.03741, 0.05376); 0.7]\rangle$$
$$q_{T_B} = \langle[(0.02872, 0.03267, 0.03962); 0.6],\\ [(0.02182, 0.03267, 0.04449); 0.7]\rangle$$
$$q_{T_C} = \langle[(0.02579, 0.03169, 0.03962)0.6],\\ \times[(0.01888, 0.03169, 0.04449); 0.7]\rangle$$
$$q_{T_D} = \langle[(0.02383, 0.03072, 0.03865); 0.6],\\ [(0.01493, 0.03072, 0.04545); 0.7]\rangle$$
$$q_{T_E} = \langle[(0.02677, 0.03267, 0.04155); 0.6],\\ [(0.01888, 0.03267, 0.04737); 0.7]\rangle$$
$$q_{T_F} = \langle[(0.02481, 0.03072, 0.03962); 0.6],\\ [(0.01888, 0.03072, 0.04545); 0.7]\rangle$$
$$q_{T_G} = \langle[(0.02383, 0.02974, 0.03865); 0.6],\\ [(0.01592, 0.02974, 0.04449); 0.7]\rangle$$
$$q_{T_H} = \langle[(0.03066, 0.03750, 0.04730); 0.6],\\ [(0.02182, 0.03750, 0.05404); 0.8]\rangle$$
$$q_{T_I} = \langle[(0.03066, 0.03750, 0.04730); 0.6],\\ [(0.02182, 0.03750, 0.05404); 0.8]\rangle$$
$$q_{T_J} = \langle[(0.03066, 0.03750, 0.04730); 0.6],\\ [(0.02182, 0.03750, 0.05404); 0.8]\rangle$$
$$q_{T_K} = \langle[(0.03066, 0.03750, 0.04730); 0.6],\\ [(0.02182, 0.03750, 0.05404); 0.7]\rangle$$
$$q_{T_L} = \langle[(0.03066, 0.03750, 0.04730); 0.6],\\ [(0.02182, 0.03750, 0.05404); 0.7]\rangle$$
$$q_{T_M} = \langle[(0.03066, 0.03750, 0.04730); 0.6],\\ [(0.02182, 0.03750, 0.05404); 0.7]\rangle$$

According to equation (7), we calculate $V(q_T, q_{T_i})$ as the following:

$$V(q_T, q_{T_A}) = 0.00068, \quad V(q_T, q_{T_B}) = 0.024,$$
$$V(q_T, q_{T_C}) = 0.03085, \quad V(q_T, q_{T_D}) = 0.03774$$
$$V(q_T, q_{T_E}) = 0.02408, \quad V(q_T, q_{T_F}) = 0.03184$$
$$V(q_T, q_{T_G}) = 0.03869, \quad V(q_T, q_{T_H}) = 0,$$
$$V(q_T, q_{T_I}) = 0, \quad V(q_T, q_{T_J}) = 0,$$
$$V(q_T, q_{T_K}) = 0, \quad V(q_T, q_{T_L}) = 0$$
$$V(q_T, q_{T_M}) = 0.$$

In conclusion, "Pin firing too short" (G) is the main reason for "automatic gun can't fire". This is also the most significant factor of influence on gun firing reliability. Therefore, at the managerial level, if we want to get higher reliability of gun firing, "pin firing too short" problem should take more concern. In other words, to improve "pin firing too short" problem is more important than other bottom events.

5 Simulations and comparison

5.1 The comparison of fuzzy fault-tree and vague fault-tree

For comparing the difference between fuzzy fault-tree and vague fault-tree, we summarize the results as Table 2. Besides, Chen's method can't be applied when $a_i \neq a_i'$ or $c_i' \neq c_i$ (as Fig. 9). The results of fuzzy fault tree are based on Cheng and Mon's method [5]. From Table 2 and Fig. 9, we can find two properties of vague fault-tree:

(1) The results of vague fault-tree are more flexible than the fuzzy fault-tree, because the left/right end points become interval values when using vague fault-tree.
(2) Fuzzy fault-tree method can't describe the uncertainty of confidence level. For example, the results of fuzzy fault-tree are the same when α is under the interval [0.6, 0.7].

5.2 Simulation example

In order to further illustrate the method of this paper and compare with other methods on fault-tree analysis, a fault tree for

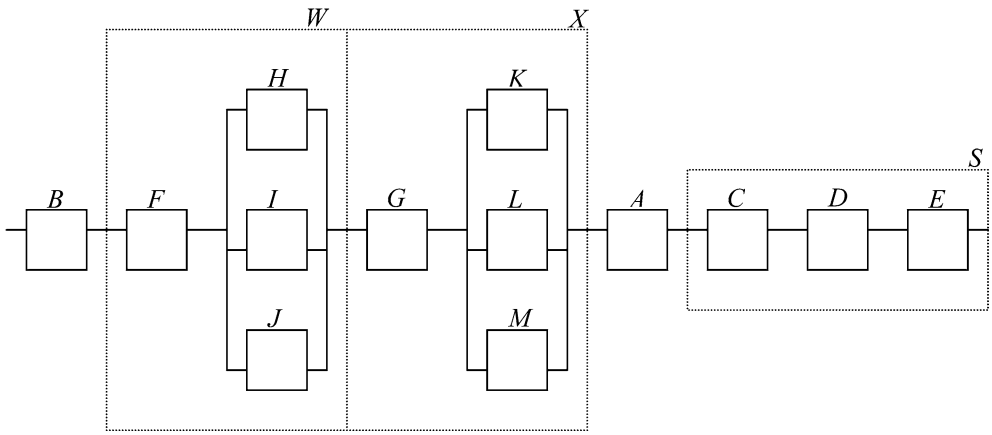

Fig. 7 Parallel and series relationship of fault tree diagram of automatic gun cannot fire

the reactor protective system [26] is adopted as simulation example as shown in Fig. 10. Let a triangle vague set A is represented as $\langle[(a', b, c'); \mu_1], [(a, b, c); \mu_2]\rangle$. For demonstration, the lower bound and upper bound of each bottom event are obtained from the point median value and the error factor (EF) of the failure probability [26]. The input values of vague set A can be obtained by equations (16), (17), (18), (19), (20).

$$a = \frac{q_i}{EF1} \tag{16}$$

$$a' = \frac{q_i}{EF2} \tag{17}$$

$$b = \quad q_i \tag{18}$$

$$c' = q_i * EF2 \tag{19}$$

$$c = q_i * EF1 \tag{20}$$

where q_i is the median value of failure probability. $EF1$ and $EF2$ are error factors of the bottom event.

The input data are given in Table 3 with the corresponding error factors. The average failure vague set for the top event of the Monte-Carlo simulation after 1200 trails is as follows (according to arithmetic operations of triangle vague set (3) to (7)):

$$q_{\text{Top}} = \langle[(2.133 \times 10^{-6}, 2.252 \times 10^{-5}, 2.720 \times 10^{-4}); 0.8], [(9.582 \times 10^{-7}, 2.252 \times 10^{-5}, 7.654 \times 10^{-4}); 0.8]\rangle$$

5.3 Comparison with other methods

In order to evaluate the proposed method, a simulation experiment is performed in Sect. 5.2. We also compare the results with Suresh et al. [26] and Huang et al.'s [10] methods. The input data of these methods is shown in Table 3.

In Suresh et al. [26], a fuzzy approach to uncertainty analysis was proposed to provide insight on the design of data and information gathering strategies that focus on the reduction of the total uncertainty. The input data of Suresh et al.' [26] method is the lower bound, upper bound and median value of bottom events, so we use the second, sixth and seventh columns values in Table 3 to calculate the failure probability of top event.

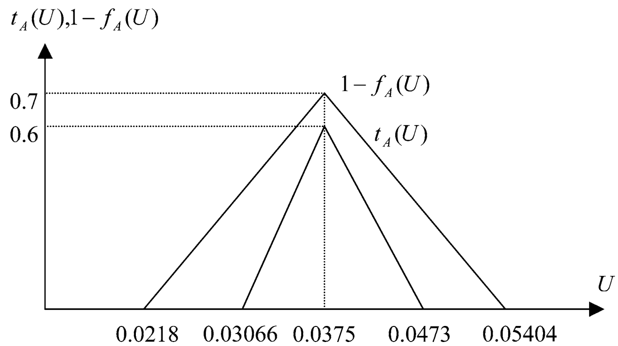

Fig. 8 Vague number for automatic gun cannot fire

Posbist fault-tree analysis (posbist FTA) was proposed by Huang et al. [10] to evaluate the failure possibility of some systems, in which the statistical data is scarce or the failure probability is extremely small. Huang et al. [10] define the AND operator and the OR operator based on the minimal cut of a posbist fault-tree.

The results of three methods are shown in Table 4 and Fig. 11. The membership function for the top event is evaluated using the α-cut method and the vague/fuzzy failure set of top event is given. From Table 4 and Fig. 11, we have some findings:

1. Suresh et al. [26] and Huang et al.'s [10] methods do not consider the confidence level of domain experts. However, the proposed method can be more flexible to present the confidence level of experts (highest confidence = 0.8).
2. The results of proposed and Suresh et al. [26] methods under different α-level ($\alpha \leq 0.8$) are approximately consistent and the trend of vague/fuzzy failure set of top event is same.
3. In Huang et al.'s [10] method, the failure possibilities of top event are all equal to 1.00E-03. Because this method is fit the statistical data is scarce or the failure probability is extremely small (recommended value is under 10^{-7} [10]).

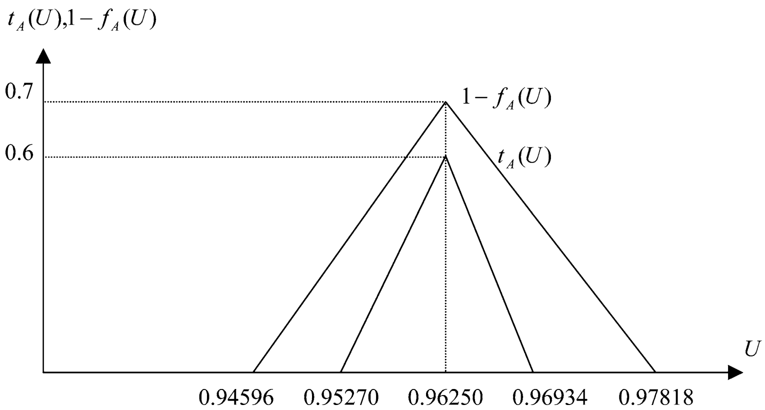

Fig. 9 The reliability interval of automatic gun can fire

Table 2 Comparison results

Fault interval by fuzzy α-cuts				Fault interval by vague α-cuts				
α	Left end point	Middle point	Right end point	a_i	a_i'	b_i	c_i'	c_i
0.7	0.037504	0.037504	0.037504	0.037504	0.037504	0.037504	0.037504	0.037504
0.6	0.037504	0.037504	0.037504	0.035264	0.037504	0.037504	0.037504	0.039866
0.5	0.036363	0.037504	0.039136	0.033024	0.036363	0.037504	0.039136	0.042229
0.4	0.035222	0.037504	0.040769	0.030784	0.035222	0.037504	0.040769	0.044592
0.3	0.034082	0.037504	0.042402	0.028544	0.034082	0.037504	0.042402	0.046955
0.2	0.032941	0.037504	0.044035	0.026304	0.032941	0.037504	0.044035	0.049317
0.1	0.031801	0.037504	0.045668	0.024064	0.031801	0.037504	0.045668	0.05168
0	0.03066	0.037504	0.047301	0.021825	0.03066	0.037504	0.047301	0.054043

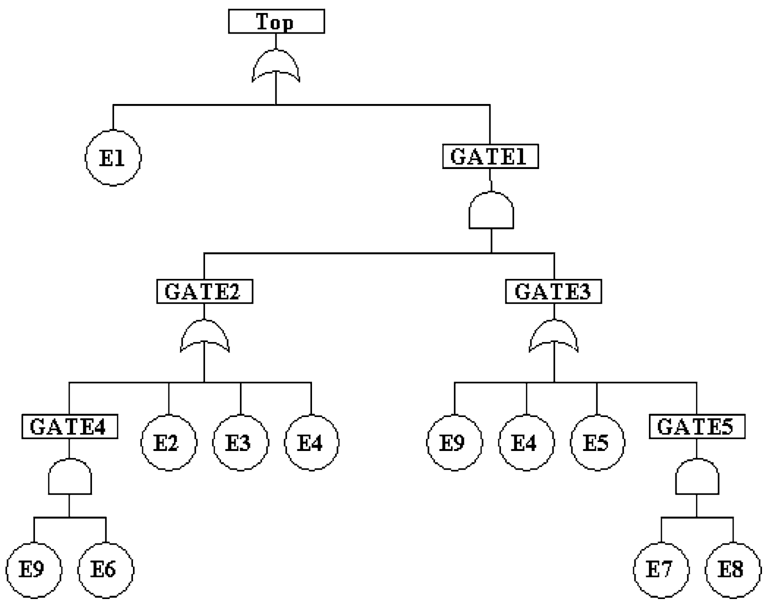

Fig. 10 Reduced fault tree for the reactor protective system

5.4 Vague fault-tree decision support system

Scott [24] first articulated the concept of decision support system (DSS) in the early 1970s. Some literatures about integrating subjective data analysis with decision support system have been published, for example, fuzzy decision support system [12,15,20,23,28,29], and vague information decision support system [1,6,19]. A few researches integrate different arithmetic operations as model for improving ability of problem solving or decision-making. On the other hand, when every calculation of system has been completed, the results can be considered as a case for analyzing and storing on the system [9]. This process infers that past case can offer a knowledge aid to solve new problems and can belong to the function of knowledge management [17].

Table 3 Input data for different fault-tree nodes

Event No.	Failure probability (median: b)	Error factor 1 ($EF1$)	Error factor 2 ($EF2$)	a	a'	c'	c	μ_1	μ_2
1	1.70E-05	20	10	8.50E-07	1.70E-06	1.70E-04	3.40E-04	0.9	0.9
2	1.00E-03	6	3	1.67E-04	3.33E-04	3.00E-03	6.00E-03	1	0.9
3	3.60E-04	6	3	6.00E-05	1.20E-04	1.08E-03	2.16E-03	1	0.9
4	1.00E-03	6	3	1.67E-04	3.33E-04	3.00E-03	6.00E-03	0.9	0.8
5	3.60E-04	6	3	6.00E-05	1.20E-04	1.08E-03	2.16E-03	0.9	0.8
6	6.10E-03	8	4	7.63E-04	1.53E-03	2.44E-02	4.88E-02	1	1
7	6.10E-03	8	4	7.63E-04	1.53E-03	2.44E-02	4.88E-02	1	0.9
8	9.70E-04	20	10	4.85E-05	9.70E-05	9.70E-03	1.94E-02	1	0.9
9	9.70E-04	20	10	4.85E-05	9.70E-05	9.70E-03	1.94E-02	0.8	0.8

Table 4 The results of comparison

$\alpha-$level	Suresh et al. [26]		$P_{oss}(T)$ [10]	Proposed method				
	Lower bound	Upper Bound		a	a'	b	c'	c
1.0	3.43E-05	3.43E-05	1.00E-03					
0.9	3.01E-05	7.36E-05	1.00E-03					
0.8	2.61E-05	1.22E-04	1.00E-03	2.25E-05	2.25E-05	2.25E-05	2.25E-05	2.25E-05
0.7	2.23E-05	1.79E-04	1.00E-03	1.98E-05	2.00E-05	2.25E-05	5.37E-05	1.15E-04
0.6	1.88E-05	2.46E-04	1.00E-03	1.71E-05	1.74E-05	2.25E-05	8.49E-05	2.08E-04
0.5	1.55E-05	3.21E-04	1.00E-03	1.44E-05	1.49E-05	2.25E-05	1.16E-04	3.01E-04
0.4	1.24E-05	4.06E-04	1.00E-03	1.17E-05	1.23E-05	2.25E-05	1.47E-04	3.94E-04
0.3	9.55E-06	4.99E-04	1.00E-03	9.04E-06	9.77E-06	2.25E-05	1.78E-04	4.87E-04
0.2	6.91E-06	6.01E-04	1.00E-03	6.34E-06	7.22E-06	2.25E-05	2.10E-04	5.79E-04
0.1	4.49E-06	7.13E-04	1.00E-03	3.65E-06	4.68E-06	2.25E-05	2.41E-04	6.72E-04
0.0	2.30E-06	8.33E-04	1.00E-03	9.58E-07	2.13E-06	2.25E-05	2.72E-04	7.65E-04

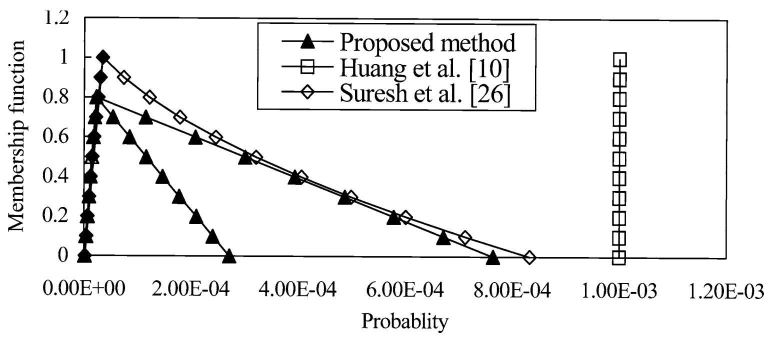

Fig. 11 Membership function for top event

In terms of providing decision support functions, this paper uses the tool of Borland C++ Builder [33] to develop a VFTDSS. The VFTDSS provides the functions that generate fault-tree, fault-tree nodes, vague fault-tree interval, traditional reliability, and vague reliability interval (see Fig. 12). In Fig. 12, the grey blocks denote "AND" gate, such as bottom event H, I, J, K, L, and M; the white blocks denote "OR" gate; clicking "Help" button in upper-left corner can show operation procedures of VFTDSS. About the results of running VFTDSS, the outputs are same as Sect. 4.1–4.2's solutions using the conditions and data of Sect. 4.1–4.2:

(1) Traditional failure rate is 0.0375 and traditional reliability is 0.9625,
(2) Vague fault rate interval, vague reliability interval (as Fig. 10),
(3) "Pin firing too short(G)" is the main reason for automatic gun can't fire.

6 Conclusion

A new vague fault-tree analysis model is proposed in this paper and it can solve the problem that experts can't express the fuzziness under confirmable confidence level when a new type of product is developed. It usually occurs in military weapon, because the new type of weapon system is usually based on previous product. This paper also modifies Tanaka et al's [27] definition on fault-tree analysis and integrates vague set arithmetic for implementing fault-tree analysis on weapon system fault diagnosis.

In order to further illustrate the method of this paper and compare with other methods on fault-tree analysis, a fault tree for the reactor protective system [26] is adopted as simulation example. We also compare the simulation results with Suresh et al. [26] and Huang et al.'s methods [10], and the proposed method can be more flexible to present the confidence level of experts. A vague fault-tree decision support system

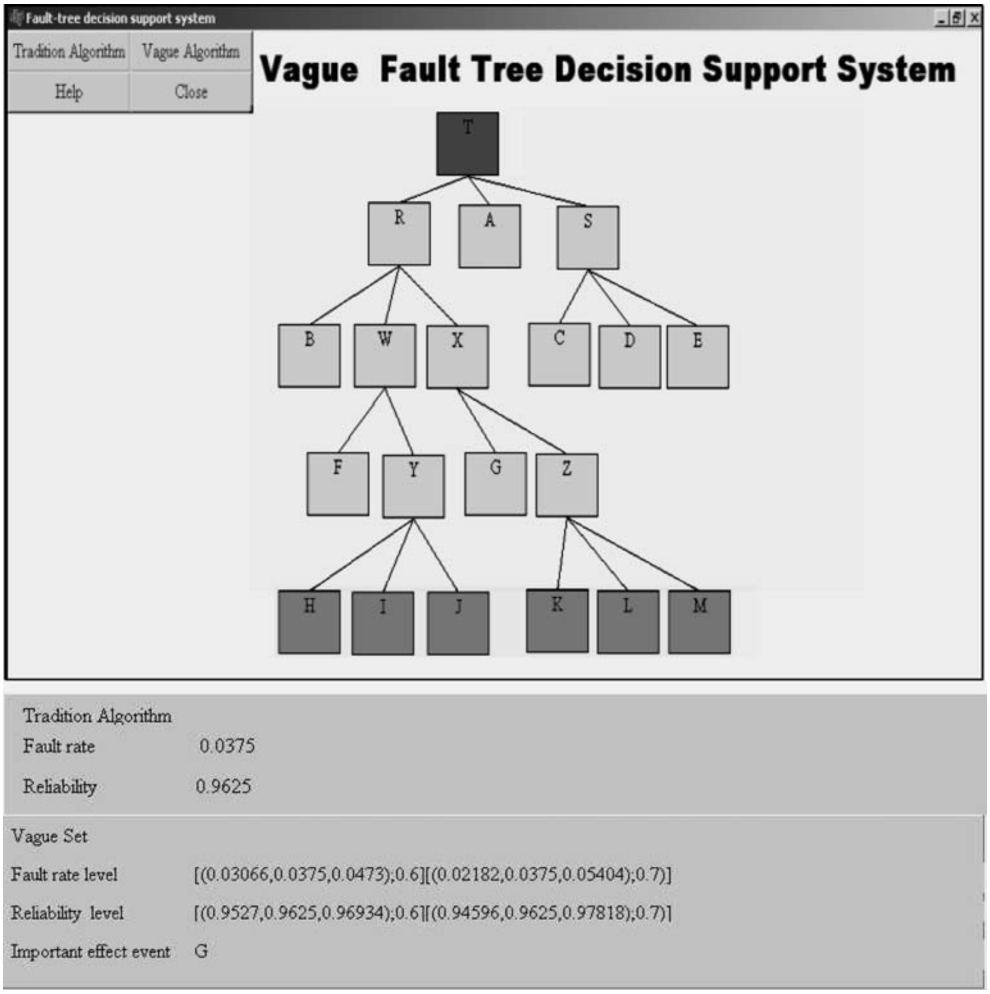

Fig. 12 Vague fault-tree decision support system

is developed to generate fault-tree nodes, vague set fault-tree interval, traditional reliability, and vague set reliability interval. This decision support system can be extended as the tool of knowledge management by integrating more information technology methods.

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