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Analysis of reservoir water quality using fuzzy synthetic evaluation

R.-S. Lu, S.-L. Lo, J.-Y. Hu

Abstract. A general methodology for fuzzy synthetic evaluation is developed and illustrated with a case study of trophic status assessment for Fei-Tsui Reservoir in Taiwan. The historical data base was collected from the management agency of Fei-Tsui Reservoir from 1987 to 1996. In fuzzy synthetic evaluation, the classification is determined by a matrix operation of the weighted vector with the fuzzy evaluation matrix. After all individual membership functions of evaluated factors have been determined, the fuzzy evaluation matrix can be established. The weighted vector is determined by the analytic hierarchy process method (AHP). The results of this investigation show that the long-term change of water quality and the overturn phenomena cannot be observed with the Carlson index from 1987 to 1992 but is expressed by fuzzy synthetic evaluation. Fuzzy synthetic evaluation is better suited than the Carlson index to rating the trophic status of self-sustaining lakes. Interpretation of the results can provide valuable information to decision makers and aid reservoir management.

Key words: eutrophication, reservoir water quality, fuzzy theory.

Introduction

Under natural conditions, lakes decrease by a depth of 3 mm/yr (Henderson-Sellers and Markland, 1987). When lakes are polluted with large amounts of nitrogen and phosphorus, however, rapid growth of algae and eutrophication occur, thereby causing the water quality to deteriorate and increasing the cost of water treatment. The trophic state index is often based on total phosphorous (TP) concentration, chlorophyll a (chl a) concentration, and Secchi disk depth (SD). Of these three factors, chl a plays the most important role, followed by TP.

Lush vegetation, a high concentration of salts, and high turbidity are the defining characteristics of eutrophication. In a single-variable index, a physico-chemical parameter that is representive of these characteristics is chosen and its threshold value is set as the trophic standard. Application of this method often produces inconsistent results when different parameters are chosen. Since eutrophication involves complex changes in the water, the results obtained from using only one parameter may easily mislead or bias the user. For this reason, the

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multivariable trophic state indexing methods were developed. The most commonly used multivariable indices are the Carlson (Carlson, 1977), Morihiri (Morihiri et al., 1981), and North Carolina indices (Weiss et al., 1985).

Multivariable indexing allows for a more thorough investigation of water quality and a more continuous description of the eutrophication process, but several problems are inherent in this method. The first problem is that geographical and atmospheric factors influence eutrophication, causing the standards for each area to differ. Consequently, the three aforementioned indices are heavily regional in nature. Another problem involves unreasonable classification standards. For example, the Carlson index gives TSI = 49 and TSI = 50 different classifications but TSI = 31 and TSI = 49 the same. A further problem with multivariables is that it gives different weights to each parameter. Thus, the scientific community has been unable to agree upon a single, reliable trophic state index.

The Fuzzy Set Theory was discovered by Zadeh in 1965 and, after 30 years of research and development, has extensive applications. Classification of river water quality using this theory began in the 1980's. The majority of research in this field centered on fuzzy synthetic evaluation and fuzzy clustering analysis (Kung et al., 1992). Synthetic evaluation is used to classify samples at a known center of classification, whereas clustering analysis is used to classify samples according to their relationships when this center is unknown. Kung et al. (1992) published the application of fuzzy clustering analysis to water quality classification. Although fuzzy clustering analysis classifies samples for unknown standards by relationship, it requires a large amount of data. The samples must have a high degree of similarity in addition to categorizing factors. Therefore, this method of analysis is better suited to regional classification.

Fuzzy synthetic evaluation classifies samples for known standards. It is a modified and corrected version of conventional synthetic evaluation, which contains defects created by binary logic. Since there is no clear, defined set of trophic state standards for water reservoirs, this study will use the standards determined by fuzzy synthetic evaluation to analyze and compare the water quality of Fei-Tsui Reservoir in past years and investigate changes and trends in its eutrophication rate.

Methodology

The Organization of Economic Cooperation Development (OECD) has investigated the water quality of 80 lakes in the world in order to set a standard for determining eutrophic status (see Table 1). In this research, total posphorous, chlorophyll a, and Secchi disk depth are the classification factors, and the membership functions of the 3 factors which are more applicable to countries such as Taiwan in the subtropic zone are developed. Eutrophic, mesotrophic, and oligotrophic standards defined by OECD are unclear; there are some fuzzy zones between the eutrophic, mesotrophic, and oligotrophic states. This is a feature of

Table 1. Trophic state as a function of nutrient levels

Trophic state	TP(μg/L)	chl a (μg/L)	SD (m)
Oligotrophic	<7.9	<2	>4.6
Oligo-meso	8–11	2.1–2.9	4.5–3.8
Mesotrophic	12–27	3.0–6.9	3.7–2.4
Meso-eutro	28–39	7.0–9.9	2.3–1.8
Eutrophic	>40	>10	<1.7

the problem which can be solved by fuzzy theory. Based on the OECD report, this study defines three membership functions for the 3 factors (TP, chl a, and SD) that are relative to the water quality index.

1 Determination of membership function

1.1 Membership function of total phosphorous

The status of the lake is eutrophic if TP is greater than 40 μg/L according to OECD standards. If TP is lower than 8 μg/L, the status of the reservoir is oligotrophic; and if TP is exactly equal to 20 μg/L, the lake is classified as mesotrophic. Several keypoints defining the value of the membership functions are listed below:

$$\mu_{pe}(40) = 1,\ \mu_{pe}(20) = 0,\ \mu_{pm}(8) = 0,\ \mu_{pm}(20) = 1,\ \mu_{pm}(40) = 0,$$
$$\mu_{po}(8) = 1,\ \mu_{po}(20) = 0$$

where p represents phosphorous; e, eutrophic; m, mesotrophic; and o, oligotrophic. After selecting the keypoints, a membership function should be selected to fit these data sets. The half-T-shaped distribution function is chosen as the TP membership function, and the parameter k must be fixed to improve the similarity between the membership function and real water quality condition. Parameter k is determined according to the OECD report. When TP ranges from 8 to 12 μg/L, it is difficult to classify the water body as either oligotrophic or mesotrophic; also, when TP ranges from 28 to 40 μg/L, it cannot be characterized as mesotrophic or eutrophic. The degree of fuzziness for the two states is assigned to a maximum value of 0.5 when TP concentration is exactly equal to 10 and 34 μg/L. Thus, parameter k is determined by equations 1 and 4. The modified TP membership functions are listed below:

$$\mu_{pe}(x) = ((x - 20)/2)^{k1} \quad \text{if} \quad 20 < x < 40 \tag{1}$$

$$\mu_{pe}(x) = 1 \quad \text{if} \quad 40 < x \tag{2}$$

$$\mu_{pm}(x) = 1 - \mu_{p1}(x) - \mu_{p3}(x) \tag{3}$$

$$\mu_{po}(x) = ((20 - x)/12)^{k2} \quad \text{if} \quad 8 < x < 20 \tag{4}$$

$$\mu_{po}(x) = 1 \quad \text{if} \quad x < 8 \tag{5}$$

$$x = 34, \quad \mu_{pm}(x) = \mu_{pe}(x) = 0.5, \quad k_1 = 1.943 \tag{6}$$

$$x = 10, \quad \mu_{pm}(x) = \mu_{po}(x) = 0.5, \quad k_2 = 3.802 \tag{7}$$

where p represents total phosphorous; e, eutrophic; m, mesotrophic; and o, oligotrophic.

The membership functions for chl a and SD are similarly given in sections 1.2 and 1.3.

1.2
Membership function of chlrophyll a

$$\mu_{ce}(x) = ((x - 5)/5)^{1.943} \quad \text{if } 5 < x < 10 , \tag{8}$$

$$\mu_{ce}(x) = 1 \quad \text{if } 10 < x \tag{9}$$

$$\mu_{cm}(x) = 1 - \mu_{ce}(x) - \mu_{co}(x) \tag{10}$$

$$\mu_{co}(x) = ((5 - x)/12)^{3.802} \quad \text{if } 8 < x < 20 , \tag{11}$$

$$\mu_{co}(x) = 1 \quad \text{if } x < 8 \tag{12}$$

where c represent chl a.

1.3
Membership function of Secchi disk depth

$$\mu_{sde}(x) = ((3 - x)/1.3)^{2.642} \quad \text{if } 1.7 < x < 3.0 \tag{13}$$

$$\mu_{sde}(x) = 1 \quad \text{if } 1.7 > x \tag{14}$$

$$\mu_{sdm}(x) = 1 - \mu_{sde}(x) - \mu_{sdo}(x) \tag{15}$$

$$\mu_{sdo}(x) = ((x - 3)/1.5)^{2.235} \quad \text{if } 3.0 < x < 4.5 \tag{16}$$

$$\mu_{sdo}(x) = 1 \quad \text{if } x > 4.5 \tag{17}$$

where sd represents secchi disk depth.

1.4
Construction of the membership matrix

A final fuzzy membership matrix used for fuzzy synthetic evaluation is derived from synthesizing the trophic membership functions of the 3 factors (TP, chl a, and SD). The matrix can be formulated as the following:

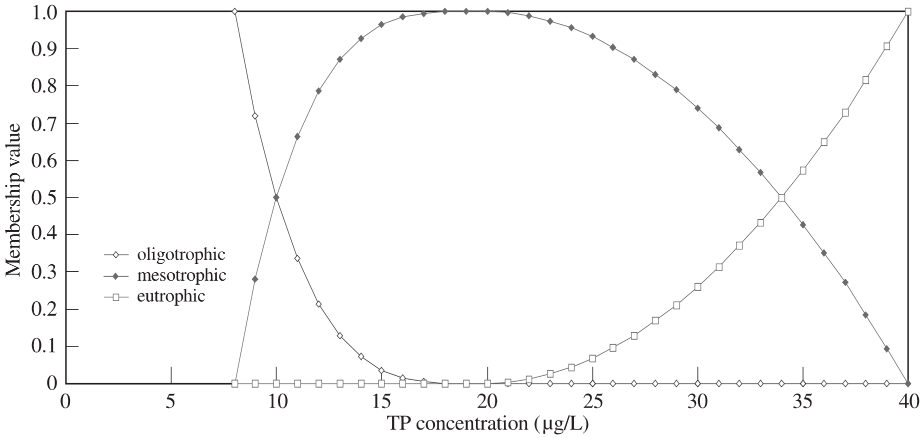

Fig. 1. The membership function for TP

$$\mathbf{R} = \begin{bmatrix} \mu_{pe} & \mu_{pm} & \mu_{po} \\ \mu_{ce} & \mu_{cm} & \mu_{co} \\ \mu_{sde} & \mu_{sdm} & \mu_{sdo} \end{bmatrix} \quad (18)$$

where p represents total phosphorous; c, chl a; sd, secchi disk depth; e, eutrophic; m, mesotrophic; and o, oligotrophic.

2
Determination of weights

Chlorophyll a is the most important factor in the determination of trophic status (Mineeva, 1993); total phosphorus is ranked as the second (Nedoma, 1993). Therefore, in this study we assume an initial judgment matrix **J** for these factors listed below:

$$\mathbf{J} = \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 2 & 1 & 3 \\ \frac{1}{2} & \frac{1}{3} & 1 \end{bmatrix} \quad (19)$$

A normalized matrix **D** is determined by normalizing the initial matrix **J** column-by-column. Then, the final weighted vector **W** can be derived from summing the elements of each row of matrix **D** and normalizing again to this vector.

$$\mathbf{D} = \begin{bmatrix} 0.2857 & 0.2727 & 0.3333 \\ 0.5714 & 0.5455 & 0.5 \\ 0.1429 & 0.1818 & 0.1667 \end{bmatrix} \quad (20)$$

$$\mathbf{W} = \left[w_p \ w_c \ w_{sd} \right] = [0.2972 \ 0.5390 \ 0.1638] \quad (21)$$

3
Fuzzy synthetic evaluation

In this study, the evaluation set **U** = {oligotrophic, mesotrophic, eutrophic} contains 3 levels, and the factor set **V** = {chl a, TP, SD} has 3 factors. The evaluation matrix of 3 factors is $\mathbf{R} = (u_{ij})_{nxm}$; the weights set is $\mathbf{W} = \{w_p, w_c, w_e\}$. When **W** and **R** are given, fuzzy synthetic evaluation can be performed following equation 22:

$$\mathbf{B} = \mathbf{W} \times \mathbf{R} = [w_p w_c w_{sd}] \times \begin{bmatrix} \mu_{pe} & \mu_{pm} & \mu_{po} \\ \mu_{ce} & \mu_{cm} & \mu_{co} \\ \mu_{sde} & \mu_{sdm} & \mu_{sdo} \end{bmatrix} = [b_o b_m b_e] \quad (22)$$

which is illustrated in Fig. 2. In this study, we use first-level fuzzy synthetic evaluation. The assessment result of the trophic status is determined by the maximum value of the 3 values (b_o, b_m, and b_e). The results of fuzzy synthetic evaluation is summed up with a eutrophication index (EI). The EI value is calculated by equation 23 can be plotted versus time to reveal the trend of water quality.

$$\mathrm{EI} = 1 \times b_o + 2 \times b_m + 3 \times b_e \quad (23)$$

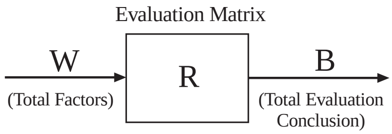

Fig. 2. Fuzzy synthetic evaluation

Case study

A case study of fuzzy synthetic evaluation of trophic status has been implemented at Fei-Tsui Reservoir, situated in northern of Taiwan. Fei-Tsui Reservoir (surface = 10.24 km^2; mean depth = 39.68 m; hydraulic retention time = 150.8 days) is one of the most detailed, surveyed reservoirs in Taiwan. The historical data base was collected from the management agency of Fei-Tsui Reservoir from 1987 to 1996. The measured data was surveyed at different water depths, but only data from the top level of the water body was used in this study. There are a total of 18 measured items, such as water temperature, pH, dissolved oxygen, suspended solid, nitrate, and nitrite. However, some of the 18 variables are not eutrophication factors. Monthly average total phosphorus (TP), chlorophyll a (chl a), and sechii depth (SD) have been used as eutrophication parameters according to the results of factors analysis.

Results and discussion

1. Synthetic evaluation results

Evaluation results for water quality data of Fei-Tsui Reservoir from January 1987 to June 1996 are summarized in Fig. 3. The pattern of eutrophication occuring around October reveals an overturn effect. Overturn is the circulation of water and nutrients in a lake or reservoir caused by a drop in temperature in the spring and the fall. Because colder water has a higher density, surface water sinks as warmer, deeper water rises to the top, causing nutrients from the bottom of the lake to rise

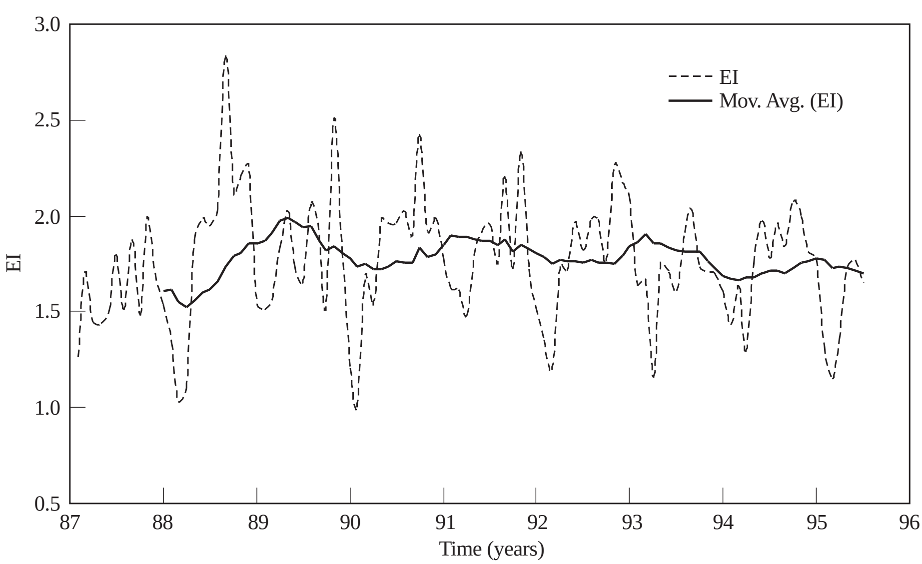

Fig. 3. The trend of eutrophication assessed by fuzzy synthetic evaluation for Fei-Tsui Reservoir

Table 2. Yearly averages of individual membership values for trophic status

Year	Individual membership value			Level
	Oligotrophic	Mesotrophic	Eutrophic	
1987	0.432	0.568	0.000	mesotrophic
1988	0.001	0.881	0.118	mesotrophic
1989	0.073	0.927	0.000	mesotrophic
1990	0.121	0.879	0.000	mesotrophic
1991	0.209	0.791	0.000	mesotrophic
1992	0.226	0.683	0.091	mesotrophic
1993	0.303	0.697	0.000	mesotrophic
1994	0.280	0.720	0.000	mesotrophic
1995	0.369	0.631	0.000	mesotrophic
1996	0.375	0.625	0.000	mesotrophic

Table 3. The relation of ambient temperature to water quality

Year	Month	Temp (°C)	Membership value			Level
			Oligotrophic	Mesotrophic	Eutrophic	
1991	9	35.0	0.297	0.685	0.018	mesotrophic
	10	21.0	0.206	0.254	0.540	eutrophic
	11	20.5	0.275	0.725	0.000	mesotrophic
	12	18.5	0.476	0.524	0.000	mesotrophic
1992	1	19.0	0.648	0.352	0.000	oligotrophic
	2	16.0	0.802	0.198	0.000	oligotrophic
	3	17.5	0.263	0.732	0.005	mesotrophic
	4	19.0	0.297	0.690	0.013	mesotrophic
	5	29.0	0.049	0.930	0.021	mesotrophic
	6	21.0	0.245	0.691	0.065	mesotrophic
	7	34.0	0.052	0.917	0.032	mesotrophic
	8	33.0	0.027	0.966	0.007	mesotrophic
	9	32.9	0.279	0.679	0.042	mesotrophic
	10	23.0	0.270	0.191	0.539	eutrophic
	11	21.0	0.363	0.098	0.539	eutrophic
	12	23.8	0.438	0.023	0.539	eutrophic

with it and thereby bringing about eutrophication. In Taiwan, winter temperatures do not fall below 4°C, so overturn occurs only in the fall. When the pollution loading is low, the effect of overturn on eutrophication becomes significant.

Construction of Fei-Tsui Reservoir was completed in 1987. Early pollution control was well-enforced by citizens living upstream of the reservoir, in which plant debris was the main pollutant. Since the nutrients in the reservoir were self-supplied, the overturn effect was very obvious in early years. Yearly averages show that the mesotrophic value of the membership function decreased while the oligotrophic value increased (see Table 2). These results reveal that good pollution control practices enable the nutrients in a reservoir to be depleted gradually and thus improve water quality. After the nutrient supply is exhausted, overturn becomes less obvious, explaining the absence of said effect for 1993 and 1994.

Table 3 shows that eutrophication of Fei-Tsui Reservoir lasted for three months (from Oct.–Dec.) in 1992. Because the reservoir experienced unnaturally high temperatures that winter (water temperatures was above 20°C), the algae that grew abundantly in October continued to flourish in the following two months and caused eutrophication to be prolonged. The water condition reverted to normal in January 1993 when the temperature decreased. The relationship between temperature and water quality is displayed in Table 3.

2. Comparison with Carlson index results

The Carlson index results for water quality data of Fei-Tsui Reservoir from January 1987 to June 1995 are summarized in Fig. 4. The highest eutrophic conditions occurred in the first 6 years (1987–1992) according to the fuzzy synthetic evaluation, peaking in August 1988 and at the end of 1992. When the conventional TSI method is applied to the data sets, the reservoir is classified as mesotrophic for October in 1988, 1990, and 1991. However, fuzzy synthetic evaluation classifies the water body as eutrophic for the same years. Moreover, the water quality for some months was classified as mesotrophic by fuzzy synthetic evaluation, whereas the Carlson index conferred an oligotrophic status. All of the conflicting data for trophic states are listed in Table 4.

A research report (Chang and Wen, 1995) reveals that a reservoir water body does not stabilize until the nutrients that existed before the reservoir began operation are exhausted. At least 5 years are needed before the water quality becomes more stable (Chang and Wen, 1995). This phenomenon is also proven by this investigation using fuzzy synthetic evaluation. Eutrophication of Fei-Tsui Reservoir caused by overturn stopped occurring after 1991, which is exactly 5 years from the first year of operation (1987). However, the Carlson index is unable to reveal the same information.

A comparison of fuzzy synthetic evaluation with Carlson indexing reveals that the latter is more sensitive to changes in SD and TP. The Carlson index provides an average index for SD, TP, and chl a but does not weigh each factor by its relative influence on eutrophication. As a consequence, the effects of SD and TP become amplified. The eutrophic status conferred on the reservoir for November

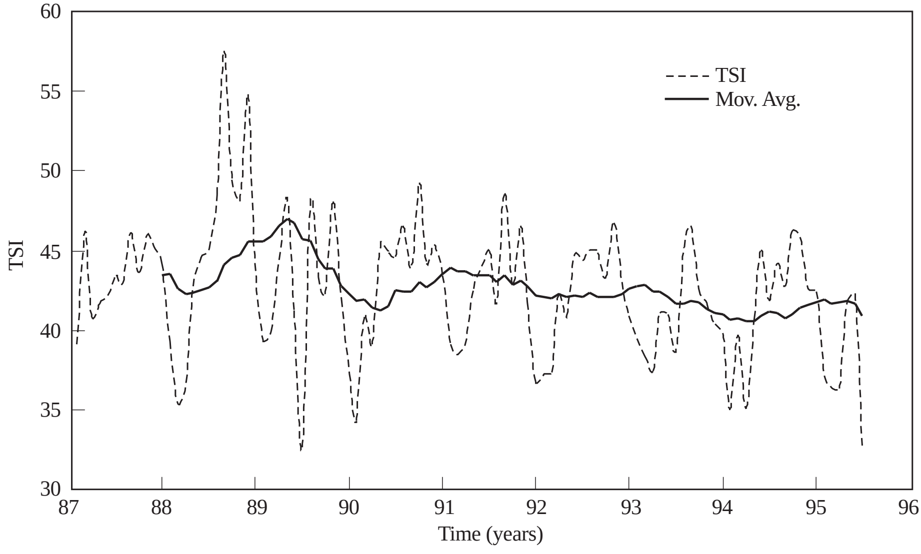

Fig. 4. The trend of eutrophication assessed by the Carlson index for Fei-Tsui Reservoir

Table 4. Comparison of the Carlson index with fuzzy synthetic evaluation

Date	Parameter			Carlson index		Fuzzy synthetic evaluation	
	SD*	TP*	chl a*	TSI	Trophic state	EI	Trophic state
Oct, 89	3.00	13.50	14.47	48.12	mesotrophic	2.51	eutrophic
Oct, 91	2.80	9.10	14.39	46.57	mesotrophic	2.33	eutrophic
Oct, 92	3.10	8.30	19.83	46.70	mesotrophic	2.27	eutrophic
Nov, 92	4.00	6.50	13.14	42.97	mesotrophic	2.18	eutrophic
Dec, 92	4.40	5.00	11.69	40.90	mesotrophic	2.10	eutrophic
Mar, 87	2.90	14.70	1.29	40.78	mesotrophic	1.45	oligotrophic
Apr, 87	3.00	14.20	1.88	41.68	mesotrophic	1.44	oligotrophic
Jan, 89	5.20	12.60	2.47	39.37	oligotrophic	1.51	mesotrophic
Jun, 93	5.96	11.30	2.90	38.72	oligotrophic	1.61	mesotrophic
Dec, 87	2.00	17.10	1.71	44.20	mesotrophic	1.54	oligotrophic
Dec, 88	2.00	17.10	1.71	44.20	mesotrophic	1.54	oligotrophic

* SD represents Secchi disk depth (m); TP, total phosphorous (μg/L); chl a, chlorophyll a (μg/L)

1988 using this index is due to the extremely low SD value. Considering individual factors alone, however, the water is mesotrophic according to TP and oligotrophic based on chl a. Therefore, the unlimited amount of influence that an individual factor may exert on the index and the failure to weigh each factor, which may create inaccurate results, are the disadvantages of this method. The low SD and high TP values obtained in November 1988 may not necessarily have been indications of eutrophication but rather were caused by a high concentration of inorganic suspended solids. In fuzzy synthetic evaluation, a comparison of membership values shows that the level of eutrophication in the reservoir decreased annually. Neither this trend nor overturn is observable with the Carlson index. Thus, fuzzy synthetic evaluation is better suited than the Carlson index to rating the trophic status of self-sustaining lakes.

Conclusions

Trophic status assessment of reservoirs or lakes in an interesting field in environmental impact assessment (EIA). For most water quality index (WQI) researchers, it is a critical issue to construct an effective and simple index or methodology so that the right information can be obtained from the measurements to illustrate spatial variations and seasonal fluctuations. In this application, an alternative tool to reservoir eutrophication index is developed and the utility of the tool is compared with the Carlson index. The conclusions are summarized as the following:

1. The result of fuzzy synthetic evaluation shows that the water quality of the Fei-Tsui Reservoir was unstable, which was caused by a short-term overturn effect from 1987 to 1992. However, the Carlson index is unable to reveal the same information.
2. Fuzzy synthetic evaluation can be used as an alternative method in reservoir trophic state assessment, especially for self-sustaining reservoirs.
3. Fuzzy synthetic evaluation is more sensitive to a variation in water quality than the Carlson index.
4. Combined with other eutrophication control technology, an expert system may be developed for providing valuable information to decision makers and for aiding reservoirs management.

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