



Prioritization of watersheds based on a picture fuzzy analytic hierarchy process and linear assignment model

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Abstract

Soil erosion is one of the most dangerous natural dangers, causing a great deal of harm in many parts of the world. In the presented study, the Gusru river watershed in Indi was divided into 14 sub-watersheds, and then 14 morphometric parameters were calculated, including drainage density (D_d), bifurcation ratio (R_b), streams frequency (F_s), average slope (S_a), form factor (R_f), circulatory ratio (R_C), elongation ratio (R_e), relative relief (R_h), ruggedness number (R_N), bifurcation ratio (R_b), texture ratio (T), length of the overland flow (L_o) compactness coefficient (C_C) and hypsometric integral (HI) were derived for each sub-watershed. Afterward, the combination of picture fuzzy-analytic hierarchy process and picture fuzzy-linear assignment model were used to assign weights to selected morphometric criteria and to rank the sub-watersheds based on the level of soil erosion susceptibility. The results of the study showed that sub-watersheds 11 and 2 were the most susceptible sub watersheds, while sub-watersheds 13 and 14 had the lowest susceptibility to soil erosion. Prioritization and ranking of sub-watersheds from the perspective of soil erosion susceptibility can be used as a powerful tool for prevention and mitigation measures.

Keywords Erosion susceptibility · Picture fuzzy · Analytic hierarchy process · Linear assignment model · Watershed prioritization

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1 Introduction

Soil erosion is a worldwide issue that causes soil loss, agricultural land loss, and crop yield reductions (Scherr and Yadav 1996; Gajbhiye et al. 2014, 2015a, b; Ghoderao et al. 2022; Assefa and Hans-Rudolf 2016; Kebede et al. 2020; Meshram et al. 2022a, b, c). But it also has significant environmental and economic consequences due to its impact on agricultural production, infrastructure, and soil–water quality (Pimentel et al. 1995; Lal 1998; Zhang et al. 2010). Each year, approximately 10 million hectares of cultivated land are lost to soil erosion, limiting the quantity of arable land accessible for food production (Pimentel 2006). Erosion not only reduces the quantity and quality of soils on-site, but it also causes serious sediment-related issues off-site. Soil erosion requires a full understanding of the numerous erosion processes, their interconnections, regulating factors, and spatial extent in order to address the problem (Meshram et al. 2017, 2018; Poesen 2019). According to FAO and ITPS studies (2015, 2019), the severe situation is continuing to deteriorate globally due to water erosion; nevertheless, 33 percent of the Earth’s soils have already been degraded, with over 90 percent of the remaining soils potentially destroyed by 2050.

Topography, climate, soil properties, soil cover, and human activities all play a role in soil erosion. Reduced soil production, degraded water quality, and increased flood danger are only a few of the effects of this occurrence (Zhou et al. 2008; Sinshaw et al. 2021; Benzougagh et al. 2022). Due to a lack of manpower and resources, implementing soil and water conservation programs for reducing soil erosion and flood problems concerns in extensive watershed areas is typically difficult. As a result, a reasonable strategy for picking watersheds for treatment based on sediment yield potential might help in the fight against this menace (Chowdary et al. 2013; Benzougagh et al. 2017, 2020).

It is vital to carry out soil and water conservation efforts in order to maintain the ecosystem and limit the harmful consequences of erosion on agriculture, infrastructure, water quality, and other factors. All soil erosion models that have an impact on soil management are designed to work on a local level (hillslope, fields). However, in order to make policy and programmatic decisions, it is also required to determine the degree of soil erosion at a larger scale.

Morphology is the study of the pattern of the earth’s surface as well as the shape and dimensions of its landforms through measurement and quantitative analysis (Clarke 1996; Benzougagh et al. 2017; Meshram and Sharma 2017; Meshram et al. 2019). This analysis can be done by measuring the linear, aerial, and relief aspects of

watersheds using remote sensing (RS) and Geographical Information Systems (GIS) techniques for the characterization and prioritization of watershed areas based on spatial computation of soil loss and other erosion-related parameters.

Prioritization of watersheds can be characterized as a way of identifying environmentally challenged sub-watersheds for priority soil conservation actions. Several empirical models based on geo-morphological parameters have previously been developed to quantify sediment yield for this purpose. The sediment yield index (SYI) proposed by Bali and Karale (1977), the universal soil loss equation (USLE) proposed by Wischmeier and Smith (1978), Picture fuzzy (PF) (1965), Linear assigning model (LAM), and the Analytical Hierarchy Process (AHP) proposed by Wind and Saaty are some of the other methods and models for prioritizing watersheds (1980). To research diverse morphometric parameters and utilize GIS approaches for watershed prioritizing, the current study used Picture Fuzzy (PF), Analytical Hierarchy Process (AHP), and Linear Assigning Models. Picture Fuzzy (PF), Analytical Hierarchy Process (AHP), and Linear Assigning Models are multi-criteria decision-making strategies that take into account both subjective and objective considerations. The AHP technique was first offered as a semi-target, multi-target, and multi-criteria method. This method is a multi-criteria decision-making methodology that selects preferences from a variety of alternatives on different scales. This is a prominent paradigm for susceptibility research, decision-making, and regional planning (Wind and Saaty 1980; Kayastha et al. 2013). There are several steps to the AHP model: Identify unstructured concerns and research objectives determine the variables that influence the situation and reorder them in a hierarchical order; to determine the relative importance of each component, rank values according to their subjective significance (Saaty and Vargas 2001).

The study’s techniques could aid in the successful implementation of development initiatives as well as the monitoring of natural resources for long-term development.

The study’s techniques could be useful in the successful implementation of development initiatives as well as the monitoring of natural resources (soil and water) for long-term development.

2 Materials and methods

2.1 Study area

Gusru is a watershed in India’s Madhya Pradesh state, in the Satna Panna districts, with longitudes of 80032’50.23’ and 80037’31.14’, and latitudes of 2406’32.75’ and

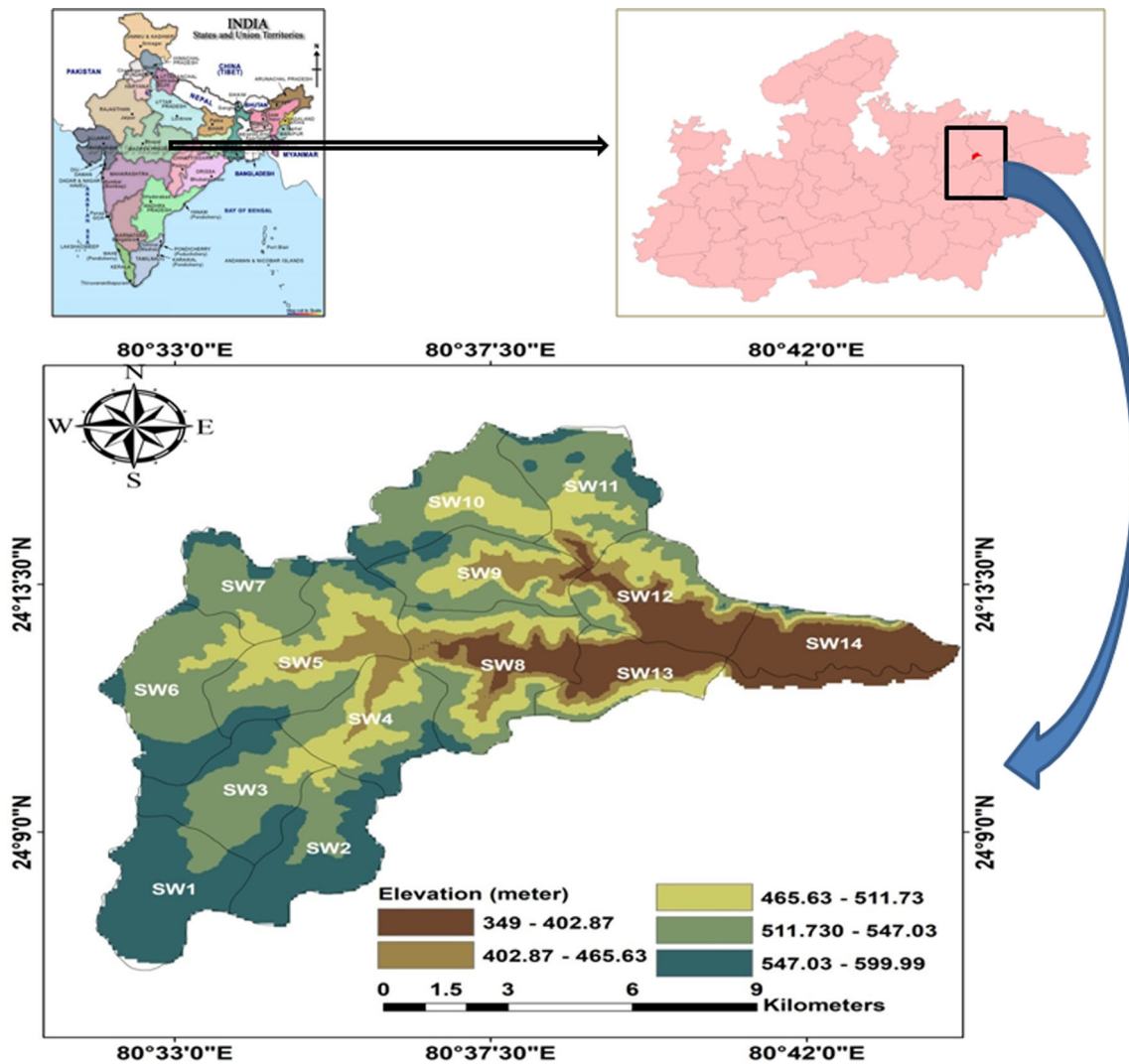


Fig. 1 Location map of the study area

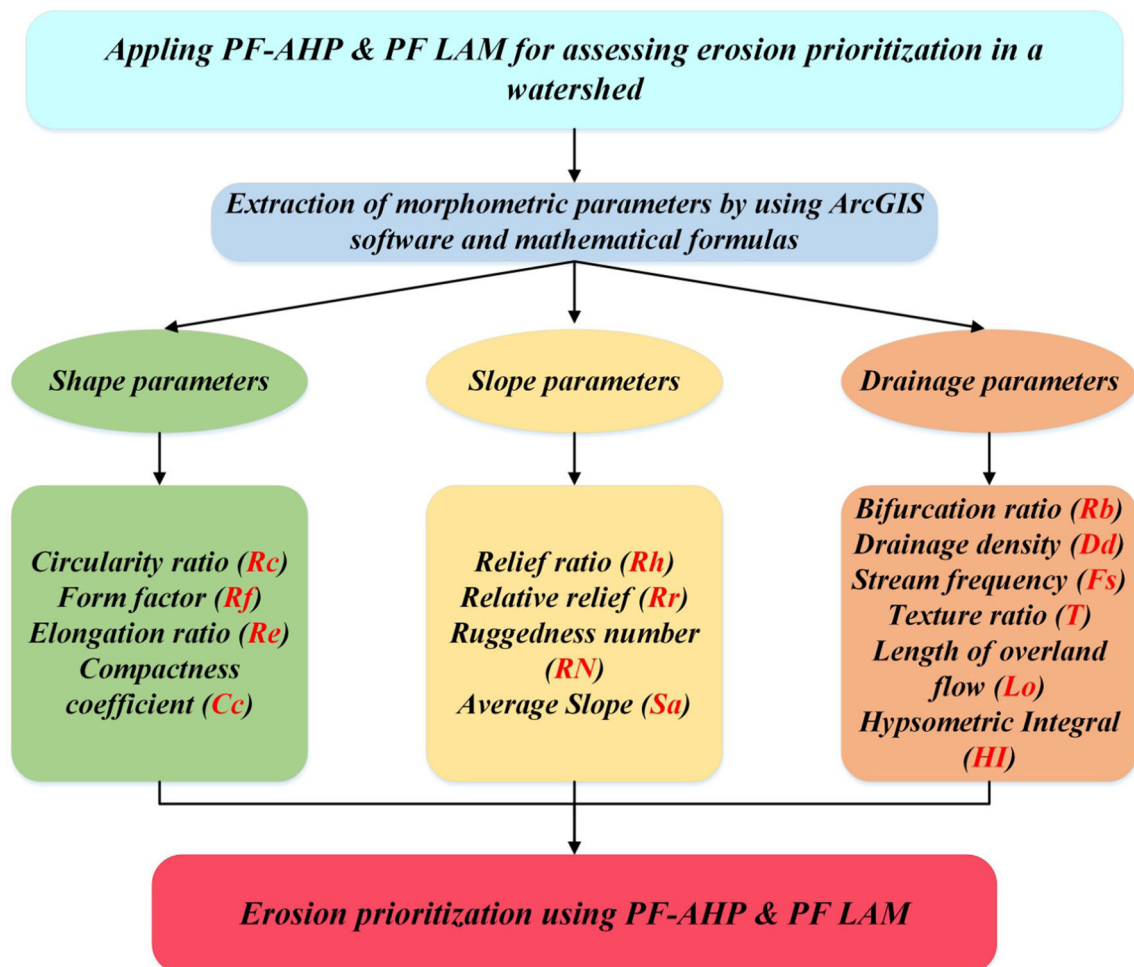
24°16'24.07'. (Fig. 1). It has a surface area of 155 km², with heights of 339 to 628 m above sea level. From east to west, the Gusru River meets the Tons River in Sagwania village. The watershed's eastern half contains a tiny check dam that serves mostly as irrigation outlet. Rainfed agriculture is the sole choice because there is no alternative source of water for irrigation. The soil structure of the watershed is predominantly sandy loam. Under rainfed and irrigated conditions, soils respond to a variety of crops and watershed management. Shale, sandstone, and calcareous rocks are the main lithological units in the watershed. The research area is located between the Bhandar escarpment and the Kaimore highlands, and descends from the Bhandar plateau.

2.2 Prioritization and mapping

The watershed boundary, sub-watershed boundary, and stream network were prepared for digitization in a GIS context. The DEM from the SRTM dataset was used to identify sub-watersheds of the Gusru watershed and create drainage maps for each. The border and drainage network of these sub-watersheds were employed for additional geomorphological investigation. The digitized coverage of the drainage network map was used to calculate the area, perimeter, stream order, stream length, stream number, and elevation morphometric parameters using GIS system. The morphological parameters of our sub-watersheds, on the other hand, were calculated using formula-based features. This study began with the determination of morphometric parameters in each sub-watershed and progressed to watershed prioritization. We compared the findings of

Table 1 Sub-watershed wise morphometric parameters (Meshram et al. 2019)

| Sub-watershed | R_h | R_r | R_N | R_b | D_d | F_s | R_e | R_f | R_c | T | L_o | C_c | S_a | HI |
|---------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|-------|
| 1 | 0.019 | 0.006 | 0.304 | 3.889 | 3.372 | 6.264 | 0.651 | 0.530 | 0.822 | 4.902 | 0.148 | 1.239 | 7.089 | 0.410 |
| 2 | 0.023 | 0.008 | 0.425 | 4.115 | 3.293 | 6.165 | 0.564 | 0.340 | 0.658 | 4.275 | 0.152 | 1.331 | 9.275 | 0.700 |
| 3 | 0.025 | 0.008 | 0.409 | 3.521 | 3.199 | 5.299 | 0.573 | 0.433 | 0.743 | 3.776 | 0.156 | 1.321 | 8.121 | 0.560 |
| 4 | 0.032 | 0.011 | 0.499 | 3.833 | 3.328 | 6.663 | 0.654 | 0.490 | 0.790 | 4.928 | 0.15 | 1.236 | 13.524 | 0.670 |
| 5 | 0.032 | 0.010 | 0.670 | 3.646 | 3.488 | 8.016 | 0.531 | 0.414 | 0.726 | 6.270 | 0.143 | 1.372 | 12.89 | 0.520 |
| 6 | 0.022 | 0.008 | 0.312 | 3.643 | 2.454 | 3.817 | 0.56 | 0.370 | 0.686 | 2.859 | 0.204 | 1.335 | 7.467 | 0.540 |
| 7 | 0.032 | 0.010 | 0.420 | 3.417 | 3.180 | 5.700 | 0.561 | 0.472 | 0.775 | 3.385 | 0.157 | 1.335 | 8.680 | 0.510 |
| 8 | 0.042 | 0.012 | 0.763 | 3.681 | 3.670 | 7.335 | 0.582 | 0.591 | 0.868 | 6.058 | 0.136 | 1.311 | 20.115 | 0.560 |
| 9 | 0.046 | 0.017 | 0.827 | 3.705 | 3.334 | 6.284 | 0.631 | 0.356 | 0.673 | 4.495 | 0.15 | 1.259 | 17.845 | 0.610 |
| 10 | 0.024 | 0.008 | 0.462 | 4.005 | 3.421 | 7.426 | 0.494 | 0.363 | 0.680 | 5.013 | 0.146 | 1.422 | 9.998 | 0.420 |
| 11 | 0.047 | 0.015 | 0.742 | 3.208 | 3.285 | 5.598 | 0.606 | 0.473 | 0.776 | 4.092 | 0.152 | 1.284 | 14.566 | 0.750 |
| 12 | 0.044 | 0.015 | 0.684 | 3.113 | 3.319 | 6.268 | 0.758 | 0.513 | 0.809 | 5.217 | 0.151 | 1.149 | 22.295 | 0.450 |
| 13 | 0.038 | 0.014 | 0.737 | 3.495 | 3.899 | 7.322 | 0.513 | 0.315 | 0.634 | 4.130 | 0.128 | 1.395 | 20.416 | 0.360 |
| 14 | 0.046 | 0.015 | 1.134 | 3.759 | 4.994 | 7.785 | 0.489 | 0.381 | 0.696 | 4.671 | 0.1 | 1.430 | 11.553 | 0.230 |

**Fig. 2** Flowchart of research methodology

those indicators using the PF-AHP and PF-LAM approaches to organize sub-watersheds. It’s worth noting that our sub-watershed prioritization process took into account a total of 14 morphometric characteristics (Table 1). We used morphometric parameter data from Meshram et al. earlier’s investigations in this research (2021). Figure 2 depicts the methods used in this investigation.

2.3 Prioritization of watersheds using PF-AHP and PF-LAM models

The AHP is a decision-making technique developed to solve complex problems, which are divided into a multi-level hierarchical structure, i.e., top-level (objectives), middle level (criteria and sub-criteria), and lower level (alternatives). This method uses a nine-point scale of preferences to express judgments of decision-makers in the form of pair-wise comparisons (Leung and Cao 2000, Wang et al. 2008, Xu and Liao 2013). Decision-makers deal with a dilemma and uncertainty to choose a specific value from this nine-point scale. Therefore, many scholars criticized the inability of conventional AHP in handling uncertainty. In recent years, to handle and reduce uncertainty in human preferences, the AHP method was combined with other methods, which are also related to fuzzy sets and their extensions. Therefore, in the presented study, the AHP technique under the picture fuzzy set (PFS) environment (one of the latest extensions of ordinary fuzzy set, which provides a larger preference domain for decision-makers to select membership functions) was used to assign weights to flood-related criteria. After assigning weights to criteria for ranking the alternatives (sub-watersheds) based on flood susceptibility, we used the linear assignment method (LAM) as one of the most used methods for ranking the alternatives in the PFS environment. The basic definitions of the PFS are as follows (Cuong and Kreinovich 2015, Dutta and Ganju 2018).

Definition 1 Single valued PFS of \tilde{A}_u of the universe of discourse U can be defined as (Eqs. 1, 2 and 3):

$$\tilde{A}_p = \left\{ u, \left(\mu_{A_p}^{\sim}(u), \mathcal{I}_{A_p}^{\sim}(u), v_{A_p}^{\sim}(u) \right) \mid u \in U \right\} \tag{1}$$

where

$$\begin{aligned} \mu_{A_p}^{\sim}(u) : U &\rightarrow [0, 1], \mathcal{I}_{A_p}^{\sim}(u) : \mathcal{U} \rightarrow [0, 1], v_{A_p}^{\sim}(u) : U \\ &\rightarrow [0, 1] \end{aligned} \tag{2}$$

and

$$0 < \mu_{A_p}^{\sim}(u) + \mathcal{I}_{A_p}^{\sim}(u) + v_{A_p}^{\sim}(u) < 1 \quad \forall u \in U \tag{3}$$

where $\mu_{A_p}^{\sim}(u)$, $\mathcal{I}_{A_p}^{\sim}(u)$ and $v_{A_p}^{\sim}(u)$ are the degree of membership, non-membership, and indeterminacy of u to \tilde{A}_p ,

respectively. Also, the degree of refusal of u can be defined as: $\chi_{A_p}^{\sim} = 1 - \left(\mu_{A_p}^{\sim}(u) + \mathcal{I}_{A_p}^{\sim}(u) + v_{A_p}^{\sim}(u) \right)$.

Definition 2 Basic operations of PFS can be defined as (Eqs. 4, 5, 6 and 7):

- Addition

$$\tilde{A}_p \oplus \tilde{B}_p = \left\{ \mu_{A_p}^{\sim} + \mu_{B_p}^{\sim} - \mu_{A_p}^{\sim} \mu_{B_p}^{\sim}, \mathcal{I}_{A_p}^{\sim} \mathcal{I}_{B_p}^{\sim}, v_{A_p}^{\sim} v_{B_p}^{\sim} \right\}. \tag{4}$$

- Multiplication

$$\tilde{B}_p \otimes \tilde{A}_p = \left\{ \mu_{A_p}^{\sim} \mu_{B_p}^{\sim}, \mathcal{I}_{A_p}^{\sim} \mathcal{I}_{B_p}^{\sim}, v_{A_p}^{\sim} + v_{B_p}^{\sim} - v_{A_p}^{\sim} v_{B_p}^{\sim} \right\}. \tag{5}$$

- Multiplication by a scalar; $\lambda > 0$

$$\lambda \cdot \tilde{A}_p = \left\{ \left(1 - \left(1 - \mu_{A_p}^{\sim} \right)^\lambda \right), \mathcal{I}_{A_p}^{\sim \lambda}, v_{A_p}^{\sim \lambda} \right\}. \tag{6}$$

- Power of \tilde{A}_p ; $\lambda > 0$

$$\tilde{A}_p^\lambda = \left\{ \mu_{A_p}^{\sim \lambda}, \mathcal{I}_{A_p}^{\sim \lambda}, \left(1 - \left(1 - v_{A_p}^{\sim} \right)^\lambda \right) \right\}. \tag{7}$$

Definition 3 Picture fuzzy weighted geometric (PFWG) mean given that, $w = \{w_1, w_2, \dots, w_n\}$, $w \in [0, 1]$, $\sum_{j=1}^n w_n = 1$ can be defined as one of the below equations (Eqs. 8 and 9).

$$PFWG_w(\tilde{A}_1, \dots, \tilde{A}_n) = \left\{ \prod_{j=1}^n \mu_{A_j}^{w_j}, \prod_{j=1}^n \mathcal{I}_{A_j}^{w_j}, 1 - \prod_{j=1}^n \left(1 - v_{A_j}^{\sim} \right)^{w_j} \right\} \tag{8}$$

$$PFWG_w(\tilde{A}_1, \dots, \tilde{A}_n) = \left\{ 1 - \prod_{j=1}^n \left(1 - \mu_{A_j}^{\sim} \right)^{w_j}, \prod_{j=1}^n \mathcal{I}_{A_j}^{w_j}, 1 - \prod_{j=1}^n v_{A_j}^{w_j} \right\}. \tag{9}$$

Definition 4 To defuzzify, rank, and compare the PFS sets, the below score (S) and accuracy (A) functions can be used (Eqs. 10, 11, 12 and 13):

$$S_1(\tilde{A}_p) = 0.5 \left(1 + 2\mu_{A_p}^{\sim} - v_{A_p}^{\sim} - \frac{\mathcal{I}_{A_p}^{\sim}}{2} \right) \tag{10}$$

$$S_2(\tilde{A}_p) = \left(2\mu_{A_p}^{\sim} - v_{A_p}^{\sim} - \frac{\mathcal{I}_{A_p}^{\sim}}{2} \right) \tag{11}$$

$$S_3(\tilde{A}_p) = \left(\mu_{\tilde{A}_p} - v_{\tilde{A}_p} \right) \tag{12}$$

$$A(\tilde{A}_p) = \mu_{\tilde{A}_p}(u) + \mathcal{I}_{\tilde{A}_p}(u) + v_{\tilde{A}_p}(u). \tag{13}$$

After calculation of the score (S) and accuracy (A) functions, the dominance rules can be as follows:

- if $S(\tilde{A}_p) > S(\tilde{B}_p)$, then $\tilde{A}_p > \tilde{B}_p$
- if $S(\tilde{A}_p) = S(\tilde{B}_p)$ and $A(\tilde{A}_p) > A(\tilde{B}_p)$, then $\tilde{A}_p > \tilde{B}_p$
- if $S(\tilde{A}_p) = S(\tilde{B}_p)$ and $A(\tilde{A}_p) < A(\tilde{B}_p)$, then $\tilde{A}_p < \tilde{B}_p$
- if $S(\tilde{A}_p) = S(\tilde{B}_p)$ and $A(\tilde{A}_p) = A(\tilde{B}_p)$, then $\tilde{A}_p = \tilde{B}_p$

There are two stages in the suggested hybrid technique. In the first stage, we use the picture fuzzy AHP approach to determine the criteria weights. The ranking of options is determined in the second stage using the picture fuzzy linear assignment method.

Stage 1: Picture fuzzy-analytic hierarchy process (PF-AHP).

- The weights of criteria, using the pair wise comparison matrices, are calculated.

In this step, the decision-makers compare the criteria relative to each other. Based on their role, they chose a value from Saaty’s nine-point scale. Then the value is transferred to corresponding picture fuzzy numbers (Table 2). Because the values in Table 2 were chosen based on expert preferences, it’s important to double-check the consistency ratio (CR) of each pair-wise comparison, which is calculated using the consistency index (CI) and random index (RI), as shown below (Eqs. 14 and 15):

$$CR = CJ/RJ \tag{14}$$

$$\text{where } CJ = \frac{(\lambda_{max} - n)}{(n - 1)} \tag{15}$$

where λ_{max} is the maximum eigenvalue of each comparison

matrix, n represents the number of criteria.

The value of RI is calculated based on randomly generating 500 matrices (Saaty 1999). The judgments of decision-makers are consistent when the value of CR is lower than 0.1. For higher values than 0.1, the decision-makers have to change their evaluations to improve the consistency index.

To aggregate decision-makers’ assessments, we used a picture fuzzy weighted geometric (PFWG) mean.

In decision-making problems, there are multiple decision-makers and, subsequently, there can be different comparison matrixes (\tilde{w}_j^{local}) . For the next stages, it is necessary to use geometric means (Eq. 16) to unify all comparison matrixes (\tilde{w}_j^{global}) . Eventually, it is necessary to calculate the final picture fuzzy weight (\tilde{w}_j^{final}) as follows (Eq. 16):

$$(\tilde{w}_j^{final}) = \tilde{w}_j^{local} \otimes \tilde{w}_j^{global}. \tag{16}$$

- Deffuzification of (\tilde{w}_j^{final}) .

In this step, to use (\tilde{w}_j^{final}) of criteria as a weight for ranking the sub-watersheds (Stage 2), it is necessary to export (\tilde{w}_j^{final}) as deffuzified value.

Stage 2: Picture fuzzy linear assignment model (PF-LAM).

- Ranking the alternatives based on the PFS

Table 2 Related Satty’s scale and Picture Fuzzy Numbers (PFNs) for linguistic terms

| Linguistic terms | Saaty’s scale | Picture fuzzy numbers (PFNs) |
|----------------------------|---------------|------------------------------|
| Absolutely more importance | 9 | (0.9,0.1,0.0) |
| Very more importance | 7 | (0.8,0.2,0.1) |
| More importance | 5 | (0.7,0.3,0.2) |
| Slightly more importance | 3 | (0.6,0.4,0.3) |
| Equally importance | 1 | (0.5,0.4,0.4) |
| Slightly low importance | 1/3 | (0.4,0.6,0.3) |
| Low importance | 1/5 | (0.3,0.7,0.2) |
| Very low importance | 1/7 | (0.2,0.8,0.1) |
| Absolutely low importance | 1/9 | (0.1,0.9,0) |

Intermediate values: used to represent compromise between the priorities listed above

This point is similar to point 1 from stage 1. However, the difference is that individual judgments of decision-makers are based on alternatives in the form of decision-matrices (Table 3).

- Based on Eq. 8, the individual decision-matrices from the previous point were aggregated, as presented in Table 4.
- Defuzzification of the aggregated matrix, using Eq. 10, to compare alternatives related to each other and their ranking.
- The dominance of alternative m on the nth criterion is demonstrated by determining the rank frequency matrix λ with associated elements (Table 5).
- Determination of the weighted rank frequency matrix Π_{ik}, which measures the contribution of the m_{th} alternative to the overall ranking (Eqs. 17 and Table 6).

$$\Pi_{ik} = w_{i1} + w_{i2} + \dots + w_{i\lambda_{mm}} \tag{17}$$

- Construction of linear assignment model based on Π_{ik} and permutation matrix P(m*m) as follows (Eq. 18):

$$\max \sum_{i=1}^m \sum_{k=1}^m \Pi_{ik} \cdot P_{ik} \tag{18}$$

$$s.t. \sum_{k=1}^m P_{ik} = 1, \forall i = 1, 2, \dots, m$$

$$\sum_{i=1}^m P_{ik} = 1, \forall i = 1, 2, \dots, m$$

P_{ik} = 0 or 1 for all i and k.

- Using Eq. 18 to obtain optimal permutation matrix (P*).
- Obtaining the rank of alternatives using multiplication

$$\text{of } P^* \otimes A = P^* \otimes \begin{bmatrix} A_1 \\ A_2 \\ \dots \\ A_m \end{bmatrix}.$$

Table 3 Individual judgments of decision-makers

| Alternative | Criteria | | | |
|----------------|-------------------------------|-------------------------------|-----|-------------------------------|
| | C ₁ | C ₂ | ... | C _n |
| A ₁ | PF ₁₁ ^k | PF ₁₂ ^k | ... | PF _{1n} ^k |
| A ₂ | PF ₂₁ ^k | PF ₂₂ ^k | ... | PF _{2n} ^k |
| ... | ... | ... | ... | ... |
| A _m | PF _{m1} ^k | PF _{m2} ^k | ... | PF _{mn} ^k |

Superscript k refers to k decision maker

Table 4 Aggregated judgments of decision-makers

| Alternative | Criteria | | | |
|----------------|--------------------|--------------------|-----|--------------------|
| | C ₁ | C ₂ | ... | C _n |
| A ₁ | PFWG ₁₁ | PFWG ₁₂ | ... | PFWG _{1n} |
| A ₂ | PFWG ₂₁ | PFWG ₂₂ | ... | PFWG _{2n} |
| ... | ... | ... | ... | ... |
| A _m | PFWG _{m1} | PFWG _{m2} | ... | PFWG _{mn} |

Table 5 Rank frequency matrix λ

| Alternative | Rank | | | |
|----------------|-----------------|-----------------|-----|-----------------|
| | 1st | 2st | ... | mth |
| A ₁ | λ ₁₁ | λ ₁₂ | ... | λ _{1m} |
| A ₂ | λ ₂₁ | λ ₂₂ | ... | λ _{2m} |
| ... | ... | ... | ... | ... |
| A _m | λ _{m1} | λ _{m2} | ... | λ _{mm} |

Table 6 Weighted rank frequency matrix Π

| Alternative | Rank | | | |
|----------------|-----------------|-----------------|-----|-----------------|
| | 1st | 2st | ... | mth |
| A ₁ | Π ₁₁ | Π ₁₂ | ... | Π _{1m} |
| A ₂ | Π ₂₁ | Π ₂₂ | ... | Π _{2m} |
| ... | ... | ... | ... | ... |
| A _m | Π _{m1} | Π _{m2} | ... | Π _{mm} |

3 Results and discussions

3.1 Morphometric analysis

Any geomorphometric investigation should begin with the identification of streams and their classification using any method such as Horton’s, Hargriv’s, or Strahler’s. In this study, the Strahler (1964) technique was used to arrange streams in the 14 sub-watersheds (Fig. 1).

Sub-watersheds 1, 6, 9, 10, 13, and 14 are long, while sub-watersheds 3, 4, 5, 7, and 11 are short and sub-watersheds 1, 8, and 12 are oval (Table 2). In an elongated basin, the hydrograph of stream flow or discharge has a smooth contour, indicating that water will take longer to travel from the watershed’s most remote point to its outlet. The compactness coefficient values for sub-watersheds 12 and 14 are 1.149 and 1.430, respectively, indicating that sub-watershed 14 is more compact than sub-watershed 12.

Sub-watershed 14 has the highest drainage density value ($D_d = 4.994 \text{ km/km}^2$). The drainage density figure for sub-watershed 6 ($D_d = 2.454 \text{ km/km}^2$) is the lowest. The bulk of sub-watersheds 1–5 and 7–13 have D_d values that are similar. Sub-watershed 6 has a low D_d value, indicating that the subsoil stratum is porous. The research area’s sub-watershed 14 has the greatest D_d score, indicating that it has a well-developed stream network. Sub-watersheds 6 and 5 had stream frequencies of 3.817 and 8.016 number/km, respectively, indicating that the watershed with the highest stream frequency has a well-developed stream network and has a greater impact on severe soil erosion. The texture ratio (T) of all sub-watersheds ranged from 2.859 (in sub-watershed 6) to 6.270 (in sub-watershed 6). In all of the Gusru watershed’s sub-watersheds, total relief (H) varies between 90 and 227 m. The relative relief (Rr) ranged from 0.006 (sub-watershed 1) to 0.017 (sub-watershed 2). The lowest and highest values of the roughness number (RN) in this study were 0.304 for sub-watershed 1 and 1.134 for sub-watershed 14. The rougher the watershed and thus the greater soil loss, the higher the RN value. The slope range of the Gusru river watersheds was discovered to be between 7.089 percent (sub-watershed 1) to 20.416 percent (sub-watershed 13). Sub-watersheds 2, 4, 9, and 11 are in an equilibrium/youthful stage, while sub-watersheds 1, 3, 5, 6, 7, 8, 10, 12, and 13 are mature, and sub-watershed 14 is monodnock.

3.2 Erodibility criteria for sub-watershed prioritization by PF-AHP and PF-LAM models

This strategy’s main features are its consistency and application, with a focus on watershed priority criteria. To rank, accept, reject, and assess a large number of best possibilities, the MCDM approach is commonly employed. Because both the PF-AHP and PF-LAM methods are MCDM ways for weighting criteria, they were used in this investigation.

This phase involved assigning weights to the selected morphometric criteria and ranking the sub-watersheds based on LAM. In this study, two decision-makers (DM) were consulted their opinions based on their research background and work experience in the study area. First, according to Table 2 and picture fuzzy, each decision-makers pair-wise comparison matrix of morphometric parameters was prepared. The consistency judgments, based on Eq. 14, were obtained. Then, using Eq. 8, i.e., subject to $w_j = \frac{1}{n} \rightarrow \frac{1}{14}$ (n is the number of morphometric parameters used), the local weight ($\tilde{w}_j^{\text{local}}$) of each criterion regarding each decision-maker was calculated. In addition, the global weight ($\tilde{w}_j^{\text{global}}$) of criteria using

Eq. 8, i.e., subject to $w_{jj} = \frac{1}{k} \rightarrow \frac{1}{3}$ (k in here is the number of decision-makers), was also calculated. Eventually, the final weight ($\tilde{w}_j^{\text{final}}$) of each criterion, using Eq. 16 was computed (Table 7).

In the following, the judgments of the mentioned decision-makers were also implemented in the PF-LAM. The selected decision-makers were used in stage 2 to provide the weights of each criterion (based on their role in flood susceptibility using Table 8 for each sub-watershed (alternative)). Then, based on Eq. 8, the consensus decision matrix was calculated using the integration of individual decision matrices. To rank the alternatives and establish the frequency matrix, Eq. 10 was used to derive the defuzzied values of alternatives (Tables 9, 10, 11).The defuzzied weight frequency rank matrix was subsequently produced (Table 12).

In the final phase, we develop the linear assignment model as follows. The goal function aims to maximize the sum of our options’ total weights by ordering them. Zero values in the scored matrix, on the other hand, imply that this rank has never had a substitute. To prevent assigning alternative values, we’ve used huge negative integers (e.g., -100) instead of zero values.

$$\begin{aligned} \text{Max } z &= -100 \text{ SW}_{11} + 0.73\text{SW}_{12} + 0.59\text{SW}_{13} + \dots + 2.4\text{SW}_{114} \\ &+ 0.59\text{SW}_{21} + 2.67\text{SW}_{22} - 100\text{SW}_{23} + \dots - 100\text{SW}_{214} \\ &+ \dots + 1.26\text{SW}_{141} + 1.35\text{SW}_{142} + \dots + 2.19\text{SW}_{1414} \\ &\text{s.t.} \\ &\text{SW}_{11} + \text{SW}_{12} + \text{SW}_{13} + \dots + \text{SW}_{114} = 1 \\ &\dots \\ &\text{SW}_{141} + \text{SW}_{142} + \dots + \text{SW}_{1414} = 1 \end{aligned}$$

By solving the above equation, the following permutation matrix was derived:

$$P^* = \begin{matrix} \text{SW}_1 \\ \text{SW}_2 \\ \text{SW}_3 \\ \text{SW}_4 \\ \text{SW}_5 \\ \dots \\ \dots \\ \dots \\ \text{SW}_{14} \end{matrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally, multiplying the permutation matrix in a vector of alternatives, i.e. $[\text{SW}_1, \text{SW}_2, \dots, \text{SW}_{14}]$, the ranking of options was determined based on their flooding susceptibility.

Table 7 Calculated weights of morphometric criteria used

| Criteria | Local weight | Global weight | Final weight | Deffuzification (score) |
|----------|---|------------------|------------------|-------------------------|
| R_h | (0.75,0.24,0.14) (0.85,0.14,0,0.05) (0.7,0.3,0.2) | (0.6,0.4,0.3) | (0.51,0.16,0.35) | 0.58 |
| R_r | (0.7,0.3,0.2) (0.76,0.24,0.14) (0.65,0.34,0.24) | (0.65,0.34,0.24) | (0.52,0.16,0.33) | 0.63 |
| R_N | (0.65,0.34,0.24) (0.7,0.3,0.2) (0.6,0.4,0.3) | (0.7,0.3,0.2) | (0.54,0.15,0.35) | 0.67 |
| ... | ... | ... | ... | 0.59 |
| C_c | (0.6,0.4,0.3) (0.65,0.34,0.24) (0.55,0.4,0.34) | (0.75,0.24,0.14) | (0.55,0.13,0.3) | 0.73 |
| S_a | (0.55,0.4,0.34) (0.6,0.4,0.3) (0.5,0.4,0.4) | (0.7,0.3,0.2) | (0.48,0.17,0.38) | 0.5 |
| HI | (0.85,0.14,0.05) (0.9,0.1,0) (0.8,0.42,0.1) | (0.85,0.14,0.05) | (0.77,0.04,0.07) | 1.45 |

Table 8 Calculated weights of alternatives (sub-watersheds) based on morphometric parameters

| Sub-watershed | R_h | R_r | R_N | ... | C_c | S_a | HI |
|---------------|------------------|------------------|------------------|-----|------------------|------------------|------------------|
| SW 1 | (0,1,1) | (0,1,1) | (0,1,1) | | (0.71,0.56,0.19) | (0,1,1) | (0.56,0.66,0.29) |
| SW 2 | (0.1,0.95,0.87) | (0.33,0.84,0.60) | (0.19,0.91,0.76) | | (0.44,0.74,0.47) | (0.19,0.91,0.76) | (0.88,0.34,0.04) |
| SW 3 | (0.16,0.92,0.8) | (0.31,0.85,0.62) | (0.17,0.92,0.79) | | (0.47,0.75,0.44) | (0.09,0.95,0.88) | (0.78,0.47,0.10) |
| SW 4 | (0.19,0.91,0.76) | (0.50,0.74,0.41) | (0.29,0.86,0.64) | | (0.67,0.60,0.23) | (0.46,0.76,0.45) | (0.86,0.37,0.05) |
| . | ... | ... | ... | | ... | ... | ... |
| SW 11 | (0.37,0.81,0.56) | (0.7,0.59,0.22) | (0.58,0.67,0.33) | | (0.61,0.64,0.28) | (0.55,0.69,0.35) | (0.9,0.3,0.03) |
| SW 12 | (0.51,0.73,0.41) | (0.7,0.59,0.22) | (0.53,0.71,0.38) | | (0.84,0.42,0.09) | (0.8,0.47,0.12) | (0.63,0.6,0.22) |
| SW 13 | (0.73,0.56,0.19) | (0.64,0.63,0.27) | (0.50,0.73,0.42) | | (0.16,0.92,0.79) | (0.68,0.6,0.23) | (0.44,0.74,0.41) |
| SW 14 | (0.6,0.66,0.31) | (0.66,0.62,0.25) | (0.77,0.52,0.15) | | (0,1,1) | (0.35,0.82,0.58) | (0,1,1) |

Table 9 Defuzzified values of alternatives

| Sub-watershed | R_h | R_r | R_N | ... | C_c | S_a | HI |
|---------------|---------|---------|---------|-----|---------|---------|---------|
| SW 1 | - 1.5 | - 1.5 | - 1.5 | | 0.951 | - 1.500 | 0.492 |
| SW 2 | - 1.140 | - 0.358 | - 0.832 | | 0.019 | - 0.840 | 1.553 |
| SW 3 | - 0.942 | - 0.431 | - 0.912 | | 0.128 | - 1.170 | 1.214 |
| SW 4 | - 0.841 | 0.214 | - 0.492 | | 0.822 | 0.090 | 1.497 |
| . | ... | ... | ... | | ... | ... | ... |
| SW 11 | - 0.227 | 0.886 | 0.494 | | 0.618 | 0.407 | 1.631 |
| SW 12 | 0.241 | 0.883 | 0.321 | | 1.377 | 1.254 | 0.738 |
| SW 13 | 1.001 | 0.701 | 0.211 | | - 0.926 | 0.843 | 0.107 |
| SW 14 | 0.567 | 0.773 | 1.121 | | - 1.500 | - 0.292 | - 1.500 |

Table 10 Ranking of alternatives

| Position | R_h | R_r | R_N | R_b | D_d | F_s | R_c | R_f | R_e | T | L_o | C_c | S_a | HI |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1th | SW 13 | SW 9 | SW 14 | SW 2 | SW 14 | SW 5 | SW 10 | SW 13 | SW 13 | SW 5 | SW 6 | SW 12 | SW 12 | SW 11 |
| 2th | SW 14 | SW 11 | SW 8 | SW 10 | SW 13 | SW 14 | SW 14 | SW 2 | SW 2 | SW 8 | SW 2 | SW 1 | SW 8 | SW 2 |
| 3th | SW 12 | SW 12 | SW 9 | SW 1 | SW 8 | SW 10 | SW 5 | SW 9 | SW 9 | SW 12 | SW 10 | SW 4 | SW 13 | SW 4 |
| 4th | SW 8 | SW 14 | SW 11 | SW 4 | SW 5 | SW 8 | SW 6 | SW 10 | SW 10 | SW 10 | SW 5 | SW 9 | SW 9 | SW 9 |
| 5th | SW 9 | SW 13 | SW 12 | SW 14 | SW 10 | SW 13 | SW 8 | SW 6 | SW 6 | SW 4 | SW 4 | SW 11 | SW 11 | SW 8 |
| 6th | SW 11 | SW 8 | SW 5 | SW 9 | SW 1 | SW 4 | SW 13 | SW 14 | SW 14 | SW 1 | SW 9 | SW 8 | SW 5 | SW 3 |
| 7th | SW 7 | SW 4 | SW 13 | SW 8 | SW 9 | SW 9 | SW 2 | SW 5 | SW 5 | SW 14 | SW 11 | SW 3 | SW 4 | SW 6 |
| 8th | SW 6 | SW 6 | SW 4 | SW 5 | SW 4 | SW 12 | SW 3 | SW 3 | SW 3 | SW 9 | SW 3 | SW 5 | SW 14 | SW 5 |
| 9th | SW 4 | SW 7 | SW 10 | SW 6 | SW 12 | SW 1 | SW 7 | SW 7 | SW 7 | SW 2 | SW 1 | SW 2 | SW 10 | SW 7 |
| 10th | SW 3 | SW 10 | SW 2 | SW 3 | SW 2 | SW 2 | SW 11 | SW 11 | SW 11 | SW 13 | SW 8 | SW 7 | SW 2 | SW 12 |
| 11th | SW 10 | SW 2 | SW 3 | SW 13 | SW 11 | SW 7 | SW 9 | SW 4 | SW 4 | SW 11 | SW 7 | SW 6 | SW 7 | SW 10 |
| 12th | SW 5 | SW 3 | SW 7 | SW 7 | SW 3 | SW 11 | SW 1 | SW 12 | SW 12 | SW 3 | SW 12 | SW 13 | SW 3 | SW 1 |
| 13th | SW 2 | SW 5 | SW 6 | SW 11 | SW 7 | SW 3 | SW 4 | SW 1 | SW 1 | SW 7 | SW 14 | SW 10 | SW 6 | SW 13 |
| 14th | SW 1 | SW 1 | SW 1 | SW 12 | SW 6 | SW 6 | SW 12 | SW 8 | SW 8 | SW 6 | SW 13 | SW 14 | SW 1 | SW 14 |

Table 11 Rank frequency of alternatives

| Sub-watershed | 1th | 2th | 3th | 4th | 5th | 6th | 7th | 8th | 9th | 10th | 11th | 12th | 13th | 14th |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|------|------|------|------|
| SW 1 | 0 | 1 | 1 | 0 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 2 | 2 | 4 |
| SW 2 | 1 | 4 | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 4 | 1 | 0 | 1 | 0 |
| SW 3 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 4 | 0 | 2 | 1 | 4 | 1 | 0 |
| SW 4 | 0 | 0 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 0 | 2 | 0 | 1 | 0 |
| SW 5 | 2 | 0 | 1 | 2 | 0 | 2 | 2 | 3 | 0 | 0 | 0 | 1 | 1 | 0 |
| SW 6 | 1 | 0 | 0 | 1 | 2 | 0 | 1 | 2 | 1 | 0 | 1 | 0 | 2 | 3 |
| SW 7 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 5 | 1 | 3 | 2 | 2 | 0 |
| SW 8 | 0 | 3 | 1 | 2 | 2 | 2 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 2 |
| SW 9 | 1 | 0 | 3 | 3 | 1 | 2 | 2 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| SW 10 | 1 | 1 | 2 | 3 | 1 | 0 | 0 | 0 | 2 | 1 | 2 | 0 | 1 | 0 |
| SW 11 | 1 | 1 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 3 | 2 | 1 | 1 | 0 |
| SW 12 | 2 | 0 | 3 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 3 | 0 | 2 |
| SW 13 | 3 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| SW 14 | 2 | 3 | 0 | 1 | 1 | 2 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 2 |

Table 12 Defuzzified weight frequency rank matrix

| Sub-watershed | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | |
|---------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| SW 1 | – 100.00 | 0.59 | 0.74 | 0.59 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | 0.74 | – 100.00 | – 100.00 | 1.97 | 0.74 | 2.41 |
| SW 2 | 0.59 | – 100.00 | 2.67 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | 0.51 | – 100.00 | 0.86 | 2.04 | 0.64 | – 100.00 | 0.59 | – 100.00 |
| SW 3 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | 1.46 | 0.74 | 1.72 | – 100.00 | 1.18 | 0.68 | 1.85 | 0.26 | – 100.00 |
| SW 4 | – 100.00 | – 100.00 | – 100.00 | 2.20 | 0.59 | 0.59 | 0.26 | 1.14 | 1.26 | – 100.00 | – 100.00 | 0.74 | – 100.00 | 0.51 | – 100.00 |
| SW 5 | 0.38 | – 100.00 | – 100.00 | 0.51 | 1.06 | – 100.00 | 1.18 | 0.74 | 2.79 | – 100.00 | – 100.00 | – 100.00 | 0.59 | 0.64 | – 100.00 |
| SW 6 | 0.48 | – 100.00 | – 100.00 | – 100.00 | 0.51 | 0.74 | – 100.00 | 1.46 | 1.22 | 0.59 | – 100.00 | 0.74 | – 100.00 | 1.18 | 0.97 |
| SW 7 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | 0.59 | – 100.00 | 3.34 | 0.74 | 1.25 | 1.27 | 0.71 | – 100.00 |
| SW 8 | – 100.00 | – 100.00 | 1.30 | 0.59 | 0.85 | 1.97 | 1.37 | 0.59 | – 100.00 | – 100.00 | 0.48 | – 100.00 | – 100.00 | – 100.00 | 0.74 |
| SW 9 | 0.64 | – 100.00 | – 100.00 | 1.41 | 2.70 | 0.59 | 1.07 | 0.85 | 0.12 | – 100.00 | – 100.00 | 0.51 | – 100.00 | – 100.00 | – 100.00 |
| SW 10 | 0.51 | – 100.00 | 0.59 | 0.74 | 0.86 | 0.59 | – 100.00 | – 100.00 | – 100.00 | 1.18 | 0.64 | 2.05 | – 100.00 | 0.74 | – 100.00 |
| SW 11 | 1.46 | – 100.00 | – 100.00 | – 100.00 | 0.68 | 1.25 | 0.59 | 0.48 | – 100.00 | – 100.00 | 1.25 | 0.71 | 0.26 | 0.59 | – 100.00 |
| SW 12 | 1.25 | – 100.00 | – 100.00 | 1.34 | – 100.00 | 0.68 | – 100.00 | – 100.00 | 0.26 | 0.59 | 1.46 | – 100.00 | 1.22 | – 100.00 | 1.10 |
| SW 13 | 1.33 | 0.59 | – 100.00 | 0.51 | – 100.00 | 0.90 | 0.51 | 0.68 | – 100.00 | – 100.00 | 0.12 | 0.59 | 0.74 | 1.46 | 0.48 |
| SW 14 | 1.26 | – 100.00 | – 100.00 | – 100.00 | 0.64 | 0.59 | 0.74 | 0.12 | 0.51 | – 100.00 | – 100.00 | – 100.00 | – 100.00 | 0.48 | 2.20 |

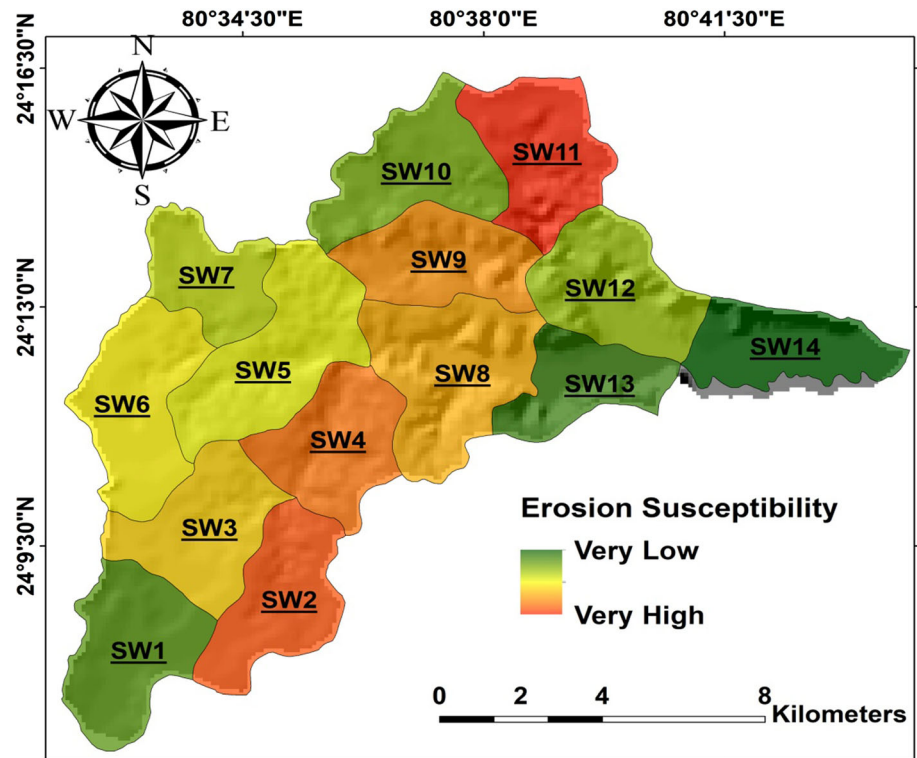
$$\text{Prioritizationrank} = \begin{bmatrix} SW_1 \\ SW_2 \\ SW_3 \\ SW_4 \\ SW_5 \\ SW_6 \\ SW_7 \\ SW_8 \\ SW_9 \\ SW_{10} \\ SW_{11} \\ SW_{12} \\ SW_{13} \\ SW_{14} \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} SW_{11} \\ SW_2 \\ SW_4 \\ SW_9 \\ SW_8 \\ SW_3 \\ SW_6 \\ SW_5 \\ SW_7 \\ SW_{12} \\ SW_{10} \\ SW_1 \\ SW_{13} \\ SW_{14} \end{bmatrix}$$

The methodology’s outcomes are depicted in Fig. 3. The most vulnerable sub-watersheds to soil erosion are sub-watersheds 11 and 2, as seen in this diagram. Sub-watersheds 13 and 14, on the other hand, exhibited the least sensitivity to soil erosion.

Evaluation of morphometric characteristics of sub-watersheds and topo-hydrological parameters concerning soil erosion is a key issue to rank and identify the critical sub-watersheds and, subsequently, the watershed management strategies. Since MCDA techniques can be used to rank and determine the most suitable alternatives, they have been at the forefront of prioritizing sub-watersheds regarding floods, erosion, landslides, or other natural hazards in recent years.

According to Smithson (1989), Sepehri et al. (2020), and Malekinezhad et al. (2021), subjectivity and objectivity are two main aspects of MCDA techniques. The main difference between these classes regards the issue of assigning weights of criteria, which are subjectively

Fig. 3 Soil erosion susceptibility map



assigned based on judgments of decision-makers. The judgments are the function of the scientific/experience work and mental states of decision-makers while, at the same time, arising the uncertainty of the MCDA technique. On the other hand, the objective methods used the natural distribution for assigning weights to the criteria. However, this issue raises one of the main weaknesses of this aspect: ignoring the relative importance of criteria to each other.

As a result, one of the study's main advantages is the combination of subjective and objective methods, which is related to the use of picture fuzzy logic, which is one of the most recent extensions of fuzzy logic to reduce judgment uncertainty by considering membership non-membership and refusal membership of criteria. Furthermore, unlike the other MCDA techniques mentioned in the Introduction section, this study's methodology uses the variability property to assign weights, i.e., for each criterion, there is an overall weight for all sub-watersheds and a unique weight for each sub-watershed based on its properties.

4 Conclusion

Soil erosion is a major cause of land loss in agricultural lands and is a major environmental hazard around the world. Fourteen of the most effective morphometric metrics for measuring soil erosion were chosen for this study. Using the PF-AHP and PF-LAM techniques, these characteristics were weighted based on their erosion effects. Soil erosion is particularly prevalent in sub-watersheds 11 and 2. Sub-watershed 14 is the least vulnerable to soil erosion. The integration of photo fuzzy with AHP and LAM has been proved to be a powerful technique for investigating soil erosion.

The picture fuzzy approach can help with other areas of MCDM. The adoption of the interval rough technique in conjunction with well-established decision-making models would significantly reduce the complexity and subjectivity of decision-making, particularly in group judgments.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethics approval and consent to participate Not applicable.

Consent for publication Not applicable.

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