

Risk analysis for water resources management under dual uncertainties through factorial analysis and fuzzy random value-at-risk

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Abstract Water resources systems are associated with a variety of complexities and uncertainties due to socio-economic and hydro-environmental impacts. Such complexities and uncertainties lead to challenges in evaluating the water resources management alternatives and the associated risks. In this study, the factorial analysis and fuzzy random value-at-risk are incorporated into a two-stage stochastic programming framework, leading to a factorial-based two-stage programming with fuzzy random value-at-risk (FTSPF). The proposed FTSPF approach aims to reveal the impacts of uncertainty parameters on water resources management strategies and the corresponding risks. In detail, fuzzy random value-at-risk is to reflect the potential risk about financial cost under dual uncertainties, while a multi-level factorial design approach is used to reveal the interaction between feasibility degrees and risk levels, as well as the relationships (including curvilinear relationship) between these factors and the responses. The application of water resources system planning makes it

possible to balance the satisfaction of system benefit, the risk levels of penalty and the feasibility degrees of constraints. The results indicate that decision makers would pay more attention to the tradeoffs between the system benefit and feasibility degree, and the water allocation for agricultural section contributes most to control the financial loss of water. Moreover, FTSPF can generate a higher system benefit and more alternatives under various risk levels. Therefore, FTSPF could provide more useful information for enabling water managers to identify desired policies with maximized system benefit under different system-feasibility degrees and risk levels.

Keywords Fuzzy random value-at-risk · Decision analysis · Uncertainty modelling · Water resources · Multi-level factorial design

1 Introduction

In water-resources system planning, uncertainties exist in many system components and parameters due to different kinds of complexities involving social, economic, environmental, political and technical factors (Loucks et al. 1981; Brink et al. 2008; Jing et al. 2016; Chalh et al. 2015; Kong et al. 2015a). For example, the economic parameters (e.g. net benefit, penalty and water loss rate during transportation) and the stream conditions (e.g. stream flow) may exhibit extensive uncertainties resulting from temporal-spatial variations in socio economic and natural systems (Marques et al. 2005; Khan and Valeo 2016; Mishra et al. 2016; Lv et al. 2010). These uncertainties in term of information quality have placed numbers of difficulties in exploring appropriate water allocation strategies in water resources systems.

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Inexact optimization methods have been developed for helping decision makers manage water resources in more efficient and environment-friendly manners under uncertainties (Verderame et al. 2010; Li et al. 2008; Beraldi et al. 2000; Nematian 2016). The interval mathematical programming, the fuzzy mathematical programming and the stochastic mathematical programming are the primary inexact optimization techniques for tackling uncertainties in water resources system planning (Huang et al. 2006; Li and Huang 2008, 2009; Karmakar and Mujumdar 2006; Singh 2012; Aggarwal and Hanmandlu 2015). For example, the inexact two-stage stochastic programming (ITSP) method proposed by Huang and Loucks (2000) has received amounts of attentions. The advantage of ITSP is that corrective actions could be undertaken by decision makers after a random event has taken place. Housh et al. (2013) investigated a limited multi-stage stochastic programming (LMSP) method, which could reduce computation for optimization problems under uncertainty through identifying and classifying decision nodes. Fan et al. (2015) developed a generalized fuzzy two-stage stochastic programming (GFTSP) method and applied it to tackle uncertainties presented as probability distributions, fuzzy sets, as well as fuzzy random variables. Solutions generated through the GFTSP method could reflect the fluctuating ranges of decision alternatives under different plausibilities. These optimization methods mainly concentrated on dealing with uncertainties presented as intervals, fuzzy sets and random numbers. However, water-related activities are often associated with many potential risks due to the limited water-resource availability, the diversity of management methods, and the restriction of financial loss etc. Therefore, identification and quantification of risks associated with different water management alternatives are desired.

Therefore, this study aims to develop a factorial-based two-stage programming with fuzzy random value-at-risk (FTSPF) for tackling the dual uncertainties and revealing the resulting risks as well as the interaction among parameters. The proposed FTSPF integrates the interactive two-stage stochastic fuzzy programming method (ITSFP), the fuzzy random value at risk (FVaR) and multi-level factorial analysis into a framework: (a) FTSPF will be proposed for dealing with multiple uncertainties in water resources management, (b) FVaR will then be introduced to reveal the risk resulting from the fuzziness and randomness, and (c) the multi-level factorial design (MFD) will be employed to identify the main factors and their interaction on the decision alternatives and the associated risk values.

2 Modeling

2.1 Interactive two-stage stochastic fuzzy programming

Allocating water resources to multiple water users efficiently is quite necessary due to limited availability of fresh water. An inexact two-stage stochastic programming (ITSP) model can be applicable to generate water allocation plans for decision makers in an interval and random environment (Huang and Loucks 2000; Wang and Huang 2011). In an ITSP model, the water allocation target to each water user could be regulated based on the regional water resources planning policies (Li et al. 2010). The purposes of these policies are: (1) to promote the sustainable planning of regional water resources, (2) to address utilization and protection of regional water resources (Otago Regional Council 2014). The ITSP approach can also deal with uncertainties presented as intervals and random variables in both objective function and constraints. However, there are two disadvantages in ITSP. One is that ITSP can hardly reflect the vague information from the subjective estimations (Wang and Huang 2011). The other is that some parameters and decision variables may not be characterized by merely one uncertainty quantification approach due to severe complexities in water resources management. These dual or multiple uncertainties cannot be treated through the ITSP method. In order to address the above issues, an interactive two-stage stochastic fuzzy programming (ITSFP) approach for water resources planning was proposed by Wang and Huang (2011):

$$\text{Maximize } \tilde{f}^{\pm} = \sum_{i=1}^m \tilde{B}_i^{\pm} X_i^{\pm} - \sum_{i=1}^m \tilde{C}_i^{\pm} \left(\sum_{j=1}^n p_j Y_{ij}^{\pm} \right) \quad (1a)$$

Subject to:

$$\sum_{i=1}^m (X_i^{\pm} - Y_{ij}^{\pm}) \left(1 + (1 - \omega) E_1^{\mp} + \omega E_2^{\mp} \right) \leq \omega E_1^{\tilde{q}_j^{\pm}} + (1 - \omega) E_2^{\tilde{q}_j^{\pm}}, \quad \forall j \quad (1b)$$

$$Y_{ij}^{\pm} \leq X_i^{\pm} \leq X_{i\max}^{\pm}, \quad \forall i, j \quad (1c)$$

$$Y_{ij}^{\pm} \geq 0, \quad \forall i, j \quad (1d)$$

where \tilde{f}^{\pm} denotes the objective function value; X_i^{\pm} and $X_{i\max}^{\pm}$ denote the water allocation target and maximum allowable water allocation, respectively; Y_{ij}^{\pm} denotes the water shortage; \tilde{B}_i^{\pm} and \tilde{C}_i^{\pm} denote the net benefit and penalty of net benefit, respectively; $\tilde{\gamma}^{\pm}$ denotes the water loss rate during transportation; \tilde{q}_j^{\pm} denotes the available water resources; E_1^{\mp} and E_2^{\mp} denote the lower and upper bound for the expected value of water loss rate during

transportation $\tilde{\gamma}^\pm$, respectively; $E_1^{\tilde{q}_i^\pm}$ and $E_2^{\tilde{q}_i^\pm}$ denote the lower and upper bound of the expected value of available water resources \tilde{q}_i^\pm , respectively; $i(i = 1, 2, \dots, m)$ is the index for water users; $j(j = 1, 2, \dots, n)$ is the index for the probability p_j ; and ω is the feasibility degree which means the degree of constraint feasibility. An interval number is defined as an interval with known upper and lower bounds but unknown distribution information (Kong et al. 2015b). For example, $x^\pm = [x^-, x^+] = \{t \in x | x^- \leq t \leq x^+\}$ is an interval number which can have the lower bound (minimum value) x^- and the upper bound (maximum value) x^+ . A fuzzy boundary interval is defined as an interval with fuzzy random variables as upper and lower bounds (Wang and Huang 2011). For example, \tilde{x}^- and \tilde{x}^+ are fuzzy random variables, corresponding to the lower and upper bounds of a fuzzy boundary interval \tilde{x}^\pm , respectively.

The ITSFP method can generate solutions under different feasibility degrees through considering the balance between the feasibility degrees of model constraints and acceptability of system benefits. However, the associated risk in the resulting solutions can hardly be revealed by ITSFP. The decision makers may also want to measure the risk of financial loss when the promised allocation targets are not reached. Moreover, the interaction among parameters and the corresponding effects on the system benefits cannot be reflected by ITSFP.

2.2 Fuzzy random value-at-risk

Value-at-risk (VaR) is a single, summary statistical measure of the possible losses on random events (Linsmeier and Pearson 2000; Engle and Manganelli 2004; Yamout et al. 2007; Katagiri et al. 2014; Moazeni et al. 2015). It was first used to characterize the risk value by major financial firms in the late 1980s. For a given risk level $\beta \in (0, 1]$, the corresponding β VaR (i.e. VaR_β) is the threshold at which the probability of a loss exceeding the threshold is equal to $1 - \beta$ (Jabr 2005; Piantadosi et al. 2008; Quaranta and Zaffaroni 2008; Mohammadi 2014; Kim et al. 2015). VaR makes it possible for decision makers to set the probability of a loss and then to find the corresponding threshold, vice versa.

According to Liu (2007; 2009) and Jin (2009), FVaR for the risk at a probability level β can be expressed as follows (see Fig. 1):

$$\xi_{FVaR}(\beta) = \inf\{x | M\{\xi \leq x\} \geq \beta\} \tag{2}$$

where ξ is the fuzzy random variable; β is the risk level; $M(\Lambda)$ is an axiomatic uncertain measure proposed by Liu (2007), which can express the chance at which uncertain event $\Lambda \in L$ occurs; L is a σ -algebra over a nonempty set Γ .

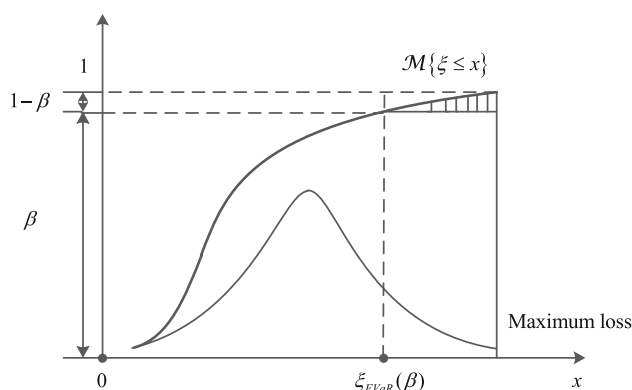


Fig. 1 Fuzzy random value-at-risk (FVaR)

FVaR is a good statistical measure for decision makers to characterize the risk value in a fuzzy random environment. Therefore, it can be widely used in water resources planning problems containing the vague information from subjective estimations.

2.3 Multi-level factorial design

MFD is a method which cannot only reflect the interaction among parameters, but also make it possible to study the relationship (including curvilinear relationship) between the design factors and the response (Montgomery 2001). Different from the common two-level factorial design, MFD is the 3^k factorial design. It consists of k factors and each factor has three levels. Three levels including low, medium and high are denoted as l_{-1} , l_0 and l_{+1} , respectively. The simplest design in the 3^k system is the 3^2 design which can be used to explore the interaction between these two factors and the relationship between the factors and the response. In order to illustrate the concept of interaction, a regression model representation of the 3^2 system of designs could be written as (Montgomery 2001):

$$y = \alpha_{-1} + \alpha_0 x_1 + \alpha_1 x_2 + \alpha_{01} x_1 x_2 + \alpha_{00} x_1^2 + \alpha_{11} x_2^2 + \varepsilon \tag{3}$$

where y is the response, α 's are parameters need to be determined, x_1 and x_2 represent the two factors, and $x_1 x_2$ represents the interaction between two factors. The addition of a third factor level allows the relationship between the response and factors to be modeled as a quadratic (Montgomery 2001). Therefore, the 3^2 design could be a possible choice in this study to pay attention to the curvature in the response function.

2.4 Factorial-based two-stage programming with FVaR method

The ITSFP method has an advantage that it can deal with dual uncertainties in both objective function and

constraints (Wang and Huang 2011). However, the risk about the second-stage penalty is not considered when the promised allocation targets are not reached. Moreover, the effects of factors and the interactions among different factors are not identified in the ITSFP model. In order to solve these problems, FVaR and MFD will be integrated into the ITSFP framework, which leading to a factorial-based two-stage programming approach with fuzzy random value-at-risk (FTSPF) method.

$$\text{Maximize } \tilde{f}^\pm = \sum_{i=1}^m \tilde{B}_i^\pm X_i^\pm - \sum_{i=1}^m \tilde{C}_i^\pm \left(\sum_{j=1}^n p_j Y_{ij}^\pm \right) \quad (4a)$$

Subject to:

$$\sum_{i=1}^m \left(X_i^\pm - Y_{ij}^\pm \right) \left(1 + (1 - \omega) E_1^{\tilde{q}_i^\pm} + \omega E_2^{\tilde{q}_i^\pm} \right) \leq \omega E_1^{\tilde{q}_i^\pm} + (1 - \omega) E_2^{\tilde{q}_i^\pm}, \quad \forall j \quad (4b)$$

$$\xi_{FVaR}^\pm \left(\sum_{i=1}^m \tilde{C}_i^\pm \left(\sum_{j=1}^n p_j Y_{ij}^\pm \right), \beta \right) \leq \tau \quad (4c)$$

$$Y_{ij}^\pm \leq X_i^\pm \leq X_{i\max}^\pm, \quad \forall i, j \quad (4d)$$

$$Y_{ij}^\pm \geq 0, \quad \forall i, j \quad (4e)$$

where τ is the maximum acceptable loss set. In this model, Eq. (4c) is an inexact FVaR constraint which indicates that $\xi_{FVaR}^\pm \left(\sum_{i=1}^m \tilde{C}_i^\pm \left(\sum_{j=1}^n p_j Y_{ij}^\pm \right), \beta \right)$ associated with high-risk events are constrained to a value $\leq \tau$. The FTSPF model can effectively allocate the available water resources to multiple users under multiple uncertainties, provide trade-offs among the satisfaction of system benefit, the risk level of water loss and the feasibility degree of constraints, and identify the effects of factors as well as their potential interactions on system benefits.

The framework of FTSPF model is shown in Fig. 2. FTSPF can hold several advantages in water resources management: (a) it can deal with multiple uncertainties including dual uncertainties; (b) it can provide tradeoffs among the satisfaction of system benefit, the risk level of water loss and the feasibility degree of constraints; (c) it can reveal interactions of factors on the decision alternative and curvature in the response function. Therefore, water resources allocation strategies under different feasibility degrees and risk levels can be generated for decision makers.

2.5 Solution method

There are two difficulties in solving FTSPF model (4). One is transformation of the FVaR constraint which is a adaption of a value-at-risk method into the fuzzy random environment. The other one is the dual uncertainties exist in the parameters and decision variables, which makes the FVaR

constraint as well as the FTSPF model more difficult to be solved. In order to overcome these difficulties, a solution algorithm is developed (Fig. 3 gives the detail process).

Step 1 Decompose the inexact FVaR constraint (i.e. Eq. 4c). For example, $\tilde{v}^\pm = \left[\left(a_\xi^-, b_\xi^-, c_\xi^-, d_\xi^- \right), \left(a_\xi^+, b_\xi^+, c_\xi^+, d_\xi^+ \right) \right]$ is denoted as an interval number with trapezoidal fuzzy random boundaries. $\tilde{v}^- = \left(a_\xi^-, b_\xi^-, c_\xi^-, d_\xi^- \right)$ and $\tilde{v}^+ = \left(a_\xi^+, b_\xi^+, c_\xi^+, d_\xi^+ \right)$ are trapezoidal fuzzy random variables, corresponding to the lower and upper bounds of \tilde{v}^\pm , respectively. The corresponding membership function $\lambda^\pm(\tilde{v}^\pm)$ can be expressed as:

$$\lambda^\pm(\tilde{v}^\pm) = \begin{cases} \frac{x^\pm - a_\xi^\pm}{b_\xi^\pm - a_\xi^\pm}, & \text{if } a_\xi^\pm \leq x^\pm \leq b_\xi^\pm \\ 1, & \text{if } b_\xi^\pm \leq x^\pm \leq c_\xi^\pm \\ \frac{c_\xi^\pm - x^\pm}{c_\xi^\pm - d_\xi^\pm}, & \text{if } c_\xi^\pm \leq x^\pm \leq d_\xi^\pm \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

The inexact FVaR function $\xi_{FVaR}^\pm(\tilde{v}^\pm, \beta)$ is formulated as follows:

$$\xi_{FVaR}^\pm(\tilde{v}^\pm, \beta) = \begin{cases} a_\xi^\pm + 2(b_\xi^\pm - a_\xi^\pm)\beta, & \text{if } \beta \leq 0.5 \\ 2c_\xi^\pm - d_\xi^\pm + 2(d_\xi^\pm - c_\xi^\pm)\beta, & \text{if } \beta > 0.5 \end{cases} \quad (6)$$

where the upper and lower bounds of inexact FVaR function correspond to the upper and lower bounds of \tilde{v}^\pm based on its characters under dual uncertainties. In detail,

$$\xi_{FVaR}^-(\tilde{v}^-, \beta) = \begin{cases} a_\xi^- + 2(b_\xi^- - a_\xi^-)\beta, & \text{if } \beta \leq 0.5 \\ 2c_\xi^- - d_\xi^- + 2(d_\xi^- - c_\xi^-)\beta, & \text{if } \beta > 0.5 \end{cases} \quad (7a)$$

$$\xi_{FVaR}^+(\tilde{v}^+, \beta) = \begin{cases} a_\xi^+ + 2(b_\xi^+ - a_\xi^+)\beta, & \text{if } \beta \leq 0.5 \\ 2c_\xi^+ - d_\xi^+ + 2(d_\xi^+ - c_\xi^+)\beta, & \text{if } \beta > 0.5 \end{cases} \quad (7b)$$

Step 2 The model (4) can be transformed into two sub-models by the two-step method. The two-step method is widely used to solve optimization model under uncertainty, whose main ideal is transforming inexact model with interval numbers to two sub-models corresponding to the upper- and lower- bound objective function value through analyzing interrelationships among parameters and variables in the objective function and constraints (Fan and Huang 2012). Let $X_i^\pm = X_i^- + \Delta X_i y_i$, where $\Delta X_i = X_i^+ - X_i^-$ and $y_i \in [0, 1]$. Model (4) can be transformed into two sub-

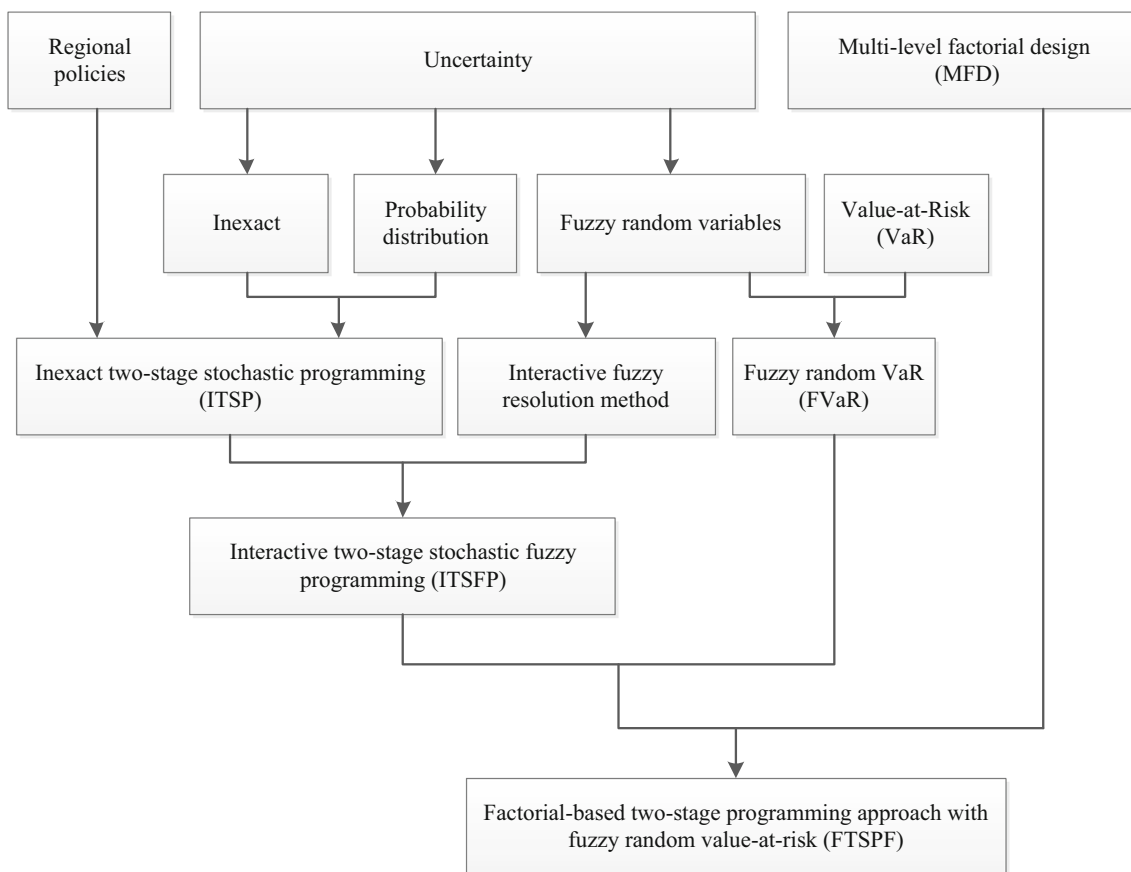


Fig. 2 The framework of FTSPF method

models based on the two-step method, do the following substeps:

Step 2.1 The sub-model corresponds to the upper bound of objective function value are formulated as follows:

$$\begin{aligned} \text{Maximize } \tilde{f}^+ &= \sum_{i=1}^m \tilde{B}_i^+ (X_i^- + \Delta X_i y_i) \\ &- \sum_{i=1}^m \tilde{C}_i^- \left(\sum_{j=1}^n p_j Y_{ij}^- \right) \end{aligned} \tag{8a}$$

Subject to:

$$\begin{aligned} \sum_{i=1}^m (X_i^- + \Delta X_i y_i - Y_{ij}^-) (1 + (1 - \omega) E_1^{\tilde{r}^-} + \omega E_2^{\tilde{r}^-}) \\ \leq \omega E_1^{\tilde{q}_j^+} + (1 - \omega) E_2^{\tilde{q}_j^+}, \quad \forall j \end{aligned} \tag{8b}$$

$$\xi_{FVaR}^-(\tilde{v}^-, \beta) = \begin{cases} a_{\tilde{\xi}}^- + 2(b_{\tilde{\xi}}^- - a_{\tilde{\xi}}^-)\beta, & \text{if } \beta \leq 0.5 \\ 2c_{\tilde{\xi}}^- - d_{\tilde{\xi}}^- + 2(d_{\tilde{\xi}}^- - c_{\tilde{\xi}}^-)\beta, & \text{if } \beta > 0.5 \end{cases} \leq \tau \tag{8c}$$

$$Y_{ij}^- \leq X_i^- + \Delta X_i y_i \leq X_{imax}^+, \quad \forall i, j \tag{8d}$$

$$Y_{ij}^- \geq 0, \quad \forall i, j \tag{8e}$$

$$0 \leq y_i \leq 1, \quad \forall i \tag{8f}$$

where Y_{ij}^- and y_i are decision variables.

Step 2.2 the sub-model corresponds to the lower bound of objective function value are formulated as follows:

$$\text{Maximize } \tilde{f}^- = \sum_{i=1}^m \tilde{B}_i^- X_{i,opt} - \sum_{i=1}^m \tilde{C}_i^+ \left(\sum_{j=1}^n p_j Y_{ij}^+ \right) \tag{9a}$$

Subject to:

$$\sum_{i=1}^m (X_{i,opt} - Y_{ij}^+) (1 + (1 - \omega) E_1^{\tilde{r}^+} + \omega E_2^{\tilde{r}^+}) \tag{9b}$$

$$\leq \omega E_1^{\tilde{q}_j^-} + (1 - \omega) E_2^{\tilde{q}_j^-}, \quad \forall j \tag{9c}$$

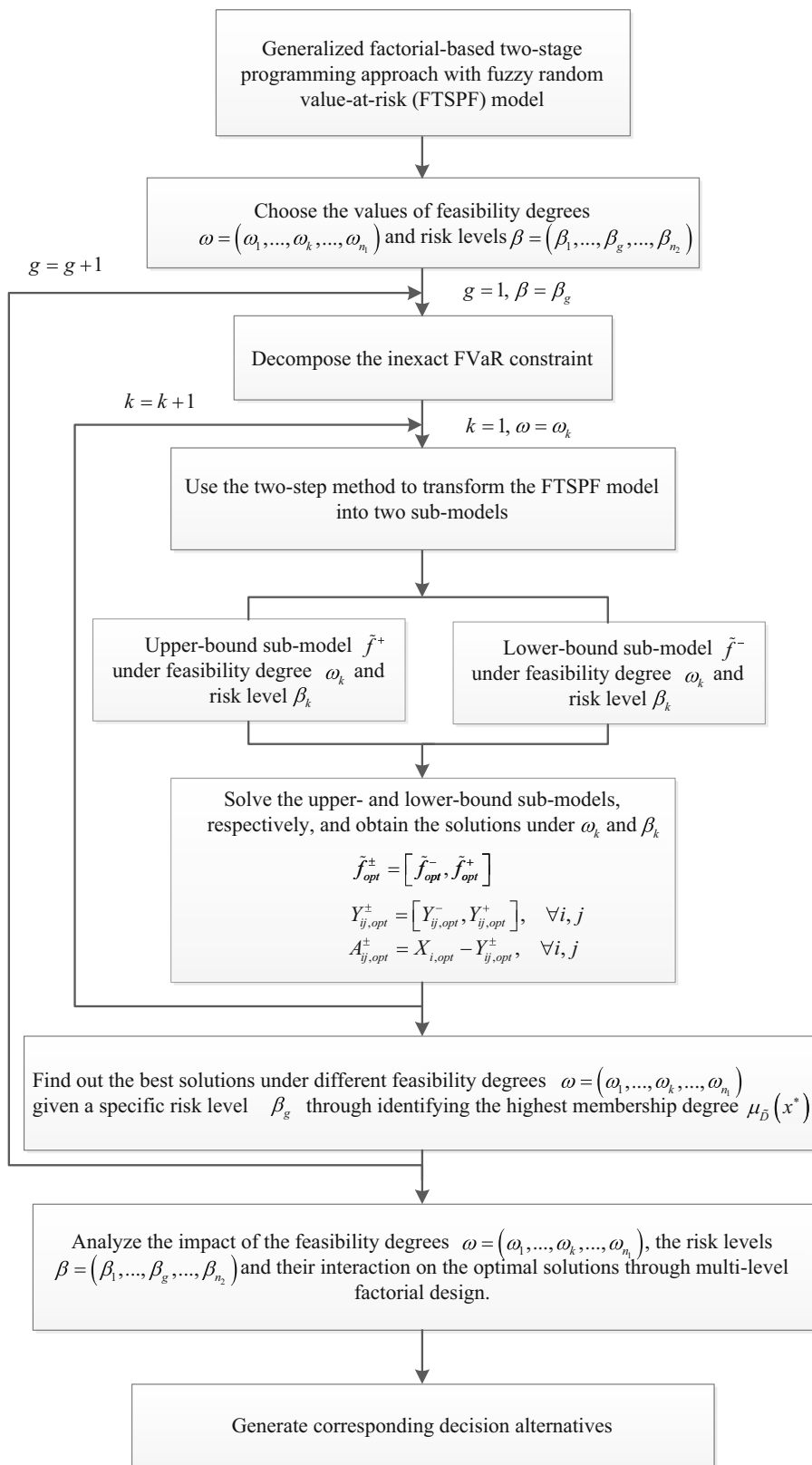
$$Y_{ij}^+ \leq X_{i,opt} \leq X_{imax}^-, \quad \forall i, j \tag{9c}$$

$$Y_{ij}^+ \geq Y_{ij,opt}^-, \quad \forall i, j \tag{9d}$$

where Y_{ij}^+ are decision variables.

Step 3 If the objective function is to be maximized, solve the sub-model corresponding to \tilde{f}^+ firstly. Thus sub-model (8) coupled with transformed FVaR constraint (7) can be

Fig. 3 The solution algorithm of FTSPF method



solved, obtaining $Y_{ij,opt}^-$, $X_{i,opt} = X_i^- + \Delta X_i y_{i,opt}$ and \tilde{f}_{opt}^+ . In the FTSPF model, the main function of FVaR constraint is to control the risk level of water loss thus identifying the

optimized water allocation targets (i.e. $X_{i,opt}$) (Shao et al. 2011). The optimized water allocation targets have been determined in sub-model (8). Therefore, the FVaR

constraint would not be involved in the sub-model (9). Through solving the sub-model (9), $Y_{ij,opt}^+$ and \tilde{f}_{opt}^- are obtained. Therefore, the solutions for the entire FTSPF model (4) are:

$$\tilde{f}_{opt}^\pm = [\tilde{f}_{opt}^-, \tilde{f}_{opt}^+] \tag{10}$$

$$Y_{ij,opt}^\pm = [Y_{ij,opt}^-, Y_{ij,opt}^+], \quad \forall i, j \tag{11}$$

$$A_{ij,opt}^\pm = X_{i,opt} - Y_{ij,opt}^\pm, \quad \forall i, j \tag{12}$$

Step 4 After solving FTSPF, plenty of solutions under different feasibility degrees of constraints ω and risk levels of water loss β are obtained. The following substeps (Jiménez et al. 2007) can be used to find out the best solutions under different feasibility degrees of constraints given a specific risk level:

Step 4.1 Introducing a fuzzy set \tilde{G} with the specified goal \bar{G} and the tolerance threshold \underline{G} , and the corresponding membership function $\mu_{(\tilde{G})}$ can be expressed below:

$$\mu_{\tilde{G}}(z) = \begin{cases} 1 & \text{if } z \geq \bar{G} \\ \frac{z - \underline{G}}{\bar{G} - \underline{G}} & \text{if } \underline{G} \leq z \leq \bar{G} \\ 0 & \text{if } z \leq \underline{G} \end{cases} \tag{13}$$

Step 4.2 Introducing the index which can indicate the degree of the satisfaction of \tilde{G} under each ω -acceptable optimal solution (i.e. optimal solution under each feasibility degree) (Yamout et al. 2007; Jiménez et al. 2007):

$$K_{\tilde{G}}(\tilde{f}^0(\omega)) = \frac{\int_{-\infty}^{+\infty} \mu_{\tilde{f}^0(\omega)}(z) \cdot \mu_{\tilde{G}}(z) dz}{\int_{-\infty}^{+\infty} \mu_{\tilde{f}^0(\omega)}(z) dz} \tag{14}$$

where the value of the objective function $\tilde{f}^0(\omega_k)$ according to the feasibility degree ω_k can be obtained by solving FTSPF model (4); $\mu_{\tilde{f}^0(\omega)}$ is the membership function (or satisfaction degree) of $\tilde{f}^0(\omega)$; and $\mu_{\tilde{G}}(z)$ is the satisfaction degree of \tilde{G} . Figure 4 shows the occurrence possibility of an objective-function value and its satisfaction degree of fuzzy goal.

Step 4.3 Applying fuzzy decision $\tilde{D} = \tilde{F} \cap \tilde{S}$ to obtain the best solutions under different feasibility degrees of constraints based on the demand of decision makers (Bellman and Zadeh 1970):

$$\begin{aligned} \mu_{\tilde{D}}(x^0(\omega_k)) &= \mu_{\tilde{F}}(x^0(\omega_k)) * \mu_{\tilde{S}}(x^0(\omega_k)) \\ &= \omega_k * K_{\tilde{G}}(\tilde{z}^0(\omega_k)) \end{aligned} \tag{15}$$

where \tilde{F} represents a fuzzy set and its membership function is $\mu_{\tilde{F}}(x^0(\omega_k))$; \tilde{S} represents a fuzzy set with the membership function shown as $\mu_{\tilde{S}}(x^0(\omega_k))$; * represents a minimum t -norm which is used to construct the intersection of two fuzzy sets (Wang and Huang 2011). Thus the final

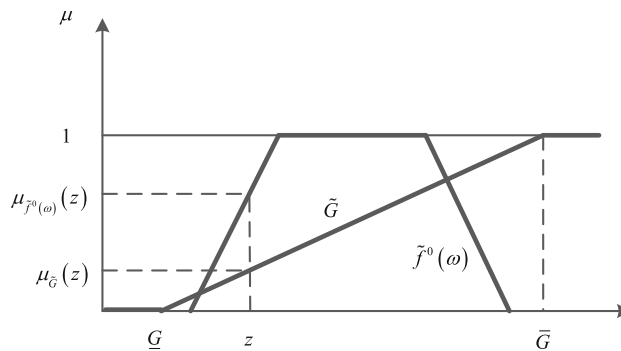


Fig. 4 Occurrence possibility of an objective-function value and its satisfaction degree of fuzzy goal

solution x^* to FTSPF can be selected by decision makers through identifying the highest membership degree expressed as:

$$\mu_{\tilde{D}}(x^*) = \max_{\omega_k \in M} \{ \omega_k * K_{\tilde{G}}(\tilde{z}^0(\omega_k)) \} \tag{16}$$

Step 5 Analyze the impact of the feasibility degree ω , the risk level β and their interaction on the optimal solutions through MFD. The geometry of the 3^2 design can be showed in Fig. 5. It consists of $3^2 = 9$ treatment combinations, where $l_{-1}l_{-1}$, $l_{-1}l_0$ and $l_{-1}l_1$ denote the treatment combinations corresponding to ω at the low level but β at the low, medium and high levels, respectively; l_0l_{-1} , l_0l_0 and l_0l_1 denote the treatment combinations corresponding to ω at the medium level but β at the low, medium and high levels, respectively; l_1l_{-1} , l_1l_0 and l_1l_1 denote the treatment combinations corresponding to ω at the high level but β at the low, medium and high levels, respectively. After generating a series of objective function value under each ω and β , jump to Step 4 to determine the optimal solution.

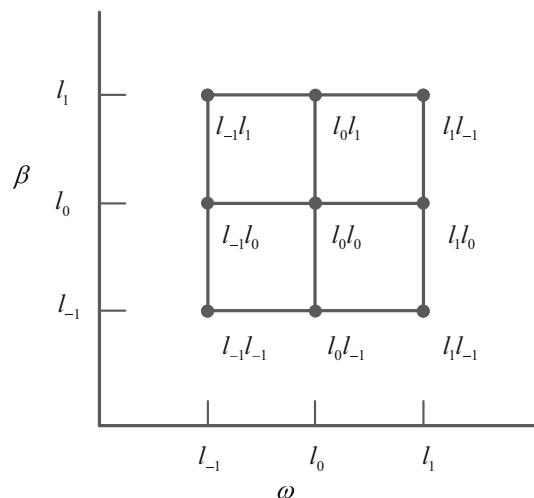


Fig. 5 The geometry of the 3^2 design

3 Application

3.1 Overview of the study system

The proposed FTSPF method is applied to a case study on water resources management. The availability water resources supply is especially important for the reason that demands from various sectors were increasing rapidly in recent years (Fan et al. 2015, 2016; Li et al. 2009; Qin et al. 2007). In this study, a region is considered where an unregulated reservoir supplies water resources to three water users, including a municipality, an industry concern and an agricultural sector.

A variety of complexities and uncertainties exist in the water resources system, such as limited water resources supply, uncertainties in economic and technical data, analysis of regional water resources policy and risk analysis in economic loss. Therefore, it is necessary to generate appropriate water resources allocation schemes for decision makers by maximizing the net system benefit under such complexities and uncertainties. The relevant data of economic factors and seasonal flow conditions are shown in Tables 1, 2 and 3. Table 1 provides the net benefit (i.e. \tilde{B}_i^\pm) and penalty (i.e. \tilde{C}_i^\pm) of the water resources system presented as fuzzy boundary intervals. The net benefits and penalties of three water users are different from each other. The Municipality earns the best benefit but pays most to the penalty when the water allocation targets are not satisfied. Although the agricultural sector earns the worst benefit, it pays least to the penalty. The available water resources from the reservoir are presented in Table 2. The seasonal flow is divided into four levels (i.e. low, low-medium, medium and high) corresponding to a certain probability. Table 3 gives the water allocation targets from the reservoir to three water users. For each water user, the maximum allowable water allocations as well as the water loss rate during transportation are the same. The water allocation targets are all presented as intervals.

In this studied water resources system, the water allocation target X_i^\pm which is promised to water user i is represented to be the first-stage decision variable. The water shortage Y_{ij}^\pm to water user i under the stream flow level j is represented to be the second-stage decision variable.

FTSPF is considered to be a feasible method for generating optimal solutions under different feasibility degrees of constraints and risk levels. The water resources system is complicated, holding multi variables, multi periods, dynamic and uncertain features. These complexities may lead to various risks for practical water resources management strategies. Therefore, the feasibility degrees of constraints and the risk levels of penalty are selected various values. In this study, 0.4, 0.5, 0.6, 0.7, and 0.8 are chosen as the representative values of feasibility degree ω , and 0.5, 0.7, 0.9 and 0.95 are chosen as the representative values of risk level β .

3.2 Results and discussion

The ω -acceptable solutions obtained from the FTSPF model are shown in Table 4. For each risk level β (i.e. 0.5, 0.7, 0.9 and 0.95), solutions under feasibility degrees 0.4, 0.5, 0.6, 0.7, and 0.8 are obtained. As shown in Table 5, the system benefits $\tilde{f}^\pm(\omega)$ under different feasibility degrees of constraints and different risk levels are presented as fuzzy boundary intervals. For example, under a risk level (i.e. β) of 0.5, the maximum system benefit under a feasibility degree $\omega = 0.4$ would be: $\tilde{f}^\pm(0.4) = [(140.14, 152.25, 164.36, 176.47), (483.17, 504.08, 524.99, 545.90)]$ ($\$10^6$) and the minimum system benefit under a feasibility degree $\omega = 0.8$ would be: $\tilde{f}^\pm(0.8) = [(130.33, 142.18, 154.04, 165.89), (468.20, 488.46, 508.71, 528.96)]$ ($\$10^6$). The results shown in Table 5 indicate that the system benefit gradually decreases as the feasibility degree increases or the risk level decreases. This is because that (1) when the risk level β decrease, the probability of a loss exceeding the threshold increases, and an increasing economic losses of water must lead to an decreasing net system benefit; (2) a strong desire to obtain a higher degree of constraint feasibility will lead to a lower objective-function value, and vice versa.

Figure 6 gives the optimal water allocation targets according to the ω -acceptable solutions obtained from the FTSPF model under a set of risk levels. The optimal water-allocation targets of the municipality as well as the industry concern would not change when the feasibility degree ω and the risk level β change. But those of agriculture sector would vary with the changes of ω and β .

Table 1 Net benefit and penalty

	Net benefit when water allocation target is satisfied \tilde{B}_i^\pm ($\$/m^3$)	Penalty when water is not delivered \tilde{C}_i^\pm ($\$/m^3$)
Municipal ($i = 1$)	[(82, 86, 90, 94), (101, 105, 109, 113)]	[(210, 215, 220, 225), (270, 275, 280, 285)]
Industrial ($i = 2$)	[(41, 44, 47, 50), (60, 63, 66, 69)]	[(55, 58, 61, 64), (84, 87, 90, 93)]
Agricultural ($i = 3$)	[(25, 27, 29, 31), (28, 30, 32, 34)]	[(43, 47, 51, 55), (64, 68, 72, 76)]

Table 2 Stream flows under different probabilistic scenarios (supply)

Flow level	Probability (%)	Stream flow \tilde{q}_j^\pm (in 10^6 m ³)
Low (L) ($j = 1$)	0.120	[(3.3, 3.4, 3.5, 3.6), (4.0, 4.1, 4.2, 4.3)]
Low-medium (Lm) ($j = 2$)	0.276	[(4.6, 4.8, 5.0, 5.2), (5.5, 5.7, 5.9, 6.1)]
Medium (M) ($j = 3$)	0.421	[(6.5, 7.7, 8.9, 10.1), (10.2, 11.4, 12.6, 13.8)]
High (H) ($j = 4$)	0.183	[(14.2, 15.0, 15.8, 16.6), (17.0, 17.8, 18.6, 19.4)]

Table 3 Water allocation targets from the reservoir to water users (demand)

	Water users		
	Municipal ($i = 1$)	Industrial ($i = 2$)	Agricultural ($i = 3$)
Maximum allowable allocation			
$X_{i\max}^\pm$ (in 10^6 m ³)	7.5	7.5	7.5
Water allocation target			
X_i^\pm (in 10^6 m ³)	[1.8, 2.4]	[2.3, 3.7]	[3.0, 5.8]
Water loss rate during transportation			
$\tilde{\gamma}^\pm$	[(0.08, 0.12, 0.16, 0.20), (0.30, 0.34, 0.38, 0.42)]		

Table 4 ω -Acceptable solutions obtained from the FTSPF model

β	ω	Shortage Y_{ij}^\pm (in 10^6 m ³)												
		Y_{11}^\pm	Y_{21}^\pm	Y_{31}^\pm	Y_{12}^\pm	Y_{22}^\pm	Y_{32}^\pm	Y_{13}^\pm	Y_{23}^\pm	Y_{33}^\pm	Y_{14}^\pm	Y_{24}^\pm	Y_{34}^\pm	
0.5	0.4	0	[2.42, 3.53]	4.47	0	[0.94, 2.45]	4.47	0	0	[0, 4.25]	0	0	0	
	0.5	0	[2.46, 3.56]	4.39	0	[1.01, 2.50]	4.39	0	0	[0, 4.39]	0	0	0	
	0.6	0	[2.50, 3.59]	4.14	0	[1.08, 2.55]	4.14	0	0	[0, 4.14]	0	0	0	
	0.7	0	[2.54, 3.62]	3.87	0	[1.15, 2.60]	3.87	0	[0, 0.42]	[0, 3.87]	0	0	0	
	0.8	0	[2.59, 3.65]	3.59	0	[1.22, 2.65]	3.59	0	[0, 0.62]	[0, 3.59]	0	0	0	
0.7	0.4	0	[2.42, 3.53]	3.88	0	[0.94, 2.45]	3.88	0	0	[0, 3.66]	0	0	0	
	0.5	0	[2.46, 3.56]	3.81	0	[1.01, 2.50]	3.81	0	0	[0, 3.80]	0	0	0	
	0.6	0	[2.50, 3.59]	3.73	0	[1.08, 2.55]	3.73	0	0	[0, 3.73]	0	0	0	
	0.7	0	[2.54, 3.62]	3.66	0	[1.15, 2.60]	3.66	0	[0, 0.42]	[0, 3.66]	0	0	0	
	0.8	0	[2.59, 3.65]	3.59	0	[1.22, 2.65]	3.59	0	[0, 0.62]	[0, 3.59]	0	0	0	
0.9	0.4	0	[2.42, 3.53]	3.73	0	[0.94, 2.45]	3.73	0	0	[0, 3.52]	0	0	0	
	0.5	0	[2.46, 3.56]	3.66	0	[1.01, 2.50]	3.66	0	0	[0, 3.66]	0	0	0	
	0.6	0	[2.50, 3.59]	3.59	0	[1.08, 2.55]	3.59	0	[0, 0.21]	[0, 3.59]	0	0	0	
	0.7	0	[2.54, 3.62]	3.52	0	[1.15, 2.60]	3.52	0	[0, 0.42]	[0, 3.52]	0	0	0	
	0.8	0	[2.59, 3.65]	3.45	0	[1.22, 2.65]	3.45	0	[0, 0.62]	[0, 3.45]	0	0	0	
0.95	0.4	0	[2.42, 3.53]	3.70	0	[0.94, 2.45]	3.70	0	0	[0, 3.48]	0	0	0	
	0.5	0	[2.46, 3.56]	3.63	0	[1.01, 2.50]	3.63	0	0	[0, 3.63]	0	0	0	
	0.6	0	[2.50, 3.59]	3.55	0	[1.08, 2.55]	3.55	0	[0, 0.21]	[0, 3.55]	0	0	0	
	0.7	0	[2.54, 3.62]	3.48	0	[1.15, 2.60]	3.48	0	[0, 0.42]	[0, 3.48]	0	0	0	
	0.8	0	[2.59, 3.65]	3.41	0	[1.22, 2.65]	3.41	0	[0, 0.62]	[0, 3.41]	0	0	0	

β is the risk level; and ω is the feasibility degree

After obtaining plenty of solutions under different feasibility degrees of constraints and risk levels, the tradeoffs between the system benefit and feasibility degree of constraints would be discussed at the first place in this study. The results in Table 5 show that a lower feasibility degree would correspond to a higher system benefit. It means that the satisfaction degree of objective-function value would

be higher if the feasibility degree of constraints is lower, and vice versa. Therefore, in order to obtain the suitable system benefit based on the demand of decision makers, the solution method proposed in this study could be used. Firstly, the membership function of fuzzy goal (\tilde{G}) under different risk levels can be calculated by using Eq. (13):

Table 5 Net system benefit obtained from the FTSPF model

β	ω	Distribution of net system benefit [\tilde{f}^{\pm} (\$10 ⁶)]
0.5	0.4	[(140.14, 152.25, 164.36, 176.47), (483.17, 504.08, 524.99, 545.90)]
	0.5	[(135.02, 146.81, 158.60, 170.39), (480.95, 501.75, 522.55, 543.35)]
	0.6	[(132.79, 144.57, 156.35, 168.13), (476.92, 497.54, 518.16, 538.78)]
	0.7	[(131.56, 143.38, 155.20, 167.02), (472.53, 492.97, 513.41, 533.85)]
	0.8	[(130.33, 142.18, 154.04, 165.89), (468.20, 488.46, 508.71, 528.96)]
0.7	0.4	[(156.14, 168.99, 181.84, 194.69), (476.74, 497.40, 518.06, 538.72)]
	0.5	[(150.93, 163.46, 175.99, 188.52), (474.55, 495.11, 515.67, 536.23)]
	0.6	[(144.03, 156.33, 168.63, 180.93), (472.39, 492.85, 513.31, 533.77)]
	0.7	[(137.17, 149.25, 161.33, 173.41), (470.27, 490.63, 510.99, 531.35)]
	0.8	[(130.33, 142.18, 154.03, 165.88), (468.20, 488.46, 508.72, 528.98)]
0.9	0.4	[(160.10, 173.13, 186.16, 199.19), (475.14, 495.75, 516.36, 536.97)]
	0.5	[(154.87, 167.59, 180.31, 193.03), (472.97, 493.47, 513.97, 534.47)]
	0.6	[(147.95, 160.43, 172.91, 185.39), (470.82, 491.22, 511.62, 532.02)]
	0.7	[(141.07, 153.33, 165.59, 177.85), (468.71, 489.00, 509.29, 529.58)]
	0.8	[(134.27, 146.31, 158.35, 170.39), (466.62, 486.81, 507.00, 527.19)]
0.95	0.4	[(161.06, 174.13, 187.20, 200.27), (474.76, 495.35, 515.94, 536.53)]
	0.5	[(155.82, 168.58, 181.34, 194.10), (472.59, 493.07, 513.55, 534.03)]
	0.6	[(148.89, 161.42, 173.95, 186.48), (470.44, 490.82, 511.20, 531.58)]
	0.7	[(142.01, 154.31, 166.61, 178.91), (468.33, 488.61, 508.89, 529.17)]
	0.8	[(135.20, 147.29, 159.38, 171.47), (466.25, 486.42, 506.59, 526.76)]

β is the risk level; and ω is the feasibility degree

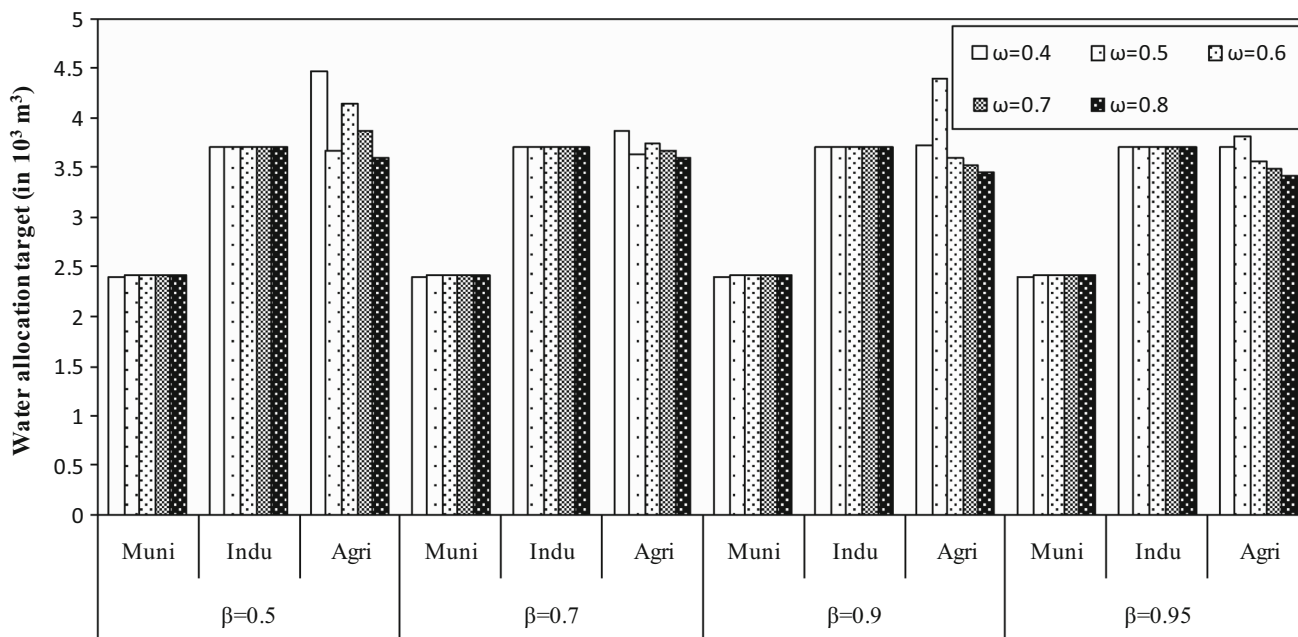


Fig. 6 Optimized water allocation targets under different feasibility degree ω and risk level β (symbol “Muni, Indu and Agri” is abbreviation of “municipal, industrial and agricultural”, respectively)

$$\mu_{G=0.5}^-(z) = \begin{cases} 1 & \text{if } z \geq 176 \\ \frac{z - 130}{176 - 130} & \text{if } 130 \leq z \leq 176 \\ 0 & \text{if } z \leq 130 \end{cases} \quad (17a)$$

$$\mu_{G=0.5}^+(z) = \begin{cases} 1 & \text{if } z \geq 546 \\ \frac{z - 468}{546 - 468} & \text{if } 468 \leq z \leq 546 \\ 0 & \text{if } z \leq 468 \end{cases} \quad (17b)$$

Equation (17) is the membership function of fuzzy goal (\tilde{G}) under the risk level $\beta = 0.5$. $\mu_{\tilde{G}\beta=0.5}^-(z)$ shown in Eq. (17a) corresponds to the lower bound system benefit and $\mu_{\tilde{G}\beta=0.5}^+(z)$ shown in Eq. (17b) corresponds to the upper bound system benefit. The parameters in Eq. (17) are supposed based on the maximum and minimum system benefits under the risk level $\beta = 0.5$ which are shown in Table 5. Therefore, the membership functions of fuzzy goal (\tilde{G}) under other risk levels are as follows:

$$\mu_{\tilde{G}\beta=0.7}^-(z) = \begin{cases} 1 & \text{if } z \geq 195 \\ \frac{z - 130}{195 - 130} & \text{if } 130 \leq z \leq 195 \\ 0 & \text{if } z \leq 130 \end{cases} \quad (18a)$$

$$\mu_{\tilde{G}\beta=0.7}^+(z) = \begin{cases} 1 & \text{if } z \geq 539 \\ \frac{z - 468}{539 - 468} & \text{if } 468 \leq z \leq 539 \\ 0 & \text{if } z \leq 468 \end{cases} \quad (18b)$$

$$\mu_{\tilde{G}\beta=0.9}^-(z) = \begin{cases} 1 & \text{if } z \geq 199 \\ \frac{z - 134}{199 - 134} & \text{if } 134 \leq z \leq 199 \\ 0 & \text{if } z \leq 134 \end{cases} \quad (19a)$$

$$\mu_{\tilde{G}\beta=0.9}^+(z) = \begin{cases} 1 & \text{if } z \geq 537 \\ \frac{z - 467}{537 - 467} & \text{if } 467 \leq z \leq 537 \\ 0 & \text{if } z \leq 467 \end{cases} \quad (19b)$$

$$\mu_{\tilde{G}\beta=0.95}^-(z) = \begin{cases} 1 & \text{if } z \geq 200 \\ \frac{z - 135}{200 - 135} & \text{if } 135 \leq z \leq 200 \\ 0 & \text{if } z \leq 135 \end{cases} \quad (20a)$$

$$\mu_{\tilde{G}\beta=0.95}^+(z) = \begin{cases} 1 & \text{if } z \geq 537 \\ \frac{z - 466}{537 - 466} & \text{if } 466 \leq z \leq 537 \\ 0 & \text{if } z \leq 466 \end{cases} \quad (20b)$$

Then, based on the membership functions of fuzzy goal (\tilde{G}) under different risk levels shown as Eqs. (17)–(20), values of index $K_{\tilde{G}}(\tilde{f}^\pm)$ can be calculated, presented in Table 6. They can indicate the satisfaction degrees of fuzzy goal \tilde{G} , and membership grades of the fuzzy decision variables Y_{ij}^\pm under each ω -acceptable optimal solution $\tilde{f}^\pm(\omega)$. Finally, the mean and deviation of membership grade for fuzzy decision (i.e. $\bar{\mu}_{\tilde{D}}(Y_{ij}^\pm)$ and $\hat{\mu}_{\tilde{D}}(Y_{ij}^\pm)$) are calculated in order to find out the best solutions under different feasibility degrees when the risk level is given.

Table 6 Fuzzy decision under each ω -acceptable optimal solution

β	ω	$K_{\tilde{G}}(\tilde{f}^-)$	$K_{\tilde{G}}(\tilde{f}^+)$	$\mu_{\tilde{D}}(Y_{ij}^-)$	$\mu_{\tilde{D}}(Y_{ij}^+)$	$\bar{\mu}_{\tilde{D}}(Y_{ij}^\pm)$	$\hat{\mu}_{\tilde{D}}(Y_{ij}^\pm)$
0.5	0.4	0.854	0.863	0.342	0.345	0.3435	0.0015
	0.5	0.702	0.821	0.351	0.411	0.3810	0.0300
	0.6	0.664	0.773	0.398	0.464	0.4310	0.0330
	0.7	0.630	0.705	0.441	0.494	0.4675	0.0265
	0.8	0.594	0.625	0.475	0.500	0.4875	0.0125
0.7	0.4	0.943	0.836	0.377	0.334	0.3555	0.0215
	0.5	0.839	0.790	0.420	0.395	0.4075	0.0125
	0.6	0.690	0.762	0.414	0.457	0.4355	0.0215
	0.7	0.556	0.716	0.389	0.501	0.4450	0.0560
	0.8	0.434	0.680	0.347	0.544	0.4590	0.0985
0.9	0.4	0.960	0.830	0.384	0.332	0.3580	0.0260
	0.5	0.870	0.778	0.435	0.389	0.4120	0.0230
	0.6	0.721	0.746	0.433	0.448	0.4405	0.0075
	0.7	0.599	0.722	0.419	0.505	0.4620	0.0430
	0.8	0.472	0.676	0.378	0.541	0.4595	0.0815
0.95	0.4	0.964	0.825	0.386	0.330	0.3580	0.0280
	0.5	0.876	0.778	0.438	0.389	0.4135	0.0245
	0.6	0.746	0.749	0.448	0.449	0.4485	0.0005
	0.7	0.608	0.704	0.426	0.493	0.4595	0.0330
	0.8	0.483	0.665	0.386	0.532	0.4590	0.0730

The bold value indicates the optimal fuzzy decision of feasibility degree under each risk level

β is the risk level; ω is the feasibility degree; $K_{\tilde{G}}(\tilde{f}^\pm)$ is the satisfaction degree of fuzzy goal; $\mu_{\tilde{D}}(Y_{ij}^\pm)$ is the membership grade of the fuzzy decision; $\bar{\mu}_{\tilde{D}}(Y_{ij}^\pm)$ is the mean of membership grade for the fuzzy decision; $\hat{\mu}_{\tilde{D}}(Y_{ij}^\pm)$ is the deviation of membership grade for the fuzzy decision

The final fuzzy decision would be the one with the highest value of $\bar{\mu}_D(Y_{ij}^\pm)$. As shown in Table 6, the best choice under the risk level $\beta = 0.5$ is the 0.8-feasibility optimal solution with a mean of 0.4875 and a deviation of 0.0125; the best choice under the risk level $\beta = 0.7$ is the 0.8-feasibility optimal solution with a mean of 0.4590 and a deviation of 0.0985; the best choice under the risk level $\beta = 0.9$ is the 0.7-feasibility optimal solution with the mean and deviation being 0.4620 and 0.0430, respectively; and the best choice under the risk level $\beta = 0.95$ is the 0.7-feasibility optimal solution with the mean and deviation respectively being 0.4595 and 0.0330. A lower feasibility degree always leads to a higher value of objective function. However, the 0.4-feasibility optimal solution which is corresponding to the highest value of objective function is not the best choice in this study. It indicates the balance between the system benefit and the degree of constraint feasibility which should be paid more attention to by the decision makers.

Table 7 shows the optimized 0.8-feasibility solutions under $\beta = 0.5$ and $\beta = 0.7$, and the optimized 0.7-feasibility solutions under $\beta = 0.9$ and $\beta = 0.95$ obtained from the FTSPF method. The optimized water allocation targets and the actual water allocation amount under each risk level for the three water users could be calculated by letting $X_{i,opt} = X_i^- + \Delta X_i y_{i,opt}$ and $A_{ij,opt}^\pm = X_{i,opt} - Y_{ij,opt}^\pm$. And they can help decision makers maximize the system benefit under various uncertainties and risk levels. For example, when the risk level is 0.5, there would be no shortage for the municipal use under all stream flow levels. Accordingly, the water allocation amount for the municipal use would be 2.400 (in 10^6 m^3) which equals to the optimized

water allocation target. For the industrial use, the shortages under the low, low-medium, medium stream flow level would be [2.586, 3.651] (in 10^6 m^3), [1.220, 2.646] (in 10^6 m^3), and [0, 0.623] (in 10^6 m^3), respectively, but no shortage happen to the industrial use under the high stream flow level. Then the water allocation amounts for the industrial use would be [0.049, 1.114] (in 10^6 m^3), [1.054, 2.480] (in 10^6 m^3), [3.077, 3.700] (in 10^6 m^3) and 3.700 (in 10^6 m^3) under each stream flow level, respectively. However, there would be no water allocation for the agricultural use under the low and low-medium stream flow level. There would be no shortage for the agricultural use under the high stream flow level. And the water allocation for the agricultural use under the medium stream flow level would depend on the realities of situation. It is because the municipal use could bring about the highest benefit when its water allocation target is satisfied as well as the highest penalty when its water allocation target is not satisfied. In order to achieve the maximum system benefit, the decision makers would promise more water allocation amount to the municipal use instead of the agricultural use.

Figure 7 presents optimized water allocation amounts to the three water users under each risk level β . Eventually, the values of the optimized system benefit under each risk level are $\tilde{f}_{\beta=0.5}^\pm(0.8) = [(130.33, 142.18, 154.04, 165.89), (468.20, 488.46, 508.71, 528.96)]$ ($\$10^6$), $\tilde{f}_{\beta=0.7}^\pm(0.8) = [(130.33, 142.18, 154.03, 165.88), (468.20, 488.46, 508.72, 528.98)]$ ($\$10^6$), $\tilde{f}_{\beta=0.9}^\pm(0.7) = [(141.07, 153.33, 165.59, 177.85), (468.71, 489.00, 509.29, 529.58)]$ ($\$10^6$), and $\tilde{f}_{\beta=0.95}^\pm(0.7) = [(142.01, 154.31, 166.61, 178.91), (468.33, 488.61, 508.89, 529.17)]$ ($\$10^6$). As discussed

Table 7 Optimized ω -acceptable solutions of the FTSPF model under each risk level β (in 10^6 m^3)

β		Water shortage ($Y_{ij,opt}^\pm$) under a stream flow level of				Allocation amount ($A_{ij,opt}^\pm$) under a stream flow level of			
		L	Lm	M	H	L	Lm	M	H
0.5	Probability (%)	0.120	0.276	0.421	0.183	0.120	0.276	0.421	0.183
	Municipal	0	0	0	0	2.400	2.400	2.400	2.400
	Industrial	[2.586, 3.651]	[1.220, 2.646]	[0, 0.623]	0	[0.049, 1.114]	[1.054, 2.480]	[3.077, 3.700]	3.700
	Agricultural	3.591	3.591	[0, 3.591]	0	0	0	[0, 3.591]	3.591
0.7	Municipal	0	0	0	0	2.400	2.400	2.400	2.400
	Industrial	[2.586, 3.651]	[1.220, 2.646]	[0, 0.623]	0	[0.049, 1.114]	[1.054, 2.480]	[3.077, 3.700]	3.700
	Agricultural	3.589	3.589	[0, 3.589]	0	0	0	[0, 3.589]	3.589
0.9	Municipal	0	0	0	0	2.400	2.400	2.400	2.400
	Industrial	[2.545, 3.622]	[1.152, 2.597]	[0, 0.417]	0	[0.078, 1.155]	[1.103, 2.548]	[3.283, 3.700]	3.700
	Agricultural	3.517	3.517	[0, 3.517]	0	0	0	[0, 3.517]	3.517
0.95	Municipal	0	0	0	0	2.400	2.400	2.400	2.400
	Industrial	[2.545, 3.622]	[1.152, 2.597]	[0, 0.417]	0	[0.078, 1.155]	[1.103, 2.548]	[3.283, 3.700]	3.700
	Agricultural	3.483	3.483	[0, 3.483]	0	0	0	[0, 3.483]	3.483

Symbol “L, Lm, M and H” are abbreviations of “low, low-medium, medium and high”, respectively

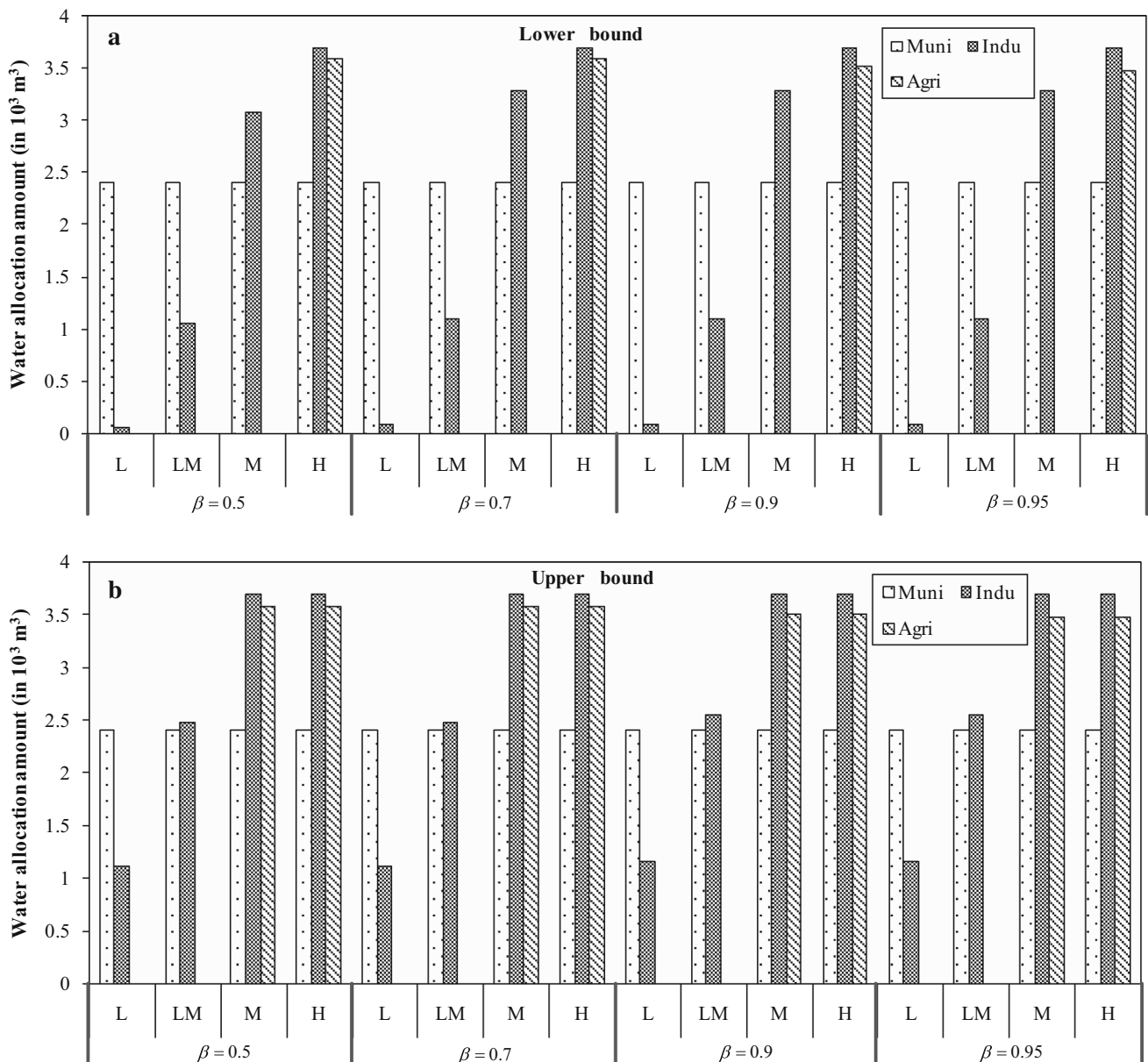


Fig. 7 Optimized water allocation amounts under each risk level β . **a** Lower bound of optimized water allocation amounts. **b** Upper bound of optimized water allocation amounts. (Symbol “Muni, Indu and Agri” is abbreviation of “municipal, industrial and agricultural”, respectively)

before, an increasing risk level means the probability of a loss exceeding the threshold decreasing, which finally leads to an increasing net system benefit. However, for the optimal solutions at the same degree of constraint feasibility, the effect of risk levels is only on the allocation of agricultural section. In detail, a higher risk level leads to a higher net system benefit achieved by decreasing the shortage of agricultural section. Therefore, the water allocation for agricultural section is the key to control the financial loss of water. The decision makes should pay more attention to water allocation amount to the agriculture use.

In order to analyze the impact of the feasibility degree ω and risk level β on the optimal solutions, a multi-level factorial design can be used in this study. The values of low, medium and high levels for ω as well as β are 0, 0.5 and 1, respectively. The results of the multi-level factorial experiment are shown in Table 8.

In this study, the final solution is determined based on the value of $\bar{\mu}_D(Y_{ij}^\pm)$. Therefore, it is necessary to explore the effects of factors to $\bar{\mu}_D(Y_{ij}^\pm)$ which has direct relationship with optimal value of objective function. Figure 8

Table 8 Multi-level factorial experiment and the corresponding optimization results

Run	Factor		Total net benefit (\$10 ⁶)
	ω	β	
1	l_{-1}	l_{-1}	[(146.89, 159.63, 172.36, 185.09), (498.12, 519.69, 541.26, 562.83)]
2	l_{-1}	l_0	[(161.20, 174.59, 187.99, 201.39), (492.37, 513.72, 535.07, 556.43)]
3	l_{-1}	l_{+1}	[(173.25, 187.20, 201.16, 215.12), (487.52, 508.69, 529.86, 551.03)]
4	l_0	l_{-1}	[(133.98, 145.73, 157.47, 169.21), (481.37, 502.18, 523.00, 543.82)]
5	l_0	l_0	[(135.02, 146.81, 158.60, 170.39), (480.95, 501.75, 522.55, 543.35)]
6	l_0	l_{+1}	[(146.74, 159.08, 171.41, 183.75), (476.24, 496.86, 517.48, 538.11)]
7	l_{+1}	l_{-1}	[(127.71, 139.63, 151.55, 163.47), (459.73, 479.62, 499.51, 513.40)]
8	l_{+1}	l_0	[(127.71, 139.63, 151.55, 163.47), (459.73, 479.62, 499.51, 513.40)]
9	l_{+1}	l_{+1}	[(127.71, 139.63, 151.55, 163.47), (459.73, 479.62, 499.51, 513.40)]

presents the effects plot for the two factors at three levels. The points in the effects plot are the mean of $\bar{\mu}_{\bar{D}}(Y_{ij}^{\pm})$ at different levels of factors. It shows that factor ω has the greater effect on the mean value of $\bar{\mu}_{\bar{D}}(Y_{ij}^{\pm})$ than factor β . The mean of $\bar{\mu}_{\bar{D}}(Y_{ij}^{\pm})$ would increase from 0 to 0.3148 and then from 0.3148 to 0.3685. This is because any change of feasibility degree of constraint would directly affect the solution of optimization, leading to a considerable variation in the system benefit.

Figure 9 shows the interaction plot for factor ω and factor β at three levels. It reveals their interactive effect on the value of $\bar{\mu}_{\bar{D}}(Y_{ij}^{\pm})$ which corresponds to optimal system benefit versus the feasibility degree for each of the three different confidence levels. The change in the value of $\bar{\mu}_{\bar{D}}(Y_{ij}^{\pm})$ differs across the three levels of ω depending on the level of β . Therefore, it is found that there exists dependence between effects of ω and β . The highest $\bar{\mu}_{\bar{D}}(Y_{ij}^{\pm})$ of 0.3976 would be obtained when ω is at its high level and β is at its low level, implying that the optimal value of objective function would occur at the point of highest feasibility degree and lowest risk level. In real-world water allocation problems, the effect the factors and their latent interactions are thus identified by using MFD then could further help make sound decisions of water allocation in the decision-making process.

3.3 Comparison of FTSPF with ITSFP

FTSPF, described in this study, and ITSFP, proposed by Wang and Huang (2011), are both methods for water resources planning problems under dual uncertainties. The above water resources system is as an example to compare these two methods.

Based on the relevant data of economic factors and seasonal flow conditions shown in Tables 1, 2 and 3, the

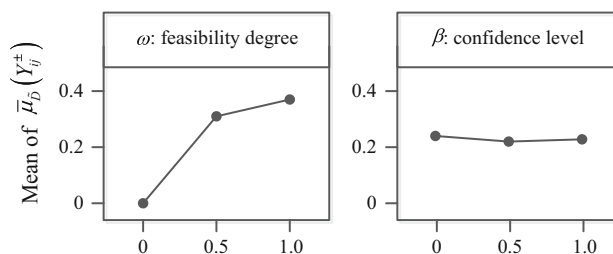


Fig. 8 Effects plot for factors at three levels

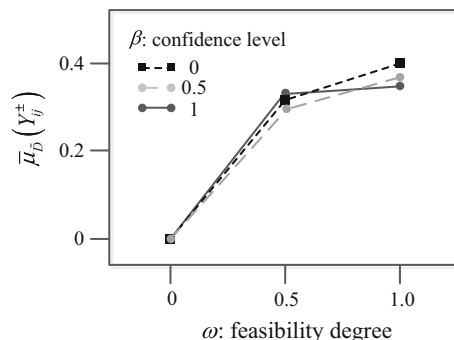


Fig. 9 Interactive plot for factors at three levels

ω -acceptable solutions can be generated by the ITSFP model shown as Eq. (1). The feasibility degrees ω are 0.4, 0.5, 0.6, 0.7 and 0.8, and the corresponding system benefits are shown in Table 9.

The net system benefits obtained by ITSFP shown in Table 9 are compared to those obtained by FTSPF shown in Table 5. In detail, the ITSFP-based values are closest to the FTSPF-based values under risk level $\beta = 0.5$ since the values obtained by two methods are the same under feasibility degree $\omega = 0.6, 0.7$ and 0.8 . Moreover, both of the optimal net benefits generated by ITSFP and FTSPF under $\beta = 0.5$ are $\tilde{f}^{\pm}(0.8) = [(130.33, 142.18, 154.04, 165.89), (468.20, 488.46, 508.71, 528.96)]$. Therefore, ITSFP is a simplified case of FTSPF proposed in this study. On the other hand, the maximum system benefit obtained from ITSFP is $\tilde{f}^{\pm}(0.4) = [(133.40, 145.20, 156.99, 168.78),$

Table 9 Net system benefit generated by the ITSFP model

ω	Distribution of net system benefit, \tilde{f}^{\pm} ($\$10^6$)
0.4	[(133.40, 145.20, 156.99, 168.78), (485.88, 506.89, 527.90, 548.91)]
0.5	[(133.98, 145.73, 157.47, 169.21), (481.37, 502.18, 523.00, 543.82)]
0.6	[(132.79, 144.57, 156.35, 168.13), (476.92, 497.54, 518.16, 538.78)]
0.7	[(131.56, 143.38, 155.20, 167.02), (472.53, 492.97, 513.41, 533.85)]
0.8	[(130.33, 142.18, 154.04, 165.89), (468.20, 488.46, 508.71, 528.96)]

The bold value indicates the optimal net benefit

ω is the feasibility degree

(485.88, 506.89, 527.90, 548.91)], and that obtained from FTSPF under $\beta = 0.5$ is $\tilde{f}_{\beta=0.5}^{\pm}(0.4) = [(140.14, 152.25, 164.36, 176.47), (483.17, 504.08, 524.99, 545.90)]$. The minimum system benefit generated by ITSFP and that generated by FTSPF under $\beta = 0.5$ are the same. It indicated that the objective function value obtained from ITSFP is less than and equal to that obtained by FTSPF. This is because compare to the ITSFP model, the FTSPF model contains a risk constraint for financial loss of water shortage, which may results a lower possible total penalties as well as a higher system benefit. The associated risk constraint is controlled by the risk level β which could be adjusted by the decision makers based on their demand and intention.

4 Conclusion

In this study, factorial analysis and fuzzy random value-at-risk approaches were introduced into a two-stage stochastic programming framework to characterize the impacts of uncertain parameters on water resources planning practices and the associated risks. The proposed method could deal with uncertainties with dual and multiple presentations and tackle problems with respect to tradeoffs among the satisfaction of system benefit, the risk level of water loss and the feasibility degree of constraints. The interactions between feasibility degrees and risk levels and the curvature in the response function could also be revealed by the proposed method.

In order to demonstrate the applicability of the proposed method, a case study of water resources planning has been provided. The results indicated that FTSPF could generate various plans of water resources allocation under multiple uncertainties and complexities. In detail, (1) optimal ω -feasibility solutions under each risk level could be determined, and the system benefit gradually decrease as the degree of constraint feasibility increase or the risk level decrease, (2) feasibility degree has the greater magnitude of the effect upon the final decision than risk level, and the highest value of objective function is not the most optimal solution as the balance between the system benefit and the

feasibility degree, (3) the effect of risk levels is only on the allocation of agricultural section for the optimal solutions at the same feasibility degree, and (4) there exists dependence between the effects of the feasibility degree and the risk level. The comparison between FTSPF and ITSFP showed that FTSPF can generate a higher system benefit with controlled risk levels. The solution obtained from FTSPF could satisfy different demands from decision makers, which has significance in the practical application of water resources planning problem.

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