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# Nonlinear dynamics and chaos in hydrologic systems: latest developments and a look forward

Bellie Sivakumar

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Abstract During the last two decades or so, studies on the applications of the concepts of nonlinear dynamics and chaos to hydrologic systems and processes have been on the rise. Earlier studies on this topic focused mainly on the investigation and prediction of chaos in rainfall and river flow, and further advances were made during the subsequent years through applications of the concepts to other problems (e.g. data disaggregation, missing data estimation, and reconstruction of system equations) and other processes (e.g. rainfall-runoff and sediment transport). The outcomes of these studies are certainly encouraging, especially considering the exploratory stage of the concepts in hydrologic sciences. This paper discusses some of the latest developments on the applications of these concepts to hydrologic systems and the challenges that lie ahead on the way to further progress. As for their applications, studies in the important areas of scaling, groundwater contamination, parameter estimation and optimization, and catchment classification are reviewed and the inroads made thus far are reported. In regards to the challenges that lie ahead, particular focus is given to improving our understanding of these largely less-understood concepts and also finding ways to integrate these concepts with the others. With the recognition that none of the existing one-sided 'extremeview' modeling approaches is capable of solving the hydrologic problems that we are faced with, the need for finding a balanced 'middle-ground' approach that can integrate different methods is stressed. To this end, the viability of bringing together the stochastic concepts and

B. Sivakumar  $(\boxtimes)$ 

Department of Land, Air and Water Resources, University of California, Davis, CA 95616, USA e-mail: sbellie@ucdavis.edu

the deterministic concepts as a starting point is also highlighted.

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## 1 Introduction

The inherent nonlinear nature of hydrologic systems and the associated processes has been known for several decades now (e.g., Izzard [1966;](#page-7-0) Amorocho [1967;](#page-7-0) Amorocho and Brandstetter [1971\)](#page-7-0). However, much of early hydrologic research (especially during 1960s–1980s), largely constrained by the lack of data and computational power, resorted to linear (stochastic) approaches (e.g., Harms and Campbell [1967](#page-7-0); Klemes [1978;](#page-8-0) Salas and Smith [1981](#page-8-0)). Although the linear approaches continue to be prevalent in hydrology, advances in computational power and data collection during the last twenty years or so have facilitated proposal and application of nonlinear approaches as viable alternatives. The nonlinear approaches include nonlinear stochastic methods, artificial neural networks, data-based mechanistic models, and deterministic chaos theory, among others. The outcomes of applications of these approaches for hydrologic modeling and prediction are certainly encouraging, especially considering the fact that we are still in the 'exploratory stage' in regards to such approaches, as opposed to the much more established linear stochastic approaches. For details on these nonlinear approaches and their applications, the reader is referred to, for example, Kavvas ([2003\)](#page-7-0) for nonlinear stochastic methods, Govindaraju [\(2000](#page-7-0)) for artificial neural networks, Young and Beven ([1994\)](#page-9-0) for data-based mechanistic

models, and Sivakumar [\(2000](#page-8-0)) for deterministic chaos theory.

Among the nonlinear approaches, deterministic chaos theory, with its philosophy that complex and randomlooking behaviors could also be the result of even simple nonlinear deterministic dynamics with sensitive dependence on initial conditions (Lorenz [1963](#page-8-0)), seems to be 'the simplest' yet also remains 'the most controversial' [see Schertzer et al. ([2002\)](#page-8-0) and Sivakumar et al. [\(2002a\)](#page-8-0) for a discussion]. Amid this controversy, however, the theory has also been finding increasing applications in hydrology in recent times. Very early studies on chaos theory applications in hydrology essentially focused on the investigation and prediction of chaos in rainfall, river flow, temperature and lake volume data in a purely single-variable data reconstruction sense (e.g., Rodriguez-Iturbe et al. [1989;](#page-8-0) Wilcox et al. [1991](#page-9-0); Berndtsson et al. [1994;](#page-7-0) Jayawardena and Lai [1994;](#page-7-0) Abarbanel and Lall [1996](#page-7-0); Koutsoyiannis and Pachakis [1996](#page-8-0); Sangoyomi et al. [1996](#page-8-0); Puente and Obregon [1996](#page-8-0); Porporato and Ridolfi [1997](#page-8-0)). Subsequent studies attempted chaos theory applications on other hydrologic problems, including scaling and data disaggregation, missing data estimation, and reconstruction of system equations (e.g., Sivakumar [2001a](#page-8-0), [b;](#page-8-0) Sivakumar et al. [2001b;](#page-8-0) Elshorbagy et al. [2002a;](#page-7-0) Zhou et al. [2002](#page-9-0)), and other processes, such as rainfall-runoff and sediment transport (e.g., Sivakumar et al. [2001a](#page-8-0); Sivakumar [2002](#page-8-0); Sivakumar and Jayawardena [2002\)](#page-8-0). They also addressed some of the important issues that had been, and continue to be, perceived to significantly influence the outcomes of chaos methods when applied to real hydrologic data, including minimum data size, data noise, presence of zeros, selection of optimal parameters, and multi-variable data reconstruction (e.g., Wang and Gan [1998;](#page-9-0) Sivakumar et al. [1999a](#page-8-0), [b](#page-8-0), [2002c;](#page-9-0) Jayawardena and Gurung [2000](#page-7-0); Porporato and Ridolfi [2001](#page-8-0); Sivakumar [2001b;](#page-8-0) Elshorbagy et al. [2002b;](#page-7-0) Jayawardena et al. [2002](#page-7-0); Phoon et al. [2002](#page-8-0)). Further, they investigated the 'superiority' of chaos theory, if any, over other theories, such as stochastic methods and artificial neural networks, for prediction purposes (e.g., Jayawardena and Gurung [2000](#page-7-0); Lambrakis et al. [2000](#page-8-0); Lisi and Villi [2001;](#page-8-0) Sivakumar et al. [2002b](#page-9-0), [c;](#page-9-0) Laio et al. [2003](#page-8-0)). Extensive reviews of these studies are already available in the literature (Sivakumar [2000,](#page-8-0) [2004a\)](#page-8-0) and, therefore, details are not reported herein.

The realization and recognition, in the aftermath of the encouraging outcomes from most of the above studies, that chaos theory could provide a new perspective and alternative avenues towards understanding the workings of hydrologic systems and processes have been important driving forces for its ever-increasing applications, despite the continuing skepticisms (sometimes valid nevertheless) being thrown away from some quarters of the hydrologic community citing possible 'blind' applications of these 'less-understood' concepts without recognizing their potential limitations for real hydrologic data (the result of which could be 'false claims'). Although this is indeed heartening, we must also not lose sight of the fact that the true potential of chaos theory in hydrology can only be realized when it is attempted to solve the more challenging problems we are faced with (e.g., hydrologic scaling and model parameter estimation problems), rather than simply chaos detection and prediction (for historical data, to be more precise). Identification of these challenging problems and evaluation of how chaos theory (either independently or in combination with others in an integrated manner) can be helpful towards solving them are crucial for true progress in hydrology. These issues are the motivation for the present study.

To address these issues in an effective manner, it is important foremost to be well aware of the latest developments in chaos theory applications in hydrology and the significant inroads we have been able to make thus far. To this end, a review of studies carried out in this area during the last few years [especially since the reviews by Sivakumar ([2000,](#page-8-0) [2004a](#page-8-0))] is first presented. With this status quo, which already identifies some of the challenging problems in hydrology and also hints at the utility and appropriateness of chaos theory (e.g., Sivakumar [2004b](#page-8-0)), potential scope and directions for further applications are then highlighted. A strong case is finally made, from both philosophical and scientific perspectives, for the urgent need to formulate a 'middle-ground approach' towards a more balanced and realistic representation of all the relevant properties of hydrologic systems and processes (linear or nonlinear, stochastic or deterministic), rather than sticking to the 'extreme views' that unfortunately prevail in our current research practice.

#### 2 Latest developments on chaos in hydrologic systems

Since the reviews by Sivakumar [\(2000](#page-8-0), [2004a\)](#page-8-0), nonlinear dynamic and chaos concepts have found their applications not only continued along the directions of earlier studies but also started in other areas of hydrology as well, including scaling, groundwater contamination, parameter optimization, and catchment classification. A brief review of these studies is presented next.

### 2.1 Scale and scale-invariance

Hydrologic processes arise as a result of interactions between climatic inputs and landscape characteristics that occur over a wide range of space and time scales. Due to the tremendous variabilities in climatic inputs (e.g. rainfall,

temperature, wind velocity) and heterogeneities in landscape properties (e.g. basin area, soil type, land use, slope), hydrologic processes may have high variability at all scales and across scales. However, hydrologic systems and processes have also been shown (e.g., Gupta and Waymire [1990;](#page-7-0) Blöschl and Sivapalan [1995;](#page-7-0) Rodriguez-Iturbe and Rinaldo [1997;](#page-8-0) Gupta [2004\)](#page-7-0) to exhibit scaling or scaleinvariance (i.e. properties of the system/process at different scales are independent of the scale of observation), which serves as an important basis for transformation of data from one scale to another, among others. In this context, Regonda et al. ([2004](#page-8-0)) investigated the type of scaling behavior (stochastic or chaotic) in the temporal dynamics of river flow, employing the correlation dimension method (Grassberger and Procaccia [1983\)](#page-7-0). Analyzing daily, 5-day, and 7-day flow data from each of three rivers in the United States, they reported the presence of chaotic scaling behavior in the flow dynamics of the Kentucky River (Kentucky) and the Merced River (California), and stochastic scaling behavior in the flow dynamics of the Stillaguamish River (Washington state). They also observed an increase in the dimensionality (or complexity) of the flow dynamics with the scale of aggregation; in other words, dynamics changing from a less complex (more deterministic) behavior to a more complex (more stochastic) behavior with aggregation in time. Similar results on the effects of scale on hydrologic process complexity (i.e. increase in complexity or change from determinism to stochasticity with increasing time scale) were also observed by a few other studies as well (e.g., Sivakumar et al. [2004,](#page-9-0) [2006,](#page-9-0) [2007;](#page-9-0) Salas et al. [2005;](#page-8-0) Sivakumar and Chen [2007](#page-8-0)), albeit in different contexts and employing different methodologies to different systems and processes (including rainfall, river flow and sediment load). There may indeed be exceptions to this situation with no trend possibly observed in the 'scale versus complexity' relationship [see Sivakumar et al. [\(2001b\)](#page-8-0) for details], since this relationship essentially depends on, for example, rainfall characteristics (e.g. intensity, duration) and catchment properties (e.g. size of basin, land use). While further investigation is obviously needed for a more reliable interpretation and conclusion on this relationship one way or another, the presence of chaotic dynamics in flow and rainfall scaling has important implications in hydrology, since it has been a common practice to employ stochastic (random) cascade approaches for scaling investigations and for data transformation (e.g. disaggregation).

#### 2.2 Groundwater contamination

As noted by Sivakumar [\(2004a\)](#page-8-0), the field of subsurface hydrology had largely eluded the attention of earlier chaos studies [with the exception of the study by Faybishenko [\(2002](#page-7-0))]. To this end, especially with the experience gained with the surface hydrologic problems and the encouraging outcomes, Sivakumar et al. [\(2005](#page-9-0)) investigated the potential use of chaos theory to understand the dynamic nature of solute transport process in subsurface formations. They analyzed, using the correlation dimension method, time series of solute particle transport in a heterogeneous aquifer medium (which was simulated using an integrated transition probability/Markov chain model, groundwater flow model, and particle transport model, for varying hydrostratigraphic conditions), with the western San Joaquin Valley aquifer system in California as a reference system. The results generally indicated the nonlinear deterministic nature of solute transport dynamics (dominantly governed by only a very few variables, on the order of three), even though more complex behavior was found to be possible under certain extreme hydrostratigraphic conditions. Later, Hossain and Sivakumar [\(2006\)](#page-7-0) studied the spatial patterns of arsenic contamination in the shallow wells (*\*150 m) of Bangladesh, employing the correlation dimension method. Particular emphasis was given to the role of regional geology (Pleistocene vs. Holocene) on the spatial dynamics of arsenic contamination. The results, with correlation dimensions ranging between 8 and 11 depending on the region, suggested that the arsenic contamination dynamics in space is a medium- to high-dimensional problem. These results were further verified using logistic regression, with an attempt to explore possible (physical) connections between the correlation dimension values and the mathematical modeling of risk of arsenic contamination (Hill et al. [2008](#page-7-0)). Eleven variables were considered as indicators of the aquifer's geochemical regime with potential influence on arsenic contamination, and a total of 2,048 possible combinations of these variables was included as candidate logistic regression models to delineate the impact of the number of variables on the prediction accuracy of the model.

#### 2.3 Parameter estimation and optimization

With the ever-increasing complexities of hydrologic models, which require more details about processes and more parameters to be calibrated, parameter estimation and optimization has become an extremely challenging problem [see, for example, Beven ([2002\)](#page-7-0) for details]. Constructive discussions and debates on this issue, especially on the identification of the best optimization technique and on the estimation of uncertainty in hydrologic models, are starting to come to the fore (e.g., Beven and Young [2003](#page-7-0); Gupta et al. [2003;](#page-7-0) Beven [2006;](#page-7-0) Sivakumar [2008b\)](#page-8-0). While these are certainly positive signs to the long-term health of hydrologic sciences, the basic problem lies essentially with our tendency (and often driven by our technological

and methodological advances) to develop more complex models than that may actually be needed (Sivakumar [2008a](#page-8-0)). In an attempt towards simplification in our modeling practice, Sivakumar ([2004b\)](#page-8-0) proposed an approach that incorporates and integrates the chaos concept with expert advice and parameter optimization techniques. The simplification was brought out essentially through the determination (using the correlation dimension method) of the 'number' of dominant variables governing the system under study, with the use of only a limited amount of data (often data corresponding to a single variable) representing the system. Hossain et al. [\(2004](#page-7-0)), in their study of Bayesian estimation of uncertainty in soil moisture simulation by a land surface model (LSM), presented a simple and improved sampling scheme (within a Monte Carlo simulation framework) to the generalized likelihood uncertainty estimation (GLUE) by explicitly recognizing the nonlinear deterministic behavior between soil moisture and land surface parameters in the stochastic modeling of the parameters' response surface. They approximated the uncertainty in soil moisture simulation (i.e. model output) through a Hermite polynomial chaos expansion of normal random variables that represent the model's parameter (model input) uncertainty. The new scheme was able to reduce the computational burden of random Monte Carlo sampling for GLUE in the range of 10–70%, and it was also found to be about 10% more efficient than the nearestneighborhood sampling method in predicting a sampled parameter set's degree of representativeness.

## 2.4 Catchment classification

The realization that hydrologic models are often developed for specific situations and thus that their extensions and generalizations to other situations are difficult has recently motivated some researchers to call for a catchment classification framework (Woods [2002](#page-9-0); Sivapalan et al. [2003](#page-9-0); McDonnell and Woods [2004\)](#page-8-0). These researchers also suggest, largely motivated by the proposal of the dominant processes concept (Grayson and Blöschl [2000](#page-7-0)), that identification of dominant processes may help in the formation of such a classification framework. With this idea, Sivakumar [\(2004b](#page-8-0)) introduced a classification framework, in which the extent of complexity or dimensionality (determined using nonlinear tools, such as the correlation dimension method) of a hydrologic 'system' was treated as a representation of the (number of) dominant processes. Following up on this, Sivakumar et al. ([2007\)](#page-9-0) explored the utility of the phase space reconstruction concept (e.g. Packard et al. [1980](#page-8-0); Takens [1981\)](#page-9-0), in which the 'region of attraction of trajectories' in the phase space was used to identify the data as exhibiting 'simple' or 'intermediate' or 'complex' behavior and, correspondingly, classify the system as potentially low-, medium- or high-dimensional. The utility of this reconstruction concept was first demonstrated on two artificial time series possessing significantly different characteristics and levels of complexity [a purely random series with independent and identically distributed numbers and a deterministic chaotic series generated using the two-dimensional Henon map (Henon [1976\)](#page-7-0)], and then tested on a host of river-related data (flow, suspended sediment concentration and suspended sediment load) representing different geographic regions, climatic conditions, basin sizes, processes and scales. The ability of the phase space to reflect the river basin characteristics and the associated mechanisms, such as basin size, smoothing, and scaling, was also observed.

## 2.5 Others

There have also been several other studies that have, in one way or another, looked into the applications of nonlinear dynamic and chaos theories in hydrology. These include: applications to yet other hydrologic processes, proposals of new ways for hydrologic data analysis, and investigations on the reliability of chaos methods to hydrologic data. A very brief account of such studies is presented next, not necessarily in any specific order.

Manzoni et al. [\(2004](#page-8-0)) studied the soil carbon and nitrogen cycles from a dynamic system perspective, wherein the system nonlinearities and feedbacks were analyzed by considering the steady-state solution under deterministic hydro-climatic conditions. Laio et al. ([2004\)](#page-8-0) employed the deterministic versus stochastic (DVS) method (e.g. Casdagli [1992](#page-7-0)) to daily river discharge from three Italian rivers in their investigation of nonlinearity in rainfall-runoff transformation. Dodov and Foufoula-Georgiou ([2005\)](#page-7-0) studied the nonlinear dependencies of rainfall and runoff and the effects of spatio-temporal distribution of rainfall on the dynamics of streamflow at flood time scales. They proposed a framework based on 'hydrologically-relevant' rainfall-runoff phase space reconstruction, but with specific acknowledgment that rainfall-runoff is a stochastic spatially extended system rather than a deterministic multivariate one. Khan et al. [\(2005](#page-8-0)), Sivakumar [\(2005b\)](#page-8-0), and Koutsoyiannis ([2006\)](#page-8-0) investigated the reliability of the correlation dimension method in the detection of chaos in hydrologic time series. They addressed, among others, the effects of data size, random and seasonal components, zeros, intermittency, and high autocorrelations. Jin et al. ([2005\)](#page-7-0) studied the nonlinear relationships between southern oscillation index (SOI) and local precipitation and temperature (in Fukuoka, Japan), by representing this joint hydro-climatic system using a nonlinear multivariate phase space reconstruction technique. Regonda et al. [\(2005](#page-8-0)) presented a nonparametric approach

based on local polynomial regression for ensemble forecast of time series, and demonstrated its effectiveness on the biweekly series of the Great Salt Lake volume, among others. Nordstrom et al. [\(2005](#page-8-0)) proposed the construction of a dynamic area fraction model (DAFM) that contains coupled parameterizations for all the major components of the hydrologic cycle involving liquid, solid and vapor phases. Using this model, which shares some of the characteristics of an Earth system model of intermediate complexity, they investigated the nature of feedback processes in regulating Earth's climate as a highly nonlinear coupled system. Still other studies of interest are those by Phillips and Walls [\(2004](#page-8-0)), Tsonis and Georgakakos [\(2005](#page-9-0)), She and Basketfield ([2005\)](#page-8-0), Gaume et al. [\(2006](#page-7-0)), Phillips  $(2006)$  $(2006)$ , and Sivakumar  $(2007)$  $(2007)$ , among others.

## 3 Challenges ahead

It must be clear by now that we have made some sincere efforts to explore the potential of nonlinear dynamic and chaos concepts for modeling and prediction of hydrologic systems and processes. The outcomes of these efforts are certainly encouraging, considering that we are still in the state of infancy in regards to these concepts when compared to the much more mature and prevalent linear stochastic concepts [this is not to say that we have achieved the 'full-fledged' status with the stochastic concepts]. The additional inroads we have made in recent years in the areas of scaling, groundwater contamination, parameter estimation and optimization, and catchment classification, among others, are significant, albeit their preliminary nature, since these are arguably some of the most important topics in hydrology at the current time.

With these positives, however, we must not forget the challenges that lie ahead on our way to further progress. Among these challenges, two are noteworthy: (1) improving our understanding of these largely less-understood chaos concepts for hydrologic applications; and (2) finding ways to integrate these concepts with the others, either already in existence or emerging in the future. The former is important for avoiding 'blind' applications of the related methods (simply because the methods exist and are there to apply!) and 'false' claims (either in support of or against their utility); and the latter is important for taking advantage of the merits of the different approaches for their 'collective utility' to solve hydrologic problems rather than for their 'individual brilliance' as perceived. The rest of this section presents some examples to the potential limitations of the above studies and to the challenges ahead.

The studies by, for example, Regonda et al. [\(2004](#page-8-0)) and Salas et al. [\(2005](#page-8-0)) provide interesting insights into the

problem of scaling and effects of data aggregation. Their message, in essence, is that complexity of the system dynamics increases (often from a more deterministic nature to a more stochastic nature) with aggregation in time scale. Although this may indeed be the case in certain situations, its generalization is often difficult to make, since the system's dynamic complexity depends on the climatic inputs and the catchment characteristics. For example, the catchment area (and, hence, the time of concentration, not to mention the rainfall characteristics) plays a vital role in defining the relationship between scale and dynamic complexity. In fact, depending upon the catchment, the dynamic complexity may increase with aggregation in time up to a certain point (probably, somewhere close to the concentration time) and then decrease with further aggregation [see Sivakumar et al. ([2001b\)](#page-8-0) for some details, in a rainfall disaggregation context].

The attempts by Sivakumar et al. ([2005](#page-9-0)) and Hossain and Sivakumar ([2006\)](#page-7-0) to search for possible nonlinear deterministic dynamics in solute transport in a heterogeneous aquifer and arsenic contamination in shallow wells are certainly interesting. However, these studies are, at best, crude one-dimensional approximations to the complex threedimensional groundwater flow and transport phenomena. They only consider the time or space (as the case may be), but what is actually needed is a spatio-temporal perspective. Moreover, although there is no 'mathematical' constraint, the 'philosophical' merit behind the use of phase space reconstruction concept in a spatial context (with its delay parameter defined in space), as is done in Hossain and Sivakumar ([2006](#page-7-0)), remains an issue to ponder.

The proposal by Sivakumar [\(2004b](#page-8-0)) on the integration of different concepts (and methods) to deal with the workings of hydrologic systems, and more specifically to simplify our modeling and parameter estimation practices, is a notable move forward, since different concepts possess different advantages and limitations. However, the utility and effectiveness of this proposal are yet to be seen through implementation. Further, speaking in a more general sense, recognition of the advantages and limitations of each of the concepts in itself is a challenging task, as such requires adequate knowledge of all of the concepts in the first place. This probably makes the idea of integration of different concepts less appealing, certainly in the context of our increasing emphasis on individual concepts in our research [see Sivakumar ([2005a](#page-8-0)) for details].

The proposal on the use of the phase space reconstruction approach for 'system classification' and also its effective demonstration and testing on synthetic and riverrelated data, as presented by Sivakumar et al. [\(2007](#page-9-0)), seem to provide strong clues to the potential of such an approach for formulation of a catchment classification framework. What remains to be studied, however, is how to incorporate

the catchment characteristics into this classification framework and how to establish connections between data (usually at the catchment scale) and the actual catchment physical mechanisms (at all scales) for this classification framework to be successful [see Wagener et al. [\(2007\)](#page-9-0) for further discussion on catchment classification framework, especially in the context of hydrologic similarity]. Moreover, nonlinearity and chaos is not just about small changes leading to large effects and complex-looking outputs coming out of simple systems, but it is also about large changes leading to small effects and simple-looking outputs coming out of complex systems. Whether or not the phase space reconstruction approach can also perform equally well for this latter situation remains to be seen.

As Koutsoyiannis ([2006\)](#page-8-0) pointed out [see also Tsonis et al. [\(1994](#page-9-0)) and Sivakumar [\(2001b](#page-8-0))], the presence of periods of zeros in a time series could result in an underestimation of the correlation dimension and (in the absence of any other analysis) could potentially lead to the conclusion that chaos exists, when actually it does not. This can indeed turn out to be a serious issue in chaos studies in hydrology, since zero values are a common occurrence in hydrologic time series (especially high-resolution rainfall). The fact that zero values are intrinsic to the system dynamics and thus must not be removed in any hydrologic analysis [possible exception being disaggregation analysis (Sivakumar et al. [2001b](#page-8-0))] makes the problem only more complicated. This does not, in any way, mean that the correlation dimension method must not be employed to hydrologic series, because dimension is simply a representation of the variability of the time series values (zeros included). What is required to realistically deal with this problem, however, is an adequate definition of what constitutes 'periods' (or a large number) of zeros; in other words, what is the 'threshold' for the number (or percentage) of zeros in a time series to obtain a reliable estimation of correlation dimension? This question is hard to answer, because determination of the 'sensitivity' of correlation dimension to the number of zeros is not straightforward, even for artificially generated time series (let alone real series). It is also important to recognize that this question is not just limited to zeros but goes well beyond that, since it is simply a problem caused by the 'repetition' of one or more values and that such repetitions may occur in many different ways depending upon the system (e.g. minimum streamflow, average temperature, maximum/minimum water level in a reservoir, water release from a reservoir, daily suspended sediment load).

Global climate change, it is believed, will have threatening consequences for our water resources during this century, and there are already noticeable indications, with an increase in abnormal events (e.g. floods and droughts) around the world. Since global climate models (GCMs) provide climate data only at much coarser spatial and temporal scales than that are required for hydrologic predictions at regional and local levels, 'downscaling' of GCM outputs is essential. The existing statistical and dynamic downscaling techniques can indeed provide some success [see, for example, Fowler et al.  $(2007)$  $(2007)$  for an extensive review of downscaling techniques], but the assumption of linearity inherent in almost all of these techniques is too simplistic and thus may greatly constrain their effectiveness, since the climate systems and the associated processes are essentially nonlinear, and possibly chaotic (e.g., Lorenz [1963](#page-8-0)). In view of this, there is increasing realization on the urgent need for formulation of nonlinear downscaling approaches (with explicit consideration given to the system's chaotic properties), but unfortunately nothing seems to have been done yet. This downscaling problem is also a complex spatio-temporal problem [and probably much more complex than groundwater flow and contamination (and also the rainfall-runoff problem), discussed above], and thus may necessitate significant modifications to the single-variable, and even multi-variable, phase space reconstruction approach that has been employed in hydrologic and climatic studies thus far. The study by Sivakumar et al. ([2001b\)](#page-8-0), proposing a nonlinear dynamic disaggregation approach for rainfall, may provide some useful clues to deal with this problem, but that too will only be of limited use and in a purely temporal sense. Although some efforts to pursue research in this direction are currently underway (e.g., Sivakumar [2008c\)](#page-8-0), there is certainly a long way to go.

Finally, the methods that have been employed by studies thus far on chaos in hydrologic systems are essentially data based, and thus their relevance to the actual physical mechanisms and system dynamics may be questioned. For example, an essential first step in any chaos method is the reconstruction of the phase space, wherein the idea is that a (nonlinear hydrologic) system is characterized by selfinteraction and that a time series of a single variable can carry the information about the dynamics of the entire multi-variable system. Although it is possible to provide convincing explanation of the relevance of this idea to some overall scenarios (e.g. input-output, rainfall-runoff), explanation as to the relevance of the parameters to specific system components is very difficult. This may be elucidated through an example, as follows. There is sufficient information in the history of runoff data about the rainfall properties and the catchment characteristics over a period of time, and thus a single-variable phase space reconstruction (with runoff) should be able, at least theoretically, to reflect the system's dynamic changes. At the same time, however, how the delay time in the embedding procedure (e.g., Takens [1981](#page-9-0)) is related to any of the system components and/or dynamics is hard to explain. The absence of consensus on the appropriate method for the selection of optimum delay time (e.g., Holzfuss and Mayer-Kress [1986](#page-7-0); Frazer and Swinney [1986](#page-7-0); Leibert and Schuster [1989;](#page-8-0) Kim et al. [1999\)](#page-8-0) makes this issue only more difficult to deal with. This issue, and similar ones, must be resolved for any hope towards establishing links between data and physics and thus to truly reflect the advances that can be made with nonlinear dynamic and chaos concepts. This situation, however, is not just specific to chaos methods; it is much more widespread, and is applicable to literally all time series methods [see, for example, Kirchner [\(2006](#page-8-0)) for some relevant details on models vs. measurements]; but that must not be a solace in any case [see Sivakumar [\(2008a\)](#page-8-0) for some details].

#### 4 Conclusion—striving for a middle ground

Every human being has his/her own perceptions about the workings of Nature, which, to a great extent, are influenced by his/her societal, cultural, economic and environmental backgrounds, among others. These perceptions, I believe, are generally the driving force for choosing his/her field of study and research and also for identifying the methods for applications. Continuation of this (trend), however, is becoming increasingly difficult these days with our society's lifestyles and pressures, largely driven by the economy. One thing seems to be emerging though: people are becoming 'specialized' and 'specialists.'

We, researchers in hydrology, are also starting to put more emphasis on applications of specific concepts and methods [including this article, I might add!] rather than on addressing the most challenging hydrologic problems affecting us all, if literature is any indication [see, for example, Sivakumar ([2005a](#page-8-0)) for details]. We are also increasingly realizing that none of the currently available tools (linear or nonlinear, stochastic or deterministic) by itself is adequate for solving our hydrologic problems to our satisfaction; for example, no model (how sophisticated it may be) has been shown to reliably capture the extreme hydrologic events at a given location, and extensions and generalizations of a model (developed for some situations) to other situations are often difficult. The following is only a small sample of the numerous questions that need to be asked about our existing research approaches and methods to tackle the challenges in hydrology: (1) How are we going to incorporate the nonlinear deterministic components that are inherent in hydrologic systems and processes into our linear (and nonlinear) stochastic approaches? (2) How are we going to address the property of sensitive dependence of hydrologic system dynamics on the initial conditions, when the initial conditions themselves cannot be known? (3) How are we going to explain the 'random'

and unpredictable system behavior using our (nonlinear) deterministic approaches? (4) How are we going to estimate the uncertainty in the parameters that serve as important inputs to our complex models and, even worse, how are we going to define 'uncertainty' in the first place [see also Sivakumar [\(2008b](#page-8-0)) for some details]? (5) How are we going to establish the 'connections' between our 'data-based' approaches and the 'process-based' approaches, especially when there are 'disconnections' (either intentional or unintentional) in our research approaches?

These are difficult questions to answer, and the only way, in my opinion, to do that is to find some 'common grounds' in our approaches to research. This does not mean that one approach has to 'give up' to make way for another, but this certainly requires some kind of 'compromise' and 'sacrifice' for the betterment of hydrologic research. To this end, bringing together the different approaches and concepts could be a good starting point, as they could supplement and complement one another (and also could be used in an integrated manner), for reliably representing all the relevant properties of hydrologic systems and processes (order or disorder, linear or nonlinear, stochastic or deterministic). This, however, is an extremely challenging task, especially with our unyielding fascination for specific concepts and methods for their 'individual brilliance' that reflects, more often than not, only our one-sided 'extreme' views. What is urgently needed, therefore, are sincere efforts to explore the 'collective utility' of different concepts for studying hydrologic systems. This warrants, as experience suggests, a change in our research paradigm and attitude [see, for example, Gupta et al. [\(2000](#page-7-0)) for some details]; that is, willingness to recognize the potential of different concepts and openness to accept the outcomes. Discussions and debates are obviously essential to this change, and constructive criticisms and useful inputs, from all sides, would certainly help towards defining and demarcating the travel path for this.

A general statement on the need for a 'middle-ground' approach is an easy thing to make, but the real challenge lies in identifying specific hydrologic problems and appropriate concepts/methods that could, in combination, yield the desired effectiveness and efficiency. It is premature, at this stage, to pinpoint these areas, mainly because of our inadequate knowledge of all the concepts [see Sivakumar  $(2005a)$  $(2005a)$  $(2005a)$  for some details] and, to some extent, of the hydrologic problems themselves [Vijay Gupta, personal communication]. However, the situation is not that bleak, and there are already some encouraging signs to these areas. For example, the study by Sivakumar et al. ([2004\)](#page-9-0) suggests that nonlinear deterministic approaches generally provide better results for streamflow disaggregation over finer resolutions than over coarser ones, while parametric and nonparametric stochastic

<span id="page-7-0"></span>approaches have been reported to provide very good results over coarser resolutions (e.g. Lin [1990;](#page-8-0) Tarboton et al. [1998\)](#page-9-0). With these observations, it might be worthwhile to couple, for example, the nonlinear deterministic approach and the linear stochastic approach for streamflow disaggregation over a much wider range of scales (e.g. between daily and annual) than the range of scales studied thus far with these approaches independently [see Sivakumar et al. [\(2004](#page-9-0)) for some details on the similarities and differences between these approaches]. Such an approach could help towards a better view of the streamflow disaggregation problem, and also provide new avenues to the study of scale problems in hydrology at large. At the end, a 'middleground' approach that can yet capture the 'extremes' would certainly be a new chapter in hydrology!

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