

Additive crown width models comprising nonlinear simultaneous equations for Prince Rupprecht larch (*Larix principis-rupprechtii*) in northern China

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Abstract

Key message Crown width models developed using nonlinear simultaneous equations with a two-step procedure provided the best performance and are recommended to predict the crown components and crown width of Prince Rupprecht larch.

Abstract Crown width (CW) is defined as an average of two crown diameters at two cardinal directions: east–west and south–north, obtained from measurements of four crown radii (crown components) at four directions: east, west, south, and north. CW is one of the important tree variables in forest growth and yield modeling, and forest management. Reliable estimates of CW are central elements of forest management. However, the additivity of CW and crown components and their inherent correlations have not been addressed in existing CW models. In this

study, two alternative procedures for nonlinear simultaneous equations (NSE) were used to develop CW models. The procedures included a disaggregation model structure with one- and two-step, proportional weighting systems, and two commonly used additivity methods, adjustment in proportion (AP) and ordinary least squares with separating regression (OLSSR). These methods were compared using data from a total of 3369 Prince Rupprecht larch (*Larix principis-rupprechtii* Mayr.) trees in 116 permanent sample plots in northern China. It was found that these methods effectively ensured that the sum of the crown components was equal to twice the total CW. The NSE accounted for the inherent correlations among the crown components and CW. The CW models developed using the NSE with the two-step procedure provided the best performance, followed by the models developed with AP and OLSSR. This methodology can be adapted to develop a system of CW models for other tree species.

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Introduction

Crown width (CW) is defined as an average of two crown diameters at two cardinal directions: east–west and south–north, obtained from measurements of four crown radii (crown components), namely east crown radius (CR_E), west crown radius (CR_W), south crown radius (CR_S), and north crown radius (CR_N) (Bragg 2001). It is a useful measure of tree vigor (Assman 1970; Hasenauer and Monserud 1997; Hynynen et al. 2002), mortality (Monserud and Sterba 1996), and above-ground biomass

(Carvalho and Parresol 2003; Fu et al. 2016). CW can be also used in ecological modeling to predict light interception in tree canopy (Oker-Blom et al. 1989; Pukkala et al. 1991). However, despite its numerous benefits, measuring the CW of every sampled tree is costly and time-consuming (Bragg 2001; Sönmez 2009; Fu et al. 2013; Sharma et al. 2016). Thus, accurate CW models are required for accurate predictions of CW.

CW estimates obtained from measurements of stand and tree characteristics serve as input information to deterministic or stochastic CW models (Biging and Wensel 1990; Baldwin and Peterson 1997; Bragg 2001; Sönmez 2009; Fu et al. 2013; Hao et al. 2015; Sharma et al. 2016). Methods of developing CW models have evolved from simple ordinary least squares (OLS) regression to linear mixed-effects (LME) modeling, and then to nonlinear mixed-effects (NLME) modeling (Sánchez-González et al. 2007; Fu et al. 2013; Hao et al. 2015; Sharma et al. 2016; Fu et al. 2017a). All existing CW models have been developed using the average of two crown diameters as a function of tree variables (e.g., diameter at breast height, total tree height, height to crown base, and the height–diameter ratio) and stand variables (e.g., dominant height, site index, and stand density measures) using OLS regression or an LME/NLME modeling approach.

For each tree in a specific stand, especially for natural forests, the differences among the crown components (CR_E , CR_W , CR_S , and CR_N) are usually very large because of vigorous competition with neighboring trees (Power et al. 2012). The CW and crown components are also strongly correlated with each other (Power et al. 2012). However, none of the CW models that were developed using existing methods have accounted for the differences among the relationships between each of four crown components and predictor variables of the CW models. In addition, correlations among the crown components have also been ignored. Thus, these CW models are not estimated efficiently, and they do not account for additivity among the crown components (Parresol 1999). The lack of additivity in existing modeling methods results in an inconsistency, as the sum of the predicted values from the crown components models does not equal the predicted value from the CW models (Kozak 1970). In addition, these CW models also have fallen short of statistical efficiency in terms of parameter estimation. These limitations might result in low prediction accuracies of the CW models (Tang et al. 2015).

A potential solution to the problem is the use of nonlinear simultaneous equations (NSEs) that not only ensure the additivity of nonlinear CW and crown components models, but also account for the correlations among the crown components (Fuller 1987; Kangas 1998; Tang and

Zhang 1998; Tang et al. 2001, 2015; Carroll et al. 2006). For example, Tang et al. (2000) proposed a disaggregation model structure using an NSE, namely a one-step, proportional weighting system, in which an above-ground biomass model is first developed, and then the estimated above-ground biomass is disaggregated into tree components (e.g., wood, bark, branch, and foliage) based on their proportions in the above-ground biomass. Furthermore, if subtotal biomass estimates are also desirable, the estimated above-ground biomass is first disaggregated into subtotals (e.g., stem and crown) based on their proportions in the above-ground biomass. Then, the estimated stem biomass is divided into wood and bark based on their proportions in the stem biomass, and the estimated crown biomass is divided into branch and foliage based on their proportions in the crown biomass. This constitutes a two-step, proportional weighting system (Tang et al. 2000). Tang et al. (2000) found that the system of biomass equations developed using this method effectively ensured additivity, and had high prediction accuracy.

In recent years, NSE has been increasingly used in forestry to develop various additive models, especially additive biomass equations (Parresol 1999; Bi et al. 2004; Dong et al. 2015, 2016; Fu et al. 2016, 2017b). It has been found that NSE ensures high additivity efficiencies of the forest models, and strong abilities of accounting for correlations among the components. NSE increases the prediction accuracy of the models, compared with other non-additive methods, such as OLS methods (Lindstrom and Bates 1990; Vonesh and Chinchilli 1997). To our knowledge, however, no study has used NSE to ensure the additivity of crown components during the estimation of CW.

In addition to NSE, two other methods, adjustment in proportion (AP) and an OLS with separating regression (OLSSR), are commonly used for forcing the additivity of a set of nonlinear models (Parresol 1999, 2001; Tang et al. 2001, 2015; Tang and Wang 2002; Fu et al. 2016). AP directly partitions the total CW of a tree into its four basic crown components (CR_E , CR_W , CR_S , and CR_N) by weighting, whereas for OLSSR, a total CW regression function is defined as the half sum of the separately calculated regression functions of the crown components. However, neither the AP nor OLSSR models account for inherent correlations among the components. Regarding the NSE, no study has used these two methods to develop a system of nonlinear CW models.

The objectives of this study were: (1) to develop nonlinear CW models using NSE under two disaggregation procedures (a one-step, proportional weighting system and a two-step, proportional weighting system) for Prince Rupprecht larch (*Larix principis-rupprechtii* Mayr.) in natural stands in northern China, and (2) to compare the AP

and OLSSR procedures with respect to their abilities to predict CW and estimate crown components.

Materials and methods

Data

We used data from 116 permanent sample plots (PSPs) that were established in natural stands of Prince Rupprecht larch in the state-owned Guandi mountain forest (67 PSPs) and the state-owned Boqiang forest (49 PSPs) of northern China (Fig. 1). Each PSP was a 0.04 ha square. The PSPs were selected in such a way that they provided representative information for a variety of stand structures and densities, tree heights and ages, and site productivity.

Within each PSP, for all standing and living trees with diameter at breast height ($D \geq 5$ cm), we measured total tree height (H), height to crown base (height above ground to crown base, HCB), and the four crown radii (CR_E , CR_W , CR_S , and CR_N). Tree HCB was defined as the height from the ground to the base of the first normal green branch as a part of the crown; this excluded the secondary branches (epicormics and adventitious) (Hasenauer and Monserud 1996). Furthermore, a single green branch was not the base of tree crown if there were at least three whorls above it (Hasenauer and Monserud 1996). The forked trees with the forks below 1.3 m were treated as separate trees and otherwise, as a single tree. The positions of four crown radii of each tree were determined by two cardinal directions (Bragg 2001). The first cardinal direction was defined as the direction from south to north (the corresponding

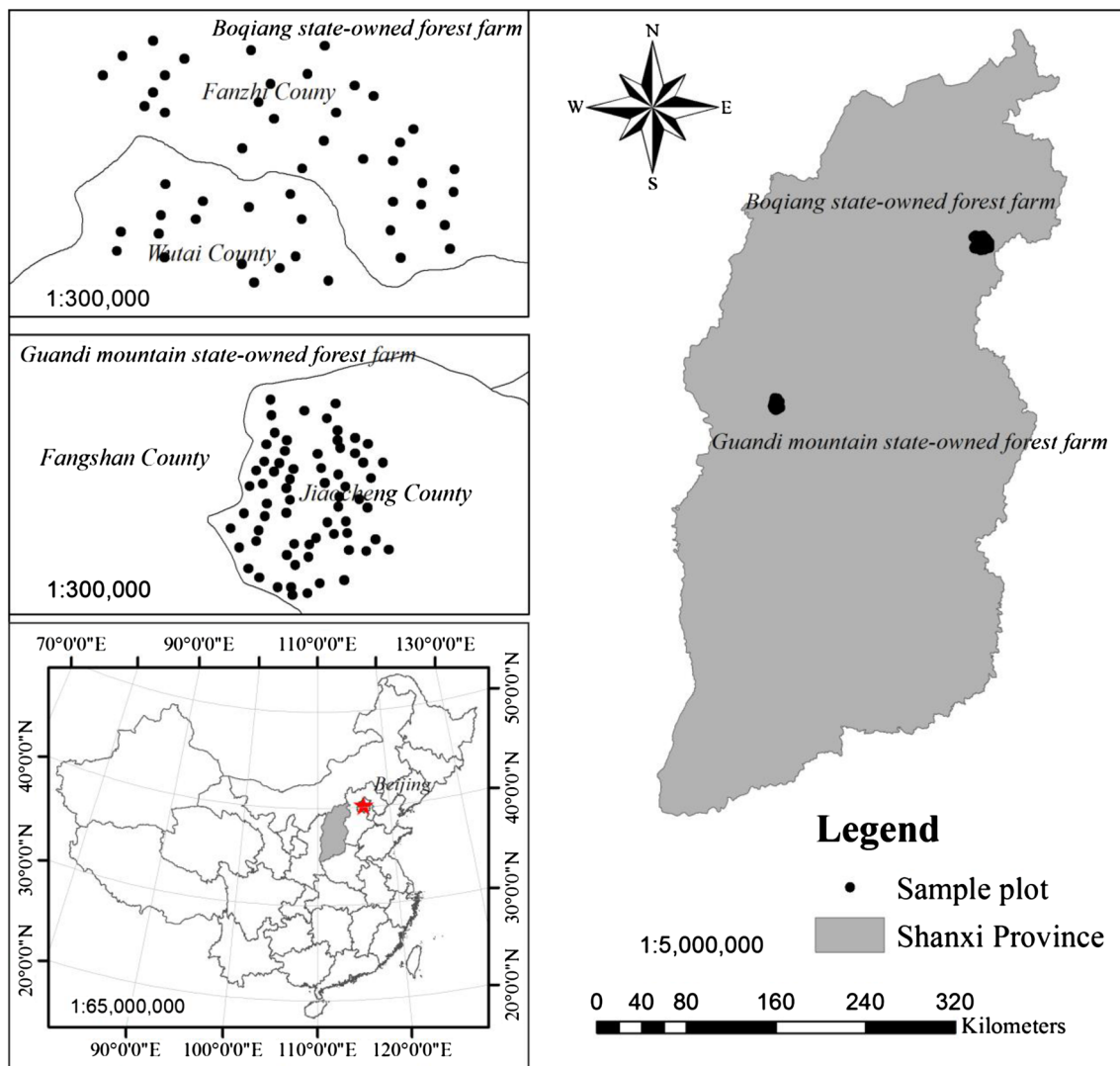


Fig. 1 Location of the study area consisting of the Guandi mountain forest and the Boqiang forest in northern China (lower left and right), and the spatial distribution of permanent sample plots (upper left)

crown width was defined as the south–north crown width, CW_{SN} , and the second cardinal direction was perpendicular to the first, namely the direction from east to west (the corresponding crown width is defined as the east–west crown width, CW_{EW}) (Fig. 2). The crown radii were measured as the horizontal distances between the center of a tree bole and the greatest extent of the crown from the bole (Fig. 2). The branch tip was located by vertical sighting with a clinometer (Marshall et al. 2003). CW was computed by $(CR_S + CR_N + CR_E + CR_W)/2$. Four dominant or codominant trees were identified and measured in each PSP (the proportion of the 100 thickest trees per ha) (Raulier et al. 2003). The ages of the selected dominant or codominant trees were recorded by counting the growth rings based on increment cores taken from the stem at a height of 0.1 m (above ground) (Rozas 2003). For each PSP, plot dominant tree diameter at breast height DD, plot dominant tree height DH, and dominant mean age DA were obtained from the averages of these attributes (Du et al. 2000). The relationships among CW, CR_E , CR_W , CW_{EW} , CR_S , CR_N , and CW_{SN} and the five tree variables, including D, H, HCB, DH, and DD, are shown in Fig. 3.

The PSPs were randomly divided into two groups: model-fitting and model-validation groups. The model-fitting group contained 2250 trees from 69 PSPs, while the

model-validation group consisted of 1119 trees from 37 plots. The statistics and relevant stand characteristics of the measurements are summarized in Table 1.

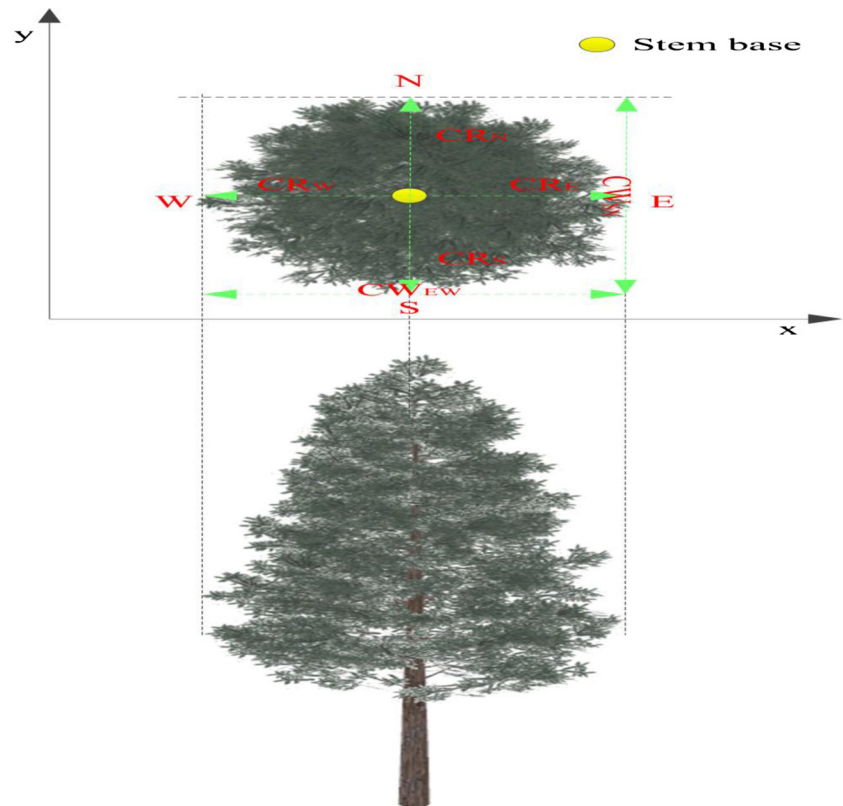
Base model

Fu et al. (2013) developed a logistic model for Chinese fir (*Cunninghamia lanceolata*) CW estimation based on D, H, HCB, and DH. They reported that their CW model provided higher predictive accuracy than other tested CW models. To account for the differences between the Guandi mountain and Boqiang forests, one additional dummy variable, P , was created; $P = 0$ denotes the Guandi mountain forest, and $P = 1$ denotes the Boqiang forest. After imposing the P on β_1 and β_2 , the modified Fu's CW model took the following form:

$$CW = f(\mathbf{x}, P, \boldsymbol{\beta}) = [\beta_1 + k_1P + (\beta_2 + k_2P)DH] / [1 + (\beta_3 + \beta_4HCB) \exp(-(\beta_5 + \beta_6H)D)] + \varepsilon \quad (1)$$

where \mathbf{x} is the covariate vector, including D, H, HCB, and DH, $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, k_1, k_2)$ is an eight-dimensional parameter vector, and ε is an error term. Model (1) was used as the base function to develop the nonlinear CW models.

Fig. 2 The positions of the east crown radius (CR_E), west crown radius (CR_W), south crown radius (CR_S), north crown radius (CR_N), south–north crown width (CW_{SN}), and east–west crown width (CW_{EW}) of a sample tree, E, W, S and N represent the east, west, south and north, respectively



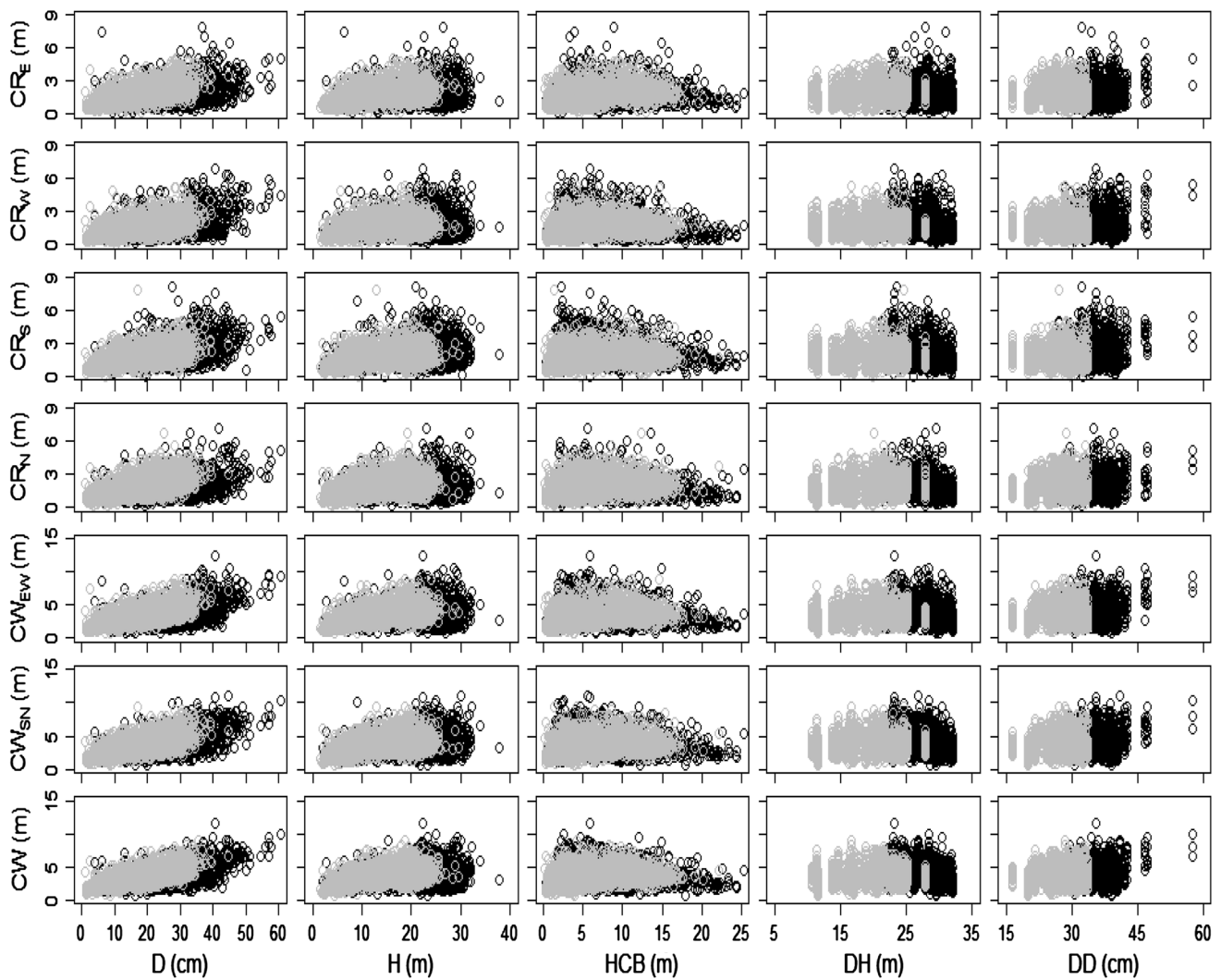


Fig. 3 The relationships among east crown radius (CR_E), west crown radius (CR_W), south crown radius (CR_S), north crown radius (CR_N), east–west crown width (CW_{EW}), south–north crown width (CW_{SN}), and crown width (CW) with five tree variables, including diameter at

breast height (D), total tree height (H), height to crown base (HCB), total height of the dominant tree (DH), and D of the dominant tree (DD) for two forests: the Guandi mountain forest denoted with black circles and the Boqiang forest denoted with gray circles

NSE

Following Tang et al. (2001), a general NSE can be formulated as follows:

$$\begin{cases} \mathbf{f}(\mathbf{y}_i, \mathbf{x}_i, \boldsymbol{\beta}) = 0, & i = 1, \dots, n \\ \mathbf{Y}_i = \mathbf{y}_i + \mathbf{e}_i \\ E(\mathbf{e}_i) = 0, & Var(\mathbf{e}_i) = \boldsymbol{\Sigma} \end{cases} \quad (2)$$

where n is the total number of observations; \mathbf{x}_i is the observed value of a q -dimensional, error-free variable (exogenous variable); \mathbf{Y}_i is the observed value of a p -dimensional, endogenous variable with error; \mathbf{y}_i is the true value of \mathbf{Y}_i ; $\boldsymbol{\Sigma}$ is the $p \times p$ dimensional, positive definite variance–covariance matrix of the error term \mathbf{e}_i , and the general expression of the matrix is given by $\boldsymbol{\Sigma} = \sigma^2 \boldsymbol{\Psi}$; σ^2 is the scaling factor for the error dispersion, which is given by

the residual variance of the model; $\boldsymbol{\Psi}$ is the $p \times p$ dimensional error structure matrix; and $\boldsymbol{\beta}$ is the $n_{par} \times 1$ dimensional parameter vector. $\mathbf{f} = (f_1, f_2, \dots, f_p)^T$, with T indicating the transpose of a matrix or vector, is a p -dimensional vector function, and in this study, it was composed of the CW and crown radii equations that were used in the base model (1) for the components.

Model (2) intrinsically ensures the compatibility of parameters for any data structure, and it does not require the specification of dependent and independent variables, as is required by the OLS model. Model (2) can be solved by a two-stage, errors-in-variables model (TSEM) algorithm (Tang and Li 2002; Fu et al. 2016). The details of NSE, TSEM estimation algorithm, and their computer implementation can be found in the studies of Tang et al. (2001, 2015).

Table 1 Summary of the statistics for the model-fitting and model-validation datasets

Variable	Model-fitting data				Model-validation data			
	Min	Max	Mean	SD	Min	Max	Mean	SD
D (cm)	5	60.5	22.2	10.1	5	57.0	21.8	9.8
H (m)	1.5	37.8	17.5	7.2	1.9	33.9	16.9	7.3
HCB (cm)	0.1	25.3	7.1	4.5	0.2	22.1	7.1	4.4
DH (m)	11.5	32.1	23.0	5.1	10.7	31.9	22.4	5.4
DD (m)	20.9	57.5	31.0	5.5	16.4	47.3	29.8	5.6
CR_E (m)	0.0	7.4	1.9	0.9	0.4	7.9	1.8	0.9
CR_W (m)	0.1	6.9	1.9	0.9	0.3	6.3	1.8	0.9
CR_S (m)	0.5	8.2	2.0	1.0	0.1	7.8	1.9	0.9
CR_N (m)	0.1	7.2	2.0	0.9	0.1	6.8	1.9	0.9
CW_{EW} (m)	0.7	12.5	3.7	1.5	0.7	10.6	3.6	1.4
CW_{SN} (m)	0.6	11.1	4.0	1.6	0.5	9.4	3.8	1.5
CW (m)	0.7	11.7	3.9	1.4	0.6	9.5	3.7	1.3

Min minimum, *max* maximum, *mean* average value, *SD* standard deviation, D diameter at breast height, H total tree height, HCB height to crown base, DH plot dominant tree height, DD plot dominant tree diameter at breast height, CR_E east crown radius, CR_W west crown radius, CR_S south crown radius, CR_N north crown radius, CW_{EW} east–west crown width, CW_{SN} south–north crown width, CW total crown width

Within the aforementioned NSE framework, there are two alternative procedures for ensuring additivity: one directly partitions the total CW of a tree into its four basic components, including CR_E , CR_W , CR_S , and CR_N , which is called a one-step, proportional weighting system; and the

the observed biomass values of CW , CR_E , CR_W , CR_S , and CR_N for the i th tree, respectively, and their values contain random errors. cw_i , cw_{Ei} , cw_{Wi} , cw_{Si} , and cw_{Ni} are the true values of CW_i , CR_{Ei} , CR_{Wi} , CR_{Si} , and CR_{Ni} , respectively. Their expressions in the NSE are given by

$$\begin{cases}
 cw_{Ei} = \frac{f_E(\mathbf{x}_i, P_i, \boldsymbol{\beta}_E)}{f_E(\mathbf{x}_i, P_i, \boldsymbol{\beta}_E) + f_W(\mathbf{x}_i, P_i, \boldsymbol{\beta}_W) + f_S(\mathbf{x}_i, P_i, \boldsymbol{\beta}_S) + f_N(\mathbf{x}_i, P_i, \boldsymbol{\beta}_N)} cw_i \\
 cw_{Wi} = \frac{f_W(\mathbf{x}_i, P_i, \boldsymbol{\beta}_W)}{f_E(\mathbf{x}_i, P_i, \boldsymbol{\beta}_E) + f_W(\mathbf{x}_i, P_i, \boldsymbol{\beta}_W) + f_S(\mathbf{x}_i, P_i, \boldsymbol{\beta}_S) + f_N(\mathbf{x}_i, P_i, \boldsymbol{\beta}_N)} cw_i \\
 cw_{Si} = \frac{f_S(\mathbf{x}_i, P_i, \boldsymbol{\beta}_S)}{f_E(\mathbf{x}_i, P_i, \boldsymbol{\beta}_E) + f_W(\mathbf{x}_i, P_i, \boldsymbol{\beta}_W) + f_S(\mathbf{x}_i, P_i, \boldsymbol{\beta}_S) + f_N(\mathbf{x}_i, P_i, \boldsymbol{\beta}_N)} cw_i \\
 cw_{Ni} = \frac{f_N(\mathbf{x}_i, P_i, \boldsymbol{\beta}_N)}{f_E(\mathbf{x}_i, P_i, \boldsymbol{\beta}_E) + f_W(\mathbf{x}_i, P_i, \boldsymbol{\beta}_W) + f_S(\mathbf{x}_i, P_i, \boldsymbol{\beta}_S) + f_N(\mathbf{x}_i, P_i, \boldsymbol{\beta}_N)} cw_i \\
 cw = f_T(\mathbf{x}_i, P_i, \boldsymbol{\beta}_T) \\
 \mathbf{Y}_i = \mathbf{y}_i + \mathbf{e}_i, \mathbf{Y}_i = (CW_{Ei}, CW_{Wi}, CW_{Si}, CW_{Ni}, CW)^T, \mathbf{y}_i = (cw_{Ei}, cw_{Wi}, cw_{Si}, cw_{Ni}, cw)^T, \\
 E(\mathbf{e}_i) = 0, \text{var}(\mathbf{e}_i) = \sigma^2 \boldsymbol{\Psi}, i = 1, \dots, n
 \end{cases} \tag{3}$$

other, which is called a two-step, proportional weighting system, first divides the total tree biomass into subtotals, CW_{EW} and CW_{SN} , and then it partitions the subtotals into four basic components.

One-step, proportional weighting system

The one-step, proportional weighting system ensures that the summation of the crown radii values from all the components is equal to twice the total CW . For $i = 1, \dots, n$, CW_i , CR_{Ei} , CR_{Wi} , CR_{Si} , and CR_{Ni} represent

where the functions $f_E(\mathbf{x}_i, P_i, \boldsymbol{\beta}_E)$, $f_W(\mathbf{x}_i, P_i, \boldsymbol{\beta}_W)$, $f_S(\mathbf{x}_i, P_i, \boldsymbol{\beta}_S)$, $f_N(\mathbf{x}_i, P_i, \boldsymbol{\beta}_N)$, and $f_T(\mathbf{x}_i, P_i, \boldsymbol{\beta}_T)$ are obtained from the base model (1) for CR_E , CR_W , CR_S , CR_N , and CW , respectively; $\boldsymbol{\beta}_E$, $\boldsymbol{\beta}_W$, $\boldsymbol{\beta}_S$, $\boldsymbol{\beta}_N$, and $\boldsymbol{\beta}_T$ are the parameter vectors for CR_E , CR_W , CR_S , CR_N , and CW , respectively; and P_i is the value of the dummy variable for the i th tree. The structure matrix, $\boldsymbol{\Psi}$, with a size of 5×5 , is used to account for the inherent correlations among the total CW and crown components (Tang et al. 2015, 2008).

Two-step, proportional weighting system

The two-step, proportional weighting system not only ensures that the sum of the crown radii values is equal to twice the total CW, but it also guarantees that the values of the basic components for CW_{EW} and CW_{SN} are summed

total CW of a tree is divided into four basic crown components, including CR_E , CR_W , CR_S , and CR_N , by proportional weights. In this approach, the base model (1) was separately fitted by nonlinear OLSs for CW, CR_E , CR_W , CR_S , and CR_N . The estimates of CW, CR_E , CR_W , CR_S , and CR_N were calculated by

$$\begin{aligned} \hat{CR}_E &= \frac{2f_E(\mathbf{x}, P, \hat{\beta}_E)}{f_E(\mathbf{x}, P, \hat{\beta}_E) + f_W(\mathbf{x}, P, \hat{\beta}_W) + f_S(\mathbf{x}, P, \hat{\beta}_S) + f_N(\mathbf{x}, P, \hat{\beta}_N)} \hat{CW} + \varepsilon_E \\ \hat{CR}_W &= \frac{2f_W(\mathbf{x}, P, \hat{\beta}_W)}{f_E(\mathbf{x}, P, \hat{\beta}_E) + f_W(\mathbf{x}, P, \hat{\beta}_W) + f_S(\mathbf{x}, P, \hat{\beta}_S) + f_N(\mathbf{x}, P, \hat{\beta}_N)} \hat{CW} + \varepsilon_W \\ \hat{CR}_S &= \frac{2f_S(\mathbf{x}, P, \hat{\beta}_S)}{f_E(\mathbf{x}, P, \hat{\beta}_E) + f_W(\mathbf{x}, P, \hat{\beta}_W) + f_S(\mathbf{x}, P, \hat{\beta}_S) + f_N(\mathbf{x}, P, \hat{\beta}_N)} \hat{CW} + \varepsilon_S \\ \hat{CR}_N &= \frac{2f_N(\mathbf{x}, P, \hat{\beta}_N)}{f_E(\mathbf{x}, P, \hat{\beta}_E) + f_W(\mathbf{x}, P, \hat{\beta}_W) + f_S(\mathbf{x}, P, \hat{\beta}_S) + f_N(\mathbf{x}, P, \hat{\beta}_N)} \hat{CW} + \varepsilon_N \\ \hat{CW} &= f(\mathbf{x}, P, \hat{\beta}) + \varepsilon \end{aligned} \tag{5}$$

and that the summations are equal to the corresponding subtotals. The expressions of the NSE for this procedure are given by

where \hat{CW} , \hat{CR}_E , \hat{CR}_W , \hat{CR}_S , and \hat{CR}_N are estimates of CW, CR_E , CR_W , CR_S , and CR_N , respectively. $\hat{\beta}$, $\hat{\beta}_E$, $\hat{\beta}_W$, $\hat{\beta}_S$,

$$\begin{cases} cw_{Ei} = \frac{f_E(\mathbf{x}_i, P_i, \beta_E)}{f_E(\mathbf{x}_i, P_i, \beta_E) + f_W(\mathbf{x}_i, P_i, \beta_W) f_{EW}(\mathbf{x}_i, P_i, \beta_{EW}) + f_{SN}(\mathbf{x}_i, P_i, \beta_{SN})} cw \\ cw_{Wi} = \frac{f_W(\mathbf{x}_i, P_i, \beta_W)}{f_E(\mathbf{x}_i, P_i, \beta_E) + f_W(\mathbf{x}_i, P_i, \beta_W) f_{EW}(\mathbf{x}_i, P_i, \beta_{EW}) + f_{SN}(\mathbf{x}_i, P_i, \beta_{SN})} cw \\ cw_{Si} = \frac{f_S(\mathbf{x}_i, P_i, \beta_S)}{f_S(\mathbf{x}_i, P_i, \beta_S) + f_N(\mathbf{x}_i, P_i, \beta_N) f_{EW}(\mathbf{x}_i, P_i, \beta_{EW}) + f_{SN}(\mathbf{x}_i, P_i, \beta_{SN})} cw \\ cw_{Ni} = \frac{f_N(\mathbf{x}_i, P_i, \beta_N)}{f_S(\mathbf{x}_i, P_i, \beta_S) + f_N(\mathbf{x}_i, P_i, \beta_N) f_{EW}(\mathbf{x}_i, P_i, \beta_{EW}) + f_{SN}(\mathbf{x}_i, P_i, \beta_{SN})} cw \\ cw = f_T(\mathbf{x}_i, P_i, \beta_T) \\ \mathbf{Y}_i = \mathbf{y}_i + \mathbf{e}_i, \mathbf{Y}_i = (CW_{Ei}, CW_{Wi}, CW_{Si}, CW_{Ni}, CW)^T, \mathbf{y}_i = (cw_{Ei}, cw_{Wi}, cw_{Si}, cw_{Ni}, cw)^T \\ E(e_i) = 0, \text{var}(e_i) = \sigma^2 \Psi, i = 1, \dots, n \end{cases} \tag{4}$$

where the functions $f_{EW}(\mathbf{x}_i, P_i, \beta_{EW})$ and $f_{SN}(\mathbf{x}_i, P_i, \beta_{SN})$ are obtained from the base model (1) for CW_{EW} and CW_{SN} , respectively. The inherent correlations among the total CW and crown components in this case are accounted for by the structure matrix Ψ with a size of 5×5 . All other variables, parameters, and variance–covariance structures in this model were as defined in the nonlinear CW model (3).

AP

The AP approach ensures that the sum of the values of all the crown components is equal to twice the total CW. The

and $\hat{\beta}_N$ are estimates of the parameters obtained by fitting the base model (1) separately for CW, CR_E , CR_W , CR_S , and CR_N , respectively.

OLSSR

The base model (1) was separately fitted by nonlinear OLSs for CR_E , CR_W , CR_S , and CR_N , and the CW estimate was obtained by obtaining the half sum of the four crown components:

$$\hat{CW} = (\hat{CR}_E + \hat{CR}_W + \hat{CR}_S + \hat{CR}_N)/2 \tag{6}$$

Table 2 Parameter estimates of the base model (1) obtained by separately fitting using nonlinear ordinary least squares for east crown radius (CR_E), west crown radius (CR_W), south crown radius (CR_S), north crown radius (CR_N), east–west crown width (CW_{EW}), south–north crown width (CW_{SN}), and total crown width (CW)

Parameter	CR _E	CR _W	CR _S	CR _N	CW _{EW}	CW _{SN}	CW
β_1	5.9471 (0.8818)	21.4953 (9.2512)	10.1567 (1.1120)	7.1215 (1.1538)	20.6796 (3.3754)	17.6719 (1.7114)	18.5692 (1.9196)
β_2	-0.0741 (0.0219)	-0.3607 (0.1650)	-0.2014 (0.0285)	-0.0859 (0.0260)	-0.3098 (0.0646)	-0.2988 (0.0408)	-0.2978 (0.0416)
β_3	4.7128 (0.6375)	12.6883 (5.5075)	4.4821 (0.5266)	5.6577 (0.8085)	6.9529 (1.1198)	5.0960 (0.4761)	5.7345 (0.5757)
β_4	0.1779 (0.0370)	0.4744 (0.1474)	0.2982 (0.0464)	0.1935 (0.0352)	0.2373 (0.0287)	0.2394 (0.0283)	0.2316 (0.0227)
β_5	0.0750 (0.0084)	0.0570 (0.0061)	0.0759 (0.0079)	0.0850 (0.0087)	0.0649 (0.0051)	0.0800 (0.0059)	0.0724 (0.0047)
β_6	-0.0006 (0.0002)	-0.0007 (0.0001)	-0.0005 (0.0002)	-0.0012 (0.0002)	-0.0007 (0.0001)	-0.0009 (0.0001)	-0.0008 (0.0001)
k_1	0.4669 (0.5613)	-3.2004 (2.1219)	-2.7163 (0.6496)	0.4205 (0.6288)	-1.2661 (1.2027)	-2.5405 (0.9105)	-1.9687 (0.8692)
k_2	0.0229 (0.0232)	0.1230 (0.0838)	0.0887 (0.0258)	0.0216 (0.0259)	0.1084 (0.0517)	0.1164 (0.0375)	0.1109 (0.0364)

β_1 – β_6 , k_1 , k_2 = model parameters

where \hat{CR}_E , \hat{CR}_W , \hat{CR}_S , and \hat{CR}_N are obtained from the base model (1) with known parameter values.

Model evaluation

We first assessed the accuracy of the base model for CR_E, CR_W, CW_{EW}, CR_S, CR_N, CW_{SN}, and CW based on the mean bias (\bar{e}), the variance of the residuals (δ), the root mean square error (RMSE), the total relative error (TRE), and the adjusted coefficient of determination (R^2_α), using both the model-fitting and model-validation datasets. Then, the fitting and predictive ability of the systems of CW models (3) and (4) and the two additive models (AP and OLSSR) were evaluated by the statistics \bar{e} , δ , RMSE, and TRE, using both the model-fitting and model-validation datasets. These evaluation statistics were defined as:

$$\bar{e} = \sum (Y_i - \hat{Y}_i) / n \tag{7}$$

$$\delta = \sum (Y_i - \hat{Y}_i)^2 / (n - 1) \tag{8}$$

$$RMSE = \sqrt{\bar{e}^2 + \delta} \tag{9}$$

$$TRE = 100 \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 / \sum_{i=1}^n \hat{Y}_i \tag{10}$$

$$R^2_\alpha = 1 - \frac{(n - 1) \sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{(n - P) \sum_{i=1}^n (Y_i - \bar{Y})^2} \tag{11}$$

where Y_i and \hat{Y}_i are the observed and predicted CW or crown radius values, respectively, for the i th observation; RMSE, which combines the mean bias and the variation of the residuals, was used as the primary criterion for the CW model evaluations. All the calculations were carried out using ForStat 2.2 software (Tang et al. 2008).

Results

Base model

The parameter estimates of the base model (1) that was separately fitted by nonlinear OLSs for CR_E, CR_W, CW_{EW}, CR_S, CR_N, CW_{SN}, and CW are listed in Table 2. Except for β_6 , all the other parameter estimates in the base model (1) for each crown component and CW were significantly different from 0 ($p < 0.05$). The fit and prediction statistics of the base model (1) for each crown component and CW are provided in Table 3.

A Student’s t test indicated that the \bar{e} statistics for the base model (1) for CR_E, CR_W, CR_S, CR_N, CW_{EW}, CW_{SN}, and CW were not significantly different from 0 ($p < 0.05$). Table 3 shows that the differences of the fit and prediction accuracies among the different crown components and CW were significant ($p < 0.05$). The base model (1) showed the best overall performance for CW prediction.

Table 3 Evaluation indices of the base model (1) for east crown radius (CR_E), west crown radius (CR_W), south crown radius (CR_S), north crown radius (CR_N), east–west crown width (CW_{EW}), south–north crown width (CW_{SN}), and total crown width (CW)

Variables	Model-fitting data					Model-validation data			
	\bar{e}	δ	RMSE	TRE	R_x^2	\bar{e}	δ	RMSE	TRE
CR_E	0.0020	0.5750	0.7583	30.4955	0.2851	-0.0943	0.5779	0.7660	31.0410
CR_W	-0.0006	0.5307	0.7285	28.5253	0.3531	-0.0953	0.5205	0.7277	28.3095
CR_S	0.0004	0.6228	0.7892	31.0224	0.3763	-0.0919	0.5622	0.7554	28.2862
CR_N	-0.0010	0.5558	0.7455	28.0731	0.3310	-0.1241	0.5894	0.7777	30.3775
CW_{EW}	0.0006	1.1059	1.0516	29.5135	0.4731	-0.1916	1.2734	1.1446	34.8162
CW_{SN}	-0.0008	1.2116	1.1007	30.3841	0.5038	-0.2173	1.2431	1.1359	32.1787
CW	-0.0001	0.8481	0.9209	21.9292	0.5664	-0.2043	0.9651	1.0034	25.9071

\bar{e} mean of the prediction error, δ variance of residuals, $RMSE$ root mean square error, TRE total relative error, R_x^2 adjusted coefficient of determination

Table 4 Evaluation indices for the system of nonlinear simultaneous equation (NSE) with both one- and two-step procedures, and the adjustment in proportion (AP) and the ordinary least squares with separating regression (OLSSR) models

Variables	Model-fitting data					Model-validation data			
	\bar{e}	δ	RMSE	TRE	R_x^2	\bar{e}	δ	RMSE	TRE
NSE with one-step procedure									
CR_E	-0.0120	0.6935	0.8329	27.0650	0.2510	-0.0175	0.5088	0.7135	26.8541
CR_W	-0.0047	0.5149	0.7176	21.4326	0.3587	-0.0569	0.4037	0.6379	19.4227
CR_S	-0.0115	0.6044	0.7775	21.8867	0.3859	-0.0733	0.4152	0.6485	20.6815
CR_N	-0.0061	0.7325	0.8559	24.2362	0.2854	-0.0894	0.4550	0.6804	23.0218
CW	-0.0162	0.9543	0.9770	18.1091	0.5372	-0.1214	0.6946	0.8422	17.8430
NSE with two-step procedure									
CR_E	-0.0017	0.2596	0.5095	24.4184	0.2837	-0.0120	0.4626	0.6803	25.1048
CR_W	-0.0021	0.3285	0.5732	17.426	0.3905	-0.0131	0.3649	0.6042	18.3002
CR_S	-0.0119	0.4427	0.6655	19.3670	0.3971	-0.0647	0.3568	0.6008	18.7875
CR_N	-0.0025	0.4813	0.6938	21.7485	0.3026	-0.0538	0.3715	0.6119	23.4323
CW	-0.0113	0.6749	0.8216	16.4427	0.5930	-0.1135	0.5530	0.7523	17.0049
AP									
CR_E	-0.0056	0.9795	0.9897	30.6119	0.1769	-0.1010	0.5706	0.7621	30.6172
CR_W	-0.0061	0.6338	0.7961	28.6063	0.3060	-0.0994	0.5087	0.7201	27.6631
CR_S	-0.0048	0.727	0.8527	31.1538	0.3385	-0.0954	0.5491	0.7471	27.6209
CR_N	-0.0087	0.8605	0.9277	28.2029	0.2322	-0.1301	0.5675	0.7645	29.2625
CW	-0.0001	0.8481	0.9209	21.9292	0.5664	-0.2043	0.9651	1.0034	25.9071
OLSSR									
CR_E	0.002	0.975	0.9874	30.4955	0.1804	-0.0943	0.5779	0.7660	31.0411
CR_W	-0.0006	0.6307	0.7942	28.5247	0.3097	-0.0953	0.5205	0.7277	28.3094
CR_S	0.0004	0.7228	0.8502	31.0224	0.3427	-0.0919	0.5622	0.7554	28.2862
CR_N	-0.001	0.8558	0.9251	28.0731	0.2378	-0.1241	0.5894	0.7777	30.3775
CW	0.0006	1.2488	1.1175	21.9498	0.4705	-0.2026	0.9657	1.0034	25.9165

CR_E east crown radius, CR_W west crown radius, CR_S south crown radius, CR_N north crown radius, CW total crown width, \bar{e} mean of the prediction error, δ variance of residuals, $RMSE$ root mean square error, TRE total relative error, R_x^2 adjusted coefficient of determination

Comparison between the different CW models

The fit statistics of CW models (3) and (4) and the AP and OLSSR models are presented in Table 4. All the models guaranteed that the sum of CR_E , CR_W , CR_S , and CR_N was equal to twice the total CW. A Student’s t test indicated

that the mean biases (\bar{e}) for all the models for CR_E , CR_W , CR_S , CR_N , and CW were not significantly different from 0 ($p > 0.05$). Except for the fact that the OLSSR under-predicted CR_E , CR_S , and CW in the model fitting, all the other predictions for each model—in terms of both model fitting and validation—showed over-predictions.

Table 5 Parameter estimates of crown width (CW) model (4)

Parameters (SE)	CW model (4)						
	CR _E	CR _W	CR _S	CR _N	CW _{EW}	CW _{SN}	CW
β_1	5.9024 (0.7431)	19.0275 (6.8164)	9.7025 (1.0849)	6.3208 (1.0241)	21.2372 (3.1059)	18.0562 (1.4913)	18.3077 (1.9012)
β_2	-0.0618 (0.0132)	-0.2136 (0.1221)	-0.1864 (0.0270)	-0.0684 (0.0249)	-0.2975 (0.0403)	-0.2550 (0.0417)	-0.2763 (0.0397)
β_3	4.5238 (0.5174)	10.6839 (4.7230)	4.1643 (0.5064)	4.8531 (0.7650)	6.4628 (1.0874)	4.4823 (0.4295)	5.8682 (0.5784)
β_4	0.1745 (0.0362)	0.5327 (0.1481)	0.3548 (0.0487)	0.2446 (0.0415)	0.1905 (0.0243)	0.2136 (0.0210)	0.2149 (0.0205)
β_5	0.0613 (0.0085)	0.0602 (0.0063)	0.0797 (0.0084)	0.0908 (0.0091)	0.0482 (0.0047)	0.0640 (0.0041)	0.0695 (0.0042)
β_6	-0.0005 (0.0002)	-0.0008 (0.0001)	-0.0006 (0.0002)	-0.0014 (0.0002)	-0.0005 (0.0001)	-0.0007 (0.0001)	-0.0007 (0.0001)
k_1	-0.7937 (0.5824)	-4.4671 (2.3045)	-4.1821 (0.6651)	-0.2245 (0.6345)	-1.4340 (1.0483)	-2.2105 (0.8449)	-1.9144 (0.8510)
k_2	0.0544 (0.0249)	0.2219 (0.1064)	0.1549 (0.0261)	0.0563 (0.0260)	0.9758 (0.0464)	0.1078 (0.0214)	0.1025 (0.0328)

CR_E east crown radius, CR_W west crown radius, CR_S south crown radius, CR_N north crown radius, CW_{EW} east–west crown width, CW_{SN} south–north crown width, CW crown width, β_1 – β_6 , k_1 , k_2 model parameters, SE standard error

CW model (4) had the better statistics for the crown components and CW, compared with CW model (3) and the AP and OLSSR models. For example, based on the fitting results, the statistics δ , RMSE, TRE, and R^2_α of CW model (4) were 29.28, 15.91 and 9.20% smaller, and 10.39% larger, respectively, than those of CW model (3); 20.42, 10.78 and 25.02% smaller, and 4.70% larger, respectively, than those of the AP model; and 45.96, 26.48 and 25.09% smaller, and 26.04% larger, respectively, than those of the OLSSR model. Based on the validation results, the values of δ , RMSE, and TRE from CW model (4) for CW were 20.39, 10.67 and 4.70% smaller, respectively, than those of CW model (3); 42.70, 25.02 and 34.36% smaller, respectively, than those of the AP model; and 42.74, 25.02 and 34.39% smaller, respectively, than those of the OLSSR model.

The parameter estimates for CW model (4) are listed in Table 5. The estimates of k_1 and k_2 in CW model (4) were significant ($p < 0.05$), indicating that there was a pronounced difference between the values of CRs from the Guandi mountain forest and the Boqiang forest. All the other parameter estimates of CW model (4) were also highly significant ($p < 0.05$), and their magnitudes and signs were also biologically plausible. Based on the validation data, the plots in Fig. 4 show that there was no serious trend in the residuals when the CW components were predicted with CW model (4); the model-fitting dataset resulted in similar patterns of the residuals.

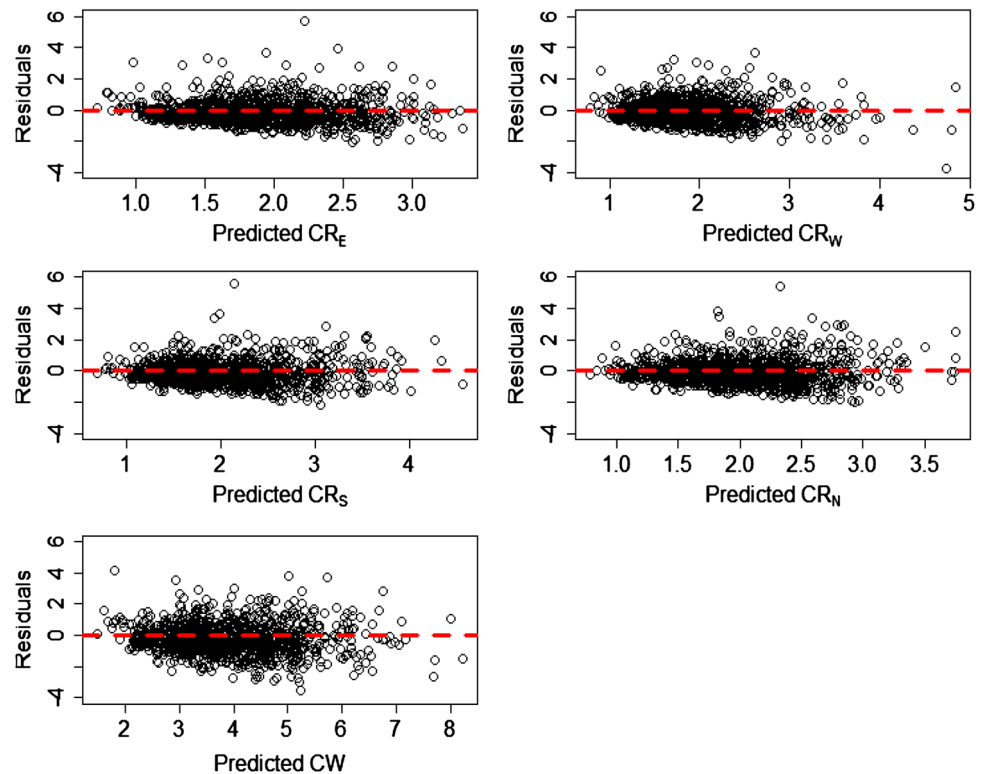
Therefore, CW model (4) is ultimately recommended for predicting the crown components and CW of Prince Ruprecht larch.

Discussion

Additivity of crown radii is a desirable characteristic for CW models used to predict CW. Moreover, if CW models can account for the inherent correlations among the crown components and CW, they will possess a great statistical efficiency (Parresol 1999, 2001). In this study, NSE was first time applied to develop CW models and also compared with other two additive models (the AP and OLSSR models) that are widely used to develop CW models. These methods effectively ensured that the sum of CR_E, CR_W, CR_S, and CR_N was equal to twice the total CW. For the NSE, the correlations among the components were effectively accounted for by the covariance matrix of random errors. In the AP and OLSSR models, however, the crown component models because of the assumption of homogeneous random error variances were fitted separately. Thus, both AP and OLSSR models could not account for the correlations.

Based on the results of Table 4, both the one- and two-step procedures maintained the property of additivity for the CW models. The prediction accuracies of each crown component and CW using the two-step

Fig. 4 Residuals graphed against predicted responses of crown width (CW) model (7) based on the model-fitting data for east crown radius (CR_E), west crown radius (CR_W), south crown radius (CR_S), north crown radius (CR_N), and total crown width (CW)



procedure were much higher than those for the corresponding components using the one-step procedure. Model specification for a two-step procedure is more complicated than that for a one-step procedure (Zeng and Tang 2010; Fu et al. 2016). Therefore, in practice, researchers prefer the use of a one-step procedure to develop a system of additive biomass equations (Zeng and Tang 2010; Fu et al. 2014, 2016). However, in this study, because the prediction accuracies of the developed CW models were not very high overall (Table 4), the model with the most powerful predictive ability with the same predictors is of greater interest to us. Thus, CW model (4) was selected to predict the crown components and CW.

In some studies that developed additive biomass equations using one- and two-step procedures, e.g., Zeng and Tang (2010), Dong et al. (2015), and Fu et al. (2016), the biomass equation of the total tree was not included, and estimated separately using OLS. These models also resulted in high prediction accuracies. In this study, we developed CW models, without including the total CW model, using one- and two-step procedures. The results showed that although the previously developed models were simpler than the CW models (3) and (4), their prediction accuracies were much lower than those of the CW models (3) and (4). In addition, the prediction accuracies of the previously developed models were similar to that of the base model (1) in this case, which is meaningless for

applying one- and two-step procedures to improve the accuracy of CW prediction.

The results in Table 4 showed that the NSE more accurately predicted the crown components and the total CW, compared with the AP and OLSSR models. Particularly, when NSE was used, the prediction accuracies of the crown components and the total CW were the highest. These results indicated that CW model (4) was a more effective additive model for crown components and total CW predictions. For the AP model, the total CW was estimated from a separately fitted model (model (1), and therefore, the prediction accuracy of the total CW was equal to that of the CW from the base model (1) (see Table 3). Similarly, in the OLSSR model, CR_E , CR_W , CR_S , and CR_N were estimated by the separately fitted base model (1), and therefore, the prediction accuracies of the crown components of the OLSSR were equal to those of the base model (1) (see Table 3). Relative to the base model (1), the prediction accuracy of the total CW obtained from the OLSSR was reduced. Over-predictions took place for all the estimations using the AP and NSE from both model-fitting and model-validation datasets and using the OLSSR from the model-validation dataset, and for the estimations of the CR_W and CR_N using the OLSSR from the model fitting dataset. This may be because the base model (1) used in this study has a characteristics of over-prediction for the total CW and crown components for trees with the corresponding large CW or crown components

(Fig. 3). Therefore, this model needs to be further improved in future study. The numbers of the PSPs allocated in both the state-owned Guandi mountain forest and Boqiang forest (Fig. 1) were approximately proportional to their contributions and total stock volumes of the natural stands of Prince Rupprecht larch in the entire northern China. The trees in the PSPs had a range of diameters from 5 to 60.5 cm. For Prince Rupprecht larch, the trees with diameter greater than 50 cm belong to super large diameter class group in the forest management (Zhang, 2008). Thus, the developed models in this study were also validated for the trees with diameter greater than 50 cm.

Measuring tree crown components (CR_E , CR_W , CR_S , and CR_N) may be subject to errors, even though stand and tree variables are commonly assumed to be measured without errors (Omule 1980; Gertner 1990). Measurement errors made by field crews or faulty instruments, or both, can be substantial (Omule 1980). For example, HCB is generally measured with a standard height measurement instrument, but when a crown is uneven, one often visually rearranges crown branches to obtain a value for HCB. There is ample evidence in the literature about the ambiguity of the visual estimation of tree variables. Nicholas et al. (1991) and Ghosh et al. (1995) highlighted the variations that could arise from the subjective measurements of tree and stand variables. In all existing CW models (Sánchez-González et al. 2007; Fu et al. 2013; Sharma et al. 2016), including those developed in this study, it is assumed that (i) CW and crown components (crown radii) are random variables, and (ii) other independent variables are fixed and measured without errors. It is well known that violations of the second assumption may lead to biased parameter estimates and standard errors, which consequently misleads the hypothesis test (Fuller 1987; Rencher and Schaalje 2008). When the covariate predictors in CW model (4) are likely to have significant errors, a new modeling approach, such as a nonlinear error-in-variable model (Fu et al. 2016), needs to be developed. We are in the process of developing CW models to solve such problems.

Conclusion

In this study, NSE, AP, and OLSSR approaches were used to develop CW models based on the CW datasets of Prince Rupprecht larch that were collected in northern China. One- and two-step procedures were applied to ensure the additivity for the NSE. We found that the model (1) proposed by Fu et al. (2013) could be used as a base model for effectively developing CW models. CW model (4) with a two-step proportional weighting system performed better than with a one-step proportional weighting system for

predictions of crown components and total CW. NSE accounted for the inherent correlations among the crown components and CW, whereas the AP and OLSSR models did not. The prediction accuracy of CW model (4) was the highest among these methods. In summary, this study developed a system of nonlinear, additive CW models in which the correlations among the crown components and CW were accounted for during the modeling and prediction of the CW of the individual trees and their crown components. The obtained results can be generalized. Thus, it is expected that this methodology can be applied to any model system in which a quantity can be partitioned into additive components.

Author contribution statement LF and ST conceived the study. LF and KH performed the analysis and wrote the initial draft of the manuscript. WX drew the figures and GW revised the MS and checked the grammatical errors in English. All authors contributed in interpreting the results and improving the manuscript.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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