Computational Mechanics 20 (1997) 551–559 © Springer-Verlag 1997

Hydrodynamic interaction between two nearby swimming micromachines

S. Nasseri, N. Phan-Thien

Abstract Numerical results for two nearby swimming micromachines, obtained by the boundary element method (BEM), are reported in this paper. Two neighbouring configurations, side by side and in tandem, are considered and the translational and rotational velocities together with the force exerted on the micromachines are given. It is demonstrated that, for both configurations, the approximate reflection method gives comparable results to the full solutions. In the side by side configuration, hydrodynamic interaction is significant when the separation distance is less than about 1.5 of the total length of the machine. A propulsion advantage in the tandem configuration is found, where the leading machine acquires a higher velocity and the results obtained by the reflection method agree well with the full BEM solution.

1 Introduction

The simplest multiparticle system for which the resistance of the particles can be exactly determined by a theoretical method is the two-sphere system. Stimson and Jeffery (1926) solved the axisymmetric problem when two spheres translate with the same velocity along their line of centres. The forces acting on the spheres were obtained only for equal-sized spheres, but later Jeffrey and Onishi (1984) calculated the resistance and mobility functions for two unequal rigid spheres in low Reynolds number flows. Davis (1969) also calculated the forces on the spheres when one is in motion along the line of centres while the other is at rest.

In order to compute the complete forces and torques acting on the spheres in the most general case, we must also consider the contributions arising from the asymmetrical parts of the motion of the spheres, i.e., the forces and torques acting when the spheres either translate or rotate relatively about the axis perpendicular to their line of centres. The calculations were carried out by Goldman et al. (1966) for equal spheres, and for unequal spheres calculations were carried out by Davis (1969). Thus, when the fluid motions caused either by the translation or rotation of the spheres are concerned, the resistance is determined, and consequently the elements of the grand resistance matrix can be found. Also, recently, a method in conjunction with the "reciprocal theorem method," is described by Phillips (1996) to calculate the motion of N spherical particles in a second-order fluid (to take into account, at least in the first approximation, the viscoelastic nature of the fluid). That method can be integrated into the Stokesian Dynamic Simulation dealing with non-Newtonian suspensions.

In 1951, Taylor analysed the swimming of two microscopic organisms executing planar sinusoidal flagellar waves. He postulated that the propulsive advantage could result, if the two neighbouring organisms beat their flagella in unison (i.e., with constant relative flagellar phase angle and the same frequency). Although this prediction was based on a simplified two-dimensional model, it was consistent with the frequent experimental observation that the flagella of neighbouring spermatozoa have a tendency to beat in this manner. Later, a model of the swimming of two organisms was proposed (in which propulsion was achieved by helical flagellar rotation) by Ramia et al. (1993). In their model, they considered two organisms swimming close and parallel to each other, and the effects of two parameters, flagellar phase angle and the separation distance were studied. It was shown that these parameters have little effects on the swimming speed, the angular velocity, the propulsive force and torque on each of the organisms. In other words, the hydrodynamic interaction would never be as significant as the more fundamental cell body/flagellar interaction. This result is quite reasonable, according to the fact that the resistance coefficients for each of two translating or rotating neighbouring spheres do not increase significantly (see Happel and Brenner 1973), unless they are very close to each other.

In a series of papers (Nasseri and Phan-Thien 1996, 1997a, 1997b), we have been considering the problem of swimming and optimising a micromachine, consisting of a head (which contains an electromechanical power source) and a tail which produces a propulsive force by rotating with respect to the head. The idea is to produce a simple profile for these machines, that can be manufactured by current, or near-future technology. After analysing different shapes for the tail for this special micromachine, rigid straight rod-shape or elastic in the form of stretching-compressing rod, we realised that a micromachine with spiral tail would be the optimal design (the spiral tail can be manufactured by twisting a plate). In this paper we examine the effect of hydrodynamic interaction between two micromachines with spiral tails, which are swimming

Communicated by S. N. Atluri, 5 June 1997

S. Nasseri, N. Phan-Thien
Department of Mechanical and Mechatronic Engineering, The University of Sydney, N.S.W. 2006, Australia

Correspondence to: S. Nasseri

in a viscous fluid in two configuration, side by side and in tandem. We also consider an approximate reflection method which produces comparable results to the full numerical BEM solutions.

1.1
Hydrodynamic interaction

The subject of hydrodynamic interaction between two or more particles in low Reynolds number has been thoroughly reviewed by Happel and Brenner (1973). A summary is outlined here for convenience. Generally, hydrodynamic interaction between the particles is governed by the following variables:

1. their shapes and sizes;
2. the distance between them;
3. their orientations with respect to each other;
4. their individual orientations relative to the direction of the gravitational field;
5. their velocities and spin relative to the fluid at infinity.

Since the Reynolds number based on a typical size of the particles is small, the local fluid motion is assumed to satisfy the quasi-static Stokes equations,

$$\mu\nabla^2\mathbf{v} = \nabla p, \qquad \nabla\cdot\mathbf{v} = 0 \tag{1.1}$$

Where $\mathbf{v}$ is the field velocity, p the dynamic pressure and μ the viscosity. Because of the linearity of the governing equations of motions and boundary conditions, two modes of motion, translation and rotation, may be separately investigated, and the results superposed. Initially, we shall restrict our attention to cases where the particle translate, without rotation, as they move through liquid (a simpler of the two possible modes of rigid body motion).

Consider two rigid particles of arbitrary shape translating through an unbounded fluid which is at rest at infinity. If we identify the particles by the label a and b, the boundary conditions are

$$\mathbf{v} = \mathbf{U}_a \quad \text{on } a \tag{1.2}$$

and

$$\mathbf{v} \to 0 \quad \text{as } r \to \infty \tag{1.3}$$

where r is measured from the centre of particle a. In addition,

$$\mathbf{v} = \mathbf{U}_b \quad \text{on } b\ . \tag{1.4}$$

The exact solution of this problem is given by Stimson and Jeffery (1926), for the slow motion of two spheres parallel to their line of centres (an axisymmetric flow). For larger collections of particles, or for pairs of nonspherical bodies, a systematic scheme of successive iterations, whereby the boundary-value problem may be solved to any degree of approximation by considering boundary conditions associated with one particle at a time, is provided by the *method of reflection* (Brenner and Happel 1958). This method is applicable for widely separated particles, whereas the distance between closest points on the surfaces is much greater than particle size. Surfaces near contact present a far more challenging problem, both from an analytical and computational viewpoint. For rigid surfaces in relative motion, the flow in the gap region dominates and *lubrication theory* provides the leading terms in an asymptotic expansion (Kim and Karrila 1991).

1.1.1
Method of reflection

Consider a system of n particles. Let $\mathbf{U}_k$ $(k = a, b, c, \ldots, n)$ denote the velocity of the kth particle. The forces, $\mathbf{F}_k$, are necessary to maintain each particle in its state of uniform motion, and the restraining torques, $\mathbf{T}_k$, required to keep the particles from rotating under the influence of the hydrodynamic stresses developed at their surfaces.

To solve the boundary-value problem posed by Eqs. 1.1–1.4, we proceed as follows: Since the equation of motion and boundary conditions are linear, the local velocity and pressure fields may be decomposed into a sum of fields, thus,

$$\mathbf{v} = \mathbf{v}^{(1)} + \mathbf{v}^{(2)} + \mathbf{v}^{(3)} + \mathbf{v}^{(4)} + \cdots \tag{1.5}$$

$$p = p^{(1)} + p^{(2)} + p^{(3)} + p^{(4)} + \cdots \tag{1.6}$$

each pair of solution, $\left(\mathbf{v}^{(j)}, p^{(j)}\right)$, separately satisfies the equations of motion and vanishes at infinity. Again, because of the linearity in the governing equations, we may further subdivide each of these into a finite sum of terms, $\left(\mathbf{v}_k^{(j)}, p_k^{(j)}\right)$, also satisfying the governing differential equations and vanishing at infinity. Now, focus attention on any particle in the system, say a, and define $\left(\mathbf{v}^{(1)}, p^{(1)}\right)$ by the boundary condition

$$\mathbf{v}^{(1)} = \mathbf{U}_a \qquad \text{on } a\ . \tag{1.7}$$

The “reflection” of this field from particle b is then defined by the boundary condition

$$\mathbf{v}_b^{(2)} = \mathbf{U}_b - \mathbf{v}^{(1)} \qquad \text{on } b\ . \tag{1.8}$$

In general, the reflection of $\mathbf{v}^{(1)}$ from any of the $n-1$ particles is defined by

$$\mathbf{v}_k^{(2)} = \mathbf{U}_k - \mathbf{v}^{(1)} \qquad \text{on } k \quad (k = b, c, \ldots, n)\ . \tag{1.9}$$

Thus, the reflection of $\mathbf{v}^{(1)}$ from all the remaining $n-1$ particles is given approximately by

$$\mathbf{v}^{(2)} = \sum_{k=b}^{n} \mathbf{v}_k^{(2)}\ . \tag{1.10}$$

The reflection process may be continued (see Happel and Brenner 1973), as far as necessary to obtain satisfaction of all boundary conditions to the desired accuracy. Except for simple arrangements, numerical evaluations will be much easier than general analytical representation of the result.

For a complete strict treatment, it is necessary to have available a solution of the creeping motion equations for the case of a single particle with an arbitrary velocity field prescribed on its surface. Good approximations, are possible, however, by assuming that when the particles are sufficiently separated:

1. the field produced by a given particle will be the same as that produced by a point force acting at the centre of the particle;
2. the drag resulting from the field reflected at a given particle can be approximated by considering the field to

be equivalent to a uniform velocity field whose magnitude and direction are the same as what actually would exist at the particle centre if it were not present.

Consider two spheres a and b, moving with instantaneous velocities $\mathbf{U}_a$ and $\mathbf{U}_b$ in an otherwise unbounded medium which is at rest at infinity (as shown in Fig. 1). We choose one axis of the reference system of coordinates along a line connecting the centres of the two particles (here z), and as the particles move in a plane, it is necessary to specify only one additional coordinate. So we will choose the xz plane. In addition to equations of motion 1.1, we require the boundary condition at infinity Eq. 1.3 and the following conditions to be satisfied:

$$\begin{aligned} \mathbf{v}^{(1)} &= \mathbf{U}_a \quad \text{on } a \\ \mathbf{v}^{(2)} &= -\mathbf{v}^{(1)} + \mathbf{U}_b \quad \text{on } b \\ \mathbf{v}^{(3)} &= -\mathbf{v}^{(2)} \quad \text{on } a \\ \mathbf{v}^{(4)} &= -\mathbf{v}^{(3)} \quad \text{on } b, \text{ etc.} \end{aligned} \tag{1.11}$$

The initial field $\mathbf{v}^{(1)}$ obviously will correspond to the setting of particle a in an unbounded fluid. Associated with this motion is the force exerted by the fluid on the particles

$$\mathbf{F}_a^{(1)} = -\mu\tilde{K}_a\,\mathbf{U}_a = -\mu\tilde{K}_a(\mathbf{i}U_{ax} + \mathbf{k}U_{az}) \tag{1.12}$$

where $\tilde{K}_a = 6\pi a$. Since particle a is assumed to be located at a relatively large distance (several diameters at least) from particle b, we may compute the translational effect of particle a by assuming that it generates the same field as would be produced by a *point force* situated at the centre of the particle (see Lamb 1932):

$$\mathbf{v}^{(1)} = -\frac{\mathbf{F}_a^{(1)}}{6\pi\mu r} - \frac{r^2}{24\pi\mu}\nabla\left(\mathbf{F}_a^{(1)}\cdot\nabla\right)\frac{1}{r} \tag{1.13}$$

and

$$p = \frac{1}{4\pi}\left(\mathbf{F}_a^{(1)}\cdot\nabla\right)\frac{1}{r} \ . \tag{1.14}$$

If we express Eq. 1.13 in Cartesian coordinates appropriate to Fig. 1, we obtain

$$\mathbf{v}^{(1)} = \frac{\tilde{K}_a U_{ax}}{8\pi r}\left(\mathbf{i} + \mathbf{r}\frac{x}{r^2}\right) + \frac{\tilde{K}_a U_{az}}{8\pi r}\left(\mathbf{k} + \mathbf{r}\frac{z}{r^2}\right) \ . \tag{1.15}$$

Since the centre of particle b has the coordinates $x = 0$, $y = 0$, $z = d$, the value of $\mathbf{v}^{(1)}$ at this point is

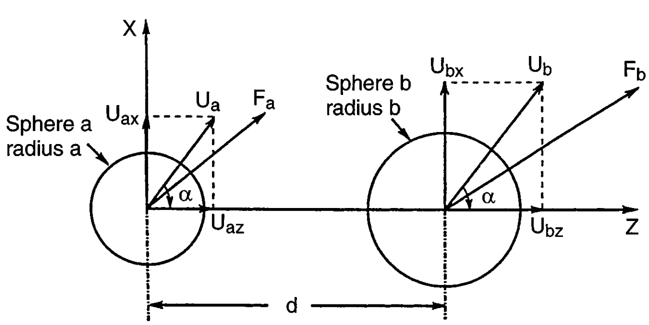

Fig. 1. Coordinate system for two particle interactions

$$\mathbf{v}_b^{(1)} = \frac{\tilde{K}_a}{8\pi d}(\mathbf{i}U_{ax} + 2\mathbf{k}U_{az}) \ . \tag{1.16}$$

where d is the centre to centre separation distance between two particles. From this we compute the force exerted on particle b

$$\mathbf{F}_{(b)}^{(2)} = -\mu\tilde{K}_b(\mathbf{U}_b - [\mathbf{v}^{(1)}]_b) = -\mathbf{i}\mu\tilde{K}_b\left(U_{bx} - \frac{\tilde{K}_a U_{ax}}{8\pi d}\right) - \mathbf{k}\mu\tilde{K}_b\left(U_{bz} - \frac{\tilde{K}_a U_{az}}{4\pi d}\right) \ . \tag{1.17}$$

Using the same technique as previously, we can calculate the velocity field generated by the force $\mathbf{F}_b^{(2)}$ acting at the location of particle b. Therefore, the origin of the coordinate system will now be at the centre of particle b, and $\mathbf{v}^{(2)}$ at particle a will be

$$\mathbf{v}_a^{(2)} = \mathbf{i}\frac{\tilde{K}_b}{8\pi d}\left(U_{bx} - \frac{\tilde{K}_a U_{ax}}{8\pi d}\right) - \mathbf{k}\frac{\tilde{K}_b}{4\pi d}\left(U_{bz} - \frac{\tilde{K}_a U_{az}}{4\pi d}\right) \ . \tag{1.18}$$

Also, by writing the force contributions $\mathbf{F}_a^{(3)}$ and $\mathbf{F}_a^{(5)}$, we will obtain the formula for $\mathbf{F}_a$ and after expressing the geometric series as a function and combining terms, we find

$$\frac{\mathbf{F}_a}{\mu\tilde{K}_a} = -\mathbf{i}\frac{U_{ax} - (\tilde{K}_b U_{bx}/8\pi d)}{1 - (\tilde{K}_a\tilde{K}_b)/(8\pi d)^2} - \mathbf{k}\frac{U_{az} - (\tilde{K}_b U_{bz}/4\pi d)}{1 - (\tilde{K}_a\tilde{K}_b)/(4\pi d)^2} \ . \tag{1.19}$$

By interchanging the subscript a and b, the force exerted on particle b, $\mathbf{F}_b$, is also obtained from Eq. 1.19.

Now consider the case of two equal-sized spheres of radii a, Eq. 1.19 then becomes

$$-\frac{\mathbf{F}}{6\pi\mu a} = \mathbf{i}\frac{U_x}{1 + (3/4)(a/d)} + \mathbf{k}\frac{U_z}{1 + (3/2)(a/d)} \ . \tag{1.20}$$

Note that the force exerted by the fluid on each particle is the same and that their motion is parallel and with the same velocity. They can move sidewise, but will maintain the same distance between each other. If the two particle move along x axis (perpendicular to their line of centres), we derive the important formula

$$F = -\frac{6\pi\mu a}{1 + (3/4)(a/d)}U \tag{1.21}$$

and if they move along their line of centres

$$F = -\frac{6\pi\mu a}{1 + (3/2)(a/d)}U \ , \tag{1.22}$$

The motion of two arbitrary particles In order to apply this approximation procedure to other than spherical particles, it is necessary not only to locate centres of the particles involved, but also to ascribe to each a characteristic "radius," which may be taken the same as that of a sphere exhibiting the same Stokes resistance as the particle. For the case of two particles with characteristic dimensions c_a and c_b, we may write Eq. 1.12 in terms of the

Stokes translation tensor in the case of an arbitrary particle which is not spherically isotropic (see Brenner 1964)

$$(\mathbf{F}_\infty)_a = -6\pi\mu c_a (\Phi_\infty)_a \cdot \mathbf{U}_a \qquad (1.23)$$

where $(\mathbf{F}_\infty)_a$ is the force which body a would experience if it moved through an unbounded fluid with a velocity U_a. The symmetric dimensionless dyadic Φ_∞ is termed the *Stokes resistance tensor* of the particle (Brenner 1963), which is independent of such factors as the size, velocity, and orientation of the particle and of the properties of the fluid through which it moves.

In particular for a sphere of radius $c_a = a$, $(\Phi_\infty)_a = \mathbf{I}$. Now, if we let $\mathbf{e}$ be a unit vector along the line of centres of the two particles, the relationship obtained by Brenner, analogous to Eq. 1.19 is,

$$\frac{\mathbf{F}_a}{6\pi\mu c_a} = -\left[(\Phi_\infty)_a^{-1} - \frac{9}{16}\frac{c_a}{d}\frac{c_b}{d}(\mathbf{I}+\mathbf{ee})\cdot(\Phi_\infty)_b\cdot(\mathbf{I}+\mathbf{ee})\right]^{-1} \cdot \left[\mathbf{U}_a - \frac{3}{4}\frac{c_b}{d}(\mathbf{I}+\mathbf{ee})\cdot(\Phi_\infty)_b\cdot\mathbf{U}_b\right], \qquad (1.24)$$

and also for $\mathbf{F}_b$, a similar equation will be obtained by interchanging a to b.

In this work, we have considered $(\Phi_\infty)_a$ to be as $\begin{bmatrix}\Phi_{\infty x} & 0 \\ 0 & \Phi_{\infty z}\end{bmatrix}_a$, $\mathbf{e}$ as $\begin{bmatrix}\sin\alpha \\ \cos\alpha\end{bmatrix}$ and hence the following formula is derived, after considerable reduction, for two particles of arbitrary shape which are moving in xz plane

$$\begin{bmatrix}F_x \\ F_z\end{bmatrix}_a = -6\pi\mu c_a \begin{bmatrix}\dfrac{(U_x)_a - \frac{3c_a}{4d}(\phi_x)_b(U_x)_b}{\frac{1}{(\phi_x)_a} - \frac{9}{16}\frac{c_a c_b}{d^2}(\phi_x)_b} \\ \dfrac{(U_z)_a - \frac{3c_a}{2d}(\phi_z)_b(U_z)_b}{\frac{1}{(\phi_z)_a} - \frac{9}{4}\frac{c_a c_b}{d^2}(\phi_z)_b}\end{bmatrix} \qquad (1.25)$$

Therefore, for two equal-sized particles the foregoing formula reduces to

$$\begin{bmatrix}F_x \\ F_z\end{bmatrix} = -6\pi\mu c_a \begin{bmatrix}\dfrac{U_x\phi_x}{(1+\frac{3c_a}{4d}\phi_x)} \\ \dfrac{U_z\phi_z}{(1+\frac{3c_a}{2d}\phi_z)}\end{bmatrix} \qquad (1.26)$$

and for two spheres

$$\begin{bmatrix}F_x \\ F_z\end{bmatrix} = -6\pi\mu a \begin{bmatrix}\dfrac{U_x}{1+\frac{3a}{4d}} \\ \dfrac{U_z}{1+\frac{3a}{2d}}\end{bmatrix} \qquad (1.27)$$

which is exactly what we obtained before (Eqs. 1.21 and 1.22) by considering $\phi_x = \phi_z = 1$ and $c_a = a$.

1.1.2
Rotational effect

Considering Eq. 1.15 we can develop the rotation of the field $\mathbf{v}^{(1)}$ using

$$\nabla \times \mathbf{v}^{(1)} = \frac{\tilde{K}_a}{4\pi r^3}\left[U_{ax}(\mathbf{k}y - \mathbf{j}z) - U_{az}(\mathbf{j}x - \mathbf{i}y)\right] \qquad (1.28)$$

In general, the fluid rotation at the location of particle b is

$$\boldsymbol{\omega}_b^{(1)} = \tfrac{1}{2}\left(\nabla \times \mathbf{v}^{(1)}\right)_b \qquad (1.29)$$

and since the location of particle b is taken at $x = 0$, $y = 0$, $z = d$, we have

$$\boldsymbol{\omega}_b^{(1)} = \mathbf{j}\frac{\tilde{K}_a U_{ax}}{8\pi d^2} \ . \qquad (1.30)$$

Now, if the particle b is not fixed it will simply rotate at the velocity $\boldsymbol{\omega}_b^{(1)}$ and the only rotational effect experienced by particle a will be that due to the point force located at particle b, the curl of which can be obtained in a similar fashion to Eq. 1.28. Thus the rotation developed at particle a by the field $\mathbf{v}^{(2)}$ will be

$$\boldsymbol{\omega}_a^{(1)} = \mathbf{j}\frac{\tilde{K}_b}{8\pi d^2}\left(U_{bx} - \frac{\tilde{K}_a U_{ax}}{8\pi d}\right) \qquad (1.31)$$

which is opposite in direction to the rotation of particle b, but the lead terms are the same in magnitude for equal-sized particles. These terms will increase in the same way as the additional point force contributions, in a geometric series, so that ultimately

$$\boldsymbol{\omega}_a = \boldsymbol{\omega}_a^{(2)} + \boldsymbol{\omega}_a^{(4)} + \cdots = \mathbf{j}\frac{\tilde{K}_b}{8\pi d^2}\left(\frac{U_{bx} - (\tilde{K}_a U_{ax}/8\pi d)}{1 - (\tilde{K}_a\tilde{K}_b)/(8\pi d)^2}\right) . \qquad (1.32)$$

The rotation of particle b, $\boldsymbol{\omega}_b$, may be obtained from the foregoing by interchanging the subscripts a and b and by changing the sign of the rotation. The rotations occur about axes perpendicular to the xz plane. For two equal-sized particle ($\tilde{K} = 6\pi K C_a$)

$$\boldsymbol{\omega} = \mp\mathbf{j}\frac{3KC_a U_x}{4d^2}\left[\frac{1}{1 + 3KC_a/4d}\right] . \qquad (1.33)$$

and if the particles are the same size and spherical, we have for a and b

$$\boldsymbol{\omega} = \mp\mathbf{j}\frac{3aU_x}{4d^2}\left[\frac{1}{1 + 3a/4d}\right] . \qquad (1.34)$$

Note that $U_x = U\sin\alpha$ ($\alpha = 90$, for particles moving side by side and $\alpha = 0$ for particles moving along their line of centres, see Fig. 1), the velocity component perpendicular to the line connecting the particles centres is the only one contributing to the rotation.

2
Swimming side-by-side

Figure 2 shows the position of two micromachines which are swimming in a highly viscous fluid. It is assumed that the machines are sufficiently distant from boundary walls for the surrounding fluid to be regarded as unbounded and the fluid at infinity is at rest. The weight of each machine can be adjusted so that the machine can be regarded as neutrally buoyant (gravity force equal to buoyancy force) and at the same time each one may lie horizontally. The principal axis of first machine is along z axis, and its centre locates at the centre of the global coordinate. The second machine swims side-by-side with the first machine, i.e., their principal axes are parallel. The two machines have equal sizes and swim with the same angular velocity $\boldsymbol{\omega}$, (both in z direction). Each machine has the

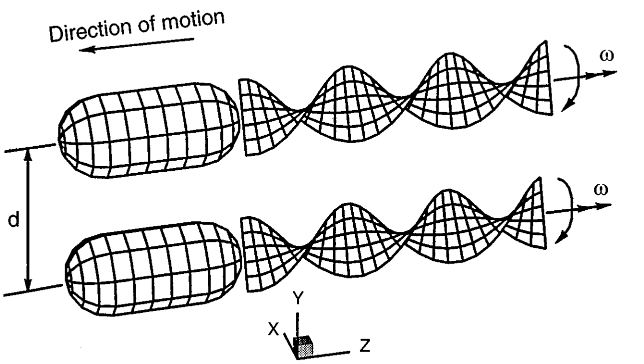

Fig. 2. Two micromachines with equal sizes, swimming side by side in viscous fluid, with a separation distance d and equal rotational velocity ω

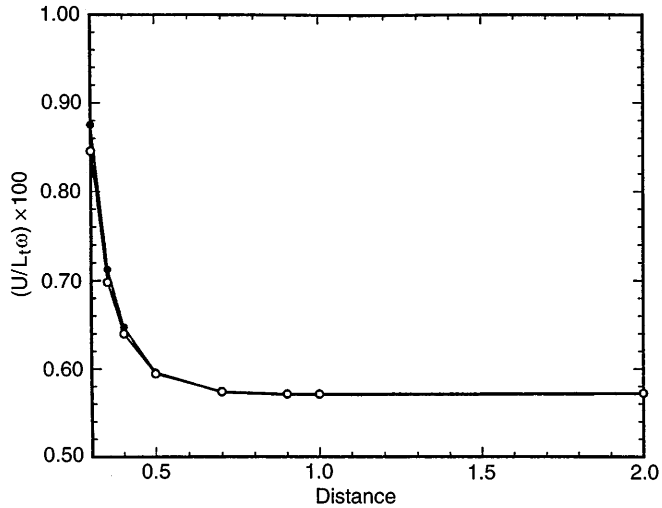

Fig. 3. The magnitude of the instantaneous translational velocities of two machines in terms of the centre to centre separation distance

optimised geometric dimensions to obtain the maximum propulsion velocity (Nasseri and Phan-Thien 1997b).

Note that in our calculations we have considered two machines, for each the tail and head are each treated as a separate individual body (i.e., closed surface). This is made possible by maintaining a small but finite separation distance ($a_h/10$) between the head and the tail (Phan-Thien et al. 1987). The viscous force $\mathbf{F}$ and torque $\mathbf{T}$ acting on a typical surface S_m, are calculated by the integrals

$$\mathbf{F} = \oint_{S_m} \mathbf{t}\, \mathrm{d}s \qquad \mathbf{T} = \oint_{S_m} (\mathbf{x} \times \mathbf{t})\, \mathrm{d}s \tag{2.1}$$

where $\mathbf{t}$ is the traction on the boundary S_m, and $\mathbf{x}$ represents the displacement vector from the head/tail joining point c, and stability requires that the force and torque on the body of each machine must vanish.

Using the BEM to solve for the boundary traction and Eq. 2.1, series of 24 numerical expressions may be carried out enabling the calculation of the elements comprising the resistance matrix in the following equation (Happel and Brenner 1973).

$$\begin{bmatrix} \mathbf{F}_1 \\ \mathbf{T}_1 \\ \mathbf{F}_2 \\ \mathbf{T}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{L}_{11}^T & \mathbf{K}_{12} & \mathbf{L}_{12}^T \\ \mathbf{L}_{11} & \mathbf{M}_{11} & \mathbf{L}_{12} & \mathbf{M}_{12} \\ \mathbf{K}_{21} & \mathbf{L}_{21}^T & \mathbf{K}_{22} & \mathbf{L}_{22}^T \\ \mathbf{L}_{21} & \mathbf{M}_{21} & \mathbf{L}_{22} & \mathbf{M}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{\Omega}_1 \\ \mathbf{U}_2 \\ \mathbf{\Omega}_2 \end{bmatrix} \tag{2.2}$$

As the velocities, forces and torques are three-dimensional vectors, each element in the above matrix will itself be a 3×3 matrix (and superscript T denotes the usual matrix transpose operation).

Figure 3 illustrates the dimensionless instantaneous translational velocity ($U/L_t\omega$), versus the distance between the centre of the two machines. These values have been obtained by using BEM for solving the traction equations on the surface of the two machines (Eq. 2.2). As explained in previous papers, L_t and ω are normalising factors (taken as unity). As it is obvious from the figure; the closer the machines get, the higher the translational velocity of both will be.

By using the method of reflection to a system of n spheres and by assuming that all velocities of the particles are in the same direction, the resulting fields involve only positive contributions (see Kim and Karrila 1991), therefore, the resultant interaction will result in increasing velocities of particles (e.g., for many particles, the larger the swarm of particles, the faster it will move).

Brenner (1964) has shown that in the case of particles which are not spherically isotropic, their trajectories will in general not be parallel, hence, the reflection technique may be generalized for two particles moving in arbitrary directions, provided that the point force approximations is applicable. Free rotation of such anisotropic particles as they settle in a fluid in the presence of each other will give rise to velocity components which cannot be represented simply by point forces and point torques. It is shown that, a rotating anisotropic particle falling under the influence of gravity will experience changes in magnitude and direction of its instantaneous velocity $\mathbf{U}$.

By using Eq. 1.33, we realized that, when two particles are moving in a fluid, supposing that the particles translate in fluid without any rotation, the velocity component perpendicular to the line connecting the particles centres, in our case velocity along z axis in Fig. 2, is the only one contributing to rotation. Therefore, as seen in Fig. 4a, each of the two machines, rotates about x axis and hence gets closer to the other one. This happens only if the separation distance is smaller than about half of the total length of the machine ($d \lesssim L/2$).

Furthermore, the spiral waves around the machines forces them to depart from the initial plane (see Fig. 4b). Therefore each machine experiences three rotations at a time (noting that rotation about z axis is more dominant). As the separation distance becomes large ($d \gtrsim 1.5L$), hydrodynamic interaction is negligible and the trajectories of two machines will be straight lines. The components and the magnitude of the rotational velocity for the machine are shown in Fig. 5 in terms of the separation distance between two machines.

Also, the numerical results from calculating the resistance force exerted on each machine if they translate with the velocities shown in Fig. 3, can be compared with the results obtained using the method of reflection (Eq. 1.26).

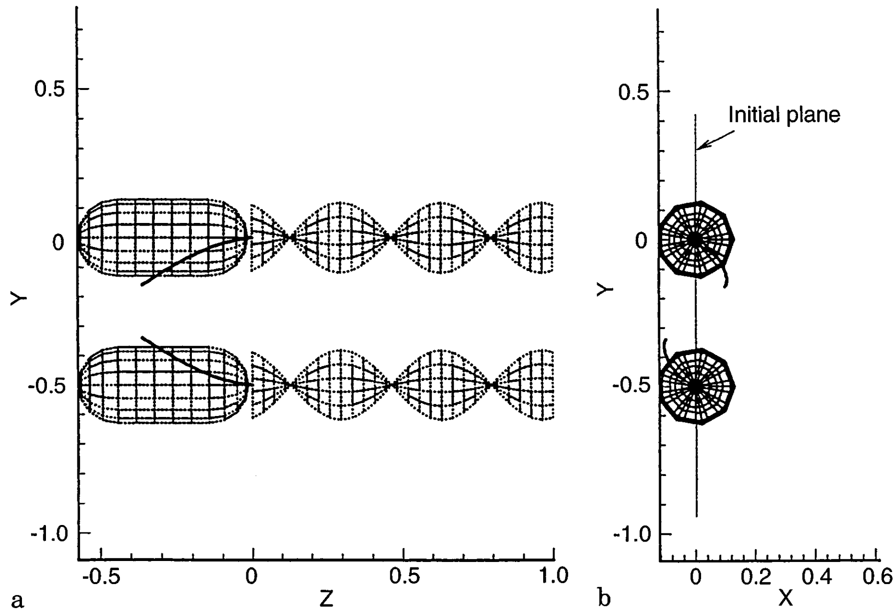

Fig. 4. Trajectories of the centre of the machines after 9 cycles of the tail rotations

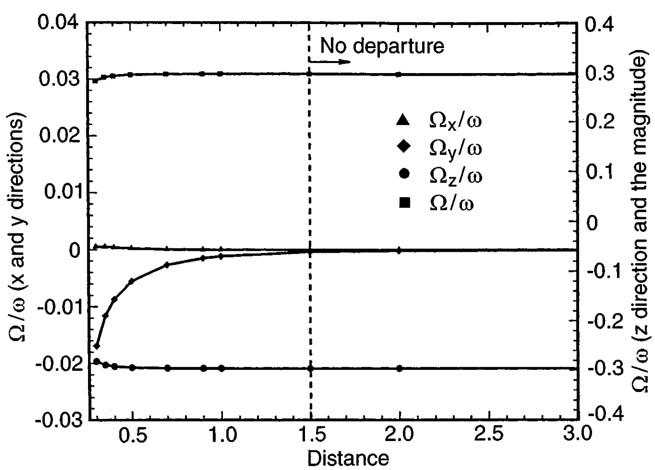

Fig. 5. The components and the magnitude of the instantaneous angular velocity for each machine in terms of the separation distance

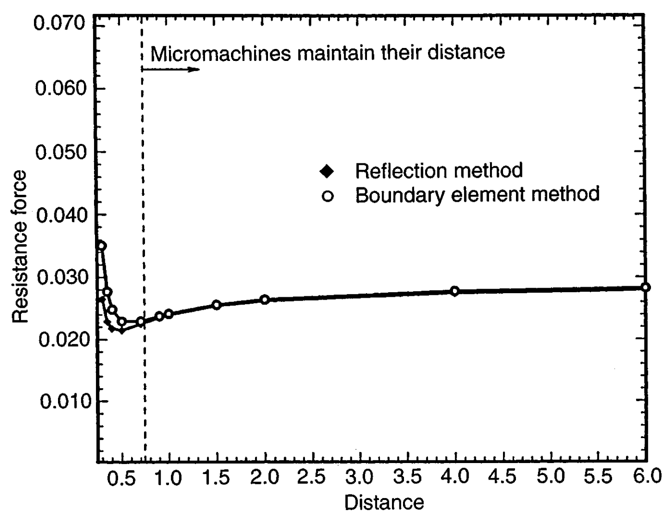

Fig. 6. Comparison between the resistance force, using BEM and method of reflection, exerted on each of two micromachines swimming side by side in terms of the separation distance. When no closure occurs, the results are the same

For two machines moving side by side and parallel to z axis (Fig. 2) we will have

$$F = -6\pi\mu \frac{UC_a\phi}{(1 + 3/4\ (C_a\phi)/d)} \ , \tag{2.3}$$

where $U = U_{ax} = U_{bx}$ and $C_a\phi$ has been derived in calculation of Stokes' law correction factor (see Appendix) and is equal to 0.268002 for this case.

Figure 6 illustrates the force exerted on each of these micromachines, using the method of reflection (Eq. 2.3) as compared to the BEM results. An excellent agreement is obtained. In other words, as long as two machines can maintain their separation distance while swimming, the method of reflection provides a good approximation to the force exerted on the machines from the surrounding fluid.

3
Swimming in tandem

Consider now two micromachines which are swimming in viscous fluid along their line of centres. As shown in Fig. 7, for this configuration the line of centres is coincident with the z axis, and similar to the previous configuration, each machine is moving along its principal axis. We wish to analyse hydrodynamic effects on the swimming characteristics of these two micromachines. From Fig. 8, it is obvious that, the front micromachine swims faster when the two machines get closer to each other. In fact, the micromachine at the back, in the wake of the front micromachine, experiences a retarding force due to hydrodynamic interaction, and consequently, has a lower velocity.

After several cycles, they get far from each other, and as the separation distance increases, hydrodynamic effects become negligible, hence they move with a constant ve-

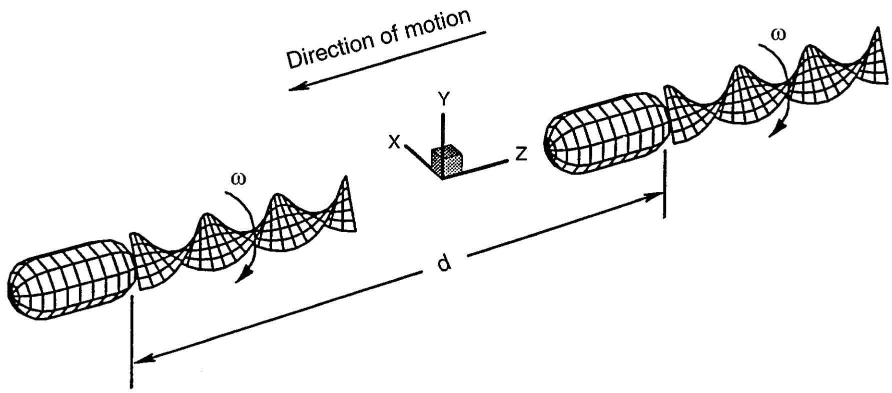

Fig. 7. Two micromachines with equal sizes, swimming along one line in viscous fluid, with separation distance d and equal rotational velocity ω

locity maintaining the distance between their centres, e.g., for $d = 1.7\ L_t$, it is needed for the tail of each machine to rotate about 30 cycles to reach to this ultimate constant velocity which in fact is the velocity each machine would have in the absence of the other one.

Furthermore, hydrodynamic interaction results in an increase in the angular velocity for the leading machine and a decrease in rotation for machine at the back.

As pointed out by Happel and Brenner (1973) the velocity component perpendicular to the line of centres, which is zero in this configuration, *i.e.*, $U_x = U \sin\alpha = 0$, referring to Eq. 1.33, is the only one contributing to rotation. They also mentioned that if two particles fall along their line of centres, no rotation will result. The result obtained here, in this regard, is in agreement with foregoing statement. Numerical results reveal that, when two machines are swimming along their line of centres no rotation occurs and they keep their straight trajectories along the line of centres.

To estimate the drag force on each machine, again by using Eq. 1.26, but knowing that in this new configuration the machines are swimming along their line of centres, we have

$$F = -6\pi\mu \frac{UC_a\phi}{(1 + 3/2\ (C_a\phi)/d)} \tag{3.1}$$

with $C_a\phi$ as given previously.

Notice that although each micromachine is considered as an anisotropic particle, but the value for the Stokes' law correction factor $(C_a\phi)$ is the same as before, because in this configuration each machine moves along its principal axis.

Figure 10, shows that when two micromachines swim along the line of centres, the method of reflection provides an excellent estimate of the drag force exerted on each machine. As the separation distance increases, the drag force exerted on each machine is equal to the force which each machine would experience in the absence of the other.

4 Conclusions

Numerical results of the swimming of two nearby micromachines in a viscous fluid are obtained by the boundary element method for two configurations: side by side and in tandem. In the side by side configuration, the method of reflection provides a reasonable estimate for the resistance

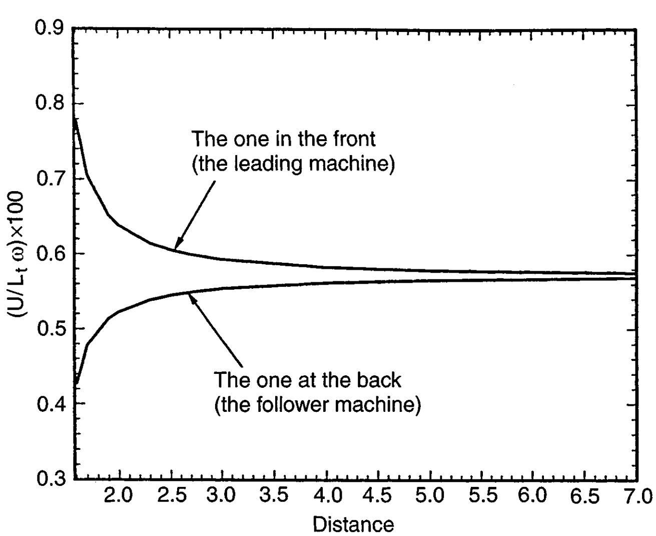

Fig. 8. The magnitude of the propulsive velocity for micromachines swimming on line of their centres versus the centre-centre separation distance. The micromachine which is in the front swims faster when the two machines are close to each other

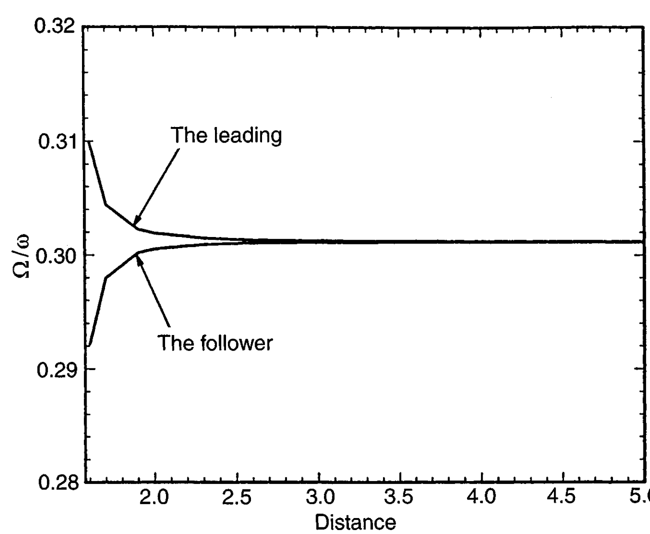

Fig. 9. The magnitude of the rotational velocity of each machine versus the separation distance. The leading machine rotates faster in close vicinity

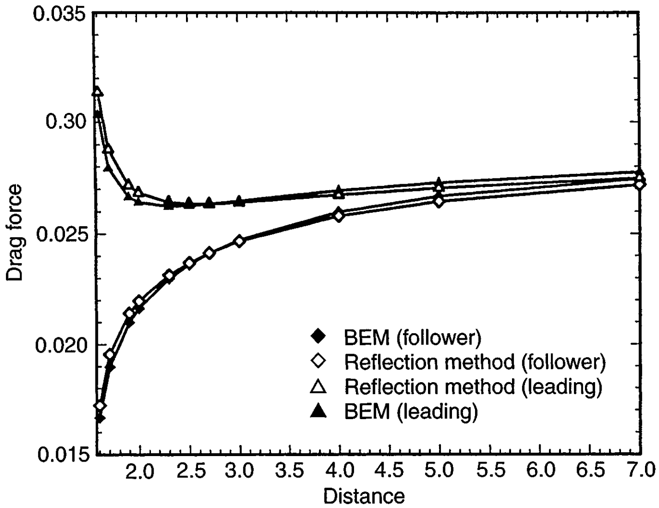

Fig. 10. The magnitude of the drag force exerted on the leading and the follower machines, using BEM in comparison with the reflection method

force, as long as the machines maintain the same relative configuration. This occurs when the two machines are separated by a distance equal or greater to half of the length of each machine. When the separation distance is smaller than about the machine's length, we have to superpose the rotation effect to Eq. 1.7 (not considered in the method of reflection). In fact, the calculation of relative velocities when the micromachines are very close to each other is a problem which, in principle, involves the unsteady form of the equations of motion, which has not been considered in this paper.

In the tandem configuration, swimming along line of centres, the method of reflection is an excellent approximation for estimating the drag force exerted on each machine, no matter how close they are swimming. The leading machine acquires higher velocity (translational and rotational), and when the separation distance becomes more than 10 times of the length of the tail or almost 6.5 times of the total length of each machine, both machines swim with the same velocity.

Appendix

Stokes' law correction factor

Here we calculate the Stokes' law correction factor for the micromachine with spiral tail and compare this to the value obtained from considering the whole machine as a prolate spheroid.

Suppose that we have a prolate spheroid, with the principal radii of a and b (b is the longest of the two semi-axes, $b > a$), which is moving parallel to its axis of revolution. To compare the resistance of a prolate spheroid with a sphere having the same equatorial radius a, we have (Happel and Brenner 1973)

$$F_z = -6\pi\mu a U K \tag{4.1}$$

where $K = K(a/b)$ is the correction to Stokes' law given by

$$K = \frac{1}{\frac{3}{4}\sqrt{\tau_0^2 - 1}[(\tau_0^2 + 1)\coth^{-1}\tau_0 - \tau_0]} \tag{4.2}$$

and

$$\tau_0 = \left[1 - \left(\frac{a}{b}\right)^2\right]^{-1/2} . \tag{4.3}$$

Some values of K are given in table 4-26.1 of Happel and Brenner (1973). If we estimate our micromachine as a prolate spheroid with principal radii of 0.125 and 0.786562, which are the radius of the head and half of the length of the machine, respectively, obtained by the optimisation procedure for analysing the motion of such machine with spiral tail (see our latest paper), we have $a/b = 0.15892$ and $K = 2.139058$.

Also, if we assume that the micromachine with optimal dimensional geometry, has a rigid body motion, moving with the maximum dimensionless translational velocity (with no rotation), we can use Stokes' law for the drag exerted by the fluid on an arbitrary particle moving in a fluid, $F = 6\pi\mu C_a \phi U$, and calculate $C_a \phi$ needed in this paper,

$$C_a \phi = \frac{F}{6\pi\mu U_{\max}} = 0.268002 \ , \tag{4.4}$$

where C_a is the characteristic length of the particle. By estimating C_a to be the radius of the head of the machine, which is 0.125, we have $\phi = 2.144018$.

This value is very close to the Stokes' law correction factor of a prolate spheroid with the above mentioned radii. In fact we can estimate our micromachine as a prolate spheroid, with error of approximately 0.2%.

References

Brenner H, Happel J (1958): Slow viscous flow past a sphere in cylindrical tube. J. Fluid Mech. 4, 195–213

Brenner H (1964): Effect of finite boundaries on Stokes resistance of an arbitrary particle part 2. Asymmetrical orientations. J. Fluid Mech. 18, 144–158

Davis MH (1969): The slow translation and rotation of two unequal spheres in a viscous fluid. Chem. Eng. Sci. 24, 1769–1776

Goldman AJ, Cox RG, Brenner H (1966): The slow motion of two identical arbitrary oriented spheres through a viscous fluid. Chem. Eng. Sci. 21, 1151–1170

Happel J, Brenner H (1973): Low Reynolds number hydrodynamics. Noordhoff International Publishing, Leyden, The Netherlands

Jeffrey DJ, Onishi Y (1984): Calculation of the resistance and mobility functions for two unequal rigid spheres in low-Reynolds number flow. J. Fluid Mech. 139, 261–290

Kim S, Karrila SJ (1991): Microhydrodynamics: Principles and selected applications. Butterworth-Heinemann, Boston, USA

Lamb H (1932): Hydrodynamics. 6th ed., Cambridge University Press, London.

Nasseri S, Phan-Thien N (1996): On the path and efficiency of two micromachines with rigid tails. J. Comp. Mech. 18 (3), 192–199

Nasseri S, Phan-Thien N (1997a): Modeling micromachines with elastic parts in a viscous environment. J. Comp. Mech. 20, 242–246

Nasseri S, Phan-Thien N (1997b): Geometric optimisation of a micromachine with spiral tail. J. Comp. Mech. 20, 267–271

Phan-Thien N, Tran-Cong T, Ramia M (1987): A boundary element analysis of flagellar propulsion. J. Fluid Mech. 184, 533–549

Phillips RJ (1996): Dynamic simulation of hydrodynamically interacting spheres in a quiescent second-order fluid. J. Fluid Mech. 315, 345–365
Ramia M, Tullock DL, Phan-Thien N (1993): The role of hydrodynamic interaction in the locomotion of microoganisms. Biophys. J. 65, 755–778
Stimson M, Jeffery GB (1926): The motion of two spheres in a viscous fluid. Proc. R. Soc. A111, 110–116
Taylor GI (1951): Analysing of the swimming of microscopic organisms. Proc. R. Soc. Lond. A 209, 447–461